Global vs. Local Competition: The Case of E-Commerce*

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Abstract

We analyze the impact of increased outside options brought to consumers by e-commerce on the classical retail market. If consumers have to choose once where to shop we show that under all forms of organizing the classical retail market, increased competition from e-commerce will crowd out variety in the retail market. However, the effect of increased competition on prices is much less clear. While it yields a price reduction under monopoly, prices increase under oligopoly. If consumers can shop in e-commerce after having benefited from inspection in the retail market, the monopoly will increase varieties in response to a decrease in prices in e-commerce.

1 Introduction

At the turn of the millennium, internet spreads around the world seemingly irresistibly. It has created a communication infrastructure ever increasing in geographical scope, and in outreach to business and consumers. In the near future, trading strategies that have proven successful in the marketplace may falter and give way to new approaches, most of which are not yet fully developed nor understood.

Possibly due to issues involving the security and reliability of trading arrangements unresolved so far, short term forecasts on trading volumes absorbed by e-commerce differ widely. Yet in the long run, e–commerce is

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expected to revolutionize *all* trading arrangements. The revolution is likely to involve both, business-to-consumer as well as business-to-business trading relationships. It comes in different ways. First, many heretofore local markets become global: both, suppliers of, and consumers demanding commodities with access to the technology are able to participate in markets of world wide extension. Second, contacts between producers and final consumers are likely to radically change, as e-commerce allows producers to circumvent intermediaries and to directly market their product.[1]

In this paper, we analyze the impacts of e-commerce on the sector likely to be affected most by e-commerce, namely classical retailing. While today's share of electronic commerce on total business-to-consumer sales in the country exhibiting the farthest development of internet activities, the U.S., is estimated to be still below one per cent, extensive investments of major classical retailers as well as of direct entrants in to the e-commerce business in technology, knowledge and marketing might change the picture sooner than expected by many. Indeed, Wilson (1998) reports from a representative sample survey that in 1998 about 50 per cent of all U.S. retailers were already present in the Web; 20 per cent planned their presence within the next 12 months; and only 10 per cent had no intention to establish a Web presence within the foreseeable future. Of the same retailers, 12 per cent were already selling on-line, and 22 per cent were planning to do so within the next 12 months. Many of them expect this to be their exclusive business in the foreseeable future.

Robertson; Stephens & Co. (1997) elucidate some key advantages of electronic over classical retailing: First, *increased width and depth of commodity assortment*. Etoys.com and The Woodwind And The Brasswind, internet based toys and children' products; and instrument and parts companies, respectively, are good examples. Second, *increased information about goods and services*. For instance, the commodity description and advice given on a new software product are of utmost benefit especially to consumers with prior knowledge about software. Third, *targeting*: internet retailers can address worldwide specialized types of customer communities, which allows them to market and to sell specialized commodities at a relatively low cost. Fourth, internet opens *new avenues for experimenting with new retailing technologies*, and in particular technologies that optimally search and match purchases to consumers' interests (Vulkan, 1999).

There are also risks involved in trading via e-commerce, to both consumers and sellers. Goods and services bought on the Web may not meet

[1] In the sequel of this introduction, we draw heavily from empirical evidence collected in Schmidt's (1999) recent Masters' Thesis.

consumers' expectations. Even liberal return policies involve consumer expenses. There is also a deep issue of consumer privacy. And hold up and security risks extend to both sides of the market. Yet all of these risks are actively confronted by the agents preparing themselves for entry into e-commerce, and proposals for solutions emerge virtually daily.

What will be the consequences of all this? Business experts such as Roberton, Stephens & Co. (1997) and Ghosh (1998) argue that marketing as well as operating expenses decrease in internet relative to conventional retailing, due to decreases in sales force, in promotional expenses as well as in expenses for brick-and-mortar stores. Yet due to the high visibility of all retailing activities in internet, most of these savings are expected to be eaten up by competition that is much fiercer than in classical retailing. *Prima facie*, all this benefits consumers.

However, increased competition should also crowd out much of conventional retailing, especially in view of e-commerce's lower cost and its global outreach. Will this also be to the consumers' benefit? This is the central question attacked with the model developed below. Before we approach this and other welfare questions, we wish to answer a number of positive questions on the effects of e-commerce on conventional retailing that, to our surprise, have not been addressed heretofore in the theoretical literature. For instance, if the typical retail market is characterized by the number and price of variety sold, how will both evolve with increased competition from e-commerce? Will product proliferation in the classical retail market be the answer to the challenge, or more competitive, decreased prices? Which type of commodities will be most affected? Convenience goods, search goods, inspection goods, experience goods? Which market organization will be better prepared to fight that competition? Is it monopolistic competition, oligopoly, or monopoly? Will the end result be more concentration, or more fragmentation in classical retailing?

Answers to some of these questions can be given without much formal reasoning. For instance, we can expect a major redistribution towards e-commerce in the sales of commodities with characteristics more or less perfectly known to consumers, in which competitive prices exert a decisive role in consumers' decisions. A standard example in which e-commerce already now assumes an important role are airline flights of a certified quality. In this situation, increased price competition unequivocally benefits consumers with access to e-commerce. Even consumers with access to classical retail outlets only can be expected to benefit, as prices for such standardized goods should decline.[2]

[2] Yet exit from classical retailing activities may harm first time buyers who are in need

However, answers to these questions are much less clear for other commodities since there is in general a benefit to *inspect* the good before purchase. Take fashion ware. While certification may remove doubts as to the craftsmanship going into the preparation of such commodities, even the best virtual reality description is a highly imperfect substitute to inspecting and trying on suits, shirts, or shoes.[3] The same is true, to a lower extend, for other commodities like books or CDs.

We therefore concentrate our analysis on the market for commodities for which *inspection prior* to purchase is valuable to the consumer. We focus on competition between a perfectly competitive e-commerce sector providing exogenous utility and one typical retail market. Similar to earlier analyses of markets for inspection commodities (e.g., Stahl 1982 or Schulz and Stahl 1996), that market is organized alternatively by an oligopolistic structure (with free entry) involving specialized firms that sell one variety of a differentiated commodity, and a monopoly selling all variants in that market place. We thus exclude from our discussion the effects of e-commerce on inter-market competition in classical retailing.

As customary, commodity variants are specified by locations on the circumference of a circle. Consumers are differentiated by ideal varieties on this circle, as well as by relative costs of accessing e-commerce vs. the retail market. The typical consumer can precisely specify his utility only upon the inspection of the commodity variants, after his decision to purchase either via internet, or in the classical market. *Ex ante*, that decision is taken on the basis of an expected (indirect) utility criterion. However, *ex post*, the consumer is faced with very different alternatives before the purchase decision is taken. While many more variants are assumed to be sold via e-commerce, he can only inspect the commodity variants on sale within the classical retailing set up.

Quite naturally, consumers' relative expected utility from patronizing the classical retail market increases as the number of variants sold increases, and the prices decrease. In the oligopolistic market, entry confers an externality on all other sellers as custom increases, and increased competition should lower prices, which again furthers the demand for all variants. This demand externality is internalized by the monopolist. As is consistent with standard markets, we assume for most of the paper that prices are not predetermined but are discovered once consumers visit the market (obviously price antici-

of expert advice some of which can only be given in personal contact. With this the introduction of new commodities may be impeded.

[3] Yet the internet may become an important means for advertising for these commodities. Improved on-line information about the off line availability of commodities indeed may further classical retailing.

pations by consumers are met in equilibrium,). We later analyze variations of the model to allow the monopolist to alleviate the hold-up problem by either pre-announcing prices (before consumers go to the market) or by facing ex-post competition from e-commerce..

One of the interesting questions arising here is which market organization will provide higher utility to the consumers: the oligopolistic market in which the externality is not internalized but competition exercises its force, or the monopolistic market in which the externality is internalized, but monopoly is only constrained by competition from e-commerce. The answer to this question has an obvious bearing on the relative survival of classical retailing, since the market providing higher utility will do better.

Our results are as follows. Within a comparative statics exercise with respect to the outside utility from consumption via e-commerce, we show that under all forms of organizing the classical retail market, increased competition from e-commerce will unequivocally crowd out variety. However, the effect of increased competition on prices is much less clear. While increased competition from e-commerce yields a price reduction under monopoly, prices *increase* under monopolistic competition! The effects on the local welfare mimic the effects on prices of an increase in the outside option of consumers: for the monopoly, welfare increases while for the oligopoly with free entry welfare decreases.

The explanations for these two results are in fact quite different. In the monopoly case, the market power of the retailer creates a hold-up problem and consumers anticipate a high price (equal to the willingness to pay of the "marginal" consumer). A larger number of varieties implies more utility for all consumers but also a higher price and, by concavity of the utility function, the smaller is the difference between the utility of the inframarginal consumers and the utility of the marginal consumer: it follows that the ex-ante surplus of a consumer is a *decreasing* function of the number of varieties. Now, in response to an increase in the outside option offered by e-commerce to consumers, the monopoly can decide to decrease the number of varieties in order to slow down its loss of market share or to increase the number of varieties in order to have a larger profit per consumer. Which strategy is optimal depends on the tradeoff between the speed at which market share is lost versus the speed at which profit per consumer increases. Whenever the distribution of types is log-concave, the first strategy is profit maximizing: demand decreases faster than profit per consumer increases when the number of varieties increases. This implies a decrease in the number of varieties (crowding out) as well as a decrease in prices. It also implies an increase in the welfare of consumers who purchase from the retail market.

With oligopoly, the ex-post hold-up problem is weakened since firms in the retail market compete for the marginal consumer and will post a price strictly lower than its utility. However, an increase in the outside option of consumers implies lower equilibrium profits for a given size of the local market. Hence, in an equilibrium with free entry there will be less varieties; but less varieties implies less competition ex-post. Under concavity of the utility function the second effect implies that prices increase.

At this point it might be tempting to assert that the difference between the two cases is linked to the magnitude of the hold-up problem: maximum with the monopoly and reduced by ex-post competition in the monopolistic competition case. As we show in the second part of the paper, this intuition is in fact incorrect. Indeed, we allow the monopoly to announce (and commit to) prices *before* consumers decide between retail and e-commerce: this gives maximum liberty for the monopoly to commit to low prices ex-post. Nevertheless, the monopoly will still choose to reduce the number of varieties and to decrease prices.

We conclude that it is not so much the magnitude of the hold-up problem that is responsible for the difference in behavior of the monopoly and monopolistic competition models but rather the *nature of competition* ex-post (once consumers are in the retail market). The monopoly faces a non-strategic competitor and the constraint that this competition imposes in equilibrium is similar to an individual rationality constraint. By contrast, in monopolistic competition firms face ex-post strategic competitors and the constraint that this imposes on equilibrium is similar to an incentive compatibility constraint.

For this reason we introduce some ex-post competition in the monopoly model by allowing consumers to inspect varieties in the retail market before buying in e-commerce. The benefit of inspection is that the consumers can precisely order a variety in e-commerce while he could not without inspection. In this case the decision for a consumer to switch ex-post from retail to e-commerce is simply a function of the relative prices in retail and e-commerce. When e-commerce becomes a strong price competitor, the ex-post competition effect forces the monopoly to increase varieties. Such a change in comparative statics does not happen in the oligopoly case.

2 The Model

There is a measure 1 of consumers on a circle of circumference 1. A consumer is identified by a pair (y, θ), where $y \in [0,1]$ is the ideal variety for the consumer and θ is her cost of accessing the retail market. Viewed as random variables, y and θ are independent; y is uniformly distributed and θ has distribution G. We assume that G is log-concave (for instance, a normal cumulative distribution is log-concave) and has a strictly positive density g on $(-\infty, +\infty)$. Note that log-concavity is equivalent to a *decreasing likelihood ratio* $\frac{g}{G}$.

Consumers want to consume at most one unit of the commodity. If a consumer with ideal variety y instead consumes variety $\widehat{y}$ where $|y - \widehat{y}| = x$, his utility is given by $h(x)$, where h is a strictly decreasing and concave function with $h'(0) = 0$.

The timing of events is as follows:

- Consumers learn their cost of going to the retail market (θ)
- Consumers anticipate that the expected utility from purchasing from e-commerce is u.
- Consumers observe the number m of varieties in the retail market.
- Consumers decide on whether to go to the retail market (at cost θ) or to buy from e-commerce.
- Prices p are set on the retail market
- Nature draws y and consumers who go to retail learn — by trying different varieties and comparing prices the variety-price pair that maximizes their utility and purchase it. y and θ are i.i.d..

Worth noting is the assumption that consumers learn first their cost θ of accessing the retail market, but learn about their best variety only after they have committed to buy from one market place. Hence, we consider a world in which agents have one opportunity to buy and cannot shift back and forth between the two market places. An extension of the model to the possibility of shifting ex-post is made in the second part of the paper.

While agents can sample from the retail market and distinguish between the varieties offered *before* purchasing, they cannot do such a sampling with e-commerce.

The ex-ante utility of a consumer who purchases from e-commerce is exogenously given by u. For instance, we could use a crude representation of e-commerce and assume that *all* varieties are sold in there. This is consistent with our vision of e-commerce as *global commerce* in which consumers can reach most producers. [4] Also consistent with this global vision, the price

[4] This assumption is only for simplicity; what matters for our results is that there is a positive mass of consumers, i.e., a positive measure of θ, that will find purchasing the commodity in e-commerce more attractive that in the retail market.

of the commodity in e-commerce is not influenced by the prices in the retail market. Letting q be the price of the commodity in e-commerce (typically q is "small" compared to the price in the retail market), we would then obtain, *if consumers cannot distinguish between the varieties*,

$$u = \int_0^1 h(x)\,dx - q.$$

As e-commerce technology improves, e.g., by offering "virtual inspections" of goods, the expected utility of agents will increase. For instance, if technological improvements imply that an agent can match with her preferred variety with probability α, and otherwise takes a random draw from the other varieties, her expected utility would be

$$u = \alpha h(0) + (1-\alpha)\int_0^1 h(x)\,dx - q.$$

For our purpose, the precise modeling of e-commerce is not crucial because there is no strategic response of e-commerce to *local* changes in the retail market. Taking u as exogenous and making comparative statics comparisons with respect to u will enable us to capture the response of local retailing to an improvement in the services offered by e-commerce. Assume that u has range $(-\infty, h(0))$.

In the retail market, there is a fixed cost $F \geq 0$ to introduce a new variety (say the lease of a space in a shopping mall) and a marginal cost $c \geq 0$ per unit sold. There are $m \geq 1$ varieties. For convenience, we treat m as a continuous variable. We assume that varieties are always symmetrically located on the circle.[5] Firms decide first to enter the retail market, then consumers decide whether or not to shop in the retail market, then firms set their prices. Consumers choose their mode of shopping anticipating the equilibrium prices in the retail market. Because the consumers learn their ideal variety *after* having decided where to shop, each consumer has the same expected utility (net of the cost θ and before prices) from shopping in the retail market. Thus, at equal prices everywhere, there exists in equilibrium a trigger value $\tilde{\theta}(m)$ such that all consumers with cost less than this value shop in the retail market and consumers with cost greater than this value shop in e-commerce.[6]

[5] This is a standard assumption. It involves some loss of generality in the oligopoly case. However, it is without loss of generality in the monopoly case.

[6] Note that a necessary condition for the retail market to attract a positive mass of consumers is that if price is equal to marginal cost and if all varieties are sold, i.e., if $\theta^{\infty} = \lim_{m\to\infty} \tilde{\theta}(m) = h(0) - \int_0^1 h(x)\,dx - c + q$, we have $G(\theta^{\infty}) > 0$. Stronger conditions are in fact necessary in order for the retail market to break even.

Now, since θ and y are i.i.d. an since the marginal cost is constant, the strategic behavior of firms is independent of the mass of consumers. It is therefore enough to find the continuation equilibria for a mass one of consumers, i.e., to find the equilibrium price $p(m)$ and the maximal acceptable distance $x(m)$ between the ideal point of a consumer and a given variety.

Here, two regimes are a-priori possible in equilibrium. In the first regime, the price is large enough so that some consumers decide not to consume after having learned their ideal variety. We will say that the market is *not covered.* In the second regime, the price is low enough so that all consumers find it optimal to consume some variety. We will say that the market is *covered.* To simplify, we assume that it is not possible to cover the market with only one variety; if only one variety is offered in the market, the consumer whose ideal variety is the farthest away, i.e. at a distance of $1/2$, has utility $h\left(\frac{1}{2}\right)$ from purchasing that variety. Since the price must be at least equal to c for the firm to break even, the market is not covered whenever

$$h\left(\frac{1}{2}\right) < c. \tag{1}$$

We consider two organizational structures—monopoly and oligopolistic with free entry—for the retail market. We will be interested in comparing the performance of each structure as e-commerce becomes a stronger competitor, i.e., as u increases.

3 Monopoly

Once m is fixed, *ex post* the monopoly chooses the segment of the market that he wants to serve, i.e., by symmetry, chooses $x \leq \frac{1}{2m}$ to maximize $x(h(x)-c)$. Let x^* be the unconstrained optimum

$$h(x^*) - c + xh'(x^*) = 0, \tag{2}$$

and let $m^* = \frac{1}{2x^*}$. We assume that fixed costs are small enough so that when $m = m^*$, the monopoly makes positive profits, i.e.[7]

$$x^*(h(x^*) - c) > \frac{F}{2}. \tag{3}$$

If $m < m^*$, the monopoly would find it optimal to set ex-post $p = h\left(\frac{1}{2m^*}\right) > h\left(\frac{1}{2m}\right)$ (remember that h is decreasing). In this case the $\theta = 0$ consumer has an expected utility of $u^L = 2m\int_0^{1/2m^*}\left[h(x) - h\left(\frac{1}{2m^*}\right)\right]dx.$

[7]The profit per variety is $2z^*(h(z^*) - c)$ while the cost is F.

Ex-ante all types $\theta \leq u^L - u$ prefer to go to the retail market and the ex-ante profit of the monopoly is

$$\pi(m) = G\left(u^L - u\right)\left(h\left(\frac{1}{2m^*}\right) - c\right)\frac{m}{m^*} - mF.$$

As long as $m < m^*$, π is the profit function in a neighborhood of m. Assuming that $\pi(m) \geq 0$, the marginal profit is

$$\pi'(m) = \left(2\int_0^{1/2m^*}\left[h(x) - h\left(\frac{1}{2m^*}\right)\right]dx\right) g\left(u^L - u\right)\left(h\left(\frac{1}{2m^*}\right) - c\right)\frac{m}{m^*} + \frac{\pi(m)}{m}.$$

Since $h(x) > h\left(\frac{1}{2m^*}\right)$ when $x < \frac{1}{2m^*}$, $\pi'(m) > 0$ and the monopoly wants to increase the segment of the market that he serves. This proves that the market is covered.

Lemma 1 *There exists m^* such that the following holds. If the monopoly enters the retail sector, the monopoly chooses a number of varieties greater than m^* and the market is covered.*

Therefore, a monopoly will choose m ex-ante in order to cover his market ex-post, i.e., $m \geq m^*$ and $p = h\left(\frac{1}{2m}\right)$. We note that in this case, the expected consumer surplus (net of θ) from purchasing from the retail market is

$$H(m) \equiv 2m\int_0^{\frac{1}{2m}}\left[h(x) - h\left(\frac{1}{2m}\right)\right]dx. \tag{4}$$

This surplus H will play a central role in the analysis of the paper. While it is difficult to sign the first derivative of H, an indirect argument shows that H is a strictly decreasing and convex function.

Lemma 2 *H is a strictly decreasing and convex function.*

Proof. Simple computations lead to

$$\begin{aligned}
H(m) &> 0 && (5)\\
\lim_{m\to\infty} H(m) &= 0 && (6)\\
H'(m) &= 2\int_0^{\frac{1}{2m}} h(x) - \frac{1}{m}h\left(\frac{1}{2m}\right) + \frac{1}{2m^2}h'\left(\frac{1}{2m}\right) && (7)\\
H''(m) &= -\frac{1}{2m^3}\left(h'\left(\frac{1}{2m}\right) + \frac{1}{2m}h''\left(\frac{1}{2m}\right)\right). && (8)
\end{aligned}$$

Since h is decreasing, $h(x) > h\left(\frac{1}{2m}\right)$ when $x \in \left(0, \frac{1}{2m}\right)$, hence (5) follows. Since h is decreasing and concave, $H'' > 0$ in (8) and H is strictly convex. Since $H > 0$, (6) and (7) are compatible with H convex only if $H'(m) < 0$. ■

Hence, the surplus of consumers is *decreasing* in the number of varieties. This illustrates in a somewhat dramatic fashion the hold-up problem created by the lack of competition ex-post. While consumers value more varieties, the reservation price of the marginal consumer is also increasing in the number of varieties. Since the monopoly will set a price equal to the reservation price of the marginal consumer, and since this reservation price is concave increasing in the number of varieties, infra-marginal consumers have less surplus when the value to the marginal consumer increases. The hold-up problem prevents the monopoly from separating the price decision from the variety decision and therefore creates a 1-1 relationship between the decision to give more surplus to the consumers and the decision to decrease varieties. Whether or not the monopoly will effectively decide to give more surplus to the consumers in response to an increase in their outside option u depends on the relative effects on the demand and on the profit per consumer.

The monopoly chooses m to solve

$$\max_{m} G\left(H(m) - u\right)\left(h\left(\frac{1}{2m}\right) - c\right) - mF \qquad \text{(P0)}$$
$$m \geq m^*.$$

We are interested by the comparative statics of a solution $m^n(u)$ with respect to u. Ignoring the constraint $m \geq m^*$, the first order condition yields

$$H'(m)\, g\left(H(m) - u\right)\left(h\left(\frac{1}{2m}\right) - c\right) - G\left(H(m) - u\right)\frac{h'\left(\frac{1}{2m}\right)}{2m^2} - F = 0. \quad (9)$$

For an interior solution, $m > m^*$, the second order condition is satisfied and the implicit function theorem implies that the sign of $\frac{dm^n(u)}{du}$ is the same as the sign of the derivative of the first order condition with respect to u, i.e.,

$$\frac{dm^n(u)}{du} \propto -H'(m)\, g'\left(H(m) - u\right)\left(h\left(\frac{1}{2m}\right) - c\right) + g\left(H(m) - u\right)\frac{h'\left(\frac{1}{2m}\right)}{2m^2}.$$

Using (9),

$$\frac{h'\left(\frac{1}{2m}\right)}{2m^2} = H'(m)\frac{g\left(H(m) - u\right)}{G\left(H(m) - u\right)}\left(h\left(\frac{1}{2m}\right) - c\right) - \frac{F}{G\left(H(m) - u\right)}$$

which upon substitution in the previous expression yields

$$\frac{dm^n(u)}{du} \propto -H'(h-c)\left[g' - \frac{g^2}{G}\right] - \frac{g}{G}F.$$

By log-concavity of G, $g' - \frac{g^2}{G} < 0$. From Lemma 2, the first term in the sum is negative and therefore $\frac{dm^n(u)}{du}$ is negative. Since the price is an increasing function of the number of varieties the unconstrained maximum number of varieties $m^n(u)$ is strictly decreasing in u. Now, as long as $m^n(u) \geq m^*$, $m^n(u)$ is the solution to P0, otherwise the solution is to set $m = m^*$. If the constraint binds at some u, it binds for all $\hat{u} > u$ and therefore the monopoly does not adjust his prices or the varieties offered when u is large enough. Whether or not the constraint binds depends on the value of $m^n(h(0))$. The astute reader would have noticed that our comparative statics result is local; a "global" proof that varieties decrease as u increases is provided in the Appendix.

Proposition 3 *As u increases, the monopolist offers less varieties and the retail price decreases.*

(i) If $m^n(h(0)) \geq m^*$, the monopoly optimizes by choosing for any u a number of varieties $m^n(u)$, strictly decreasing with u.

(ii) If $m^n(h(0)) < m^*$, there exists $u^* \in (-\infty, h(0))$ such that for all $u < u^*$ the solution is $m^n(u)$, and for all $u \geq u^*$ the solution is m^*.

Proof. See the Appendix for the global argument for comparative statics. In the absence of e-commerce ($u \to -\infty$), the unconstrained optimum solves

$$-h'\left(\frac{1}{2m}\right) = 2m^2F.$$

From (2), $-h'(x^*) = \frac{h(x^*)-c}{x^*}$. From (3), $\frac{h(x^*)-c}{x^*} > \frac{F}{2x^{*2}}$; therefore, $-h'(x^*) > \frac{F}{2x^{*2}}$. Since the left hand side is increasing in x (follows from $h' < 0$ and $h'' < 0$) and the right hand side is decreasing in x, the first order condition can be satisfied at $x < x^*$, i.e., at $m > \frac{1}{2x^*}$. Hence, when e-commerce is not a serious competitor ($u \to -\infty$), the solution to P0 is to set $m^*(u)$, i.e., the constraint $m \geq \frac{1}{2x^*}$ does not bind.

(i) If $m^*(h(0)) \geq \frac{1}{2x^*}$, $m^*(u) > \frac{1}{2x^*}$ for all u since m^* is decreasing in u.

(ii) If $m^*(h(0)) < \frac{1}{2x^*}$, since $m^*(-\infty) > \frac{1}{2x^*}$, the constraint does not bind for low values of u and by monotony of m^* there exists a unique value u^* such that the constraint binds if and only if $u \geq u^*$. ■

Remember that the surplus of consumers is *decreasing* in the number of varieties offered. If the monopoly wants to give more utility to his consumers, he needs to decrease the number of varieties. Faced with a stronger competitor—in the form of a larger u—the monopolist will trade off the "compensating effect" needed not to lose consumers, which is equal to the variation of u with the resulting loss in profits when these consumers show up in his shops. The proposition shows that the "demand effect" dominates, i.e., that the monopoly will offer a better deal to his consumers and decrease varieties.

4 Oligopoly with Free Entry

We identify a variety with a "firm" (think of a shop owner). With respect to the monopoly situation, there is now an ex-post *competitive effect*. This new effect prevents the retail market from extracting too much rent from the consumers.

We are looking for a symmetric free-entry equilibrium, i.e., we need to find a number of firms m such that the marginal firm makes zero profits. Once m is fixed, the ex-post price is determined by standard competition on the circle. Let $p(m)$ be the ex-post equilibrium price corresponding to m firms. Replicating arguments we gave for the monopoly case, we can show that m is always chosen in such a way that the market is covered.

Lemma 4 *In a free-entry equilibrium, the equilibrium number of firms m is such that the ex-post equilibrium price is $p(m) \leq h\left(\frac{1}{2m}\right)$, i.e., the market is covered.*

Proof. Clearly if there exists a symmetric price equilibrium $p(m)$ for which the market is not covered, the consumer whose ideal type is in the middle of an arc between two varieties prefers not to consume any variety, i.e.,

$$h\left(\frac{1}{2m}\right) - p(m) < 0. \tag{10}$$

Now, when the market is not covered, each seller enjoys monopoly power on his natural market, i.e., the set of consumers for which $h\left(\frac{1}{2m}\right) \geq p(m)$. A necessary and sufficient condition is then that the maximum monopoly profit is attained at a price satisfying (10). A necessary and sufficient condition for a symmetric price equilibrium for which the market is not covered is then that the function that $m^* > m$. In this case the price is $h\left(\frac{1}{2m^*}\right)$. The rest of the argument follows the argument for the monopoly case. ■

$>$From now on we can assume that $m \geq m^*$. Let $p(m) \leq h\left(\frac{1}{2m}\right)$ be a candidate symmetric price equilibrium when there are m symmetrically located firms. If i and j are two adjacent firms on the circle, a necessary and sufficient condition for equilibrium is that firm i does not gain by deviating to price $p \neq p(m)$. Measuring distances with respect to firm j, the marginal consumer is at a distance x where

$$h(x) - p = h\left(\frac{1}{m} - x\right) - p(m).$$

Firm $i's$ ex-post payoff is $\left(h(x) - h\left(\frac{1}{m} - x\right) + p(m) - c\right)x$; simple computations show that this function is locally concave in p at $p = p(m)$, [8] and the first order condition for $p = p(m)$ to maximize this function is

$$\begin{aligned} p(m) &= c - \frac{x}{dx/dp} \\ &= c - \frac{h'\left(\frac{1}{2m}\right)}{m}. \end{aligned} \tag{11}$$

Note that the equilibrium price is a *decreasing* function of the number of varities:

$$\begin{aligned} \frac{dp(m)}{dm} &\propto \frac{h''\left(\frac{1}{2m}\right)}{2m} + h'\left(\frac{1}{2m}\right) \\ &< 0. \end{aligned} \tag{12}$$

Hence, decreasing the number of varieties will increase the equilibrium price.

Hence while the utility of each consumer decreases, the competition for marginal consumers is less intense and this effect dominates the first effect. Here the price is related to the *marginal* utility of the "marginal" consumer, and this marginal utility decreases with the number of varieties (by concavity). By contrast, in the monopoly case the price-cost margin is related to the *total* utility of the "marginal" consumer and this utility is increasing in the number of varieties.

The expected utility of a consumer entering the retail market is then

$$\begin{aligned} \hat{H}(m) &= 2m\int_0^{\frac{1}{2m}} h(x)\,dx + \frac{h'\left(\frac{1}{2m}\right)}{m} - c \\ &= H(m) + \frac{h'\left(\frac{1}{2m}\right)}{m} + h\left(\frac{1}{2m}\right) - c \end{aligned}$$

[8] Indeed, $\pi''(p) = 2\frac{dz}{dp} + (p-c)\frac{d^2z}{dp^2}$. Now, $\frac{d^2z}{dp^2} \propto -\frac{dz}{dp}\left\{h''(z) - h''\left(\frac{1}{m} - z\right)\right\}$ where the right hand side equal to zero at $p = p(m)$ since $z(p(m), p(m)) = \frac{1}{2m}$. The implicit function theorem implies that $\frac{dz}{dp} = \frac{1}{h'(z) + h'\left(\frac{1}{m} - z\right)} < 0$ and it follows that profits are locally concave at $p = p(m)$.

Note that $\hat{H}(m)$ is increasing in m. Indeed, $\hat{H}'(m) = \left\{ 2\int_0^{\frac{1}{2m}} h(x)\,dx - \frac{h\left(\frac{1}{2m}\right)}{m} \right\} - \frac{\frac{h''\left(\frac{1}{2m}\right)}{2m} + h'\left(\frac{1}{2m}\right)}{2m^2}$, the term in brackets is positive since $h' < 0$ and the last term is negative since $h' < 0$ and $h'' < 0$. This is in sharp contrast with the monopoly case since it suggests that in order to give more utility to the consumers the retail market should *increase* the number of varieties sold.

Since the market must be covered, $h\left(\frac{1}{2m}\right) \geq c - \frac{h'\left(\frac{1}{2m}\right)}{m}$ or $h(x) - c \geq -2xh'(x)$.

Lemma 5 *Let* $\underline{m}$ *be the unique solution of the equation* $h\left(\frac{1}{2m}\right) \geq c - \frac{h'\left(\frac{1}{2m}\right)}{m}$. *In equilibrium,* $m \geq \underline{m}$.

Proof. Letting $x = \frac{1}{2m}$, the covering condition is $h(x) - c \geq -2xh'(x)$..The left hand side is decreasing and concave while the right hand side is increasing and convex (if h''' is small enough or is negative); hence there exists a unique value $\overline{x}$ such that $h(x) - c \geq -2xh'(x)$ if and only if $x \leq \overline{x}$. Letting $\underline{m} = \frac{1}{2\overline{x}}$ proves the lemma. ■

Given m and the anticipated value of $p(m)$, the demand facing the retail market is $G(\theta(m;u))$, where $\theta(m;u) = \hat{H}(m) - u$. From Lemma 5, $\hat{H}(m) > H(m)$ in equilibrium. This illustrates the fact that—due to the ex-post competition—the oligopoly can *at a lower cost* provide surplus to the consumers.Finally the ex-ante equilibrium profit of a firm *net of the entry cost* F is

$$\pi(m;u) = \frac{G(\theta(m;u))}{m}\left(-\frac{h'\left(\frac{1}{2m}\right)}{m}\right)$$

An *equilibrium* is then defined by a *number of firms* m^* such that

$$\begin{aligned} \pi(m^*;u) &\geq F \\ \pi_m(m^*;u) &\leq 0. \end{aligned}$$

i.e., firms make nonnegative profits and no additional firm wants to enter.

Clearly, an equilibrium can exist only if $\max_m \pi(m;u) \geq F$. This condition is also sufficient for existence as the following establishes.

Lemma 6 *Let* $\overline{\pi}(u) = \max_m \pi(m;u)$. *An equilibrium exists in the oligopoly model if, and only if,* $\overline{\pi}(u) \geq F$.

Proof. Clearly if $\overline{\pi}(u) < F$, no firm wants to enter. Suppose that $\overline{\pi}(u) \geq F$. Note that π is continuous in m and that $\pi(0;u) = 0$ and $\lim_{m\to\infty} \pi(m;u) = 0$. Hence, if $\overline{\pi}(u) > F$, there exists m such that $\pi(m;u) =$

F and $\pi_m(m;u) < 0$. If $\bar{\pi}(u) = F$, choose the largest value of m such that $\pi(m;u) = \bar{\pi}(u)$. ■

Intuitively, the equilibrium condition $\pi'(m) \leq 0$ states that the per-firm demand (i.e., the mass $\frac{G(\theta(m;u))}{m}$) elasticity is less than the (absolute value) of the price-cost margin elasticity. The price-cost margin elasticity captures the direct strategic effect of entry while the demand elasticity captures the consumers response to entry anticipating this strategic effect. As we showed, the demand elasticity is positive (more varieties and lower prices make the retail market more attractive) while the price-cost margin elasticity is negative (more competition decreases prices ex-post).

An equilibrium level of varieties satisfies[9]

$$G(\theta(m;u))\left(-h'\left(\frac{1}{2m}\right)\right) = m^2 F \tag{13}$$

$$\theta_m g(\theta(m;u))\left(-h'\left(\frac{1}{2m}\right)\right) + G(\theta(m;u))\left(\frac{h''\left(\frac{1}{2m}\right)}{2m^2} + \frac{2h'\left(\frac{1}{2m}\right)}{m}\right) \leq 0 \tag{14}$$

$$m \geq \underline{m}. \tag{15}$$

(13) is the zero profit condition, (14) is the free entry condition $\Pi_m(m;u) \leq 0$[10] and (15) is the bound found in Lemma 5 corresponding the "covering the market condition".

The implicit function theorem applied to (13) implies that at the equilibrium value $m(u)$,

$$\frac{dm(u)}{du} = -\frac{\Pi_u(m;u)}{\Pi_m(m;u)}.$$

>From (14), the denominator is non positive. Hence, since $\Pi_u(m;u) = -g(\theta(m;u))\left(-h'\left(\frac{1}{2m}\right)\right) < 0$, it follows that

$$\frac{dm(u)}{du} < 0. \tag{16}$$

Now,

$$\frac{dp(u)}{du} = \frac{dm(u)}{du}\frac{dp(m)}{dm}.$$

Therefore, (16) and (12) yield $\frac{dp(u)}{du} > 0$.

[9] Note that $\Pi_m\left(m;u^E\right) = \pi\left(m;u^E\right) - F + m\pi_m\left(m;u^E\right)$ and therefore $\Pi_m\left(m;u^E\right) \leq 0$ is equivalent to $\pi_m\left(m;u^E\right) \leq 0$ when $\pi\left(m;u^E\right) = F$.

[10] After making use of (13) to substitute for mF.

Proposition 7 *In a free entry equilibrium, as u increases the number of varieties decreases and the price increases.*

As the monopoly the retail market has two basic choices in response to an increase in u. First, in order to keep its demand, it can "compensate" the consumers by increasing the value they will obtain in the retail market, this would come at the expense of the ex-post profits however and if the initial situation is one of zero profits such a solution is not sustainable. Second, it accepts a decrease in demand but then ex-post profits must increase in order to induce firms to enter the retail market. Ex-post profits can increase only if firms face less competition, i.e., only if the number of varieties decreases.

5 Monopoly : The Value of Pre-Announcing Prices

The monopoly now offers (m, p), where p is the price at which any of the m varieties is sold on the retail market. Given (m, p), the marginal consumer is the consumer of type

$$\tilde{\theta}(m, p) = 2m \int_0^{\frac{1}{2m}} h(x)\, dx - p - u. \tag{17}$$

Note that $\tilde{\theta}$ is linear in p, in fact $\tilde{\theta}_p = -1$ and that

$$\begin{aligned} \tilde{\theta}_m(m, p) &= 2\int_0^{\frac{1}{2m}} h(x)\, dx - \frac{h\left(\frac{1}{2m}\right)}{m} \\ &= \frac{H(m)}{m} \end{aligned}$$

is positive and is decreasing in m (since H is decreasing in m).

Here also the monopoly chooses optimally to cover the market.

Lemma 8 *If the monopoly enters the retail market, the market is covered.*

Proof. Suppose that $p > h\left(\frac{1}{2m}\right)$. The profit of the monopoly is

$$\pi = G\left(\tilde{\theta}(m, p)\right)(p - c)\, 2mh^{-1}(p) - mF$$

where $2mh^{-1}(p) < 1$ since $p > h\left(\frac{1}{2m}\right)$. Differentiating with respect to m yields

$$\begin{aligned} \pi_m &= \tilde{\theta}_m(m, p)\, g\left(\tilde{\theta}(m, p)\right)(p - c)\, 2mh^{-1}(p) + 2G\left(\tilde{\theta}(m, p)\right) x(p) - F \\ &= \tilde{\theta}_m(m, p)\, g\left(\tilde{\theta}(m, p)\right)(p - c)\, 2mh^{-1}(p) + \frac{\pi}{m}. \end{aligned}$$

Since $\tilde{\theta}_m > 0$, as long as $\pi \geq 0$, $\pi_m > 0$ (indeed $\pi \geq 0$ implies that $p > c$ when $F > 0$). ■

>From now, we assume that $p \leq h\left(\frac{1}{2m}\right)$ and we write the problem of the monopoly as

$$\max_{m,p} \pi(m,p) = G\left(\tilde{\theta}(m,p)\right)(p-c) - mF$$

$$\tilde{\theta}(m,p) = 2m \int_0^{\frac{1}{2m}} h(x)\,dx - p - u \tag{18}$$

$$h\left(\frac{1}{2m}\right) - p \geq 0 \tag{19}$$

It is convenient to make a change of variable. Let $v = 2m\int_0^{\frac{1}{2m}} h(x)\,dx - p - u$, hence $p = H(m) + h\left(\frac{1}{2m}\right) - v - u$ and the problem can be rewritten as[11]

$$\max_{m,v} \pi(m,v) = G(v)\left(H(m) + h\left(\frac{1}{2m}\right) - v - u - c\right) - mF \tag{P'}$$
$$m \geq H^{-1}(v+u).$$

The solution can be in one of two regimes: either the constraint is binding, in which case the solution coincides with that of the model of the first part of the paper, or the constraint is not binding in which case the two first order conditions $\pi_m = 0$ and $\pi_v = 0$ are satisfied.

As long as we are in the first regime, our previous comparative statics results apply, that is as u increases varieties and price decrease.

Consider now the non binding regime. The first order conditions imply

$$\pi_m = 0 : G(v)\frac{H(m)}{m} - F = 0 \tag{20}$$

$$\pi_v = 0 : g(v)\left(H(m) + h\left(\frac{1}{2m}\right) - v - u - c\right) - G(v) = 0. \tag{21}$$

π is not necessarily globally concave. However, we have the following.

Lemma 9 *(i) π is concave in m*

(ii) For each m, π is single-peaked in v, i.e., has a unique extremum that is a maximum.

[11] Since Δ is decreasing and convex, H^{-1} is also decreasing and convex. Hence, the constraint can be written $H(m) \leq v+u$ or $m \geq H^{-1}(v+u)$. Note that the "covering" constraint $m \geq H^{-1}(v+u)$ *does not* impose that $m \geq m^*$ since the monopoly can ex-ante announce prices and even when $m < m^*$, the monopoly can adjust the price p in order to satisfy the constraint.

Proof. (i) $\pi_{mm} \propto G(v) h'(1/2m) < 0$.
(ii) Note that

$$\pi_v(m,v) = g(v)\left\{H(m) + h\left(\frac{1}{2m}\right) - u - c - \left[\frac{G(v)}{g(v)} + v\right]\right\}. \qquad (22)$$

By log-concavity the term in brackets is increasing in v. Hence, if there is v such that $\pi_v(m,v) = 0$, this value is unique and furthermore π_v is positive for smaller values and negative for larger values. This proves the result. ■

If we represent the graphs of $\pi_m = 0$ and of $\pi_v = 0$ in a (v,m) space, both of these graphs are increasing. Indeed, since $\frac{H(m)}{m}$ is a decreasing function of m, when m increases, it is necessary that v increases in order to satisfy (20). Since $H(m) + h\left(\frac{1}{2m}\right)$ is increasing in m, it is necessary that v increases in order to satisfy (22). We can show that for a given u these graphs intersect only once and moreover that the graph of $\pi_v = 0$ intersects the graph of $\pi_m = 0$ from below.

Lemma 10 *(i) if $\pi_m(m,v) = 0$ then $(\hat{m} - m)\pi_m(\hat{m},v) < 0$ and $(\hat{v} - v)\pi_m(m,\hat{v}) > 0$;*

(ii) if $\pi_v(m,v) = 0$ then $(\hat{m} - m)\pi_v(\hat{m},v) > 0$ and $(\hat{v} - v)\pi_v(m,\hat{v}) < 0$

(iii) The graph of $\pi_v = 0$ intersects the graph of $\pi_m = 0$ only once and "from below": if (m,v) satisfies $\pi_m = \pi_v = 0$, then when $\hat{v} < v$, $\pi_v(\hat{m},\hat{v}) = 0 \Rightarrow \pi_m(\hat{m},\hat{v}) > 0$ and when $\hat{v} > v$, $\pi_v(\hat{m},\hat{v}) = 0 \Rightarrow \pi_m(\hat{m},\hat{v}) < 0$.

Proof. Appendix. ■

These properties are represented in the following picture where the arrows indicate the direction of increasing profits.

As u increases the graph of $\pi_m = 0$ does not change while the graph of $\pi_v = 0$ moves to the left. Hence, if we are in a regime in which the covering constraint is not binding, v and m will decrease. This is established formally in the Appendix. Hence, locally we recover the results of the first part of the paper.

Now a potential difficulty is when there is a regime change, i.e., when we go from a situation in which the constraint binds to a situaion in which the constraint does not bind. Now, when the constraint binds we are back to the monopoly problem of the first part of the paper and by Lemma 10, we can deduct that this optimum must be in the region $\pi_v \leq 0$ and $\pi_m \leq 0$, as represented by the bracket in the figure above. In fact, the optimum is in the regime of binding constraint if, and only if, the graph of $m = H^{-1}(v+u)$ intersects the area defined by $\pi_v \leq 0$ and $\pi_m \leq 0$. Now, as u increases the graph of $m = H^{-1}(v+u)$ moves to the left and locally the property is preserved; we therefore have the same comparative statics as in the first part

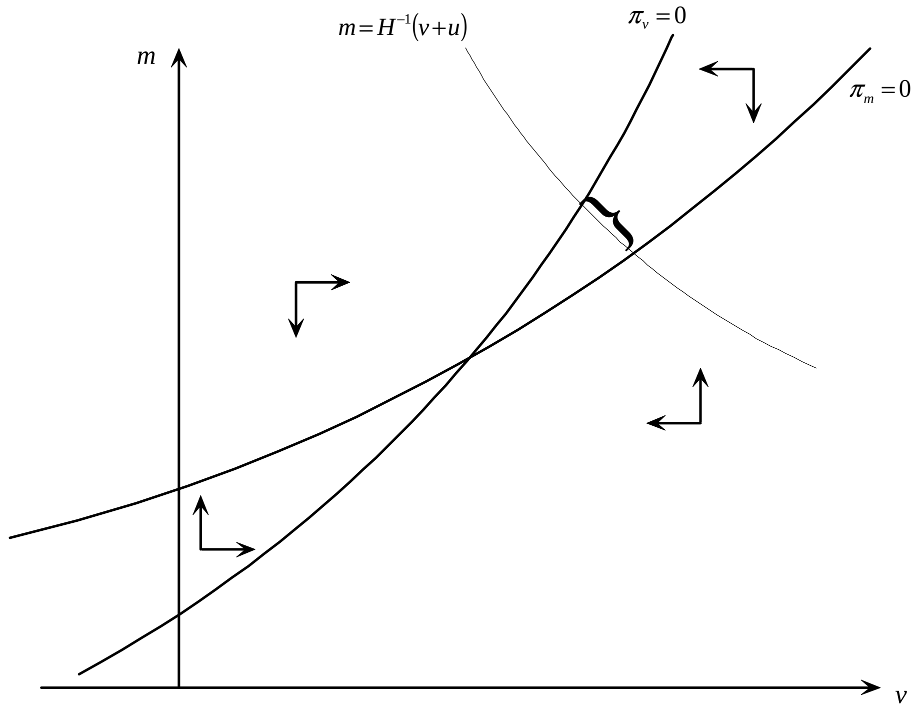

Figure 1:

of the paper. If the graph of $m = H^{-1}(v+u)$ moves to the left "faster" than the graph of $\pi_v = 0$ then we enter the second regime where the covering constraint is not binding.

It is simple to show that as u is small ($u \to -\infty$), there is no value for the monopoly to pre-announce prices, i.e., to reduce the hold-up problem, while for u large ($u \to h(0)$) there is value to pre-announcing prices.

Corollary 11 *For u sufficiently small there is no value to pre-announcing prices, i.e., the solution to (??) is such that $v + u = H(m)$.*

Proof. Appendix. ■

6 Ex-Post Competition from e-Commerce

The initial comparative static results could have suggested that the difference in response of the monopoly and the oligopoly were due to the fact that the monopoly faces a hold-up problem while the oligopoly does less so. However as the previous section has showed even if the monopoly can eliminate his hold-up problem we still obtain the same comparative statics as when he cannot. The difference between the two organizational forms is therefore due to the nature of the competition rather than to the magnitude of the hold-up problem.

For both of the monopoly cases that we considered, the nature of competition was similar to an *individual rationality* constraint that the monopoly has to take into account. For the oligopoly, there was also this (ex-ante) individual rationality constraint but also an (ex-post) *incentive compatibility* constraint since two firms compete for the marginal consumer.

This suggests that by introducing some (ex-post) competition we could have a different response of the monopoly to an increase in the outside option of consumers. A simple way to do so is to allow consumers to first visit the retail market and then go buy the variety that they have identified to be the "best fit" on e-commerce. For instance, while it is difficult to describe ex-ante the characteristics of a running shoe, it is possible after having tried on different shoes in a store to identify the exact model and size that is best; it is then easy to order such an item from e-commerce. To analyze this model and avoid trivialities, it is necessary to distinguish between the cost of accessing the retail market, that is θ, and the cost of accessing e-commerce, that we denote by α. We assume that θ has distribution G (log-concave) and that α is a constant.

The new timing is as follows.

- Consumers learn θ and observe the number of varieties in the retail market;
- If the consumer shops on e-commerce his expected utility is $u = \underline{h} - q - \alpha$;[12]
- If the consumer goes to the retail market, he learns by sampling the m varieties which variety gives him the "best fit" $h(x)$;
  - he can then buy this variety on the retail market and his utility is $h(x) - p-$,
  - or he can buy this variety from e-commerce and his utility is $h(x) - q - \theta - \alpha$.

There are now three options for consumers:

1. Buy from e-commerce immediately: surplus is u
2. stay in the local market: ex-post surplus is $h(y) - p - \theta$ or 0 is the consumer decides not to buy.
3. Go to the retail market and buy from e-commerce: ex-post if the best fit is $h(x)$, the surplus is $h(x) - q - \alpha - \theta$ (since the consumer has to pay the two costs).

The share of the market of the monopoly is the mass of consumers choosing the second option.

Note that the choice between 2 and 3 depends on whether α is greater or lower than $p - q$: if $\alpha < p - q$, all consumers purchase from e-commerce. Hence we need[13]

$$\alpha \geq p - q. \tag{23}$$

This is a simple representation of the ex-post price competition from e-commerce. Clearly in equlibrium this constraint is satisfied and therefore consumers should anticipate in equilibrium a price

$$p(m) = \min\left\{\alpha + q, h\left(\frac{1}{2m}\right)\right\}.$$

[12] See the introduction for possible stories for $\underline{h}$; as we will see it is important to distinguish between prices and utility here.

[13] Therefore, no consumers will buy ex-post from e-commerce; this is due to the assumption that the cost α is common to all consumers.

>From an ex-ante perspective, the consumers who decide to go to the retail market are those whose type is

$$\theta \leq \tilde{\theta}\left(m, pm^{a}\right) = 2m \int_{0}^{1/2m} h\left(x\right) dx - p\left(m\right) - u,$$

where $p\left(m\right)$ is the price that the consumers anticipate on the retail market. Once the customers show up at the retail market, the monopolist chooses the price. The problem is then

$$\max_{m} G\left(\left(\tilde{\theta}\left(m, p\left(m\right)\right)\right)\right)\left(p\left(m\right) - c\right) - mF.$$

If the solution is in the region $\left\{m : \alpha + q > h\left(\frac{1}{2m}\right)\right\}$, the solution is identical to the case in the first part of the paper; the comparative static results follow. If the solution is in the region $\left\{m : \alpha + q \leq h\left(\frac{1}{2m}\right)\right\}$, m solves the first order condition

$$\frac{H\left(m\right)}{m} g\left(\tilde{\theta}\left(m, \alpha + q\right)\right)\left(\alpha + q - c\right) = F$$

Note that $\tilde{\theta}\left(m, \alpha + q\right) = H\left(m\right) + h\left(\frac{1}{2m}\right) - \underline{h}$. Hence, the first order condition is

$$\frac{H\left(m\right)}{m} g\left(H\left(m\right) + h\left(\frac{1}{2m}\right) - \underline{h}\right)\left(\alpha + q - c\right) = F$$

Here the static comparative must distinguish the source of the increase in u : either because e-commerce allows better search or inspection ($\underline{h}$ increases) or because the expected total price $\left(\alpha + q\right)$ decreases or because the total variation $\underline{h} - q - \alpha$ decreases.

If there is an improvement in inspection on e-commerce, then

$$\frac{dm}{d\underline{h}} \propto -g'\left(\tilde{\theta}\left(m, \alpha + q\right)\right)$$

can be positive if $g' < 0$, which typically happens when $\tilde{\theta}$ is large.

If however the improvement is due to a lowering of costs

$$\frac{dm}{d\left(\alpha + q\right)} = \frac{H\left(m\right)}{m} g\left(H\left(m\right) + h\left(\frac{1}{2m}\right) - \underline{h}\right)$$

and varieties unambiguously increase.

Note that these results are not so surprising since price competition is eliminated! The only strategic variable of the monopoly is the number of varieties and because there is no hold-up problem (the ex-post price is a constant), the way to give more surplus to retail consumers is to give them more varieties.

However, this result is suggestive of a possible pattern in variety and price competition in a monopolized retail market. When e-commerce offers a small outside option to consumers the monopoly responds to an increase of this option by decreasing varieties and prices while when e-commerce offers a large outside option, the monopoly retailer will shift to variety competition and will offer more varieties.

Note that the logic for the monopolistic competitive retail market is the same as before: allowing consumers to inspect varieties on the retail market will create even more competition ex-post and will decrease oligopoly profits; there will be therefore less varieties in response to a stronger e-commerce.

7 Conclusion

Our analysis has showed that e-commerce will crowd out varieties from classical retailing, independently of the market structure in retailing. Comparing the effects of a stronger e-commerce on monopoly and oligopoly prices, we have found that while prices will decrease in the monopoly they will increase in the oligopoly. The main intuition is that the oligopoly can partially commit to low prices because there is ex-post competition between firms. However, less varieties decreases competition ex-post and therefore yield to higher prices. When we allow the monopoly to pre-announce prices we find that the monopoly *will not use the option of committing* when e-commerce is a weak competitor and will use the option otherwise; comparative statics on prices and varieties are similar however to the case where the monopoly cannot pre-announce prices.

Things are different for the monopoly when the consumers can benefit from information obtained in the retail market before shoping in e-commerce. In this case, the monopoly is subject to ex-post price competition and when e-commerce becomes a strong competitor on this dimension the optimal response of the monopoly is to increase the number of varieties that is offered.

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8 Appendix

8.1 Proof of Proposition 3

Let $u > \hat{u}$ and assume that the optimal solution for u is m and the optimal solution for $\hat{u}$ is $\hat{m}$. Assume by way of contradiction that $m > \hat{m}$. Let $v = \Delta(m)$, $\hat{v} = \Delta(\hat{m})$, $h = h\left(\frac{1}{2m}\right)$ and $\hat{h} = h\left(\frac{1}{2\hat{m}}\right)$. By Lemma 2, $v < \hat{v}$ and since h is a decreasing function $h > \hat{h}$.

Since $v < \hat{v}$ and $u > \hat{u}$, log-concavity of G implies

$$\log G(v-\hat{u}) - \log G(v-u) > \log G(\hat{v}-\hat{u}) - \log G(\hat{v}-u),$$

adding $\log h - \log h$ to the left hand side and adding $\log \hat{h} - \log \hat{h}$ to the right hand side we preserve the inequality and obtain

$$\log G(v-\hat{u})\,h - \log G(v-u)\,h > \log G(\hat{v}-\hat{u})\,\hat{h} - \log G(\hat{v}-u)\,\hat{h}. \tag{24}$$

Revealed preferences imply

$$\begin{aligned} G(v-u)\,h - mF &\geq G(\hat{v}-u)\,\hat{h} - \hat{m}F && (25)\\ G(\hat{v}-\hat{u})\,\hat{h} - \hat{m}F &\geq G(v-\hat{u})\,h - mF. && (26)\end{aligned}$$

Since $m > \hat{m}$, (25) implies

$$G(v-u)\,h > G(\hat{v}-u)\,\hat{h} \tag{27}$$

and by adding (25) and (26) and rearranging we have

$$0 \leq G(v-\hat{u})\,h - G(v-u)\,h \leq G(\hat{v}-\hat{u})\,\hat{h} - G(\hat{v}-u)\,\hat{h}. \tag{28}$$

Let $\delta \geq 0$ be equal to the difference between the right and left hand sides of (28). Since log is a concave function, (27) implies

$$\begin{aligned}\log G(v-\hat{u})\,h - \log G(v-u)\,h &< \log\left\{G(\hat{v}-u)\,\hat{h} - \delta\right\} - \log G(\hat{v}-\hat{u})\,\hat{h}\\ &\leq \log G(\hat{v}-u)\,\hat{h} - \log G(\hat{v}-\hat{u})\,\hat{h}\end{aligned}$$

which contradicts (24). This proves that when $u > \hat{u}$, the optimal value of m decreases.

8.2 Proof of Lemma 10

(i) and (ii) follow directly from the discussion in the text. For instance, since $\pi_m = G(v)\frac{H(m)}{m} - F$, if $\pi_m(m,v) = 0$, then for $\hat{m} > m$, $\frac{H(\hat{m})}{\hat{m}} < \frac{H(m)}{m}$ and $\pi_m(\hat{m}, v) < 0$ and $\hat{v} > v$, $G(\hat{v}) > G(v)$ and $\pi(m, \hat{v}) > 0$.

We prove (iii) in a series of steps.

Step 1: We show that at a local maximum the graph of $\pi_v = 0$ interesects the graph of $\pi_m 0$ from below. Since the function $\frac{H(n)}{m}$ is strictly decreasing, it has an inverse function Φ that is also strictly decreasing. The first order condition $\pi_m = 0$ can be then written as

$$m = \Phi\left(\frac{F}{G(v)}\right).$$

Since this is a necessary condition for an interior solution, we can substitute this expression for m in the profit function and we have the new problem

$$\max_v \hat{\pi}(v) = G(v)\left(h\left(\frac{1}{2\Phi\left(\frac{F}{G(v)}\right)}\right) - v - u - c\right),$$

where $\hat{\pi}$ is a function of v only while π is a function of m and v.

It follows that

$$\begin{aligned}
\hat{\pi}_v(v) &= g(v)\left(h\left(\frac{1}{2\Phi\left(\frac{F}{G(v)}\right)}\right) - v - u - c\right) \\
&\quad + G(v)\left(\frac{Fg(v)}{2G^2(v)}\frac{\Phi'\left(\frac{F}{G(v)}\right)}{\Phi^2\left(\frac{F}{G(v)}\right)}h'\left(\frac{1}{2\Phi\left(\frac{F}{G(v)}\right)}\right) - 1\right)
\end{aligned}$$

If the monopoly makes positive profits, the first term in the sum is positive. The second term is positive or negative.[14] Assuming an interior local optimum, $\hat{\pi}_{vv} < 0$, and differentiating the first order condition $\hat{\pi}_v(v) = 0$ yields $\frac{dv}{du} = \frac{g(v)}{\hat{\pi}_{vv}(v)} < 0$. Hence *locally* if u increases the optimal value of v decreases.

Now, by construction, a solution to $\hat{\pi}_v = 0$ solves $\pi_m = \pi_v = 0$, i.e., the optimal v is found at the intersection of the graphs of $\pi_m = 0$ and $\pi_v = 0$.

[14] A solution—i.e., where $\pi_v \leq 0$—exists. Indeed, we cannot have $\pi_v > 0$ everywhere for then $v = h(0)$ would be the solution, and this is possible only if $u = 0$, $m = \infty$ and $p = 0$, which is incompatible with positive profits.

Assume that the graph of $\pi_v = 0$ intersects the graph of $\pi_m = 0$ from above. Since when u increases the graph of $\pi_v = 0$ moves to the left, the intersection shifts to the righ, that is the optimal v increases as u increases which contradicts the previous observation. Hence at an interior optimum the graph of π_v must intersect the graph of π_m from below.

Step 2: We show that there is no local minimum.

>From Lemma 9, whenever the graphs of $\pi_m = 0$ and $\pi_v = 0$ intersect, the point (m, v) is a maximum.

Step 3: We show that there is a unique local maximum.

>From the previous two steps, the graph of $\pi_v = 0$ cannot intersect the graph of $\pi_m = 0$ from above. Hence, the graph of $\pi_v = 0$ can "cut" the graph of $\pi_m = 0$ only once, and possibly be tangent *to* the graph of $\pi_m = 0$ at other points, e.g., we could have a situation like in the figure below where there are two local maxima. howeverm this case is not possible because by increasing u we shift the graph $\pi_v = 0$ upward and therefore we generate a situation where the graph of $\pi_v = 0$ interesects the graph of $\pi_m = 0$ from above. The same reasoning prevents a siuation in which the two graphs are tangent at a unique point.

8.3 Proof of Corollary 11

There is no value to pre-announcing prices if the constraint in (**??**) binds, i.e., if $\pi_v(m, H(m) - u) < 0$. Now,

$$\pi_v = g(v)\left(H(m) + h\left(\frac{1}{2m}\right) - v - u - c\right) - G(v)$$

as $u \to -\infty$, $v \to +\infty$ (since $v + u \geq 0$). Now, by log-concavity of G, as $v \to +\infty$, $g(v) \to 0$. Since $H + h - v - u - c$ is bounded above, the first term on the right hand side goes to zero as u goes to $-\infty$ while $G(v) \to 1$. Hence for any m, $\pi_v < 0$ which proves the result.

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