Market Equilibrium as a MCP

(Mixed Complementarity Problem)

Specification of an Equilibrium Model

The description of an economic system in terms of a general equilibrium problem requires the specification of different entities and their relationships:

(i) definition of economic agents (firms, households, government, abroad,...)

(ii) definition of signals/decision variables for agents (prices, elasticities, ...)

(iii) description of agents through rules which reflect their behavior depending on the perceived signals (profit/utility maximization, ...)

(iv) definition of equilibrium mechanisms and market structure (flexible prices in competitive markets, "sticky" prices and rationing, ...)

Definition of an Economic Equilibrium - Naming Conventions

i	=	index of the i-th good ($i=1..n$)
j	=	index of the j-th activity ($j=1..m$)
h	=	index of the h-th household ($h=1..k$)
p	=	vector of prices (p_i)
a	=	netput vector (a_j) or netput-coefficient (a_{ij})
A	=	netput technology matrix for input-output-coefficients consistent with unit production ($A(p)= [a_1(p), ...,a_m(p)]$)
b	=	vector of endowments (b_{ih})
Inc	=	vector of household incomes (Inc_h)
y	=	vector of activity levels (y_j)
ξ	=	vector of excess demand functions ($\xi_i(p)$: market excess demand, $\xi_{i,h}(p)$: household excess demand)
d	=	vector of commodity demands by households (d_{ih})

Definition of an Economic Equilibrium - Equilibrium conditions

Market equilibrium in the competitive (Arrow-Debreu) economy is defined by non-negative vectors in prices *p*, activities *y* and incomes *Inc* which satisfy the following conditions:

(GE-1) No sector (activity) earns a positive profit (**zero profit**):
$- \Sigma_{i \in I} a_{ij} p_i \geq 0 \quad \forall j \in J$

(GE-2) Supply minus demand for every commodity is non-negative (**market clearance**):
$\Sigma_{j \in J} a_{ij} y_j \geq \xi_i(p,Inc) = \Sigma_{h \in H} \xi_{ih}(p,Inc) = \Sigma_{h \in H} (d_{ih}(p,Inc) - b_{ih}) \quad \forall i \in I$

(GE-3) Endowment income determines expenditures (**budget constraint**):
$Inc_h = \Sigma_{i \in I} b_{ih} p_i = \Sigma_{i \in I} d_{ih} p_i \qquad \forall h \in H$

(GE-4) Producer responses functions $a_j(p)$ maximize profit of sector *j* at given market prices *p*.

(GE-5) Household excess demand functions $\xi_h(p,Inc)$ maximize utility at market prices *p* and expenditure level *Inc*. Non-satiation assures that demands exhaust income.

Complementarity Features of an Economic Equilibrium

(i) Assumption of non-satiation implies Walras' law: $\Sigma_{i \in I} p_i \xi_i = 0$
The sum of the values of market excess demand equals zero.

(ii) The equilibrium determines only relative prices because the excess profit functions as well as the excess demand for each household are linearly homogeneous in prices. This explains the need for a numeraire (absolute prices are meaningless).

(iii) In the equilibrium Walras' law implies complementarity between:

- market equilibrium and market prices:
  $(\Sigma_{j \in J} a_{ij} y_j - \xi_i(p, Inc)) \cdot p_i \quad = 0 \quad \forall i \in I$
- activity levels and excess profit functions:
  $(\Sigma_{i \in I} a_{ij} p_i) \cdot y_j \quad = 0 \quad \forall j \in J$
- income levels and income definition:
  $(Inc_h - \Sigma_{i \in I} p_i b_{ih}) \cdot Inc_h \quad = 0 \quad \forall h \in H$

===> Complementarity features motivate the formulation of equilibrium problems in the complementarity format.

Equilibrium in MCP Format

The mixed complementarity format accommodates the formulation of equilibrium conditions and features in the most general way through the simultaneous and explicit treatment of equalities/ inequalities as well as decision variables:

The complementarity problem

(CP) Find $z \in R^n$ s.t. $f(z) \geq 0$, $z \geq 0$ and $z^T f(z) = 0$

is equivalent to the equilibrium conditions by letting

$z = [y, p]^T$ and $f(z) = [-A^T p, b + Ay - d(p)]$.

Advantages of MCP approach as compared to traditional approaches:

+ inequality constraints (square system of equations)
+ non-integrabilities (mathematical programming)

Formulation of a GE Model in the MCP Format

The model is formulated as a nonlinear system of (weak) inequalities corresponding to the three classes of equilibrium conditions associated with an Arrow-Debreu GE:

- supply minus demand for every commodity is nonnegative,
- no sector earns a positive profit,
- {income equals expenditure}.

The fundamental unknowns of the system are three vectors:

- non-negative prices,
- activity levels (production indices),
- {consumer incomes}.

In equilibrium each of these variables is linked to one inequality condition:

- a commodity price with a market clearance condition,
- an activity level with an exhaustion of product constraint,
- {a consumer income variable with an income definition equation}.

The standard Arrow-Debreu framework can be extended to reflect market restrictions (imperfections) such as administered prices and quantities. These constraints are associated with endowment rationing instruments or endogenous tax rates. The latter indicate the price wedge between the marginal cost and the market price of a commodity caused by quantity administration.

GAMS Solution Report

• Solution values for the central variables are returned as GAMS variable level values.

• The level value returned for a "consumer variable" is an income level, not a welfare index.

• Level values for slacks are returned as "marginals" for the associated variables.

Examples:

- marginal of prices	--> excess supply
- negative marginal of activities	--> excess profit
- marginal of incomes	--> excess income

• Complementarity implies that in equilibrium either the level value of a variable will be positive or the marginal value will be positive, but not both.

Benchmark Replication Check and Debugging

• Benchmark check: If we start from balanced data, the model should replicate the initial equilibrium.

• Benchmark replication provides an important check on consistent data treatment (in p-fields and q-fields), but does not assure that the model makes economic sense.

• To do a benchmark check, set ITERLIM = 0. The level values passed to the solver are unaltered because the iteration limit is zero.

• Market excess supplies and zero profit checks are returned in the "marginals" of the associated commodity prices and activity levels respectively.

• If the model does not calibrate, one or several market equilibrium conditions do not hold. Check these conditions in the listing file.

Benchmark Replication Check and Debugging

(1) Start with zero-profit conditions - the marginals of the activity levels indicate excess profits.

(2) Next check market clearance - the marginals of prices indicate excess demands.

(3) It might be necessary to check zero-profit or market clearance explicitly:

```
PARAMETER       CHK (*,*);              CHK (“PRF”,S) = .....
ABORT$ABS(SMAX(S,CHK (“PRF”,S)) GT 0) "zero profit condition violated", CHK;
```

(4) Check income balance - fix household budget (income balance will be excluded from computation)

==> Income level variable tells the BMK deviation between income and expenditure