Computable General Equilibrium Analysis: An Introduction

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Wide-spread use of CGE models

- Literature (JEL: D58)
  - trade (e.g. Shoven and Whalley 1984)
  - public finance (e.g. Peireira and Shoven 1988)
  - energy and environment (e.g Conrad 1999, 2001)
- Users:
  - universities/research institutes
  - international organisations (OECD, world bank, EU commission)
  - ministries

Reservations in Science and Practise

- Ideal assumptions:

  Walras-Mechanismus

- Lack of empirical foundation:

  calibration of functional forms

- Lack of transparency:

  „black-box“

„Curse“ of Numerical Analysis

“jack of all trades”

Shoven und Whalley (JEL, 1984, S.1047)

Modelers (users) must know:

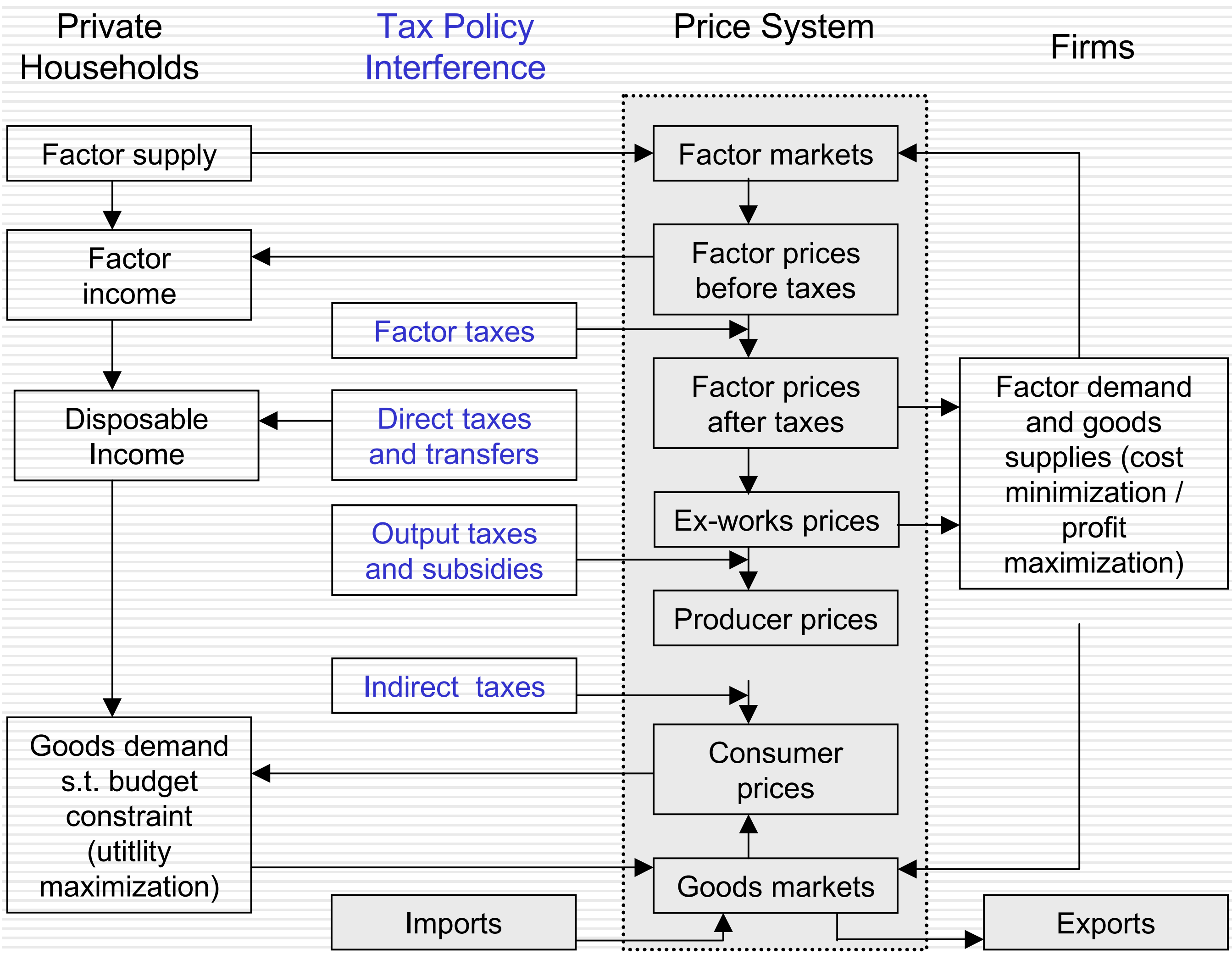
- general equilibrium theory so that their models have a sound theoretical basis,
- how to solve their models,
- they need to be able to program,
- they must understand the policy issues on which they work,
- they have to know about data sources and all their associated problems,
- and they have to be conversant with relevant literature, especially that on elasticities.

„New approaches to Equilibrium“

- State-of-the-art instruction for potential modelers:
  - Mathematical formulation (MCP, share form)
  - Software packages (Implementation: GAMS/PATH),
- Convenient interactive access for economists: (without programming skills):
  - Economic intuition (e.g. teaching)
  - Policy decision support (sensitivity analysis)

Basic Structure of a GE Model

- Motivation
- **Basics**
- The 2x2x1-Model
- Implementation
- Applications
- Conclusion



Steps in Applied CGE Analysis

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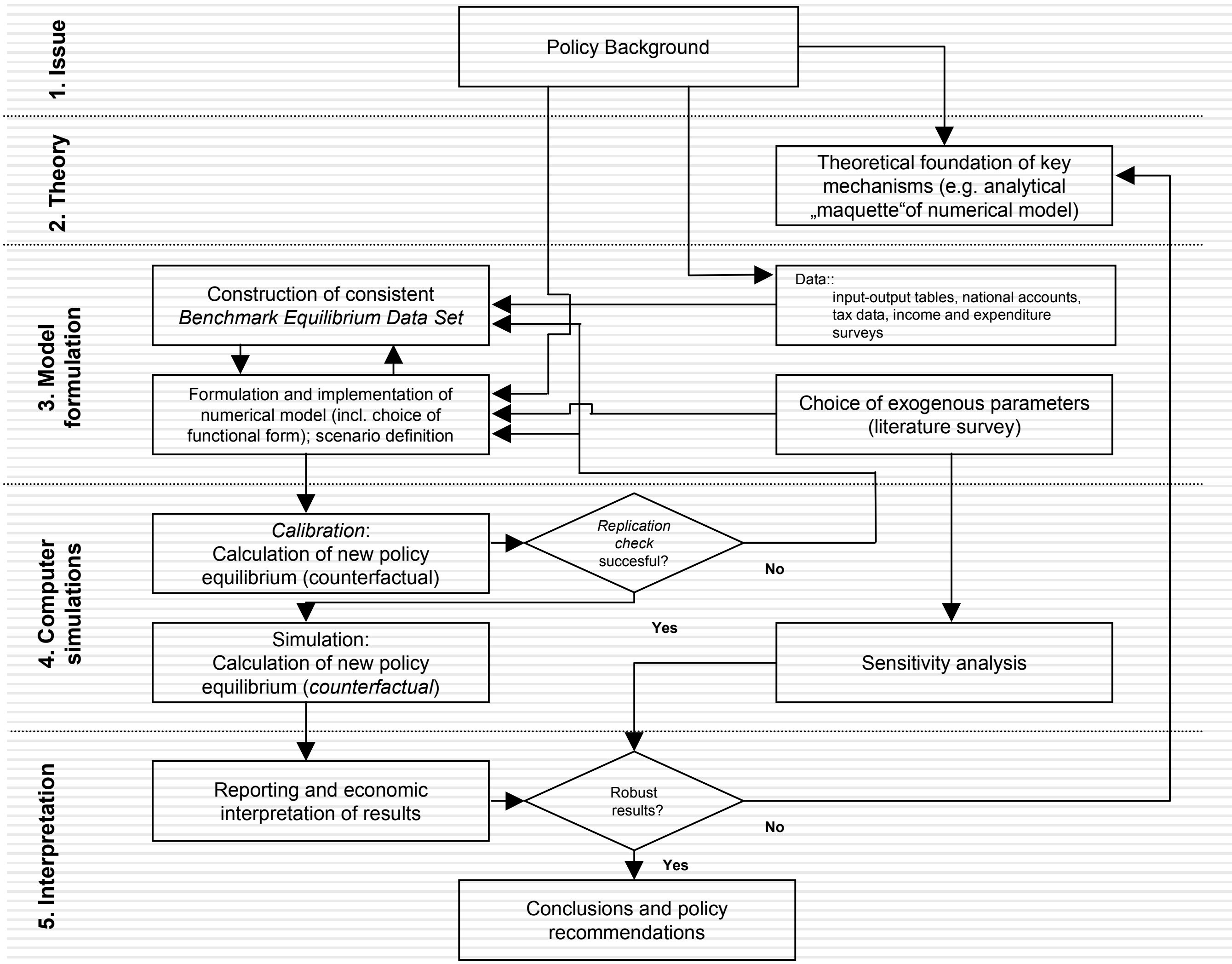

The 2x2x1 - Model

Equilibrium conditions for competitive 2x2x1-economy:

Zero profit:	$p_i = r K_i^y(r,w) + w L_i^y(r,w)$	$i=1,2$
Capital demand:	$K_i = K_i^y(r,w) Y_i = \frac{\partial p_i}{\partial r} Y_i$	$i=1,2$
Labor demand:	$L_i = L_i^y(r,w) Y_i = \frac{\partial p_i}{\partial w} Y_i$	$i=1,2$
Market clearance:	$Y_i = X_i$	$i=1,2$
Goods markets:	$X_i = X_i(p_1, p_2, M)$	$i=1,2$
Capital market:	$\sum_{i=1}^{2} K_i^y(r,w) Y_i = \bar{K}$	
Income definition:	$M = r\bar{K} + w\bar{L}$	
Numéraire:	$\mathrm{w} = 1$	

$\Rightarrow$ **System of 12 nonlinear equations in 12 variables**

N.B.: implicit variables $\Rightarrow$ ***K_i, L_i, X_i, M***

Mixed Complementarity Problem - Format (MCP)

$$Given: f: R^n \rightarrow R^n$$

$$Find: z \in R^n$$

$$subject\ to: f(z) \geq 0,\ z \geq 0,\ z^T f(z) = 0$$

Mixed:	Mixture of equalities and inequalities
Complementarity:	Complementarity between system variables and system conditions

Advantages:

- activity analysis: discrete regime shifts between alternative activities
- weak inequalities (e.g. minimum wages)

Example: Fixed Wages and Unemployment

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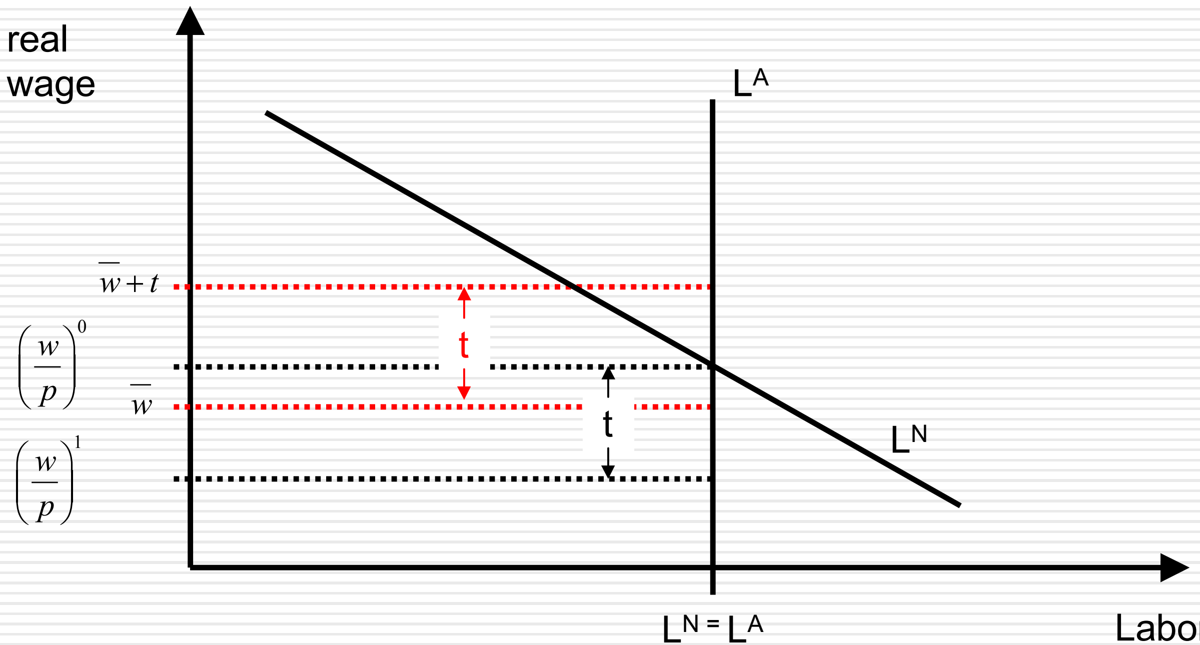
Labor market equilibrium: $\sum_{i=1}^{2} L_i^y(r,w)Y_i = \bar{L}$

Wage restriction: $\frac{w}{p} \geq \frac{\bar{w}}{p} \perp u; \left(\frac{w}{p} - \frac{\bar{w}}{p}\right)u = 0; u \geq 0 \qquad p = f(p_1, p_2)$

Rationed equilibrium: $\sum_{i=1}^{2} L_i^y(r,w)Y_i = \bar{L}(1-u)$



The Arrow-Debreu-Model as MCP

p := a non-negative n-vector of prices for all goods and factors ($I=\{1,\ldots,n\}$)

y := a non-negative m-vector of activity levels for CRTS production sectors ($J=\{1,\ldots,m\}$)

M := a non-negative k-vector of incomes ($H=\{1,\ldots,k\}$)

Zero profit condition for CRTS producers:

$$-\Pi_j(p) = C_j(p) - R_j(p) \geq 0 \quad \forall j$$

Market clearance for all goods and factors:

$$\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} \geq \sum_h d_{ih} \quad \forall i$$

Budget constraints for households:

$$\sum_h p_i b_{ih} = M_h \geq \sum_h p_i d_{ih} \quad \forall h \qquad d_{ih}(p, M_h) \equiv \arg\max \left\{ U_h(x) \middle| \sum_i p_i x_i = M_h \right\}$$

Complementarity Features of Economic Equilibrium

Walras‘ law („Non-satiation“) yields:

$$\sum_j y_j \Pi_j(p) = 0 \qquad \text{bzw.} \qquad y_j \Pi_j(p) = 0 \quad \forall j$$

$$p_i \left(\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} - \sum_h d_{ih} \right) = p_{i\xi} \xi_i = 0 \quad \forall i$$

$$M_h \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) = 0 \quad \forall h$$

Ergo: The problem of solving the economic equilibrium corresponds to a MCP where:

$$z = [y, p, M] \qquad \text{bzw.} \qquad f(z) = \left[\Pi_j(p), \xi_i, \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) \right]$$

Coefficient Form versus Calibrated Share Form

	CES coefficient form:	CES calibrated share form:
Production:	$y=\gamma\left(\sum_i \alpha_i x_i^{\rho}\right)^{1/\rho}$	$y=\bar{y}\cdot\left[\sum_i\left(\theta_i\cdot\left(\frac{x_i}{\bar{x}_i}\right)^{\rho}\right)\right]^{1/\rho}$
Cost:	$C=\gamma^{-1/\sigma}\left[\sum_i \alpha_i^{\sigma}\cdot\gamma^{(\sigma-1)\cdot\rho}\cdot w_i^{1-\sigma}\right]^{1/(1-\sigma)}\cdot y$	$C=\bar{C}\cdot\left[\sum_i \theta_i\cdot\left(\frac{w_i}{\bar{w}_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}\cdot\frac{y}{\bar{y}}$
Demand:	$x_i=\gamma^{\sigma-1}\cdot\left(\frac{\alpha_i p}{w_i}\right)^{\sigma}\cdot y$	$x_i=\bar{x}_i\cdot\frac{y}{\bar{y}}\cdot\left(\frac{c}{\bar{c}}\cdot\frac{\bar{w}_i}{w_i}\right)^{\sigma}$

Advantage of *calibrated share form*:

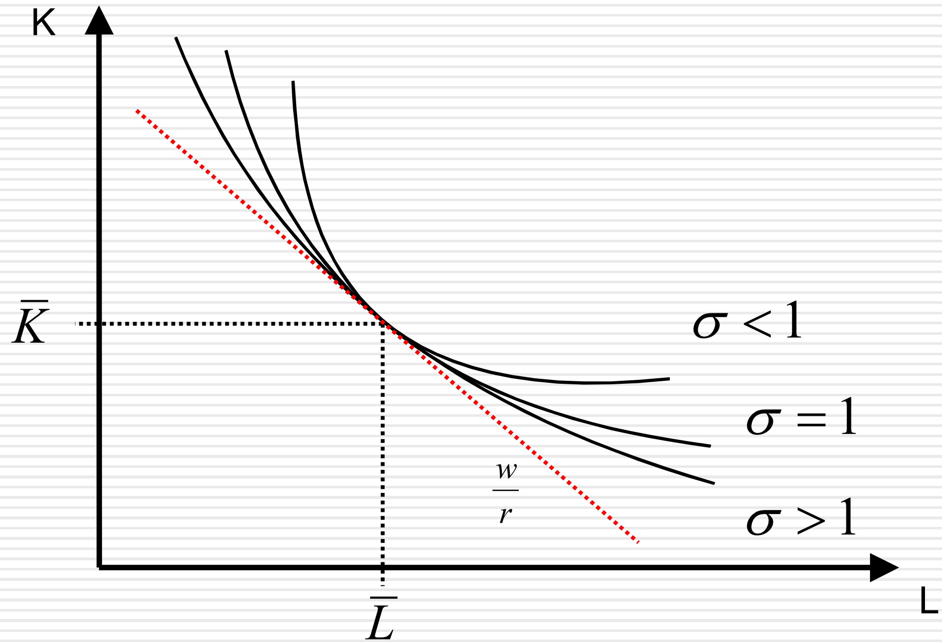
No messy inverting:

$\Rightarrow$ Direct calibration from benchmark values

Calibration - The Basics

CES function is determined by:

- Quantities (Zeroth order approximation - anchor point)
- Prices (First order approximation - slope)
- Elasticity (Second order approximation - curvature)



Calibration - Microconsistent Dataset

Benchmark equilibrium:

Price convention: $p_1 = p_2 = r = w = 1$

	Y_1	Y_2	Household	$\sum$
Y_1	40	–	-40	0
Y_2	–	40	-40	0
$\overline{K}$	-20	-30	50	0
$\overline{L}$	-20	-10	30	0
$\sum$	0	0	0	

- Zero profit: column sum
- Market clearance: row sum

} input-output table

- Budget constraint

→ Social Accounting Matrix (SAM)

MCP-Implementation of 2x2x1 - Model

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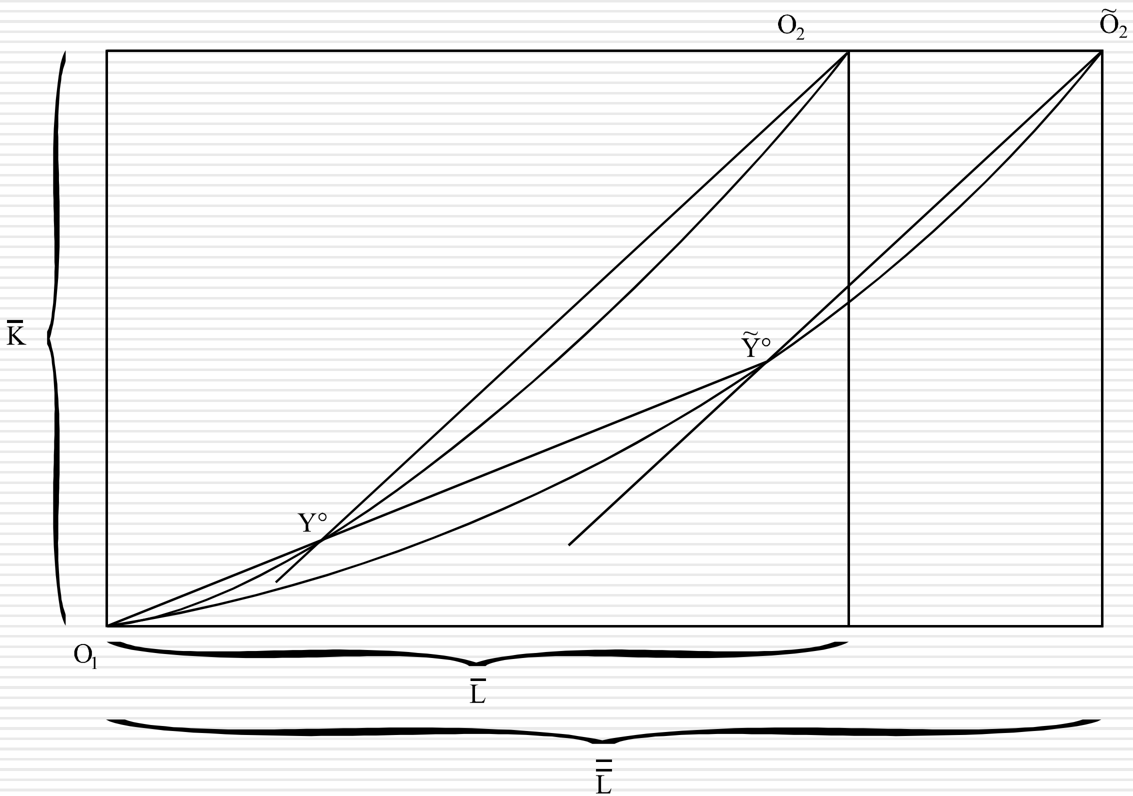
Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variables**	
$r^{0.5}w^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5}w^{0.5} - p_1\right)y_1 = 0$
$r^{0.75}w^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(r^{0.75}w^{0.25} - p_2\right)y_2 = 0$
Market clearance	**Price variable**	
$40\,y_1 \geq 40\frac{M}{80}\frac{1}{p_1}$	$p_1 \geq 0$	$\left(40\,y_1 - 40\frac{M}{80}\frac{1}{p_1}\right)p_1 = 0$
$40\,y_2 \geq 40\frac{M}{80}\frac{1}{p_2}$	$p_2 \geq 0$	$\left(40\,y_2 - 40\frac{M}{80}\frac{1}{p_2}\right)p_2 = 0$
$30 \geq 20\,y_1\frac{p_1}{w} + 10\,y_2\frac{p_2}{w}$	$w \geq 0$	$\left(30 - \left(20\,y_1\frac{p_1}{w} + 10\,y_2\frac{p_2}{w}\right)\right)w = 0$
$50 \geq 20\,y_1\frac{p_1}{r} + 30\,y_2\frac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\,y_1\frac{p_1}{r} + 30\,y_2\frac{p_2}{r}\right)\right)r = 0$
Budget constraint	**Income variable**	
$30w + 50r \geq M$	$M \geq 0$	$\left((30w + 50r) - M\right)M = 0$

Graphical Impact Analysis

Increase in labor endowment (e.g. EU Enlargement)



Numerical Impact Analysis

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	Variable	Benchmark	RYBCZINSKI	Counterfactual
Prices	w	1.0	1.0	1.0
	r	1.0	1.0	1.17
	p_1	1.0	1.0	1.08
	p_2	1.0	1.0	1.12
Quantities	Y_1	40.0	55.5	43.20
	X_1	40.0	42.5	43.20
	Y_2	40.0	30.0	41.57
	X_2	40.0	42.5	41.57
	K_1	20.0	27.5	20.00
	K_2	30.0	22.5	30.00
	$\overline{K}$	50.0	50.0	50.00
	L_1	20.0	27.5	23.33
	L_2	10.0	7.5	11.67
	$\overline{L}$	30.0	35.0	35.00
	M	80.0	85.0	93.33

„Visual“ Analysis - User Interface:

- Use without programming skills
- Separation between modeler/programmer and user
- Simple scenario management (data base)

Variants:

- „online“ internet e.g. http://brw.zew.de
  - \+ no modeling environment (programming language, solver, ...)
  - \+ control
  - – maintenance / proctection
- „decentral“ e.g. ftp://ftp.zew.de/pub/zew-docs/div/M2x2x1.exe
  - \+ „off“ the internet
  - – additional solves require software (e.g. GAMS)

Activity Analysis

Price convention: $p_1 = p_2 = r = w = 1$					
	Y_1	Y_2	Household	$\sum$	Y_3
Y_1	40	–	-40	0	-
Y_2	–	40	-40	0	40
$\overline{K}$	-20	-30	50	0	-30 (1+λ)
$\overline{L}$	-20	-10	30	0	-10 (1+λ)
$\sum$	0	0	0	0	-40 (1-λ)

Y_3:
- Perfect substitute for production activity Y_1
- slack activity in benchmark: λ > 1 („non market“ data)
  - black market
  - smuggling

Policy simulation:

- Ad-valorem production taxes on Y_1: $t^{Y_1} \in \{0.01, \ldots, 0.25\}$
- Laffer-curve with(out) slack activity

Results

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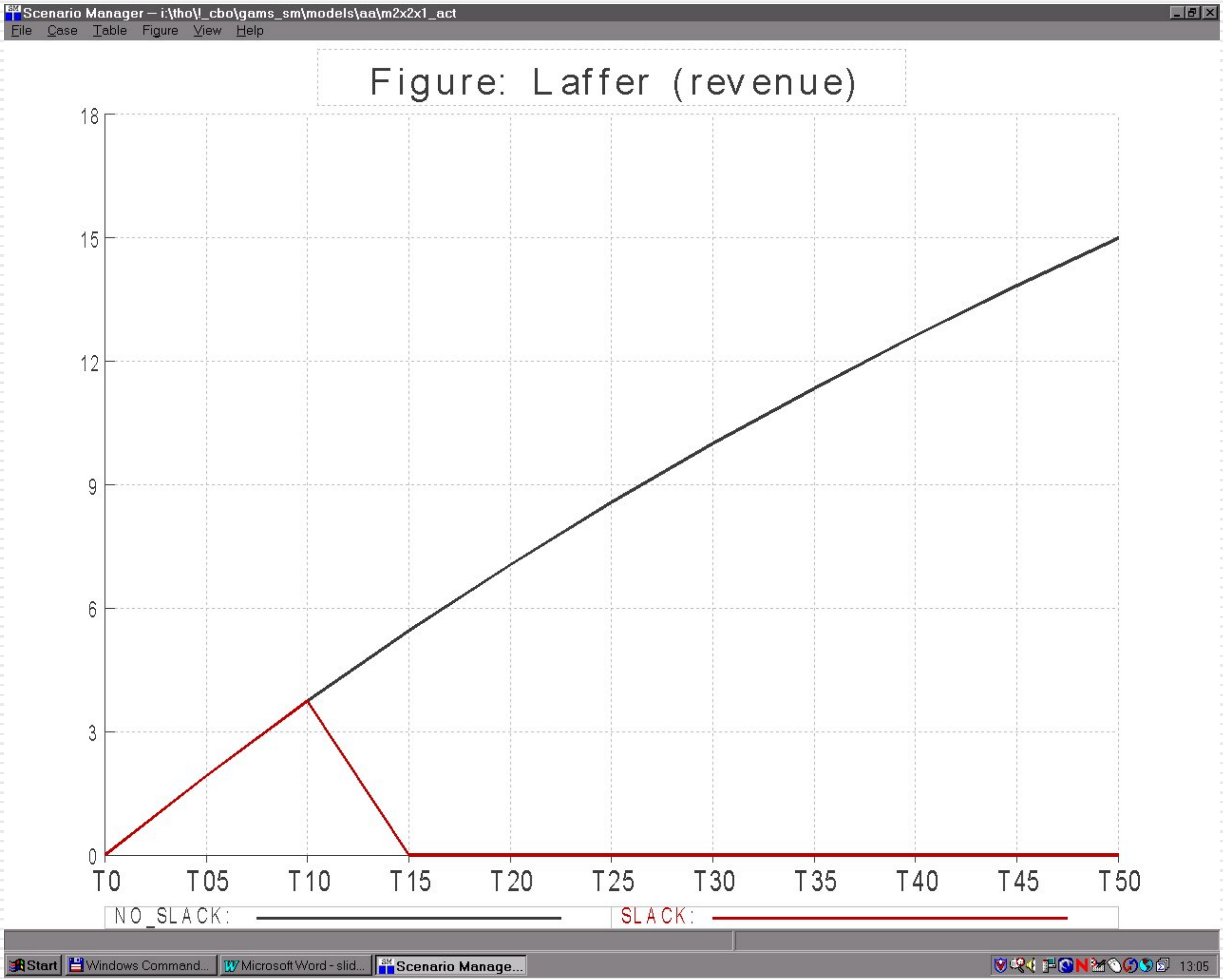

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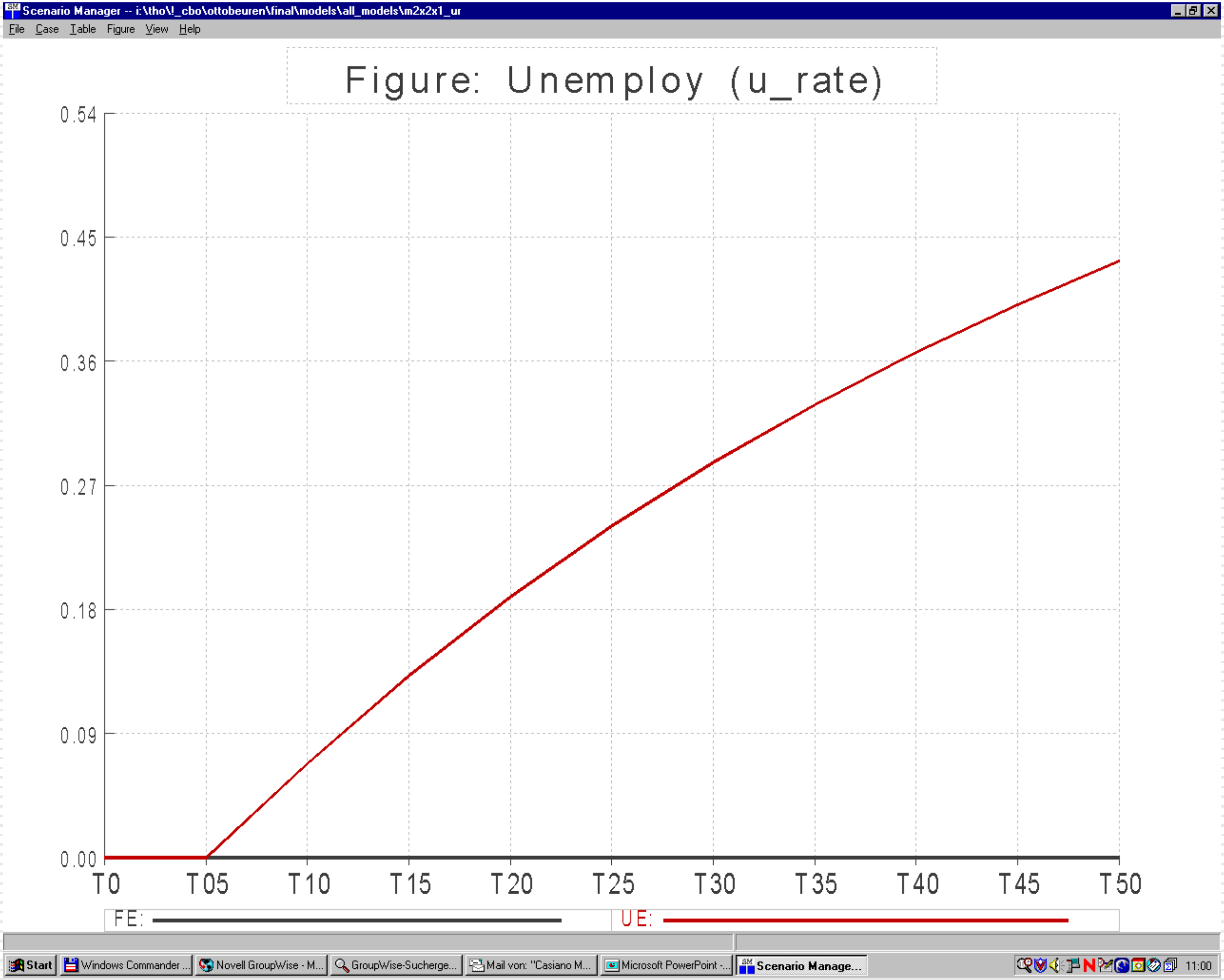

Summary

Ex-ante:

- Getting compex problems „at work"(efficiency / equity issues)
- Systematic policy assessment (sensitivity analysis)
- Microconsisten framework (test for intuition)

Ex-post:

- jack of „some" trades
- Flexible and efficient implementation
  (Meta programming language, MCP and calibrated share form)
- Visual tools(GAMSsm)
  - ⇨ Economics: bridge between theorie and applied work
  - ⇨ Teaching: economic intuition
  - ⇨ Policy: efficient scenario management