Innovation Diffusion with Endogenous Word of Mouth Communication

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Abstract: In this article, we argue that information diffusion plays a central role in the diffusion of a new technology. We explicitly model social communication channels through a word of mouth mechanism. We suppose that only consumers with similar tastes can exchange information and that the probability of a word of mouth contact is positively related to the ratio of quality to price. Lower prices play two roles. On the one hand, they increase the speed of word of mouth communication so that a firm can capture newly informed consumers. On the other hand, a firm charging a low price can extend its market share to the segment of informed consumers with lower willingness to pay for quality.

Keywords: Information diffusion; innovation diffusion; word of mouth; vertical differentiation; social network; optimal control; differential game.

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1. Introduction

Innovation diffusion is an important economic process that determines the intra- and inter-industry dynamics as well as promote economic growth. New technologies share two main features. First, the diffusion of the information about the existence and the characteristics of an innovation takes time. Hence, information diffusion determines the pattern of market diffusion. Second, an innovation often appears as a higher quality alternative to the existing technologies.

First, as Rogers (1993) notes, information diffusion precedes innovation diffusion. We are interested in two aspects of information diffusion: how do economic agents become informed and what are the factors that influence the speed of information diffusion? We believe that agents interact according to the social network to which they belong. A typical network specifies who interacts with who and the direction of the interaction. For instance, one can assume that all agents have the same probability to interact with any other agent. This global interaction generates a logistic information diffusion curve. In the beginning of the diffusion process, few agents are informed about the existence and the characteristics of a new technology, while many are not informed, so that information diffusion is slow. Diffusion is also slow at the end of the process as the number of informed agents is large but the number of uninformed agents is small. Diffusion is fastest in the middle of the process and the aggregate diffusion curve follows an S-shaped trajectory over time. However, the assumption that all agents are identical in term of interaction probability and that they interact with all other agents is too strong. As a matter of fact, the literature on social networks, reviewed by Scott (1991), stresses the importance of local interactions and argues that only agents that are close to each other can interact. This notion of proximity can be interpreted in a flexible way. On the one hand, proximity can correspond to geographic distance. On the other hand, it can represent the distance between different social classes. We use this last notion of proximity in the remaining of the article. We therefore assume that communication costs are relatively independent of geographical distance and have a social nature. Indeed, agents from a social category communicate more easily with agents from the same category. Once the social network is defined, we must specify what are the factors that influence the speed of information diffusion. One such factor is advertisement through old and new media such as newspapers, radio and television broadcasting and the internet. Another method is information diffusion through word of mouth, the process by which a person *voluntarily* communicates a piece of information to another person. This process is especially efficient when the cost of paid advertisement is high and ineffective, when the product or service is intangible and complex, or when the degree of personal involvement in the purchase decision is high. It is mostly relevant for new products. Indeed, habit formation prevents economic agents to easily switch to new technologies as noted by Mokyr (1992).

The power of the word of mouth process is related to five characteristics.[1] First, personal influences can bypass self-defense mechanisms that slow mass media information diffusion. Secondly, face to face personal contacts are flexible and interactive. Thirdly, in contrast to mass media, word of mouth is efficient when there is some degree of social control, as personal sources can offer rewards for compliance with recommendations, as well as punishments. Fourth, word of mouth is effective through informal channels that offer high credibility as well as customized and unique information. Fifth, word of mouth can provide a social support to an innovation. Indeed, new product technologies that suffer a negative word of mouth will reach new consumers with great difficulty, as producers who are building a new brand may want remove these products form the market. We focus of information diffusion through word of mouth, since we want to stress the effect of social networks in the innovation process. What are the economic factors that increase the speed of information diffusion? It seems obvious that the probability that an agent communicates a good piece of information (positive word of mouth) depends on price and quality and that the diffusion of innovation is faster when the ratio of quality to price is interesting.

Second, an innovation often appears as a higher quality alternative to the existing technologies and not all informed consumers purchase a new technology of higher quality because it may simply be too expensive. It is therefore sensible to study information and innovation diffusion in a model in which products are differentiated and consumers have a taste for quality so that they do not necessarily purchase the higher quality product. In this context, it seems therefore important to make a distinction between information and market diffusion through price and quantity dynamics, as informed consumers do not necessarily purchase the new technology if they do not value quality enough.

In the static model of vertical differentiation studied by Gabszewicz and Thisse (1979), which has been extensively used in the industrial organization literature and on endogenous growth models with a quality ladder, firms produce goods of different qualities and consumers are indexed according to their taste for quality. However, in this static framework, the market for the high quality good always includes all consumers with the highest willingness to pay for quality. Hence as a new product becomes available on the market, information diffusion is instantaneous and there is no room for diffusion in this model. However, the diffusion of innovations takes time since agents need time to acquire information about the new technology, are heterogeneous and have different attitudes with respect to technological change, there are large switching costs associated with the innovation, and so on. It seems relevant to extend the static model of vertical differentiation in order to allow for diffusion in

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[1] According to Arndt (1967).

general and to take account of informational issues in particular. Indeed, consumers with large willingness to pay for quality are not necessarily the first to obtain useful information about the new technology.

In this article, we argue that information diffusion plays a central role in the diffusion of a new technology. We explicitly model social communication channels through a word of mouth mechanism. We believe that in a complex environment, face to face communication is often the most effective way to communicate useful information with respect to the existence and the characteristics of a new product technology. We define the social distance between any two agents as the proximity of their taste for quality.[2] We suppose that only consumers with similar tastes can exchange information and that the probability of a word of mouth contact is positively related to the ratio of quality to price. We study the role of these local interactions on information and market diffusion in two stages. In the first stage, we analyze the intertemporal pricing strategy of a monopoly who sells the product innovation with a constant marginal cost and then with a decreasing average cost. We can, in this way, extend the static model of vertical differentiation by taking account of the information diffusion process among consumers with large willingness to pay for high quality products. Lower prices play two roles. On the one hand, they increase the speed of word of mouth communication through a higher ratio of quality to price so that the firm can capture newly informed consumers. On the other hand, a firm charging a low price can extend its market share to the segment of informed consumers with lower willingness to pay for quality when average cost is decreasing. In the second stage, we analyze prices and quantities dynamics in a duopoly in which one firm produces the new technology, and the other produces the old

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[2] We note that the taste for quality of an agent is closely related to her revenue. See for instance Tirole (1988). Hence, social classes defined with respect to taste for quality or revenues are equivalent.

technology. We characterize the open-loop strategies of the firms. That is we assume that both firms can commit to a fixed sequence of prices. Hence, equilibrium strategies only depend on time and not on relevant state variables, contrary to closed-loop or Markov Perfect Strategies.[3] This is a clear limitation of the analysis, but we can fully characterize transitional dynamics with open-loop strategies while using feedback strategies would introduce effects that would complicate the interpretation of the results. In this differential game, we show that the equilibrium trajectory corresponds to an introductory price type dynamics in which information travels fast in the beginning of the diffusion curve and then slows towards the steady state. However, prices charged by both firms can have different dynamics according to whether information diffusion is fast or not and whether the difference of quality between the two products is large or not.

Alhough the important role of word-of-wouth communication on innovation diffusion as been acknowledged in the marketing literature for a long time [see for instance Arndt (1967b), Martilla (1971), Czepiel (1974) and Dodson and Muller (1978)], there are few studies in the economic literature that explictily model information diffusion through communication networks and study their effect on innovation diffusion [although there exists an emerging literature on interactions between economic agents; see Cohendet et al. (1998) and Brock and Durlauf (2000) for recent surveys]. The earliest article on innovation diffusion and social networks is Allen (1982) who studies the global phase of a random Markov field in which the probability that an agent adopts a new technology depends on whether the other agents in her neighborhood have already adopted the innovation or not. However, there is no distinction between information and innovation diffusion as prices are not explicitly taken into account.

[3] For a discussion of these two types of equilibria, see Fudenberg and Tirole (1991).

Information diffusion and intertemporal competition have been separately studied in the vertical differentiation model with durable goods by Vettas (1997) and Deneckere and de Palma (1998). When products are durable, an additional issue arises; consumers not only have to decide what to buy but also when to buy. Moreover, they have to anticipate the intertemporal pricing strategies of the firms. The analysis of this type of model is complex, has not yet explicitly taken the communication network into account and does not make a clear distinction between information and innovation diffusion. For instance, in Vettas (1997), there exists a probability at any given period that all consumers become informed about the true quality of the new durable good and a complementary probability that all consumers who have not purchased the product before remain uninformed. But as soon as they receive the information, newly informed consumers make a purchase. The probability of a "word-of-mouth" is proportional to the volume of past sales. As it is mentioned in a footnote of the article, this probability is more easily interpreted as the probability that a reviewer writes an article that is instantaneously read by all consumers. The probability is larger when the volume of sales is high. However this communication network is global, while it is more intuitive to think about word-of-mouth communication as resulting from local interactions. In Deneckere and de Palma (1998), strategic interactions are studied in an intertemporal framework, but there are no interactions among consumers and therefore no local information diffusion.

The remaining of the article is organized as follows. We describe the framework in Section 2. We consider the intertemporal pricing strategy of a monopoly selling a high quality product innovation, first with constant, and then with decreasing average cost in Section 3. We study the differential game in Section 4. Finally, section 5 concludes the article.

2. The model

We consider the standard model of vertical differentiation that we extend to study product diffusion. Consumers have a utility function $u = \theta s - p$, where θ represents taste for quality and s is the quality of a product sold at price p. We assume that

$$\theta \in [\theta_0, \theta_1] \text{ with density } f.$$

In the remaining of the article we will work with a distribution of θ that is uniform on its support, i.e. $f(x) \propto 1/(\theta_1 - \theta_0)$ for all $x \in [\theta_0, \theta_1]$.

We assume that agents locally interact and that each informed agents has a probability to voluntarily communicate a good or bad piece of information through word of mouth to consumers who have a similar taste for quality. Information diffusion can occur upwards as well as downwards. We first consider upward information diffusion.[4] Let $y(t)$ denote the number of informed agents at time t. Between t and $t + \mathrm{d}t$, the informed consumer with the highest willingness to pay for quality can communicate his piece of information to a consumer located at a distance dt above on the taste for quality ladder (dt is small and should be interpreted as $\mathrm{d}t = 1$) with a probability x and no communication occurs with the complementary probability $1 - x$. Hence,

$$y(t + \mathrm{d}t) - y(t) = \begin{cases} \mathrm{d}t & \text{with probability } x \\ 0 & \text{with probability } 1 - x \end{cases}$$

So, $\mathrm{E}\, y'(t) = \mathrm{E} \lim\, (y(t + \mathrm{d}t) - \mathrm{y}(t)) / \mathrm{d}t = x$ gives the instantaneous increase in the number of informed consumers. We use this linear diffusion mechanism to study the role of this simple social network on innovation diffusion.

[4] The reasoning is similar for downward diffusion.

The consumer who is indifferent between purchasing the new and the old technology has a taste for quality equal to $\theta^* = (q - p) / \Delta s$, where q is the price of the high quality product, p is the price of the low quality product, and Δs is the difference between the quality of the high and the low quality product. We assume that the word of mouth process is faster when the ratio of incremental quality to incremental price is high. Indeed, the absolute level of quality of a product is not important to the consumers. What matters is the difference in quality with respect to the existing competition. Again the increase in quality is meaningless when it is not reported to the difference in price. We thus specify a positive relation between the probability of communication and this ratio of quality to price: $x = x(\Delta s / (q - p)]$ with $\partial x / \partial(\Delta s / (q - p))$ > 0. We use the following simple linear specification:

$$\begin{aligned} y' &= x(\cdot) = f - g\,(q - p) / \Delta s \quad && \text{if } y < y^* \\ y' &= 0 && \text{otherwise} \end{aligned} \tag{1}$$

where y' denotes the derivative of y with respect to t, f and g are two numbers that influence the speed of information diffusion, and $y^* = (\theta_1 - j) / h$ is the upper limit on y. We also note, that prices charged by both firms must be equal to the values given in the static equilibrium.

3. Monopoly

We consider a monopoly labeled firm 1 producing the high quality good facing a competitive market for the old product technology sold at price p. We assume that the initial segment of informed consumers is $[j - i\, y(0), j + h\, y(0)]$ with $h + i = 1$.[5] Demand faced by firm 1 is thus:

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[5] For instance if both h and i are equal to 1/2, the initial segment of size y is centered around j. Different values of parameters h and i can account for a speed of information diffusion that is different when information is exchanged upwards or downwards.

$j + h\,y - (q - p) / \Delta s$

There are several restrictions. First, we necessarily have $j + h\,y \leq \theta_1$. Next, information diffusion can not be negative, that is, we always have $y' > 0$.

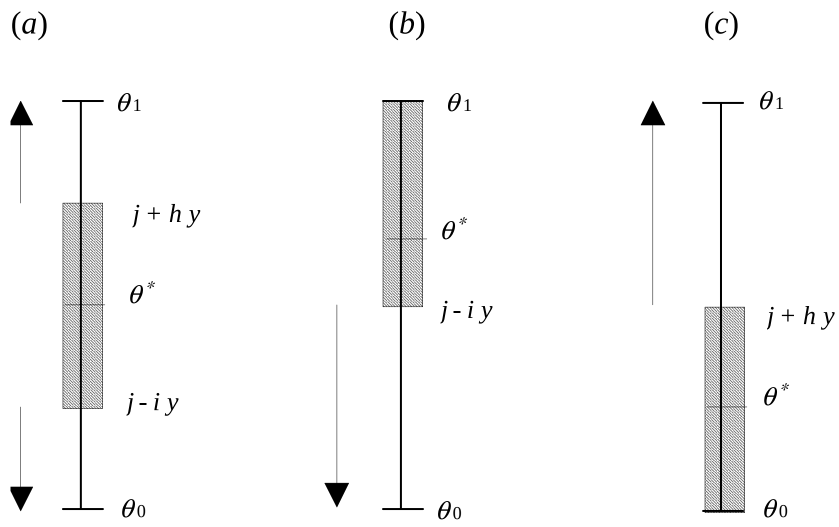

Figure 1.

In Figure 1 we have represented information diffusion with three different initial conditions. The dark zone in panels (a), (b) and (c) indicates the segment that is initially informed. The arrows point to the direction of information diffusion. In panel (*a*), consumers with an average taste for quality are informed first, while in panels (b) and (c), consumers with high and low willingness to pay for quality are respectively the first agents informed.

It is natural to assume that consumers with large willingness to pay are not informed in the first place. Indeed, if these consumers were the first to come in contact with the new technology, there is no room for information diffusion about the new technology. Indeed, firm 1 can not capture newly informed (for demand does not depend on y anymore and information spreads downwards, towards consumers who do not purchase the high quality good anyway). By adopting the situation depicted in panel (*a*) or (*c*), we implicitly assume that technophiles are not necessarily the wealthiest consumers (or with the highest willingness to pay for quality).

We pause and make three remarks at this point. First, the indifferent consumer is not necessarily informed. This will be the case if she is located below the informed segment. Demand must then be restricted. We assume that we can always achieve an interior solution, that is we assume that the initial values of the parameters that affect the speed of information diffusion are such that the indifferent consumers is always strictly inside the informed segment at any time. Secondly, information can spread downwards. The new product is thus not necessarily an experienced good that consumers must purchase first before assessing the quality. In fact, word of mouth communication can substitute for direct experience, and informed consumers can transmit information without purchasing the good.[6] Thirdly, positive and negative word of mouth mechanisms are symmetric. To account for an asymmetry (which is relevant if negative word of mouth travels faster than positive word of mouth), we can define a critical value such that information diffusion is slower if quality to price is low and faster if this ratio is high.

3.1. Constant marginal cost

We first assume that marginal cost of producing the high and the low quality products c is constant and equal to zero. Price charged by the old technology producers is normalized to $p = c = 0$. The intertemporal program of firm 1 can be stated as follows:

$$\text{Max} \int_0^\infty e^{-rt} (j + h\, y - q / \Delta s)\, q$$

[6] Consumers who have received a piece of information thourgh word-of-mouth are not necessarily experts on new product technologies. What they should become informed of is where they can find the relevant information such as a specialized magazine, a newspaper article or a review posted on an internet site.

Subject to $y' = (f - g\, q / \Delta s)$

$q \geq 0$

The current value Hamiltonian for this maximization problem is:

$$\mathcal{H} = (j + h\, y - q / \Delta s)\, q + \lambda\, (f - g\, q / \Delta s)$$

where λ is the current co-state variable. First order conditions are:

$$j + h\, y - 2\, q / \Delta s - \lambda\, g / \Delta s = 0 \qquad (2)$$

$$h\, p = r\, \lambda - \lambda' \qquad (3)$$

From (2), we obtain:

$$\lambda = [j + h\, y - 2\, (q / \Delta s)]\ \ [\Delta s / g]$$

$$\lambda' = [h\, y' - 2\, (q' / \Delta s)]\ [\Delta s / g]$$

Substituting these two expressions in (3) and substituting the expressions of y', we obtain:

$$q' = \Delta s\, (2\, g)^{-1}\, [- 2\, r\, q + r\, \Delta s\, j + r\, h\, \Delta s\, y - h\, f\, \Delta s] \qquad (4)$$

Hence the ($q' = 0$) schedule can be represented by a line in the (y, q) system:

$$q = (2\, r)^{-1}\, [r\, \Delta s\, j - h\, f\, \Delta s + r\, h\, \Delta s\, y]$$

Finally, the steady state value of the price that corresponds to full information diffusion is given by $q^* = \Delta s\ \theta_1 / 2$ and y can not grow above $y^* = (j - \theta_1) / h$. The situation is represented in figure 2, where arrows point to the direction of motion implied by (1)-(4). The stable arm is upward sloping. Thus the only equilibrium trajectory starts from $q(0)$ smaller than q^* and converges to this steady state value.[7] Along this trajectory price and the number of informed consumers increase, as stated in Proposition 1.

<u>Proposition 1</u>. q increases over time.

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[7] It is easy to check that $q(0) < q^*$.

This says that the optimal strategy of a firm selling the high quality product is to propose introductory prices to create a mass effect in the beginning of the diffusion curve. As prices start to increase, information diffusion becomes slower and the market covered by the high quality producer shifts upward together with the indifferent consumer.

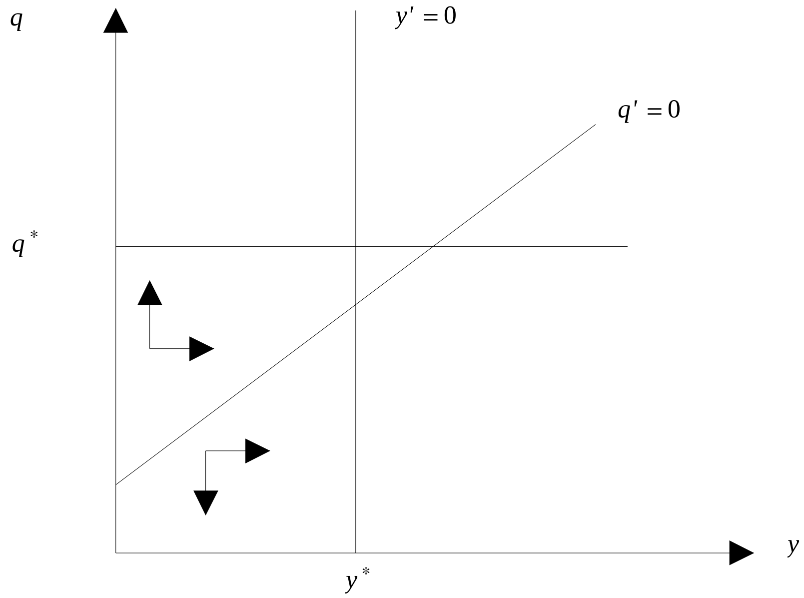


Figure 2.

3.2. Decreasing average cost

We now assume that average cost is given by

$$C(e) / e = b - c\, e \qquad (5)$$

where b and c positive constants. Price in the static model is obtained by solving the program Max $\pi = (q - C(e) / e)\, e$, where $e = \theta_1 - (q / \Delta s)$. First order condition give

$$(-1 / \Delta s)\,(q + b - c\,(\theta_1 - q / \Delta s) + (\theta_1 - q / \Delta s)\,(1 - c / \Delta s) = 0$$

which yields the steady state price

$$q^* = [\Delta s / (2\,(1 - c / \Delta s))]\,[b / \Delta s + \theta_1\,(1 - 2\,c / \Delta s)] \qquad (6)$$

Le $e = j + h\,y - q / \Delta s$. The Hamiltonian of the dynamic optimization program is

$$\mathcal{H} = (j + h\,y - q / \Delta s)\,(q - b\, + c\,(j + h\,y - q / \Delta s)) + \lambda\,(f - g\,q / \Delta s)$$

The problem is concave if $\partial^2 \pi / \partial q^2$ is negative. This condition is satisfied if

$$c < \Delta s$$

We suppose that this condition is always satisfied in the remaining of this section. First order conditions are:

$$-\Delta s^{-1}\,(q\, + b\, - c\,(j + h\,y - q / \Delta s)) + (j + h\,y - q / \Delta s)\,(1 - c / \Delta s) + \lambda\,\gamma / \Delta s = 0 \quad (7)$$

$$h\,(q - b + c\,(j + h\,y - q / \Delta s)) + c\,h\,(j + h\,y - q / \Delta s) = r\,\lambda - \lambda' \quad (8)$$

$$\lim \mathrm{e}^{-rt}\,y = 0 \quad (t \to \infty) \quad (9)$$

From equation (7), we obtain

$$\lambda = [\Delta s / g]\,[b / \Delta s - 2\,q / \Delta s\,(1 - c / \Delta s) + (1 - 2\,c / \Delta s)\,(j + h\,y)] \quad (10)$$

$$\lambda' = -\,2\,q\,(1 - c / \Delta s) / g + \Delta s\,h\,(1 - 2\,c / \Delta s)\,(f - g\,q / \Delta s) / g \quad (11)$$

Substituting (10) and (11) in (8), we obtain the equation of motion of q'. The ($q' = 0$) schedule can be represented by a line of equation:

$$q = [b\,(g\,h + r) - s\,f\,h + 2\,c\,f\,h + (r\,s - 2\,c\,(g\,h + r))\,(j + h\,y)]\;/\;[2\,r\,(1 - c / \Delta s)]$$

Let q^{**} denote the intersection of the ($q' = 0$) line with the line $y = y^*$. There are four cases that depend on the values of q^{**} with respect to q^*. The slope of the line, is related to the sign of

$$B = r\,s - 2\,c\,(g\,h + r)$$

This parameter is negative when the degree of increasing returns represented by c is large. On the other hand, we have

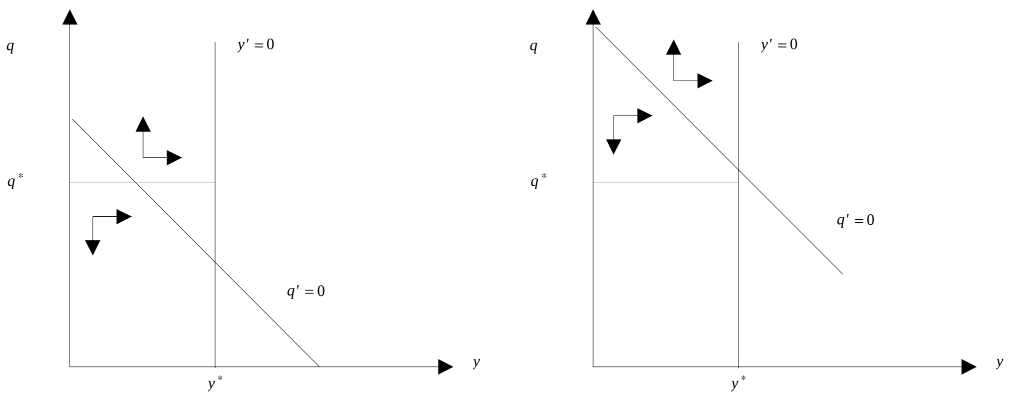
$$q^{**} - q^* = [h\,s / 2\,(1 - c / \Delta s)]\,A$$

with $A = f\,(2\,c - s) + b\,g - 2\,g\,c\,\theta_1$. This parameter is positive when the constant part of the marginal and average cost, b, is large.

The phase diagrams are represented in Figure 3 for different configurations of the values of A and B. In panels (b) and (c), the equilibrium trajectory starts above θ^* and along the trajectory, prices decrease, while the number of informed consumers increases. In this situation, the monopoly can extend his market share at both end of the covered segment. Indeed, lower prices mean faster information diffusion and newly informed consumers. The speed of information diffusion increases as the market reaches the steady state. Moreover, with lower prices, the monopoly can reach informed consumers with lower willingness to pay for quality. Hence, the market covered by the monopoly extends upwards and downwards.

In panels (a) and (d), the phase diagrams can be interpreted as Figure 2.

(a) $A < 0, B < 0$ (b) $A > 0$, $B < 0$



(c) $A < 0, B > 0$ (d) $A > 0, B > 0$

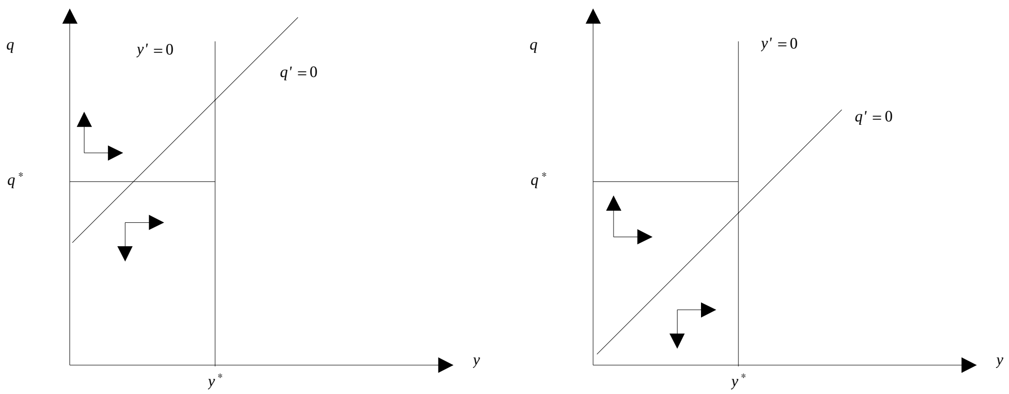


Figure 3.

4. Duopoly

We now consider two firms competing in prices. Both firms produce at constant marginal costs normalized to zero. We assume that the market is covered, that is $\theta_0 > p/s_0$, where s0 is quality of the old technology. We can use the following figure to build the demand functions.

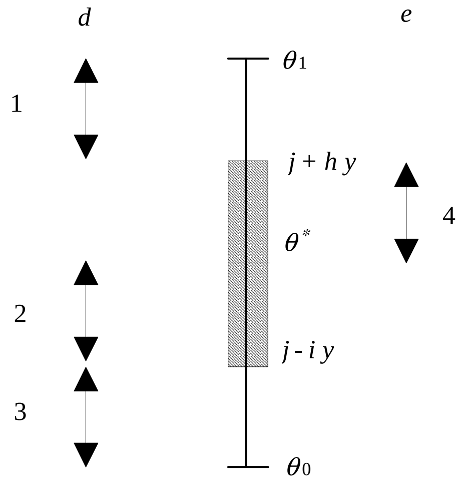


Figure 4.

Demand faced by firm 0, *d*, is represented on the left-hand side. It is the sum of the demand from uninformed consumers with high willingness to pay (1), plus the demand of the informed consumers with a low willingness to pay for quality (2), and the demand form uninformed consumers with a low taste for quality (3). Demand faced by firm 1, *e*, is represented by arrow (4) on the right-hand side of Figure 4, which denotes the demand of informed consumers with a high willingness to pay for quality. We assume that the consumer indifferent between buying the low and the high quality product is strictly inside the segment $(j - i\,y(0), j + h\,y(0))$. In this case,

$$d = (\theta_1 - (j + h\,y) + (q - p) / \Delta s - \theta_0$$

$$e = (j + h\,y) - (q - p) / \Delta s$$

We analyze the open-loop strategies, that is, we assume that firm can commit to a time path of strategic variables. Given that the number of consumers is eventually finite, there is no steady-state growth rate of information diffusion. We are thus looking at transitional dynamics. The current value Hamiltonian for firm 0 is

$$\mathcal{I} = (\theta_1 - (j + hy) + (q - p)/\Delta s - \theta_0)\, p + \lambda\, (f - g\, (q - p)/\Delta s) \tag{12}$$

For firm 1, the Hamiltonian is :

$$\mathcal{J} = (j + hy - (q - p)/\Delta s)\, q + \mu\, (f - g\, (q - p)/\Delta s) \tag{13}$$

First order conditions for intertemporal profit maximization for firm 0 are

$$\theta_1 - (j + hy) + (q - 2p)/\Delta s - \theta_0 + \lambda\, g/\Delta s = 0 \tag{14}$$

$$-\, hp = r\, \lambda - \lambda' \tag{15}$$

We note that the transversality conditions are always satisfied because the system reaches a stead-state in finite time. Solving for λ in (14) yields

$$\lambda = -\Delta s\ \{ \theta_1 - (j + hy) + (q - 2p)/\Delta s - \theta_0\}$$

and differentiating with respect to time yields

$$\lambda' = -\Delta s/g\ \{ -hy' + (q' - 2p')/\Delta s\}$$

Now substituting these expressions together with (1) into (15) and rearranging yields

$$p' = g/2\ \{hp - \Delta s\ r/g\ (\theta_1 - \theta_0 - (j + hy) + (q - 2p)/\Delta s) - \Delta s\ h/g\ (f - g\ (q - p)/\Delta s)\}$$
$$+\, q'/2 \tag{16}$$

Similarly, first-order conditions for intertemporal profit maximization for firm 1 are

$$j + hy - (2q - p)/\Delta s - \mu\, g/\Delta s = 0 \tag{17}$$

$$hq = r\, \mu - \mu' \tag{18}$$

Solving for μ in (17) yields

$$\mu = \Delta s/g\ \{j + hy - (2q - p)/\Delta s\}$$

$$\mu' = \Delta s/g\ \{hy' - (2q' - p')/\Delta s\}$$

Substituting these two expressions and (1) into (18) , we obtain :

$$q' = g/2\ \{hq - r\ \Delta s/g\ (j + hy - (2q - p)/\Delta s) + \Delta sh/g\ (f - g(q - p)/\Delta s\} + p'/2 \qquad (19)$$

We have a system with three endogenous variables. We can reduce it to two dimensions by working with $\theta = (q - p)/\Delta s$. Subtracting (16) from (19), rearranging and dividing by Δs, we obtain:

$$\theta' = 1/3\ \{(3r + gh)\ \theta - 2r\ (j + hy) + r(\theta_1 - \theta_0) + 2h\ (f - g\theta)\} \qquad (20)$$

It is important to note that the quality difference between the two products, Δs, does not influence the dynamics of the indifferent consumer. Now the ($\theta' = 0$) schedule can be written as:

$$\theta = (3r - gh)^{-1}\ [2r\ (j + hy) - r(\theta_1 - \theta_0) - 2hf] \qquad (21)$$

To analyze the phase diagram, we must first compute the static equilibrium position of the indifferent consumer. It is given by:

$$\theta^* = (\theta_0 + \theta_1)/3 \qquad (22)$$

We check the following to claims in the Appendix. Let y^{**} denote the point at which the right hand side of (21) is equal to θ^*.

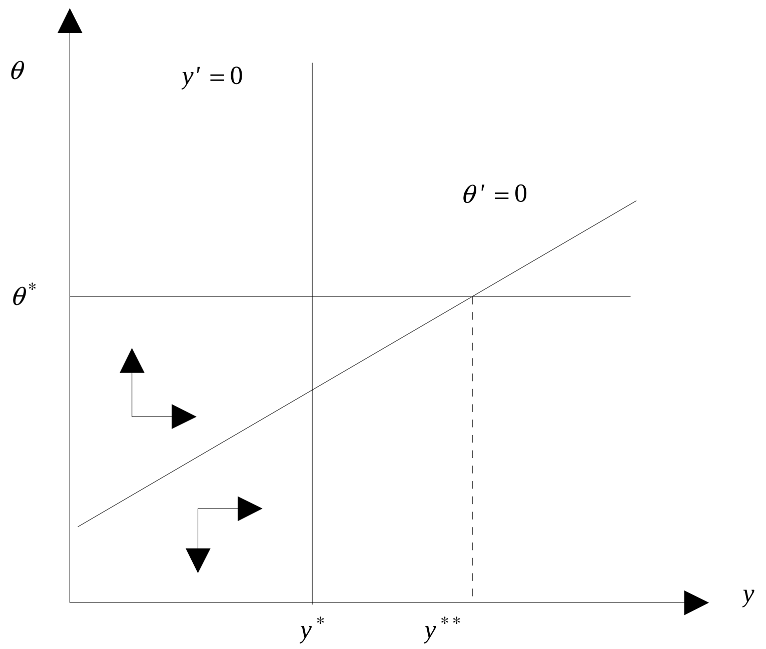
<u>Claim 1</u>: $y^{**} > y^*$

Next, let $A = [2rj - r(\theta_1 - \theta_0) - 2\ hf] / [3r - gh]$ and $B = 2\ rh / (3r - gh)$. The following properties of these numbers hold.

<u>Claim 2</u>: when $B > 0$, $A < \theta^*$ and when $B < 0$, $A > \theta^*$

Hence, there are two cases to analyze.

(a) $B > 0$, $A < \theta^*$



In this case, the initial values of y and θ must be such that θ is above the schedule ($\theta' = 0$).

Then the value associated with the indifferent consumer increases to its steady state value θ^{*}.

(b) $B < 0$, $A > \theta^{*}$

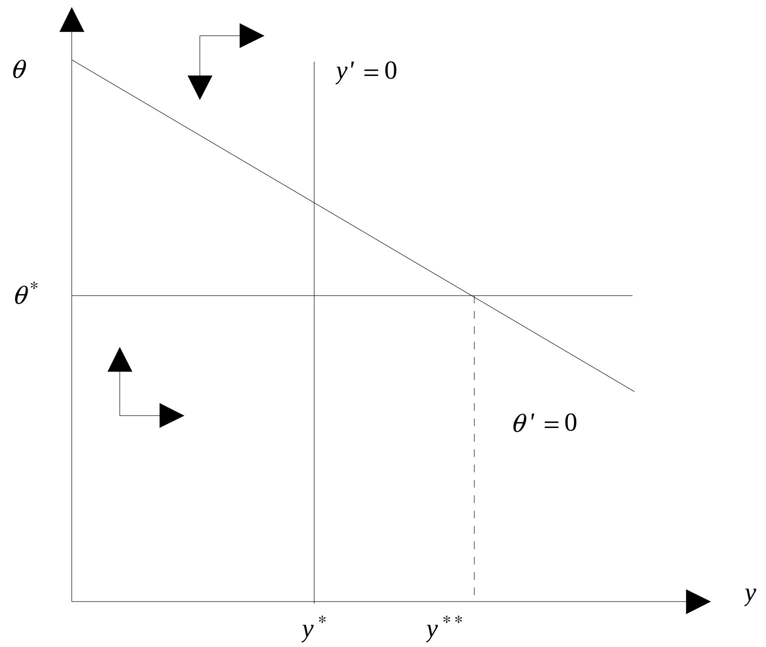


Figure 5.

When $B < 0$, that is, when the interest rate is close to 0, then the initial value of θ must lie below the ($\theta' = 0$) schedule. In this case, the value associated with the indifferent consumer also increases to its steady state value.

To sum up in both cases, information diffusion is fastest in the beginning of the process and slows as the system reaches its steady state. During the transitional dynamics, the segment covered by the firm producing the new technology shifts upwards.

We can further characterize the dynamics of the model by studying how prices change over time. We use the following claim that we check in the Appendix.

Claim 3:

(i) There exist $\Delta s^* \in \mathrm{R}_+$ such that if $\Delta s < \Delta s^*$, $p' > 0$ and $p' < 0$ otherwise

(ii) if $fh > r(2\theta_1 - \theta_0)$ then $q' > 0$ and otherwise there exists $\Delta s^{**} \in \mathrm{R}_+$ such that

if $\Delta s < \Delta s^{**}$, $q' > 0$ and $q' < 0$ otherwise

(iii) $\Delta s^* < \Delta s^{**}$ when such numbers exist.

This claim is illustrated in Figure 6.

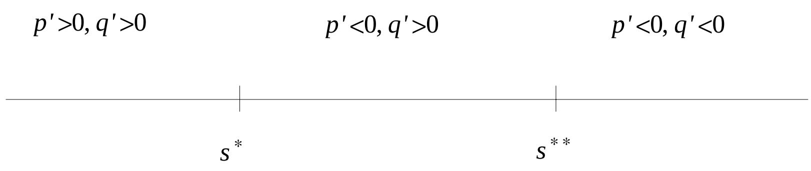


Figure 6

The condition stated in part (ii) is equivalent to saying that information travels fast enough. In this case there exists a threshold level for the quality differential, s^*, such that when quality increment is large enough, $p' < 0$ and $q' > 0$. This is an interesting situation. Indeed, if we consider two points in time, and reason with static reaction functions, we would have expected that when a manufacturer lowers (increases) its price, then the other manufacturer lowers (increases) its price too (product are strategic complements). But here, prices are moving in opposite directions when information travels fast and the difference between the quality of both manufacturers is large enough.

5. Conclusion

We have studied the role of information diffusion on the diffusion of a new product technology in a vertically differentiated industry. We have defined a simple linear social network in which consumers interact by word of mouth with other agents who have a similar willingness to pay for quality. The dynamics of this model allow a firm selling a high quality product to extend his market share in both directions of the network. This result holds with and without strategic interactions.

References

ARNDT J., *Word of Mouth Advertising: a Review of the Literature,* Advertising Research Foundation, 1967a.

Arndt, J., "Role of Product-Related Conversations in the Diffusion of a New Product Technology," *Journal of Marketing Research* 4 (1967b), 291-295

COHENDET, P., LLERENA P., STAHN H. and G. UMBHAUER (eds.), *The Economics of Networks: Interactions and Behaviors*, Springer-Verlag, 1998

CZEPIEL M., "Word of mouth processes in the diffusion of a major technological innovation," *Journal of Marketing Research*, 11 (1974), 172-180.

DENECKERE R.J. and A. DE PALMA, "The Diffusion of Consumer Durables in a Vertically Differentiated Oligopoly," *Rand Journal of Economics*, 29 (1998), 4: 750-771

DODSON, J.A.A and E. MULLER, "Models of New Product Diffusion through Advertising and Word-of-Mouth," *Management Science* 24 (1978), 1568-1578

FUDENBERG D. And J. Tirole, *Game Theory*, MIT Press, 1991

GABSZEWICZ J. And J.F. THISSE, "Price Competition, Quality and Income Disparities," *Journal of Economic Theory* 20 (1979): 464-473

MARTILLA, J.A., "Word of Mouth Communication in the Industrial Adoption Process," *Journal of Marketing Research* 8 (1971), 173-178

MOKYR J., "Technological Inertia in Economic History," *Journal of Economic History* 52 (1992), 325-338.

ROGERS E., *Diffusion of Innovations*, Free Press, 1993.

SCOTT J., *Social Network Analysis*, London, Sage, 1991.

TIROLE J., *The Theory of Industrial Organization*, MIT Press, 1988.

VETTAS N., "On the Informational Role of Quantities: Durable Goods and Consumers' Word-of-Mouth Communication", *International Economic Review*, 38 (1997), 4: 915-944

APPENDIX

Claim 1: $y^{**} > y^{*}$

Check: to check this result we first compute y^{**}. We obtain $y^{**} = [(3r - gh)\,\theta^{*} + r(\theta_1 - \theta_0) + 2hf - 2rj] / 2rh$. Substituting $\theta^{*} = (\theta_0 + \theta_1)/3$ in this expression and noting that $y^{*} = (\theta_1 - j) / h$, we can write:

$$y^{**} = y^{*} + (f - (g/2)\,(\theta_0 + \theta_1)/3)/r$$

The second term on the right hand side of the previous equation is always positive because we have assumed that $f - g\,\theta^{*} > 0$. ■

Claim 2: (*i*) $B > 0$ implies $A > \theta^{*}$ and (*ii*) $B < 0$ implies $A < \theta^{*}$.

Check: (*i*) suppose that $B > 0$, then $A > \theta^{*}$ if and only if

$$6rj - 3r(\theta_1 - \theta_0) - 6hf < (3r - gh)\,(\theta_0 + \theta_1) \qquad \text{(A1)}$$

After arranging and simplifying the previous equation, this condition can be stated as $j < \theta_1 + h\,(f - (g/2)\,(\theta_0 + \theta_1)/3)$ which is always satisfied as $j < \theta_1$ and $f - g\,\theta^{*} > 0$.

(*ii*) When $B < 0$, the inequality (A1) is reversed and we must have $A < \theta^{*}$. ■

Claim 3: For a given state (p, q, r) (*i*) there exists $s^{*} \in R^{+}$ such that $p' > 0$ if $s < s^{*}$, and $p' < 0$ otherwise; (*ii*) If $fh > r(2\theta_1 - \theta_0)$ then $q' > 0$, otherwise there exists $s^{**} \in R^{+}$ such that $q' > 0$ if $s < s^{**}$, and $q' < 0$ otherwise; (*iii*) $s^{*} < s^{**}$.

Check: Solving (16) and (19) for p' and q', we obtain:

$$p' = (1/3)\,\{(3r + gh)p + 2ghq - fhs + rs(j + hy) - 2rs\,(\theta_1 - \theta_0)\} \qquad \text{(A2)}$$

$$q' = (1/3)\,\{(3r + gh)q + 2ghp + fhs - rs(j + hy) - rs\,(\theta_1 - \theta_0)\} \qquad \text{(A3)}$$

(i) from (A2), we have $p' > 0$ if and only if

$$s < s^{*} = [(3r + gh)\, p + 2ghq] \,/\, [fh - r(j + hy) + 2r\,(\theta_1 - \theta_0)]$$

It is easy to check that $s^{*} > 0$ as the numerator is positive and the denumerator is also positive. To see this we write

$$fh - r(j + hy) + 2r(\theta_1 - \theta_0) > fh + r(\theta_1 - 2\theta_0) > 0$$

where the first inequality comes from the fact that $j + hy < \theta_1$ and the second inequality is due to the assumption $\theta_1 > 2\theta_0$.

(ii) from (A3) if $fh > r(2\theta_1 - \theta_0)$ then q' > 0 because $j + hy < \theta_1$. Otherwise there exists $s^{**} = [(3r + gh)\, q + 2\, ghp] \,/\, [\, r(j + hy) - r(\theta_1 - \theta_0)] > 0$ such that if $s < s^{**}$, then $q' > 0$ and $q' < 0$ otherwise.

(iii) During the transitional dynamics, $\theta' > 0$. Hence, $q' > p'$. Therefore, we must necessarily have $s^{*} < s^{**}$.■