Financing and Product Market Competition: Optimal Contracts with Venture Capitalists

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Abstract

We consider the provision of venture capital in a dynamic agency model. In particular, we focus on the interaction between venture capital financing and product market competition: A young firm with a risky innovation project attempts to enter a market where it faces two periods of price competition with an incumbent firm. Since the young firm is wealth-constrained, it seeks equity financing from a venture capital company. The allocation of funds and learning about the project's quality are both subject to moral hazard. We analyze the provision of capital under (i) short-term and (ii) long-term contracting, and compare the results.

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1 Introduction

Venture capital is an important source of financing for young high-tech firms that spur innovation and induce economic growth. While empirical literature on venture capital is abundant, theoretical studies on venture capital contracting have only seen a recent increase in interest. These theoretical models on venture capital financing have, so far, concentrated on the bilateral relationship between a young firm and a venture capital company. Surprisingly, the competitive environment in which the new firm operates has been neglected. To bridge this gap, the present paper investigates the interaction between venture capital financing and product market competition. More precisely, we study how the market structure of an industry and the strategic behavior of competitors will affect the financial contracting between a young firm and a venture capital company.

If a young firm successfully innovates and introduces its new product into a market, it will typically face competition with existing firms. Incumbent firms of that industry will certainly react to the market entry of the new firm: either by softly accommodating entry, or by strategically engaging in competition and predation. Expected returns from the innovation project are thus determined by the competitive environment in the product market.

The main results of our paper are as follows.

- The riskier the innovation project and the fiercer the competition in the product market, the more likely is it that venture capital financing is only provided via short-term contracts.
- Venture capital companies are reluctant to finance pure start-up projects and prefer to engage in expansion stage financing.
- The young firm, on the other hand, seeks to obtain venture capital financing for both investment periods and, therefore, prefers a long-term venture capital contract.

Our model is characterized by two essential features:

(i) *Market entry and product market competition*

We consider a young firm that develops a two-period innovation project. The innovation project consists of a startup stage and, potentially, a market expansion stage. During the startup stage, the firm attempts to enter the market with a new product variant. If entry is successful, the young firm faces price competition with a monopolist. In the subsequent expansion stage, the young firm attempts to acquire additional market shares by lowering the product price. In each period, the young firm has to invest in R&D expenditures in order to develop its project. The outcome of these innovation activities is uncertain and can be either success or failure.

(ii) *Financial contracting*

The young firm is wealth-constrained. Therefore, it seeks equity financing from a venture capital company in order to finance its R&D investments. In exchange for the capital provided, the venture capitalist obtains a share of stochastic returns from the innovation project. We analyze the provision of venture capital under short-term and long-term financing. Under short-term contracting, the venture capital company supplies funds for just one period, and a different contract is written for each investment stage. Under long-term contracting, the venture capital company commits to finance the R&D expenditures in both periods.

The financial relationship between the young firm and the venture capital company is affected by conflicts of interest, i.e. moral hazard, and learning. Though the venture capital company is perfectly informed about the profits in each period, it cannot observe the truthful investment of funds. The young firm is, therefore, able to divert the funds to its private ends, and a (dynamic) moral hazard problem arises. Moreover, the quality of the innovation project is initially uncertain and more information arrives by developing the project. If the firm misuses the venture funds and the project consequently fails, this will also affect the updating of beliefs about the project's quality. The young firm thus gets access to an information rent, which imposes additional agency costs on the financial contracting.

The existing theoretical literature on venture capital financing can be classified into two main groups: The first group of papers investigates the distribution of control rights in venture capital contracting: The central question is who should manage the project – the entrepreneur or the venture capital company? Papers from this area include the work of Amit, Glosten, and Muller (1990), Chan, Siegel, and Thakor (1990), Berglöf (1994), Marx (1998), and Hellmann (1998). The second group of papers focuses on the distribution of ownership rights and stands in the tradition of the principal-agent theory: The venture capital company as the principal provides equity capital for the R&D investment, while the entrepreneur as the agent has to truthfully allocate these funds. The principal typically cannot observe the investment decision of the agent and, due this information asymmetry, a problem of moral hazard arises. Most papers on venture capital contracting investigate this moral hazard problem in a static environment (see e.g. Hansen, 1991; Cornelli and Yosha, 1997; Trester, 1998; Admati and Pfleiderer, 1994; and for a double-sided moral hazard problem, Repullo and Suarez, 1998; Casamatta, 1999). The first authors who investigated venture capital financing in a true dynamical context were Bergemann and Hege (1998). Their model is characterized by a long-term financial relationship in which venture capital is provided for a multi-period innovation project. The allocation of funds is subject to dynamic moral hazard from part of the entrepreneur. In contrast to our model, however, the analysis is carried through without taking product market competition into account. The project's returns are thus exogenously given, and might be either too optimistic or too pessimistic.

Our paper is much more closely related to the second group of papers. It extends the Bergemann and Hege (1998) framework by an entry and competition game. We explicitly model product innovation and subsequent price competition between the new firm and an existing firm. Moreover, since venture capital companies typically invests in projects only for a limited amount of time, we deviate from the Bergemann and Hege (1998) multi-period framework and restrict ourselves, instead, to a two-period model which still enables us to take intertemporal aspects into account.

The remaining of the paper is organized as follows: In section 2, we present the two-period innovation-, market entry- and competition game where the young firm self-finances its innovation project. In section 3, we assume that the firm needs external equity financing for its R&D expenditures. We analyze the provision of venture capital under short-term (3.1) and long-term (3.2) contracting. In section 4, we investigate the strategic reactions of the incumbent and how they influence the financial contracting between the young firm and the venture capital company. In section 5, we draw some final conclusions.

2 Innovation, market entry, and competition

Consider a market in which an incumbent enjoys monopoly profits. The incumbent produces a special product of high consumer value and is the only one who has knowledge about the production technology. The young firm has an idea how to produce a similar product, with which it attempts to reap a share of the monopoly profits. If the product innovation is successful, the monopoly is replaced by duopolistic price competition with heterogeneous products. In the subsequent period after market entry, the young firm attempts to innovate again in order to reduce its marginal production costs, which enables it to lower the price and to further expand the new product into the market.

2.1 The basic model

To study the entry and competition game, we use a well-known model of industrial organization, the circular city (Salop 1979).[1] The model consists of two stages: First, firms enter a market and choose their location on the circle. As a second step, the firms engage in price competition with their horizontally differentiated products. Consumers are uniformly distributed along the circular city and all travel occurs around the circle. However, we modify the Salop-model by extending the single-period framework to a two-period time horizon. Moreover, instead of analyzing simultaneous entry, we consider the subsequent entry of a young firm after a monopolist has already established himself in the market. All agents are risk-neutral.

[1] A concise introduction to competition models in heterogeneous oligopolies is given by Tirole (1988).

2.1.1 Market demand

In addition to the product price, consumers have to bear disutilities T if the product variant of their choice is not supplied. Consumers wish to buy one unit of the good per period of time. Given that the young firm successfully enters, the market demand is divided between the two firms: The young firm will place itself exactly opposite the incumbent firm in order to attract the customers whose preferences deviate most from the existing product.[2] Depending on the own price p_i, the rival's price p_y, and the consumers' preferences T, firm i (i = incumbent) faces a demand of:

$$D_i(p_i, p_y, T) = \frac{p_y - p_i + T/2}{T}, \tag{1}$$

while firm y (y = young firm) serves the rest of the market, $D_y(p_i, p_y, T) = 1 - D_i(p_i, p_y, T)$. Thus, the firm-specific demand increases the lower the own price and the higher the rival's price.

2.1.2 Profit possibilities

If the incumbent serves the entire market, his market share equals $D_i=1$. The monopoly profits Π^M are given by:

$$\Pi^M(p_i^M, c_i, T) = (p_i^M - c_i)\cdot 1 = \bar{s} - \tfrac{1}{2}T - c_i \tag{2}$$

where $\bar{s}$ represents the consumers' reservation value and c_i the marginal production costs.
If the young firm fails to enter and does not produce, it will realize gross profits of zero. In this case, the incumbent continues to earn monopoly profits. If the young firm successfully enters the market, the two firms will compete in the product market. Each firm operating in the market produces with constant marginal costs c. Fixed production costs are assumed to be zero throughout the paper. The profit function for firm i now equals (see Appendix 1):

$$\Pi_i(p_i, p_y, c_i, T) = (p_i - c_i)D_i = (p_i - c_i)\left(\frac{p_y - p_i + T/2}{T}\right), \tag{3}$$

and for firm y, analogously. Both firms choose their prices such as to maximize their per-period profits. We distinguish between two different cost situations: If both the incumbent and the young firm produce with the same marginal costs, i.e. $c_y = c_i$, they choose an (identical) optimal price, $p_i^* = p_y^*$. In this case, market shares equal one half and the profits are

$$\Pi_i(T) = \Pi_y(T) = T/4. \tag{4}$$

[2] Principle of maximal differentiation (cf. D'Aspremont, Gabczewicz, Thisse (1979). The model shows that for profit maximizing reasons, firms locate equidistantly. The firms differentiate their products in order to soften price competition.

We see that these profits depend on the size of the consumers' preferences, T , but not on the marginal costs, c. In the other case, the young firm and the incumbent produce with different marginal costs. We express this cost difference by $\Delta c = c_i - c_y$. The firm with the cost-advantage will demand a lower product price, and thus attracts a higher market share, i.e. $D^A > D^D$. Its profits (superscript A) are given as:

$$\Pi^A(\Delta c, T) = T/4 + \Delta c/3 + (\Delta c/3)^2/T \,. \tag{5}$$

We see that the profit, in the case of cost-advantage, depends directly on the cost-difference Δc between the two firms. At the same time, the firm with the cost-disadvantage realizes profits (superscript D) of:

$$\Pi^D(\Delta c, T) = T/4 - \Delta c/3 + (\Delta c/3)^2/T \,. \tag{6}$$

Due to the effect of the second term in (6), the profits of this firm are negatively influenced by the cost-difference. Independent of the cost-situation, however, the young firm enters the market more easily if consumers' preferences T are strong: The firm can thus attract a large market share by offering a different product variant more suitable to many customers.

2.1.3 Innovation activities and learning

The quality of the innovation project is initially unknown to the young firm (and to any other party of the game, such that there is symmetric non-information). Every agent believes that the project is either "good" with prior probability α_1 or "bad" with prior probability $1-\alpha_1$. If the project is "good", then in every period t, there is a certain probability that the innovation is successfully realized. The probability of success in period t, conditional on the project being good, is denoted by θ_t. The probability θ_t, in turn, is an increasing function of the R&D expenditures. Inversely, a success probability θ_t requires an R&D investment of $r(\theta_t)$. We assume these research costs $r(\theta_t)$ to be linearly increasing in the success probability θ_t: $r(\theta_t) = g\theta_t$, with the cost parameter $g>0$. However, there exists an upper limit $g\theta^{\max}$, above which additional funds do not increase the success probability $\theta^{\max} < 1$ of the project. We use this linear R&D technology for pure simplicity reasons (see Bergemann and Hege (1998) for a similar approach). The same R&D technology is valid if the firm switches from product to process innovation and attempts to reduce marginal production costs in the second period. This research cost function is equivalently given as $r(\theta_2^+) = g\theta_2^+$ for $\theta_2^+ \in [0;\theta^{\max}]$. If the project is "bad", then it always fails and yields a zero return independent of the amount of R&D expenditures invested.

The uncertainty of the project is (partially) resolved over time. During the startup process, the young firm and any potential investor learn more about the prospects of the innovation project. If no success has occurred in the first period, all participants in the project update their beliefs about the project's quality according to Bayes-rule: The posterior belief α_2 of developing a good project in the second period equals the probability of having a good, yet unsuccessful project, divided by the total probability of failure in the past period

$$\alpha_2 = \frac{\alpha_1(1-\theta_1)}{\alpha_1(1-\theta_1)+1-\alpha_1} \le \alpha_1. \qquad (7)$$

The revised beliefs α_2 of developing a good project are smaller than the initial beliefs α_1 (for $\theta_1 \neq 0$) (see also Appendix 2).

2.1.4 The time-line of the basic game

The basic game of innovation and competition in our model consists of two steps: In the first stage, the young firm invests in R&D in order to realize a product innovation. Nature, then, decides whether the outcome of the project is a success or a failure. If it is a success, then in the second stage, both firms choose their optimal prices, sell their product variants and realize their respective profits. If the project fails, the young firm has to stay out and the incumbent continues to realize his monopoly profits. The time-line of this one-shot innovation-, entry- and competition game is as follows:

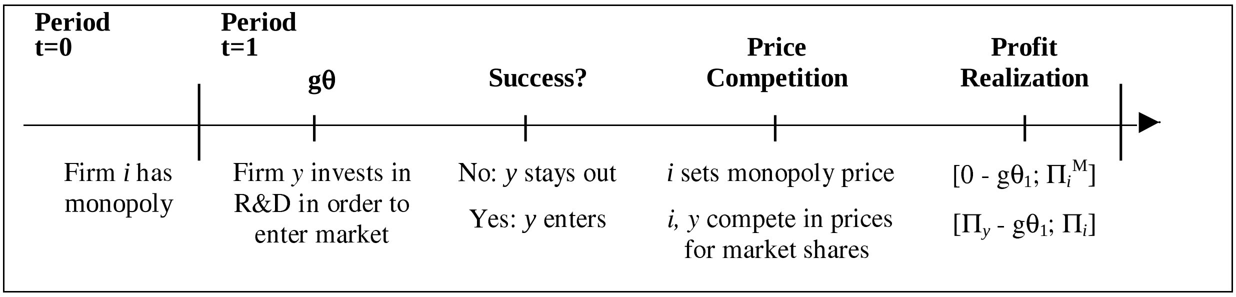

Figure 1: The time-line of the basic entry model

We suppose that the incumbent has no way to deter the entrance of the young firm.[3] We solve this game by backward induction: First, both firms choose their optimal prices. Then, secondly, the young firm determines her level of R&D expenditures. As for the price competition stage, we suppose that if entry is successful, the young firm and the incumbent produce with the same marginal costs. This implies that both firms charge the same optimal product price,[4] the market splits into two equal shares, and the firms realize symmetric profits, $\Pi_i=\Pi_y$. If entry fails, the incumbent charges the monopoly price, while the young firm does

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3 We will discuss this assumption in section 4 below.

4 In Milgrom and Roberts's (1982) limit pricing model, the entrant's marginal costs can be either high or low, such that the young firm supplies either at a higher or lower product price. Since in the Milgrom and Roberts's world, however, consumer's preferences are identical (no transportation costs), the price competition in the homogeneous duopoly results in that only one firm (entrant or incumbent) finally supplies to the market. In contrast to this, our model assumes symmetric market shares after the young firm has entered.

not offer its product and realizes a gross return of zero. Given these expectations about the prices and profit levels, the young firm calculates its optimal R&D expenditures $g\theta_1$ and indirectly, her optimal success probability, θ_1. The success probability of market entry is $\alpha_1\theta_1$, which depends on both the estimated quality of the project and the level of R&D investments. The probability of failure is given by $1-\alpha_1\theta_1$. Therefore, the expected value of the innovation project in the first period equals:

$$V(\theta_1) = \alpha_1\theta_1\Pi_y - g\theta_1 . \tag{8}$$

The firm maximizes the project value over the success probability θ_1:

$$\partial V(\theta_1)/\partial\theta_1 = \alpha_1\Pi_y - g \; . \tag{9}$$

Since the project value is linear in θ_1, the firm will invest the maximal amount $g\theta_1^{\max}$ in R&D activities as long as $\alpha_1\Pi_y \geq g$ or, equivalently, as long as $\alpha_1[\frac{1}{4}T] > g$. For $\alpha_1\Pi_y < g$, however, the firm will spend nothing at all on R&D activities, such that $g\theta_1=0$.

2.2 Innovation, entry and competition in the two-period framework

We now extend our model to two periods of innovation and competition. The idea is that, even if the young firm failed to enter in the first period, it attempts to yield a breakthrough in the subsequent period and to again challenge the monopolist. Thus, the firm has another opportunity to spend on R&D activities and to enter. However, if the young firm has already entered in the first period, it will not cease to innovate, but will - in contrast to the Bergemann and Hege (1998) framework - continue to develop its product: In the second period, it spends again on R&D activities in order to reduce the marginal production costs and to gain a competitive advantage over the incumbent. In this manner, the young firm intends to lower its product price, to further expand its product into the market and to increase the profit level. By extending the time horizon to a second period of innovation and either entry or repeated competition, our model thus combines product innovation with subsequent process innovation. Empirically, this phenomenon is often observed in the life-cycle of technology-intensive products (see e.g. Pfirrmann, Wupperfeld, and Lerner, 1997, on the development of new technology based firms in the IT, software, and biomedical sectors). The extended form of this two-period innovation-, entry- and competition game is as follows:

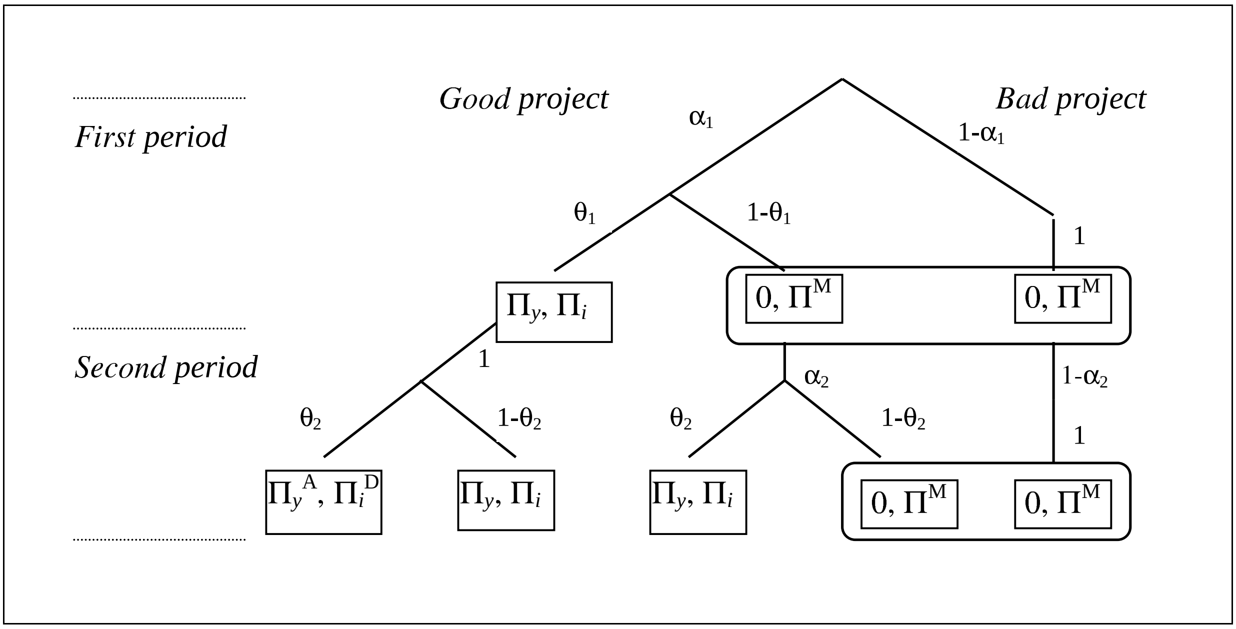

Figure 2: The two-period game of innovation and competition

We see that at the end of period two, some projects (the left-hand side subgame of the second period) established themselves well in the market and became true shooting stars, some projects achieved market entry only in the second period and thus obtained a mediocre return, whereas the remaining fraction of projects failed completely and earned zero gross profits. In that case, the entrepreneur does not know whether the losses are due to the fact that a good project didn't succeed twice or whether it invested in a bad project. Thus, the uncertainty about the project's quality is not completely resolved at the end of the time horizon. The distribution of profits in our model corresponds to the stylized fact that returns of high-risk innovation projects vary widely, ranging from payoffs greater than ten times costs to projects experiencing a total loss (Sahlman, 1990).

Given the profit levels from the price competition (see subsection 2.1.2), we start at the end of the second period and derive the optimal R&D expenditures for each subgame. Starting with the second period expansion stage, the firm maximizes the value of the innovation project in this subgame as:

$$\max_{\theta_2^+} \ V(\theta_2^+) = \theta_2^+ \Pi_y^A + (1-\theta_2^+)\Pi_y - g\theta_2^+ . \tag{10}$$

Since (10) is a linear function of θ_2^+, we see that for $\Pi_y^A - \Pi_y \geq g$, the firm will invest the maximal amount of R&D expenditures, such that $\theta_2^+ = \theta^{\max}$, whereas for $\Pi_y^A - \Pi_y < g$, the firm will not invest at all, i.e. $\theta_2^+ = 0$.

In the other second-period subgame, the second attempt of market entry, the firm maximizes the expected project value as:

$$\max_{\theta_2} \ V(\theta_2) = \alpha_2 \theta_2 \Pi_y - g\theta_2 . \tag{11}$$

Again, since (11) is a linear function of the success probability θ_2, the firm will invest the maximal amount of R&D expenditures, $g\theta_2 = g\theta^{\max}$, as long as $\alpha_2\Pi_y \geq g$ holds, and nothing ($\theta_2 = 0$) otherwise.

Stepping back to the first period of innovation, we start by calculating the total expected value of the venture. Here, we have to take into account that the expected value of the expansion stage is conditioned on the probability of market entry in the first period, $\alpha_1\theta_1$; and that the expected value of the second-period market entry stage is conditioned on the probability of failure in the first period, $1-\alpha_1\theta_1$. The total expected value of the innovation project is, thus, (no discounting):

$$\begin{aligned} V(\theta_1,\theta_2,\theta_2^+) = {} & \alpha_1\theta_1\Pi_y - g\theta_1 \\ & + \alpha_1\theta_1[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y - g\theta_2^+] \\ & + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - g\theta_2]. \end{aligned} \tag{12}$$

It consists of the expected net return in the first period, plus the returns if the firm tries to capture additional market shares in the second period of competition, plus the returns if entry occurs after the second investment round. Rearranging terms, the total value can also be expressed as:

$$\begin{aligned} V(\theta_1,\theta_2,\theta_2^+) = {} & \alpha_1\sum_{t=1}^{2}(\theta_t\Pi_y - g\theta_t)(1-\theta_1)^{t-1} \\ & -(1-\alpha_1)\sum_{t=1}^{2} g\theta_t \\ & + \alpha_1\theta_1[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y - g\theta_2^+]. \end{aligned} \tag{13}$$

Thus, total expected value of the project can be seen as the sum of net returns if entry occurs in the first or in the second period, which is reduced by the sum of R&D investments wasted on bad projects (= second term), and increased by additional returns if the firm further develops its product in the expansion phase (= third term).

How does the expected value of the venture project depend on the R&D investment in the first period? Taking the derivative of (12) with respect to θ_1 and substituting (7) for α_2, we obtain:

$$\frac{\partial V(\theta_1,\theta_2,\theta_2^+)}{\partial \theta_1} = \alpha_1\Pi_y - g + \underbrace{\alpha_1[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y - \theta_2\Pi_y]}_{>0}. \tag{14}$$

We see that the value of the venture project is again linear in θ_1 and, therefore, it is optimal to either allocate the maximal amount of capital or not to allocate any capital at all. Thus, if the marginal expected returns exceed the marginal research costs in equation (14), the firm will invest the maximal amount in R&D expenditures, such that $\theta_1 = \theta^{\max}$. Due to the additional term on the right hand side, expression (14) is less restrictive than the condition from the single-period case (8). Moreover, because $\alpha_2 < \alpha_1$, expression (14) is also less restrictive than the investment condition (11) from the second-period market entry stage. Therefore, the two most binding restrictions for the maximal R&D investment in the multi-period framework are the ones from equation (11), $\alpha_2 \Pi_y > g$, and from equation (10), $\Pi_y^A - \Pi_y > g$. We thus can state:

Proposition 1 (Investment Policy):

If the project is profitable (i.e. the investment conditions $\alpha_2 \Pi_y > g$ *and* $\Pi_y^A - \Pi_y > g$ *are fulfilled), it is optimal to invest the maximal amount of capital in each period into the R&D project, such that* $\theta_t = \theta^{\max}$ *for t=1,2.*

3 Venture capital financing and product market competition

In this section, we now suppose that the young firm does not have sufficient funds to develop its product. Moreover, the firm is subject to limited liability. In order to circumvent the financial restrictions, it seeks equity capital from a venture capital company. High risk-high growth firms that are characterized by significant intangible assets and tremendous uncertainties are unlikely to receive debt financing, which makes venture capital the only source of funding (Fenn and Liang, 1998; Gompers and Lerner, 1999). In exchange for the provision of capital, the venture capital company obtains a share of uncertain, but potentially high returns from the project. We assume that the venture capital industry is characterized by perfect competition. This reflects the fact that there is abundant capital available seeking profitable investment (Reid, 1996; Gompers, 1998). The young firm suggests financial contracts to any of the venture capital companies. The bargaining power is on the side of the firm. The selected venture capitalist then decides whether to accept or reject the financial contract. A venture capital company agrees to finance the project if it receives a net expected profit of zero.[5] Yet, even though venture capital companies usually spend a considerable amount of time to screen their potential projects, we exclude the due diligence process of project selection. We suppose instead that the venture capital company has gone through the whole evaluation process. At the beginning of the financial relationship, both the venture capital company and the young firm believe that the project is good with probability α_1, such that there is symmetric non-information. More information arrives by developing the project. Berger and Udell (1998) point out that informational opaqueness is extremely acute in young start-up firms. This implies that, even though the financial relationship between the venture capital company and the young firm is very close, entrepreneurs still have private information about the projects they develop: Venture capitalists receive regular financial reports and have

[5] As in Diamond's (1984) model of delegated monitoring, the depositors who provide the original funds for the venture capital company require a net return of R at the minimum (see also Ramakrishnan and Thakor (1984)). In our model, we assume that this return R equals zero.

monthly meetings with their portfolio company, but they typically do not become involved in the day-to-day management of the firm (Gompers, 1995). Thus, the entrepreneur may behave opportunistically and expend insufficient effort or exhibit expense preference behavior. In our model, we assume that a problem of moral hazard with hidden action exists. Upon receiving the venture funds, the entrepreneur can either allocate the funds truthfully into research and development of the product, or she can misuse the funds and divert them to her private ends. The venture capital company is unable to observe the proper investment of funds, because high monitoring costs prevent a detailed understanding of the technical position of the young firm (Gompers and Lerner, 1996). We assume, however, that the venture capital company is informed about (i) the level of gross profits realized at the end of each financing period $[0, \Pi_y, \Pi_y^A]$, (ii) the number of firms operating in the industry (monopoly or duopoly), and (iii) how many times the young firm has tried to innovate so far.[6]

3.1 Short-term contracting

We begin by analyzing the procurement of venture capital under short-term financing at time $t=1,2$, i.e., via one-period contracts. To make best use of the information available, the firm offers a different financial contract for each scenario: In the first period, the contract states that the firm obtains the share S_1 of the expected profits, whereas the venture capital company obtains the remaining share, $(1-S_1)$. In the second period, the firm obtains the share S_2 of the realized profits in the second attempt of market entry stage, and S_2^{Duo} in the expansion stage, while the venture capital company receives the remaining shares, $(1-S_2)$ and $(1-S_2^{Duo})$, respectively. Thus, the optimal short-term contracts are simple share contracts between the young firm and the venture capitalist.

3.1.1 First period contracting

We start with the financial contract for the first period market entry stage. The young firm's objective function is to maximize the expected value of its share of profits:

$$\max_{\theta_1, S_1} \; V_{t=1}^{ST} = S_1 \alpha_1 \theta_1 \Pi_y \,. \tag{15}$$

Here, the superscript ST stands for short-term financing. Recall that, in the case of external financing, the young firm does not make any monetary investments into the innovation project. However, the firm must have sufficient incentives to truthfully invest the funds. Therefore, the share S_1 of expected profits has to exceed the amount of research expenditures invested in this period. The incentive compatibility constraint (IC) for the young firm is, therefore, given by:

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[6] One can imagine that after each financing period, the firm receives a time-label for identification.

$$\alpha_1\theta_1 S_1\Pi_y - g\theta_1 \geq 0 \ . \qquad \text{(IC)} \qquad (16)$$

Rearranging terms, we solve for the minimal share of profits that the entrepreneur requires for truthful investment in the first period:

$$S_1 \geq \frac{g}{\alpha_1\Pi_y}. \qquad (17)$$

We see that for $\theta_1 \neq 0$, this minimal share is independent of the innovation success probability which is due to the fact that the R&D cost function is linear.

The venture capital company, on the other hand, is willing to accept the financial engagement, if its share $(1\text{-}S_1)$ of expected profits exceeds the amount of funds invested. The venture capital company decides whether or not to participate in the financial contract based upon the following restriction:

$$(1-S_1)\alpha_1\theta_1\Pi_y - g\theta_1 \geq 0. \qquad \text{(PC)} \qquad (18)$$

Since all bargaining power is on the side of the firm, the expected profits of the venture capital company are reduced to zero. Therefore, the participation constraint (PC) in (18) is binding. Rearranging terms, we see that the minimal share of expected profits that the venture capitalist requires in $t=1$ for participating in the financial contract is:

$$(1-S_1) = \frac{g}{\alpha_1\Pi_y} \ . \qquad (19)$$

The young firm maximizes the value of its expected share of profits (15) under the incentive constraint (16) and the participation constraint of the venture capital company (18). The financial contract is conditioned on the level of profits (Π_y in case of success and 0 in case of failure) as well as on the respective shares of the contracting parties, S_1 and $(1\text{-}S_1)$.

Adding up the two constraints (16), (18), we see that the expected profits of the venture project must equal at least

$$\alpha_1\theta_1\Pi_y - 2g\theta_1 \geq 0 \qquad (20)$$

in order for the firm to obtain financing in the first period. [7] Thus, the more expensive the R&D costs $g\theta_1$, or the lower the beliefs α_1 about the project's quality, the higher the profit level of potential market entry must be to obtain financing. Profits are high if the consumers have strong preferences for the product variants supplied. The success probability of the good project has no influence on the minimum threshold of the profit level as long as the R&D cost function is linear.

[7] The competing claims emanating from the investment problem of the venture capital company and the agency problem of the entrepreneur lead to a conflict of interest if profits cannot cover the total remuneration for the participation and incentive compatibility constraints. Thus, the financing of the venture project resembles a team problem à la Holmström (1982), where both parties contribute and the budget is not balanced.

3.1.2 Second period market entry stage

Proceeding to the second period market entry stage, i.e. the case in which the firm has failed to innovate in the first period but attempts, again, to enter in the second period, the firm's maximization problem has exactly the same structure as the one above. The firm maximizes

$$\begin{aligned} &\max_{\theta_2, S_2} \; V_{t=2}^{ST} = S_2\alpha_2\theta_2\Pi_y \\ s.t. \quad &(\text{IC}) \qquad S_2\,\alpha_2\theta_2\Pi_y - g\theta_2 \geq 0, \\ &(\text{PC}) \qquad (1-S_2)\alpha_2\theta_2\Pi_y - g\theta_2 \geq 0. \end{aligned} \tag{21}$$

Since the updated beliefs α_2 are smaller than the first-period beliefs α_1, the incentive and participation constraints from (21) become more restrictive than in the first-period problem. Adding up the two second-period constraints (IC) and (PC) of (21), we derive that the profit level for the project to obtain financing for second period market entry must be:

$$\alpha_2\theta_2\Pi_y - 2g\theta_2 \geq 0. \tag{22}$$

This minimum threshold of profits in the second period must be higher than in the first period since $\alpha_2 < \alpha_1$. It is intuitively clear that if a project has failed once, it is harder to obtain financing for this project in the subsequent period. Therefore, the share of profits accruing to the venture capital company ($1\text{-}S_2$) also has to be larger than under first period contracting.

3.1.3 Second period expansion stage

Finally, if the firm needs external financing for the expansion stage,[8] the firm's maximization problem and the respective incentive compatibility and participation constraints in this subgame are given by:

$$\begin{aligned} &\max_{\theta_2, S_2^{Duo}} \; V_{t=2Duo}^{ST} = S_2^{Duo}[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y] \\ s.t. \quad &(\text{IC}) \qquad S_2^{Duo}[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y] \geq g\theta_2^+ + S_2^{Duo}\Pi_y \\ &(\text{PC}) \qquad (1-S_2^{Duo})[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y] - g\theta_2^+ \geq 0. \end{aligned} \tag{23}$$

We recall that the venture capital company is able to distinguish between the two profit levels realized, Π_y and Π_y^A. Therefore, the financial contract can be conditioned upon these profit levels. The incentive constraint in this subgame states that the share S_2^{Duo} of profits accruing to the firm under truthful investment must be higher than the sum of diverted research expenditures $g\theta_2^+$ plus the share of certain profits $S_2^{Duo}\Pi_y$ that the entrepreneur obtains if, in the second period expansion stage, no innovation is realized. Rearranging terms leads to:

[8] The profits of the first period could be high enough such that the firm could self-finance the subsequent innovation and market expansion activities. In this case, the firm would not offer a financial contract for the expansion phase.

$$S_2^{Duo} \geq \frac{g}{(\Pi_y^A - \Pi_y)}. \tag{24}$$

Additionally, rearranging the participation constraint of the venture capital company leads to:

$$(1 - S_2^{Duo}) \geq \frac{g\theta_2^+}{\theta_2^+ \Pi_y^A + (1-\theta_2^+)\Pi_y}. \tag{25}$$

Adding up the incentive (24) and the participation (25) constraints of this problem, we derive the net expected profits in the expansion stage:

$$\theta_2^+ \Pi_y^A + (1-\theta_2^+)\Pi_y - S_2^{Duo}\Pi_y - 2g\theta_2^+ \geq 0. \tag{26}$$

The expected profits of the expansion stage, less the share of symmetric profits accruing to the firm for incentive reasons, must be higher than twice the amount of R&D expenditures such that financing is obtained and funds are invested truthfully. We summarize our analysis of the short-term contracts in the following proposition.

Proposition 2 (Short-term contracting):

Under short-term contracting, the firm writes a different financial contract Γ_t^{ST}, t=1,2, for each stage. Conditional on the fact that a minimum threshold of profits is attained, the financial contracts for the first period market entry stage, the second period market entry stage, and the second period expansion stage, respectively, are given by:

$$\Gamma_{t=1}^{ST}\left[S_1,(1-S_1)\middle|\Pi_y^{1\min}\right], \qquad \textit{with } S_1 \geq \frac{g}{\alpha_1 \Pi_y};\ (1\text{-}S_1) = \frac{g}{\alpha_1 \Pi_y};\ \Pi_y^{1\min} = \frac{2g}{\alpha_1},\quad \theta_1,\alpha_1 \neq 0.$$

$$\Gamma_{t=2}^{ST}\left[S_2,(1-S_2)\middle|\Pi_y^{2\min}\right], \qquad \textit{with } S_2 \geq \frac{g}{\alpha_2 \Pi_y};\ (1\text{-}S_2) = \frac{g}{\alpha_2 \Pi_y};\ \Pi_y^{2\min} = \frac{2g}{\alpha_2},\ \theta_2,\alpha_2 \neq 0.$$

$$\Gamma_{t=2Duo}^{ST}\left[S_2^{Duo},(1-S_2^{Duo})\middle|\Pi_y^A,\Pi_y,\theta_2^+\right], \quad \textit{with } S_2^{Duo} \geq \frac{g}{\Pi_y^A - \Pi_y};$$

$$(1 - S_2^{Duo}) = \frac{g\theta_2^+}{\theta_2^+ \Pi_y^A + (1-\theta_2^+)\Pi_y};\ \theta_2^+ \Pi_y^A + (1-\theta_2^+)\Pi_y - S_2^{Duo}\Pi_y - 2g\theta_2^+ \geq 0.$$

We state again that in each financial contract, the venture capital company obtains no more than the minimal share of profits that it requires for participation in the financial contract. The firm, on the other hand, obtains at least the minimal share of profits necessary for incentive reasons, as well as all additional profits in case of financial slack, i.e. if the project's profits surpass the minimum threshold level.

3.1.4 Comparison of the short-term contracts

In addition to the analysis of the minimal shares and profit levels, we want to know in which stage it is most difficult for the young firm to obtain short-term financing. Therefore, we compare the total project financing restrictions of the first (20) and second period (22) market entry stages with the restriction of the expansion stage (26):

$$\alpha_1\theta_1\Pi_y - 2g\theta_1 \geq 0, \tag{20}$$

$$\alpha_2\theta_2\Pi_y - 2g\theta_2 \geq 0, \tag{22}$$

$$\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y - S_2^{Duo}\Pi_y - 2g\theta_2^+ \geq 0. \tag{26}$$

As we have stated above, due to $\alpha_2 < \alpha_1$, project financing for a firm which has not yet successfully entered the market, is harder to obtain in the second than in the first period. Thus, condition (22) of the second period market entry stage is more restrictive than its counterpart (20) from the first period.

Next, we compare these market entry stage restrictions with the expansion stage condition (26). Financing is easier to obtain in the expansion stage, i.e. after the firm has entered, than before market entry, if the expected profits are higher:

$$\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y - S_2^{Duo}\Pi_y \geq \alpha_t\theta_t\Pi_y \qquad \textit{for } t=1,2, \tag{27}$$

where $\theta_t = \theta_2^+ = \theta^{max}$. After rearranging terms, we can, therefore, state:

Proposition 3 (Financial restrictions):

The ranking of the financial contracts shows that for $\Pi_y^A/\Pi_y \geq 1+\alpha_1 - (1-S_2^{Duo})/\theta$ *:*

i) Short-term financing is easier to obtain in the expansion phase (after successful market entry) than in the startup stage (before market entry).

ii) Short-term financing is most difficult to obtain for the second startup phase after the young firm has already failed once to enter the market in the first period.

Proposition 3 stands in accordance with the empirical fact that many venture capital companies prefer to finance the expansion stage of young firms, whereas only few venture capitalists specialize in startup financing. In Germany, only 15 per cent of venture funds are devoted to seed and startup financing, while expansion stage financing accounts for 55 per cent of the investments (Bundesverband deutscher Kapitalbeteiligungsgesellschaften, 2000). Similarly, the OECD reports for Europe that the lion's share of venture capital investments are dedicated to later stage investments in established businesses and management buyouts/buyins (Organisation of Economic Co-operation and Development, 1996). The US venture capital industry, however, has shifted from 15 per cent startup financing, 65 per cent later stage investment, and 20 per cent leveraged buyouts and acquisition deals during the 1980s (Sahlman, 1990) towards a higher investment into early stage businesses. Black and Gilson (1998) state that US venture capital companies, nowadays, invest in almost as many early stage as later-stage companies, which they suggest is due to the more complete development of the US venture capital markets.

However, since short-term contracts do not allow for intertemporal transfers, they restrict financing to projects with very high expected returns. In the next subsection we, therefore, analyze whether long-term contracting will improve the financing possibilities for the young firm.

3.2 Long-term contracting

Empirically, the financing horizon of venture capital projects is rather medium or long-term than short-term: Venture capital companies typically provide funds for a period of three to five years (Gompers, 1998). Therefore, we now investigate how long-term commitment of the venture capital company changes the financial contracting. The venture capital company hereby commits to supplying funds to the firm for two subsequent periods. Thus, the various short-term contracts are replaced by a single long-term contract, $\Gamma^{LT}(S_1^{LT}, S_2^{LT}, S_{2Duo}^{LT}, \theta_1, \theta_2, \theta_2^+)$, which starts in the first period and covers the whole project horizon. The financial contract is a time-varying share contract which prescribes the participation and incentive compatible shares according to the relevant scenario in each period.[9]

Furthermore, the long-term financial contract provides room for intertemporal transfers: While under short-term financing, the young firm does not always obtain a new contract for the second period, e.g. if the expected profits are drawn from the interval $[2g\theta_2 > \alpha_2\theta_2\Pi_y \geq g\theta_2]$, the project would be profitable under self-financing, but is denied venture capital under short-term financing in the second period, while long-term contracting permits the firm to transfer shares of expected profits between periods and scenarios. This means that, instead of breaking even in each period, the venture capital company accepts to engage in the project as long as the expected total repayments cover the sum of the research and development costs in both periods. The sequence of participation constraints is thus replaced by a single intertemporal participation constraint. If the venture capital company agrees to the long-term financial contract, refinancing for the young firm is guaranteed with certainty. This means that if market entry failed in the first period, the firm has yet another chance to innovate and to enter in the next period. However, due to the intertemporal structure of the financial arrangement, the young firm has more opportunities to divert the funds. Firstly, the firm can divert the provided funds in both investment periods. Secondly, the firm

9 This stands in contrast to the results of Admati and Pfleiderer (1994), who show that a time-invariant share contract is optimal for venture projects under uncertainty.

can divert the funds today and can bet on a positive realization of the innovation project tomorrow. In this case, the learning process about the project's quality will become asymmetric: The venture capital company, on the one hand, updates its beliefs because the project return after the first period is zero. The entrepreneur, on the other hand, knows that the project failed as a consequence of her hidden action and does not adjust her beliefs about the project's quality. Information becomes asymmetric, and the entrepreneur is, therefore, granted an information rent (to be specified below).

The maximization problem of the entrepreneur under long-term contracting, if the funds are invested truthfully and if the venture capital company agrees to finance the project, is given as follows:

$$\underset{\substack{\theta_1,\theta_2 \\ S_1^{LT}, S_2^{LT}, S_{2Duo}^{LT}}}{Max} \quad V^{LT} = S_1^{LT}\alpha_1\theta_1\Pi_y + \alpha_1\theta_1 S_{2Duo}^{LT}[\theta_2^+\Pi_y^A + (1-\theta_2^+)\Pi_y] + (1-\alpha_1\theta_1)[\alpha_2\theta_2 S_2^{LT}\Pi_y]. \tag{28}$$

We state again that the firm does not make any monetary investment expenditures during the entire project horizon. Equation (28) represents the total expected value of the project accruing to the entrepreneur, if she is willing to allocate the funds truthfully over the whole project horizon. Since under long-term contracting, the entrepreneur has different possibilities to divert the funds during the whole financing horizon, we have to account for four different incentive compatibility conditions, (29 a-d):

$$S_{2Duo}^{LT}[\theta_2(\Pi_y^A - \Pi_y) + \Pi_y] \geq g\theta_2 + S_{2Duo}^{LT}\Pi_y. \qquad \text{(IC)} \tag{29a}$$

Condition (29a) prevents the firm from misuse of funds in the second period if it has truthfully invested in the first period and market entry has occurred.

$$\alpha_2 S_2^{LT}\theta_2\Pi_y \geq g\theta_2. \qquad \text{(IC)} \tag{29b}$$

Condition (29b) prevents the firm from misuse of funds in the second period if the firm has truthfully invested in the first period, but no entry has occurred.

$$V^{LT} \geq g\theta_1 + S_2^{LT}\alpha_1\theta_2\Pi_y \;. \qquad \text{(IC)} \tag{29c}$$

Condition (29c) states that the expected value under truthful investment should be higher than first-period diversion of funds and subsequent truthful investment in the second period. Note that the aposteriori beliefs on the right hand side of this equation are equal to α_1 instead of α_2 because the entrepreneur does not updated her beliefs after deviation.

$$V^{LT} \geq g\theta_1 + g\theta_2. \qquad \text{(IC)} \tag{29d}$$

Finally, condition (29d) shall prevent the entrepreneur from the misuse of funds in both periods. Moreover, the intertemporal participation constraint for the venture capital company is given as follows:

$$\begin{aligned}(1-S_1^{LT})\alpha_1\theta_1\Pi_y + \alpha_1\theta_1(1-S_{2Duo}^{LT})[\theta_2\Pi_y^A + (1-\theta_2)\Pi_y] \\ + (1-S_2^{LT})\alpha_2\theta_2\Pi_y(1-\alpha_1\theta_1) \geq 2g\theta.\end{aligned} \qquad \text{(PC)} \tag{30}$$

The shares of profits that the venture capital company obtains in both periods must be larger than the sum of funds invested in the two subsequent financing periods.

We analyze the maximization problem in two steps: First we look for the minimal incentive compatible shares for which the entrepreneur is willing to truthfully allocate the funds over the whole project financing horizon. Then, in a second step, we check under which conditions the participation constraint of the venture capital company (30) is fulfilled.

3.2.1 The incentive problem

We solve the incentive problem of the firm by backward induction. The two second period conditions (29a) and (29b) must be binding, since we look for the minimal incentive compatible shares:

$$S_{2Duo}^{LT} = g/(\Pi_y^A - \Pi_y), \tag{29a$'$}$$

$$S_2^{LT} = g/\alpha_2 \Pi_y. \tag{29b$'$}$$

Thus, the second period incentive compatibility conditions under long-term contracting are identical to the ones under short-term contracting (cf. equations (21) and (24)).
Next, by inserting the binding expression (29b′) into condition (29c), we get:

$$V^{LT} \geq g\theta_1 + [g\theta_2]\cdot(\alpha_1/\alpha_2). \tag{29c$'$}$$

Finally, by comparing the above expression with (29d), we see that expression (29c′) is more restrictive, due to the term $(\alpha_1/\alpha_2) \geq 1$. This captures the fact, that under long-term financing, in addition to the sum of funds provided, $g\theta_1 + g\theta_2$, the entrepreneur gets access to an informational rent. We conclude that condition (29d) can be eliminated from the optimization problem. Thus, to prevent the entrepreneur from moral hazard, the share of profits under truthful investment (28) must be higher than or equal to the payoffs under deviation (29c′):

$$\alpha_1\theta_1 S_1^{LT}\Pi_y + \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + (1-\alpha_1\theta_1)(\alpha_2\theta_2 S_2^{LT}\Pi_y) \;\geq\; g\theta_1 + \frac{\alpha_1}{\alpha_2}(\alpha_2\theta_2 S_2^{LT}\Pi_y). \tag{31}$$

The solution to this intertemporal incentive problem is summarized as follows:

Proposition 4 (Share contract):

The minimum shares of profits that the entrepreneur requires for truthful investment under long-term contracting are given by $S_1^{LT}, S_2^{LT}, S_{2Duo}^{LT}$, *with*

$$S_1^{LT} = \underbrace{\frac{g}{\alpha_1 \Pi_y}}_{I} + \underbrace{\frac{g\theta_2}{\Pi_y}}_{II} - \underbrace{S_{2Duo}^{LT} \frac{E(\Pi_{t=2})}{\Pi_y}}_{III} + \underbrace{\left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right) \frac{g\theta_2}{\alpha_1 \Pi_y}}_{IV}, \tag{32}$$

$S_2^{LT} = g / \alpha_2 \Pi_y$, and

$S_{2Duo}^{LT} = g / (\Pi_y^A - \Pi_y)$.

Proof. See Appendix 3.

The minimum share S_1^{LT} incorporates the intertemporal aspects of the financial contract and ensures that the firm employs the capital in each period towards the discovery process and the improvement of the product. Equation (32) may seem rather inaccessible at first, but can be decomposed into different aspects of the agency problem between the venture capital company and the young firm:

I) Static agency costs: We see that term *I* of (32) is identical to the static incentive compatibility condition (16) from the short-term financing section above, i.e.,

$$\frac{g}{\alpha_1 \Pi_y} \equiv S_1. \tag{16'}$$

This would be the minimal share for the entrepreneur if the innovation project were financed during a single period only.

II) Intertemporal agency costs: If the venture capital company agrees to long-term financing but could observe the development of beliefs such that the entrepreneur would not get access to the informational rent, the minimum share would have to be modified in two ways: On the one hand, the minimum share has to be increased by the second term *II*, which reflects the option for the entrepreneur to withhold financing for a single period, but switch to truthful investment in the next period:

$$+\frac{g\theta_2}{\Pi_y}.$$

III) Competition effect: On the other hand, the minimum share has to be reduced by the share of profits the entrepreneur could have gained in the expansion stage (term *III*). Thus, the opportunity to increase potential profits in the second period of competition - if funds are employed appropriately in the first period - helps to realign the incentives of the entrepreneur:

$$-S_{2Duo}^{LT} \frac{E(\Pi_{t=2})}{\Pi_y}.$$

We denote the above term as the "competition effect". Moreover, we see that the effects *II* and *III* work in opposite directions: The withholding option, $g\theta_2/\Pi_y$, increases the minimal share of profits, S_1^{LT}. By contrast, the competition effect, $-S_{2Duo}^{LT}E(\Pi_{t=2})/\Pi_y$, reduces the minimal share. This negative effect of the foregone opportunities to gain additional profits in the expansion stage may even dominate the positive effect of the withholding option!

IV) Information rent: The informational agency costs to be added are expressed by term *IV*:

$$+\left(\frac{\alpha_1}{\alpha_2}-\alpha_1\right)\frac{g\theta_2}{\alpha_1\Pi_y}.$$

This term represents the development of the informational advantage of the entrepreneur, conditioned on her amount of control in the next period, $g\theta_2$. The informational agency costs depend on the difference between the ratio of beliefs α_1/α_2 and the original beliefs, α_1. We therefore interpret the term in brackets as an "informational" mark-up factor.

As a next step, we investigate which of these dynamical effects under long-term contracting has the strongest influence on S_1^{LT}. As we have seen in equation (32), the intertemporal profit share S_1^{LT} consists of the short-term first-period share S_1, as well as of three additional effects.[10] We have to distinguish between two scenarios: On the one side, the first-period share under long-term contracting can be higher than (or equal to) its equivalent under short-term contracting, $S_1^{LT} \geq S_1$. In this case, the increase in agency costs and the information rent together dominate the negative competition effect of (32). On the other side, however, the negative competition effect might have a stronger impact on S_1^{LT} than the additional agency costs and the informational rent. In that case, the first-period share of the entrepreneur under long-term contracting will actually be lower than the one under short-term contracting, $S_1^{LT} < S_1$. We thus derive the following corollary:

Corollary 1 (Intertemporal incentive problem):

i) *In case $S_1^{LT} \geq S_1$, the intertemporal agency costs and the information rent have a dominating influence on S_1^{LT}, and the firm, therefore, obtains a higher first-period share of profits under long-term contracting than under short-term contracting ("normal case").*

ii) *In case $S_1^{LT} < S_1$, the firm actually obtains a lower first-period share of profits under long-term contracting. This surprising result is due to the strong impact of the competition effect on S_1^{LT}, which helps to realign the incentives of the entrepreneur under long-term contracting ("disciplinary case").*

[10] The size of these three effects depends on the profit levels Π_y^A and Π_y which, in turn, are influenced by the demand parameter *T*, by the marginal production costs *c*, and by the difference of the marginal production costs between the two firms, Δc (see section 2.1.2 above).

The competition effect becomes stronger with higher potential profits from the second period market expansion $E(\Pi_{t=2})$ and with a higher share S_{2Duo}^{LT} accruing to the entrepreneur in that stage. In the special case where the competition effect is zero, our results include Proposition 4 of Bergemann and Hege (1998), which states that under long-term contracting the first-period share of the entrepreneur increases due to intertemporal agency costs and an informational rent.[11]

3.2.2 The intertemporal participation problem

We have stated above that, under long-term contracting, the venture capital company faces a single intertemporal participation constraint instead of the various static participation constraints. Presumably, financing for the young firm becomes easier now, since the intertemporal participation constraint allows to exchange profit shares between all three scenarios. The intertemporal participation constraint of the maximization problem above is given as:

$$[(1-S_1^{LT})\alpha_1\theta_1\Pi_y - g\theta_1]+\alpha_1\theta_1[(1-S_{2Duo}^{LT})E(\Pi_{t=2})-g\theta_2^+]+(1-\alpha_1\theta_1)[(1-S_2^{LT})\alpha_2\theta_2\Pi_y - g\theta_2] \geq 0. \tag{30'}$$

The first term of (30′) is identical to the first-period participation constraint under short-term financing, except for the expression (1-S_1), which has been substituted by the long-term share (1-S_1^{LT}). The second and third terms represent the respective participation constraints of the second period expansion and market entry phase, which are identical to those under short-term contracting, but weighted with their entrance probabilities $\alpha_1\theta_1$ and $1-\alpha_1\theta_1$.

3.2.3 Project value under long-term contracting

Next, we insert the entrepreneur's minimal incentive compatible shares (29a′), (29b′) and (32) into the respective first and second-period shares of the intertemporal participation constraint of the venture capital company (30) in order to solve for the minimum project value under long-term contracting (see Appendix 4):

$$\underbrace{[\alpha_1\theta_1\Pi_y - 2g\theta_1]}_{I} - \underbrace{\left(\frac{\alpha_1}{\alpha_2}-\alpha_1\right)g\theta_1\theta_2}_{II} + \alpha_1\theta_1\underbrace{[E(\Pi_{t=2})-2g\theta_2^+]}_{III}+(1-\alpha_1\theta_1)\underbrace{[\alpha_2\theta_2\Pi_y - 2g\theta_2]}_{IV} \geq 0 \tag{33}$$

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[11] If $-S_{2Duo}^{LT}\frac{E(\Pi_{t=2})}{\Pi_y}=0$, then the first period share is equal to $S_1^{LT}=S_1+\frac{g\theta}{\Pi_y}+(\frac{\alpha_1}{\alpha_2}-\alpha_1)\frac{g\theta}{\alpha_1\Pi_y}$, which implies that $S_1^{LT}>S_1$, and that the scenario of Corollary 1*i*) is in place. This result is analogous to Bergemann and Hege (1998), who show that in an intertemporal set-up, the first-period share increases to a maximal value (p.718 and Figure 2).

Term I of (33) represents the net profits of the first period, which is identical to the minimum profit condition under short-term financing. Term II incorporates the informational rent (slightly modified) granted to the entrepreneur in the intertemporal context. Term III consists of the net profits of the expansion stage. Finally, term IV reflects the expected net profits of the second-period market entry stage, which is again equivalent to the second period minimum profit condition under short-term financing. The project obtains long-term financing as long as the above expression is greater than zero.

We compare the project value under long-term contracting with its counterpart under short-term contracting. Given that short-term contracting is granted for all three scenarios, its total value is composed of (20) and the weighted equations (22) and (26):

$$[\alpha_1\theta_1\Pi_y - 2g\theta_1] + \alpha_1\theta_1[E(\Pi_{t=2}) - S_2^{Duo}\Pi_y - 2g\theta_2^+] + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2]\ . \quad (34)$$

Subtracting equation (34) from the project value under long-term contracting (33) yields:

$$\alpha_1\theta_1 S_2^{Duo}\Pi_y - \alpha_1(1/\alpha_2 - 1)g\theta_1\theta_2 \geq 0\ . \quad (35)$$

Substituting $S_2^{Duo} = g/(\Pi_y^A - \Pi_y)$ leads to:

$$\alpha_1\theta_1 g\left[\frac{\Pi_y}{\Pi_y^A - \Pi_y} - \theta_2(1/\alpha_2 - 1)\right] \geq 0\ . \quad (36)$$

The term in the brackets is positive because the profit ratio is close to one while the information term $\theta_2(1/\alpha_2\text{-}1)$ is relatively close to zero. Thus, for $\alpha_1, \theta_1, \theta_2^{(+)}, g, \neq 0$, long-term contracting is more efficient than short-term contracting.

Now suppose for a moment that under short-term contracting the project is not profitable any more in the second period market entry stage and that refinancing is denied. Recall, however, that the project's profitability conditions under external financing are stricter than under self-financing due to the impact of the moral hazard problem; e.g. $\alpha_2\theta_2\Pi_y - 2g\theta_2 \geq 0$ instead of $\alpha_2\theta_2\Pi_y - g\theta_2 \geq 0$. Thus, if the expected second period market entry profits are drawn from the interval $[2g\theta_2 > \alpha_2\theta_2\Pi_y \geq g\theta_2]$, it is socially desirable that the entrepreneur obtains financing for this stage too. A long-term contract can help to circumvent the financial restriction, if the following relation holds:

$$\alpha_1\theta_1 g\left[\frac{\Pi_y}{\Pi_y^A - \Pi_y} - \theta_2(1/\alpha_2 - 1)\right] \geq -(1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2], \quad (37)$$

i.e. if the surplus from long-term contracting - derived in (36) - exceeds any potential losses from the second period entry stage. Since we restrict our analysis to projects with a maximal loss of $[-g\theta_2]$, we can substitute $[\alpha_2\theta_2\Pi_y - 2g\theta_2]$ in the right hand side of (37) by $[-g\theta_2]$ and obtain, after rearranging terms and simplifying:

$$\frac{\Pi_y}{\Pi_y^A - \Pi_y} \geq \theta_2(1/\alpha_2 - 1) + (1 - \alpha_1\theta_1)/\alpha_1 \qquad \text{for } \alpha_1, \theta_1, \theta_2, g, \neq 0\,. \tag{38}$$

Thus, if condition (38) is fulfilled, a long-term contract enables the entrepreneur to realize a second attempt of market entry which would not be granted under short-term financing. From a social point of view, a long-term contract therefore increases market efficiency. We summarize our results in the following proposition.

Proposition 5 (Long-term project financing):

i) Long-term contracting is more efficient than short-term contracting if the project's profitability conditions are met in each stage.
ii) Long-term contracting helps to circumvent financial restrictions of innovation projects that are stopped too early under short-term contracting.

Proposition 5 indicates that a long-term financial contract actually improves the financing situation of the young firm. The entrepreneur is better off since, not only top innovation projects, but also projects of slightly lower expected value will now obtain financing in both investment periods. Our results correspond to the findings of Bergemann and Hege (1998, their Proposition 5), that long-term contracts reduce inefficiencies in financial contracting which are caused by information problems between the venture capitalist and the young firm. Our results also explain the wide-spread use of long-term contracts in equity financing relationships: Venture projects are characterized by staged financing, in which prospective projects obtain more than one financing round (Sahlmann, 1990; Gompers, 1995; Cornelli and Yosha, 1997).

3.3 Comparison of the results between short-term and long-term financing

We repeat that under short-term contracting, it is most difficult to obtain project financing for the second period market entry stage, followed by the request of financing for the first-period market entry stage and, finally, followed by the financing demand for the second-period expansion stage. Translating this into a ranking of required minimum profit levels, we state that the absolute profits must be highest in the second period market entry stage.

Thus, it can happen that a firm obtains venture capital financing for the first period of innovation, but is denied second-period financing after its project has failed. This is due to the updating of beliefs and, therefore, to the lower expected value of the project in the second period. Moreover, the innovation project might, likewise, be stopped too early if (i) the initial beliefs about the project's quality, α_1, suddenly decrease; if (ii) the profit level Π_y declines due to a reduction in consumers' preferences, T; or if (iii) the required investment volume,

$g\theta_t$, increases. The problem can be circumvented with a long-term financial contract: The venture capital company hereby guarantees refinancing for the second period in exchange for potential profits from the expansion stage.

On the other hand, short-term financial contracting might be better for the young firm if its profits Π_y after the first period of innovation are rather high: In case the innovation project shows significant upside potential, the young firm might be able to self-finance the second period research expenditures, $g\theta_2$. This situation is given if profits are sufficiently high (i) to induce incentive-compatible investment, (ii) to compensate the venture capital company for the capital provided, and (iii) if sufficient financial slack remains to finance the research expenditures of the second period: i.e., if $\Pi_y - 2g/\alpha_1 - g\theta_2 \geq 0$. This situation is more likely to occur if the initial beliefs α_1 are high, if the marginal research costs g are low, and if the required investment volume $g\theta_2$ is low as well. And, even in case the financial slack is insufficient to completely self-finance the second R&D investment, the young firm, nevertheless, might find a "cheaper" financing alternative than venture capital: Since in the expansion stage the project will realize positive profits with certainty, the firm could apply for mezzanine funds or a short-term financial institution loan (Berger and Udell, 1998). This, however, is beyond the scope of our model.[12]

Finally, let us say a few words about the industry into which the young firm attempts to enter: Suppose that the industry is characterized by either strong consumers' preferences or distinctive product variants, which implies that this industry's parameter T is very high.[13] In this case, financing becomes easier to obtain, and both short-term and long-term financing should be available to the young firm. If, by contrast, the industry's parameter T is low, the price competition between the firms is more intense, and the expected profits from market entry will be lower. Financing now becomes harder to obtain. The young firm might, therefore, switch from short-term to long-term contracting.

As far as the size of the innovation is concerned, it is always favorable to the young firm to achieve a better cost-effectiveness than its competitor: If the innovation is drastic (Δc is large), the firm will quickly expand its market shares and increase its profit. Financing - especially for the second-period expansion stage - becomes easier to obtain.

[12] Gompers (1998) points out that venture capital is a very costly source of funding. Thus, as soon as any tangible assets are available or a steady cash flow is realized, the young firm will switch to "cheaper" debt financing. In our model, however, the venture capital company realizes zero profits due to the perfect competition in the venture capital market. Its expected share of profits in the expansion stage just equals the investment costs, $g\theta_2^+$. This implies that, within our framework, there exists no "cheaper" financing alternative. Thus, we abstract from analyzing convertible securities, although recent literature has focused on these hybrid financial instruments mainly for incentive reasons (see e.g. Cornelli and Yosha, 1997).

[13] Sutton (1998, chapter 6) describes the market for flowmeters as an example of an industry in which high T-values are present. The different flowmeter types are characterized by their physical principles employed in measurement (electromagnetic, ultrasonic,...). A firm which discovers an alternative principle of measurement can easily enter the market and, in case of low price and production costs, will attract a large market share and realize high profits.

4 Strategic reactions of the incumbent

So far, we have assumed that only the young firm, but not the incumbent, is able to innovate and to reduce marginal production costs. However, the incumbent - instead of being passive - will certainly react to the potential market entry of the young firm. In this situation, he can pursue different strategies: He can either invest $r(\mu_t) = g\mu_t$ with $\mu_t \in [0; \mu^{\max}]$, $t=1,2$ in R&D activities, too, such that the innovation and competition game is extended to a simultaneous move game where both firms invest in R&D activities, successfully innovate with probability $\theta^{max} = \mu^{max}$, and then subsequently compete in prices for market shares. The innovation activities of the incumbent reduce the expected profits of the young firm in the market entry as well as the expansion stage. Here, the young firm's profits decline as the success probability of the incumbent μ^{max} increases, and the higher the potential cost-advantage of the incumbent, Δc. Thus, in all three subgames, it becomes more difficult for the young firm to obtain venture capital financing.

Another possibility is that the incumbent strategically invests in predation activities which will, in turn, reduce the innovation success probability of the young firm from θ_t to θ^{pred} and, therefore, increase his own chances of remaining a monopolist (see Snyder (1996) for a similar approach). The incumbent chooses to prey if the predation costs are lower than his additional monopoly profits: $K^{pred} < \alpha_t(\theta_t - \theta^{pred})(\Pi^M - \Pi_i)$. This strategy has a clear negative impact on the profit situation of the young firm; its chances to obtain venture capital financing are likewise reduced. Thus, the predation strategy of the incumbent stands in tradition of the long purse story (Telser 1966): the better access to financial resources has a strategic effect on the competitive position of both firms, and allows the incumbent to deter entry or to drive his competitor out of the market.

Defining welfare as the sum of consumers' and producers' surplus, we see that welfare increases if the young firm enters the market. Moreover, if the incumbent also invests in R&D activities and the young firm still enters the market, production costs and product prices are expected to be lower and welfare, thus, increases. In contrast, if the incumbent invests in predatory activities and successfully defends his monopoly position, welfare will decline.

5 Conclusion

We have presented a dynamic agency model of venture capital financing, where a young firm attempts to enter a market with an innovative product. In our model, we have explicitly formalized the market entry of the young firm and the subsequent price competition with an incumbent. In contrast to the existing literature, we have thus endogenized the stochastic returns from the venture project. By controlling for industry characteristics as well as for the size of innovation, we have shown that expected profits from the project are higher the stronger the consumer's preferences and the higher the potential cost-advantage of the new product.

In the case where the innovation project is financed via short-term contracts, we derived the following hierarchy of financial contracting: (i) Venture capital financing is easiest to obtain for the market expansion stage, because then the project has already proven to be good. (ii) It is more difficult to obtain venture capital for the first-period market entry stage, since the project's quality is yet unknown. (iii) It is most difficult to obtain venture capital financing for a second attempt of market entry, after a negative signal (i.e., zero profits in the first period) has been obtained. These theoretical findings stand in accordance with the empirical evidence that the majority of venture capital is invested into expansion stage projects and not into seed and start-up financing that are associated with higher risk. Under short-term contracting, unsuccessful projects are presumably stopped after the first period. On the other hand, it is quite likely that good, yet unsuccessful projects are denied follow-up financing for the second period. This is due to the imperfect information about the project's quality and the moral hazard problem between the firm and the venture capital company. It implies that a fraction of good innovation projects will be stopped prematurely which, in turn, increases the incumbent's chances of remaining a monopolist. Therefore, our model explains how financial market restrictions reinforce concentration tendencies in certain industries.

The early stopping problem can be circumvented via a long-term financial contract, because it allows for an intertemporal share trade-off. In this case, refinancing is guaranteed for all R&D investments in the second period. The long-term relationship, however, imposes additional agency costs on the financial contract. Surprisingly, though, competition in the product market actually helps to realign the incentives of the entrepreneur: If this "competition effect" is strong enough, the moral hazard problem under long-term contracting is reduced. This is a new result in the financing literature. We, therefore, strongly suggest that future research on venture capital contracting should take the competitive environment of the young, innovative firm into account. Otherwise, incentive problems in dynamic financial contracting might appear too strong.

As long as the innovation project is profitable in all scenarios, long-term contracts are more efficient than short-term contracts. If, however, the incumbent strategically reacts to the potential entry of the young firm, expected returns from the innovation project will decrease, and/or the risk of project failure will increase. Under these circumstances, a long-term venture capital contract may become unavailable. Then, short-term venture capital financing will be the only source of funding for the young firm.

References

Admati, A., Pfleiderer, P., 1994. Robust financial contracting and the role of venture capitalists. Journal of Finance 49, 371--402.

Amit, R., Glosten, L., Muller, E., 1990. Entrepreneurial ability, venture investments, and risk sharing. Management Science 36, 1232--1245.

Bergemann, D., Hege, U., 1998. Venture capital financing, moral hazard and learning. Journal of Banking and Finance 22, 703--735.

Berger, A., Udell, G., 1998. The economics of small business finance: the roles of private equity and debt markets in the financial growth cycle. Journal of Banking and Finance 22, 613--673.

Berglöf, E., 1994. A control theory of venture capital finance. Journal of Law, Economics, and Organization 10, 247--267.

Black, B., Gilson, R., 1998. Venture capital and the structure of capital markets: banks versus stock markets. Journal of Financial Economics 47, 243--277.

Bundesverband deutscher Kapitalbeteiligungsgesellschaften, 2000. Jahrbuch 1999. Berlin.

Casamatta, C., 1999. Financing and advising: optimal financial contracts with venture capitalists. Working paper, University of Toulouse.

Chan, Y., Siegel, D., Thakor, A., 1990. Learning, corporate control and performance requirements in venture capital contracts. International Economic Review 31, 365--381.

Cornelli, F., Yosha, O., 1997. Staged financing and the role of convertible debt. CEPR Discussion Paper 1735.

D'Aspremont, C., Gabczewicz, J., Thisse, J., 1979. On Hotelling's stability in competition. Econometrica 17, 1145--1151.

Diamond, D., 1994. Financial intermediation and delegated monitoring. Review of Economic Studies 51, 393--414.

Fenn, G., Liang, N., 1998. New resources and new ideas: private equity for small businesses. Journal of Banking and Finance 22, 1077--1084.

Gompers, P., 1995. Optimal investment, monitoring, and the staging of venture capital. Journal of Finance 50, 1461--1489.

Gompers, P., 1998. Venture capital growing pains: should the market diet?. Journal of Banking and Finance 22, 1089--1104.

Gompers, P., Lerner, J., 1996. The use of covenants: an empirical analysis of venture partnership agreements. Journal of Law and Economics 39, 463--498.

Gompers, P., Lerner, J., 1999. What drives venture capital fundraising?. NBER Working Paper # 6906.

Hansen, E., 1991. Venture capital finance with temporary asymmetric learning. London School of Economics Financial Market Group Discussion Paper 112.

Hellmann, T., 1998. The allocation of control rights in venture capital contracts. Rand Journal of Economics 29, 57--76.

Holmström, B., 1982. Moral hazard in teams. Bell Journal of Economics 13, 324--340.

Marx, L., 1998. Efficient venture capital financing combining debt and equity. Review of Economic Design 3, 371--387.

Milgrom, P., Roberts, P., 1982. Limit pricing and entry under incomplete information: an equilibrium analysis. Econometrica 50, 443--460.

Organisation for Economic Co-operation and Development, 1996. Venture capital and innovation. Organisation for Economic Co-operation and Development, Paris.

Pfirrmann, O., Wupperfeld, U., Lerner, J., 1997. Venture capital and new technology based firms: an US-German comparison. Heidelberg, Physica.

Ramakrishnan, S., Thakor, A., 1984. Information reliability and a theory of financial intermediation. Review of Economic Studies 51, 415--432.

Reid, G., 1996. Fast growing small entrepreneurial firms and their venture capital backers: an applied principal-agent analysis. Small Business Economics 8, 235--248.

Repullo, R., Suarez, J., 1998. Venture capital finance: a security design approach. Working paper, University of Madrid.

Salop, S., 1979. Monopolistic competition with outside goods. Bell Journal of Economics 10, 141--156.

Sahlman, W., 1990. The structure and governance of venture capital organizations. Journal of Financial Economics 27, 473--521.

Snyder, C., 1996. Negotiation and renegotiation of optimal financial contracts under the threat of predation. Journal of Industrial Economics 44, 325--343.

Sutton, J., 1998. Technology and Market Structure. Cambridge, MIT Press.

Telser, L., 1966. Cutthroat competition and the long purse. Journal of Law and Economics 9, 259--277.

Tirole, J., 1988. The Theory of Industrial Organization. Cambridge, MIT Press.

Trester, J., 1998. Venture capital contracting under asymmetric information. Journal of Banking and Finance 22, 675--699.

Appendix 1: Price Competition, Market Shares and Profits

The profit function of firm *i* equals:

$$\Pi_i(p_i, p_y, c_i, T, F) = (p_i - c_i)(\frac{p_y - p_i + T/2}{T}) - F \quad . \qquad \text{(A1)}$$

The first-order condition and the price reaction function are given by:

$$\frac{\partial \Pi_i(p_i, p_y, c_i, T, F)}{\partial p_i} = \frac{p_y + T/2 - 2p_i}{T} + \frac{c_i}{T} \overset{!}{=} 0, \text{ and } \ p_i = \tfrac{1}{2}\cdot(p_y + T/2 + c_i). \qquad \text{(A2a)}$$

The price reaction function of the young firm y is analogously given by

$$p_y = \tfrac{1}{2}\cdot(p_i + T/2 + c_y). \qquad \text{(A2b)}$$

The optimal prices are found at the intersection of the reaction functions:

$$p_i^*(c_i, c_y, T) = \tfrac{1}{2}T + \tfrac{1}{3}(2c_i + c_y), \text{ and} \qquad \text{(A3a)}$$

$$p_y^*(c_i, c_y, T) = \tfrac{1}{2}T + \tfrac{1}{3}(2c_y + c_i). \qquad \text{(A3b)}$$

Resubstitution into the firm-specific demand function gives us the respective market shares:

$$D_i(c_i, c_y, T) = \frac{p_y + T/2 - p_i}{T} = \frac{\tfrac{1}{2}T + \tfrac{1}{3}c_y - \tfrac{1}{3}c_i}{T} = \frac{1}{2} + \frac{1}{3T}(c_y - c_i) \quad , \qquad \text{(A4a)}$$

$$D_y(c_i, c_y, T) = 1 - D_i(c_i, c_y, T). \qquad \text{(A4b)}$$

Finally, the gross profits in reduced form are derived as:

$$\Pi_i(c_i, c_y, T) = (p_i - c_i)D_i(c_i, c_y, T) - F. \qquad \text{(A5)}$$

If both firms produce with symmetric costs $c_y = c_i$, optimal product prices are given by

$$p^*(c_i, c_y, T) = \tfrac{1}{2}T + c; \qquad \text{(A6)}$$

market shares equal one half,

$$D_i(c_i, c_y, T) = D_y(c_i, c_y, T) = 1/2; \qquad \text{(A7)}$$

and profits are given by,

$$\Pi_i(c_i, c_y, T) = (\tfrac{1}{2}T + \tfrac{1}{3}(2c_i + c_y) - c_i)\cdot\tfrac{1}{2} - F = \Pi_y(c_i, c_y, T) = \tfrac{1}{4}T - F. \qquad \text{(A8)}$$

Therefore, the profits in the symmetric case depend only on the size of the consumers' preferences, *T*.

If the two firms produce with different marginal costs, we denote this cost difference by Δc. Then, the profits of the firm who produces with lower marginal costs (= firm *y*) are equal to:

$$\Pi_y^A(c_i,c_y,T)=(p_y-c_y)D_y-F=[\tfrac{1}{2}T-\tfrac{1}{3}(-\Delta c)][\tfrac{1}{2}+(\Delta c/3)/T]-F$$
$$= T/4+\Delta c/3+(\Delta c/3)^2/T-F. \tag{A9}$$

Thus, the profits for the firm with the cost-advantage are positively influenced by Δc.

The profits for the firm who has a cost-disadvantage (presumably firm *i*) are equal to:

$$\Pi_i^D(c_i,c_y,T)=(p_i-c_i)D_i-F=[\tfrac{1}{2}T-\tfrac{1}{3}(\Delta c)][\tfrac{1}{2}-(\Delta c/3)/T]-F$$
$$= T/4-\Delta c/3+(\Delta c/3)^2/T-F \tag{A10}$$

This profit level is negatively influenced by the cost-difference between the two firms.

Appendix 2: Learning Process

The updating of the beliefs is accomplished according to the Bayes-rule:

$$\alpha_2=\frac{\alpha_1(1-\theta_1)}{\alpha_1(1-\theta_1)+1-\alpha_1}=\frac{\alpha_1(1-\theta_1)}{1-\alpha_1\theta_1}. \tag{A11}$$

Rearranging terms shows that

$$1-\alpha_1\theta_1=\frac{\alpha_1}{\alpha_2}(1-\theta_1). \tag{A12}$$

Solving for (α_1/α_2) and taking the inverse ratio gives us:

$$\frac{\alpha_1}{\alpha_2}=\alpha_1+(1-\alpha_1)\frac{1}{(1-\theta_1)}. \tag{A13}$$

(A13) again shows that the initial beliefs, α_1, are larger than the updated beliefs, α_2.

Appendix 3: Minimum share for truthful investment

To prevent the entrepreneur from moral hazard under long-term contracting, the share of profits under truthful investment must be at least equal to the payoffs under deviation. Inserting the full expressions into (29c′), we obtain equation (31):

$$\alpha_1\theta_1S_1^{LT}\Pi_y+\alpha_1\theta_1S_{2Duo}^{LT}E(\Pi_{t=2})+(1-\alpha_1\theta_1)(\alpha_2\theta_2S_2^{LT}\Pi_y)\overset{!}{=}g\theta_1+\frac{\alpha_1}{\alpha_2}(\alpha_2\theta_2S_2^{LT}\Pi_y). \tag{31}$$

Rearranging terms leads to:

$$\alpha_1\theta_1S_1^{LT}\Pi_y=g\theta_1-\alpha_1\theta_1S_{2Duo}^{LT}E(\Pi_{t=2})+(\alpha_2\theta_2S_2^{LT}\Pi_y)\left[\frac{\alpha_1}{\alpha_2}-(1-\alpha_1\theta_1)\right]$$
$$=g\theta_1-\alpha_1\theta_1S_{2Duo}^{LT}E(\Pi_{t=2})+(\alpha_2\theta_2S_2^{LT}\Pi_y)\frac{\alpha_1}{\alpha_2}\left[1-\frac{(1-\alpha_1\theta_1)\alpha_2}{\alpha_1}\right]. \tag{A14}$$

Substituting α_2 by (A11) for the term in the brackets yields:

$$\alpha_1\theta_1 S_1^{LT}\Pi_y = g\theta_1 - \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + (\alpha_2\theta_2 S_2^{LT}\Pi_y)\frac{\alpha_1}{\alpha_2}\theta_1 . \qquad \text{(A15)}$$

Moreover, we multiply the entrepreneur's value under deviation from (29c)

$$V^D(\alpha_1) = g\theta_1 + \frac{\alpha_1}{\alpha_2}(\alpha_2\theta_2 S_2^{LT}\Pi_y), \qquad \text{(29c)}$$

by θ_1, and rearrange terms in order to obtain the following expression:

$$\frac{\alpha_1}{\alpha_2}\theta_1(\alpha_2\theta_2 S_2^{LT}\Pi_y) = \theta_1 V^d(\alpha_1) - g\theta_1^{\ 2} . \qquad \text{(A16)}$$

Inserting this expression into the above equation (A15), we get:

$$\begin{aligned}\alpha_1\theta_1 S_1^{LT}\Pi_y &= g\theta_1 - \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + \theta_1 V^d(\alpha_1) - g\theta_1^{\ 2} \\ &= (1-\theta_1)g\theta_1 - \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + \theta_1 V^d(\alpha_1).\end{aligned} \qquad \text{(A17)}$$

Next, we take the modified profits under deviation, $V^d = g\theta_1 + g\theta_2(\alpha_1/\alpha_2)$ from (29c′). We substitute for (α_1/α_2) by (A13) and obtain:

$$V^d(\alpha_1) = g\theta_1 + g\theta_2\left[\alpha_1 + \frac{(1-\alpha_1)}{(1-\theta_1)}\right]. \qquad \text{(A18)}$$

Inserting this expression for $V^d(\alpha_1)$ into equation (A17), we derive:

$$\alpha_1\theta_1 S_1^{LT}\Pi_y = (1-\theta_1)g\theta_1 - \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + \theta_1\left[g\theta_1 + g\theta_2\alpha_1 + g\theta_2\frac{(1-\alpha_1)}{(1-\theta_1)}\right]. \qquad \text{(A19)}$$

After rearranging and collecting terms, we obtain:

$$\alpha_1\theta_1 S_1^{LT}\Pi_y = g\theta_1 + \alpha_1\theta_1 g\theta_2 - \alpha_1\theta_1 S_{2Duo}^{LT} E(\Pi_{t=2}) + \theta_1 g\theta_2\frac{(1-\alpha_1)}{(1-\theta_1)} . \qquad \text{(A20)}$$

We divide the above equation by $\alpha_1\theta_1\Pi_y$ to solve for the minimum share S_1^{LT} of profits that the entrepreneur requires for truthful investment in the first period of the long-term contract:

$$S_1^{LT} = \frac{g}{\alpha_1\Pi_y} + \frac{g\theta_2}{\Pi_y} - S_{2Duo}^{LT}\frac{E(\Pi_{t=2})}{\Pi_y} + \frac{(1-\alpha_1)}{\alpha_1\Pi_y}\cdot\frac{g\theta_2}{(1-\theta_1)} . \qquad \text{(A21)}$$

Finally, we substitute (A13) into the last term of the equation above and obtain:

$$S_1^{LT} = \frac{g}{\alpha_1\Pi_y} + \frac{g\theta_2}{\Pi_y} - S_{2Duo}^{LT}\frac{E(\Pi_{t=2})}{\Pi_y} + \left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right)\frac{g\theta_2}{\alpha_1\Pi_y} . \qquad \text{(A22)}$$

Q.E.D.

Appendix 4: Project value under long-term contracting

To calculate the expected minimum project value that is required under long-term contracting, we insert the incentive compatible shares of the entrepreneur that we derived in Proposition 4 into the intertemporal participation constraint of the venture capital company:

$$[(1-S_1^{LT})\alpha_1\theta_1\Pi_y - g\theta_1] + \alpha_1\theta_1[(1-S_{2Duo}^{LT})E(\Pi_{t=2}) - g\theta_2^+] + (1-\alpha_1\theta_1)[(1-S_2^{LT})\alpha_2\theta_2\Pi_y - g\theta_2] \geq 0. \tag{30'}$$

As a first step, we insert the second period minimum share $S_2^{LT} = g/\alpha_2\Pi_y$ from (29b′) into the last term of (30′) and obtain after simplifying:

$$[(1-S_1^{LT})\alpha_1\theta_1\Pi_y - g\theta_1] + \alpha_1\theta_1[(1-S_{2Duo}^{LT})E(\Pi_{t=2}) - g\theta_2^+] + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2] \geq 0. \tag{A23}$$

Next, we substitute $S_{2Duo}^{LT} = g/(\Pi_y^A - \Pi_y)$ from (29a′) in the second term of (A23) and rearrange to get:

$$[(1-S_1^{LT})\alpha_1\theta_1\Pi_y - g\theta_1] + \alpha_1\theta_1\left[E(\Pi_{t=2}) - 2g\theta_2^+ - g\frac{\Pi_y}{\Pi_y^A - \Pi_y}\right] + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2] \geq 0. \tag{A24}$$

Finally, we have to insert the first period minimum share from (32)

$$S_1^{LT} = \frac{g}{\alpha_1\Pi_y} + \frac{g\theta_2}{\Pi_y} - S_{2Duo}^{LT}\frac{E(\Pi_{t=2})}{\Pi_y} + \left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right)\frac{g\theta_2}{\alpha_1\Pi_y}$$

into the first term of (A24). Before doing this, we substitute $S_{2Duo}^{LT} = g/(\Pi_y^A - \Pi_y)$ in (32) as well and write $E(\Pi_{t=2}) = \theta_2^+(\Pi_y^A - \Pi_y) + \Pi_y$ in full length in order to get (32′)

$$S_1^{LT} = \frac{g}{\alpha_1\Pi_y} + \frac{g\theta_2}{\Pi_y} - \frac{g}{(\Pi_y^A - \Pi_y)}\frac{[\theta_2^+(\Pi_y^A - \Pi_y) + \Pi_y]}{\Pi_y} + \left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right)\frac{g\theta_2}{\alpha_1\Pi_y}.$$

Inserting this into (A24) we obtain:

$$\left[\alpha_1\theta_1\Pi_y - g\theta_1 - g\theta_1 - g\theta_2\alpha_1\theta_1 + \frac{g}{(\Pi_y^A - \Pi_y)}[\theta_2^+(\Pi_y^A - \Pi_y) + \Pi_y]\alpha_1\theta_1 - \left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right)g\theta_2\theta_1\right]$$
$$+\alpha_1\theta_1\left[E(\Pi_{t=2}) - 2g\theta_2^+ - g\frac{\Pi_y}{\Pi_y^A - \Pi_y}\right] + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2] \geq 0. \tag{A25}$$

If $\theta_1 = \theta_2^{(+)} = \theta^{\max}$, simplifying yields straightforwardly equation (33):

$$[\alpha_1\theta_1\Pi_y - 2g\theta_1] - \left(\frac{\alpha_1}{\alpha_2} - \alpha_1\right)g\theta_1\theta_2 + \alpha_1\theta_1[E(\Pi_{t=2}) - 2g\theta_2^+] + (1-\alpha_1\theta_1)[\alpha_2\theta_2\Pi_y - 2g\theta_2] \geq 0 \tag{33}$$

Q.E.D.