Catching-up or leapfrogging? The effects of competition on innovation and growth

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Abstract

The main objective of this paper is to analyze the links between product market competition, innovation and growth. We capture the idea that firms innovate in order to try to escape -albeit temporarily - from the pressure of competition exerted on them by their rivals. There are two ways in which competitive pressure can be thought of as a driving force to innovate. In leveled industries where all the firms have access to the same technological knowledge, the greater is the intensity of competition between the neck by neck firms the lower will be their current profits. Thus, as the competitive pressure increases, these firms will devote a higher R&D effort to obtain a leadership and escape from the unprofitable state. In unleveled industries, where one firm has obtained a technological lead, the greater is the intensity of competition, the lower will be the current profit of the laggard firm. This should increase the incentive of this firm to eliminate its disadvantage by catching-up or leapfrogging the current leader. We assume that if a laggard firm succeeds in innovating, it will either leapfrog the leader with some probability or catch-up its technology with the complementary probability. The dynamics of industry are thus more complex than in pure leapfrogging models. By using a quadratic R&D cost function, we investigate how innovation and growth are affected in the stationary state by the intensity of competition and by the probability of leapfrogging.

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1 Introduction

The link between competition and innovation has been at the heart of much of the economic theory of innovation, going back to the classic works of Schumpeter (1942), Arrow (1962) and Dasgupta and Stiglitz (1980). There has been a resurgence of interest in this question following the recent developments in endogenous growth theory. The particular aspect in which we are interested in this paper is the link between product market competition, innovation and growth.

The consensus emerging from much of the recent literature on endogenous growth is that increased product market competition is bad for growth - see for example Aghion and Howitt (1992, 1998), Grossman and Helpman (1991). However these models are based on the Schumpeterian idea of growth taking place through a process of creative destruction. This is captured in these models by assuming that progress takes the form of what we will call strong leapfrogging. This contains two ideas:

(L1) whichever firm successfully innovates becomes the technological leader;

(L2) the previous leader is driven out of the market.

The corollary of these two assumptions is that each industry is characterized by persistent monopoly - with the identity of the monopolist continually changing.

In the context of such models the only meaning one can give to increased product market competition is that there is some exogenous change in, say, demand conditions that lowers the level of profits that the monopolist can earn. Thus increased product market competition takes the form of what we will call a more competitive environment. It is clear that this will reduce the returns to R&D and hence both the pace of innovation and the rate of growth. So this form of competition is indeed bad for growth.

The problem with this line of argument is that it fails to capture the idea that firms innovate in order to try to escape - albeit temporarily - from the pressures of competition being exerted on them by their rivals . The notion of competitive pressure being referred to here relates to the intensity of competition between firms within any given market environment (demand conditions, etc.) rather than the competitiveness of the environment within which they are competing.

There are two ways in which this type of competitive pressure can be thought of as driving firms to innovate.

(a) In situations where firms are very equal - what is known as neck-and-neck competition - then the greater is the intensity of competition between firms the lower will be their profits. If, by innovating, a firm could obtain some advantage - a better product or a better technology than its rivals - then the greater is the intensity of competition between firms the more the firm will be able to exploit this advantage to increase its profits. Intuitively, for both these reasons, an increase in the intensity of competition should increase the incentives of firms to escape from neck-and-neck situations.

(b) In situations where one firm has obtained an advantage, then the greater is the intensity of competition between firms the lower will be the profits of the firms that are disadvantaged. This should increase the incentive of these firms to eliminate their disadvantage by catching up or overtaking the current leaders.

Implicit in this discussion is the idea that an increase in the intensity of competition is being defined by the two properties:

(i) when firms are neck-and- neck the profits of all firms fall;

(ii) when firms are in a leader-follower situation then:

- the profits of the leader increase;
- the profits of the follower fall;
- industry profits increase.

Now it is clear that it is impossible to formally investigate the effects on innovation of this type of increase in competition within the usual Schumpeterian framework. Since that is characterized by permanent monopoly the intensity of competition is constant. In order to investigate the effects on innovation of this type of increase in competition, one needs a model in which, in equilibrium, many firms can co-exist in the same industry. In a recent paper Aghion, Harris and Vickers (1997) use a catch up or step-by-step innovation to explore this issue. They also, briefly, examine what we will call a weak-leapfrogging that is characterized by L1 but not L2 - creation without the destruction.

In this paper we propose a much more general class of models within which to investigate the effects of an increase in the intensity of competition on innovation. This will incorporate both catch-up and weak leapfrogging as special cases.

As in Aghion, Harris, Vickers our model has just two firms and we assume that the maximum gap between the firms is always 1. To explain this latter assumption and the way our model embraces both leapfrog and catch up, consider the case where the industry is in a leader-follower situation, with one firm on the current technological frontier and the other firm one step behind. If the leader succeeds in innovating it will move one step ahead to a new technological frontier. However we assume that patents protect only latest technological frontier, so the follower is also able to move up one step onto the technological frontier, leaving the leader's gap as 1. We assume that if the follower innovates then, with probability μ, $(0 \le \mu \le 1)$, it will actually understand the ideas at the current frontier. This progress will take it onto a new frontier and so it will leapfrog the current leader. However, with probability $1 - \mu$, the follower will not master the ideas at the current frontier, and so its 1 step of progress will just take it onto the existing frontier alongside the follower, and so into a situation of neck-and-neck competition. Thus our model allows both weak leapfrogging and catch up, and reduces to the pure forms of these in the cases where respectively $\mu = 1$ and $\mu = 0$.

An important feature of the model is that the dynamics of movement from the leader-follower situation are more complex than in pure leapfrogging models, since the industry can move from leader-follower to neck-and-neck or to the next leader follower situation with the respective roles of the leader and follower being reversed.

We use this general framework to investigate:

(i) how innovation and growth are affected by an increase in the intensity of competition as defined above;

(ii) how innovation and growth are affected by μ.

However the model also enables us to address another important issue. Since the primary factor driving firms to innovate is to try to escape intense competitive pressure, it is interesting to ask whether they actually succeed. In other words does innovation result in industries which are frequently in the leader-follower situation or frequently in the neck-and-neck situation?

We can also ask how the frequency with which the industry is in the neck-and-neck situation is affected by firms having to work in a more competitive environment or by an increase in the intensity of competition. Put somewhat differently - is competition good for competition?

This latter question is particularly pertinent since it helps us understand whether positions of dominance are indeed the natural outcome of the innovative behavior of firms trying to escape intense competitive pressure - as Bill Gates would have us believe - or whether they reflect some lack of competitive pressure in the market.

The paper is organized as follows. The basic set-up is described in section II. The determination of the value functions and the rate of growth at the stationary state are derived in section III. A quadratic R&D cost function is specified in section III and the main results are derived in this case. The concluding remarks appear in section IV.

2 The basic set-up

2.1 The consumption side

There exists a continuum of final goods in the economy. Each good is produced in a specific industry indexed by $i \in [0;1]$: Consumption of output from industry i at time t is denoted by $c_i(t)$. Time is continuous. We suppose that the representative consumer is infinitely lived and has a separable intertemporal utility function given by :

$$U = \int_0^{\infty} \ln C(t) e^{-\rho t} dt \tag{1}$$

where :

$C(t)$ represents an index of overall consumption at date t and is defined as a Cobb-Douglas instantaneous utility function on the continuum of final goods :

$$\ln C(t) = \int_0^1 \ln c_i(t) di \tag{2}$$

$\rho > 0$ is the rate of time preference.

We suppose that financial markets are perfect and characterized by an instantaneous interest rate r_t: Since the intertemporal rate of substitution is constant and equal to unity in (1), the utility maximization of the consumer under an intertemporal budget constraint leads to the standard Ramsey equation:

$$\frac{\dot{E}(t)}{E(t)} = \rho - r_t \tag{3}$$

where:

$E(t)$ is the instantaneous global expenditure at date t : $E(t) \equiv P(t)C(t)$;

$P(t)$ is the general price index at date t defined as: $\ln P(t) = \int_0^1 \ln p_i(t)di$; $p_i(t)$ is the price in industry i at date t.

We choose the normalization rule:

$$E(t) \equiv P(t)C(t) = 1 \quad \forall t \in [0; \infty[\tag{4}$$

According to (3) and (4), the interest rate r_t is thus equal to the discounting rate ρ:

$$r_t = \rho \quad \forall t \in [0; \infty[\tag{5}$$

We will denote r the constant rate of interest.

According to (2), each final good has the same weight in the instantaneous utility function. Thus the normalization rule leads also to a uniform spending in each industry:

$$p_i(t)c_i(t) = 1 \quad \forall t \in [0; \infty[; \forall i \in [0; 1] \tag{6}$$

2.2 The productive side

We suppose that in each industry there are two firms which are involved both in production and R&D. In each industry, the rival firms can be at different technology levels. A firm at technology level k can produce γ^k units of output per unit of labour employed, where the exogenous parameter γ ($\gamma > 1$) represents the labour productivity when $k = 1$. Thus, the cost per unit of output of a firm in technology k is $w\gamma^{-k}$ where w is the wage rate. As our model is written in a partial equilibrium framework, we treat w as an exogenous variable and we choose $w = 1$. The total production $q_i(t)$ of industry i ($i \in [0; 1]$) at date t is equal to the consumption of the corresponding good: $q_i(t) = c_i(t)$. The level of production $q_i(t)$ depends on the technology levels of both firms in industry i and on the intensity of competition between the two firms. We describe now how the technology levels evolve in time.

At each date t, the current state of an industry can be described by a pair of technology levels $(k; k - n)$, where k is the level of the technological leader and $k - n$ is the technological level of the follower. Thus, n is the technological gap between the two firms. This technological gap will be treated as a state variable in what follows. According to (6); a constant proportion of income is spent on the product supplied by each industry. Thus, whatever the nature of product market competition is, firms' equilibrium profit flows derived from competition in the product market depend only upon the gap n and not upon the level k: We denote by π_n the equilibrium profit flow of a firm which is, from a technological aspect, n steps ahead of it's rival (or $-n$ steps behind it if n is negative).

As in A.H.V.(1997), we suppose that the technological gap between firms cannot exceed one step. This assumption, made only in order to simplify the model and to allow analytical solutions, can be justified in three ways. First, it may be too costly (in terms of R&D effort) to a firm to get more than one step ahead of it's rival. Second, imitation by the follower might become easy once the leader is more than one step ahead. Third, patents protect much more the latest technological knowledge than the previous ones. The consequence of this assumption is that, at any time, each of the two firms in an industry can be in one and only one of the following three states $n \in \{-1; 0; 1\}$. A firm which is in the state $n = -1$ at some date is the technological follower while it's rival in the same industry, which is in the state $n = 1$; is the technological leader. The corresponding unleveled industry is characterized as being of the follower-leader type. When a firm is in the state $n = 0$; both firms are at the same technological level and the corresponding leveled industry is of the head-to-head type. An industry can be, at any time, in just one of these two types. But, as time elapses, the type of an industry changes permanently. We suppose indeed that there exists an innovation process that leads to an increase of the labour productivity by a parameter φ ($\varphi > 1$) (equivalent to a reduction of unit cost). We suppose that, by employing $\gamma(p)$ units of labour in R&D, a firm moves one step ahead with Poisson hazard rate p: The R&D cost function $\gamma(p)$ is supposed to be increasing, continuous and convex. We also assume that $\gamma(0) = 0$.

We are now in position to describe how an industry evolves in time. Consider a period $[t; t + dt]$: Two cases must be considered according to the type of the industry at the beginning of the period.

1. If an industry starts at date t in the head-to-head state, neither firm has a gap over the other, and both are at the existing cutting edge of technology. Three situations can occur during the period $[t; t + dt]$. First, if both firms innovate during the period, neither will create a gap over the other and the

industry will end the period as it began it in the head-to-head state. Second, the same is true if neither innovates. Third, if only one …rm innovates, it will open a unit gap over its rival and the industry ends the period in the leader-follower type.

2. If an industry starts at date t in the leader-follower type, then one …rm (the leader) has a unit gap over its rival (the follower) and is at the cutting edge of technology. Because the leader is at the cutting edge, if it succeeds in innovating, it can only lower its cost by the speci…ed amount: For the follower, the situation is slightly di¤erent. We assume that with probability μ, $0 \leq \mu \leq 1$, the follower, if it succeeds in innovating, will be able to acquire an understanding of the technology at the cutting edge, and so will be able to achieve exactly the same technology as the leader would acquire if it innovated. Thus, when its innovation succeeds, the follower can leapfrog the preceding leader with probability μ. However, with probability $1 - \mu$, the follower will not acquire the understanding of the technology at the cutting edge, and so, if it succeeds in innovating, it will acquire only the technology currently used by the leader. In this case, there is only a catching-up of the current leader's technology by the follower. The model captures thus the two polar cases corresponding respectively to weak leapfrogging ($\mu = 1$) and to step-by-step innovation ($\mu = 0$). It captures also all the intermediate cases where leapfrogging occurs with probability μ and catching-up occurs with probability $1 - \mu$: Moreover, note that contrary to the Schumpeterian leapfrogging models where monopoly is typically a universal market structure, our assumptions allow the possibility of a richer market structure. When its innovation succeeds, a technological laggard can advance either by leapfrogging the previous leader or by catching-up it. The possibilities for industry evolution, when starting from a leader-follower type, are thus more complex.

Suppose the follower does not succeed in innovating. If the leader innovates, it will open up a gap of 2, but according to our assumption of a maximal gap of 1, the follower gets access to the previous leader's technology and the industry ends the period in the follower-leader position. If the leader does not succeed in innovating, the industry ends the period as it began it, namely in a follower-leader position.

Suppose the follower succeeds in innovating from the existing cutting edge. If the leader also innovates, the gap between them will be reduced to zero during the period and the industry will end the period in the head-to-head position. However, if the leader fails to innovate, then the previous follower will have become the new leader and the industry will end the period in the leader-follower type (but with the identity of …rms being reversed).

Suppose the follower succeeds in innovating, but not from the cutting edge. Then if the leader also innovates, it will maintain its gap of 1, and the industry ends the period as it began it, namely in the leader-follower type. However, if the leader fails to innovate, then its technological lead will be eliminated, and the industry will end the period in the head-to-head position.

Innovative advance and hence economic growth occur at a rate determined by R&D e¤orts. By allocating $°(p_0)$ units of labour to the R&D activity, a …rm at the technological frontier which is level with its rival, moves one step ahead with Poisson hazard rate p_0: Similarly, by allocating $°(p_{¡1})$ units of labour in R&D, a follower succeeds in innovating with Poisson hazard rate $p_{¡1}$. Conditional to its success, it leapfrogs the leader with Poisson hazard rate $p_{¡1}µ$ and it catches-up with Poisson hazard rate $p_{¡1}(1 ¡ µ)$: Finally, by employing $°(p_1)$ units of labour in R&D, a leader succeeds in innovating with Poisson hazard rate $p_{1:}$

We can now determine the system of equations whose solutions are the R&D e¤orts $p_{¡1}$; p_0, p_1.

3 The stationary state equilibrium

3.1 The value functions of the markovian game at the steady state

We focus on the determination of equilibrium in Markov strategies at the stationary state of the economy. A Markov strategy speci…es a …rm's choice of its R&D e¤ort as a function of the current gap[1] in the corresponding industry, leading thus to the symmetric equilibrium values of $p_{¡1}$; p_0 and p_1 in terms of the parameters $¼_{¡1}; ¼_0; ¼_1; r; µ$; for a given R&D cost function $°(p)$:

Consider the value functions for a …rm whose rival's strategy is given by $(\overline{p}_{¡1}; \overline{p}_0; \overline{p}_1)$: Let V_n denote the value function of a …rm stating from state n:

[1] Note that a …rm which is ahead (n = 1) has no incentive do undertake R&D due to our assumption of a maximal gap of 1. Thus, we will …nd that at equilibrium, $p_1 = 0$:

This value gives the firm's expected discounted payoff in the game starting from the state in which it is n steps ahead (or behind if $n < 0$) of its rival ($n = -1; 0; 1$):

In order to obtain the Bellman equations satisfied by the value functions, consider for instance the situation of a technological follower which is in the state $n = -1$ at the beginning date of a period $[t; t + dt]$. By spending $\gamma(p_{-1})$ in R&D, it obtains an innovation during this period with Poisson hazard rate $p_{-1}dt$. As it has been explained, we suppose that this innovation may lead the follower either to the cutting edge of the technology, leapfrogging thus the leader with probability $p_{-1}\theta dt$ or to the technology currently used by the leader, catching-up it with the probability $p_{-1}(1\text{-}\theta)dt)$: With the complementary probability $1 - p_{-1}dt$; the follower does not obtain any innovation during the period $[t; t + dt]$:

During the same period, the leader's innovation occurs with the probability $\bar{p}_1dt$. Figure 1 describes the corresponding game during the period $[t; t + dt]$: It also gives the discounted payoffs (net of R&D costs) of the follower at the end of this period.

The value function V_{-1} of a follower satisfies thus the following Bellman equation :

$$\begin{aligned} V_{-1} = \underset{p_{-1} \geq 0}{Max} \{ (\pi_{-1} - \gamma(p_{-1}))dt + e^{-rdt}[V_1 p_{-1}\theta dt + V_0 p_{-1}(1 - \theta)dt \\ + V_{-1}\bar{p}_1 dt + V_{-1} - V_{-1}(p_{-1} + \bar{p}_1)dt] \} \quad (7) \end{aligned}$$

By using the first order approximation $e^{-\rho dt} \simeq 1 - \rho dt$; and by keeping only the first order terms in dt; one obtains the equivalent Bellman equation:

$$rV_{-1} = \underset{p_{-1} \geq 0}{Max} \{ \pi_{-1} - \gamma(p_{-1}) + p_{-1}\theta V_1 + p_{-1}(1 - \theta)V_0 - p_{-1}V_{-1} \} \quad (8)$$

The corresponding first order condition leads to (if p_0 is strictly positive):

$$\gamma'(p_{-1}) = \theta(V_1 - V_0) + (V_0 - V_{-1}) \quad (9)$$

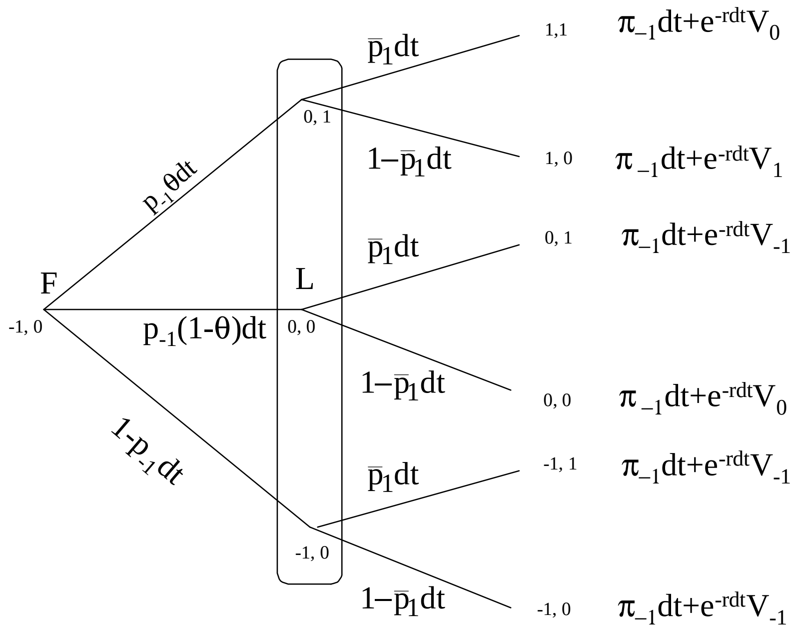

Figure 1: Determination of the follower's value function

The interpretation of condition (9) is straightforward. The LHS is the R&D marginal cost of the follower. The RHS gives the expected marginal revenue which is equal to $\theta(V_1 - V_{-1}) + (1 - \theta)(V_0 - V_{-1})$.

In the same way, one obtains the Bellman equations giving the the values of a firm starting the period $[t; t + dt]$ at the respective states n=1 and n=0, and the corresponding first order conditions:

$$rV_1 = \max_{p_1 \geq 0} \left\{ \pi_1 - \gamma(p_1) - \bar{p}_{-1}(V_1 - V_0) - \bar{p}_{-1}\theta(V_0 - V_{-1}) \right\} \tag{10}$$

$$\gamma'(p_1) \geq 0 \text{ and } p_1\gamma'(p_1) = 0 \tag{11}$$

$$rV_0 = \max_{p_0 \geq 0} \left\{ \pi_0 - \gamma(p_0) - p_0(V_0 - V_1) - \bar{p}_0(V_0 - V_{-1}) \right\} \tag{12}$$

$$\gamma'(p_0) = V_1 - V_0 \qquad (13)$$

The symmetric-equilibrium conditions $\bar{p}_{-1} = p_{-1}$; $\bar{p}_0 = p_0$, $\bar{p}_1 = p_1$ are added to the preceding system of six equations ((8)-(13)). The unknowns of this system as the three value functions $V_{-1}; V_0; V_1$ and the three intensity R&D efforts $p_{-1}; p_0; p_1$: They depend of the following parameters:

r = the interest rate,

θ = the probability of leapfrogging by the follower,

π_{-1} = the current profit of a technological follower,

π_0 = the current profit of a neck by neck technological firm,

π_1 = the current profit of a technological leader,

$\gamma(p)$ = the R&D cost (in units of labor) of a firm moving one technological step ahead with Poisson hazard rate p:

Note that, according to the first order condition (11), if $\gamma'(p_1) \neq 0$, then, $p_1 = 0$: This results from the assumption of a maximal gap of 1.

3.2 The determination of industry structures and rate of growth

The steady-state distribution of industry structures between leveled and unleveled industries is endogenous. Let denote α the proportion of industries that are of the head-to-head type in the steady state. In order to determine the value of α, consider a period $[t; t + dt]$: During this time interval, two types of evolutions do occur.

In industries which are of the head-to-head type (proportion α), successful innovation made by only one of the two neck by neck firms leads to an evolution towards industries of the leader-follower type. This occurs with the probability $2p_0dt(1 - p_0dt) \simeq 2p_0dt$:

In industries which are of the leader-follower type (proportion $1 - \alpha$), successful innovations made either by both the leader and a follower who reaches the cutting edge of technology or by only the follower who just happen to catch-up the leader's technology leads to an evolution towards industries of the head-to-head type. This occurs with the probability $(p_{-1}\theta dt p_1 dt) + (p_{-1}(1 - \theta)dt(1 - p_1dt)) \simeq p_{-1}(1 - \theta)dt$:

Since the distribution of industries remains stationary over time in the steady state, we must have:

$$2p_0\alpha dt = p_{-1}(1-\theta)(1-\alpha)dt. \tag{14}$$

This last equation leads to the value of the proportion of industries that are of the head-to-head type in the steady state. Note that his value depends directly and indirectly on the parameter θ since the values of p_0 and p_{-1} depend themselves on θ.

$$\alpha(\theta) = \frac{(1-\theta)p_{-1}(\theta)}{2p_0(\theta) + (1-\theta)p_{-1}(\theta)} \tag{15}$$

We thus have the following lemma:

Lemma 1 In the steady state of the economy, the proportion of industries that are head-to-head is given by
$\alpha(\theta) = \frac{(1-\theta)p_{-1}(\theta)}{2p_0(\theta)+(1-\theta)p_{-1}(\theta)}$, where θ is the probability to leapfrog the leader, conditional to the success of innovation by the follower, $p_0(\theta)$ is the R&D effort by a neck-by neck firm and $p_{-1}(\theta)$ is the R&D effort by a follower. For a given θ in $(0,1)$ the proportion of industries that are head to head in the steady state is an increasing function of $p_{-1}(\theta)$ and a decreasing function of $p_0(\theta)$.

Note that for $\theta = 0$, we obtain the same result as in A.H.V. (1997). For $\theta = 1$, we have $\alpha(1) = 0$, since, as we will check later $p_0(1) \neq 0$. This means that in a weak leapfrogging situation, where a successful innovation by the follower gives it a leadership position, there are no industries that are of the head-to-head type.

Let us now determine the rate of growth of the economy in the steady state. Consider again a period $[t, t+dt]$. The growth rate g is defined by

$$g = \frac{d}{dt}\ln Q(t) = \frac{d}{dt}\ln C(t). \tag{16}$$

The rate of growth of an industry in a complete cycle is given by $\ln\varphi$, which is the amount by which is increased the log of it's output. A complete cycle occurs in two ways.

Firstly, by the evolution from a head-to head type towards the next head-to head type, where each firm has access to the new cutting edge of the technology. The complete cycle can be decomposed in this case into a two-stage cycle. The first stage corresponds to an evolution from a head-to head type to a leader follower type. Let denote $\ln \varphi_1$ the rate of growth of an industry in this first stage. The probability that an industry of the head-to head type (proportion α) moves to an industry of the leader-follower type during the period $[t; t + dt]$ (first stage of the complete cycle) is given by $2p_0dt(1 - p_0dt) \simeq 2p_0dt$. The second stage corresponds to an evolution from a leader follower type to a head-to head type. Let denote $\ln \varphi_2$ the rate of growth of an industry in this second stage. The probability that an industry of the leader-follower type (proportion $1 - \alpha$) moves to an industry of the head-to head type during the same period (second stage of the complete cycle) is given by $p_{-1}(1 - \theta)dt(1 - p_1dt) \simeq p_{-1}(1 - \theta)dt$. Of course, $\ln \varphi = \ln \varphi_1 + \ln \varphi_2$, since the industry output increases by a factor of φ after a complete cycle.

Secondly, a complete cycle occurs also during the evolution from a leader-follower type to the next leader-follower type, where the follower succeeds in leapfrogging the leader and the previous leader does not succeed in innovating. This evolution, which reverses the identity roles of the leader and follower, gives rise to a rate of growth of φ since one of the two firms (the follower) has increased its technological level by 2 steps. The probability that an industry of the leader-follower type (proportion $1 - \alpha$) moves to an industry of the next follower-leader type during the same period (complete cycle) is given by $p_{-1}\theta dt(1 - p_1dt) \simeq p_{-1}\theta dt$.

The average growth rate of final output is thus given by:

$$gdt = 2\alpha p_0 dt \ln \varphi_1 + p_{-1}(1 - \theta)(1 - \alpha)dt \ln \varphi_2 + p_{-1}\theta(1 - \alpha)dt \ln \varphi \qquad (17)$$

Note that g depends again directly and indirectly (via the variables α; p_0 and p_{-1}) on the parameter θ. By using the relationship $\ln \varphi = \ln \varphi_1 + \ln \varphi_2$; we obtain finally the following expression of the rate of growth of the economy:

$$g(\theta) = (2\alpha(\theta)p_0(\theta) + \theta p_{-1}(\theta)(1 - \alpha(\theta))) \ln \varphi \qquad (18)$$

By substituting the value of $\alpha(\theta)$ given in the previous lemma, one obtains:

$$g(\theta) = \left(\frac{2p_0(\theta)p_{-1}(\theta)}{2p_0(\theta) + (1 - \theta)p_{-1}(\theta)}\right) \ln \varphi \qquad (19)$$

We have thus proved the following lemma:

Lemma 2 For any $\mu \in [0;1]$ the growth rate at the the steady state of the economy $g(\mu)$ is given by (19). It is an increasing function of both the R&D effort $p_{-1}(\mu)$ of a follower in an unleveled industry and the R&D effort $p_0(\mu)$ of a neck-by-neck firm in a leveled industry.

Note that for $\mu = 0$; which corresponds to the step by step technological progress situation where a follower must catch-up the leader before becoming a leader itself, we obtain the same expression for the rate of growth as in A.H.V.(1997) : $g(0) = 2®(0)p_0(0)\ln ' = \frac{2p_{-1}(0)}{2p_0(0)+p_{-1}(0)} \ln '$: So, $g(0)$ is an increasing function of $p_{-1}(0)$ and a decreasing function of $p_0(0)$: Intuitively, one expects that $p_{-1}(0)$ decreases and $p_0(0)$ increases with the degree of competititive pressure, so that the effect of competition on $g(0)$ is a-priori ambiguous. For $\mu = 1$; which corresponds to the weak leapfrogging situation, where a successful follower obtains a technological leadership, we have $®(1) = 0$ and thus $g(1) = p_{-1}(1) \ln '$: In this case, the rate of growth is directly proportional to the follower effort in R&D. Intuitively, one expects that $p_{-1}(1)$ is an increasing function of the degree of competitive pressure.We return to these two points later.

We have now to compute the solution of the non linear system $(8 - 13)$: [2]

In what follows, we solve this system for a quadratic specification of the R&D cost function $°(p)$; having in mind the two following questions:

1. How the rate of growth of the economy is affected by the parameter μ which defines the degree of leapfrogging ?

2. How the innovative process and the growth of the economy are affected by the degree of competitive pressure between the two firms, given that rivalry occurs both on the product market and on the innovation side?

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[2] Note that a solution of this system exists if the R&D cost function is continuous and convex. To get a sketch of the proof of this existence, consider the vector $X = (p_{-1}; p_0; p_1; V_{-1}; V_0; V_1)$ and write the system $(8 - 13)$ as $F(X) = 0$, where F is a continuous and convex function from R^{6+} to R^6: Choose a convex compact set $B \subset R^{6+}$ sufficiently large to insure that F is defined in B and have values in B. Now, consider the function $G(X) = F(X) + X$: By the Brouwer fixed point theorem, there exists a value of X such that $G(X) = X$. Such a value of X is a solution of the system $(8 - 13)$.

4 The solution for a quadratic R&D cost function

Suppose that the R&D cost function is given by: $\varphi(p) = \frac{1}{2}dp^2$. After tedious but straightforward substitutions, the system $(8-13)$ leads to a system of two equations having the variables $p_0(\theta)$ and $p_{-1}(\theta)$ as solutions:

$$2rp_0 + (1+2\theta)(p_0)^2 + 2\theta(p_{-1})^2 - 2\theta^2 p_0 p_{-1} - 2a/d = 0 \tag{20}$$

$$2r(p_{-1} - \theta p_0) - (1+2\theta)(p_0)^2 + (p_{-1})^2 + 2p_0 p_{-1} - 2b/d = 0 \tag{21}$$

In order to simplify we consider the case $r = 0$. Let us introduce the parameters $a = \pi_1 - \pi_0$ and $b = \pi_0 - \pi_{-1}$ which play an important role in what follows. These parameters measure the profit flow increments associated, respectively, with gaining the lead and catching up. They are directly associated to the short term determinants of the industry evolution (Budd, Harris and Vickers (1993)). The values of these parameters depend on φ and on the nature of product market competition (for instance Cournot vs. Bertrand competition). Any model of imperfect competition leads to the following relationships: $a > b \geq 0$. Moreover and more important, these profit flows increments are monotonic functions of the intensity of competition in the product market, whatever one defines this intensity : a is increasing, b is decreasing and $a + b$ is increasing with the intensity of competition[3]. By transforming the preceding system, one obtains:

$$p_0 = \frac{2(a+b)/d - (p_{-1})^2(1+2\theta)}{2p_{-1}(1-\theta^2)} \tag{22}$$

[3] The comparison between Cournot and Bertrand competition illustrates these results. With Cournot competition in the product market, one obtains $a^{Cournot} = (\frac{\varphi}{1+\varphi})^2 - \frac{1}{4}$ and $b^{Cournot} = \frac{1}{4} - \frac{1}{(1+\varphi)^2}$. With Bertrand competition, the corresponding values are $a^{Bertrand} = \frac{\varphi-1}{\varphi}$ and $b^{Bertrand} = 0$. One can verify immediately the following implications: $\varphi > 1 \Rightarrow a^{Cournot} < a^{Bertrand}$, $b^{Cournot} > b^{Bertrand}$, $(a+b)^{Cournot} < (a+b)^{Bertrand}$, $a^{Bertrand} > (a+b)^{Cournot} > a^{Cournot}$.

$$p_{-1} = \frac{(1 + 2\mu)^2 (p_0)^2 - 2(a - 2\mu b)/d}{2\mu(\mu + 2)p_0} \tag{23}$$

Finally, after substituting the value of p_{-1} in p_0; we obtain a fourth order polynomial whose p_0 is the root. The corresponding equation has the form :

$$A(p_0)^4 + B(p_0)^2 + C = 0 \tag{24}$$

where:

$$A = d^2(-1 - 16\mu - 44\mu^2 - 32\mu^3 + 4\mu^4 + 8\mu^5) \tag{25}$$

$$B = d(4a + 32a\mu - 8b\mu + 24a\mu^2 - 32b\mu^2 - 16b\mu^3 + 8b\mu^4) \tag{26}$$

$$C = -4a^2 + 16ab\mu - 16b^2\mu^2 \tag{27}$$

The fourth order equation (29) can be solved in p_0. We keep only the positive root p_0 for which the value of p_{-1}, given by (28) is also positive. However, rather than work with complex analytic expressions, we use different simulations for different values of the parameters.

In order to examine the effects of μ, we take first the following values of the parameters $a = 3; b = 0.1; d = 2$ and $' = e$. Table 1 gives the values of $p_0(\mu)$; $p_{-1}(\mu)$; ®(μ); and $g(\mu)$ for different values of μ in $[0; 1]$ with a grid of 1/10.

μ	$p_0(\mu)$	$p_{-1}(\mu)$	$\alpha(\mu)$	$g(\mu)$
0	1.7321	0.7378	0.1756	0.6082
1/10	1.5558	0.7734	0.1828	0.6320
2/10	1.4225	0.8038	0.1844	0.6556
3/10	1.3188	0.8311	0.1807	0.6809
4/10	1.2365	0.8566	0.1721	0.7092
5/10	1.1703	0.8812	0.1584	0.7416
6/10	1.1168	0.9058	0.1396	0.7794
7/10	1.0735	0.9311	0.1151	0.8239
8/10	1.0389	0.9575	0.0844	0.8767
9/10	1.0118	0.9858	0.0464	0.9400
1	0.9915	1.0165	0	1.0165

Table 1: The values of $p_0(\mu)$, $p_{-1}(\mu)$, $\alpha(\mu)$, and $g(\mu)$ for the following values of the parameters: $a = 3$, $b = 0.1$, $d = 2$, $' = e$, $r = 0$.

Different other values of the parameters have been tested and they all give rise to the same qualitative results as those which appear in table 1. They are summarized in the following proposition:

Proposition 3 Suppose that the R&D cost function is quadratic, $\gamma(p) = \frac{1}{2}dp^2$. For small values of the interest rate r, the R&D effort $p_0(\mu)$ by a neck-by-neck firm decreases with μ, while the R&D effort $p_{-1}(\mu)$ by a follower increases with μ. The graph of $p_{-1}(\mu)$ intersects the graph of $p_0(\mu)$ from below and, as μ goes to 1, $p_{-1}(\mu)$ becomes higher to $p_0(\mu)$. The proportion $\alpha(\mu)$ of industries that are head-to head in the steady state has an inverted U-form. It first increases with μ for small values of μ, then it decreases with μ, becoming equal to 0 for $\mu = 1$. The rate of growth $g(\mu)$ increases with μ.

Two messages emerge from this proposition. The main one is quite intuitive: an increase of the probability μ of leapfrogging by a laggard firm leads to an increase of the growth rate $g(\mu)$ of the economy. The importance of this effect is illustrated in table 1: the growth rate increases by more than 40% when μ increases from 0 to 1. The second message is that the R&D effort of a follower overruns the R&D effort of a neck by neck firm for sufficiently high values of μ. This illustrates the importance of the role of technological followers in the innovation process.

In order to analyze the effect of competitive pressure on the intensity of innovation and on the rate of growth, we begin by examining what happens for $\theta = 1$: From (25) and (26), one obtains directly the values of $p_{-1}(1) = \sqrt{\frac{2(a+b)}{3d}}$ and $p_0(1) = \frac{1}{3}\left(\sqrt{\frac{2(a+b)}{3d}} + \sqrt{\frac{2(a-5b)}{3d}}\right)$. These two R&D efforts increase with the degree of competition on the product market, since $a = \pi_1 - \pi_0$ is an increasing function of of the intensity of competition, $b = \pi_0 - \pi_{-1}$ is a decreasing function of competition and $a + b = \pi_1 - \pi_{-1}$ increases with competition. Thus, in the weak leapfrogging situation, the intensity of the innovation process, measured by the values of the R&D efforts $p_0(1)$ and $p_{-1}(1)$, increases with the intensity of competition in the product market . Moreover, since the rate of growth for $\theta = 1$ is given by $g(1) = p_{-1}(1) \ln \varphi$, (see (19)), it appears that the economy's rate of growth in the weak leapfrogging situation ($\theta = 1$) is also an increasing function of the intensity of competition in the product market[4]. The intuition is clear. By substituting to the usual strong leapfrogging assumption, according to which an industry is always monopolized by the last innovator, a more realistic one which allows both a laggard and a lead-up firm to coexist at any time, one opens the way for the analysis of the link between competition in the product market and growth. In the weak leapfrogging situation, more competition in the product market gives to the follower more incentives to invest in R&D, in order to leapfrog the leader, leading thus to a higher rate of growth.

What happens for other values of θ[5]? Let us introduce a parameter x measuring the intensity of competition in the product market, with $0 \leq x \leq 1$, such that higher values of x correspond to higher intensity of competition. Let us take for instance as a parametrization of the profit flow increments

[4] The same effect is obtained with a linear R&D cost function, and this confirms the robustness of the result.

[5] For $\theta = 0$, we obtain easily from (25) and (26) ; $p_0(0) = \sqrt{\frac{2a}{d}}$ and $p_{-1}(0) = \sqrt{\frac{2(2a+b)}{d}} - \sqrt{\frac{2a}{d}}$. Thus, the rate of growth $g(0) = \frac{2p_0(0)p_{-1}(0)}{2p_0(0)+p_{-1}(0)} \ln \varphi$ (from (19)) is equal in this case to $g(0) = \frac{2\sqrt{4a(2a+b)}-4a}{\sqrt{d}(\sqrt{2}\sqrt{2a+b}+\sqrt{2a})} \ln \varphi$. A.H.V.(1997) compare in this case the growth rates obtained with Bertrand and Cournot competition in the product market. In order to do that, it is sufficient to replace a and b with their corresponding values (depending on φ) obtained in these two types of competition (see footnote 3). The finding is that $g^{Bertrand}(0) > g^{Cournot}(0)$ if $\varphi < 7.26$. Thus, unless φ is large, the growth rate for $\theta = 0$ is higher with Bertrand than Cournot, which means that a higher degree of rivalry leads to a higher growth rate.

the two following linear functions $a(x) = 3(x + 1)$ and $b(x) = 2(1 - x)$: For a soft competition in the product market, as described for instance by the Cournot behavior $(x = 0)$; we have $a(0) = 3$ and $b(0) = 2$, while for a tough competition as described for instance by the Bertrand behavior $(x = 1)$, we have $a(1) = 6$ and $b(1) = 0$: The following figures give the graphs of respectively $p_0(\theta)$, $p_{-1}(\theta)$; $\alpha(\theta)$; and $g(\theta)$ where θ appears in the horizontal axis. In each figure, the graphs are plotted for the two extreme values $x = 0$ and $x = 1$: The value of the parameter d is normalized to 1:

The first graph (figure 2) depicts $p_0(\theta)$. The upper curve corresponds to $x = 1$ and the lower curve to $x = 0$: It appears clearly that a higher intensity of competition in the product market leads to a higher R&D effort of a neck by neck firm in a leveled industry. Moreover the gap between $p_0(\theta)$ for $x = 1$ and $p_0(\theta)$ for $x = 0$ increases with θ:

In the second graph (figure 3), the curves of $p_{-1}(\theta)$corresponding to $x = 1$ and to $x = 0$ intersect for some value of θ in $[0; 1]$: For small values of θ; the curve corresponding to $x = 1$ is below the curve corresponding to $x = 0$: It appears that it is only for high values of θ that a higher intensity of competition in the product market leads to a higher R&D effort of a laggard firm. This result is important. It asserts that it is only when the probability to leapfrog is sufficiently high that the intensity of the competition in the product market gives a higher incentive to innovate in order to escape from competition.

In the third graph (figure 4), the curve $\alpha(\theta)$ corresponding to $x = 1$ is below the curve $\alpha(\theta)$ corresponding to $x = 0$ for all values of θ: Thus, a higher intensity of competition in the product market leads to a lower proportion of leveled industries in the stationary state. Equivalently, this means that the proportion of industries of the leader-follower type increases with the intensity of competition in the product market.

Finally, in the thourth graph giving $g(\theta)$ (fig 5), the curve corresponding to $x = 1$ is above the curve corresponding to $x = 0$ for all values of θ. This means that a higher intensity of competition in the product market leads to a higher growth rate in the stationary state and the contribution of the competition to the growth rate is all the more important since the probability to leapfrog θ is high.

All these results are summarized in the following proposition

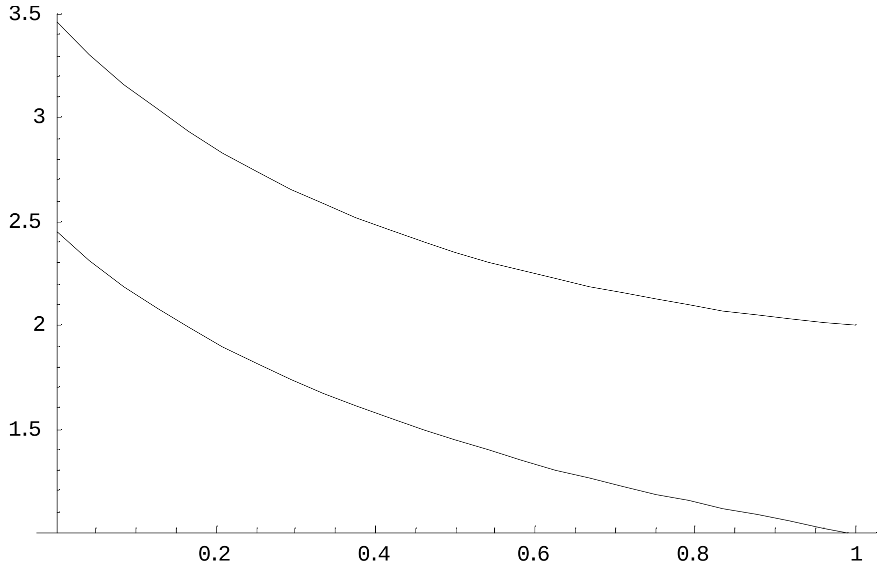

Figure 2: The R&D e¤ort $p_0(\mu)$ of a neck by neck …rm according to 2 values of the competitive pressure x = 0 and x = 1

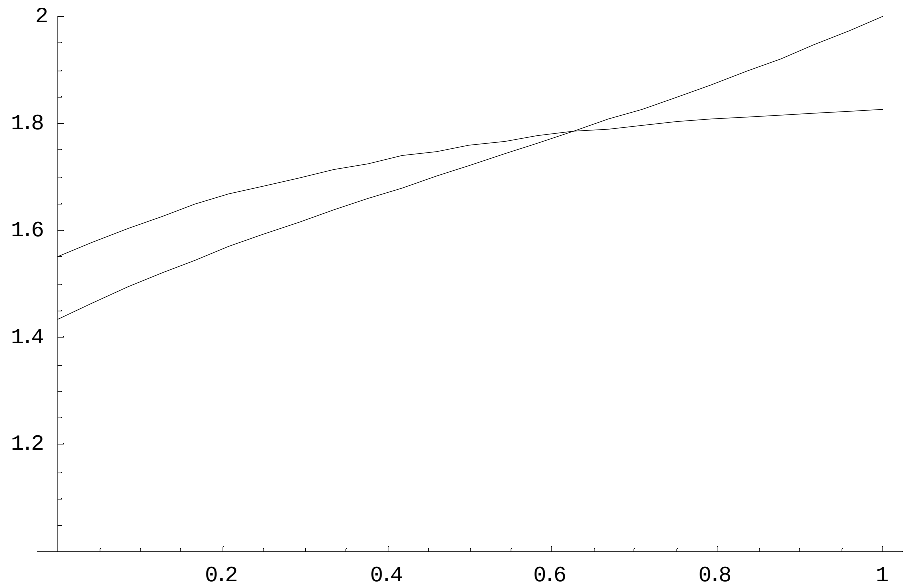

Figure 3: The R&D e¤ort $p_{¡1}(\mu)$ of a laggard …rm according to 2 values of the competitive pressure x = 0 and x = 1

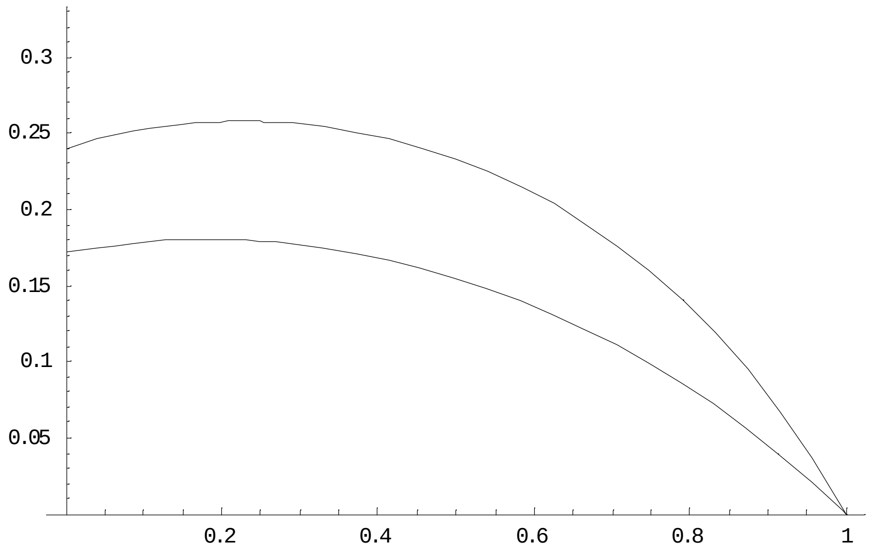

Figure 4: The proportion ® (µ) of leveled industries according to 2 values of the competitive pressure x = 0 and x = 1

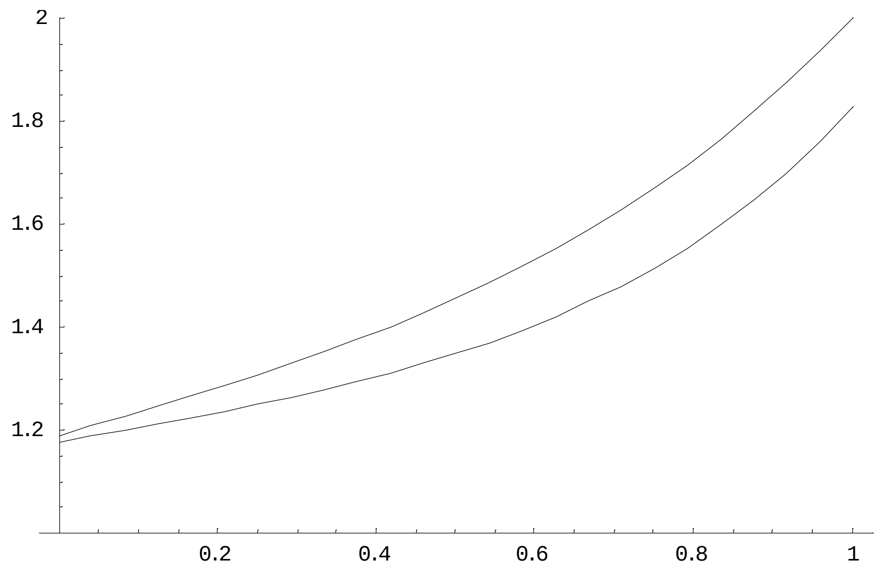

Figure 5: The growth rate g (µ) according to 2 values of the competitive pressure x = 0 and x = 1

Proposition 4 An increase in the intensity of competition leads to a higher R&D e¤ort $p_0(\mu)$ by a neck-by-neck …rm for all values of μ, to a higher R&D e¤ort $p_{i1}(\mu)$ by a follower only when μ:is su¢ciently high, to a lower proportion ®(μ) of industries that are head-to head in the steady state, and to a higher growth rate $g(\mu)$ of the economy.

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