Natural concentration in industrial research collaboration

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Abstract

Empirical work suggests that the network of research and development alliances is asymmetric, with a small number of firms being involved in the majority of alliances. The paper relates the structure of the collaboration network to a fundamental characteristic of the demand for research output: the indivisibility in the use of research output creates private and social incentives for a concentration of collaborative activities. I investigate two models on the formation of bilateral collaborative links in a socially managed industry and in an oligopolistic industry respectively. I find that a concentrated network is a typical equilibrium and a socially efficient structure.

1 Introduction

The literature on research and development (R&D) collaboration has focused on the pros and cons of an industry-wide agreement as compared to the case of independently operating research labs (e.g. d'Aspremont and Jacquemin, 1988; Kamien et al., 1992). Yet, empirical work on the high-tech sectors suggests that not all companies, even not the direct competitors, are equally active in collaborating. Hagedoorn and Schakenraad (1992), for example, identify seven key players in the information and communication technology industries of the 1980s (AT&T, IBM, Siemens, Philips, Fujitsu, NEC, and Olivetti), who are involved in many of the collaborative partnerships in these industries. These companies belong, at the same time, to the list of top-ten sellers in their respective industry. Similarly, for the global biotech and pharmaceutical industries of the 1990s, Powell et al. (2005) find that a group of 24 well-known players had each formed more than twenty strategic alliances, while the majority of firms had two or less alliances.

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†This research is part of the UU High-Potential program 'Dynamics of Cooperation, Networks and Institutions' under the direction of Vincent Buskens and Stephanie Rosenkranz. I would like to thank my supervisors as well as Thomas P. Gehrig, José Luis Moraga-Gonzáles, Jurjen Kamphorst, Berno Büchel, and participants at the NAKE Research Day 2006 in Amsterdam, the GEABA Symposium 2007 in Tübingen, and the GREQAM Summer School 2007 in Aix-en Provence for helpful comments and suggestions.

The observations raise a number of questions. Can we explain the emergence of a concentrated market structure in a micro-economic model on R&D collaboration? Is concentration welfare-efficient? Finally, in the light of the favorable treatment of research joint ventures in the U.S. and in Europe, is there a policy lesson to learn from such a model? For example, should R&D collaboration be encouraged between the firms at the margin or rather between the central players in an industry?

Despite of the empirical evidence, issues like these have not been sufficiently addressed in the literature. Katz (1986) was the first to study a less than industry-wide collaborative agreement. More recent contributions come from Bloch (1995) and Yi (1998), who investigate the set of industry partitions into research coalitions, and Goyal and Moraga-Gonzáles (2001) and Goyal and Joshi (2003), who study *networks* of bilateral R&D alliances. A common finding is that less than industry-wide agreements are typical market structures in equilibrium. Of particular interest are the analyses of some specific market settings in Goyal and Moraga-Gonzáles (2001) and Goyal and Joshi (2003). The authors find that equilibrium networks are typically highly asymmetric, with some firms being completely excluded from any collaborative activity. Moreover, they obtain the surprising result that those structures do, under certain conditions, welfare-dominate symmetric networks.

In this paper, I argue that concentration in the market structure of R&D collaboration arises in a wide range of market settings and is typically efficient. My argument builds on a fundamental property of the demand for R&D investments. The seminal articles by Arrow (1962) and Dasgupta and Stiglitz (1980a) point already to the *indivisibility* in the use of research output and to its implications for the market structure. An invention, for example one that enables a firm to reduce its marginal production costs, can be applied to every unit of the firm's product, regardless of its scale of production. The returns to investment are therefore higher, the larger the production scales.

This motivates two conjectures about the investment profile in an oligopolistic industry: first, in an intra-industry comparison the returns to investment are increasing in a firm's current stock of R&D investments. Second, a reallocation of investments from a firm with a small investment stock to an already R&D intensive firm is socially desirable. Likewise, there are two good reasons for why concentration is also favored in the market structure of collaborative agreements. First, every reallocated joint venture offers new investment possibilities for an R&D intensive firm. Second, the members of a joint venture are likely to enjoy larger spillovers from their partners than nonmembers. Hence, a reallocation increases the potentially attainable spillovers for a R&D intensive firm.[1]

This paper scrutinizes the reasoning by means of two models on the formation of collaborative R&D agreements. I use a graph-theoretical framework that depicts an agreement as a *bilateral link* in a network. The framework captures important stylized facts: the majority of R&D alliances is bilateral and embedded in a network of similar ties. In the Thomson SDC Platinum database, for example, the share of multilateral agreements is only 14% of the total of newly formed alliances over the period 1990–2002. In this network,

[1] The empirical literature on spillovers within joint research projects and the complementarities between different projects suggests that both effects are actually at work (Henderson and Cockburn, 1996).

some firms have multiple alliances with different partners, stressing the *non-exclusivity* in the formation of collaborative agreements.

The first model investigates the formation of a collaboration network, given the linking and production decisions are managed by a social planner. In the model, a set of firms produces a homogenous good. Moreover, the firms are each endowed with a distinct 'intellectual asset' of identical value. The formation of a link allows the involved firms to exchange their assets, which leads to an exogenously specified reduction in their marginal production costs. My main finding is the following: given the planner has only a limited number of collaboration links at his disposal he prefers a *star* network, in which all links originate from a single firm. The reason is that since the center firm of the star accumulates all the intellectual assets in the industry, there is no other network that enables a more cost-efficient production. Thus, I find that the welfare-efficient network exhibits maximal concentration.

The result is the stepping stone for a second, more challenging model. Here, the social planner loses control of the production decisions, which are now made by the firms. The further modifications are that (i) the marginal cost reductions require endogenously determined and link-specific R&D investments, (ii) the firms produce differentiated products, and (iii) also the number of collaborative links is endogenous. I analyze a two-stage non-cooperative game, where in the first stage the firms and the social planner, respectively, form a collaboration network and make link-specific R&D investments. In the second stage, all firms, even the collaborators, compete in the product market.

I start with an investigation of the R&D investments, taking a collaboration network as given. Using a specific but not uncommon R&D production function, I find that the investments of the firms and the social planner are directly determined by the structure of the network. In particular, the same amount is invested on every link.[2] I use this to show that the two-stage game is equivalent to a simpler game of link formation. In this game and like in the first model, a link leads to an automatic decline of the marginal production costs. The equivalence greatly simplifies the characterization of the equilibrium and the welfare-efficient networks of the two-stage game.[3]

Concerning the efficient network, my first result pertains its architecture. I find that the efficient network may be the empty network, in which no firm collaborates, or the complete network with a collaborative agreement between any pair of firms. But if it is neither of both, the efficient network has a *dominant group* or a *core-periphery* architecture, both of which exhibit a strong asymmetry between firms.

Second, I examine the *density* and the *degree variance* in the efficient network. While the density captures the welfare contribution from the mere number of collaborative link,

[2]The R&D production function is specified as follows: (i) there are perfect spillovers within the project on a link, but no spillovers between different projects; (ii) the returns to research investments are constant up to a certain threshold value, from which on no further investment pays off; and (iii) the R&D production function exhibits constant returns with respect to the number of links a firm is involved in.

[3]The equivalence of the two games also motivates the models of Bloch (1995), Yi (1998), and Goyal and Joshi (2003), who assume from the outset that the affiliation to a collaborative agreement is aligned with an exogenous cost reduction.

the degree variance depicts the contribution from their dispersion among the firms. It turns out that the social welfare in a network is a function of density and degree variance only. Moreover, among the networks of the same density the network with the highest degree variance maximizes social welfare. Hence, like in the first model, it is efficient to maximally concentrate a given number of collaboration links.

Third, I determine conditions on the demand and cost parameters, for which the efficient network exhibits concentration. Because a general analysis is very complicated, I work with two examples of a small market with seven firms and of a large market with an infinite number of firms respectively. It turns out that although the exact architecture of the efficient network depends on the characteristics of the market and the R&D technology, I find in both cases a generic set of parameters that supports a concentrated network.

A characterization of the equilibria in the game provides a similar insight: the equilibrium networks are typically concentrated as well. I therefore compare the extents of concentration in the example of seven firms. I find that if the degree of product differentiation is on an intermediate level, the equilibrium networks are of efficient density but too little concentrated. If, on the other hand, products are strongly (weakly) differentiated the equilibrium networks are too dense (too sparse). This suggests that the extent of concentration is of particular relevance for markets with differentiated goods.

Before introducing the models, I briefly relate them to the literature. First, there is the clear link to the literature on R&D collaboration. Two rather well-understood themes are the free-rider problems in collaborative agreements (Kamien et al., 1992; Goyal and Moraga-Gonzáles, 2001) and the fact that the current members of a joint venture may exclude the entry of rivals to their agreement in order to maintain market power (Bloch, 1995; Yi, 1998). The current study is to the best of my knowledge the first to point out that a concentration of collaborative R&D activities around a small group of firms is socially efficient, because these firms accumulate the output of costly joint research. A contribution to the studies by Bloch (1995) and Yi (1998) is to show that an exclusion of some firms from collaborating is often socially desired. The authors point to the welfare losses from an asymmetric market structure in a game, where the costs of collaboration are insignificantly small and the industry-wide agreement is efficient. The current study, on the other hand, investigates the situation of large costs, for which the complete network is typically inefficient.

I should mention that the articles by Goyal and Moraga-Gonzáles (2001) and Goyal and Joshi (2003) provide comparable network characterizations. Goyal and Moraga-Gonzáles show in a Cournot oligopoly of three firms that an asymmetric network can be strategically stable and efficient. Goyal and Joshi find for a more general Cournot oligopoly and significant (but not too large) costs aligned with the collaborative links that a dominant group is a typical equilibrium architecture. Moreover, they show for Bertrand competition and small linking costs that the efficient network has a core-periphery architecture. My analysis indicates that all of their findings carry over to markets with an arbitrary number of firms and differentiated products as well as to a wide range of cost levels.

Second, the property that the output of an investment can be applied to any scale of production is not specific to collaborative R&D, but one of many investments involving high fixed costs. Other investments with the same property are, for example, in-house R&D projects, technology licensing, investments into human capital or into the organization. My analysis therefore has some overlap with the studies investigating the market structure in industries with high fixed costs (e.g., Dasgupta and Stiglitz, 1980a; Spence, 1984).

The remainder of the paper is organized as follows. Section 2 introduces some network terminology. Section 3 investigates the collaboration network in a socially managed industry. Section 4 analyzes a market with imperfect competition. I characterize the efficient collaboration network and compare it with the networks that arise from the strategic formation of links. The section ends with a robustness check of the main findings with respect to market exit and spillovers. Section 5 concludes.

2 Networks

Consider a set of firm, $N = \{1, 2, ..., n\}$ with $n > 2$. For any distinct firms $i, j \in N$, the pair-wise relationship between the firms is depicted by a *bilateral* link ij. A *network* $g = \{ij : i, j \in N, i \neq j\}$ is the complete collection of the links between firms. Denote by $N_i(g)$ the set of firms, with which firm i has a link in network g; firm i's *degree* is the cardinality of this set, $\eta_i(g) = |N_i(g)|$. A network partitions the set of firms according to their degrees. We say that distinct $i, j \in N$ are members of the same group $h_l(g)$, $0 \leq l \leq m$, of the degree partition $\{h_0(g), h_1(g), ..., h_m(g)\}$, if and only if $\eta_i(g) = \eta_j(g) = l$. The *degree distribution* of network g assigns to every nonnegative integer l the frequency weight $n_l(g) = |h_l(g)|/n$. Two important characteristics of the degree distribution are:

- the *density* $D(g) = \bar{\eta}(g)/(n-1)$, where $\bar{\eta}(g) = \sum_{i \in N} \eta_i(g)/n$ denotes the *average degree*, and
- the normalized *degree variance* $C(g) = V(g)/V_{max}$, where the degree variance, $V(g) = \sum_{i \in N}(\eta_i(g) - \bar{\eta}(g))^2/n$, is normalized by its maximum given n firms, $V_{max} = (3n-2)^2 (n-2)(3n+2)/256n^2$.

I now define the architecture of a network and introduce some relevant examples. Any two networks g and g' share the same *architecture*, if there exists a permutation of firms, P, such that $g' = \{P(i)P(j)|ij \in g\}$. Thus, network g' should be attainable from g by just relabeling the firms. A *regular* network architecture of density D is such that every firm has a degree of $\eta_i(g) = (n-1)D$. Two special cases are the *empty* network, g^e, with $D = 0$ and the *complete* network, g^c, with $D = 1$. An important irregular network architecture is the *dominant group* of size n', $g^{n'}$, with $2 \leq n' \leq n-1$. The dominant group consists of two groups of firms, one in which everybody is linked to everybody else in that group (denoted by $h_{n'-1}$), and another group consisting of isolate firms (denoted by h_0). The *core-periphery* architecture, g^x, on the other hand induces a degree partition, where there are two or more groups of firms with a positive degree. Moreover, every firm in group h_m (the core group) is linked to every other firm with at least one link. Hence, let $i \in h_m(g^x)$. If $\eta_j(g^x) \geq 1$ it holds $ij \in g^x$. The *star*, g^s, is a special case of a core-periphery architecture.

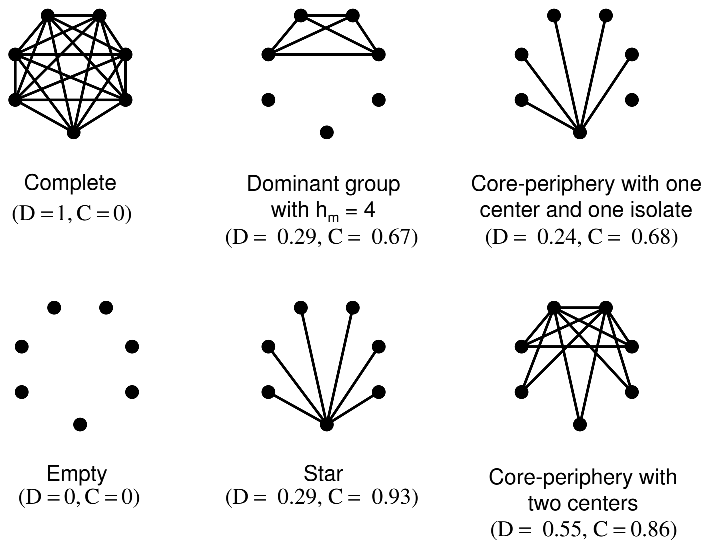

Figure 1: Network structures

It induces a two-point degree partition of the set of firms, $\{h_1, h_{n-1}\}$, with $|h_{n-1}| = 1$. The defined network architectures are illustrated in Figure 1.

3 Concentration in a socially managed industry

In this section, I investigate a socially managed industry. A social planner chooses the scales of production at a set of firms. At the same time, he may form links between pairs of firms in order to exchange their distinct 'intellectual assets'. The question is at which firms to produce and how to organize the network of collaborative ties efficiently.

Consider a set of initially *identical* firms, $N = \{1, 2, ..., n\}$ with $n > 2$. Each firm produces a homogeneous product at zero fixed costs and without any capacity restrictions. Let each firm be endowed with a distinct intellectual asset, such as a blueprint for a sophisticated component of the product. A firm may not use the asset on its own, but it may share the asset with the other firms, in which case the recipient's marginal production costs decline. If firm i receives the assets of y_i other firms its variable production costs are given by $C(q_i, y_i)$. It is a natural assumption in the context of intellectual assets that the rate of the unit-cost reduction is the same for every unit produced at a firm, regardless of its scale of production. I therefore specify $C(q_i, y_i) = (\gamma_0 - y_i)q_i$, where γ_0 denotes the initial marginal production costs, $\gamma_0 > n - 1$. Let Q denote the total quantity of the commodity and q_i the quantity produced by firm i. The net social benefit of production is given by

$$W(q_1, q_2, ..., q_n, y_1, y_2, ..., y_n) = U(Q) - \sum_{i \in N} (\gamma_0 - y_i) q_i$$

where $U'(Q) > 0$. I assume that two firms automatically share their assets, if they are involved in a collaborative link. Hence, in network g it holds $y_i = \eta_i(g)$. The formation of a link shall be free of any costs, but I suppose that the social planner has only a limited number of y links, $2 \leq y \leq n-1$, at his disposal.

The solution to the planner's problem is the following. Concerning the efficient production program, consider any network g. Denote the group of firms with the lowest marginal production costs by $h_m(g)$, with $h_m(g) \subseteq N$, and suppose without loss of generality that $h_m(g) = \{1, 2, ..., m\}$, $1 \leq m \leq n$. Suppose the planner chooses an output profile, $(q_1, q_2, ..., q_n)$, such that $q_i > 0$ for some $i > m$. Clearly, he can save production costs at an amount of $q_i \left(\eta_m(g) - \eta_i(g)\right)$ by shifting production from firm i to firm m. Hence, it is efficient to produce exclusively at the firms with the most acquired assets.

Concerning the efficient collaboration network, consider any positive total output Q. Suppose the planner chooses a network g consisting of $y = n-1$ links with $|h_m(g)| > 1$. Network g is not welfare-optimal. The planner may remove some links from the network until he arrives at a core-periphery network g^x, where $|h_m(g^x)| = 1$ and for any $i \in N\backslash\{j\}$ with $\eta_i(g^x) > 0$ it is $ij \in g^x$ if and only if $j \in h_m(g^x)$. He may produce output Q at firm j for the same total production costs as before and we have $W(g^x) = W(g)$. Furthermore, the planner may add the previously removed ties to network g^x and link firm j to every other firm that is not yet connected to it. In the resulting star network g^s, firm j accumulates $n-1$ assets and the planner saves production cost at the amount of Q for every reestablished link. Hence, we find that $W(g^s) > W(g)$ and that the concentration of collaborative links is efficient, because of the benefits that arise from an accumulation of intellectual assets.

4 Concentration under imperfect competition

4.1 The model

The preceding analysis highlights the efficiency of a concentrated collaboration network in a socially managed industry. In the following, I repeat the analysis in a more challenging setting, where (i) the social planner loses control of the production decisions, which are in the hand of the profit-maximizing firms; (ii) the firms produce differentiated products; (iii) the amount of assets exchanged on a link is endogenously determined by a link-specific R&D investment; and (iv) next to the degree of concentration in the network, also the number of links is endogenously determined.

I investigate a two-stage game, where in the first stage the firms and a social planner, respectively, form a collaboration network and make link-specific R&D investments in order to reduce the marginal production costs. In the second stage, all firms, even the collaborators, compete in the product market. I first introduce the model and solve its second stage. I will then examine the equilibrium and the efficient R&D investments of the first stage, taking as given an arbitrary network. Subsequently, I analyze the efficient collaboration network and compare it to the equilibrium networks. Finally, I discuss some of the crucial assumptions underlying the analysis. I now describe the model in more detail.

Market competition. Consider a set of initially *identical* firms, $N = \{1, 2, ..., n\}$, $n > 2$. In the second stage of the game, each firm sells a single, possibly differentiated, product to a continuum of homogeneous consumers, who consume each a *numeraire* good I in addition. The representative consumer maximizes the quasi-linear utility function

$$U(I, q_1, ..., q_n) = I + \alpha \sum_{i\in N} q_i - \frac{1}{2}\sum_{i\in N} q_i^2 - \frac{\beta}{2}\sum_{i\in N}\sum_{j\neq i} q_i q_j \tag{1}$$

under the constraint $I \leq -\sum_{i=1}^{n} p_i q_i$, where q_i denotes the quantity and p_i the price of good i. Let α, the market size, be positive throughout, and let β, the degree of substitutability between products, come from the interval $(0, 1]$, which implies a focus on markets, where the collaborating firms are competitors in the product market. The case of $\beta = 1$ depicts a market, where firms sell perfect substitutable goods, and $\beta = 0$ the case of firms operating in independent markets. From utility maximization with respect to quantities, we obtain the linear inverse demand functions $p_i = \alpha - q_i - \beta \sum_{j\neq i} q_j$, $i \in N$. A firm's profit, gross of any costs aligned with its R&D and linking activity, shall be given by $\pi_i = (p_i - c_i)q_i$. As in Section 3, the marginal production costs are firm-specific and assumed to be independent of the output.

To model market interaction, I suppose that all firms compete either in quantities or in prices, but I disregard the case of perfect Bertrand competition. Thus, I shall assume that $\beta < 1$, if the firms compete in prices.[4] The existence of a unique Nash equilibrium follows from the sub-(super-)modularity of the Cournot game (the Bertrand game with differentiated products) and the linearity of the best-response functions (Amir, 1996). Using superscript q (p) to denote competition in quantities (prices), the Nash equilibrium quantities and prices are given by

$$\mu^q q_i^q = \frac{\alpha}{2 + (n-1)\beta} - \frac{1}{2-\beta} c_i + \frac{\beta}{\big(2 + (n-1)\beta\big)(2-\beta)} \sum_{j\in N} c_j \tag{2}$$

$$\mu^p q_i^p = \frac{(1-\beta)\alpha}{2 + (n-3)\beta} - \frac{1 + (n-1)\beta}{2 + (2n-3)\beta} c_i + \frac{\beta(1 + \beta(n-2))}{\big(2 + (n-3)\beta\big)\big(2 + (2n-3)\beta\big)} \sum_{j\in N} c_j \tag{3}$$

and $p_i = \mu q_i + c_i$, where $\mu^q = 1$ and $\mu^p = (1-\beta)\,(1 + (n-1)\beta)\,/\,(1 + (n-2)\beta)$. The gross profit of a typical firm is $\pi_i = \mu q_i^2$. The following assumption guarantees that, regardless of the profile of research investments and the network of collaborative ties, no firm exits the product market.

[4] For an analysis of collaboration networks under perfect Bertrand competition, see Goyal and Joshi (2003).

(A1) *No market exit: for any profile of marginal production costs, induced by the R&D investments and the links between firms, it holds* $q_i(c_i, \sum_{j\neq i} c_j) > 0$ *for any* $i \in N$.[5]

Link-specific R&D. Consider a given network g. For any $ij \in g$, denote the research effort by firm i invested into the joint project with firm j by $x_{i,j}$, and denote an industry profile of R&D investments by $x = (x_{i,j}, x_{j,i})_{ij\in g}$. The total investments on the link ij are given by $x_{ij} = x_{i,j} + x_{j,i}$. Taking into account all the possible technological spillovers between firms, the unit costs of a typical firm is a function of all the R&D investments in the industry, $c_i = c(x_{1,2}, x_{1,3}, ..., x_{2,1}, x_{2,3}, ..., x_{n,n-1})$, where $x_{j,k} = 0$ if $jk \notin g$. Let a firm's initial marginal production costs be $c(\mathbf{0}) = \gamma_0$, with $0 < \gamma_0 < \alpha$. The R&D technology shall satisfy the following assumptions:

(A2) *Perfect spillovers within a joint project and no involuntary spillovers: for any* $ij \in g$ *and* $k \in N\backslash\{i,j\}$, *it holds* $\partial c_k/\partial x_{ij} = 0$. *For any* $ij \in g$, *it is* $\partial c_i/\partial x_{i,j} = \partial c_j/\partial x_{i,j}$ *and* $\partial c_i/\partial x_{j,i} = \partial c_j/\partial x_{j,i}$.
(A3) *Constant returns up to a threshold level: for any* $ij \in g$ *and* $k \in \{i,j\}$, *it is*

$$\frac{\partial c_k}{\partial x_{i,j}} = \frac{\partial c_k}{\partial x_{j,i}} = \begin{cases} -\gamma & \textit{if } x_{ij} \leq 1 \\ 0 & \textit{otherwise} \end{cases}$$

where $\gamma > 0$.

I therefore write more conveniently $c_i = \gamma_0 - \gamma \sum_{j\in N_i(g)} \min\{x_{ij}\ , 1\}$.

The choice of the R&D production function needs some motivation. Though atypical it is nothing but a specification from the class of commonly used production functions. First, (A2) is an extreme form of the widely used assumption that the spillovers between the partners in a joint venture are larger than between the members of different joint ventures (e.g., Katz, 1986; Kamien et al., 1992). Second, also (A3) is nothing but a specification of the standard assumption that the R&D production function of a joint venture is concave (d'Aspremont and Jacquemin, 1988; Kamien et al., 1992). The threshold investment value can be motivated as follows. Suppose the second input in a joint research project is an idea and suppose the idea is essential for R&D investments to take effect. A collaborative partnership enables the firms to pool their distinct capabilities in order to create a new idea. Moreover, they may exploit their idea in a joint project. Let the idea be exploited, if the the total R&D investments on a link exceed the threshold value.

Concerning the division of labor on a link, denote by f the cost per unit of R&D effort invested, with $f > 0$. I assume that:

(A4) *Equal cost sharing: for any* $ij \in g$, *the investment costs borne by firm* i *is* $f x_{ij}/2$.

Given a network g and an investment profile x, a firm's profit is given by $\Pi_i(g,x) = \pi\left(q_i(g,x)\right) - \frac{1}{2} f \sum_{j\in N_i(g)} x_{ij}$.

[5] The precise condition depends on the mode of competition and the productivity of research investments. This condition will be specified in footnote 7.

Equilibrium and efficiency. Concerning an *equilibrium* in the first stage of the game, I adopt the rules by Goyal et al. (2005). In this game, the firms decide *non-cooperatively* about whether to form a new link or to terminate some of their existing links and, at the same time, about their R&D investments. A strategy of firm i is to announce a set of intended collaboration links, $s_i = \{ij : j \in N \backslash \{i\}\}$, and R&D efforts, $e_i = (e_{i,j})_{j \in N \backslash \{i\}}$. A link is established between two firms and the announced efforts come into realization, $e_{i,j} = x_{i,j}$ and $e_{j,i} = x_{j,i}$, if and only if $ij \in s_i$ and $ij \in s_j$. I assume that there are, next to the R&D investment costs, *small* but positive costs of establishing a link.

Let (s, e) be a strategy profile, then is $g(s)$ the induced network by the link announcements and $x(s, e)$ the profile of R&D investments. In the following, I omit the dependence on the underlying strategies for expositional simplicity. An equilibrium is defined as follows. Write $g + ij$ for the network that is obtained from network g by adding the link ij.

A network and a profile of R&D investments, $(\widehat{g}, \widehat{x})$, constitutes a pair-wise equilibrium:
(i) if $(\widehat{s}, \widehat{e})$ constitutes a Nash equilibrium.
(ii) For $\widehat{g} + ij$ and any $x_{i,j}, x_{j,i} \geq 0$:
if $\Pi_i(\widehat{g} + ij, x_{i,j}, x_{j,i}, \widehat{x}_{-ij}) > \Pi_i(\widehat{g}, \widehat{x}) \Rightarrow \Pi_j(\widehat{g} + ij, x_{i,j}, x_{j,i}, \widehat{x}_{-ij}) \leq \Pi_j(\widehat{g}, \widehat{x})$.

Condition (i) requires the link and effort announcements to be stable against any unilateral deviation by a firm. Condition (ii) ensures that no single additional joint research project pays off, given that all other projects remain in place and all other effort announcements are unchanged.

Concerning an *efficient* state, the analysis comprises both social welfare concepts, the consumer surplus (net of research costs) and the total surplus. A profile (g^*, x^*) is therefore efficient, if respectively $U(g^*, x^*) - f \sum_{ij \in g^*} x^*_{ij} \geq U(g, x) - f \sum_{ij \in g} x_{ij}$ and $U(g^*, x^*) + \sum_{i \in N} \Pi_i(g^*, x^*) \geq U(g, x) + \sum_{i \in N} \Pi_i(g, x)$ for all g, $x_{i,j} \geq 0$, and $x_{j,i} \geq 0$.

4.2 R&D investments

In the first step of the analysis, I examine the R&D investments, taking a network as given. I will take advantage of the fact that the investments are subject to a convexity property, which has a strong implication for the efficient investment levels and the pair-wise equilibrium investments on a link.

Lemma 1. *Suppose the pair-wise equilibrium $(\widehat{g}, \widehat{x})$ exists. For any $ij \in \widehat{g}$, it must be $\widehat{x}_{ij} = 1$. Moreover, suppose the efficient profile (g^*, x^*). For any $ij \in g^*$, it must be $x^*_{ij} = 1$.*

The proof is given in Appendix A. The essential ingredient in the proof is the fact that the marginal profit as well as the marginal social benefit of a unit research effort increase in the current stock of investments in a project. Due to this *convexity* property, if the firms i and j and the social planner, respectively, invest on the link ij they prefer the maximal attainable investment of *one*. The optimal R&D strategy is therefore a question of 'all or nothing'. Suppose, however, a network containing the link ij with $x_{ij} = 0$. It is clear that for any arbitrarily small linking costs the firms i and j and the social planner, are better off by deleting it. Hence, for every link of an equilibrium network and of an efficient network, respectively, the total invested effort is one.[6]

Together with (A4), the lemma has a strong implication that simplifies the following analysis. Because the R&D investment costs are shared equally on a link, the division of labor between the partners does not matter for profits or social welfare. In fact, it holds

$$\Pi_i(g, x_{i,j}, x_{j,i}, x_{-ij}) = \Pi_i(g, x'_{i,j}, x'_{j,i}, x_{-ij}) \quad \text{for any } ij \in g \text{ if and only if } x_{ij} = x'_{ij}.$$

It follows that all economically relevant information about the investment profiles x^* and $\widehat{x}$ is contained in the structure of a network, since additionally in equilibrium and in an efficient state it holds $x_{ij} = 1$. In the following, it is therefore sufficient to investigate a simpler game of link formation introduced by Goyal and Joshi (2006).

In this game, a link induces an automatic change of the partners' research and production costs that depicts all the effects of their investment and link formation decisions. The marginal production costs of firm i in network g is given by a linear function of its degree, $c_i(g) = \gamma_0 - \gamma\eta_i(g)$, and the term $\frac{1}{2}f$ represents a constant *cost of linking*. The net profit is given by $\Pi_i(g) = \pi_i[\eta_i(g), \sum_{j\neq i}\eta_j(g)] - \frac{1}{2}f\eta_i(g)$, and the utility of the consumer by $U(g) = U\left[(\eta_i(g))_{i\in N}\right]$. A *pair-wise equilibrium network* in this game satisfies the following conditions: (i) $\widehat{s}$ constitutes a Nash equilibrium; and (ii) for $\widehat{g} + ij$: if $\Pi_i(\widehat{g} + ij) > \Pi_i(\widehat{g})$, it follows $\Pi_j(\widehat{g} + ij) \leq \Pi_j(\widehat{g})$. A network g^* is *efficient*, if respectively $U(g^*) - \frac{1}{2}f\sum_{i\in N}\eta_i(g^*) \geq U(g) - \frac{1}{2}f\sum_{i\in N}\eta_i(g)$ and $U(g^*) + \sum_{i\in N}\Pi_i(g^*) \geq U(g) + \sum_{i\in N}\Pi_i(g)$ for all g.[7] For expositional simplicity, I denote the total surplus and the consumer surplus in a network by $W(g)$ throughout the following, unless a distinction is necessary.

The following proposition shows that the link formation game can be interpreted as a reduced form of the model in this section.

Proposition 1. *A network $\widehat{g}$ is a pair-wise equilibrium network if and only if there exists a profile $\widehat{x}$, with $\widehat{x}_{ij} = 1$ for any $ij \in \widehat{g}$, such that $(\widehat{g}, \widehat{x})$ is a pair-wise equilibrium. Moreover, a network g^* is efficient if and only if there exists a profile x^*, with $x^*_{ij} = 1$ for any $ij \in g^*$, such that (g^*, x^*) is efficient.*

The proof is provided in Appendix A.

[6] Another important ingredient in the proof of the lemma is (A3), hence constant returns to R&D investments up to a threshold value. If we relax the assumption and apply a strictly concave production function instead, the firms i and j might prefer an interior investment level. Intuitively, if the production function is sufficiently concave also the functions of profits and social benefits become concave. A study that follows this path is Goyal et al. (2005). Because in their model the investment levels on the links depend on the whole structure of a network, the authors encounter difficulties in determining the equilibrium and the welfare-efficient networks.

[7] To satisfy (A1), we need to require that in a dominant group architecture g^{n-1}, where only firm i is isolated, $q_i(g^{n-1}) > 0$. This implies that under quantity competition $\alpha - \gamma_0 > [\gamma(n-1)(n-2)\beta] / [2-\beta]$ and under price competition $\alpha - \gamma_0 > [\gamma(1+(n-2)\beta)(n-1)(n-2)\beta] / \left[2+(2n-5)\beta-(2n-3)\beta^2\right]$.

4.3 The efficient collaboration network

The analysis of the R&D investments has shown that the model in this section is equivalent to a simpler game of link formation. In the following, I investigate the efficient collaboration network of this link formation game. I show that the social welfare function is *convex* and *submodular* with respect to collaborative ties. I use these properties to characterize the efficient network and find that it is typically highly concentrated. For a market with a very large number of firms, I determine conditions on the cost and market parameters that are sufficient for a concentrated network to be welfare-dominant. I see two implications of the analysis. First, the accumulation of intellectual assets is also efficient in a market with imperfect competition. Second, because the degree variance in a network is related to the *Herfindahl* index, the market dominance of a few firms is not necessarily detrimental from a societal perspective.

Convexity and submodularity. In the model, the formation of a collaborative tie costs society a constant amount of money. Although I assume away technological spillovers, the marginal social returns to any two links are interrelated by the fact that the social benefit to a link ij depends on the market shares of the firms i and j, prior to the link. Firm i's and j's market shares are, however, again determined by all prior investments being made in the industry. This 'market-interdependence' has two faces: adding link ij to network g becomes more beneficial the larger the degrees of firms i and j in network g (convexity). On the other hand, the social returns to ij deteriorate with the degree of a firm $k \in N \backslash \{i, j\}$ (submodularity). This is shown in the following result.

Lemma 2. *The total surplus and the consumer surplus are:*
(i) convex in links: $W(g+ij+ik)-W(g+ik) > W(g+ik)-W(g)$ *for any* g *with* $ij, ik \notin g$, *if additionally* $\eta_j(g) \geq \eta_k(g)+1$, *and*
(ii) submodular in links: $W(g+ij+kl)-W(g+kl) < W(g+ij)-W(g)$ *for any* g *with* $ij, kl \notin g$ *and* $k, l \in N \backslash \{i, j\}$, *if additionally, in case that firms compete in quantities and* $W(g)$ *represents the consumer surplus,* $\beta < (n-4)/(n-1)$.

The proof can be found in Appendix B.

Network architecture. The lemma allows for a marginal check, of whether a network is socially efficient or whether an additional collaboration tie between firms increases welfare.

Proposition 2. *If the efficient network* g^* *has a regular architecture, then it must either be the empty network,* g^e, *or the complete network,* g^c. *If the efficient network* g^* *is irregular, it has either a dominant group architecture,* $g^{n'}$, *for some* $0 < n' < n$, *or has a core-periphery architecture,* g^x.

Proof. I first characterize an efficient architecture, which is regular. Suppose a regular network g that is neither empty nor complete. Thus, there exist distinct firms i, j, and k with $ij \in g$ and $ik \notin g$. For efficiency of g, it must hold for any $ij \in g$ that $W(g) -$

$W(g-ij) \geq 0$. In network $g-ij$, however, $\eta_k(g-ij) = \eta_j(g-ij)+1$ and therefore, from *convexity in links*, $W(g+ik) - W(g) > W(g) - W(g-ij) \geq 0$. This is a contradiction to network g having an efficient architecture. Hence, if the efficient network is regular it must either be the *empty* network or the *complete* network.

I turn to a characterization of irregular architectures. Suppose first that network g^* with an efficient and irregular architecture induces a two-point degree partition, $\{h_0, h_m\}$ with $0 < m < n-1$, and suppose $|h_m| = 2$. Then we have a *dominant group*. Assume next a network g that induces a two-point degree partition with $|h_m| > 2$, and in which for every firm $i \in h_m$ it holds $\eta_i(g) < |h_m| - 1$. Hence, there exist distinct firms $i, j, k \in h_m$, with $ik \in g$ and $ij \notin g$. For g having an efficient architecture, it must hold $W(g) - W(g-ik) \geq 0$. However, in network g it is $\eta_j(g-ik) = \eta_k(g-ik)+1$. Hence, *convexity in links* applies and therefore $W(g+ij) - W(g) > W(g) - W(g-ik)$. This is a contradiction to g having an efficient architecture. Thus, if an efficient network g^* induces a two-point partition with $|h_m| > 2$ then $\eta_i(g^*) = |h_m| - 1$ for every $i \in h_m$. This is a *dominant group*.

Suppose next that a network g^* with an efficient architecture induces a degree partition, $\{h_{l_1}, h_{l_2}, ..., h_m\}$, with at least two groups of firms with a positive degree. I show that for every firm i with $\eta_i(g^*) > 0 : ij \in g^*$, if $j \in h_m$. Suppose not and take a network g that induces a degree partition with more than one group of positive degree and with distinct i and j, such that $\eta_i(g) > 0$ and $j \in h_m$ but $ij \notin g$. In this network, there is a firm k with $ik \in g$. However, since $\eta_k(g-ik) \leq \eta_j(g-ik)-1$, *convexity in links* applies and therefore $W(g+ij) - W(g) > W(g) - W(g-ik)$. This is a contradiction to g being efficient. In an efficient network g^*, it therefore holds for any $j \in h_m$ that $ij \in g^*$ for every firm i with $\eta_i(g^*) > 0$. This is a *core-periphery* network. □

The proof bases exclusively on the convexity property of Lemma 2. An efficient network has one of the architectures illustrated in Figure 1. A commonality of all the irregular networks in the figure is that they have a high degree variance, and hence are characterized by a high degree of concentration. This is the result of the interplay between convexity and submodularity. Convexity makes it socially beneficial to form a link between those firms that are already involved in a lot of ties. Submodularity, on the other hand, dilutes the benefits from linking the active firms also to the poorly connected firms.[8] This suggests that a disparity between firms might actually be a desirable characteristic of a network. It is still an open question, however, whether and when the irregular architectures welfare-dominate the empty network and the complete network. This is further investigated below.

Density and degree variance. The following result shows that social welfare in a network is intimately related to two simple properties of the degree distribution.

[8]Although the proof of Proposition 2 does not take explicit advantage of submodularity, it plays an important role as I show elsewhere. If the welfare function is super- instead of submodular in links, which is the case when the firms produce complimentary products or sell on independent markets, the efficient network is either only the empty network or the complete network (Westbrock, 2007).

Lemma 3. *The total surplus and the consumer surplus in network g can be written as functions of the density and the degree variance. In particular, $W(g) = Y[D(g), C(g)]$ with*

$$Y[D(g), C(g)] = \xi_1 + \xi_2 D(g) + \xi_3 D(g)^2 + \xi_4 C(g), \tag{4}$$

where $\xi_1, \xi_3 > 0$ and $\xi_4 \geq 0$, with strict inequality, if additionally, in case that firms compete in quantities and $W(g)$ represents the consumer surplus, $\beta < 1$.

The proof and the precise specification of the parameters are given in Appendix B. The lemma implies that the welfare-related properties of a network are completely captured by the density and the degree variance. The density captures the contribution to welfare from the total amount of collaborative activity. This contribution can be positive or negative depending on the sign of ξ_2 and the size of ξ_3. The final summand in equation (4) captures the welfare effects from the dispersion of collaboration ties among the firms. Since ξ_4 is typically strictly larger zero, it follows that, having fixed the network density, social welfare increases with the concentration of collaborative activities.

Proposition 3. *For any $0 \leq D \leq 1$, an efficient network g^* with $D(g^*) = D$ is maximally concentrated.*

This confirms our findings from the socially managed industry in Section 3, however, in a more challenging setting: maximal concentration is efficient, even though the firms are engaged in imperfect competition and even though they sell differentiated products. The underlying intuition is the following.

Suppose a regular network, g^r, where marginal production costs are given by $c_i = c_j$ and prices by $p_i = p_j$ for any $i, j \in N$. Suppose we remove a link ij and connect firm i to another firm k instead. In this way, concentration increases. The representative consumer is affected as follows: given the products of the firms are not homogenous, p_j increases and p_k declines, both at the same absolute rate. Hence, in the more concentrated network, the consumer can still afford the old bundle of goods. However, it is well known from consumer theory that he can save costs and still obtain the same utility level by shifting some consumption from good j to k. Furthermore, with any further price reduction of p_k, compensated by a price increase of another good, the consumer can save even more costs, because his expenditure function is concave in prices (see e.g., Deaton and Muellbauer, 1980). The utility-maximizing network is therefore a 'corner solution'. Similar, also the industry profit-maximizing network is maximally concentrated, because the function of joint profits is concave in the marginal production costs.[9]

A different interpretation of Proposition 3 is that it places a necessary restriction on the structure of an efficient network. The proposition implies that we can limit our search to

[9]Note that the degree of concentration is irrelevant for consumers ($\xi_4 = 0$), if the firms sell a homogeneous product. The reason is that all firms need to choose the same price to stay in the market, regardless of the asymmetry in the network. Moreover, since it is a feature of the model that any two networks with the same density are aligned with the same average market price, the distribution of links does not matter.

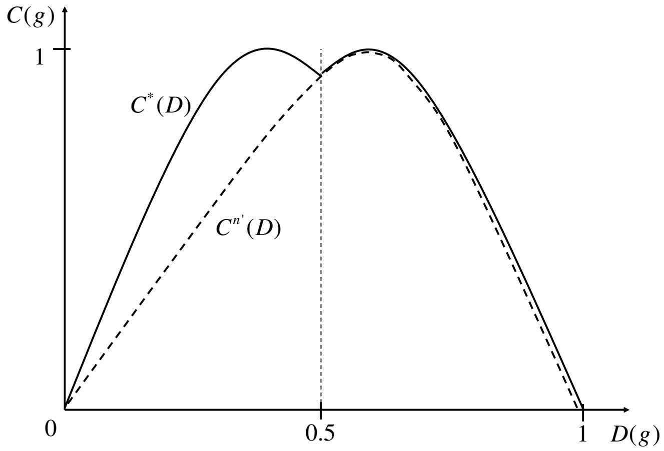

Figure 2: Network concentration as a function of density

the set of maximally concentrated networks. More precisely, define the function $C^*(D) = \max\{C(g) : D(g) = D\}$ for any $0 \leq D \leq 1$. Denote the set of maximally concentrated network given $G^* = \{g : C(g) = C^*(D)\}$. The concentration in an efficient network is now equivalently determined by $C(g^*) = C^*(D^*)$, where

$$g^* = \underset{g \in G^*}{argmax}\{W(g)\} \quad \text{and} \quad D^* = \underset{0 \leq D \leq 1}{argmax}\{Y[C^*(D), D]\}.$$

Hence, the search for the density and the concentration in an efficient network simplifies to a optimization problem in density only. Snijders (1981) determines the architectures of the networks in G^*. Unfortunately, the term $C^*(D)$ is rather complicated for general n and D. Therefore, I investigate the case of a large industry, for which $C^*(D)$ can be approximated by a much simpler expression. An analysis of the efficient network in an exemplary small industry is delegated to Section 4.5.

Example of a large industry. Appendix C shows that $C^*(D)$ has a limiting value given by

$$\lim_{n \to \infty} C^*(D) = \frac{256}{27} \begin{cases} (1-D)^{\frac{3}{2}}\left(1-(1-D)^{\frac{1}{2}}\right) & \text{if } 0 \leq D \leq \frac{1}{2} \\ D^{\frac{3}{2}}\left(1-D^{\frac{1}{2}}\right) & \text{if } \frac{1}{2} \leq D \leq 1 \end{cases}$$

The expression is illustrated in Figure 2. For $\frac{1}{2} \leq D \leq 1$, this is the concentration in a dominant group, and for $0 \leq D \leq \frac{1}{2}$ it is the concentration in a core-periphery architecture. Appendix C derives moreover a limiting value for the normalized $Y[C^*(D), D]$. The existence of this limiting value, yet, requires that the costs of linking, f, are proportional to the number of firms in the industry. Otherwise, $Y\left[C^*(D), D\right]$ is uniquely maximized for $D = 1$. Assuming differentiability of the limit value in addition, we obtain the following result:

Proposition 4. *In a large industry and for $f = n\phi$, with $\phi > 0$, the normalized total surplus and the normalized consumer surplus, net of linking costs, have limiting values, $Y_\infty\left[C^*(D(g)), D(g)\right] = \lim_{n\to\infty}[W(g) - W(g^e)]/n^3$. Suppose $Y_\infty[\cdot]$ is continuously differentiable, there exist $\phi \in (\phi_1, \phi_2)$, with $\phi_1 > 0$, such that*

$$\frac{\partial}{\partial D} Y_{\phi,\infty}\left[0, 0\right] > 0 \quad \textit{and} \quad \frac{\partial}{\partial D} Y_{\phi,\infty}\left[0, 1\right] < 0,$$

if additionally, in case that firms compete in quantities and $Y_\infty[\cdot]$ represents the consumer surplus, $\beta < 1$.

Thus, for $\phi \in (\phi_1, \phi_2)$ the efficient network in a large industry is certainly asymmetric. The proof and the precise $Y_\infty\left[C^*(D), D\right]$ are given in Appendix C. The proposition confirms the prevalence of concentration, but also provides a new insight on its nature: the extent of concentration is sensitive to the costs of forming collaborative ties. This becomes more clear, when we specify $Y_\infty\left[C^*(D), D\right]$. For example, let the term represent total welfare and let the firms compete in quantities. Moreover, let us use the parameter A, with $A > 1$, to depict the market size in a large industry.[10] The requirement for an asymmetric efficient network is

$$\gamma^2 \frac{4A(2-\beta) - 3 + \beta}{4\left(2-\beta\right)^2} < \frac{1}{4}\phi < \gamma^2 \frac{4A(2-\beta) + 3 - \beta}{4\left(2-\beta\right)^2}.$$

For ϕ being large, both the left-hand side (LHS) and the right-hand side (RHS) are smaller than the middle term (M). Hence, the empty network maximizes welfare. On the other hand, for small ϕ, both terms are greater (M), meaning the complete network is efficient. Asymmetry is desired in industries with intermediate levels of linking costs. The comparative statics with respect to the other project and market characteristics is the following: since (LHS) and (RHS) increase in A, γ^2, and β, it follows that, adapting the argument from above, concentration is typically favored in markets of medium size, for medium R&D productivity, as well as medium levels of product substitutability.

This suggests that concentration is efficient, when collaborative links are moderately productive and competition is moderately intense. For example, the inequalities are simultaneously satisfied for $A = 2$, $\gamma = 1$, and $\beta = 0.5$, if additionally $4.22 < \phi < 6.44$. However, we can verify that the difference between (RHS) and (LHS) is larger under price competition than under quantity competition. Thus, efficient concentration seems to be a more common phenomenon under the more intense price competition.

[10]More precisely, it is the effective market size, $\alpha - \gamma_0$, A times the minimum effective market size determined by (A1).

Concentration of market shares. Proposition 3 has another implication. The distribution of collaborative links also determines the firms' market shares and hence concentration in the product market. Denoting by $m_i = q_i / \sum_{i \in N} q_i$ the market share of firm i and by $\overline{m} = 1/n$ the average market share, we can write for any network g

$$m_i(g) - \overline{m}(g) = \frac{\chi_1}{\chi_2 + \chi_3 \overline{\eta}(g)} \left[\eta_i(g) - \overline{\eta}(g) \right],$$

where the χ's depend on the mode of competition, but can shown to be always strictly positive. It follows that, for given n and $\overline{\eta}(g)$, a typical measure of product market concentration, like the *Herfindahl* index, is proportional to the degree variance in a network. Proposition 3 therefore suggests that, if it is for the purpose of concentrating the R&D production around a group of firms, concentration in the product market is a tolerable side effect. Note, however, that the gains from concentration are not only appropriated by the firms, because it is in the consumers' interest as well.

4.4 Collaboration networks in equilibrium

A main finding of the previous analysis is that in markets with imperfect competition an efficient network typically exhibits some concentration of collaborative links. This puts the observations by Hagedoorn and Schakenraad (1992) and Powell et al. (2005) of highly concentrated networks in several high-tech industries in a rather positive light. I now investigate the pair-wise equilibrium networks of the link formation game. A first question is how well the model does in 'matching' the data. Moreover, the following characterization results will be used in a subsequent comparison of the equilibrium and the efficient networks.

Convexity and strategic substitutes. Because the number of collaborative ties determines a firm's competitive position in the market, we obtain the following complement to Lemma 2.

Lemma 4. *A firm's profit is:*
(i) convex in links: $\Pi_i(g + ij + ik) - \Pi_i(g + ik) > \Pi_i(g + ik) - \Pi_i(g)$ *for any* g *with* $ij, ik \notin g$, *and*
(ii) links are strategic substitutes: $\Pi_i(g + ij + kl) - \Pi_i(g + kl) < \Pi_i(g + ij) - \Pi_i(g)$ *for any* g *with* $ij, kl \notin g$ *and* $k, l \in N \backslash \{i\}$.

The proof is omitted.

Network architecture. The lemma allows for a characterization of the pair-wise equilibrium networks. It suffices to apply a proposition by Goyal and Joshi (2006) showing that network asymmetry is a typical outcome under convexity of the profit function.

Proposition 5 (Prop. 3.1 Goyal and Joshi (2006)). *Suppose the profit function in a link formation game satisfies convexity and market-wide externalities. A pair-wise equilibrium network exists and is either empty, complete, or has a dominant group architecture.*

A dominant group architecture can entail a high concentration of collaborative ties. As illustrated in Figure 2, in particular the dense dominant groups, with $0.5 < D(g^{n'}) < 1$, attain maximal concentration.

Sufficient condition for concentration. The following question is under which conditions on costs and market characteristics concentration is a common characteristic of every equilibrium network. This is the result.

Proposition 6. *Suppose $\beta > \beta^*$, with $\beta^* \leq 1/2$, but regardless of whether the firms compete in quantities or in prices. It exist $f \in (f_1, f_2)$, with $f_1 > 0$, such that every pair-wise equilibrium network has a dominant group architecture.*

Proof. For the empty network to be supported as a pair-wise equilibrium network, necessarily $\frac{1}{2}f \geq \pi_i[1,1] - \pi_i[0,0]$. Otherwise, condition (ii) is violated. For the complete network to be supported as a pair-wise equilibrium network, necessarily $\frac{n-1}{2}f \leq \pi_i[(n-1),(n-1)^2] - \pi_i[0,(n-1)(n-2)]$. Otherwise, condition (i) is violated. In combination with Proposition 5, the proposition follows, if and only if we can find f such that

$$\frac{1}{n-1}\left(\pi_i[(n-1),(n-1)^2] - \pi_i[0,(n-1)(n-2)]\right) < \frac{1}{2}f < \pi_i[1,1] - \pi_i[0,0]. \qquad (5)$$

This requires $\beta > \frac{2}{n+1}$ under quantity competition and $\beta > [n-5+\sqrt{9n^2-10n+1}]/[2\,(n^2-3)]$ under price competition. $\square$

Hence, if the products of the firms are sufficiently close substitutes there exist some intermediate levels of linking costs, for which every equilibrium network exhibits concentration. A comparative static analysis of the inequalities in (5) with respect to the other market characteristics provides the following insights: first, concentration typically occurs for moderate levels of research productivity and for a moderate market size. This is because (LHS) and (RHS) are both increasing in γ and $\alpha - \gamma_0$. Second. since (LHS) is decreasing in n and (RHS) first increases and then decreases as well, concentration is a phenomenon of medium-sized industries. Finally, a comparison of price and quantity competition produces a ambiguous results. For small β and n, the difference between (RHS) and (LHS) is larger under price competition, and concentration seems therefore more typical there. On the other hand, for β and n being large, the opposite is true. To summarize, just as for the efficient networks, in particular markets of medium size and with a medium research productivity appear to be prone to concentration. On the other hand, since we require $\beta > \beta^*$, concentration is a more common equilibrium phenomenon the more intense the competition in a market is.

Data matching. Here, I shortly compare the model predictions with the empirical findings on alliance networks in three high-tech industries. A first interesting property of a dominant group architecture is that, as shown by Goyal and Joshi (2006), the firms in the central group attain higher profits and have a larger market share than the firms at the fringe. This is consistent with the findings of Hagedoorn and Schakenraad (1992) that the leading companies in the information technology industries are also central in the network. Second, I also tried to match the model predictions on the network concentration. Appendix C provides the formula for the concentration in a dominant group network as a function of density. The respective figures for the real networks are taken from Powell et al. (2005) and Duysters and Vanhaverbeke (1996). The comparison produces, however, a rather unclear picture. While for the biotech and pharmaceutical firms in the 1990s it is $C(g^{bio}) = 1.36 \times 10^{-4}$ at a density of $D(g^{bio}) = 2 \times 10^{-3}$, a dominant group of the same density has $C = 8.1 \times 10^{-4}$, which is about six times higher.[11] On the other hand, Duysters and Vanhaverbeke (1996) calculate for the DRAM industry of the 1980s $C(g^{dram}) = 4.25 \times 10^{-2}$ and $D(g^{dram}) = 6.96 \times 10^{-2}$ and for the RISC industry $C(g^{risc}) = 8.1 \times 10^{-1}$ and $D(g^{risc}) = 2.5 \times 10^{-1}$. The dominant groups of the same density have $C = 7.14 \times 10^{-2}$ and $C = 5.88 \times 10^{-1}$ respectively. Hence, the real networks are in one case substantively more and in two cases substantively less concentrated.

4.5 Match of Equilibrium and Efficiency

We have seen that both the equilibrium networks and the efficient networks typically exhibit a high degree of concentration of collaborative ties. In the following, I investigate for a small market with seven firms and various levels of linking costs whether or not an equilibrium network coincides with an efficient structure. The analysis focuses on the empirically most relevant sparse networks.

Consider a Cournot-oligopoly of seven firms and fix $\gamma = 1$ and $\alpha - \gamma_0 = 60\beta/(2-\beta)$. For some moderate degree of product substitutability ($\beta = 0.8$), density and concentration in the equilibrium networks and the efficient networks are illustrated in Figure 3. The figure shows that the density of both networks decreases in linking costs, whereas concentration is non-monotonic in costs. Moreover, because the efficient network is asymmetric and actually highly concentrated for $f \in [12.5\,,\,16.5]$, we can confirm Proposition 4 for this exemplary small industry. Finally, the figure points to a tension between equilibrium and efficiency: any sparse equilibrium network is too little concentrated. A comparison of densities does, on the other hand, produce a rather unclear picture.

To make the point clear, I focus on the regime $f \in [14.5\,,\,16]$, for which the empirically most relevant sparse but non-empty equilibrium networks are supported. The left graph shows that in this cost interval there exists an equilibrium network that is denser than the efficient structure as well as another sparser equilibrium network. On the other hand, take the right graph and $f \in [15.25\,,\,16]$. It becomes clear that any equilibrium network in this cost interval is too little concentrated as compared to the efficient network.

I have repeated the exercise for various degrees of substitutability and found the same kind of tension between equilibrium and efficiency, as long as $\beta \in (0.69\,,\,0.83)$. For larger

[11] The figures for the biotech and pharmaceutical industry are calculated on basis of Figure 3 in Powell et al. (2005).

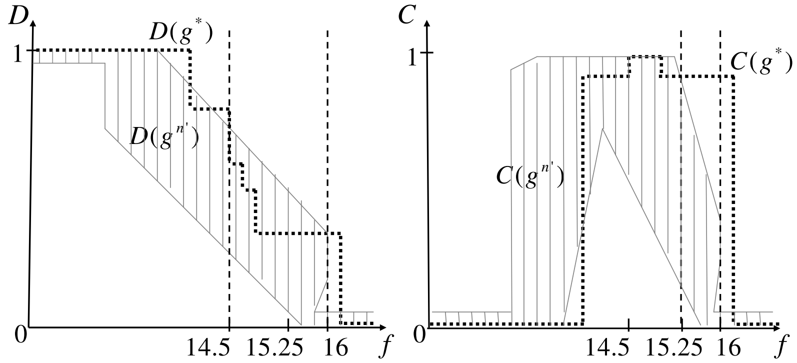

Figure 3: Density and concentration for moderate product substitutability

NOTE. The figure illustrates the density (D) and the concentration (C) of the pair-wise equilibrium networks and the efficient network for various linking costs (f). For any cost level on the x-axis, the grayish areas indicate the range of densities and concentration, respectively, that are supported in a pair-wise equilibrium network. The dotted lines show the density and concentration of the efficient network.

(smaller) β, the sparse equilibrium networks are denser (sparser) than the efficient structure, which is then the empty (complete) network. This suggests that, from a welfare viewpoint, the dispersion of collaborative ties matters only for moderate market rivalry, where the equilibrium networks suffer from too little concentration. Although they are too concentrated under intense or weak market rivalry, the main problem is a discrepancy in the amount of collaborative activities.

4.6 Robustness

The analysis so far has assumed that all firms produce positive output in any possible network and that there are no involuntary technological spillovers between firms. Yet, both issues might seriously violate the favorable assessment of a highly concentrated structure. I discuss here the robustness of the results on their welfare-superiority. Moreover, I check the implications of adding in-house R&D projects to the model.

Market exit. According to (A1), no firm is forced out of the market by occupying only a peripheral position in the network and thereby having a comparative cost disadvantage. Here, I relax the assumption and investigate an implication for the efficient network of the link formation game.

Suppose $n > 3$. In the spirit of Proposition 3, let us compare total welfare in a regular network, g^r, and in a dominant group, $g^{n'}$, of the same density. To incorporate the effect of market exit, let (A1) be violated for the $n-n'$ firms that are isolated in network $g^{n'}$. Hence, for any $i \in h_0(g^{n'})$ it shall be $q_i(g^{n'}) \leq 0$. Their quantities are set to zero. Moreover, the remaining firms $j \in h_{n'-1}(g^{n'})$ compete in the market, given n' firms. Suppose competition is in quantities. The market exit condition becomes $(2-\beta)(\alpha-\gamma_0) \leq \beta n'(n'-1)\gamma$. In network g^r, total welfare can be written as

$$W(g^r) = n\big(\alpha - \gamma_0 - \frac{n'(n'-1)}{n}\gamma\big) q_i(g^r) - \frac{n}{2}\big(1+(n-1)\beta\big) q_i(g^r)^2 - \frac{1}{2}n'(n'-1)f,$$

which is under quantity competition

$$= \frac{(n(\alpha-\gamma_0) + n'(n'-1)\gamma)^2}{2n(2+(n-1)\beta)} \frac{(3+(n-1)\beta)}{(2+(n-1)\beta)} - \frac{1}{2}n'(n'-1)f.$$

Note first that for $n = n'$, network g^r is the dominant group network and therefore $W(g^r) = W(g^{n'})$. Second, write $n = n' + (n-n')$ and differentiate $W(g^r)$ with respect to $(n-n')$. It is $(3+(n-1)\beta)/(2+(n-1)\beta)$ positive, but declining in $(n-n')$. Moreover, also the first factor in $W(g^r)$ is positive and its derivative is equal to

$$[n(\alpha-\gamma_0) + n'(n'-1)\gamma] \frac{n\,(2-\beta)\,(\alpha-\gamma_0) - (2+(2n-1)\beta)\,n'(n'-1)\gamma}{(2+(n-1)\beta)^2\, n^2}.$$

Note now that $(2+(2n-1)\beta)\,n'(n'-1)\gamma > (2n-1)\beta n'(n'-1)\gamma$, which is again greater $n(\alpha-\gamma_0)(2-\beta)$ due to the market-exit condition. Thus, the derivative is negative and therefore $W(g^r) < W(g^{n'})$ for any $n' < n$. Moreover, the dominant group performs better the larger the difference $n-n'$, and hence the higher the concentration in $g^{n'}$. This suggests that highly concentrated networks are also efficient under market exit.

Spillovers. The focus of a large body of the literature on research collaboration are the effects of knowledge spillovers to the whole industry (d'Aspremont and Jacquemin, 1988) or via the network (Goyal and Moraga-Gonzáles, 2001). Due to the complications involved in the analysis of the second type, I focus here on a robustness check with respect to industry-wide spillovers.[12] A modification of (A2) in this direction is the following. Suppose firm i's marginal cost of production in network g is given by

$$c_i(g) \;=\; \gamma_0 - \gamma\big[(1-\theta)\sum_{j\in N_i(g)} \min\{x_{ij},1\} + \theta \sum_{kl\in g} \min\{x_{kl},1\}\big],$$

[12] See Deroïan (2005) for an investigation of network-dependent spillovers.

with $0 < \theta < 1$. Hence, the unit costs are additionally reduced by collaborative projects not in the neighborhood of firm i. It turns out that the results on the efficient network carry over to this modified model, as long as the rate of spillovers is not too high.

A unit R&D investment has now the following impact on the second-stage quantities

$$\begin{aligned}
\frac{\partial q_i}{\partial x_{ij}} &= -\gamma\left(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_i}{\partial c_j} + \theta \sum_{k\in N\setminus\{i,j\}} \frac{\partial q_i}{\partial c_k}\right) \\
\frac{\partial q_i}{\partial x_{jk}} &= -\gamma\left(2\frac{\partial q_i}{\partial c_j} + \theta\frac{\partial q_i}{\partial c_i} + \theta \sum_{l\in N\setminus\{i,j,k\}} \frac{\partial q_i}{\partial c_l}\right).
\end{aligned}$$

Suppose θ is sufficiently small such that $\partial q_i/\partial x_{ij} > 0$. For example, under Cournot competition with homogenous products this is satisfied for any $\theta < 1$. It follows that total welfare and profits are convex with respect to R&D investments and, if the additional requirements of Lemmas 2 (i) and 4 (i) are met, collaborative links. Thus, Lemma 1 and Propositions 1, 2, and 5 carry immediately over for the total surplus and profits. For the convexity of the consumer surplus, we require a somewhat lower spillover rate (under Cournot competition with $\beta = 1 : \theta < 0.95$).

Furthermore, let $\partial q_i/\partial x_{jk} < 0$ ($\theta < 0.66$). The total welfare is then submodular in links and links are strategic substitutes. Hence, a dominant group or a core-periphery are typical architectures of an efficient network, and Proposition 6 applies to the equilibrium networks. For the consumer surplus, we require again a lower spillover rate (under Cournot competition with $\beta = 1$, the conditions for submodularity cannot be met).

Finally, since the terms $\partial q_i/\partial x_{ij}$ and $\partial q_i/\partial x_{jk}$ are independent of network g, the social welfare can be expressed as a function of density and concentration. Thus, we can apply Lemma 3 to the case of spillovers. Moreover, if the requirements on ξ_4 of Lemma 3 are satisfied but for any $\theta < 1$, total welfare and consumer surplus increase in the degree of concentration in the network. Proposition 3 therefore holds as well.

In-house R&D. I have excluded in-house R&D projects from the analysis. Enabling a firm to do a single in-house project does not change any of the main results. Suppose the same research technology as for a cooperative project ((A2) and (A3)). Since profits and social welfare are then convex with respect to in-house R&D investments, Lemma 1 applies to these projects as well. Moreover, collaborative links still satisfy convexity, submodularity, and strategic substitutability. Thus, Lemmas 2 and 4 and Propositions 2 and 5 carry over. The limitation of this modification is that an empty network can be supported in a pair-wise equilibrium with no R&D investments at all as well as with a single firm investing in its in-house project. Proposition 1 and Lemma 3 therefore need to be adjusted.

5 Concluding remarks

In his seminal article, Arrow (1962) points to the imperfections aligned with the demand and the production of the commodity information, and to the consequences for the market structure in innovating industries. I have studied two models on the formation of an R&D collaboration network and investigated the consequences of the indivisibility in the use of R&D output. The fact that the output of a joint project can be applied to any unit of the firms' products, independent of their production scales, creates private and social incentives for a concentration of costly research output.

The first aim of the paper is to explain the fact that in several high-tech industries the industry leaders are also involved in a large number of R&D alliances. Although the fit of the precise model predictions with three exemplary alliance networks is rather weak, my analysis is qualitatively confirmative: the equilibrium networks typically exhibit a concentration of collaborative links. Moreover, the well-connected firms attain a larger market share and higher profits in equilibrium.

The second question is how the empirical pattern can be assessed from a societal viewpoint. My analysis of the welfare-efficient networks suggests that the collaborative production of R&D should preferably be organized in a network dominated by a small number of firms. This conclusion turns out to be robust to a host of challenges. In particular, concentration is efficient despite the adverse effect on competition in the product market and despite consumers' tastes for differentiated products. I have also checked the robustness with respect to a setting, where concentration leads to market exit of the firms in the periphery of the network.

Another interesting finding of this analysis is that, for any two industries with the same number of firms and the same number of collaborative links, the extent of network concentration is proportional to the Herfindahl index of product market concentration. This has a surprising implication for the index. In contrast to the common interpretation, the extent of product market concentration may be positively related to the welfare generated in a market.

Furthermore, the models show that the welfare in a network increases in the extent of concentration of collaborative links. This might have an interesting implication for the selection of partner firms in a policy program to foster collaborative R&D activity, like the European EUREKA. The result suggests that collaboration should not be encouraged between the small and young enterprises, which get a favorable treatment in the program, but between the large and already very actively collaborating companies.

As a caveat to the normative conclusions, there are some restrictive assumptions underlying the models in this paper. I have confined my analysis to collaborative projects bearing an incremental innovation, meaning that a project aims at improving the current products of the partners. On the other hand, a joint project may also enable a drastic innovation that replaces their old generation of products. The welfare assessment of an asymmetric market structure is a completely open issue in this context. Other issues, which are highly relevant but have only shortly been addressed in this study, are market exit and network-dependent spillovers.

A R&D investments

I begin by deriving a convexity property of R&D investments:

Convexity property. Because the unit-production costs of a firm are linear in R&D efforts and moreover since $\pi_i = \mu q_i^2$, $\frac{\partial U}{\partial p_i} = -q_i$, and $p_i = \mu q_i + c_i$, for any network g, $0 \leq x_{ij} \leq 1$, and $ij \in g$:

$$\frac{\partial^2 \pi_i}{\partial^2 x_{ij}} = 2\mu \left(\frac{\partial q_i}{\partial x_{ij}}\right)^2 \tag{6}$$

$$\frac{\partial^2 \sum_{j \in N} \pi_j}{\partial^2 x_{ij}} = 2\mu \left[\sum_{k \in \{i,j\}} \left(\frac{\partial q_k}{\partial x_{ij}}\right)^2 + \sum_{l \in N \setminus \{i,j\}} \left(\frac{\partial q_l}{\partial x_{ij}}\right)^2\right] \tag{7}$$

$$\frac{\partial^2 U}{\partial^2 x_{ij}} = \mu \left[\sum_{k \in \{i,j\}} \frac{\partial q_k}{\partial x_{ij}} \left(\frac{\gamma}{\mu} - \frac{\partial q_k}{\partial x_{ij}}\right) - \sum_{l \in N \setminus \{i,j\}} \left(\frac{\partial q_l}{\partial x_{ij}}\right)^2\right] \tag{8}$$

where $\frac{\partial q_i}{\partial x_{ij}} = -\gamma(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_i}{\partial c_j})$ greater zero and $\frac{\partial q_i}{\partial x_{jk}} = -2\gamma \frac{\partial q_i}{\partial c_j}$ smaller zero. The convexity of the firm profits follows immediately from the fact that (6) and (7) are both positive. Although the two summands in (8) are of opposite sign, their sum is positive regardless of the mode of competition between firms. This follows from the fact that (8) becomes under quantity competition

$$2\gamma^2 \frac{4 + (4n-8)\beta + (n^2 - 8n + 9)\beta^2 - (n^2 - 4n + 3)\beta^3}{(2 + (n-1)\beta)^2 (2-\beta)^2}$$

and under price competition

$$\frac{2\gamma^2}{\mu^p} \frac{(1 + (n-2)\beta)\left(4 + (8n-16)\beta + (5n^2 - 24n + 25)\beta^2 + (n^3 - 9n^2 + 21n - 15)\beta^3\right)}{(2 + (n-3)\beta)^2 (2 + (2n-3)\beta)^2}.$$

Both are positive for $n > 2$ and $\beta \in (0, 1]$ under quantity competition and for $n > 2$ and $\beta \in (0, 1)$ under price competition respectively.

I continue with an implication for the efficient and the pair-wise equilibrium R&D investments.

Proof of Lemma 1. Suppose a network g with $0 < x_{ij} < 1$ for some $ij \in g$. Suppose moreover, without loss of generality, $x_{i,j} > 0$. For $(g, x_{i,j}, x_{j,j}, x_{-ij})$ to constitute a pair-wise equilibrium:

$$\pi_i(g, x_{i,j}, x_{j,i}, x_{-ij}) - \frac{1}{2} f x_{i,j} \geq \pi_i(g, 0, x_{j,i}, x_{-ij}).$$

Due to the convexity of profits, we have, however, $\pi_i(g, 1-x_{j,i}, x_{j,i}, x_{-ij}) - \frac{1}{2} f(1-x_{j,i}) > \pi_i(g, x_{i,j}, x_{j,i}, x_{-ij}) - \frac{1}{2} f x_{i,j}$. Hence, firm i is better off choosing $\widehat{x}_{i,j} = 1 - x_{j,i}$. Profile $(g, x_{i,j}, x_{j,i}, x_{-ij})$ is therefore not a pair-wise equilibrium. Suppose now $ij \in g$ with $x_{ij} = 0$. For any arbitrarily small linking costs, firm i is better off by deleting the link. Profile $(g, 0, 0, x_{-ij})$ therefore does not constitute an equilibrium either. It follows that in a pair-wise equilibrium $(\widehat{g}, \widehat{x})$ for any $ij \in \widehat{g}$: $\widehat{x}_{ij} = 1$. A similar argument applies to the efficient R&D investments on a link, and we therefore find that the maximal attainable investment of *one* is individually as well as socially rational. □

Another related implication is the following:

Lemma 5. *Consider any given $(s_{-i}, e_{-i}) = (s_j, e_j)_{j \in N \setminus \{i\}}$. It is $(\widehat{s}_i, \widehat{e}_i)$ a 'best response' strategy of firm i, if for any $ij \in g(\widehat{s}_i, s_{-i})$ it holds $\widehat{x}_{i,j} = 1 - x_{j,i}$. Moreover, it is (s_i^*, e_i^*) 'welfare dominant', if for any $ij \in g(s_i^*, s_{-i})$ it holds $x_{i,j}^* = 1 - x_{j,i}$.*

Proof. Suppose a network $g(\widehat{s}_i, s_{-i})$ with $0 \leq x_{ij} < 1$ for some $ij \in g$. For $(\widehat{s}_i, e_i)$ to be a 'best response' to (s_{-i}, e_{-i}):

$$\pi_i(g, x_{i,j}, x_{j,i}, x_{-ij}) - \frac{1}{2} f x_{ij} > \pi_i(g - ij, 0, 0, x_{-ij})$$

This cannot be satisfied for $x_{ij} = 0$, because of the linking costs. Strategy $(\widehat{s}_i, e_i)$ is therefore not a best response. For any $0 < x_{ij} < 1$, we have $\pi_i(g, 1 - x_{j,i}, x_{j,i}, x_{-ij}) - \frac{1}{2} f > \pi_i(g, x_{i,j}, x_{j,i}, x_{-ij}) - \frac{1}{2} f x_{ij}$ due to the convexity of a firm's profit function. Hence, firm i is better off by choosing $\widehat{x}_{i,j} = 1 - x_{j,i}$. Strategy $(\widehat{s}_i, e_i)$ is therefore not a best response. It follows that $(\widehat{s}_i, \widehat{e}_i)$ requires $\widehat{x}_{i,j} = 1 - x_{j,i}$ for any $ij \in g(\widehat{s}_i, s_{-i})$. A similar argument applies to the efficient R&D investments on a link. □

We can now verify the following:

Proof of Proposition 1. I begin with the equivalence of the set of equilibria. Let us depict a strategy profile s of the link formation game by a strategy profile $(s, \bar{x})$ of a game of link formation and R&D investments, Γ, where $\bar{x}$ is such that $\bar{x}_{i,j} = \bar{x}_{j,i} = 1/2$ if $ij \in g$. Using the specification of the profit functions in this section, clearly $\Pi_i(g) = \Pi_i(g, \bar{x})$ for any $s \in S$ and $i \in N$. Hence, the set of equilibrium networks in the two games are equivalent.

Let us now relate the set of equilibria of Γ to the set of equilibria of the model described in this section. Suppose $(\widehat{g}, \widehat{x})$ constitutes a pair-wise equilibrium in the model of this section. By Lemma 1 and (A4), we have $\Pi_i(\widehat{g}, \widehat{x}) = \Pi_i(\widehat{g}, \bar{x})$ for any $i \in N$. Moreover, define $X_i = \prod_{j \in N_i(g)} X_{i,j}$. The strategy set S_i of Γ is a strict subset of the strategy set $S_i \times X_i$ of the model in this section. Hence, $(\widehat{g}, \bar{x})$ constitutes a pair-wise equilibrium of Γ.

Suppose, to the converse, that $(\widehat{g}, \bar{x})$ is a pair-wise equilibrium of Γ. It is immediately clear that $\bar{x}$ satisfies $x_{ij} = 1$ for $ij \in \widehat{g}$. Moreover, for any $s \in S$ and $i \in N$: $\bar{x}_i = (\bar{x}_{i,j})_{j \in N_i(g)}$ satisfies the requirement of a best response strategy according to Lemma 5. It follows that $(\widehat{g}, \bar{x})$ constitutes a pair-wise equilibrium of the model in this section. A similar argument can be made for the equivalence of the set of efficient states. □

B Efficient collaboration network

This appendix contains the proofs of Lemma 2 and Lemma 3.

Proof of Lemma 2. I begin with part (i). Because the unit-production costs of firm i are linear in the degree of firm i in network g and moreover since $\pi_i = \mu q_i^2$, $\frac{\partial U}{\partial p_i} = -q_i$, and $p_i = \mu q_i + c_i$, for any network g with $ij, ik \notin g$:

$$\begin{aligned}\sum_{i \in N} \Pi_i(g + ij + ik) - 2\Pi_i(g + ik) + \Pi_i(g) &= 2\mu \Big[(\frac{\partial q_i}{\partial x_{ij}})^2 + \sum_{l \in N \setminus \{i,j,k\}} (\frac{\partial q_l}{\partial x_{ij}})^2 \\ &\quad + \frac{\partial q_k}{\partial x_{ik}} (q_j(g + ik) - q_k(g)) + \frac{\partial q_j}{\partial x_{ik}} (q_k(g + ik) - q_j(g)) \Big] \end{aligned} \tag{9}$$

$$\begin{aligned} U(g + ij + ik) - 2U(g + ik) + U(g) &= \mu \Big[\frac{\partial q_i}{\partial x_{ij}} (\frac{\gamma}{\mu} - \frac{\partial q_i}{\partial x_{ik}}) - \sum_{l \in N \setminus \{i,j,k\}} (\frac{\partial q_l}{\partial x_{ij}})^2 \\ &\quad + (\frac{\gamma}{\mu} - \frac{\partial q_k}{\partial x_{ik}})(q_j(g + ik) - q_k(g)) - \frac{\partial q_j}{\partial x_{ik}} (q_k(g + ik) - q_j(g)) \Big] \end{aligned} \tag{10}$$

where $\frac{\partial q_i}{\partial x_{ij}} = -\gamma(\frac{\partial q_i}{\partial c_i} + \frac{\partial q_i}{\partial c_j})$ and $\frac{\partial q_i}{\partial x_{jk}} = -2\gamma \frac{\partial q_i}{\partial c_j}$. If $\eta_j(g) = \eta_k(g) + 1$, (9) and (10) simplify to (7) and (8), respectively, since then $q_j(g + ik) - q_k(g) = \frac{\partial q_j}{\partial x_{ij}}$ and $q_k(g + ik) - q_j(g) = \frac{\partial q_j}{\partial x_{ik}}$. The convexity in links follows immediately from the *convexity property* in Appendix A. Moreover, it can be verified that for $\eta_j(g) > \eta_k(g)+1$, (9) is even greater (7) and (10) is greater (8) (for $\eta_j(g) < \eta_k(g)+1$, on the other hand, there are parameters that do not satisfy the convexity property).

I turn to part (ii). For any network g with $ij, kl \notin g$ and $k, l \in N\backslash\{i,j\}$:

$$\begin{aligned} \sum_{i\in N} \Pi_i(g+ij+kl) - \Pi_i(g+kl) - \Pi_i(g+ij) + \Pi_i(g) &= 2\mu\Big[\sum_{u\in\{i,j,k,l\}} \frac{\partial q_u}{\partial x_{ij}} \frac{\partial q_u}{\partial x_{kl}} \qquad (11)\\ &\quad + \sum_{v\in N\backslash\{i,j,k,l\}} \Big(\frac{\partial q_v}{\partial x_{ij}}\Big)^2 \Big] \\ U(g+ij+kl) - U(g+kl) - U(g+ij) + U(g) &= \mu\Big[\sum_{u\in\{i,j\}} \frac{\partial q_u}{\partial x_{kl}} (\frac{\gamma}{\mu} - \frac{\partial q_u}{\partial x_{ij}}) \qquad (12)\\ &\quad - \sum_{v\in\{k,l\}} \frac{\partial q_v}{\partial x_{ij}} \frac{\partial q_v}{\partial x_{kl}} - \sum_{w\in N\backslash\{i,j,k,l\}} \Big(\frac{\partial q_w}{\partial x_{ij}}\Big)^2 \Big] \end{aligned}$$

The summand in the parentheses of (11) is under quantity competition $-4\frac{(4+n\beta)\beta}{(2+(n-1)\beta)^2(2-\beta)^2}$ and under price competition $-\frac{4}{(\mu^p)^2}\frac{(1+(n-2)\beta)(4+(5n-8)\beta+(n^2-4n+2)\beta^2)\beta}{(2+(n-3)\beta)^2(2+(2n-3)\beta)^2}$. Both are negative for any $\beta \in (0,1)$. The summand in the parentheses of (12) is under quantity competition $-4\frac{(n-4-(n-1)\beta)\beta^2}{(2+(n-1)\beta)^2(2-\beta)^2}$, which is negative for $n > 4$ and $\beta \in (0, \frac{n-4}{n-1})$. Under price competition, the summand becomes $-\frac{4}{(\mu^p)^2}\frac{(1+\beta n-2\beta)(n-4+(n^2-5n+7)\beta)\beta^2}{(2+(n-3)\beta)^2(2+(2n-3)\beta)^2}$, which is negative for $\beta \in (0,1)$. □

Proof of Lemma 3. I will show that:

$$\begin{aligned} U(g) &= \lambda_3\big\{\lambda_1 + n\,(n-1)\,D(g)\,[\lambda_2 + D(g)]\big\} + \lambda_5 C(g) \\ \sum_{i\in N}\Pi_i(g) &= \lambda_4\big\{\lambda_1 + n\,(n-1)\,D(g)\,[\lambda_2 + D(g)]\big\} + \lambda_6 C(g) - \frac{1}{2} f n\,(n-1)\,D(g) \end{aligned}$$

with λ_1 to λ_6 as defined below, which immediately implies the lemma. Let us express the welfare in network g by

$$W(g) = [W(g) - W(g^r)] + [W(g^r) - W(g^e)] + W(g^e).$$

The first summand depicts the welfare contribution from the degree variance, whereas the second summand contains the contribution from the density (note that $D(g) = D(g^r)$ and $C(g^r) = C(g^e) = 0$). I derive each summand in turn and begin with $W(g^r) - W(g^e)$.

Using $U = \sum_{i\in N}\Big[q_i\Big(\alpha - c_i - (\mu+\frac{1}{2})q_i - \frac{\beta}{2}\sum_{j\neq i} q_j\Big)\Big]$ and $\sum_{i\in N}\pi_i = \mu\sum_{i\in N} q_i^2$, we can write

$$\begin{aligned} U(g^r) - U(g^e) &= n\,(\alpha-\gamma_0+\gamma\bar\eta)\,q_i(g^r) - n(\alpha-\gamma_0)q_i(g^e) \qquad (13)\\ &\quad - n\frac{2\mu+1+(n-1)\beta}{2}\left(q_i(g^r)^2 - q_i(g^e)^2\right) \\ \sum_{i\in N}\Pi_i(g^r) - \sum_{i\in N}\Pi_i(g^e) &= n\mu\left(q_i(g^r)^2 - q_i(g^e)^2\right) - \frac{1}{2}fn\bar\eta, \qquad (14) \end{aligned}$$

where $\bar\eta$ denotes the average degree in network g^r. It can be verified that

$$q_i(g^r) - q_i(g^e) = \bar\eta(\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}}) \quad \text{and} \quad q_i(g^e) = \frac{(\alpha-\gamma_0)}{\gamma}(\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}}).$$

For example, concerning the left-hand term, note that in network g^r a firm i is involved in $\bar\eta$ links more than in the empty network, and the other firms, $j \in N\backslash\{i\}$, are involved in $\bar\eta(\frac{n}{2}-1)$ more links.

After substituting and using the facts that $q_i(g^r)^2 - q_i(g^e)^2 = [q_i(g^r) - q_i(g^e)]\,[q_i(g^r) - q_i(g^e) + 2q_i(g^e)]$ and $\bar{\eta} = (n-1)D(g^r)$, we can simplify (13) and (14):

$$\begin{aligned} U(g^r) - U(g^e) &= n(n-1)D(g^r)\big[\lambda_2\lambda_3 + \lambda_3 D(g^r)\big] \\ \sum_{i\in N}\Pi_i(g^r) - \sum_{i\in N}\Pi_i(g^e) &= n(n-1)D(g^r)\big[\lambda_2\lambda_4 + \lambda_4 D(g^r) - \frac{1}{2}f\big] \end{aligned}$$

with

$$\begin{aligned} \lambda_2 &= \frac{2(\alpha-\gamma_0)}{\gamma(n-1)} \\ \lambda_3 &= (n-1)(\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}})\frac{2\gamma - [2\mu+1+(n-1)\beta][\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}}]}{2} \\ \lambda_4 &= (n-1)\mu(\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}})^2. \end{aligned}$$

The reader might check that $(\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}}) > 0$ and $2\gamma - [2\mu+1+(n-1)\beta][\frac{\partial q_i}{\partial x_{ij}} + (\frac{n}{2}-1)\frac{\partial q_i}{\partial x_{jk}}] > 0$, for any $\beta \in (0,1]$ and regardless of whether competition between firms is in quantities or prices.

Let us turn to the derivation of $W(g) - W(g^r)$. I start from the expressions

$$\begin{aligned} U(g) - U(g^r) &= \sum_{i\in N}\{q_i(g)\,[\alpha-\gamma_0+\gamma\eta_i(g)]\} \,-\, n(\alpha-\gamma_0+\gamma\bar{\eta})q_i(g^r) \qquad (15) \\ &\quad - (\mu+\frac{1}{2})\sum_{i\in N}q_i(g)^2 \,-\, \frac{\beta}{2}\sum_{i\in N}\sum_{j\neq i}[q_i(g)q_j(g)] \\ &\quad + n\frac{2\mu+1+(n-1)\beta}{2}q_i(g^r)^2 \\ \sum_{i\in N}\Pi_i(g) - \sum_{i\in N}\Pi_i(g^r) &= \mu\sum_{i\in N}[q_i(g)^2 - q_i(g^r)^2]. \qquad (16) \end{aligned}$$

It can be verified that $q_i(g) - q_i(g^r) = (\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}})(\eta_i(g) - \bar{\eta})$, because the difference $\eta_i(g) - \bar{\eta}$ measures, in how many links firm i is more (less) involved in network g as compared to g^r, and accordingly in how many links less (more) another firm $j \in N\backslash\{i\}$ is involved in. The following properties help to simplify (15) and (16):

(i) $\sum_{i\in N}\{[\alpha-\gamma_0+\gamma\eta_i(g)]\,q_i(g^r)\} - n(\alpha-\gamma_0+\gamma\bar{\eta})q_i(g^r) = 0$,
(ii) $q_i(g)q_j(g) - q_i(g^r)^2 = [q_i(g)-q_i(g^r)]\,[q_j(g)-q_i(g^r)] \,+\, q_i(g^r)\,[q_i(g)-q_i(g^r)+q_j(g)-q_i(g^r)]$,
(iii) $\sum_{i\in N}(\eta_i(g)-\bar{\eta}) = 0$,
(iv) $\sum_{i\in N}\sum_{j\neq i}(\eta_i(g)-\bar{\eta}) = 0$,
(v) $\sum_{i\in N}\eta_i(g)(\eta_i(g)-\bar{\eta}) = nV(g)$
(vi) $\sum_{i\in N}\sum_{j\neq i}(\eta_i(g)-\bar{\eta})(\eta_j(g)-\bar{\eta}) = -nV(g)$.

Applying all of them, we get

$$\begin{aligned} U(g) - U(g^r) &= \lambda_5 C(g) \\ \sum_{i\in N}\Pi_i(g) - \sum_{i\in N}\Pi_i(g^r) &= \lambda_6 C(g) \end{aligned}$$

with

$$\begin{aligned} \lambda_5 &= V_{max}n(\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}})\frac{2\gamma - (2\mu+1-\beta)(\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}})}{2} \\ \lambda_6 &= V_{max}n\mu(\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}})^2. \end{aligned}$$

The reader might check that $(\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}}) > 0$ and $2\gamma - (2\mu + 1 - \beta)(\frac{\partial q_i}{\partial x_{ij}} - \frac{\partial q_i}{\partial x_{jk}}) \geq 0$ for any $\beta \in (0,1]$ and regardless of the competition mode, with equality only if the firms compete in quantities and $\beta = 1$. To complete, it can be verified that $U(g^e) = \lambda_1\lambda_3$ and $\sum_{i\in N}\Pi_i(g^e) = \lambda_1\lambda_4$, where $\lambda_1 = n(\alpha - \gamma_0)^2/(\gamma(n-1))$. □

C Network density and concentration

Here, I derive density and concentration of the equilibrium and the efficient networks (in a large industry) and provide the proof of Proposition 4.

The normalized degree variance in a dominant group network of size n' is given by

$$\begin{aligned} C(g^{n'}) &= \frac{n'(n'-1-\frac{n'(n'-1)}{n})^2 + (n-n')(\frac{n'(n'-1)}{n})^2}{n}/V_{\max} \\ &= \frac{(n'-1)(n-n')}{n}\frac{n'(n'-1)}{n}/V_{\max}. \end{aligned}$$

Snijders (1981) determines the architecture of the variance maximizing networks of density D, $0 < D < 1$, which is obtained as follows: define the *complementary* network, g_c, to network g as $g_c = \{ij : i, j \in N, i \neq j, ij \notin g\}$. The complementary network has its links exactly where the other does not. Note that $D(g_c) = 1 - D(g)$ and $V(g_c) = V(g)$ for any g. A variance-maximizing network $\tilde{g}$ of density D has the following properties:

(i) either $\tilde{g}$ or $\tilde{g}_c$ partitions the set of firms into a four-point *degree partition* $\{h_0, h_{l_1}, h_{l_2}, h_{l_3}\}$, for which $l_1 = |h_{l_3}|$, $l_2 = |h_{l_2}| + |h_{l_3}| - 1$, and $l_3 = |h_{l_1}| + |h_{l_2}| + |h_{l_3}| - 1$ (see Section 4.1 for the definition of a degree partition). Moreover, $|h_{l_1}| = 1$ and $|h_{l_2}| + |h_{l_3}|$ is the largest integer I with $I(I-1)/n(n-1) < D$, if $\tilde{g}$ induces the partition of firms, and the largest integer I with $I(I-1)/n(n-1) < 1 - D$, if $\tilde{g}_c$ induces the partition, and

(ii) $\tilde{g} = \underset{g'}{argmax}\{\max[V(g'), V(g'_c)]\}$, where $g' \in \{g : i, j \in N, i \neq j, D(g) = D\}$.

According to property (i), either $\tilde{g}$ or $\tilde{g}_c$ has a dominant group-like architecture, where the size of the group of connected firms is maximized given D. The remaining links originate from the single firm in h_{l_1}, which is linked to every firm in h_{l_3}. Property (ii) then ensures that we choose a variance-maximizing network from the networks under (i).

For n being finitely large, determining the maximum variance, $V(\tilde{g})$, is cumbersome. However, for $n \to \infty$, we can exploit the fact that $\lim\left[D(g^{n'+1}) - D(g^{n'})\right] = \lim\left[\frac{n'(n'+1)}{n(n-1)} - \frac{n'(n'-1)}{n(n-1)}\right] = \lim\frac{2n'}{n(n-1)} = 0$. Hence, as an approximation, either $\tilde{g}$ or $\tilde{g}_c$ has a dominant group architecture for every $0 < D < 1$. Suppose that for n large, $V(\tilde{g})$ is a continuous function of D, which I denote by $V^*(D)$. It follows

$$\begin{aligned} \lim C^*(D) = \lim\left[V^*(D)/V_{max}\right] &\approx \lim\left[\max\left\{V(g^D), V(g^{1-D})\right\}/V_{max}\right] \\ &= \max\left\{D^{\frac{3}{2}}(1-\sqrt{D}), (1-D)^{\frac{3}{2}}(1-\sqrt{1-D})\right\}\frac{256}{27}, \end{aligned}$$

where in the first line, g^D and g^{1-D} denote dominant group networks of density D and $1-D$ respectively. For the second line, I use

$$\lim\frac{1}{n^2}V(g^D) = \lim\frac{1}{n^2}\frac{(n'-1)(n-n')}{n}\frac{n'(n'-1)}{n} = D^{\frac{3}{2}}(1-\sqrt{D}).$$

This is because $D(g^D) = [n'(n'-1)]/[n(n-1)]$, hence we have $n' = \frac{1}{2} + \frac{1}{2}\sqrt{(1+4Dn(n-1))}$ and $\lim\frac{n'(n'-1)}{n^2} = D$. Let us approximate n' by $\frac{1}{2} + n\sqrt{D}$, which is a good approximation for n large and D not too small. We obtain $\lim\frac{(n'-1)(n-n')}{n^2} = \sqrt{D}(1-\sqrt{D})$. Moreover, I use $\lim\frac{n^2}{V_{\max}} = \frac{27}{256}$.

For the following proof of Proposition 4, I specify the effective market size, $\alpha - \gamma_0$, by a parameter A, with $A > 1$. More precisely, under quantity competition $\alpha - \gamma_0 = A[\gamma\beta\,(n-1)\,(n-2)]/[2-\beta]$ and under price competition $\alpha - \gamma_0 = A[\gamma\beta\,(1+(n-2)\beta)\,(n-1)\,(n-2)]/[2+\beta(2n-5)-\beta^2(2n-3)]$, where the second factors on the right-hand sides are the minimum effective market sizes of footnote 7.

Proof of Proposition 4. Suppose a large industry, with $n \to \infty$, and suppose $f = n\phi$. Using the specification of $Y\,[C^*(D(g)), D(g)]$ from Lemma 3 and L'Hospital's rule, we get under quantity competition:

$$\begin{aligned}\lim \frac{U(g)-U(g^e)}{n^3} &= \gamma^2 \frac{A}{2-\beta} D(g) + \gamma^2 \frac{27}{512} \frac{(1-\beta)}{(2-\beta)^2} \lim C(g) \\ \lim \frac{\sum_{i\in N} \Pi_i(g) - \Pi_i(g^e)}{n^3} &= -\frac{1}{4}\phi D(g) + \gamma^2 \frac{54}{512\,(2-\beta)^2} \lim C(g),\end{aligned}$$

and under price competition:

$$\begin{aligned}\lim \frac{U(g)-U(g^e)}{n^3} &= \gamma^2 \frac{A}{2-2\beta} D(g) + \gamma^2 \frac{27}{2048(1-\beta)} \lim C(g) \\ \lim \frac{\sum_{i\in N} \Pi_i(g) - \Pi_i(g^e)}{n^3} &= -\frac{1}{4}\phi D(g) + \gamma^2 \frac{54}{2048(1-\beta)} \lim C(g).\end{aligned}$$

The efficient density is therefore determined by $D(g^*) = argmax\{\lim \frac{W(g)-W(g^e)}{n^3}\}$, with $\lim C(g) = \lim C^*(D)$. Suppose the limiting value of $Y\,[\cdot]$ is differentiable with respect to D. The first-order derivative evaluated at $D \in \{0,1\}$ is under quantity competition approximately:

$$\begin{aligned}\left(\lim \frac{U(g)-U(g^e)}{n^3}\right)'\Big|_D &\approx \gamma^2[4A(2-\beta)+(1-2D)(1-\beta)]/[4\,(2-\beta)^2] \\ \left(\lim \frac{\sum_{i\in N} \Pi_i(g) - \sum_{i\in N} \Pi_i(g^e)}{n^3}\right)'\Big|_D &\approx \gamma^2[2-4D]/[4(2-\beta)^2] - \frac{1}{4}\phi\end{aligned}$$

and under price competition approximately:

$$\begin{aligned}\left(\lim \frac{U(g)-U(g^e)}{n^3}\right)'\Big|_D &\approx \gamma^2[8A+1-2D]/[16(1-\beta)] \\ \left(\lim \frac{\sum_{i\in N} \Pi_i(g) - \sum_{i\in N} \Pi_i(g^e)}{n^3}\right)'\Big|_D &\approx \gamma^2[2-4D]/[16(1-\beta)] - \frac{1}{4}\phi.\end{aligned}$$

For example, a network that maximizes consumer surplus, net of linking costs, under quantity competition has a positive degree variance, if

$$\gamma^2 \frac{4A(2-\beta)-1+\beta}{4\,(2-\beta)^2} < \frac{1}{4}\phi < \gamma^2 \frac{4A(2-\beta)+1-\beta}{4\,(2-\beta)^2}.$$

□

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