Patent Protection, Takeovers, and Startup Innovation: A Dynamic Approach

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Abstract

The impact of IP protection on the innovation incentives of startup firms is examined in a dynamic model where an incumbent faces a sequence of potential startups and the incumbent's chance of winning an infringement lawsuit increases with the size of its patent portfolio. It is shown that takeover deals generate extra benefits for the incumbent via its enhanced future bargaining positions, a part of which accrues to the current startup as an increased bargaining share. This increased bargaining share can be large enough to justify the startup's innovation activity that would not have taken place otherwise. This effect may be greatest under moderate levels of IP protection, because the increase in the bargaining share, being proportional to the marginal benefits brought by the last patent added to the portfolio, would be too small if the protection was too weak while it would taper off too quickly if the protection was excessive.

Keywords: Patent litigation, takeovers, patent portfolios.

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1 Introduction

It is widely acknowledged that the creation in 1982 of the Central Appellate Court for the Federal Circuit (CAFC), a centralized patent court in the US that lacks an EU equivalent, strengthened intellectual property (IP) rights. As a consequence, firms became to rely more heavily on patents as a means to protect their IP, a shift from prior practice that has lead to an upsurge in patenting and has contributed to the creation of large patent portfolios by dominant US firms. Proponents of this change view stronger IP rights as having provided a fertile ground on which US technological superiority has spawned, contrary to the situation in the EU. Critics, on the other hand, acknowledge a drop in patent quality and frequent overlapping patents that have considerably increased costly patent litigation both in time and money. In addressing these concerns policy makers have introduced the Patent Reform Act of 2007, whose main role is to reduce patent litigation.

Following this trend economists recognized the need to reassess the rationale for patents given that new innovations increasingly tend to be cumulative/sequential, building upon previously-patented prior art. In such environments, due to the inevitable conflict in providing incentives to current and future innovators, the dynamic effects of patent systems can be fundamentally different from the conventional wisdom, as forcefully argued recently by Bessen and Maskin (2007) in a dynamic game between rival innovators, and by Hopenhayn *et al.* (2006) from the social planner's optimal patent design perspective. The current paper contributes to this debate by furthering our understanding of the impact of different levels of IP protection on the long-run innovation dynamics of startup firms. This shift in focus from established patentees to startup innovators is in recognition of the extent to which the latter's innovativeness has shaped modern hi-tech industries, and of their frequent inability to defend their IP rights against dominant rivals (owing to the considerable legal fees and the uncertainty surrounding patent litigation).

In the US where startups are most active, since the foundation of CAFC there has been a steady increase of takeover activities (even after discounting the surge of such activities shortly before 2001, that is attributable to the internet bubble) as shown in Figures 1-2, where established firms ventured to acquire smaller ones. True as it may be that the reasons behind such takeovers are diverse, many were driven by the desire to expand a firm's technological horizons via the purchase of innovative startup firms.[1]

[1] Restricting the argument to a few prominent examples, in the 1984 to 2001 period G.E. and Siemens acquired more than 110 and 170 firms respectively; see Dessyllas and Hughes (2005).

This drive, coined by H. Chesbrough as *open innovation*, gained prominence among firms such as Intel and Cisco; see Chesbrough (2003). The prospects of such takeovers serve as a secondary market for ideas, in addition to the NASDAQ, offering entrepreneurial entrants an extra option for reaping the benefits of their inventions. This co-evolution of the court's attitude and takeover activities is consistent with the intuition behind this paper.

Operating under the constant threat of infringement allegations from an incumbent firm, a startup's innovation incentive is influenced by the level of IP protection that the legal system would provide, in the form of upholding the incumbent's patent infringement allegation against the startup. Along these lines, in a model where an incumbent faces a sequence of potential startups and the incumbent's chance of winning an infringement lawsuit increases with the size of its patent portfolio, we show that takeover deals (out of court settlements) generate extra benefits for the incumbent via its enhanced bargaining positions in future settlement deals, and a part of such extra benefits accrues to the current startup as an increased bargaining share. This increased bargaining share may actually be large enough to justify the startup's innovation activity that would not have taken place otherwise. This effect may be greatest under moderate levels of IP protection, because the increase in the bargaining share, being proportional to the marginal benefits brought by the last patent added to the portfolio, would be too small if the protection was too weak while it would taper off too quickly if the protection was excessive.

In particular, in hi-tech industries where technology is complex and cumulative, a new innovation is likely to infringe on existing patents. Therefore, an entrant firm is likely to face the threat of litigation from a competing incumbent patent holder of a significantly larger size. Commentators argue that when IP protection is limited, and the courts have lenient views regarding alleged infringement, an entrant will abstain from innovating because its innovation will not be adequately protected from the incumbent. However, the flip side of this logic also suggests that if IP protection is strong, and the courts take a tough stance on alleged infringement, the fear of a difficult litigation battle would equally diminish the entrant's incentives to innovate.

Implicit in the argument above appears to be a presumption we adopt, namely, that an incumbent firm has accumulated a patent portfolio on a set of technologies that are interrelated and advance on a common theme (having a central idea as a backbone) and hence, form a unified technological territory. Although in reality large firms' patent portfolios tend to range across many and frequently diverging technologies, for the purpose of this paper a patent portfolio means a subset of a firm's patents that are on a specific technological terrain.[2] Thus, the larger is a firm's patent portfolio, the better described, entrenched, and protected becomes its technological territory, enhancing the firm's ability to prevail in court over infringement disputes concerning newer ideas that are perceived as having built upon the same technological terrain. This association between patent portfolios and the ability to protect an individual patent is supported by empirical findings, e.g., Lanjouw and Schankerman (2004) explained in Section 2.

As will be shown later, if IP protection is moderate, allowing the courts a balanced approach towards alleged infringement, the benefit of a takeover for the incumbent goes beyond commercializing the new innovation. This is because the incumbent capitalizes on the enhanced bargaining position that the current takeover will bring in all potential future deals by incorporating the current startup's patented ideas to its own patent portfolio, which allows it to better barricade its technological territory. Since this prospect of future surplus for the incumbent hinges on the current takeover, a part of the surplus accrues to the current startup, enlarging its bargaining share. We show that this dynamic effect of moderate IP protection can motivate the startups' innovation activities that would not take place without it, and as a consequence, increase the social welfare. We emphasize, however, that for maximum effect the level of IP protection should be selected carefully because excessive IP protection would accumulate the incumbent's bargaining power too quickly, killing off the innovation incentives for startups prematurely.

The existing literature on cumulative innovations includes Scotchmer (1996), Green and Scotchmer (1995), and Chang (1995), that focus on a single follow-on innovation; and O'Donoghue, *et al.* (1998), Hopenhayn, *et al.* (2006), and Bessen and Maskin (2007), on multiple sequential innovations. The main feature that differentiates our paper is that we explicitly model the uncertain nature of court rulings in infringement suits, based on how dissimilar firms (incumbents *vs.* startups), having amassed patent portfolios to indifferent depths, defend their IP rights. Our approach shares some key features of, and is complementary to, Bessen and Maskin (2007) who examine the dynamic interaction between equally dominant firms, with an important distinction that we do not

[2] Chemical firms frequently form such portfolios. For example, upon the invention of NYLON DuPond patented all the chemical formulae bearing resemblance to its core technology. More recently, with regard to the wireless market contended by tech giants such as Apple Inc., *Forbes* wrote "The end game: futuristic gizmos controlled by gestures that are tied wirelessly to the world around them, *protected by a broad portfolio of patents,* $\cdots$" (emphasis added); see http://www.forbes.com/2008/04/30/apple-iphone-3g-tech-wire-cx_bc_0501apple.html?partner=yahootix.

have "complementarities" between innovation efforts that appear to be essential in their analysis.

Defining boundaries of intellectual properties is inherently imperfect and as a result, disputes are inevitable and the court plays an active role in the way the patent system operates.[3] The effects of patent litigation have been studied by Meurer (1989), Choi (1998), Aoki and Hu (1999), Crampes and Langinier (2002), and Llobet (2003), however these authors mostly dealt with a single patent to protect and thus, their foci of analysis differ from ours: In a model where the incumbent's patent portfolio can evolve over time, we examine the feedback effect that the prospect of such evolution may have on the startup's innovation incentives, and discuss the long-run welfare implications thereof.

The paper continues as follows. Section 2 is a brief review of the patent system that provides some background for our analysis. Section 3 presents a static model as a benchmark. Section 4 provides the main analysis of the dynamic model and characterizes the unique equilibrium. Section 5 discusses some implications of our findings using simulations. Section 6 contains some concluding remarks.

2 A closer look at the patent system

Patents are monopoly-grants that hold for a time period of 20 years. During this period no one, apart from the patent holder, may freely make use of the technology embodied in the patent's claims. Nevertheless, occasionally new ideas arrive, whose technological domain may well rest in a technological territory vaguely entrenched by the patent's claims, in which case the issue of possible infringement arises, frequently accompanied by a counter claim of validity. This infringement differs from a direct copying and re-branding of one's patented ideas, in as much as it progresses the prior art. The question of how and by how much the novel idea progresses prior art finds no equivalent in other forms of material-property. This is because, asserting property rights on one's ideas is far from simple. An idea, contrasting land, can never be fully barricaded or entrenched. Therefore, the issue of infringement is largely a subjective one, resting on the decision of courts and it is up to the innovator, through litigation, to prove the merits of her innovation. In other words, patents convey imperfect property rights to technologies, rights that can only be asserted

[3] "According to U.S. patent law, the issuance of a patent does *no more than* confer a patent right that is "presumed" valid (35 U.S.C.A. Sec. 282) in that the final responsibility for validating or invalidating the patent resides with the courts." (Choi, 1998, p.1249).

by courts. It has thus been argued that patents can be thought of as lotteries that carry some probability of being verified by a court; see Lemley and Shapiro (2005).

Two factors indisputably affect the enforcement probability of a patent. The first one is the scope (or breadth) of the patent, describing, through the claims that the patent office allows the patent holder to include, the patent's technological territory. The more extensive the claims are the more powerful a patent is, for it is harder to bypass it (e.g., by innovating around it) in developing a better and more advanced technology. Nonetheless, the claims of the patent themselves lack the ability to self-enforce. The power to enforce is vested in courts, which are the ultimate judges of the patents' merits. Therefore, the courts' attitude towards infringement is also a key determinant of patent validity.

The above factors are far from static. As far as the US is concerned, we have seen a drastic change in the past 20 years, both in court attitudes and patent scope. The catalyst for these changes was the Federal Courts Improvements Act that allowed for the formation of the CAFC. The CAFC raised the evidentiary standards required to challenge patent validity and broadened the interpretation of patent scope. In doing so the CAFC tilted the table towards patents making it easier to assert infringement. The data is revealing. Koenig (1980) provided validity and infringement data for district and circuit courts litigated patents during the years 1953-1978. He found that courts upheld the validity of patents in about 35% of the cases in which validity was an issue. This pattern was to drastically change with the introduction of the CAFC. Specifically, Harmon (1991) finds that from 1982 to 1990, the CAFC affirmed 90% of district court decisions holding patents to be valid and infringed, and reversed 28% of the judgments of invalidity and non-infringement. As a result, the overall probability that a litigated patent will be held to be valid has risen to 54%; see Allison and Lemley (1998).

This pro-patent stance is not cost-free. Merges (1999) points out that the expenses of a patent infringement court case can range from $1 million to several millions, although Farrell and Merges (2004) show that as more money is at risk in the suit, litigation costs rise sharply. Taking a closer look at the total cost of patent litigation, Lerner (1994) reports that, from July 1989 to June 1990, 1318 patent related suits were initiated in the US Federal Court and approximately 3900 procedures within the US Patents and Trademarks Office (PTO). He estimates, based on historical costs, that these cases will involve legal expenditures of about one billion 1991 dollars, which should be compared with expenditure on basic research of 3.7 billion by US firms in 1991.

It appears that litigation is an easier and less costly path to follow for firms with large patent portfolios. For example, Lanjouw and Schankerman (2004) find that having a larger portfolio of patents reduces the probability of filing a suit on any individual patent in the portfolio. As they note, *"for a (small) domestic unlisted company with a small portfolio of 100 patents, the average probability of litigating a given patent is 2%. For a company with a similar profile but with a moderate portfolio of 500 patents the figure drops to 0.5%. Thus, it is easier (less costly) to protect any given patent when that patent is part of a larger portfolio of patents"*. Furthermore, as they indicate, large firms (with large patent portfolios) have the experience and the ability to settle disputes by pooling or trading intellectual property. Therefore, if imperfect capital markets limit the capacity of smaller firms to finance litigation, larger firms may be able to extract better terms because they pose more credible litigation threats in confronting smaller firms.

On the other side of the Atlantic, even though the European Patent Office (EPO) grants patents using largely similar requirements to the PTO, it is stricter in granting a patent[4] and it allows, through a post grant opposition mechanism, any interested party to challenge a patent at the EPO up until 9 months after the patent is granted. In fact, 8.2% of all EPO patents are challenged, and about one third are revoked; see Harhoff and Reitzig (2004). This procedure does not undermine the power of the member states' courts, and if an opposing party, having lost its EPO opposition, wants to pursue its case at a national level it is free to do so, at a cost of anything between 50,000€ and 500,000€, depending on the country and the complexity of the case; see Ropski (1995). These differences limit the scope of EPO patents and reduce legal opposition.

3 A static approach

We consider two firms in the same industry, operating under a single line of patented, cumulative technology. Firm 1 is an established incumbent, holding an extensive patent portfolio. Firm 2 is an entrant startup that is capable of developing a promising new technology by investing an R&D cost $C > 0$, and thereby, obtaining 1 single-claim patent as a testimony to its innovativeness. We assume that, due to the cumulative nature of technology, firm 2's patent will be perceived as infringing on one or more of the incumbent's patents. By commercializing the new technology firm 2 can generate a profit of $V > C$. With its greater marketing experience, however, firm 1 would be able to enlarge

[4] As Graham and Harhoff (2006) suggest, even the most valuable US patents, those that are inviting costly litigation in US courts, are not being granted EPO patent protection in about 20% of cases.

the market value of this technology and generate a total extra profit of $V^* \geq V$ if it commercializes the technology in place of firm 2. For this, though, firm 1 should seek access to firm 2' technology either through court or by a technology sharing agreement.

If firm 1 files a suit alleging that 2's technology is infringing on its patents, the outcome of the suit is uncertain. Following Choi (1998), we capture this uncertainty by the probability p with which 1 wins the case. We also interpret a higher p as reflecting a stronger stance of the court toward IP protection. If 2's technology is found as trespassing on the technological territory of firm 1, it can only follow that the patent granted on 2's technology must be invalid, that is, within its single claim it did not put forth a new idea that, in the court's view, is diversified enough from the existing subject matter as to deliver a non-infringing technology. In this case, considering the high cost of patent litigation and the lack of a final product, firm 2 can only exit the market. If the court finds 2's technology non-infringing, on the other hand, firm 2 commercializes it and reaps a profit of V.[5]

Contrary to property law, where barriers between properties are well defined, there is uncertainty over barriers between ideas. As a result, aspects of the startup's infringing idea can be re-bundled by more experienced parties in a patent whose aspects may be deemed as valid and non-infringing by a later court hearing. In this case, when 1 wins it allows aspects of the infringing idea to find uses by potential rivals, lowering the actual value of the idea to firm 1 down to bV^*, $b \in (0, 1)$. The notion that an invalid patent can lead rivals to freely appropriate aspects of the idea is similar to Farrell and Shapiro (2006), who suggest that invalid patents can be used by rivals due to a US court precedent.[6] [7]

Instead of litigation, firm 1 may also consider to negotiate a technology sharing agreement with 2 (i.e. a takeover deal, or a licensing/cross-licensing agreement). In line with Crampes and Langinier (2002), we model this process as a Nash bargaining one, where the disagreement/threat points are the expected surpluses when firm 1 files an infringement lawsuit. Our aim is to find the bargaining surplus of each firm when they decide

[5] In our model the usual counter accusation of invalidity is not modeled because, contrary to an individual patent, it is next to impossible to invalidate a patent portfolio consisting of many patents.

[6] Blonder-Tongue Labs, Inc. v. University of Illinois Foundation, 402 U.S. 313, 350 (1971).

[7] In the real world of multi claim patents a verdict of infringement can compromise the validity of many claims. Since frequently startups cannot survive a lost litigation battle they exit the market. In this case patent trolls usually pick up their patents. Depending on their strategies these patents can be licensed. However, due to a court precedent (see the previous footnote) all invalid claims and the methods leading to these may be freely appropriated by rivals.

on a technology sharing agreement, and to examine how IP protection affects innovation incentives via its impact on the bargaining outcome.

An option that we do not consider further is a preliminary injunction, which, as Lemley and Shapiro (2006) note, enhances the plaintiff's negotiating power. Here such an enhancement can be delivered via a greater p. Since p and its determinants (both in this and in the following sections) constitute the main exogenous variables of our model, preliminary injunctions can be captured by increases in IP protection that broaden the plaintiff's negotiating power. Moreover, to simplify the argument, we operate under the assumptions that: a) litigation cost is zero, and b) justice is swift. Assumption b) is not unsubstantiated, as the overwhelming majority of IP related cases are settled swiftly out of court. The swiftness of a final ruling (either in or out of court) allows the model to abstain from elaborating on the damages that the losing party needs to pay.[8]

To summarize, the order of moves is as follows. First, firm 2 decides whether to innovate by investing C or not. If 2 does not innovate, the game is over with zero payoffs to both firms. If 2 innovates, 1 decides whether to file a suit or pursue a technology-sharing agreement *a la* Nash bargaining. If a suit is filed, with probability p firm 1 wins and gets a surplus of bV^*; and with probability $1-p$ firm 2 wins and gets a surplus of V. The losing party has a surplus of 0. If an agreement is pursued, the Nash bargaining outcome results over the total surplus of V^*. The structure of the game is common knowledge.

The equilibrium of this model follows from the Nash bargaining outcome as explained below. Specifically, the disagreement/threat points of firms 1 and 2 are, $d_1 = pbV^*$ and $d_2 = (1-p)V$. Since V^* is the maximum value of the technology, the Nash bargaining set is defined as $B = \{(\overline{s}_1, \overline{s}_2) \in \Re^2_+ \mid \overline{s}_1 + \overline{s}_2 \leq V^*\}$ where $\overline{s}_i$ denotes the bargaining share of firm $i = 1, 2$ (the bar above s_i is derogatory of the static framework). Given that B is compact and convex, there is a unique Nash bargaining outcome $(\overline{s}_1, \overline{s}_2)$ that solves $\max_{(\overline{s}_1,\overline{s}_2)\in B}(\overline{s}_1 - d_1)(\overline{s}_2 - d_2)$, expressed as the following functions of p where $r = V/V^* \in [0, 1]$:

$$\overline{s}_1(p) = \frac{V^* + d_1 - d_2}{2} = \frac{1 + p(b+r) - r}{2}V^* \tag{1}$$

$$\overline{s}_2(p) = \frac{V^* - d_1 + d_2}{2} = \frac{1 - p(b+r) + r}{2}V^*. \tag{2}$$

[8] The yardstick used by courts in deriving damages is either the accumulated royalties resulting from a hypothetical licensing agreement (this is usually a per-period payment of 1-2% of the product's value), or the foregone profits from the sale of the infringing good. Both of these are minimal if justice is swift. Specifically, since a final product is yet to be developed there are no foregone profits, plus any foregone royalty payments cannot be central to the paper's argument because they have yet to accumulate.

Note that $\overline{s}_1(p) > pbV^*$ and $\overline{s}_2(p) > (1-p)V$ hold if $1-r > p(b-r)$, which is indeed the case as $b, p < 1$. Hence, both firms would find it optimal to pursue a technology sharing agreement *a la* Nash bargaining, instead of litigation, once an innovation took place. Anticipating such an agreement, a startup would invest in R&D iff $\overline{s}_2(p) \geq C$. The next proposition summarizes the findings in the static approach. Noting that the ownership of the patent *per se* does not alter the maximum value of the technology V^*, the bargaining agreement need not take the form of a takeover, as it can equally well be attributed to licensing/cross-licensing.

Proposition 1: *In the static model, the Nash bargaining over a startup's innovation splits the total surplus V^* into $\overline{s}_1(p)$ for the incumbent and $\overline{s}_2(p)$ for the startup, as expressed in (1)-(2). Hence, a startup invests in R&D if and only if $\overline{s}_2(p) \geq C$. Stronger IP protection, i.e., a higher p, therefore, decreases (increases) the share of the startup (incumbent) via weakening (strengthening) its bargaining position, reducing the startup's innovation incentives.*

4 A dynamic approach

In this section we elaborate on the issues arising from the cumulative nature of technology by extending the model to infinite periods. In each period a new startup firm (firm 2) enters the market with an idea that can be developed into an innovation, described through one single-claim patent, if the startup invests $C > 0$ on R&D.[9] However, due to the cumulative nature of technology, 2's patent is perceived as infringing one or more of the incumbent's patents. The value of the new technology is $V > C$ when commercialized by the startup, while if the incumbent (firm 1), who is long-lived, commercializes the technology its value becomes $V^* \geq V$. To avoid the *replacement effect*, as in Bessen and Maskin (2007), we assume these values as incremental. We stress here that, as will be illustrated in Section 5, our results are not driven by the disparity between V^* and V but rather by the additional bargaining power that expanding patent portfolios allow for.

If the incumbent acquires new patents through takeover deals, the technological territory covered by its patent portfolio expands and thus, as argued in the Introduction, the likelihood increases that it will prevail in patent-infringement suits. In this regard, we assume that legal power increases as the portfolio size gets bigger, but at a decreasing

[9] Our results extend straightforwardly to the cases that investing C leads to an innovation with a known probability less than (rather than equal to) 1.

rate. That it increases at a decreasing rate is attested by Bessen and Meurer (2005) who observe decreasing returns to scale between the size of a software firm's patent portfolio and the probability of winning a patent litigation suit. Moreover, it is also a logical consequence of the fact that the chance of prevailing in court is bounded above by 1.

To capture this we re-define p, the probability of firm 1 winning an infringement suit, as a function of the degree of IP protection, denoted by $z > 0$, and the size of 1's portfolio, measured by the number of patents in firm 1's portfolio. Specifically, through z (which can be considered as patent breadth) we focus on the court's attitude towards infringement, where an increase in z implies a tougher stance on infringement, increasing p.

At this point, to facilitate presentation, we make two indexing conventions. First, since the continuation game is fully described by the size of 1's portfolio at the beginning of that period, with slight abuse of terminology we index the period by the size of 1's portfolio. Second, since what matters in the analysis is the accumulation of patents on top of the incumbent's initial portfolio, we index the size of the initial portfolio as the base size of 1, and each patent added to it increases the portfolio size by one. Hence, period 1 designates the initial period (of portfolio of size 1) and period $t > 1$ designates any period prior to which firm 1's portfolio size has reached t but no higher, i.e., firm 1 has added $t - 1$ patents to its initial portfolio. So long as firm 1 has added one patent every period from the initial period, our indexing coincides with the natural indexing of periods by natural numbers. Two consecutive periods are indexed the same, however, if the incumbent's portfolio did not grow in the first of the two periods. Thus, p is a function of z and t, which we denote as $p_z(t)$. As discussed earlier, we assume that

$$\partial p/\partial z > 0, \quad \partial p/\partial t > 0, \quad \text{and} \quad \partial^2 p/\partial t^2 < 0.$$

Our core argument starts with the observation that the benefits of a takeover venture beyond current bargaining for firm 1, as the added bargaining power (caused by the expansion of 1's portfolio) may mean better future deals. This suggests that the total surplus to bargain over can be larger than in the static model. Consequently, firm 2 may rationally anticipate a larger bargaining share (compared to the static model), suggesting that dynamic incentives may induce innovations that would not have been possible in a static setting. This dynamic argument, unlike in the static model, implies that a takeover may be preferred to licensing because licensing (or cross-licensing) does not allow for the additional incremental value caused by the expansion in 1's patent portfolio.

The argument is not yet complete, because as takeover deals go on, i) firm 2's bargaining power weakens, and ii) the extra benefits of a takeover that accrues from future deals dwindles to zero due to decreasing returns to scale, rendering the total surplus/pie to bargain over to shrink to V^*, the size of the pie in the static model. Therefore, restricting, for example, the argument to innovations that would not materialize in the static setting (i.e. $C > \overline{s}_2(p)$), these two effects dictate that at some point firm 2 abstains from innovating as it would not recoup C. Such an end period upsets the potential equilibrium. To see this denote the last period that innovation is supposed to take place in equilibrium as T. Considering that in period $T+1$ firm 2's bargaining share, if it innovated, would not cover C, the absence of future innovation reduces the bargaining surplus down to V^* in period T and consequently, firm 2's bargaining share would not cover C if it invested in period T. This implies no innovation in period T and, in addition, backward induction implies no innovation in all preceding periods, either.

This glitch is an artifact of the assumption that all startups have the same R&D cost C. We relax this assumption minimally by assuming that in each period there is a probability $\eta > 0$ that the arrived startup has an R&D cost smaller than C, normalized to 0 for expositional ease, and the realized value of R&D cost is private information but η is common knowledge. This is in line with Bessen and Maskin (2007) and amounts to assuming the existence of *Silicon Valley* startups that, contrasting their high-cost counterparts, innovate with minimum cost. As such firms will engage in R&D whenever they arrive, an innovation comes forth with at least probability η in every future period. Thus, in the last period T in which firm 2 would invest in R&D regardless of its R&D cost, the total surplus to bargain over is larger than V^* by at least a certain amount, and firm 2 may have an incentive to invest even when it is a high-cost firm.

To summarize, the order of moves in each period t is as follows. First, firm 2 arrives and decides whether to innovate or not contingent on its R&D cost. If 2 does not innovate, nothing happens until the next period starts. If 2 innovates, 1 decides whether to file a suit or pursue a technology-sharing agreement *a la* Nash bargaining. If a suit is filed, with probability $p_z(t)$ firm 1 wins and gets a surplus of bV^*; and with probability $1 - p_z(t)$ firm 2 wins and gets a surplus of V. The losing party has a surplus of 0. If an agreement is pursued, the Nash bargaining outcome results over the total surplus of V^*, plus, in case of a takeover, the additional benefits that would accrue to firm 1 in future deals due to its enlarged portfolio. We present our main analysis presuming that any technology-sharing agreement takes the form of a takeover, then explain how the results change when licensing agreements are allowed. The startup in each period maximizes the expected surplus of that period, net of innovation cost when relevant. The incumbent maximizes the expected present value of the stream of surpluses with a discount factor $\delta \in (0,1)$.

Remark: Note that we do not explicitly model the possibility that the startup tries to build up its own portfolio via takeover deals with future startups. As long as we consider startups facing an incumbent with a large established portfolio, the value of pursuing this route would be low because the startup will continue to compete with the incumbent in the product market as well as in future takeover deals, both of which will reduce the expected surplus, and consequently, the startup would find it optimal to use this option as a threat to extract the best takeover deal from the incumbent. In so far as this is the case, our argument remains valid with V interpreted as including this option value.[10]

We now provide a formal analysis of the dynamic model and characterize the (subgame-perfect) equilibrium.[11] As we will show, there is a unique equilibrium and it largely exhibits the features elucidated above, namely, that the takeover deal *a la* Nash bargaining provides innovation incentives for high-cost startups, during an early stage of innovation dynamics at least. Such dynamic effects of inducing high-cost innovations would be best illustrated if a high-cost innovation would never be possible in a static situation. Hence, we first present our analysis in such environments, and then discuss other environments.

Thus, first we consider the case that $\overline{s}_2(p_z(1)) < C$, or equivalently,

$$\overline{s}_2(p_z(t)) < C \quad \text{for all} \quad t \geq 1. \tag{3}$$

where $\overline{s}_2(\cdot)$ is firm 2's bargaining share in the static model as defined in (2). Let T denote, in an arbitrary equilibrium, the last period in which a high-cost startup innovates with a positive probability, allowing for the possibility that $T = 0$, i.e., a high-cost startup never innovates. T is our point of departure in the analysis, and for notational purposes, in the sequel a hat on top of a variable is derogatory of all $t \leq T$ periods, and absence of a hat denotes all $t > T$ periods. Given that $T < \infty$ exists (indicating that in equilibrium a high-cost startup would not innovate indefinitely) as is proved in Proposition 3 below, for $t \geq T+1$, let $X(t)$ denote the value of firm 1 at the beginning of period t. Then,

$$X(t) = (1-\eta)\delta X(t) + \eta(s_1(t) + \delta X(t+1)) \tag{4}$$

[10] Admittedly, occasionally startups become dominant firms, however, these firms tend to rise to dominance in newly forming markets (e.g., Google, Genetech), rather than "win" it over from established incumbents.

[11] Firm 1 decides to litigate or bargain without knowing the R&D cost of firm 2, so technically speaking the subgame-perfectness does not require rationality of the incumbent's decision. However, the R&D cost of the startup is sunk at this point, so it does not affect the continuation game. Hence, in the spirit of subgame-perfectness, we require that each choice of the incumbent be optimal in the continuation game.

because, a) if a high-cost firm arrives with probability $1-\eta$, there is no innovation and firm 1's value in the next period is the same as that in the current period (i.e. $X(t)$) and, b) if a low-cost firm arrives with probability η, firm 1 captures the bargaining surplus over the current innovation, $s_1(t)$, plus its value in the next period which is $X(t+1)$.

Focusing on equation (4), when $t > T$ the total surplus that a startup's innovation generates is maximized when firm 1 commercializes it, adding it to its portfolio. The total surplus it brings forth in this case is $V^* + \delta(X(t+1) - X(t))$, which is the size of the pie on the bargaining table. If the case is litigated, since both parties must accept the court's decision, there is no takeover deal. Therefore, the threat points are the court outcomes, $d_1 = p_z(t)bV^*$ and $d_2 = (1-p_z(t))V$. Since the Nash bargaining set in this case is $B(t) = \{(s_1, s_2) \in \Re^2_+ \mid s_1 + s_2 \leq V^* + \delta(X(t+1) - X(t))\}$, the Nash bargaining outcome (s_1, s_2) that solves $\max_{(s_1,s_2)\in B(t)}(s_1 - d_1)(s_2 - d_2)$ is calculated as,

$$s_1(t) = \frac{1 + p_z(t)(b+r) - r}{2}V^* + \frac{\delta(X(t+1) - X(t))}{2} \tag{5}$$

$$s_2(t) = \frac{1 - p_z(t)(b+r) + r}{2}V^* + \frac{\delta(X(t+1) - X(t))}{2} \tag{6}$$

where $r = V/V^* \in [0,1]$. Plugging $s_1(t)$ back into equation (4) and rearranging, we get

$$X(t+1) - X(t) = \frac{2(1-\delta)}{3\delta\eta}X(t) - \frac{1 + p_z(t)(b+r) - r}{3\delta}V^*, \tag{7}$$

a difference equation that characterizes the sequence $X(t)$ for $t > T$. Since the value of additional patent diminishes to 0 as $t \to \infty$, it turns out that this sequence increases and converges, as formalized in the next result. Although $X(t)$ is pertinent for $t > T$, it proves useful to treat it as a function defined for all natural numbers $t \geq 1$.

Proposition 2: *The sequence $\{X(t)\}$ defined by (7) is unique, monotonically increases at a decreasing rate, i.e., $X(t) - X(t-1) > X(t+1) - X(t) > 0$ for all $t > 1$, and converges to*

$$X(\infty) = \frac{1 - r + p_z(\infty)(b+r)}{2(1-\delta)}V^*\eta \quad \text{as} \quad t \to \infty. \tag{8}$$

Proof: See Appendix.

Proposition 3: *If (3) holds, in any equilibrium there exists $T < \infty$ such that a high-cost startup does not innovate in any period $t > T$.*

Proof: See Appendix.

Let $\hat{X}(t)$ denote firm 1's value at the beginning of period t for $t \leq T$. Presuming that a high-cost startup innovates for sure in period T, firm 1's value at the beginning of T is

$$\hat{X}(T) = \hat{s}_1(T) + \delta X(T+1) \tag{9}$$

where $\hat{s}_1(T)$ denotes the bargaining share that it derives over the current innovation. Given that the total surplus to bargain over is $V^* + \delta(X(T+1) - \hat{X}(T))$ and the threat points are $d_1 = p_z(T)bV^*$ and $d_2 = (1 - p_z(T))V$ in period T, we calculate the Nash bargaining outcome $(\hat{s}_1(T), \hat{s}_2(T))$ as

$$\hat{s}_1(T) = \frac{1 + p_z(T)(b+r) - r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T))}{2} \quad \text{and} \tag{10}$$

$$\hat{s}_2(T) = \frac{1 - p_z(T)(b+r) + r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T))}{2}. \tag{11}$$

Plugging $\hat{s}_1(T)$ into equation (9), we derive $\hat{X}(T)$ in terms of $X(T+1)$ as

$$\hat{X}(T) = \Big(1 + \frac{\delta}{2}\Big)^{-1} \cdot \Big(\frac{3\delta}{2} X(T+1) + \frac{1 + p_z(T)(b+r) - r}{2} V^*\Big). \tag{12}$$

Furthermore, rearranging equation (7) we get

$$X(T) = \Big(1 - \delta + \frac{3\delta\eta}{2}\Big)^{-1} \cdot \Big(\frac{3\delta}{2} X(T+1) + \frac{1 + p_z(T)(b+r) - r}{2} V^*\Big)\eta. \tag{13}$$

Since $1 - \delta + \frac{3\delta\eta}{2} > (1-\delta)\eta + \frac{3\delta\eta}{2} = (1 + \frac{\delta}{2})\eta$, it follows from (12) and (13) that $\hat{X}(T) > X(T)$, hence $\hat{s}_2(T) < s_2(T)$. If $\hat{s}_2(T) \geq C$, then a high-cost startup would innovate in period T as presumed. But, it is also possible that $\hat{s}_2(T) < C < s_2(T)$, in which case a high-cost startup would not innovate in period T. This problem is resolved when mixed strategies are considered: if a high-cost startup invests with an appropriate probability, $\hat{X}(T)$ gets reduced, pushing up $\hat{s}_2(T)$ to a level equal to C so that the startup is indifferent between investing and not. Specifically, if $\hat{s}_2(T) < C < s_2(T)$ we redefine $\hat{X}(T)$ and $\hat{s}_1(T)$ as $\hat{X}(T,a)$ and $\hat{s}_1(T,a)$ that solve

$$\hat{X}(T,a) = (\eta + a)(\hat{s}_1(T,a) + \delta X(T+1)) + (1 - \eta - a)\delta\hat{X}(T,a), \tag{14}$$

$$\hat{s}_1(T,a) = \frac{1 + p_z(T)(b+r) - r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T,a))}{2}, \quad \text{and} \tag{15}$$

$$\hat{s}_2(T,a) = \frac{1 - p_z(T)(b+r) - r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T,a))}{2}, \tag{16}$$

for some $a \in (0, 1-\eta)$ so that, in particular,

$$\hat{X}(T,a) = \Big(1 - \delta + \frac{3\delta(\eta + a)}{2}\Big)^{-1} \cdot \Big(\frac{3\delta}{2} X(T+1) + \frac{1 + p_z(T)(b+r) - r}{2} V^*\Big)(\eta + a). \tag{17}$$

As a increases from 0 to $1-\eta$, $\hat{X}(T,a)$ increases from $X(T)$ of equation (13) to $\hat{X}(T)$ of equation (12). Analogously, $\hat{s}_2(T,a)$ decreases from $s_2(T)$ to $\hat{s}_2(T)$. Since $\hat{s}_2(T) < C < s_2(T)$, it follows that there exists a unique value of $a \in (0, 1-\eta)$, denoted by $\hat{a}(T)$, such that $\hat{s}_2(T, \hat{a}(T)) = C$. Thus, if a high-cost startup were to invest C with probability $\frac{\hat{a}(T)}{1-\eta}$ in period T, its bargaining share would be $\hat{s}_2(T, \hat{a}(T)) = C$, ensuring that a high-cost startup is indifferent between innovating and not and thus, justifying the mixed strategy.

We have specified above the unique equilibrium behavior in period T, according to which a high-cost startup innovates with a positive probability. This, however, does not imply that an innovation takes place for sure in all preceding periods $t < T$, so we need to apply analogous reasoning to periods $T-1$, $T-2$, and so on, and recursively find the equilibrium strategies. The process being analogous, we explain it only for $T-1$.

For expositional ease, let $\hat{a}(T) = 1-\eta$ and $\hat{X}(T, 1-\eta) = \hat{X}(T)$ if $\hat{s}_2(T) \geq C$. Presuming $\hat{s}_2(T-1) \geq C$, we have the following equilibrium conditions:

$$\begin{aligned} \hat{X}(T-1) &= \hat{s}_1(T-1) + \delta \hat{X}(T, \hat{a}(T)), \\ \hat{s}_1(T-1) &= \frac{1+p_z(T-1)(b+r)-r}{2} V^* + \frac{\delta(\hat{X}(T,\hat{a}(T)) - \hat{X}(T-1))}{2}, \\ \hat{s}_2(T-1) &= \frac{1-p_z(T-1)(b+r)+r}{2} V^* + \frac{\delta(\hat{X}(T,\hat{a}(T)) - \hat{X}(T-1))}{2}. \end{aligned} \tag{18}$$

Solving the first two equations simultaneously, we get

$$\hat{X}(T-1) = \Big(1+\frac{\delta}{2}\Big)^{-1} \cdot \Big(\frac{3\delta}{2}\hat{X}(T,\hat{a}(T)) + \frac{1+p_z(T-1)(b+r)-r}{2}V^*\Big). \tag{19}$$

From equation (19) and equation (13) evaluated at $T-1$, we deduce that $\hat{X}(T-1) > X(T-1)$, which implies that $\hat{s}_2(T-1) < s_2(T-1)$ because $\hat{X}(T,\hat{a}(T)) > X(T)$ and $(1+\frac{\delta}{2})^{-1} > (1-\delta+\frac{3\delta\eta}{2})^{-1}\eta$. Hence $\hat{s}_2(T-1) < C < s_2(T-1)$ may still hold, in which case we solve, analogously to before, the simultaneous equations,

$$\hat{X}(T-1,a) = (\eta+a)(\hat{s}_1(T-1,a) + \delta\hat{X}(T,\hat{a}(T))) + (1-\eta-a)\delta\hat{X}(T-1,a), \tag{20}$$

$$\hat{s}_1(T-1,a) = \frac{1+p_z(T-1)(b+r)-r}{2}V^* + \frac{\delta(\hat{X}(T,\hat{a}(T)) - \hat{X}(T-1,a))}{2}, \tag{21}$$

$$\hat{s}_2(T-1,a) = \frac{1-p_z(T-1)(b+r)+r}{2}V^* + \frac{\delta(\hat{X}(T,\hat{a}(T)) - \hat{X}(T-1,a))}{2} = C, \tag{22}$$

for $a \in (0, 1-\eta)$ and $\hat{X}(T-1,a)$ so that, in particular, we have,

$$\hat{X}(T-1,a) = \Big(1-\delta+\frac{3\delta(\eta+a)}{2}\Big)^{-1} \cdot \Big(\frac{3\delta}{2}\hat{X}(T,\hat{a}(T)) + \frac{1+p_z(T-1)(b+r)-r}{2}V^*\Big)(\eta+a).$$

Since $(1-\delta+\frac{3\delta\eta}{2})^{-1} \cdot \frac{3\delta\eta}{2} < 1$ and $\hat{X}(T,\hat{a}(T)) > X(T)$, this implies that $\hat{X}(T,\hat{a}(T)) - \hat{X}(T-1,0) > X(T) - X(T-1)$, and consequently, $\hat{s}_2(T-1,0) > s_2(T-1) > C$. Since

$\hat{X}(T-1, a)$ increases in a, lowering $\hat{s}_2(T-1, a)$ to a level below C at $a = 1-\eta$ as presumed above, there is a unique value of $a \in (0, 1-\eta)$, denoted by $\hat{a}(T-1)$, such that $\hat{s}_2(T-1, a) = C$. Thus, we have derived a unique equilibrium probability, $\frac{\hat{a}(T-1)}{1-\eta} \in (0,1)$, of innovating during period $T-1$ and the corresponding equilibrium value of firm 1, $\hat{X}(T-1, \hat{a}(T-1))$. It is straightforward to verify that an analogous recursive process uniquely determines the equilibrium strategy in each period all the way back to $t = 1$. This completes characterization of the unique equilibrium for the cases that satisfy (3).

We now discuss the alternative case that $\bar{s}_2(p_z(1)) \geq C$. There are two subcases to consider, namely, $\bar{s}_2(p_z(\infty)) < C$ and $\bar{s}_2(p_z(\infty)) \geq C$. When $\bar{s}_2(p_z(\infty)) < C$ it is straightforward to see that there exists a unique equilibrium analogous to the one characterized above. Specifically, high-cost startups innovate for sure in all periods t such that $\bar{s}_2(p_z(t)) \geq C$, because $\hat{X}(t)$ increases in t and consequently,

$$\hat{s}_2(t) = \bar{s}_2(p_z(t)) + \frac{\delta(\hat{X}(t+1) - \hat{X}(t))}{2} > C. \tag{23}$$

Then, since the increase in $\hat{X}(t)$ slows down and, for some t, $\bar{s}_2(p_z(t))$ will eventually dip bellow C, high-cost startups stop innovating from a certain period. If $\bar{s}_2(p_z(\infty)) \geq C$, on the other hand, a high-cost startup innovates in every period in the unique equilibrium of the dynamic model because $\hat{s}_2(t) > \bar{s}_2(p_z(t))$ as per (23) and $\bar{s}_2(p_z(t)) > \bar{s}_2(p_z(\infty))$. Now we can characterize the unique equilibrium of the dynamic model in the next theorem.

Theorem 4: *The dynamic model has a unique equilibrium. If $\bar{s}_2(p_z(\infty)) \geq C$, in this equilibrium a startup innovates for sure in every period regardless of its R&D cost; If $\bar{s}_2(p_z(\infty)) < C$, on the other hand, there is a critical period $0 \leq T < \infty$ such that a high-cost startup innovates with a positive probability in every period $t \leq T$ but not in periods $t > T$, while a low-cost startup innovates for sure in every period. In either case, when there is an innovation the innovator reaches a takeover deal with the incumbent.*

An interesting policy-relevant question is what is the optimal level of IP protection, z, that provides the innovation incentives for startups for longest. Although an algebraic answer is hard to obtain due to the recursive nature of the solution and the discontinuity at $t = T$, our simulation results (summarized in Section 5) confirm the following intuition: If z is excessive, the marginal protective power that an extra patent brings to the incumbent is large initially but quickly dwindles as a result of accumulating its power too rapidly, killing off the positive effect on startup innovation prematurely. If z is flimsy, on the other hand, the marginal protective power of an extra patent is small and its impact on the startup's innovation incentives is limited. Consequently, the optimal level of IP protection tends to be at a moderate level.

We have carried out our analysis presuming that any technology-sharing agreement is restricted to a takeover deal, i.e., licensing was not considered. Since licensing (lacking the added advantages accruing to firm 1 from future dealings) fails to increase the innovation's total value beyond V^*, one can see that the Nash bargaining outcome of a licensing deal is the same as the static model's bargaining outcome, $\bar{s}_1(p_z(t))$ and $\bar{s}_2(p_z(t))$. Thus, if $\bar{s}_2(p_z(1)) < C$ licensing in any period t would not cover C for the startup, allowing only low-cost innovations, while if $\bar{s}_2(p_z(\infty)) \geq C$ it would allow high-cost innovations in every period t. In either case, reaching a takeover deal instead of licensing would not affect the startups' innovation decisions. However, with takeover deals firm 1 anticipates a larger surplus due to its enhanced future bargaining positions and, furthermore, a part of this extra surplus accrues to the current startup firm (at the expense of the startups in future deals). Therefore, the equilibrium outcome does not change even if licensing is allowed.

If $\bar{s}_2(p_z(1)) \geq C > \bar{s}_2(p_z(\infty))$, on the other hand, there is room for licensing. For example, after expanding its portfolio size via takeovers to the size T, the incumbent may obtain access to new technology through licensing in all subsequent periods, so that startups innovate forever regardless of their R&D cost. Relative to when licensing is disallowed, this would bring about more innovations but the bargaining share of the incumbent could be smaller for the periods in which licensing deals will be reached. Foreseeing this, once the portfolio grows to size T the incumbent may want to selectively release some of the patents in its portfolio to avoid discouraging startup innovations by becoming too powerful a potential plaintiff. This practise is reminiscent of the recent trend of patent donations: in the last few years firms such as DuPont, Lubrizol, Eastman Chemicals, and General Motors have given away patents with an estimated value of hundreds of millions of dollars. An alternative interpretation of this practice (given that, in this context, patents last a fixed number of years and every period a patent is added to the incumbent's portfolio) is to treat T as the optimal patent length, constraining the portfolio size to T, allowing for the arrival of high-cost innovations *ad infinitum.*

5 Simulation and comparative statics

In light of the diminishing marginal protective power of patents underlying our theoretical results, prior to the simulation we need to address how $p_z(t)$ changes with t and z. Empirical estimates of the marginal protective ability of patents are scarce and inconclusive. Lanjoun and Schankerman (2004), who look at how patent portfolios help reduce a firm's probability of facing litigation, are one of the few that examine how patent portfolios affect litigation. They find the marginal protective power to be positive but slowing down. We capture this through $p_z(t) = 1 - (1 - z)^t$. To provide an example (in line with the magnitudes of z we find), when $z = .007$ a firm with a portfolio made up of 100 patents stands a 50% chance of winning its case, and an increase of 1 patent raises this by .34%.

Normalizing $V^* = 1$, we argue our case for $r = 1$, $b = .5$ and $C = 1.0001$. An $r = 1$ allows for results that are not driven by the disparity between V and V^*, and $b < 1$ means that takeover deals will be pursued over litigation. Additionally, since $\overline{s}_2(p) \leq \overline{s}_2(0) = 1$, by choosing $C > 1$ we ensure that a) innovation by high-cost startups may only be possible in a dynamic model, and b) that IP protection is necessary for such innovation because $p_z(t)$ is constant at 0 for all t if $z = 0$, erasing any dynamic effect. In terms of δ and η, we initially simulate the model for $\delta = .97$ and $\eta = .2$ and then for $\delta = .99$ and $\eta = .8$.

Concentrating on the $t > T$ periods, for given z, we simulate the unique sequence $\{X(t)\}$ defined by (7), which converges to (8). From this sequence, through equation (6), is derived a convergent sequence $s_2(t)$, as illustrated in Figure 3 for $z = .007$. Then, the period T is obtained by identifying the last period for which $s_2(t) \geq C$.

To examine how z affects T, we find the values of T for z's between .0001 and .01 in 20 steps of .0005. Figure 4 shows how T changes as z increases when $\delta = .97$ and $\eta = .2$ (the lower graph) and when $\delta = .99$ and $\eta = .8$ (the upper graph).

Both graphs are quasiconcave, in particular, T initially increases with z, then decreases as z increases further. Specifically, if z is high then $X(t)$ converges quickly, driving the future benefits from an extra takeover to nil and thus, halting the positive effect on startup innovation prematurely. For small z's, on the other hand, an extra patent increases $p_z(t)$ and $X(t)$ only marginally, failing to sufficiently increase $s_2(t)$ as to allow for a high T. Needless to say, the precise relationship between z and T changes as other details of the specification change. However, the quasiconcavity with an interior peak prevails in all our simulations so long as $\overline{s}_2(0) < C$ and high-cost innovation is possible at all, as partly demonstrated in Figure 4 explained below. Furthermore, in our analysis both firms are assumed to have equal bargaining power, however, endowing different bargaining power would lead to different levels of T. Lowering C would also lead to a higher T.

Turning our attention to $t \leq T$, for $\delta = .97$, $\eta = .2$ (and for the same z's used above) we employ equations (11)-(12) to derive $\hat{X}(T)$ and $\hat{s}_2(T)$. If $\hat{s}_2(T)$ is less than C, using mixed strategies, we find $\hat{a}(T)$ by equating (16) to C, and derive $\hat{X}(T, \hat{a}(T))$ from equation (17). The equilibrium strategies for the remaining periods until $t = 1$ can be recursively derived as follows: For $T-1$ we use equations (18)-(19) to derive $\hat{s}_2(T-1)$ and $\hat{X}(T-1)$. If $\hat{s}_2(T-1) < C$, using mixed strategies, we find the value of $\hat{a}(T-1)$ by equating (22) to C. Using the same routine we derive the equilibrium value of $\hat{a}(t)$ for each $t \leq T$, as reported in Figure 5 for each of the 20 different values of z. Due to the discreteness, a range of z provides the highest T as shown in Figure 5. Among these values of z, those with higher values of $\hat{a}(t)$ induce more startup innovation on average in each period $t \leq T$. In Figure 5, the middle values in the range of z for the highest T tend to have higher values of $\hat{a}(t)$ although no single value of z is pinned down as having the highest $\hat{a}(t)$ for all $t \leq T$.

Finally, we report some comparative statics results by examining how b, η, and δ affect the graphs in Figure 4.[12] Starting with b, Figure 6 plots the graphs as we vary b from .1 to 1 in 10 steps (changing z as before), which indicate that increase in b does not affect T but it increases the range of z's for which T is highest. Next, Figure 7 plots the graphs as δ increase from .9 to .99 in 10 steps, which indicate that T increase as δ increases. Lastly, Figure 8 plots the graphs as η varies from .1 to 1 in 10 steps, which indicate that T increases (at a decreasing rate) as η increases.

6 Conclusions

In the 1990's a vibrant literature analyzed the rate of IP protection (in terms of patent breadth *vs.* length) that minimizes the effects of the monopoly that patents imply while offering sufficient R&D incentives. In this paper we depart from this tradition focusing instead on the dynamic effect of IP protection on startup innovation. We argue that positive but not excessive IP protection may foster takeover agreements between the incumbent and startup innovator, the prospect of which motivates the startup's entrepreneurial activity in the first place. Since the benefits of a takeover venture beyond the current invention via strengthened bargaining position in future takeover deals due to an enlarge patent portfolio, innovation activities may be less active when only licensing agreements are allowed or when there is no IP protection. On the other hand, excessive

[12] We do not change r as any decrease in r reduces $s_2(t)$ below C for all t, dictating a need to change C as well, in which case comparisons are void. Nonetheless, for $r = 1/3$, $V^* = 1.5$ and $C = 1.0001$ we derive a graph almost identical to the one we find when we use, $\delta = .97$, $\eta = .2$ and $b = .5$.

IP protection accumulates the incumbent's bargaining power too quickly and kills off the startup's innovation activities prematurely. We demonstrate this intuition by simulation results that exhibit an inverse U-shape relationship between the number of sustainable innovations and the level of IP protection.

Our theory can help explain the increase in takeovers we have witnessed since the 1980's, an increase that coincides with a major shift in US patent policy, namely, the formation of a single patent's court, in place of many appellate courts with diverging attitudes towards infringement. Such a legal apparatus is absent from the EU, where questions of enforcement, validity and revocation are dealt with by national courts that have varying attitudes towards infringement. The EU Commission has been advocating the creation of a central patent dispute court, in the hope of simplifying the IP framework in the EU. Our analysis supports this initiative suggesting that, provided the court keeps a balanced approach, it can spur innovation activities by startup entrepreneurs.

We have labored under the assumption that there is only one firm interested in the startup. The struggle between Microsoft and Google for a share of Facebook suggests that this is not always the case. How the bargaining will get resolved between a startup and competing incumbents is unclear, because (setting aside competition law issues) there is a variety of different strategies that the competitors may follow, which venture beyond the scope of this paper. Subsequently, we set aside such a question for further research.

Appendix

Proof of Proposition 2: First, note that $X(t)$ is bounded below (by 0) and above because maximum surplus in each period is bounded and $\delta < 1$. If $X(t+1) \leq X(t)$, then the right hand side of equation (7) would be non-positive and, furthermore, its value would strictly decrease when evaluated for $t+1$ because $X(t+1) \leq X(t)$ and $p_z(t+1) > p_z(t)$. This would mean that $X(t+2) - X(t+1) < X(t+1) - X(t) \leq 0$. Applying the same argument repeatedly, we deduce that if $X(t+1) \leq X(t)$ then the sequence should decrease forever at an increasing rate after t, which is a contradiction because the sequence is bounded below. Hence, we conclude that $X(t+1) - X(t) > 0$ for all t. Since the sequence is bounded above, it further follows that it must converge. The limit value, $X(\infty)$ in (8), is obtained by setting $X(t+1) = X(t)$ and $p_z(t) = p_z(\infty)$ in equation (7) and solving for $X(t)$.

To show uniqueness, suppose to the contrary that there are two sequences, $\{X(t)\}$

and $\{X'(t)\}$, that satisfy (7), such that $X'(t') = X(t') + \gamma$ for some $\gamma > 0$ and t'. By (7), we have $X'(t'+1) = X(t'+1) + (1 + \frac{2(1-\delta)}{\delta\eta})\gamma > X(t'+1) + \gamma$ and by repeating the same calculation, $X'(t) > X(t) + \gamma$ for all $t \geq t'$. This is impossible because both sequences should converge to the same limit as proved above, proving the uniqueness.

Finally, to show that $X(t) - X(t-1) > X(t+1) - X(t)$, note from equation (7) that

$$X(t+1) - X(t) - (X(t) - X(t-1)) = \frac{2(1-\delta)}{3\delta\eta}(X(t) - X(t-1)) - \frac{(p_z(t) - p_z(t-1))(b+r)}{3\delta}V^*. \tag{24}$$

If $X(t+1) - X(t) \geq X(t) - X(t-1)$ for some t, it would follow from equation (24) that $X(t+2) - X(t+1) \geq X(t+1) - X(t)$ because $0 < p_z(t+1) - p_z(t) < p_z(t) - p_z(t-1)$ due to the assumption that $\partial^2 p/\partial t^2 < 0$. Furthermore, $X(t+1) - X(t)$ would increase in t by repeated application of the same argument. This is impossible because the sequence $X(t)$ converges as shown above, hence we conclude that $X(t) - X(t-1) > X(t+1) - X(t)$. *Q.E.D.*

Proof of Proposition 3: To reach a contradiction, suppose to the contrary that in an equilibrium there is an arbitrarily large t such that a high-cost startup innovates with a positive probability in period t. Note that a takeover deal will be reached if an innovation takes place in period t, for otherwise a high-cost startup would not innovate because $d_2 = (1 - p_z(t))V < \bar{s}_2(p_z(t)) < C$. Let $\hat{X}_t$ denote firm 1's value at the beginning of period t, and let α_t denote the probability that an innovation takes place in period t. Since a low-cost startup always innovates, $\alpha_t = \eta + (1-\eta)a_t \geq \eta$ where a_t is the probability that a high-cost startup innovates in period t. Then, $\hat{X}_t = \alpha_t(\hat{s}_{1t} + \delta\hat{X}_{t+1}) + (1 - \alpha_t)\delta\hat{X}_t$ where $\hat{s}_{1t} = \frac{1+p_z(t)(b+r)-r}{2}V^* + \frac{\delta(\hat{X}_{t+1}-\hat{X}_t)}{2}$, so that

$$\hat{X}_t = \frac{\alpha_t}{1 - \delta + \frac{3}{2}\delta\alpha_t}\Big(\frac{3}{2}\delta\hat{X}_{t+1} + \bar{s}_1(p_z(t))\Big). \tag{25}$$

If $\hat{X}_t \geq \hat{X}_{t+1}$, firm 2's bargaining share in period t, $\hat{s}_{2t} = \bar{s}_2(p_z(t)) + \frac{\delta(\hat{X}_{t+1}-\hat{X}_t)}{2}$, is less than C because $\bar{s}_2(p_z(t)) < C$ by (3), and consequently, $\alpha_t = \eta$. Since $\alpha_{t+1} \geq \eta$ and $p_z(t+1) \geq p_z(t)$, therefore, $\hat{X}_t \geq \hat{X}_{t+1} < \hat{X}_{t+2}$ would imply

$$\frac{\alpha_t}{1 - \delta + \frac{3}{2}\delta\alpha_t}\Big(\frac{3}{2}\delta\hat{X}_{t+1} + \bar{s}_1(p_z(t))\Big) < \frac{\alpha_{t+1}}{1 - \delta + \frac{3}{2}\delta\alpha_{t+1}}\Big(\frac{3}{2}\delta\hat{X}_{t+2} + \bar{s}_1(p_z(t+1))\Big),$$

contradicting the presumption that $\hat{X}_t \geq \hat{X}_{t+1}$ according to (25). Hence, we deduce that if $\hat{X}_t \geq \hat{X}_{t+1}$ then $\alpha_t = \eta$ and $\hat{X}_{t+1} \geq \hat{X}_{t+2}$, and by repeatedly applying the same logic, $\alpha_{t'} = \eta$ for all $t' > t$. Since this contradicts to the supposed equilibrium, we conclude that $\hat{X}_t < \hat{X}_{t+1}$ for all t. Since firm 1's value is bounded above, it further follows that $\hat{X}_{t+1} - \hat{X}_t \to 0$ as $t \to \infty$, which in turn implies that $\hat{s}_{2t} \to \bar{s}_2(p_z(\infty))$ as $t \to \infty$, contradicting the presumption that a high-cost startup innovates with a positive probability indefinitely. Q.E.D.

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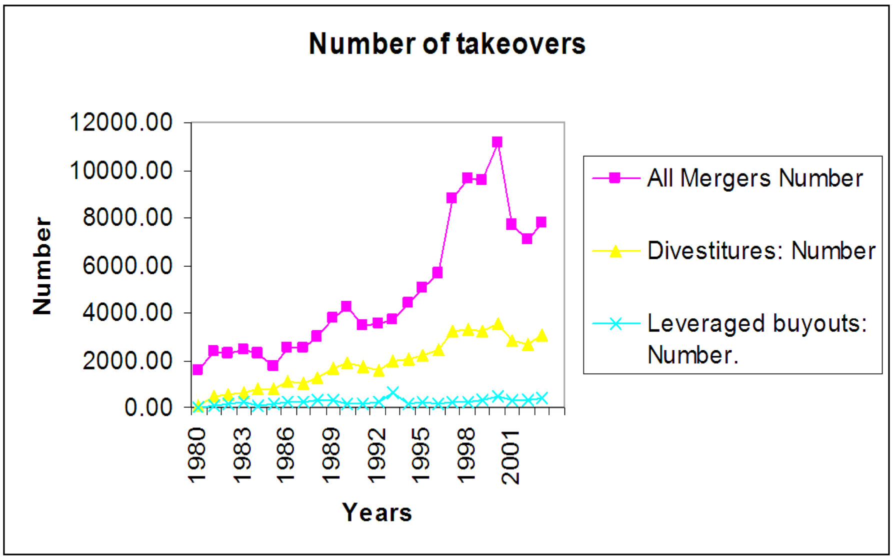


Figure 1: Source: Statistical Abstracts of the United States.

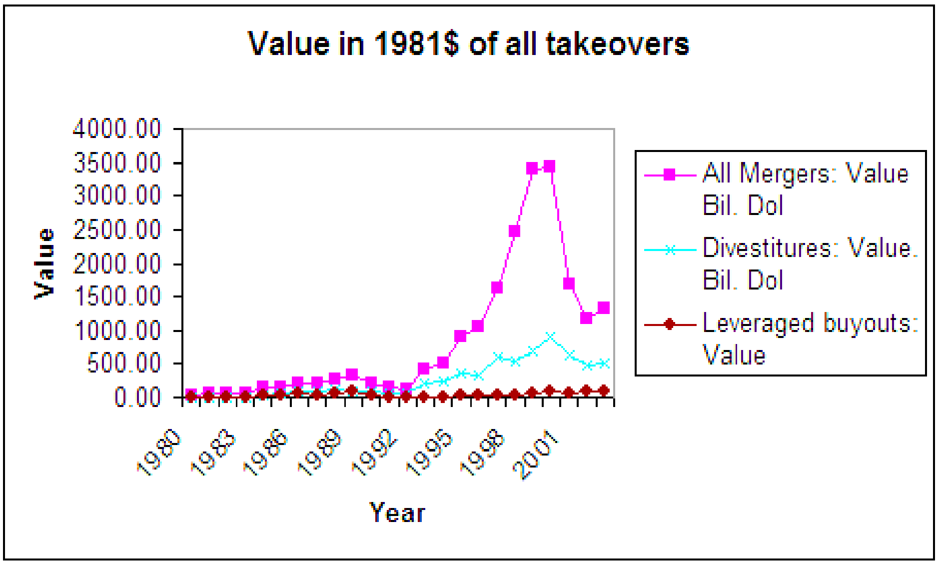
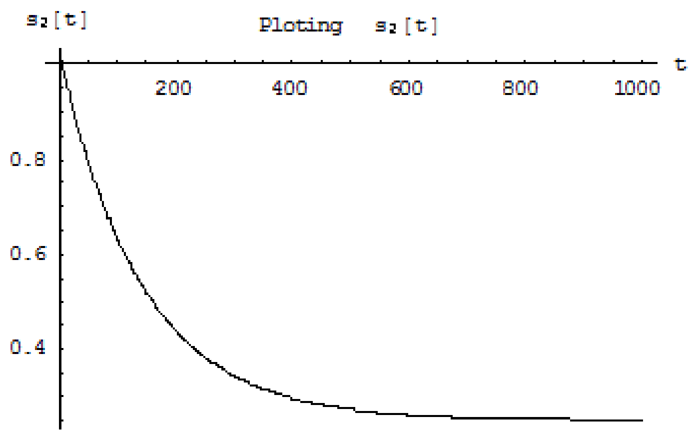


Figure 2: Source: Statistical Abstracts of the United States.



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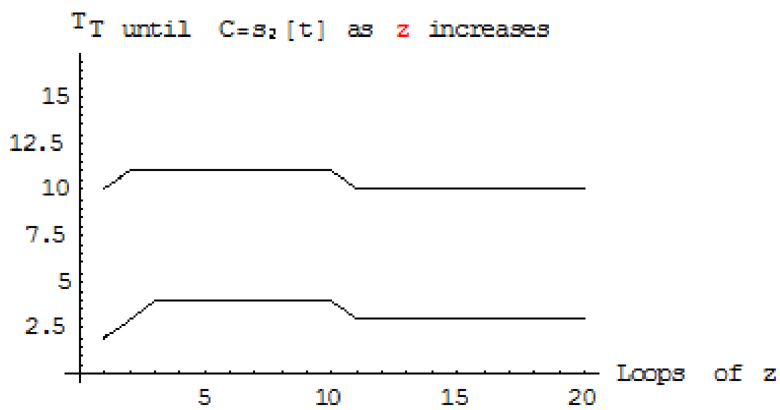

Figure 4:

The alphas

0.0005	2	0.272055	0.0580671		
0.001	3		0.283294	0.0717768	
0.0015	**4**		0.321421	0.0816182	0.0015245
0.002	**4**		0.335476	0.0851063	0.0028399
0.0025	**4**		0.341747	0.0861406	**0.0030948**
0.003	**4**		0.344559	**0.0862831**	0.0289699
0.0035	**4**		**0.345539**	0.0859166	0.0024669
0.004	**4**		0.345439	0.0852552	0.0019035
0.0045	**4**		0.344654	0.0844097	0.001258
0.005	**4**		0.34341	0.0834435	0.0005595
0.0055	3		0.341893	0.0824194	
0.006	3		0.340308	0.0814193	
0.0065	3		0.33856	0.0803811	
0.007	3		0.33669	0.0793155	
0.0075	3		0.334729	0.0782306	
0.008	3		0.332697	0.0771321	
0.0085	3		0.330611	0.0760243	
0.009	3		0.328486	0.0749104	
0.0095	3		0.326329	0.0737932	
0.01	3		0.32415	0.0726744	

Figure 5:

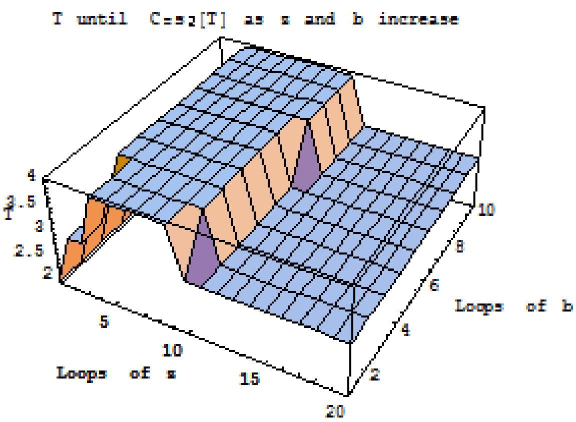

Figure 6:

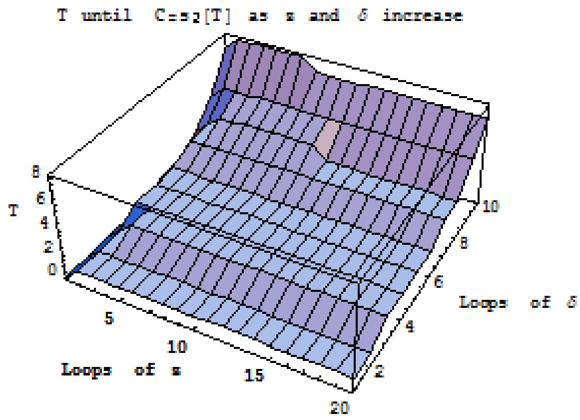

Figure 7:

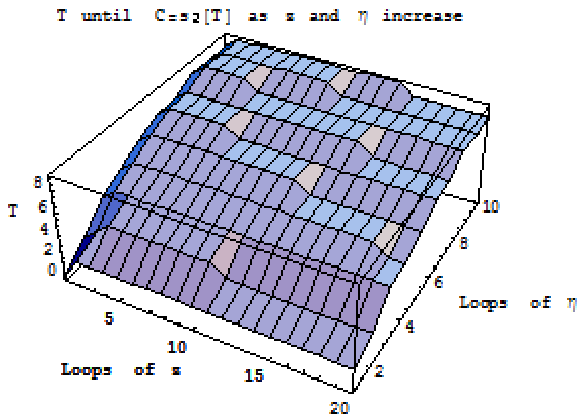

Figure 8: