Financially Constrained Innovation, Patent Protection, and Industry Dynamics*

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Abstract

This paper assesses the importance of intellectual property (IP) protection for innovation in the context of a model of the industry dynamics in which business niches can be monopolized by the holders of valuable IP. Successfully developed innovations add to the stock of valuable IP but also detract from it by turning some older IP invaluable. IP may also turn invaluable as a result of imitation. We consider the case in which innovations are generated by financially constrained entrepreneurs who are partly motivated by the search for independent business success. We find that the protection against subsequent innovators is counterproductive for innovation and welfare, while some (generally not full) protection against imitation is good in both dimensions. We also find that the net welfare gains from increasing IP protection are increasing with the tightness of financial constraints.

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1 Introduction

This paper assesses the importance of intellectual property (IP) for innovation and welfare in the context of a model of industry dynamics where innovations are generated by financially constrained entrepreneurs. The increase in IP protection and the development of alternatives for the financing of high-tech start-ups (most notably, venture capital) are in the list of factors that may have facilitated the unprecedented prosperity of innovative entrepreneurial activities since the early 1990s, which also includes the opportunities and technological changes associated with the information technology (IT) revolution, the reduction in setup costs and other barriers to the creation and development of new firms, and the emergence of a new "entrepreneurial culture" that has increased the social recognition and other rents associated with being and succeeding as an entrepreneur.

Regarding IP protection, it is commonly understood that, in the United States, the creation of a unique Court of Appeals of the Federal Circuit in 1982 strengthened the position of patent holders against potential infringers. Other legislative changes, such as the 1984's Semiconductors Act or the extension of patent duration to 20 years, has contributed to this increase in protection. However, there is considerable empirical and theoretical controversy on whether these reforms actually promote innovation.[1] Some quantitative assessments based on US data conclude that higher protection would induce more innovation (Denicolò (2007)), while others suggest that the greater protection brought about by recent reforms may have actually reduced innovation (Levin et al. (1985), Hall and Ziedonis (2001)). Practitioners express their doubts regarding the role of IP protection by referring to issues such as the "tragedy of the anti-commons" that deems strategic patenting and patent stacking as obstacles to innovation (Heller and Eisenberg (1998)). At a theoretical level, advice against excessive IP protection can be found in papers such as Boldrin and Levine (2002), Hunt (2004) or Bessen and Maskin (2006). None of these papers, however, makes explicit reference to the innovators' financing problem and the

[1] See Gallini (2002) for a review of the reforms and their effect on patenting activity.

direction in which financial constraints might change their normative prescriptions.

Regarding the financing of innovative start-ups, it is widely admitted that the access to informal sources of capital (such as private equity financing from friends and relatives, or from *business angels*) and venture capital are very important, since these start-ups typically lack the collateral required for the access to more conventional financing sources (such as bank loans).[2] But the availability and degree of sophistication of these sources of capital vary notably across industries, countries, and time periods, so the incidence of financial constraints may also vary a lot.[3] Most papers on entrepreneurial financing consider the traditional partial equilibrium setup of corporate finance and focus on understanding microeconomic issues such as the staging of finance (Gompers (1995) and Neher (1999)), the use of convertible securities (Casamatta (2003) and Schmidt (2003)), or optimal contracting when venture capitalists play an advising role (Repullo and Suarez (2004)). Some papers, including Holmstrom and Tirole (1997), Inderst and Muller (2004), and Michelacci and Suarez (2004), examine the equilibrium implications of financial constraints, but make no explicit reference to IP protection.

In this paper we construct a model of industry dynamics that allows us to judge the effects of IP protection and financial constraints on the equilibrium levels of innovation, competition, entrepreneurship, and social welfare. We consider an industry made up of a continuum of business niches. The successful developers of new products contribute to welfare and appropriate temporary monopoly profits like in a standard quality ladder model with limit pricing. Temporary monopolies are based on the protection granted by IP and are threatened by the entry of the developers of newer products, as well as imitators. The success of the developers of new products is compromised by the competition coming from other developers and by the opposition of the incumbent monopolists, who use their IP to fight imitators and innovators alike. In non-monopolized niches, the hurdle

[2]See Gaston (1989), Gompers (1999), and Gompers and Lerner (2001).

[3]For instance, it has been argued that the lower development of European private equity markets may be the reason why Europe lags behind the US in terms of entrepreneurship and innovation (see Bottazzi and Da Rin (2002)).

for innovative entry is lower since the incumbents have less incentives and capability to defend their IP.[4]

Formally, we model the generation of new products as an uncoordinated costly-entry process subject to congestion. From the perspective of a niche (and its occupants) innovation and imitation are random arrival processes and we assume that IP provides incumbent monopolists with (random) protection against them, thereby affecting their survival as monopolists and the barriers faced by their potential challengers.[5]

Adjusting IP protection in this setup involves several non-trivial dynamic trade-offs. From the perspective of the developer of a new product, stronger IP protection implies a larger expected duration of the monopoly obtained conditional on entry, but a lower probability of successful entry. Stronger protection against imitation also lengthens the expected duration of the monopoly obtained by innovative entrants, but, at the industry level, increases the fraction of business niches monopolized by holders of IP and, hence, the hurdle for successful entry.

We assume that financial constraints strike innovating entrepreneurs in their entry process, when trying to develop their innovations into new products. As in Holmstrom and Tirole (1997), a simple moral hazard problem affects the relationship between entrepreneurs and their external financiers. Instead of considering the development investments as part of a single non-transferable investment project, we assume that each entrepreneurial innovation has a continuum of possible development paths that are patented and can be separately developed by either the entrepreneur or a licensee. We assume that an entrepreneur can only reach some rents associated with independent success by developing a new product herself, but we explore whether the licensing of some of the development

[4]The different opposition faced across monopolized and non-monopolized niches can arise because of at least three reasons. First, with competing incumbents, the entrant may obtain a license for one of the existing technologies at a lower price. Second, previously successful imitation may mean that the patent of the previous monopolist was invalid or had expired. Finally, to the extent that court damages due to patent infringement are related to foregone profits, the entrant may expect to reach a more favorable settlement with the incumbent(s) when the pre-entry profits in a niche are low. The evidence in Cockburn and MacGarvie (2006) for the software industry is consistent with these views.

[5]This modeling allows us to abstract from the traditional distinction between patent length and patent breadth; see Scotchmer (2004) for a review of the treatment of these issues in the literature.

paths of her innovation to outsiders ("partial licensing") may help in solving her financing problem.[6]

The analysis of the effects of IP protection in the context of our model yields a number of interesting insights. We show that the protection of incumbents against future innovation is unambiguously detrimental to the steady-state rate of innovation and social welfare. Its net discouraging effect on innovation is due to essentially two reasons. First, the protection of the incumbents makes entrepreneurs less likely to succeed in the development of new products and, hence, implies the loss, in expectation, of some of the rents from independent business success that the society might, otherwise, mobilize so as to encourage entrepreneurs to innovate. Second, success probabilities and expected monopoly durations after success are substitutes in the compensation of innovation, but the contribution of the latter is discounted relative to the contribution of the former (and similarly the impact of IP protection on each of them).

As for the protection of IP against imitation, it turns out that an intermediate level is generally good for innovation and welfare. On the one hand, the standard argument in the literature applies and capturing more rents from success entices innovation. On the other, imitation facilitates entry, since it increases the proportion of competitive niches, where the hurdle for innovative entrants is lower.

We identify circumstances in which financial constraints justify the use of partial out-licensing as part of entrepreneurs' strategy for the financing of the development of their innovations. Licensing some of the paths of the innovations has the double effect of providing entrepreneurs with royalty income and reducing the size of their in-house development investments, which results in higher internal financing ratios and, hence, ameliorates the moral hazard problem vis-a-vis external financiers. At an industry level,

[6]We focus on this novel rationale for (partial) licensing while abstracting from many of the issues analyzed in the existing literature on technology transfers. Those issues include the strategic concerns that shape the patent licensing contract (see, for instance, Kamien (1992), Muto (1993), and Wang (1998)), the disclosure strategy of the inventor (Anton and Yao (2002), Bhattacharya and Guriev (2006)), and various dimensions of the competition and cooperation among patentholders with complementary innovations (such as cross-licensing agreements in Fershtman and Kamien (1992), patent pools in Lerner and Tirole (2004), and royalty staking in Lemley and Shapiro (2004)).

financial constraints reduce the fraction of entrepreneurs among the final developers of new products and they increase (decrease) the net welfare gains (losses) from increasing IP protection. This implies that an IP policy that is correct for a given industry, region and time period might need to change as institutions for entrepreneurial financing develop: if financial constraints get relaxed, IP protection should diminish.

The rest of the paper proceeds as follows. In Section 2 we introduce the entrepreneur's financing problem. Section 3 embeds this problem in an industry setup. Section 4 analyzes its equilibrium and steady-state properties. Section 5 discusses some welfare implications and Section 6 concludes. All proofs are relegated to the Appendix.

2 An Innovator's Financing Problem

In this section, we study the financing problem of an entrepreneur who has engendered and patented an innovation. Developing this innovation into a marketable new product requires financial resources that the entrepreneur does not have. A moral hazard problem limits the capacity of the entrepreneur to raise the funds needed for the full in-house development of the innovation. As an alternative, the entrepreneur may license some (or all) of the possible development *paths* of the innovation to established firms (which are assumed to be able to finance the development investments internally). Licensing some paths, however, implies sacrificing some of the extra rents that independently reaching success might give to the entrepreneur.

In the two subsections that follow, we first complete the description of the environment and then characterize the optimal licensing and financing arrangement. In Section 3 we embed this problem into a fully dynamic industry equilibrium setup, endogenizing the value of some key variables with respect to which entrepreneurs behave parametrically (and that are treated as exogenous in this section).

2.1 Setup of the Model

All agents are risk-neutral and discount the future by the same discount factor $\beta < 1$. So far we consider two dates, $t = 0, 1$. At date $t = 0$, the entrepreneur obtains and patents an innovation. The innovation can be developed using a measure-one continuum of alternative paths. At most one of these paths can lead to a new marketable product at $t = 1$ and ex-ante all paths are equally likely to lead to such a product. The successfully developed product generates verifiable profits with an expected present value of $v > 0$ at $t = 1$.

Researching each path requires one unit of investment. The probability of success along each path, conditional on that path being the one that leads to the marketable product, is $p > 0$ under *diligent* management and zero under *negligent* management. Under negligent management, however, the developer obtains a non-verifiable private benefit of $b > 0$.[7] Path-development technologies combine linearly in the sense that if a share $\xi \in [0, 1]$ of the paths are researched with diligent management and the rest with negligent management, the total probability of generating a new marketable product is ξp and the developers providing negligent management appropriate total private benefits of $(1-\xi)b$. The quality of management in each path is chosen by the corresponding developer and is unobservable to everybody else.

When E succeeds in developing a new product in-house, she obtains *entrepreneurial rents* of $C \geq 0$. This one-time non-pecuniary payoff captures the value of the independence, reputation, status, and control rents associated with entrepreneurial success. The successful development of the new product by an incumbent firm does not produce this type of rents.[8] As a result, efficiency would call for the innovation to be fully developed

[7]The normalization of the probability of success under negligent management to 0 is only used to simplify the exposition. The same results would hold if instead success occurred with probability $(1-\Delta)p$ for $\Delta \in (0, 1]$. Formally, in such a setup, all final equations would be the same except for the fact that our parameter b would have to be replaced by b/Δ.

[8]This is an innocuous simplification. All that we require is that the incremental utility of business success is higher for an entrepreneur who succeeds for the first time than for an established firm that, arguably, has already succeeded in previous businesses.

in-house by the entrepreneur.

However, E has no wealth and is protected by limited liability, so the unobservability of the management decision creates a moral hazard problem for the external financing of in-house development. In order for such a problem to be relevant, we make the following assumptions regarding the returns from developing any given research path:

Assumption 1 $\beta pv - 1 > 0$.

Assumption 2 $\beta pv > b > \beta pC$.

Assumption 1 states that, under diligent management, the verifiable cash flows associated with the development investment have a positive expected net present value (while under negligent management they have a net present value of -1). Assumption 2 states that the incremental net present value associated with diligent management exceeds the private benefits associated with negligent management, but these private benefits are larger than the incremental entrepreneurial rents associated with diligent management. So investing under diligent management is overall efficient but the sole ambition of the entrepreneurial rents does not solve E's moral hazard problem vis-a-vis her financiers.

In addition to undertaking the development exclusively in-house, E can partially (or fully) license the innovation to incumbent firms. When licensing, E relinquishes some development paths to a licensee or pool of licensees who can take charge of the corresponding investments and management, and, if successful, appropriate the profits generated by the resulting new product. Notice that, given the linearity of the external development technology, the division of paths across (one or more) licensees is irrelevant.[9] So E's licensing decision can be described by the proportion of out-licensed paths, $\alpha \in [0, 1]$, and the total royalties obtained in exchange, T.

We assume that incumbent firms have sufficient internal funds or collateral so as to guarantee the diligent development of the research paths under their control. Furthermore,

[9]This linearity does not apply across in-house and external development due to the entrepreneurial rents C that E appropriates if she discovers the new product.

we assume that incumbents compete among themselves to become licensees or buyers of the innovation. For this reason, the equilibrium royalty equals the whole expected net present value (to outsiders) of α research paths, $T = \alpha(\beta pv - 1)$. As a result, licensing may help the entrepreneur in two ways. First, by reducing the size of the in-house investment and, second, by allowing her to use the royalties T for the internal financing of such an investment.[10]

The sequence of events regarding E's financing problem can be summarized in the following timeline, where R denotes the amount that E promises to the external financiers of her in-house development investments.

$t = 0$ Stage 0	Stage 1	Stage 2	Stage 3	$t = 1$ Stage 4
Entrepreneur obtains an innovation ↓ Patent	Fraction α of paths licensed ↓ Royalty T	In-house investment $1 - \alpha$ financed ↓ Promised repayment R	E & licensees develop their paths ↓ Diligent development?	Development uncertainty resolves ↓ New product?

The statement and solution of the problem leading to the characterization of E's optimal licensing and financing arrangement is the focus of the next subsection.

2.2 The Optimal Licensing and Financing Arrangement

The best licensing and financing arrangement that allows the entrepreneur to remain involved in the in-house development of her innovation can be found as the solution to the following optimization problem:

[10]Due to the extra revenue (and, additionally, to the possibility of using it for internal financing), the licensing of the paths that E does not develop in-house clearly dominates the alternative of leaving some paths undeveloped; so this possibility can be safely ignored in the rest of the paper.

$$
\left.
\begin{array}{lll}
\max\limits_{\{\alpha,\ M,\ R\}} & V \equiv (1-\alpha)\beta p(v+C-R) + [\alpha(\beta pv - 1) - M] & \text{(OBJ)} \\
\text{s.t.:} & (1-\alpha)\beta p(v+C-R) \geq (1-\alpha)b & \text{(C1)} \\
 & (1-\alpha)\beta pR \geq (1-\alpha) - M & \text{(C2)} \\
 & 0 \leq \alpha \leq 1 & \text{(C3)} \\
 & M \leq \alpha(\beta pv - 1) & \text{(C4)} \\
 & 0 \leq R \leq v. & \text{(C5)}
\end{array}
\right\} \quad (1)
$$

The decision variables in (1) are the proportion of licensed paths, α, the part of the licensing revenues that E contributes to the in-house development investments, M, and the amount R that E promises to repay to financiers for their contribution to those investments. The objective function V is the present value of E's expected pecuniary and non-pecuniary payoffs. Its first term reflects the payoffs associated with the in-house development of $1 - \alpha$ paths of the innovation: when E succeeds in developing a new product at $t = 1$, she receives the fraction $v - R$ of the pecuniary value of the new product and the entrepreneurial rents C. The second term is the licensing revenue that E does not invest at $t = 0$.[11]

E's incentive compatibility constraint, (C1), requires that the present value of her expected pecuniary and non-pecuniary payoffs are no lower under diligent management than under negligent management. Financiers' participation constraint, (C2), guarantees that the repayment R that they receive when E succeeds compensates them for the initial contribution of $(1 - \alpha) - M$. The constraint (C3) sets $[0, 1]$ as the admissible range for α. Constraint (C4) accounts for the fact that licensing proceeds are the only funds that E can devote to the internal financing of her investment.[12] Finally, (C5) captures the

[11] In this formulation, we assume, without loss of generality, that E limits her liability vis-a-vis financiers to the pecuniary returns of her development investments. As shown below, under this formulation, it is optimal to set $M = \alpha(\beta py - 1)$. One could easily check that the equilibrium payoffs and licensing decisions would be identical under an alternative formulation in which E does not directly invest in the development of the innovation ($M = 0$) but is allowed to pledge all or a part of $\alpha(\beta py - 1)$ as collateral for the repayment of R.

[12] We do not constrain M to be positive because, in principle, E could borrow against the future value

limited liability protections of both the entrepreneur ($R \leq v$) and her financier ($R \geq 0$) at $t = 1$.

In the following proposition, we summarize the solution to the in-house development problem and provide a detailed expression for α^*.

Proposition 1 *If $b \leq \beta p(v + C) - 1$, the entrepreneur develops her innovation fully in-house, obtaining a net payoff $\beta p(v + C) - 1$. Otherwise, she out-licenses a fraction*

$$\alpha^* = 1 - \frac{\beta pv - 1}{b - \beta pC} \tag{2}$$

of the development paths and develops the remaining fraction in-house, obtaining a net payoff $V^* = (1 - \alpha^*)b$.

To understand Proposition 1, notice that, subject to feasibility and because of the presence of the entrepreneurial rents C, the entrepreneur prefers the maximum in-house development of her innovation. However, the moral hazard problem requires E to appropriate a minimum fraction of the pecuniary returns on the paths that she develops, which limits her capacity to repay the financiers. Under Assumption 2, licensing allows E to increase the internally financed *proportion* of the in-house development investment, restoring her incentives for diligent management.

Notice that the parameter related to the moral hazard problem, b, plays a crucial role in Proposition 1. When b is low relative to E's net payoff under diligent management, full in-house development is feasible and, hence, optimal. As b grows, the licensing of more and more paths may become necessary. Intuitively, partial licensing is a second-best solution to the moral hazard problem and, hence, it is less needed whenever the net present value of the pecuniary returns of the investment, $\beta pv - 1$, or the entrepreneurial rents, βpC, increase.

of her business. As a matter of fact, since E and her financiers are both risk neutral and have the same discount factor, this possibility turns out to be irrelevant.

3 The Industry Model

Assume that firms operate in an infinite horizon industry where time is indexed by t. The industry consists of a measure-one continuum of business niches. Each niche can be interpreted as the market for a different product.[13] At each date t there is a measure $x_t \in [0, 1]$ of niches monopolized by producers protected by an active patent; the remaining niches are either empty or occupied by (symmetric) firms that compete a la Bertrand and make zero profits.[14] For the duration of their incumbency, monopolists obtain a profit or cash flow $a > 0$ per period.

Active patents become worthless whenever their niche is successfully occupied by an imitator or the holder of a patent on a newer product.[15] At each date t, each monopolized niche is challenged by at least an imitator with a probability $\delta > 0$, which is exogenous and independent across niches. When challenged by imitators, a patent grants the incumbent producer a probability λ_1 of preserving his niche. When this protection fails, the niche becomes Bertrand competitive.

The entry of the developers of new products occurs once the imitation process is complete and makes each niche (monopolized or not) to be challenged by at least one new product with probability q_t. In monopolized niches, patent protection allows the incumbent to preserve his niche with probability λ_2 when facing future innovators. Otherwise, one of the successful developers becomes the new monopolist (whose patent joins the stock of active patents). Hence, the value of an active patent at date t, which is the present value of the monopoly profits that yields to its holder, can be recursively written as

$$v_t = a + \beta[1 - (1 - \lambda_1)\delta][1 - (1 - \lambda_2)q_{t+1}]v_{t+1}, \tag{3}$$

where the two terms in brackets represent the probability of surmounting the entry of

[13]This simplification allows us to abstract from cross-product competition and to focus on competition related with concomitant and future entry into each niche.

[14]Symmetry is a consequence of the imitation process described below.

[15]In Section 5, we interpret the introduction of newer products in terms of a standard quality ladder model with limit pricing in which a is the quality improvement brought about by each successful innovation in the corresponding niche.

imitators and innovators, respectively, at date $t+1$.

At any date t, the stock of active patents equals the mass of monopolized niches, whose law of motion can be written as

$$x_t = [1-(1-\lambda_1)\delta]x_{t-1} + \{1-[1-(1-\lambda_1)\delta]x_{t-1}\}q_t. \quad (4)$$

The first term in the right hand side accounts for the niches that, being monopolized at $t-1$, remain monopolized after the entry of imitators at t; the second term accounts for those non-monopolized niches that after such a process become monopolized by the subsequent entry of a new patented product at t.[16]

We assume that at each date t there is an unlimited number of agents who may become entrepreneurs and engender an innovation by incurring a non-pecuniary cost $\Phi < b$.[17] Entrepreneurs and/or their licensees develop new products according to the process described in Section 2. For simplicity, we assume that, in equilibrium, it is optimal for the entrepreneurs to license a fraction $\alpha_t \in (0,1)$ of the development paths of their innovations. As shown in Proposition 1, an entrepreneur's expected payoff under partial licensing is $V_t = (1-\alpha_t)b$, so having $\Phi < b$ is a necessary condition for the equilibrium entry rate to be positive.

The developer of a new product can only monopolize a business niche after overcoming the competition of the developers of alternative new products that covet the same niche, as well as the opposition of incumbent producers. To model the former, let $e_t \in [0,\infty)$ denote the mass of innovations subject to development between dates $t-1$ and t if they exist (which coincides with the mass of entrepreneurs entering at $t-1$). We postulate that each of these innovations becomes the challenging product of a niche with an identical and independent probability $1/(1+e_t)$, so that the probability of success goes to one as the measure of simultaneously developed innovations goes to zero. With this function, we

[16] The entry of newer products in already monopolized niches implies the replacement of previously active patents with new ones but this is inconsequential to the size of the stock x_t.

[17] We make this cost non-pecuniary (e.g., an opportunity cost) for the sole purpose of focusing entrepreneurs' financing problem on the funding of their in-house development investments. In practice, financial needs appear in both stages, but the required investments are typically much higher in the development stage.

capture that innovation is uncoordinated and, hence, subject to congestion.[18] Hence, like in a reduced-form patent race among symmetric contestants, the probability of success of any given innovation declines with the number of competing innovations.[19]

Obviously, the probability q_t with which a business niche is challenged by at least one developer at date t must equal the product of the number of innovations subject to development at that date, e_t, and the probability with which each of them gives raise to a challenger product, $1/(1+e_t)$. Thus, we must have $q_t = e_t/(1+e_t)$, which is increasing in e_t and we take as an alternative measure of the *entry flow*.

To simplify the analysis, we assume that when the developer of a new product reaches an empty or competitive niche, it immediately becomes its monopolist. In an already monopolized niche, however, the entrant faces opposition (based on a legal dispute on patent rights) from the incumbent and only becomes its new monopolist with probability $1-\lambda_2$. Thus, the ex-ante probability of success in the development of an innovation between dates $t-1$ and t is

$$p_t = \{1-\lambda_2[1-(1-\lambda_1)\delta]x_{t-1}\}(1-q_t), \tag{5}$$

where we have used the equality $1/(1+e_t) = 1-q_t$ to rewrite the probability that the innovation becomes a challenging product. The term $[1-(1-\lambda_1)\delta]x_{t-1}$ reflects the fraction of niches that remain monopolized when developers reach them.

Finally, notice that at dates in which entrepreneurs' entry is strictly positive, their free-entry condition under partial licensing reads

$$\beta p_t[v_t + (1-\alpha_t)C] - (1+\Phi) = 0, \tag{6}$$

while (2) establishes

$$\alpha_t = 1 - \frac{\beta p_t v_t - 1}{b - \beta p_t C}. \tag{7}$$

[18] Of course, coordination and congestion problems could be modeled in many other ways. For example, the explicitly probabilistic *urn-ball process* postulated by the literature on random matching would imply a success probability of $[1-\exp(-e)]/e$ for each innovation. Our formulation is simply more tractable.

[19] As opposed to classical models in the patent-race literature such as Loury (1979) or Lee and Wilde (1980), we do not model the timing of innovation and our interpretation corresponds to a one-shot game.

But (6) and (7) together imply $1 - \alpha_t = \phi$, where $\phi \equiv \Phi/b < 1$. Hence, using (5) and considering the possibility that the net gains from becoming an entrepreneur are negative at dates in which no entrepreneur enters, free entry can be summarized by two conditions, the inequality

$$\beta\{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}\}(1 - q_t)(v_t + \phi C) - (1 + \Phi) \leq 0, \tag{8}$$

and the complementary slackness condition

$$q_t\{\beta\{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}\}(1 - q_t)(v_t + \phi C) - (1 + \Phi)\} = 0. \tag{9}$$

The purpose of (9) is to guarantee that no entrepreneur enters ($q_t = 0$) when (8) holds with strict inequality.

4 Analysis of the Equilibrium

In this section we describe the dynamic industry equilibrium and analyze its steady-state properties. We start by defining what we denote as a *candidate equilibrium with partial licensing.*

Definition 1 *Given an initial condition x_0, a* candidate equilibrium with partial licensing *is a sequence of non-negative triples (q_t, x_t, v_t), for $t = 1, ...\infty$, that satisfy the valuation equation (3), the law of motion (4), the free-entry inequality (8), and the complementary slackness condition (9).*

The triples (q_t, x_t, v_t) mentioned in Definition 1, describe the entry flow, the stock of active patents, and the value of a patent at each date t. This definition refers to a "candidate" equilibrium with partial licensing because it does not explicitly impose the condition that guarantees that entrepreneurs use partial licensing at all dates. As shown in Proposition 1, the additional condition boils down to requiring that the parameter that captures the severity of the moral hazard problem, b, is large enough relative to the net present value of the development investment:

Definition 2 *A candidate equilibrium with partial licensing is an* equilibrium *if* $b > \beta p_t(v_t + C) - 1$ *for* $t = 1, ...\infty$.

We have checked numerically that there is a wide range of parameter values and initial conditions x_0 for which the candidate partial licensing equilibrium sequences obtained from the conditions stated in Definition 1 satisfy the additional condition imposed by Definition 2. One such parameterization is presented in the simulations that appear later in the paper.

When entry is strictly positive along the equilibrium sequence, the set of equilibrium conditions described in Definition 1 can be reduced to a bidimensional first-order non-linear system of difference equations in x_t and v_t. Specifically, equation (4) can be used to solve for q_t and substitute the resulting expression into the remaining three equilibrium conditions. But when entry is positive, (9) implies that (8) must hold with equality. Substituting the expression for q_t in this equality and in (3), respectively, yields the two difference equations of the reduced system:

$$\beta(1 - x_t)\frac{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}}{1 - [1 - (1 - \lambda_1)\delta]x_{t-1}}(v_t + \phi C) - (1 + \Phi) = 0, \tag{10}$$

$$\beta[1 - (1 - \lambda_1)\delta]\frac{1 - (1 - \lambda_2)x_t - \lambda_2[1 - (1 - \lambda_1)\delta]x_{t-1}}{1 - [1 - (1 - \lambda_1)\delta]x_{t-1}}v_t - v_{t-1} + a = 0. \tag{11}$$

Although it would be possible to generalize these equations to accommodate the case in which entry is zero ($q_t = 0$) at some dates, they are sufficient to describe the dynamics of the system in the neighborhood of a steady-state (SS) equilibrium with a positive stock of active patents.[20]

When (10) and (11) are evaluated in a steady-state equilibrium with $x_t = x_{t-1} = x_{ss}$ and $v_t = v_{t-1} = v_{ss}$, we obtain

$$\beta(1 - x_{ss})\frac{1 - \lambda_2[1 - (1 - \lambda_1)\delta]x_{ss}}{1 - [1 - (1 - \lambda_1)\delta]x_{ss}}(v_{ss} + \phi C) - (1 + \Phi) = 0, \tag{12}$$

$$\left[1 - \beta[1 - (1 - \lambda_1)\delta]\frac{1 - [1 - \lambda_2(1 - \lambda_1)\delta]x_{ss}}{1 - [1 - (1 - \lambda_1)\delta]x_{ss}}\right]v_{ss} - a = 0. \tag{13}$$

[20] Notice that in a steady-state equilibrium we must have $(q_t, x_t, v_t) = (q_{ss}, x_{ss}, v_{ss})$ for all t, but then $x_t = x_{t-1} = x_{ss} > 0$ requires $q_{ss} > 0$, by (4).

The steady-state entry variable q_{ss} can be obtained as a function of x_{ss} using (4):

$$q_{ss} = \frac{(1-\lambda_1)\delta x_{ss}}{1-[1-(1-\lambda_1)\delta]x_{ss}}. \tag{14}$$

In the next lemma we provide conditions for the existence of a candidate steady-state equilibrium with partial licensing. We also show that, if it exists, the steady state is unique and locally stable.

Lemma 1 *There exists a unique candidate steady-state equilibrium with partial licensing if and only if*

$$\beta\left\{\frac{a}{1-\beta[1-(1-\lambda_1)\delta]} + \phi C\right\} - 1 \geq \Phi. \tag{15}$$

This equilibrium is locally stable and exhibits monotonic convergence in the state variable x_t and saddle-path convergence in the jump variable v_t.

The steady-state stock of active patents x_{ss} and the steady-state value of a patent v_{ss} can be described as the coordinates of the intersection between the two curves depicted in Figure 1, which are defined by equations (12) and (13). As shown in the proof of the previous lemma, equation (12), related to free entry, defines an increasing relationship between x_{ss} and v_{ss}. Quite intuitively, this curve reflects that, when the stock of active patents is larger, the developers of new products are more likely to find opposition from incumbents and, thus, less likely to enter successfully, so a larger (contingent-on-success) value of patents is necessary to encourage entrepreneurs to innovate. Equation (13) expresses the value of a patent as a discounted sum of the one-period monopoly profits a and establishes a negative relationship between x_{ss} and v_{ss}. The reason behind the negative slope is that, as shown in (14), steady-state entry is positively related to x_{ss}, and entry increases the risk that a patent becomes worthless.[21] The existence condition provided in the lemma is equivalent to requiring that the intercept of the free-entry curve (12) is lower than the intercept of the present-value curve (13).

[21] Notice that steady-state entry has to be sufficient for the additions to the stock of active patents to offset the substrations due to imitation, which are proportional to x_{ss}.

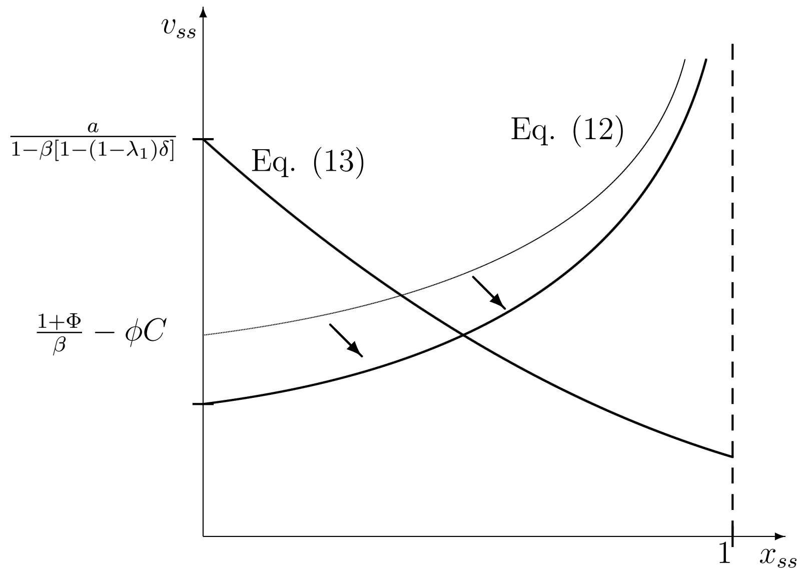

Figure 1: Characterization of the Steady State. It is easy to study the comparative-statics implications of the model for most parameters. Here, for example, we display the effect of an increase in C.

Figure 1 is also useful to perform comparative statics regarding the effects of most parameters on x_{ss} and v_{ss}. For some of them, graphical arguments are ambiguous and an analytical proof is required. The next proposition summarizes these effects.[22]

Proposition 2 *The effects of the parameters of the model on the steady-state variables* q_{ss}, x_{ss}, *and* v_{ss} *have the signs shown in Table 1.*

	a	β	δ	λ_1	λ_2	b	C	Φ
q_{ss}	+	+	?	?	−	−	+	−
x_{ss}	+	+	−	+	−	−	+	−
v_{ss}	+	+	−	+	+	+	−	+

Table 1: Comparative statics.

As expected, stronger protection of intellectual property rights (an increase in either λ_1 or λ_2) results in an increase in the value of each active patent, as it reduces the proba-

[22]Because of the way they enter all expressions, the effects of δ are colinear to the effects of λ_1, but with the opposite sign. We report them below for completeness, but the proofs refer to the effect of λ_1 only.

bility that the patent becomes worthless because of imitation or innovation. Interestingly, however, λ_1 and λ_2 have opposite implications for the stock of patents. Protection against imitation, λ_1, merely expands the expected valuable life of each patent, resulting in an increase in x_{ss}, while more protection against innovative entry, λ_2, has the additional effect of weakening the incentives for would-be entrepreneurs to enter. When λ_2 increases, the entry preemption effect dominates for two reasons. First, because from the perspective of a potential entrant the future protection granted by a larger λ_2 is discounted vis-a-vis the extra hurdle to entry that it imposes. Second, because a larger λ_2 reduces turnover within the stock of active patents and hence, at a societal level, it implies a lower "mobilization" of the rents from independent entrepreneurial success (C) as a reward for innovation.

Reductions in the moral hazard problem that determines entrepreneurs' financing constraints (a decrease in b), a greater valuation of independent business success (an increase in C), and reductions in entrepreneurs' costs of entry (a decrease in Φ) have all qualitatively similar effects: they make innovative entry more likely, which increases the stock of active patents but reduces the expected duration of the monopoly granted by each patent and, thus, its value.[23]

Given the positive relationship between x_{ss} and q_{ss} described by (14), the effect of most parameters on steady-state entry is of the same sign as their effect on the steady-state stock of patents. The exceptions are the parameters that determine patents' effective risk of demise by imitation, $(1-\lambda_1)\delta$. These parameters have an indirect effect through x_{ss} but also a direct effect on q_{ss}. Mathematically, one can immediately see that the direct effect is positive because steady-state entry has to be sufficient to offset the subtractions to the stock of active patents due to imitation; in contrast, the indirect effect is negative since, as already explained, imitation risk reduces x_{ss}. Economically, the opposite sign of the effects is explained by the fact that imitation, on the one hand, erodes the expected profits of the successful developer of a new product but, on the other hand, it also increases the

[23]To complete the description of the comparative statics, notice that the monopoly rents a and the discount factor β have the standard positive effects on the value of innovation and, thus, increase both x_{ss} and v_{ss}.

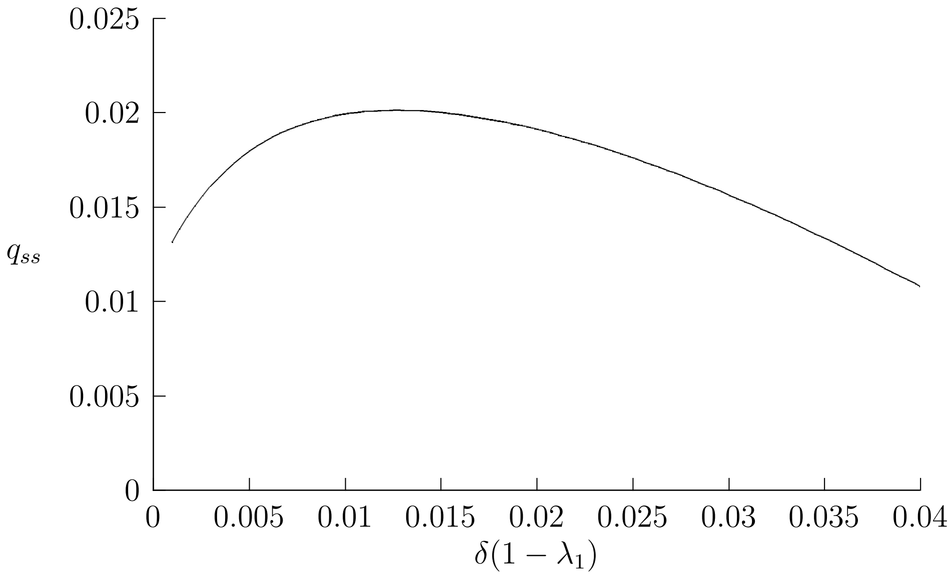

Figure 2: Steady-state entry and imitation risk. This graph depicts q_{ss} (vertical axis) as a function of $(1-\lambda_1)\delta$ (horizontal axis). The underlying parameter values are $a = 0.1$, $\beta = 0.96, \lambda_2 = 0.5$, $b = C = 0.3$, and $\Phi = 0.15$.

fraction of competitive niches, which facilitates entry.

We have verified, by numerical simulation, that it is possible to find examples in which either the positive effect or the negative effect dominates. In many cases, as in the parameterization illustrated in Figure 2, entry is maximized at some interior value of the probability $(1-\lambda_1)\delta$. This example suggests that the trade-offs involved in the choice of an innovator's protection against imitators are not trivial. And they would be even less so it were not possible to separately manage the protection against imitators and that against innovators. We come back to these issues below, when discussing the welfare implications of our analysis.

Before closing this section, it is worth to briefly comment on a significant case in which, in the transition towards a steady state with partial licensing, the equilibrium is not characterized by (10) and (11). Suppose that the initial stock of active patents x_0 is well above its steady-state value x_{ss}. How will the steady-state be reached? If the initial proportion of monopolized niches is too large, there are a few periods during which entry is not profitable, inducing $q_t = 0$ and allowing incumbents to enjoy the excess profits of a below-normal entry threat. As time passes, however, imitation erodes the stock of active patents up to a point where entry is reestablished and the steady-state described by (10) and (11) is reached.[24]

5 Welfare Implications

So far we have not explicitly referred to the demand side of the industry. In this section we fill this gap in order to perform a meaningful analysis of the welfare and policy implications of the model. We will interpret the innovation process in terms of a standard quality ladder model with limit pricing. The demand configuration and the proposed welfare measure are inspired in Hopenhayn et al. (2006), that applies them in a similar sequential innovation setup. Notice that, since in equilibrium entrepreneurs, financiers, and innovation developers operate under zero-net-value or free-entry conditions, their direct net contribution to aggregate social surplus is zero so we can focus our analysis on consumers' net utility.

Suppose that there is a unit mass of infinitely-lived homogeneous consumers willing to buy at most one unit of the product from each niche $j \in [0, 1]$ at each date t. Utility is additive across goods and dates, the intertemporal discount factor is $\beta < 1$, and the net utility flow from buying good j at price P_{jt} is $U_{jt} = A_{jt} - P_{jt}$, where A_{jt} is the quality of the good. Suppose that the successful entry of an innovation in a given niche improves the quality of the best good available in that niche by a units, while the successful entry of an imitator in the niche makes the production technology of the best quality good freely available to him (as well as to the previous monopolist). Finally, suppose, for simplicitly, that production costs are zero.

How are goods priced in each niche? How does consumers' utility evolve over time? To answer these questions, notice that active monopolists are always able to charge a price $P_{jt} = a$ that captures the full quality advantage of their product vis-a-vis the best

[24]In this transition, the reduction in the stock of monopolized niches will typically lead to a situation with $x_t > x_{ss} > [1 - (1 - \lambda_1)\delta]x_t$ just one period before the steady-state would have been reached.

competing product. So the quality improvement associated with an innovation does not *directly and immediately* translate into an increase in consumers' net utility flow. If the innovation falls on a non-monopolized niche, consumers will enjoy the greater quality of the new good but will also pay a higher price and their net utility gain will be zero. The increase in consumers' net utility occurs when the monopolized niche experiences the entry of a competitor of either equal quality (imitator) or greater quality (innovator). Consumers will then enjoy an extra surplus of a per period for all periods ahead either because of the smaller price (zero) paid for the same old good (after imitation) or for enjoying (after innovation) a better quality good at the same price as before.

Clearly, in this setup, consumers' net utility in the steady state equilibrium grows linearly over time, so it seems adequate to measure the social welfare associated with a steady state, W_{ss}, through the present value of consumers' *incremental* net utility flows due to the imitation and innovation processes completed in a typical date:[25]

$$W_{ss} = x_{ss}\{(1-\lambda_1)\delta + [1-(1-\lambda_1)\delta](1-\lambda_2)q_{ss}\}\frac{a}{1-\beta}. \tag{16}$$

To explain (16), notice that utility additions only occur over monopolized business niches, whose measure is x_{ss}, and are associated with either imitation, which occurs at rate $(1-\lambda_1)\delta$ over those niches, or innovation, which occurs at rate $(1-\lambda_2)q_{ss}$ over the remaining proportion of monopolized niches $1-(1-\lambda_1)\delta$. Any of these entry processes imply a perpetual addition of a to the consumers' net utility flow and $a/(1-\beta)$ is just the discounted value of such a perpetuity.

Expression (16) allows us to decompose the total effect of any model parameter θ on social welfare in up to a direct effect and two indirect effects channeled through the steady-state variables q_{ss} and x_{ss}:

$$\frac{dW_{ss}}{d\theta} = \frac{\partial W_{ss}}{\partial \theta} + \frac{\partial W_{ss}}{\partial q_{ss}}\frac{dq_{ss}}{d\theta} + \frac{\partial W_{ss}}{\partial x_{ss}}\frac{dx_{ss}}{d\theta},$$

where $\partial W_{ss}/\partial q_{ss} = [1-(1-\lambda_1)\delta](1-\lambda_2)x_{ss}a/(1-\beta) > 0$ and $\partial W_{ss}/\partial x_{ss} = W_{ss}/x_{ss} > 0$.

[25] Compared to models in the endogenous growth literature, here steady-state welfare increases linearly. As a result, our welfare measure does not require to discount utility using the equilibrium growth rate

Direct inspection of (16) and the results in Proposition 2 allow us to construct the following table:

	a	β	δ	λ_1	λ_2	b	C	Φ
$\frac{\partial W_{ss}}{\partial \theta}$	+	+	+	−	−	0	0	0
$\frac{\partial W_{ss}}{\partial q_{ss}} \frac{dq_{ss}}{d\theta}$	+	+	?	?	−	−	+	−
$\frac{\partial W_{ss}}{\partial x_{ss}} \frac{dx_{ss}}{d\theta}$	+	+	−	+	−	−	+	−
$\frac{dW_{ss}}{d\theta}$	+	+	?	?	−	−	+	−

Table 2: Decomposition of the model parameters' welfare effects.

The effects of parameters such as a, β, b, C, and Φ are self-explanatory, once we account for their role in determining the steady-state level of innovation and the stock of active patents and after noting that their direct effects on our welfare measure are either of the same sign as the indirect effects (a and β) or zero (b, C, and Φ). More intriguing are the effects of the parameters related to the effective risk of imitation, $(1-\lambda_1)\delta$, and the protection that IP grants against subsequent innovations, λ_2. In Subsection 5.1 we discuss them first separately and then when the changes in λ_1 and λ_2 are linked to the general strength of IP protection. In Subsection 5.2 we discuss how the optimal design if IP protection in each of these cases gets modified when the tightness of entrepreneurs' financial constraints (as captured by b) changes.

5.1 IP protection

Increasing the protection of patent holders against subsequent innovations, λ_2, has an unambiguously negative total effect on social welfare, since both the direct effect and the indirect effects shown in Table 5 are negative.

The total effect on welfare of imitation risk, $(1-\lambda_1)\delta$, is ambiguous and numerical examples show that it may have an inverted-U shape. The ambiguity has a double source. First, the sign of the indirect effect channeled through q_{ss} is ambiguous since imitation risk has an unclear and potentially non-monotonic effect on steady-state entry, which was

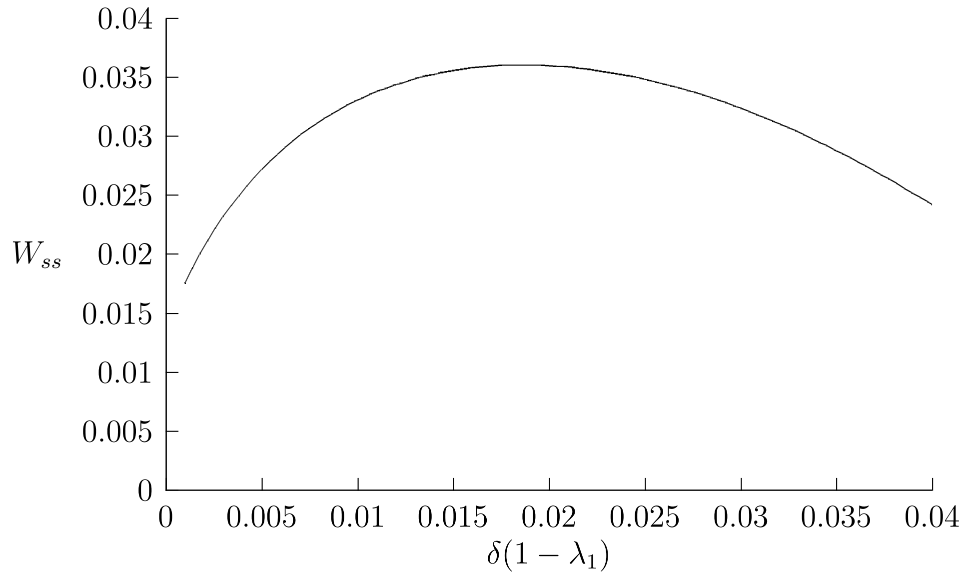

Figure 3: Steady-state welfare and imitation risk. This graph depicts W_{ss} (vertical axis) as a function of $(1-\lambda_1)\delta$ (horizontal axis). The underlying parameter values are $a = 0.1$, $\beta = 0.96, \lambda_2 = 0.5$, $b = C = 0.3$, and $\Phi = 0.15$.

commented at the end of Section 4 and illustrated in Figure 2. Second, unless such an effect on q_{ss} is positive and large enough to offset the negative indirect effect channeled through x_{ss}, the direct effect of imitation risk is positive and, thus, generates further ambiguity. Numerical examples show that the resolution of the ambiguity can point in any direction: depending on parameter values, W_{ss} can reach a maximun at $(1-\lambda_1)\delta = 0$, at $(1-\lambda_1)\delta = 1$, or at some interior level. Figure 3 shows that, under the parameterization previously used in Figure 2, the socially optimal level of imitation risk is interior.

So far we have discussed the effects of λ_1 and λ_2 separately. But it may be argued that IP protection cannot be tailored to make such a clear distinction between imitation and innovation. As a result, the patent statute is likely to hinder both kinds of entry in a related manner. Figure 4 provides an example of the welfare implications of our model in one such case: when we impose $\lambda_1 = \lambda_2 = \lambda$ under the parameterization used before and with $\delta = 0.05$. In this case, the overall degree of IP protection has an inverted-U shaped effect on social welfare and, as one might expect, social welfare is maximum at a level of protection between zero (which would be the optimal value for an independently set λ_2)

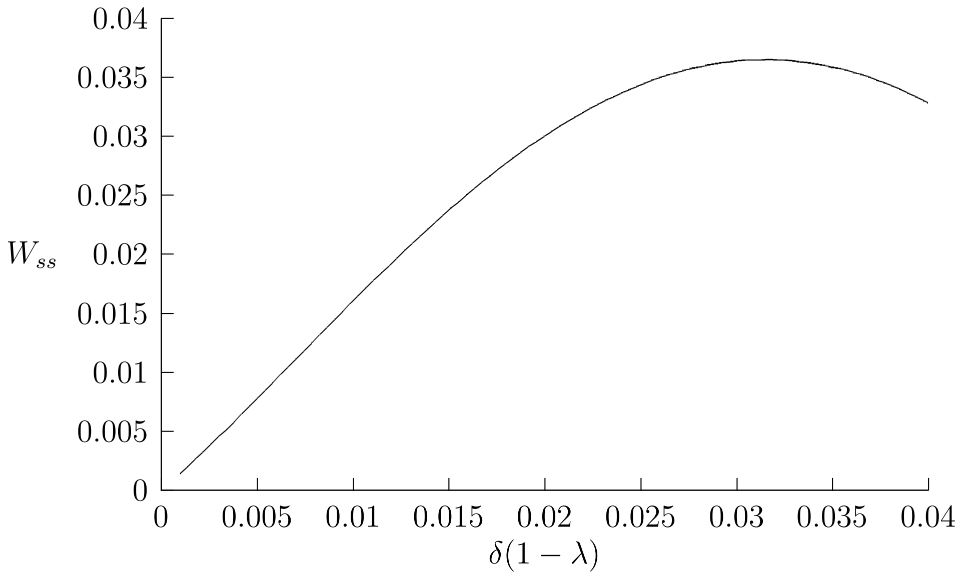

Figure 4: Steady-state welfare and IP protection for $\lambda_1 = \lambda_2 = \lambda$ and $\delta = 0.05$. Other parameters are as in Figure 3.

and the optimal λ_1 of Figure 3.

5.2 Financial constraints and IP protection

Finally, we study how financial constraints interact with IP protection. For brevity we focus on the case with $\lambda_1 = \lambda_2 = \lambda$. Taking the same underlying parameterization as in previous figures, Figure 5 displays the relationship between $\delta(1-\lambda)$ and social welfare in three regimes that differ in the tightness of financial constraints, as determined by the moral hazard parameter b. Holmstrom and Tirole (1997) model the monitoring role of financial intermediaries as a reduction in b. Along the same lines, in an innovation financing context, one could interpret a reduction in b as the result of the monitoring provided by expert venture capitalists.

The intermediate curve in Figure 5 is the same as in Figure 4. The top curve only differs in that $b = 0$, in which case entrepreneurs can develop their innovations fully in-house, so $\phi = 0$ in the relevant equations. The bottom curve exhibits the limit case in which the moral hazard problem is so severe that the innovation is fully licensed to outside developers, $\phi = 1$. In addition to illustrating the detrimental effect of financial

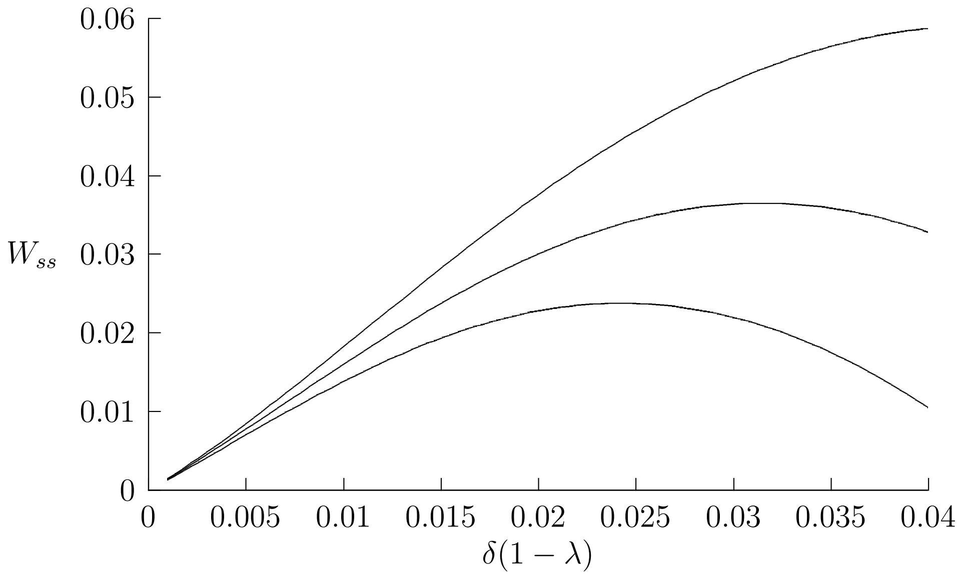

Figure 5: Changes in welfare and the optimal level of IP protection for different tightness of the financial constraints. The underlying parameters are as in Figure 4.

constraints, the figure clearly shows that they increase the optimal level of IP protection. Hence, a level of IP protection that is optimal under tight financial constraints may become excessive once the development of institutions such as venture capital financing or an improvement in monitoring technologies lessen the relevant financial constraints.

The reason why this effect occurs is different from what transpires of a typical static analysis. Remember that, with the dynamic effects captured in the model, increasing λ_2 has the net effect of discouraging innovation so one cannot immediately argue that increasing λ seeks to compensate for the negative effect of financial constraints on innovation. However, increasing λ would approach the protection against imitation λ_1 to what, if separately fixed, would be its socially optimal level (which, in addition, is increasing in the tightness of financial constraints). Under tighter financial constraints, the tension between approaching λ to zero or to the optimal level of λ_1 resolves more favorably to the latter because tightening the constraints reduces the opportunity cost of increasing λ_2. Specifically, it forces the entrepreneurs to out-license a larger proportion of development paths of their innovations and, hence, reduces the size of the rents from entrepreneurial success associated with each potential innovation. Reducing the turnover of actual innovators (via a larger protection of incumbents) is then less of a waste for the society's system of innovation incentives.

6 Concluding Remarks

Innovation is considered key to industry dynamics. Entry, exit, and innovation are complex interrelated phenomena in every industry, and especially so in the youngest and more technology-intensive or knowledge-intensive industries. Many of these industries rely on intellectual property (IP) as the source of temporary monopoly power that allows the successful innovators to obtain a return for their previous research and development (R&D) investments. IP protection, however, is a double cutting edge knife for the dynamics of innovative industries, as the protection of incumbent innovators may be an obstacle to the success of novel innovators. This paper contributes to the growing literature that analyzes the role of IP protection in an industry dynamics setting by explicitly considering the implications of financial constraints, with relevance to entrepreneurial innovators that is out of doubt but has received almost no attention in the IP literature.

In our model we have made a clear distinction between innovative entry and imitative entry, and the protection against each of them granted by IP to incumbents. This feature has allowed us to identify some novel trade-offs concerning the role of imitation. Specifically, we find that the pro-competitive effect of imitation may make imitation overall beneficial for innovation and welfare, since the hurdle for the entry of innovators is lower when there are less incumbent monopolists defending their business niches. For a wide range of parameter values, the relationship between imitation risk and welfare (as well as innovation) has an inverted-U shape, and such a shape extrapolates to the relationship between IP protection and welfare. Qualitatively, this effect resembles the type of relationship between competition and innovation identified by Aghion et al. (2001) by opposing an "escape competition" effect to the standard rent-reducing "Schumpeterian effect". The mechanisms underlying their story and ours are, however, very different: in theirs, incumbents tend to innovate more so as to take some distance from the competitive fringe of the industry; in ours, competition eliminates the incumbent monopolies, reducing the barriers for the entry of future innovators.

We have shown that financial constraints provide a rationale for the use of the partial out-licensing of innovations as part of entrepreneurs' strategy for the financing of their R&D investments. At an industry level, this microeconomic insight implies that financial constraints reduce the fraction of entrepreneurs among the final developers of new products. We have found that such an effect alters the trade-offs underlying the choice of a socially optimal degree of IP protection. It turns out that, with a lower entrepreneurial fraction of innovators, the rents associated with entrepreneurial success play a smaller role in the system of incentives to innovate and the desirability of a high turnover among incumbent innovators gets reduced. In these conditions, the protection against imitation (and, more generally, IP protection) becomes comparatively more desirable for the society. The reverse argument applies if financial constraints get relaxed: IP protection should diminish.

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Appendix

Proof of Proposition 1: To find the solution to the problem in (1), it is convenient to first ignore the limited liability constraints in (C5) and later verify that the resulting solution satisfies them. We characterize the solution to the maximization of (OBJ) subject to (C1)-(C4) in three steps:

1. *Financiers' participation constraint is binding.* Notice that R appears in (OBJ), (C1) and (C2). (OBJ) is decreasing in R and (C1) gets clearly relaxed as R decreases, while (C2) gets tightened as R increases. But then (C2) must hold with equality since, otherwise, R could be reduced, improving (OBJ) without compromising feasibility.

2. *Value maximization entails the minimization of α.* Since (C2) is binding, we can use it to substitute for R in (OBJ). After some reordering, the term in M cancels out and we get

$$V = (\beta pv - 1) + (1 - \alpha)\beta pC, \tag{17}$$

   where the only decision variable is α and $\partial V/\partial\alpha = -\beta pC < 0$. Thus, the objective of the problem can be reformulated as one of minimizing α.

3. *Constraints (C1), (C2), and (C4) can be summarized as one.* Since (C2) is binding, we can substitute for R in (C1) and obtain, after some reordering,

$$(1 - \alpha)(\beta pv - 1) + (1 - \alpha)\beta pC + M \geq (1 - \alpha)b, \tag{18}$$

   which gets relaxed by increasing M. But the reformulated objective function does not directly depend on M so we can focus, without loss of generality, on solutions in which (C4) is binding.[26] After replacing M in (18), we obtain

$$(\beta pv - 1) + (1 - \alpha)\beta pC \geq (1 - \alpha)b, \tag{19}$$

   which, under Assumption 2, gets tightened when α decreases. Hence, the solution to the optimization problem can be described as the minimum $\alpha \in [0, 1]$ that satisfies (19), say α^*. From it, using the fact that (C4) and (C2) hold with equality, we can recursively solve for M^* and R^*.

Notice that when $b \leq \beta p(v + C) - 1$, the incentive compatibility constrained written in (19) holds for $\alpha = 0$ and, thus, the first-best allocation (full in-house development) is feasible and, resultingly, optimal, yielding $V = \beta p(v + C) - 1$.

When $b > \beta p(v + C) - 1$, and under Assumptions 1 and 2, there always exists a unique $\alpha^* \in (0, 1)$ for which (19) holds with equality; any other feasible α would be larger and, from the arguments given in the text, suboptimal. Profits under α^* can be computed as

$$V = (\beta pv - 1) + (1 - \alpha)\beta pC = (1 - \alpha^*)b,$$

[26]It is that arrangements in which E does not devote all her licensing revenues to the development investment are strictly suboptimal. Those arrangements, if feasible, would be dominated by an also feasible arrangement in which E reduces her licensing α and stops using the corresponding revenue for consumption at $t = 0$.

where the last equality arises, again, from (19). ■

Proof of Lemma 1: This proof has two parts. First we discuss the uniqueness and existence of a SS equilibrium with partial licensing. Then we discuss the local stability of the SS equilibrium. *Existence and uniqueness of the SS equilibrium:* For brevity. it is convenient to eliminate the subscripts from x_{ss} and v_{ss} and rewrite (12) and (13) abstractly as:

$$f_1(x, v; \theta) = 0, \tag{20}$$
$$f_2(x, v; \theta) = 0, \tag{21}$$

where θ is the vector of parameters of the model. We will save on notation by refering to a single parameter $\psi \equiv (1-\lambda_1)\delta$ rather than δ and λ_1 separately.

To establish the sign of the monotonic relationship between x_{ss} and v_{ss} in each of the equations, notice that

$$\frac{\partial f_1}{\partial v} = \beta(1-x)\frac{1-\lambda_2(1-\psi)x}{1-(1-\psi)x} > 0,$$

and

$$\frac{\partial f_1}{\partial x} = \beta(v+\phi C)\frac{\lambda_2(1-\psi)[2x-(1-\psi)x^2]-(1-\lambda_2)\psi-\lambda_2}{[1-(1-\psi)x]^2}.$$

The numerator in the last expression is increasing in x and, hence, maximum at $x=1$, but if we evaluate the numerator at $x=1$ we obtain

$$-\lambda_2\psi^2-(1-\lambda_2)\psi < 0,$$

so $\frac{\partial f_1}{\partial x} < 0$ for all x. This implies that (12) defines an upward slopping curve in (x_{ss}, v_{ss}) space. Moreover, it is immediate to check that v_{ss} goes to infinity as x_{ss} approaches one.

As for (13), it can be verified that

$$\frac{\partial f_2}{\partial x} = \frac{\beta\psi(1-\psi)(1-\lambda_2)}{1-(1-\psi)x}v > 0$$

and

$$\frac{\partial f_2}{\partial v} = 1-\beta(1-\psi)\frac{1-(1-\lambda_2\psi)x}{1-(1-\psi)x} > 0,$$

so (13) describes a downward slopping curve. Obviously, the upward and downward sloping curves just described can intersect at most once and such an intersection, if it exists, defines the unique SS equilibrium. Since (12) has a vertical asymptote at $x=1$, the necessary and sufficient condition for existence of the SS equilibrium is that the intercept of (12), $a/[1-\beta(1-\psi)]$, is lower than the intercept of (13), $(1+\Phi)/\beta-\phi C$, which explains condition (15).
Stability of the SS equilibrium: To analyze the local stability of the system around steady state, we proceed to log-linearize (10) and (11) around the SS point (v, x). Log-linearizing (10) yields

$$-\frac{1}{1-x}dx_t + \frac{(1-\psi)(1-\lambda_2)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1} + \frac{1}{v+\phi C}dv_t = 0.$$

Log-linearizing (11) yields

$$-\frac{1-\lambda_2}{1-(1-\lambda_2\psi)x}dx_t + \frac{1}{v}dv_t - \frac{1}{v-a}dv_{t-1} + \frac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1} = 0$$

These expressions can be written as the following system of equations

$$\begin{aligned}
dx_t - \frac{1-x}{v+\phi C}dv_t &= \frac{(1-\psi)(1-\lambda_2)(1-x)}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1}, \\
-\frac{(1-\lambda_2)v}{(1-(1-\lambda_2\psi)x)}dx_t + dv_t &= -\frac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}dx_{t-1} + \frac{v}{v-a}dv_{t-1},
\end{aligned}$$

or in matrix form as

$$\begin{bmatrix} 1 & w_{12} \\ w_{21} & 1 \end{bmatrix}\begin{bmatrix} dv_t \\ dx_t \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} dv_{t-1} \\ dx_{t-1} \end{bmatrix} \tag{22}$$

where

$$\begin{aligned}
w_{11} &= 1, & z_{11} &= \tfrac{(1-\psi)(1-\lambda_2)(1-x)}{[1-\lambda_2(1-\psi)x][1-(1-\psi)x]} > 0, \\
w_{12} &= -\tfrac{1-x}{v+\phi C} < 0, & z_{12} &= 0, \\
w_{21} &= -\tfrac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} < 0, & z_{21} &= -\tfrac{(1-\psi)(1-\lambda_2)(1-x)v}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} < 0, \\
w_{22} &= 1, & z_{22} &= \tfrac{v}{v-a} > 1.
\end{aligned}$$

Finding the inverse of matrix W and premultiplying both sides of (22) by it, the system becomes

$$\begin{bmatrix} dv_t \\ dx_t \end{bmatrix} = Y \begin{bmatrix} dv_{t-1} \\ dx_{t-1} \end{bmatrix}$$

with

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \equiv \frac{1}{1-w_{12}w_{21}}\begin{bmatrix} z_{11}-w_{12}z_{21} & -w_{12}z_{22} \\ -w_{21}z_{11}+z_{21} & z_{22} \end{bmatrix}.$$

The two eigenvalues, μ_1 and μ_2, of matrix Y can be found as the solutions of the equation

$$\det(Y-\mu I) = 0$$

where I is the identity matrix of rank 2. Proving saddle-path convergence towards SS in the log-linearized system amounts to showing that, in absolute value, one of the eigenvalues of matrix Y is greater than 1 and the other is less than 1. We will further show that both eigenvalues are positive.

Since the function $D(\mu) \equiv \det(Y-\mu I)$ describes a parabola that tends to infinity when μ tends to both plus and minus infinity, then showing that $D(0) > 0 > D(1)$ would be enough for our proof. Consider first the sign of

$$D(0) = \det(Y) = \frac{z_{11}z_{22}}{1-w_{21}w_{12}}.$$

Clearly, $z_{11}z_{22} > 0$, so proving that $D(0) > 0$ boils down to showing that

$$1-w_{12}w_{21} = 1 - \frac{1-x}{v+\phi C}\frac{(1-\lambda_2)v}{1-(1-\lambda_2\psi)x} > 1 - \frac{(1-\lambda_2)(1-x)}{1-(1-\lambda_2\psi)x} = \frac{\lambda_2[1-(1-\psi)x]}{1-(1-\lambda_2\psi)x} \in (0,1). \tag{23}$$

Now, as for

$$D(1) = \det(Y-I) = \frac{(y_{11}-1)(y_{22}-1)-y_{12}y_{21}}{(1-w_{21}w_{12})^2},$$

notice that we can ignore the denominator and prove the negativity of

$$\begin{aligned}
(y_{11}-1)(y_{22}-1)-y_{12}y_{21} &= [z_{11}-w_{12}z_{21}-(1-w_{12}w_{21})][z_{22}-(1-w_{12}w_{21})]-w_{12}w_{21}z_{11}z_{22}+w_{12}z_{22}z_{21} \\
&= (1-w_{12}w_{21})\left[w_{12}(z_{21}-w_{21})-(z_{11}-1)(1-z_{22})\right].
\end{aligned}$$

We already know, from (23), that $(1 - w_{12}w_{21}) > 0$. Moreover, from their expressions above, we clearly have $w_{12} < 0$, $1 - z_{22} < 0$, and

$$z_{21} - w_{21} = \frac{(1-\lambda_2)\psi v_{ss}}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]} > 0.$$

It only remains to show that $z_{11} - 1 < 0$, where

$$z_{11} - 1 = \frac{-\psi - (1-\psi)\lambda_2 + 2\lambda_2(1-\psi)x - \lambda_2(1-\psi)^2x^2}{[1-(1-\lambda_2\psi)x][1-(1-\psi)x]}.$$

The denominator of this expression is clearly positive, while the numerator is maximized at $x = 1$. But at $x = 1$ the denominator becomes $-\psi[1–\lambda_2(1–\psi)] < 0$, so the denominator must be negative for all x.■

Proof of Proposition 2: Following Figure 1, changes in parameters like C, b, F or a entail an unambiguous movement of only one of the steady state conditions and therefore the effect on the SS variables x and v (written without the subscript SS, unless needed, to save on notation) is immediate. For the sake of brevity we omit these proofs (we provide, however, some examples in Figure 1) and we focus instead on the effects of the remaining parameters.
Effect of λ_2*:* The effect on v is immediate, since an increase in λ_2 entails an upward shift of the two curves depicted in Figure 1. Regarding the effect on x, define $v_2(x;\theta)$ from the equation $f_2(x, v_2(x;\theta);\theta) = 0$, recalling that f_2 is the left hand side of (13). Also, define

$$g(x;\theta) \equiv f_1(x, v_2(x;\theta);\theta),$$

so that x_{ss} solves $g(x;\theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that g is decreasing in both x and λ_2. With respect to the first,

$$\frac{\partial g}{\partial x} = \frac{\partial f_1}{\partial x} + \frac{\partial f_1}{\partial v}\frac{\partial v_2}{\partial x} < 0$$

since $\partial v_2/\partial x = -(\partial f_2/\partial v)/(\partial f_2/\partial x) < 0$. Regarding the second, we obtain

$$\frac{\partial g}{\partial \lambda_2} = -\beta(1-x)x(1-\psi)\left[\frac{(1-\beta)[1-(1-\psi)x]a}{\{1-(1-\psi)x-\beta(1-\psi)[1-(1-\lambda_2\psi)x]\}^2} + \frac{\phi C}{1-(1-\psi)x}\right] < 0.$$

Effect of β*:* The effect on x is immediate, since an increase in β entails an upward shift of the curve defined by (12) and a downward shift of the curve defined by (13) in Figure 1. Regarding the effect on v, let us implicitly define $x_2(v;\theta)$ from the equation $f_2(x_2(v;\theta), v;\theta) = 0$, recalling that f_2 is the left hand side of (13). Also, define

$$h(v;\theta) \equiv f_1(x_2(v;\theta), v;\theta), \tag{24}$$

so that v_{ss} solves $h(v;\theta) = 0$. Using the Implicit Function Theorem, it is enough for the result to show that h is increasing in v and decreasing in β. With respect to the first,

$$\frac{\partial h}{\partial v} = \frac{\partial f_1}{\partial v} + \frac{\partial f_1}{\partial x}x_2'(v;\theta) > 0$$

since $\partial x_2/\partial v = -(\partial f_2/\partial x)/(\partial f_2/\partial v) < 0$. Regarding the second, we obtain

$$\frac{\partial h}{\partial \lambda_2} = -\frac{\lambda_2(v+\phi C)\{[v-a-\beta(1-\psi)v]^2 + \beta^2\psi(1-\psi)(1-\lambda_2)v^2\}}{(1-\psi)[v-a-\beta(1-\psi)v]^2} < 0.$$

Effect of λ_1*:* Similarly to the case of β, the effect on x is immediate from the upward shift of the curve defined by (12) and the downward shift of the curve defined by (13). Regarding the effect on v_{ss}, and using the function h defined in (24) it is enough to show that $\partial h/\partial\lambda_1 < 0$. In particular, this derivative can be written as

$$\frac{\partial h}{\partial \lambda_1} = \delta \frac{(v+\phi C)[a-(1-\beta)v]\{-(v-a)[a-(1-\beta)v]+\beta v_{ss}[\beta v_{ss}\lambda_2^2(1-\psi)^2-(v-a)(1+\lambda_2-2\varphi\lambda_2)]\}}{(1-\psi)^2 v[v-a+\beta(1-\varphi\lambda_2)v]^2}$$

Notice that $x \in [0,1]$ and $v \in [a/[1-\beta(1-\psi)], a/[1-\beta(1-\psi)\lambda_2]]$, so $a-(1-\beta)v > 0$. Moreover, the last term in the expression in curly brackets will be negative as long as

$$v \geq \frac{a}{1-\dfrac{\beta\lambda_2^2(1-\psi)^2}{1+\lambda_2-2\varphi\lambda_2}},$$

which is true since

$$\frac{a}{1-\dfrac{\beta\lambda_2^2(1-\psi)^2}{1+\lambda_2-2\varphi\lambda_2}} < \frac{a}{1-\beta\lambda_2(1-\psi)} < v.$$

Effect of Φ*:* The parameter Φ only operates through equation (12). It can be shown that

$$\frac{\partial f_1}{\partial \Phi} = -1 + \frac{\beta C(1-x)}{b}\frac{1-\lambda_2(1-\psi)x}{1-(1-\psi)x} = -1 + \frac{\beta p_{ss} C}{b} < 0$$

where the last equality uses (5) and the last inequality arises directly from Assumption 2. As a result, increases in Φ shift upward the curve defined by (12) in Figure 1, resulting in an increase in v and a decrease in x.■