Starting a R&D project under uncertainty*

Sabien Dobbelaere† Roland Iwan Luttens‡ Bettina Peters§

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Abstract

We model a two-stage R&D project with an abandonment option. Two types of uncertainty influence the decision to start R&D. Demand uncertainty is modelled as a lottery between a proportional increase and decrease in demand. Technical uncertainty is modelled as a lottery between a decrease and increase in the cost to continue R&D. Both lotteries become more divergent when the difference between the outcomes of the lottery increases. A potential entrant is endowed with a superior technology and threatens to drive the incumbent out of the market. The incumbent has a time lead over the entrant and can obtain the same superior technology by completing the R&D project before the entrant can enter the market. We derive under which lottery probabilities more divergent demand and supply lotteries positively or negatively affect the decision to start R&D. We test the derived hypotheses using a unique dataset containing proxies for demand and technical uncertainty as well as perceived entry threat for about 4000 German firms in manufacturing and services (CIS IV). Strongly confirming our model predictions, we find that for firms facing lotteries where the good outcome is more likely to prevail (i) a 10% increase in the degree of divergence of the demand lottery increases the likelihood of undertaking R&D by 1.3% and (ii) a 10% increase in the degree of divergence of the supply lottery increases the likelihood of undertaking R&D by 1.5%. For firms facing a demand lottery where the bad outcome is more likely to prevail, a more divergent demand lottery decreases the probability of undertaking R&D significantly.

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†Ghent University, K.U.Leuven, IZA Bonn. Postdoctoral Fellow of the Research Foundation - Flanders (FWO). Corresponding author: Sabien.Dobbelaere@UGent.be

‡SHERPPA, Ghent University and CORE, Université Catholique de Louvain. Postdoctoral Fellow of the Research Foundation - Flanders (FWO).

§Centre for European Economic Research (ZEW).

1 Introduction

The decision to start a Research and Development (R&D) project is one of the most challenging firm decision problems. R&D projects usually take time to complete, their investments are irreversible and therefore represent sunk costs and above all, they are highly uncertain. Models of investment decisions in an uncertain environment have permeated different parts of the investment literature, ranging from the neoclassical theory of investment, over the real options approach to oligopolistic settings explicitly accounting for strategic interaction.

The neoclassical theory of investment postulates that an investment project should be undertaken whenever its net present value (NPV) is positive (Jorgenson, 1963; Eiser and Nadiri, 1968). This theory assumes that the project is either completely reversible or a now-or-never investment opportunity. However, the values of most projects evolve dynamically over time as they are contingent on fluctuating input and output prices. Hence, the opportunity to change the timing of the investment carries an intrinsic value. This observation laid the foundation of the real options approach.

The main insight of the real options approach is that it is only optimal to invest when the present value of the expected cash flow exceeds the cost of investment by a strictly positive amount to compensate for the loss of the option value to delay (we refer to Dixit and Pindyck, 1994 for an excellent survey). The first papers in this field focus on individual discrete projects (Brennan and Schwartz, 1985; McDonald and Siegel, 1985; McDonald and Siegel, 1986). The seminal paper of Pindyck (1988) studies incremental capacity investment, followed by Bertola and Caballero (1994) and Dixit (1995). By modelling the effect of two types of frictions, adjustment costs and irreversibility, on firms' investment decisions, Abel and Eberly (1994) and Abel et al. (1996) link the adjustment cost approach and the real options approach in a unified framework.

The neoclassical as well as the real options theory consider optimal investment behavior of a firm in isolation from its competitors. Typically, competitive interactions in the product market are prevalent. Contributions linking the framework of irreversible investment to the competitive equilibrium with industry-wide uncertainty are Leahy (1993) and Pindyck (1993a). However, many markets are characterized by a relatively small number of firms. In such oligopolistic settings, strategic interaction must be explicitly accounted for. Grenadier (2000) summarizes studies combining game theoretical analysis with the real options methodology.

While most of the literature considers uncertainty in input and output prices, for R&D projects other sources of uncertainty are possible. Grenadier and Weiss (1997) and Farzin et al. (1998) focus on uncertainty in the technological progress. Besides input cost uncertainty, Pindyck (1993b) also considers a second type of cost uncertainty, namely technical uncertainty. It implies that, although the input prices are known, the firm does not know at the beginning the amount of time, effort and materials ultimately needed to complete the project. Importantly, this type of cost uncertainty can only be solved by starting the R&D project. Market uncertainty is related to the future value of the innovation which is strongly determined by market demand (Tyagi, 2006). For example, if firms have successfully developed the new product or production technology, uncertainty still exists about market acceptance and hence innovation rents.

Empirical studies focusing on the effect of uncertainty on *aggregate* investment use the unconditional standard deviation of a time series —such as past changes in inflation, in real exchange rates or in the risk premium— or a more complicated prediction model as uncertainty proxies (see among others Ferderer, 1993b [US]; Episcopos, 1995 [US]; Darby et al., 1999 [France, Germany, US]; Calcagnini and Saltari, 2000 [Italy]; Goel and Ram, 2001 [9 OECD countries]).

Empirical work examining the impact of uncertainty on investment using *industry* data include Goldberg (1993) [US], Huizinga (1993) [US], Caballero and Pindyck (1996) [US], Ghosal and Loungani (1996) [US], Bell and Campa (1997) [EU, US], Ghosal and Loungani (2000) [US] and Temple et al. (2001) [UK]. For example, Caballero and Pindyck (1996) measure uncertainty by the variance of the marginal revenue product of capital. They find that increased uncertainty positively affects the investment threshold that spurs irreversible investment but the quantitative effect is modest. Ghosal and Loungani (1996) provide evidence of price uncertainty exerting a negative effect on investment in the relatively competitive industries. Huizinga (1993) finds that wage and material cost uncertainty adversely affects investment while price uncertainty has the opposite effect.

Empirical evidence on the impact of uncertainty on *firm*-level investment is given by Dorfman and Heien (1989) [US], Ferderer (1993a) [US], Leahy and Whited (1996) [US], Guiso and Parigi (1999) [Italy], Pattillo (1998) [Ghana], Ogawa and Suzuki (2000) [Japan], Green et al. (2001) [Poland], Peeters (2001) [Belgium, Spain], Henley et al. (2003) [UK], Bloom et al. (2007) [UK] and Czarnitzi and Toole (2008) [Germany]. In general, the uncertainty variable is measured in three ways: (i) as a forward-looking indicator based on objective data, (ii) as a measure of the subjective perception of risk of the entrepreneur or (iii) as a measure based on the firm's past experience. Among others, Leahy and Whited (1996) belong to the first group. They use the variance of the firm's daily stock returns as a measure of uncertainty, arguing that this variance captures higher demand or factor price volatility. The authors find that higher uncertainty lowers investment. Guiso and Parigi (1999) belong to the second group. They use survey data on managers' subjective distributions of future demand growth to estimate the variance of firm-level demand shocks. The authors find that the negative effect of uncertainty on investment is stronger for firms with more irreversible investment and for firms with substantial market power. Czarnitzki and Toole (2008) belong to the third group. They proxy uncertainty by the variance of the share of the firm's sales with new products in a pre-sampled period. The authors find that product market uncertainty reduces R&D investments with the negative effect being dampened if firms receive R&D subsidies.

Our paper contributes theoretically and empirically to the literature reviewed above. From a theoretical point of view, we study R&D decisions in the presence of entry threat under two types of uncertainty, demand uncertainty and technical uncertainty. Our model is a generalization of Lukach et al. (2007). We model a R&D project as a two-stage game where the incumbent must decide at the first stage to start and at the second stage to continue R&D. The decision to start is influenced by on the one hand demand uncertainty modelled as a lottery between a proportional increase (=good state) and decrease (=bad state) in demand and on the other hand technical uncertainty modelled as a lottery between a decrease (=good state) and increase (=bad state) in the cost to continue R&D. In comparing two lotteries, a lottery is more favorable (more unfavorable) than another lottery if the probability of the good state of the former is higher (lower) than the probability of the good state of the latter. In comparing two lotteries with equal probabilities, a lottery is more divergent (less divergent) than another lottery if the difference between the good and the bad state is larger (smaller) in the former than in the latter. A potential entrant is endowed with a superior technology and threatens to drive the incumbent out of the market. The incumbent has a time lead over the entrant and can obtain the same technology by completing the R&D project before the entrant can enter the market. Strategic interaction takes the form of preemptive behavior, where the investment of the incumbent discourages the entrant to enter the market. In our model, two forces weaken the incumbent's option to delay the investment: the incumbent's option to abandon R&D in the second stage (see also Bar-Ilan and Strange, 1996; Alvarez and Keppo, 2002) and preemptive competition (see also Kulatilaka and Perotti, 1998; Cottrel and Sick, 2002). We derive under which lottery probabilities more divergent demand and supply lotteries positively or negatively affect the decision to start R&D. We find that a more divergent demand (supply) lottery cannot negatively affect the probability of starting R&D when the lotteries are such that an increase in demand or a decrease in the cost are more likely to prevail than a decrease in demand or an increase in the cost respectively. Under mild assumptions, relating (in the absence of technical uncertainty) the cost of starting R&D to the cost of continuing R&D and the total cost of the R&D project to the profit gain, we find that the more unfavorable the lotteries become, i.e. the more likely it is that the firm experiences a decrease in demand or an increase in the cost, the more divergent the demand (supply) lottery must be in order to positively affect the probability of starting R&D.

From an empirical point of view, we test the hypotheses derived from the theoretical model using data from the fourth Community Innovation Survey (CIS) data in Germany. The uniqueness of our data lies in the availability of proxies for the degree of divergence of the demand and supply lotteries as well as perceived entry threat for about 4000 firms to explain actual and planned R&D investments. We strongly believe that exploiting this kind of firm heterogeneity is the only way to credibly provide empirical evidence of the uncertainty-R&D investment relationship at the firm level. This belief is motivated by the observation that the results about the effect of uncertainty on investment, both quantitatively and qualitatively, greatly vary across studies as soon as the analysis is performed using more aggregated data or taking less firm heterogeneity regarding uncertainty into account. To the best of our knowledge, there is no large-scale econometric study analyzing the effect of two types of uncertainty, one on the demand side and one on the supply side, on the decision to invest in R&D projects. Our main results, strongly confirming our model predictions, are that for firms facing lotteries where the good state is more likely to prevail (i) a 10% increase in the degree of divergence of the demand lottery increases the likelihood of undertaking R&D by 1.3% and (ii) a 10% increase in the degree of divergence of the supply lottery increases the likelihood of undertaking R&D by 1.5%. For firms facing a demand lottery where the bad state is more likely to prevail, a more divergent demand lottery decreases the probability of undertaking R&D significantly.

The remaining part of the paper is organized as follows. Section 2 provides a theoretical analysis of R&D decisions under uncertainty. The comparative statics of section 3 allow us to derive testable hypotheses on the relation between demand and technical uncertainty and the decision to start R&D. Section 4 presents the empirical analysis. Section 5 concludes.

2 A theoretical analysis of R&D decisions under uncertainty

2.1 The model

The incumbent is producing a homogeneous good at unit cost $c \in [0, P]$, where $P \in [0, 1]$ denotes the normalized output price. A potential entrant is endowed with a superior technology that, for simplicity, allows him to produce at a zero unit cost. He faces an entry cost equal to $\omega \in \mathbb{R}_{++}$. Upon entry, both firms engage in Bertrand competition.

We model a R&D project as a two-stage game where the incumbent must decide at the first (second) stage to start (continue) R&D. This captures more realistically R&D outcomes as a sequence of successive decisions rather than as a result of an irreversible one-shot decision. Furthermore, by allowing the incumbent to abandon the R&D project in the second stage, we are able to study the effect of an abandonment option on optimal investment decisions. In our model, two types of uncertainty, one on the demand side and one on the supply side, influence the decision to start. The incumbent has a time lead over the potential entrant. When the incumbent starts *and* continues R&D, he obtains the same superior technology as the potential entrant before the latter can enter the market. Figure 1 illustrates the game tree.

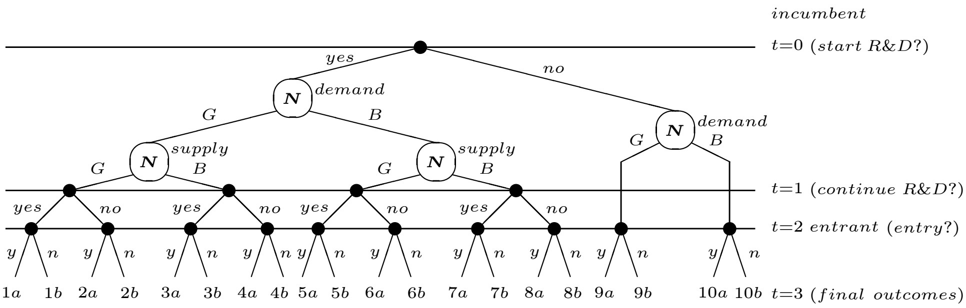

Figure 1 **Game tree.** At t=0, the incumbent decides whether to start R&D. Before t=1, nature (**N**) reveals the good/bad state (G/B) on the demand and supply side (the true state on the supply side is of no influence when the incumbent decides not to start R&D). At t=1, the incumbent decides whether to continue R&D. At t=2, the potential entrant, fully informed about the incumbent's decisions, decides whether to enter. At t=3, final outcomes are realized.

At time zero, the incumbent has to decide whether to start R&D at a *known* cost $I_0 \in \mathbb{R}_{++}$ but under an *unknown* state of the world. There are four possible states of the world, depending on the combination of a good/bad state on the demand and supply side. On the demand side, the good/bad state manifests itself as a proportional increase or decrease in demand, parameterized by $\theta \in [0, 1]$. *A priori*, true demand is a lottery, i.e. the inverse market demand function $D(P, \theta)$ equals $(1 + \theta)(1 - P)$ with probability $p_\theta \in [0, 1]$ and $(1 - \theta)(1 - P)$ with probability $(1 - p_\theta)$. On the supply side, the good/bad state manifests itself as a decrease or an increase in a known cost $I_1 \in \mathbb{R}_{++}$ to continue R&D, parameterized by $\lambda \in [0, I_1]$. *A priori*, the true cost to continue R&D is a lottery, i.e. equal to $(I_1 - \lambda)$ with probability $p_\lambda \in [0, 1]$ and $(I_1 + \lambda)$ with probability $(1 - p_\lambda)$. We assume that all parameters are known beforehand and that both lotteries are independent. Before time one, nature (**N**) reveals the true state of the world.

At time one, the incumbent makes the decision whether to continue R&D.

At time two, the incumbent obtains the superior technology if he continued R&D. Having perfect knowledge about the incumbent's decisions, the potential entrant makes his entry decision. Upon a positive entry decision, the entrant enters the market, producing at a zero unit cost.

At time three, the final market structure is realized and the game ends.

2.2 Optimal entry decision and payoffs

Optimal entry decision

The final market structure is never a duopoly. Indeed, if the incumbent does not possess the superior technology, the potential entrant can push the incumbent out of the market by setting the price slightly under the incumbent's unit production cost. However, entry is only optimal when monopoly profits are higher than or equal to the entry cost. If the potential entrant does not enter, the incumbent stays a monopolist. If the incumbent does possess the superior technology, entry is never optimal. The potential entrant knows that in equilibrium price equals marginal cost and hence profits equal zero, which does not cover the entry cost.

Payoffs

In equilibrium, the monopolist sets $P(c) = \frac{1+c}{2}$ and the corresponding profits are $\pi(c) = \frac{(1-c)^2}{4}$ for all $c \in [0, P]$.

In order to characterize the optimal R&D decisions of the incumbent, we present the incumbent's payoffs that correspond with the bottom row outcomes of Figure 1.

Under scenarios 1, 3, 5 and 7, the incumbent possesses the superior technology and entry is never optimal. Therefore, we only present the incumbent's payoffs under b, which equal:

$$1b : (1+\theta)\,\pi(0) - I_0 - (I_1 - \lambda) \qquad 5b : (1-\theta)\,\pi(0) - I_0 - (I_1 - \lambda)$$
$$3b : (1+\theta)\,\pi(0) - I_0 - (I_1 + \lambda) \qquad 7b : (1-\theta)\,\pi(0) - I_0 - (I_1 + \lambda)$$

Under scenarios 2, 4, 6, 8, 9 and 10, the incumbent does not possess the superior technology. Hence, entry can be optimal. Therefore, we present the incumbent's payoffs valid under a (when entry is optimal ($\pi(0) \geq \omega$)) and b (when entry is not optimal ($\pi(0) < \omega$)).

$$2a : -I_0 \qquad 2b : (1+\theta)\,\pi(c) - I_0$$
$$4a : -I_0 \qquad 4b : (1+\theta)\,\pi(c) - I_0$$
$$6a : -I_0 \qquad 6b : (1-\theta)\,\pi(c) - I_0$$
$$8a : -I_0 \qquad 8b : (1-\theta)\,\pi(c) - I_0$$
$$9a : 0 \qquad 9b : (1+\theta)\,\pi(c)$$
$$10a : 0 \qquad 10b : (1-\theta)\,\pi(c)$$

2.3 Optimal R&D decisions

We determine the optimal R&D decisions of the incumbent by backward induction. We start at $t = 1$. We denote the four possible states of the world by $\{GG, GB, BG, BB\}$, where the first character reflects the good (G) or bad (B) demand state and the second character reflects the good (G) or bad (B) supply state. Let the incumbent's profit gain from innovation be $\Delta\pi = \pi(0) - \pi(c)$. This profit gain is higher when the entrant enters the market than when the entrant does not enter the market, since $\pi(c) = 0$ for the incumbent in the former case, whereas $\pi(c) > 0$ for the incumbent in the latter case. For each possible state of the world $s \in \{GG, GB, BG, BB\}$, we calculate Δ^s_{NPV}, i.e. the difference between the net present value (NPV) of continuing R&D and the NPV of not continuing R&D:

$$\Delta^{GG}_{NPV} = (1+\theta)\,\Delta\pi - (I_1 - \lambda)$$
$$\Delta^{GB}_{NPV} = (1+\theta)\,\Delta\pi - (I_1 + \lambda)$$

$$\Delta_{NPV}^{BG} = (1-\theta)\,\Delta\pi - (I_1 - \lambda)$$
$$\Delta_{NPV}^{BB} = (1-\theta)\,\Delta\pi - (I_1 + \lambda).$$

The incumbent continues R&D if and only if this difference is positive under the true state of the world, taking the entrant's entry decision into account.

Optimal decision to continue r&d: *For each possible state of the world $s \in \{GG, GB, BG, BB\}$, the incumbent continues R&D if and only if $\Delta_{NPV}^{s} \geq 0$.*

Let $\boldsymbol{\psi} = (\psi_{GG}, \psi_{GB}, \psi_{BG}, \psi_{BB})$, where $\psi_s = 1$ when $\Delta_{NPV}^{s} \geq 0$ and $\psi_s = 0$ when $\Delta_{NPV}^{s} < 0$ for all $s \in \{GG, GB, BG, BB\}$, be the vector that comprises the optimal decision to continue R&D under every possible state of the world. Notice that $\Delta_{NPV}^{GG} \geq \Delta_{NPV}^{s} \geq \Delta_{NPV}^{BB}$ for $s \in \{GB, BG\}$. Therefore $\boldsymbol{\psi} \in \Psi = \{\,(1,1,1,1)\,,(1,1,1,0)\,,(1,1,0,0)\,,(1,0,1,0)\,,(1,0,0,0)\,,(0,0,0,0)\,\}$.

At $t = 0$, for every $\boldsymbol{\psi} \in \Psi$, we calculate $\Delta_{NPV}^{\boldsymbol{\psi}}$, i.e. the difference between the NPV of starting R&D and the NPV of not starting R&D. For every $\boldsymbol{\psi} \in \Psi$, we determine the NPV of starting R&D by calculating the weighted sum of the incumbent's payoffs when starting R&D in every possible state of the world (using the probabilities of a good/bad state on the demand and supply side as weights). We determine the NPV of not starting R&D by calculating the weighted sum of the incumbent's payoffs when not starting R&D (using the probabilities of a good/bad state on the demand and supply side as weights). The NPV of not starting R&D is the same for every $\boldsymbol{\psi} \in \Psi$.

Hence, we get:

$$\begin{aligned}
\Delta_{NPV}^{(1,1,1,1)} &= p_\theta p_\lambda\left[(1+\theta)\,\pi(0) - I_0 - (I_1 - \lambda)\right] \\
&\quad + p_\theta\,(1-p_\lambda)\left[(1+\theta)\,\pi(0) - I_0 - (I_1 + \lambda)\right] \\
&\quad + (1-p_\theta)\,p_\lambda\left[(1-\theta)\,\pi(0) - I_0 - (I_1 - \lambda)\right] \\
&\quad + (1-p_\theta)\,(1-p_\lambda)\left[(1-\theta)\,\pi(0) - I_0 - (I_1 + \lambda)\right] \\
&\quad - \left[p_\theta\left[(1+\theta)\,\pi(c)\right] + (1-p_\theta)\left[(1-\theta)\,\pi(c)\right]\right] \\
&= p_\theta p_\lambda \Delta_{NPV}^{GG} + p_\theta\,(1-p_\lambda)\,\Delta_{NPV}^{GB} + (1-p_\theta)\,p_\lambda \Delta_{NPV}^{BG} \\
&\quad + (1-p_\theta)\,(1-p_\lambda)\,\Delta_{NPV}^{BB} - I_0.
\end{aligned}$$

From this, we calculate:

$$\begin{aligned}
\Delta_{NPV}^{(1,1,1,0)} &= \Delta_{NPV}^{(1,1,1,1)} - (1-p_\theta)\,(1-p_\lambda)\left[(1-\theta)\,\pi(0) - I_0 - (I_1 + \lambda)\right] \\
&\quad + (1-p_\theta)\,(1-p_\lambda)\left[(1-\theta)\,\pi(c) - I_0\right] \\
&= \Delta_{NPV}^{(1,1,1,1)} - (1-p_\theta)\,(1-p_\lambda)\,\Delta_{NPV}^{BB} \\
&= p_\theta p_\lambda \Delta_{NPV}^{GG} + p_\theta\,(1-p_\lambda)\,\Delta_{NPV}^{GB} + (1-p_\theta)\,p_\lambda \Delta_{NPV}^{BG} - I_0.
\end{aligned}$$

Similarly, we get:

$$\Delta_{NPV}^{(1,1,0,0)} = p_\theta p_\lambda \Delta_{NPV}^{GG} + p_\theta\,(1-p_\lambda)\,\Delta_{NPV}^{GB} - I_0,$$

$$\Delta_{NPV}^{(1,0,1,0)} = p_\theta p_\lambda \Delta_{NPV}^{GG} + (1-p_\theta)\, p_\lambda \Delta_{NPV}^{BG} - I_0,$$
$$\Delta_{NPV}^{(1,0,0,0)} = p_\theta p_\lambda \Delta_{NPV}^{GG} - I_0,$$
$$\Delta_{NPV}^{(0,0,0,0)} = -I_0.$$

Clearly, $\Delta_{NPV}^{(0,0,0,0)} < 0$ and the incumbent does not start R&D.

The incumbent starts R&D if and only if there exists a positive $\Delta_{NPV}^{\boldsymbol{\psi}}$ for $\boldsymbol{\psi} \in \Psi \backslash \{ (0,0,0,0) \}$. Note that these $\Delta_{NPV}^{\boldsymbol{\psi}}$'s cannot be ordered. For example, take $\Delta_{NPV}^{(1,1,1,1)}$ and $\Delta_{NPV}^{(1,1,1,0)}$. We can write $\Delta_{NPV}^{(1,1,1,1)} = \Delta_{NPV}^{(1,1,1,0)} + (1-p_\theta)(1-p_\lambda)\,\Delta_{NPV}^{BB}$. If $\Delta_{NPV}^{BB} > 0$, then $\Delta_{NPV}^{(1,1,1,1)} > \Delta_{NPV}^{(1,1,1,0)}$ and it is possible to have $\Delta_{NPV}^{(1,1,1,1)} > 0$, while $\Delta_{NPV}^{(1,1,1,0)} < 0$. On the other hand, if $\Delta_{NPV}^{BB} < 0$, then $\Delta_{NPV}^{(1,1,1,1)} < \Delta_{NPV}^{(1,1,1,0)}$ and it is possible to have $\Delta_{NPV}^{(1,1,1,1)} < 0$, while $\Delta_{NPV}^{(1,1,1,0)} > 0$. A similar argument can be made for any other comparison.

Therefore, let $\Phi = \max\{\Delta_{NPV}^{(1,1,1,1)}, \Delta_{NPV}^{(1,1,1,0)}, \Delta_{NPV}^{(1,1,0,0)}, \Delta_{NPV}^{(1,0,1,0)}, \Delta_{NPV}^{(1,0,0,0)}\}$.

Optimal decision to start r&d: *The incumbent starts R&D if and only if* $\Phi \geq 0$.

3 Comparative statics

3.1 Motivation

In the previous section, we derive that it is optimal for the incumbent to start R&D if and only if $\Phi \geq 0$ in a model incorporating both demand and technical uncertainty. This R&D decision depends on the vector of parameters $(c, I_0, I_1, \theta, p_\theta, \lambda, p_\lambda)$.

In this section, we investigate how changes in demand and technical uncertainty, i.e. changes in demand and supply lotteries, affect the incumbent's decision to start R&D. We therefore assume that entry is not optimal, because if entry were optimal, the entrant would drive the incumbent out of the market (cfr. section 2.2). Throughout the remaining analysis, we use the following terminology. A lottery is defined to be *favorable (unfavorable)* if the probability of the good state is higher than or equal to (lower than) the probability of the bad state. In comparing two lotteries, a lottery is defined to be *more favorable (more unfavorable)* than another lottery if the probability of the good state of the former is higher (lower) than the probability of the good state of the latter. However, we do not only distinguish between lotteries in terms of probabilities but also in terms of outcomes. In comparing two lotteries with equal probabilities, a lottery is defined to be *more divergent (less divergent)* than another lottery if the difference between the good and the bad state is larger (smaller) in the former than in the latter. In our model, the degree of divergence depends on θ and λ: a demand (supply) lottery becomes more divergent than another demand (supply) lottery when, *ceteris paribus*, θ (λ) increases and a demand (supply) lottery becomes less divergent than another demand (supply) lottery when, *ceteris paribus*, θ (λ) decreases.

Our motivation to focus explicitly on changes in demand and technical uncertainty is twofold. Theoretically, embedding θ and λ in a model explaining R&D decisions distinguishes our work from previous contributions (cfr. introduction) and allows a richer description of the firm's R&D decision problem. Empirically, the firm heterogeneity in our unique dataset is exactly reflected by proxies for the variables θ and λ. The comparative statics of this section allow us to derive testable hypotheses for the empirical analysis of the next section.

3.2 Approach

We show that a more divergent lottery influences the probability to start R&D differently depending on whether the lottery is favorable or unfavorable. We explicitly focus on how the effect of an increase in θ depends, *ceteris paribus*, on p_θ. A completely similar reasoning, here omitted for reasons of parsimony, holds for how the effect of an increase in λ depends, *ceteris paribus*, on p_λ.

An increase from θ to θ' can, *ceteris paribus*, either have one of the three effects on the decision to start:

(i) a positive effect, i.e. when $\Phi(\theta) < 0$ and $\Phi(\theta') \geq 0$,
(ii) a negative effect, i.e. when $\Phi(\theta) \geq 0$ and $\Phi(\theta') < 0$ or
(iii) no effect, i.e. when $\Phi(\theta) < 0$ and $\Phi(\theta') < 0$ or $\Phi(\theta) \geq 0$ and $\Phi(\theta') \geq 0$.

Our approach aims at comparing $\Phi(\theta)$ and $\Phi(\theta')$ for any $\theta, \theta' \in [0,1]$ where $\theta < \theta'$. We want to make explicit which effects are found for every $p_\theta \in [0,1]$, while restricting the parameter space of $(c, I_0, I_1, \lambda, p_\lambda)$ as little as possible.

Ceteris paribus, it is impossible to compare $\Phi(\theta)$ and $\Phi(\theta')$ for any $\theta, \theta' \in [0,1]$ where $\theta < \theta'$ and never find no effect, since $\Phi(\theta)$ is a continuous function in θ.

Our first proposition states that a more divergent demand lottery never positively affects the decision to start R&D as long as the demand lottery is most unfavorable. In other words, when we compare $\Phi(\theta)$ and $\Phi(\theta')$ for any $\theta, \theta' \in [0,1]$ where $\theta < \theta'$, we never find a positive effect if $p_\theta = 0$. Our second proposition states that a more divergent demand lottery never negatively affects the decision to start R&D when the demand lottery belongs to the set of favorable demand lotteries. In other words, when we compare $\Phi(\theta)$ and $\Phi(\theta')$ for any $\theta, \theta' \in [0,1]$ where $\theta < \theta'$, we never find a negative effect if $p_\theta \in [\frac{1}{2}, 1]$. Remember that the same results are obtained by replacing p_θ and θ by p_λ and λ respectively. All proofs are relegated to Appendix A.

Proposition 1: *If $p_\theta = 0$, there does not exist a $\theta, \theta' \in [0,1]$, where $\theta < \theta'$, such that $\Phi(\theta) < 0$ and $\Phi(\theta') \geq 0$ for all $(c, I_0, I_1, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}^3_{++} \times [0,1]$.*

Proposition 2: *If $p_\theta \in [\frac{1}{2}, 1]$, there does not exist a $\theta, \theta' \in [0,1]$, where $\theta < \theta'$, such that $\Phi(\theta) \geq 0$ and $\Phi(\theta') < 0$ for all $(c, I_0, I_1, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}^3_{++} \times [0,1]$.*

Both Propositions 1&2 hold over the complete parameter space of $(c, I_0, I_1, \lambda, p_\lambda)$. The intuition behind Propositions 1&2 is straightforward. For a demand lottery that excludes the good state to happen, an increase in θ corresponds to a worsening of the bad state, which can never positively affect the decision to start. For demand lotteries where the good state is more likely to happen than the bad state, an increase in θ *a priori* increases the attractiveness of the R&D project and hence never affects the decision to start negatively.

However, it remains to show how more divergent demand lotteries affect the decision to start R&D when the demand lottery is unfavorable. From Proposition 1, the open question is from which value of p_θ on, it is possible to find a positive effect. Similarly, from Proposition 2, the question remains from which value of p_θ on, it is not possible to find a negative effect. In other words, we aim at extending Propositions 1&2 by respectively finding the minimal values $x \in (0, 1]$ and $y \in [0, \frac{1}{2}]$ such that the following results hold:

If $p_\theta \in [0, x)$, there does not exist a $\theta, \theta' \in [0, 1]$, where $\theta < \theta'$, such that $\Phi(\theta) < 0$ and $\Phi(\theta') \geq 0$.

If $p_\theta \in [y, 1]$, there does not exist a $\theta, \theta' \in [0, 1]$, where $\theta < \theta'$, such that $\Phi(\theta) \geq 0$ and $\Phi(\theta') < 0$.

Answering these questions give us the following additional insights. If x were between 0 and $\frac{1}{2}$, we would determine the subset of unfavorable demand lotteries (namely those demand lotteries with $p_\theta \in [0, x)$) for which a more divergent demand lottery never positively affects the decision to start R&D. If x were between $\frac{1}{2}$ and 1, we would conclude that for all unfavorable demand lotteries a more divergent demand lottery never positively affects the decision to start R&D and we would determine the subset of favorable demand lotteries (namely those demand lotteries with $p_\theta \in [\frac{1}{2}, x)$) for which a more divergent demand lottery always has no effect on the decision to start R&D. In determining y, we identify the subset of unfavorable demand lotteries (namely those demand lotteries with $p_\theta \in [y, \frac{1}{2}]$) for which a more divergent demand lottery never negatively affects the decision to start R&D. If x were smaller than y, we would identify the subset of unfavorable demand lotteries (namely those demand lotteries with $p_\theta \in [x, y)$) for which a more divergent demand lottery could have a positive or a negative effect on the decision to start.

The additional question becomes over which domains these extensions of Propositions 1&2 hold. Necessary conditions to obtain a positive (negative) effect are that, *ceteris paribus*, there exists a $\theta \in [0, 1]$ such that $\Phi(\theta) \geq (<)0$. Obviously, these necessary conditions cannot be fulfilled over the complete parameter space of $(c, I_0, I_1, \lambda, p_\lambda)$. The intuition is that if the total cost of undertaking the R&D project —which depends on $(I_0, I_1, \lambda, p_\lambda)$— exceeds by far (is much smaller than) the total gain of the R&D project —which depends on (c, θ, p_θ)—, then Φ will always be negative (positive).

In the following section we extend Propositions 1&2 by determining x and y under a restricted parameter space of $(c, I_0, I_1, \lambda, p_\lambda)$. In relating different demand lotteries to the decision to start the R&D project, we deliberately do not want to restrict the set of lotteries on the supply side. In other words, in determining x and y, we choose from the total set of supply lotteries (i) that particular lottery for which we obtain the smallest interval $p_\theta \in [0, x)$ of demand lotteries for which a more divergent demand lottery cannot positively affect the decision to start R&D and (ii) that particular lottery for which we obtain the smallest interval $p_\theta \in [y, 1]$ of demand lotteries for which a more divergent demand lottery cannot negatively affect the decision to start R&D. Larger intervals than $[0, x)$ and $[y, 1]$ would be obtained if one excluded these particular supply lotteries from the total set. All results also hold for any strictly positive value of c. When c equals zero, the incumbent never starts the R&D project. Summing up, we only restrict the parameter space of (I_0, I_1) and all results hold over the complete parameter space of (c, λ, p_λ).

A completely similar exercise is performed to relate changes in λ and values of p_λ to changes in Φ under the complete parameter space of (c, θ, p_θ). More specifically, we aim at finding respectively the minimal values $v \in (0, 1]$ and $w \in [0, \frac{1}{2}]$ such that the following results hold:

If $p_\lambda \in [0, v)$, there does not exist a $\lambda, \lambda' \in [0, 1]$, where $\lambda < \lambda'$, such that $\Phi(\lambda) < 0$ and $\Phi(\lambda') \geq 0$ for all $(c, \theta, p_\theta) \in [0, 1]^3$.

If $p_\lambda \in [w, 1]$, there does not exist a $\lambda, \lambda' \in [0, 1]$, where $\lambda < \lambda'$, such that $\Phi(\lambda) \geq 0$ and $\Phi(\lambda') < 0$ for all $(c, \theta, p_\theta) \in [0, 1]^3$.

3.3 Extending Propositions 1 & 2

Benchmark case

In the benchmark case, we impose two assumptions on the model, relating (in the absence of technical uncertainty) the cost of starting R&D to the cost of continuing R&D and the total cost of the R&D project to the profit gain. The impact of each of these two assumptions is discussed in the sensitivity analysis. We assume that (i) the two cost components of R&D would be the same in the two periods when $\lambda = 0$ and (ii) the total cost of R&D would equal the profit gain of R&D when $\lambda = 0$.

Assumption 1: $I_0 = I_1 = I$.
Assumption 2: $I_0 + I_1 = \Delta\pi$.

Under Assumptions 1-2, we obtain Propositions 3a&3b for the minimal values x, v and Proposition 4 for the minimal values y, w respectively; all proofs are relegated to Appendix A:

Proposition 3a: *Under Assumptions 1-2, if* $p_\theta \in [0, \frac{1}{4})$*, there does not exist a* $\theta, \theta' \in [0, 1]$*, where* $\theta < \theta'$*, such that* $\Phi(\theta) < 0$ *and* $\Phi(\theta') \geq 0$ *for all* $(c, \lambda, p_\lambda) \in [0, 1] \times \mathbb{R}_{++} \times [0, 1]$.

Proposition 3b: *Under Assumptions 1-2, if* $p_\lambda \in [0, 0.28)$*, there does not exist a* $\lambda, \lambda' \in [0, 1]$*, where* $\lambda < \lambda'$*, such that* $\Phi(\lambda) < 0$ *and* $\Phi(\lambda') \geq 0$ *for all* $(c, \theta, p_\theta) \in [0, 1]^3$.

Proposition 4: *Under Assumptions 1-2, Proposition 2 is not extended: both y and w equal $\frac{1}{2}$ for all $(c, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}_{++} \times [0,1]$ and for all $(c, \theta, p_\theta) \in [0,1]^3$ respectively.*

From Proposition 3a it follows that for the subset of unfavorable demand lotteries with $p_\theta \in [0, \frac{1}{4})$, a more divergent demand lottery never positively affects the decision to start R&D. From the determination of y in Proposition 4 we learn that for all unfavorable demand lotteries, we can not exclude that a more divergent demand lottery negatively affects the decision to start R&D. From Proposition 3b it follows that for the subset of unfavorable supply lotteries with $p_\lambda \in [0, 0.28)$, a more divergent supply lottery never positively affects the decision to start R&D. From the determination of w in Proposition 4 we learn that for all unfavorable supply lotteries, we can not exclude that a more divergent supply lottery negatively affects the decision to start R&D.

For the subset of unfavorable demand lotteries for which a more divergent demand lottery can positively or negatively affect the decision to start R&D (namely those demand lotteries with $p_\theta \in [\frac{1}{4}, \frac{1}{2})$), we obtain an important additional insight. There is a trade-off between the unfavorability and the degree of divergence of the demand lottery. The more unfavorable the demand lottery becomes, the more divergent the demand lottery has to become in order to positively affect the decision to start R&D. For example, setting λ equal to $\lambda_{\max}$ and p_λ to 1 (cfr. the proof of Proposition 3a), we derive that for $p_\theta \in [\frac{1}{4}, \frac{1}{2})$, a positive effect on the decision to start R&D is found when we move from a demand lottery with $\theta < \frac{1}{2p_\theta} - 1$ to a demand lottery with $\theta \geq \frac{1}{2p_\theta} - 1$. Hence, the more p_θ goes to $\frac{1}{4}$, the more θ has to go to 1, in order to positively affect the decision to start R&D. A similar result can be derived for the subset of unfavorable supply lotteries for which a more divergent supply lottery can positively or negatively affect the decision to start R&D (namely those supply lotteries with $p_\lambda \in [0.28, \frac{1}{2})$).

Sensitivity analysis

In the sensitivity analysis, we investigate the impact of Assumptions 1-2 alternately; all proofs are relegated to Appendix A.

Relaxing Assumption 1

We relax Assumption 1, setting $I_1 = aI_0$, where $a \in \mathbb{R}_{++}$. Hence, we assume that, in the absence of technical uncertainty ($\lambda = 0$) the cost of continuing the R&D project in the second period equals a times the cost of starting the R&D project in the first period. Remember that in our benchmark case above, $a = 1$. We obtain the following result. The higher (lower) the cost of continuing R&D compared to the cost of starting R&D, the smaller (larger) the subset of unfavorable demand (supply) lotteries for which a more divergent demand (supply) lottery never positively affects the decision to start R&D. In other words, relaxing Assumption 1 alters the minimal values x and v. Hence, Propositions 3a and 3b are generalized in the following way:

Sensitivity result 1a: *Under Assumption 2, if* $p_\theta \in [0, \frac{1}{2(1+a)})$*, there does not exist a* $\theta, \theta' \in [0,1]$*, where* $\theta < \theta'$*, such that* $\Phi(\theta) < 0$ *and* $\Phi(\theta') \geq 0$ *for all* $a \in \mathbb{R}_{++}$ *and for all* $(c, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}_{++} \times [0,1]$.

Sensitivity result 1b: *Under Assumption 2, if* $p_\lambda \in [0, \min\{\frac{1}{1+a}, \frac{-3+\sqrt{9+8a}}{4a}\})$*, there does not exist a* $\lambda, \lambda' \in [0,1]$*, where* $\lambda < \lambda'$*, such that* $\Phi(\lambda) < 0$ *and* $\Phi(\lambda') \geq 0$ *for all* $a \in \mathbb{R}_{++}$ *and for all* $(c, \theta, p_\theta) \in [0,1]^3$.

In Figure 2, we depict the minimal values $x = \frac{1}{2(1+a)}$ (left panel) and $v = \min\{\frac{1}{1+a}, \frac{-3+\sqrt{9+8a}}{4a}\}$ (right panel).

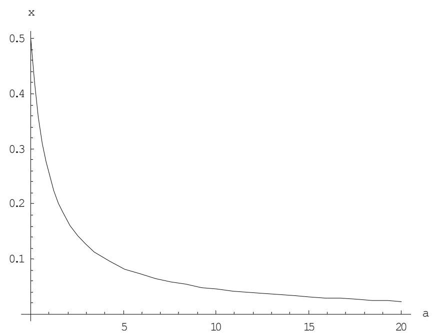

Figure 2 **Relaxing Assumption 1.** Depicted are the minimal values x (left panel) and v (right panel) as a function of a.

Compared to the benchmark case, lower (higher) values of x and v are simulated when the cost of starting R&D is relatively lower (higher) than the cost of continuing R&D. This is because I_1 never has a higher negative weight than I_0 in every element of Φ (cfr. section 2.3). Intuitively, the incumbent's decision to start R&D is more influenced by the cost of starting R&D than by the cost of continuing R&D. Hence, a higher cost of starting R&D necessitates a more favorable demand/supply lottery in order to effectively start R&D.

We also provide an intuition why if a 'goes to' ($\rightarrow$) zero (remember that $a \in \mathbb{R}_{++}$), $x \rightarrow \frac{1}{2}$ and $v \rightarrow \frac{1}{3}$. When $a \rightarrow 0$, $I_1 \rightarrow 0$, $\lambda_{\max} \rightarrow 0$ and there is no technical uncertainty. By Assumption 2, $I_0 \rightarrow \Delta\pi$. Then, for any unfavorable demand lottery $p_\theta \in [0, \frac{1}{2})$ and for any $\theta \in (0,1]$, the total gain of the R&D project is lower than I_0. The R&D project is never started and hence $x \rightarrow \frac{1}{2}$. Now suppose an infinitely small increase of $\lambda_{\max}$ from zero to ε, an infinitely small positive number. This increase will change Φ negligibly. From the proof in Appendix A, we derive that when $a \rightarrow 0$, $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ for $\lambda = \lambda_{\max}$ and $\theta \in [0,1]$ and that an increase in λ can have a positive effect on the decision to start if and only if $p_\theta \leq \frac{p_\lambda}{1-p_\lambda}$. From these observations, consider the most favorable, most divergent demand lottery, i.e. the demand lottery where $\theta = 1$ and $p_\theta = \frac{p_\lambda}{1-p_\lambda}$. For this demand lottery, $\Delta_{NPV}^{GG} = 2\Delta\pi$, $\Delta_{NPV}^{GB} \rightarrow 2\Delta\pi$ and

$\Delta_{NPV}^{BG} = 0$. The incumbent only starts R&D when $\Delta_{NPV}^{(1,1,1,0)}(\lambda_{\max}) \geq 0$, which, for $\theta = 1$ and $p_\theta = \frac{p_\lambda}{1-p_\lambda}$, implies that $p_\lambda \geq \frac{1}{3}$. Hence $v \to \frac{1}{3}$.

An additional result is that relaxing Assumption 1 leaves the minimal values y and w unchanged. In other words, Proposition 4 does not change:

Sensitivity result 2: *Under Assumption 2, Proposition 2 is not extended: both y and w equal $\frac{1}{2}$ for all $a \in \mathbb{R}_{++}$ and for all $(c, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}_{++} \times [0,1]$ and all $(c, \theta, p_\theta) \in [0,1]^3$ respectively.*

Sensitivity result 2 indicates for all unfavorable demand/supply lotteries, we cannot exclude that a more divergent demand/supply lottery negatively affects the decision to start R&D, whatever the relative importance of the two cost components I_0 and I_1. We provide an intuition for this result. Start from a situation where there is no demand uncertainty ($\theta = 0$) and no technical uncertainty ($\lambda = 0$). Since by Assumption 2, $I_0 + I_1 = \Delta\pi$, it is easy to see that in this situation $\Phi = 0$ and the incumbent starts the R&D project. Now suppose the demand lottery becomes more divergent ($\theta > 0$), while there is still no technical uncertainty. Then $\Phi < 0$ as long as $p_\theta < \frac{1}{2}$, i.e. as long as the demand lottery is unfavorable. Hence, $y = \frac{1}{2}$. Similarly, suppose the supply lottery becomes more divergent ($\lambda > 0$), while there is still no demand uncertainty. Then $\Phi < 0$ as long as $p_\lambda < \frac{1}{2}$, i.e. as long as the supply lottery is unfavorable. Hence, $w = \frac{1}{2}$.

Relaxing Assumption 2

We relax Assumption 2 by expressing the total cost of R&D as a proportion $b \in \mathbb{R}_{++}$ of the profit gain of R&D when $\lambda = 0$, i.e. $I_0 + I_1 = b\Delta\pi$. Remember that in our benchmark case above, $b = 1$. We obtain the following results. Relaxing Assumption 2 alters the minimal values x and v as well as the minimal value y. However, relaxing Assumption 2 leaves the minimal value w unchanged at $\frac{1}{2}$. Notably, x is found to be a linear function of b. Propositions 3a and 3b are generalized in the following way:

Sensitivity result 3a: *Under Assumption 1, if $p_\theta \in [0, \min\{\frac{b}{4}, 1\})$, there does not exist a $\theta, \theta' \in [0,1]$, where $\theta < \theta'$, such that $\Phi(\theta) < 0$ and $\Phi(\theta') \geq 0$ for all $b \in \mathbb{R}_{++}$ and for all $(c, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}_{++} \times [0,1]$.*

Sensitivity result 3b: *Under Assumption 1, if $p_\lambda \in [0, \min\{\max\{\frac{-4+b+\sqrt{9b^2-8b+16}}{4b}, \frac{b}{4}\}, 1\})$, there does not exist a $\lambda, \lambda' \in [0,1]$, where $\lambda < \lambda'$, such that $\Phi(\lambda) < 0$ and $\Phi(\lambda') \geq 0$ for all $b \in \mathbb{R}_{++}$ and for all $(c, \theta, p_\theta) \in [0,1]^3$.*

In Figure 3, we depict the minimal values $x = \min\{\frac{b}{4}, 1\}$ (left panel) and $v = \min\{\max\{\frac{-4+b+\sqrt{9b^2-8b+16}}{4b}, \frac{b}{4}\}, 1\}$ (right panel).

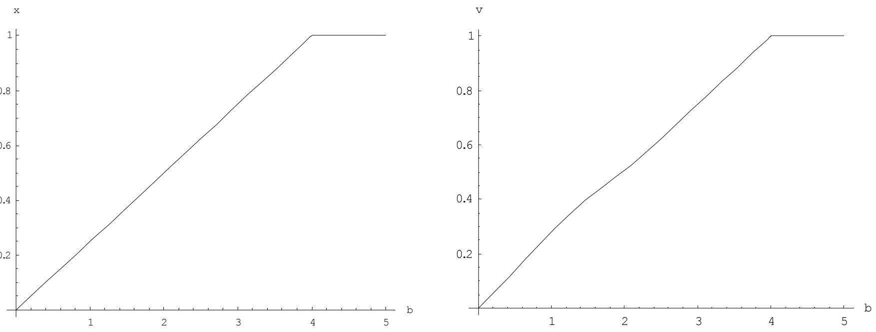

Figure 3 **Relaxing Assumption 2.** Depicted are the minimal values x (left panel) and v (right panel) as a function of b.

The intuition behind Sensitivity results 3a&3b is straightforward. The lower the profit gain of the R&D project compared to the total cost, the more favorable the demand/supply lottery has to become in order to start R&D. Note that if the incumbent knows that both the demand lottery and the supply lottery are most favorable and most divergent, the R&D project will be started as long as the cost of starting R&D does not more than four times exceed the profit gain of the R&D project.

Propositions 4 is generalized as follows:

Sensitivity result 4: *Under Assumption 1, if* $p_\theta \in [\min\{\frac{b}{4(1-b)}, \frac{1}{2}\}, 1]$, *there does not exist a* $\theta, \theta' \in [0,1]$, *where* $\theta < \theta'$, *such that* $\Phi(\theta) \geq 0$ *and* $\Phi(\theta') < 0$ *for all* $b \in \mathbb{R}_{++}$ *and for all* $(c, \lambda, p_\lambda) \in [0,1] \times \mathbb{R}_{++} \times [0,1]$. *Under Assumption 1, Proposition 2 is not extended for w: w equals* $\frac{1}{2}$ *for all* $b \in \mathbb{R}_{++}$ *and for all* $(c, \theta, p_\theta) \in [0,1]^3$.

In Figure 4, we depict the minimal value $y = \min\{\frac{b}{4(1-b)}, 1\}$.

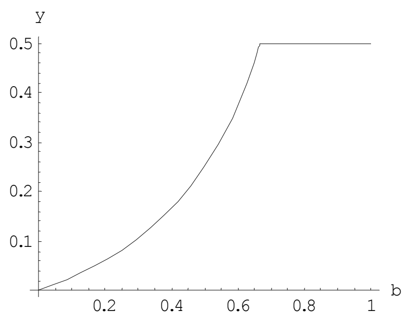

Figure 4 **Relaxing Assumption 2.** Depicted is the minimal value y as a function of b.

We provide an intuition why if $b \to 0$, $y \to 0$ but $w = \frac{1}{2}$. When $b \to 0$, $I_0 + I_1 \to 0$, $I_0 \to 0$, $I_1 \to 0$, $\lambda_{\max} \to 0$ and there is no technical uncertainty.

Then $\Phi \geq 0$ for $p_\theta \geq 0$ and $\theta \in [0,1]$, i.e. the R&D project is always started. Hence, $y \rightarrow 0$. Now consider again an infinitely small increase of $\lambda_{\max}$ from zero to ε. Suppose the demand lottery is most unfavorable and most divergent ($p_\theta = 0$ and $\theta = 1$). Then, when $\lambda_{\max} = 0$, $\Phi = 0$ and the R&D project is started. However, when $\lambda_{\max} = \varepsilon$, $\Phi < 0$ as long as $p_\lambda < \frac{1}{2}$, i.e. as long as the supply lottery is unfavorable. Hence, $w = \frac{1}{2}$.

4 An empirical analysis of the optimal decision to undertake R&D under uncertainty

4.1 Data

In the previous section, we analyze under which circumstances a more divergent demand lottery (increase in θ) or a more divergent supply lottery (increase in λ) positively or negatively affects the incumbent's decision to undertake R&D. To test the derived propositions, we use data from the 2005 official innovation survey in the German manufacturing and services industries which constitute the German part of the European-wide harmonized fourth Community Innovation Surveys (CIS 4).[1] The CIS data provide rich information on firms' innovation behavior. The target population consists of all legally independent firms with at least 5 employees and their headquarters located in Germany.[2] The survey is drawn as a stratified random sample and is representative of the corresponding target population. The stratification criteria are firm size (8 size classes according to the number of employees), industry (22 two-digit industries according to the NACE Rev.1 classification system) and region (East and West Germany). The survey is performed by mail and in 2005 data on 4776 firms were collected (*total sample*), corresponding to a response rate of about 20%.[3] In order to control for a response bias in the net sample, a non-response analysis was carried out collecting data on 4000 additional firms. A comparison shows that the innovation behavior of respondents and non-respondents does not differ significantly. The share of innovators is 63.9% in the former group and 62.2% in the latter group.[4]

For estimation purposes we exclude firms with incomplete data for any of the relevant variables (which are discussed in section 4.2), ending up with a sample of 3681 firms. As illustrated in Table B.1 in Appendix B, our sample (*full sample*) reflects total-sample distributional characteristics very well and does not give

[1]The innovation surveys are conducted by the Centre for European Economic Research (ZEW), Fraunhofer Institute for Systems and Innovation Research (ISI) and infas Institute for Applied Social Sciences on behalf of the German Federal Ministry of Education and Research (BMBF). A detailed description of the data is given in Peters (2008).

[2]A firm is defined as the smallest combination of legal units operating as an organizational unit producing goods or services.

[3]This rather low response rate is not unusual for surveys in Germany and is due to the fact that participation is voluntary.

[4]The p-value of the Fisher-test on equal shares in both groups amounts to 0.108.

any obvious cause for selectivity concerns. About 53.8% of the observed firms are in manufacturing.

4.2 Econometric model and testable hypotheses

Econometric model

In our theoretical model, the incumbent has to decide whether to undertake a R&D project which aims at obtaining the same superior production technology as the potential entrant.[5] The optimal decision to undertake R&D depends, *ceteris paribus*, on the degree of divergence of the demand and supply lotteries. Empirically, we operationalize this optimal decision as follows.

Let y_i^* denote firm i's maximal difference between the NPV of undertaking R&D and the NPV of not undertaking R&D, which cannot be observed. Exploiting the firm heterogeneity in our unique dataset, we assume that for firm i this difference depends on θ_i and λ_i, some other observable characteristics summarized in the row vector $\mathbf{x}_i$ and unobservable factors captured by ϵ_i:

$$y_i^* = \alpha\theta_i + \gamma\lambda_i + \mathbf{x}_i\boldsymbol{\beta} + \epsilon_i \tag{1}$$

In section 2.3, we derive that it is optimal for incumbent i to undertake R&D if and only if y_i^* is larger than or equal to zero:

$$y_i = \begin{cases} 1 & \text{if} \quad y_i^* \geq 0 \\ 0 & \text{if} \quad y_i^* < 0 \end{cases} \tag{2}$$

where y_i denotes the observed binary endogenous variable. We estimate equation (2) using the probit estimator.

Testable hypotheses

Table 1 gives the descriptive statistics of all variables used in the econometric analysis and Table B.2 in Appendix B provides detailed definitions of all variables. We proxy the observed binary endogenous variable (y_i) by three variables. The first proxy indicates whether the firm has performed R&D in the period 2002-2004 ($R\&D$). Table 1 shows that 48% of the firms in the full sample undertook R&D projects. However, over the same period, we observe θ_i and λ_i, our measures reflecting uncertainty on the demand and the supply side respectively. Due to the short time-span of our data, we cannot use lagged values as instruments for the uncertainty measures to encounter the possible endogeneity problem. Instead, we employ as an alternative proxy an *expected* decision, indicating whether the firm plans to introduce a new production technology in the next year 2005 ($PROCESS$). We find that 46% of the firms in the full sample planned to introduce a process innovation. Although our theoretical model is expressed in terms of cost-reducing process innovations, our analysis might also apply for studying R&D decisions with respect to product innovation. Imagine a potential entrant who is able to offer a new product of higher quality and

[5] In what follows, the notions *firm* and *incumbent* are used interchangeably.

assume that only the firm producing the product with the highest quality will stay in the market. Therefore, the third proxy denotes whether the firm plans to introduce a product innovation in the next year 2005 ($PRODUCT$). Around 55% of the firms in the full sample planned to introduce a product innovation.

<Insert Table 1 about here>

In our theoretical model, demand uncertainty stems from the two components in the lottery on the demand side: the degree of divergence (represented by θ) and the probability (p_θ) of facing a good demand state. The variable θ is measured by the average of the absolute percentage change in sales over the last two years 2002-2003 and 2003-2004 ($THETA$). Table 1 reveals that the absolute change in sales was on average about 14 % in the last two years. In our benchmark estimations, we assume that p_θ is the same for all firms. Our dataset enables us to relax this assumption later on.

Similarly, technical uncertainty is represented by the two components in the lottery on the supply side: the degree of divergence (parameterized by λ) and the probability (p_λ) of facing a good supply state. For the full sample, λ is proxied by two dummy variables ($LAMBDA1$ and $LAMBDA2$). The first equals 1 if an innovation project was extended due to the lack of technological information in the period 2002-2004, while the second equals 1 if the extension was due to high innovation costs in the period 2002-2004. The motivation for using this information is that an unexpected delay of an innovation project is presumably associated with *unexpected* higher costs. Around 5% and 19% of the firms were confronted with a severe extension of an innovation project due to technical or cost reasons, respectively. Alternatively, we use a third proxy for λ ($LAMBDA3$) which is defined as the absolute deviation between on the one hand the R&D expenditures for 2004 expected in 2003 and on the other hand the realized R&D expenditures in 2004. The virtue of this measure is that it most closely corresponds to the way we model λ in our theoretical analysis. The defect is that we can apply it only to a subset of enterprises since we have to use the prior wave of the innovation survey to construct this variable.[6] However, this subsample is representative for the full sample as can be inferred from Table B.1 in Appendix B. The average absolute deviation between expected and realized innovation expenditure comes to 2.4 mill. Euro. The deviation turns out to be highly skewed. We therefore use a logarithmic transformation of this variable in the econometric analysis. In all our estimations, we assume that p_λ is the same for all firms. Our dataset does not allow to relax this assumption.

The probabilities p_θ and p_λ are determined as follows. To calculate p_θ using the full sample, we derive that 56.9% of the firms experienced a positive growth in sales between 2002 and 2003 and 61.8% between 2003 and 2004. No information is available to calculate p_λ from the full sample. However, we are able to

[6] In Germany, the innovation surveys are conducted annually and they are designed as a panel (so called Mannheim Innovation Panel). Unfortunately, the overlap between the 2004 and 2005 survey only amounts to almost 40% due to a major refreshment and enlargement of the gross sample.

determine p_λ from the subsample. More specifically, we observe that for 59.4% of the firms, realized innovation expenditure in 2004 turns out to be lower than expected in 2003. Given the representativeness of the subsample, we assume that the calculated p_λ is also valid for the full sample.

Assuming that p_θ and p_λ are the same for all firms and given that p_θ and p_λ are calculated to be larger than $\frac{1}{2}$, we postulate from Proposition 2 the following hypotheses.

Hypothesis 1: The probability of undertaking R&D does not decrease with a more divergent demand lottery.

Hypothesis 2: The probability of undertaking R&D does not decrease with a more divergent supply lottery.

In our theoretical model, the incumbent is challenged by a potential competitor. Our data reveal that about 91% of the firms perceive a threat of its own market position due to the potential entry of new competitors. In the estimations, we therefore control for potential entry by including 3 dummy variables indicating whether the firm perceives a high, medium or low threat.

We also control for the following factors found to be important in the literature. Two main determinants explaining innovation activities go back to Schumpeter (1942), who states that large firms in concentrated markets have an advantage in innovation. Therefore, we include firm size ($SIZE$) and market structure ($NUMCOMP$). Firm size is measured by the logarithm of the number of employees in 2003 and we expect a positive relationship. Market structure is captured by 3 dummy variables indicating the number of competitors. Schumpeter stresses a negative relationship between competition and innovation. His argument is that *ex ante* product market power on the one hand increases monopoly rents from innovation and on the other hand reduces the uncertainty associated with excessive rivalry. Recently, Aghion et al. (2005) find evidence for an inverted U-relationship between competition and innovation. For low initial levels of competition an escape-competition effect dominates (i.e. competition increases the incremental profits from innovating, and, thereby, encourages innovation investments) whereas the Schumpeterian effect tends to dominate at higher levels of competition.

The incentive to engage in R&D may further depend on the type of competition ($COMP$). We include 5 dummy variables indicating whether firms primarily compete in prices, product quality, technological lead, product variety or product design.

The innovation literature stresses that certain firm characteristics —such as the degree of product diversification, the degree of internationalization, the availability of financial resources and technological capabilities— are likewise crucial for explaining innovation (see, e.g., the references cited in Peters, 2008). More diversified firms possess economies of scope in innovation. As they have more opportunities to exploit new knowledge and complementarities among their diversified activities, they tend to be more innovative. We measure product diversification by the share of turnover of the firm's most important product in 2004 ($DIVERS$). Therefore, we expect a negative coefficient since more diversified firms exhibit lower values for this proxy.

The more a firm is exposed to international competition, the more likely the firm engages in R&D activities. The degree to which a firm is exposed to international competition is captured by a dummy variable taking the value of 1 if the firm sells its products to international markets ($EXPORT$).

The availability of financial resources is proxied by an index of creditworthiness ($RATING$). A lower creditworthiness implies less available and more costly external funding to finance R&D projects. Since the index ranges from 1 (best rating) to 6 (worst rating), we expect a negative coefficient for this proxy.

Innovative capabilities are determined by the skills of employees. We take into account the share of employees with a university degree ($HIGHSKILLED$), a dummy variable being 1 if the firm has not invested in training its employees ($NOTRAIN$) and the amount of training expenditure per employee ($TRAINEXP$) if the firm has invested in training. Since information on training expenditure is missing for 11.3% of the firms, we do not drop these observations but rather set the expenditure to zero and include a dummy variable indicating the missing value status ($MVTRAIN$).

We also include variables reflecting whether the firm is located in East Germany ($EAST$) and whether the firm is part of an enterprise group ($GROUP$). *A priori*, the effect of these variables is unclear. Finally, industry dummies are included in all regressions.

4.3 Results

4.3.1 Firms facing equal lottery probabilities

Table 2 reports the marginal effects of the probit estimates for the full sample, assuming that all firms face the same probabilities in the demand and supply lotteries. For each of the three endogenous variables, the first column reports the results for a parsimonious specification —including only $SIZE$ and industry dummies in addition to demand uncertainty, technical uncertainty and entry threat— whereas the second column employs the full set of control variables described in the previous section.

<Insert Table 2 about here>

Hypothesis 1, postulating that the probability of undertaking R&D does not decrease with an increase in θ, is strongly confirmed for our two main endogenous variables (*R&D* and $PROCESS$). Focusing on our preferred specification (*R&D* (2)), our results indicate that a 10% increase in θ increases the likelihood of undertaking R&D by 1.3%. The positive effect of demand uncertainty is not significant for planned product innovations ($PRODUCT$) in the specification including all control variables.

Hypothesis 2, postulating that the probability of undertaking R&D does not decrease with an increase in λ, is strongly confirmed. This result is robust across the different endogenous variables and holds when additional control variables are incorporated. We estimate that a 10% increase in λ increases the likelihood of undertaking R&D by 1.5%.

Entry threat does not significantly influence the decision to undertake R&D. As $R\&D$ and $THREAT$ are measured over the same period, an endogeneity problem might arise as the decision to perform R&D could reduce the perceived entry threat. This explanation is supported by the fact that entry threat does significantly positively affect the decision to undertake process innovations in the next year. A similar result is, however, not found for planned product innovations.

Regarding the impact of the other control variables, most results are in line with expectations. Firm size exerts a significantly positive impact. Market structure has a non-linear effect on innovation. Firms in oligopoly markets have a higher likelihood of undertaking R&D or introducing new products compared to monopolists or firms with more competitors. Hence, our results support evidence in favor of the inverted U-relationship between competition and innovation as suggested by Aghion et al. (2005). Another striking and robust finding is that firms acting on markets where competition is mainly settled through prices are less likely to innovate. On the contrary, innovation activities are stimulated if competitive advantage can be achieved by technological leadership. Firms being exposed to international competition as well as more diversified firms have a higher likelihood of undertaking R&D and introducing new products. There is, however, no significant impact on process innovation. Finally, the results highlight the important role of innovative capabilities. Firms employing a higher share of high-skilled workers or firms investing in training are likely to be more innovative.

For the subsample, Table 3 presents in columns (2), (4) and (6) the estimates using our preferred measure for technical uncertainty ($LAMBDA3$). For reasons of comparison, columns (1), (3) and (5) show the subsample results employing $LAMBDA1$ and $LAMBDA2$. In general, the results are very similar to the full sample. Hypothesis 2 is also strongly confirmed using $LAMBDA3$. Since we measure this variable in logarithm, a value of 0.017 implies that an increase in the deviation of expected and actual R&D expenditure by 1 percent increases the propensity to undertake R&D by 1.7 percent.

<Insert Table 3 about here>

4.3.2 Firms facing different demand lottery probabilities

In this section we relax the assumption that the probability of facing a good demand state is the same for all firms. We approximate p_θ by looking at the firms' sales histories in the past two years. We define three groups of firms (see Table B.2 in Appendix B for exact definitions). Group 1 ($G1$) comprises all firms that experienced a decrease in sales in 2002-2003 as well as in 2003-2004. The idea is that these firms face an unfavorable demand lottery reflected by a low value of p_θ. These firms are much more likely to face a bad demand state than a good demand state. Group 2 ($G2$) consists of all firms that experienced one yearly decrease and one yearly increase in sales during the period 2002-2004. On the basis of this observation, we assume that these firms face equal probabilities of a good/bad demand state and therefore have a p_θ around $\frac{1}{2}$. All firms in group 3 ($G3$) experienced an increase in sales in 2002-2003 as well as in 2003-2004. The assumption is that these firms face a favorable demand lottery reflected by a high value of p_θ.

Assuming that firms in group 1 have a p_θ smaller than $\frac{1}{4}$, firms in group 2 have a p_θ around $\frac{1}{2}$ and firms in group 3 have a p_θ larger than $\frac{1}{2}$, we postulate from Proposition 3a and Proposition 2 respectively the following hypotheses.

Hypothesis 3: For firms in group 1, the probability of undertaking R&D does not increase with a more divergent demand lottery.

Hypothesis 4: For firms in group 2 and group 3, the probability of undertaking R&D does not decrease with a more divergent demand lottery.

Table 4 presents the results of distinguishing the effect of a more divergent demand lottery across groups of firms facing different demand lottery probabilities. Confirming hypothesis 3, we find that for firms in group 1 the effect of an increase in θ is significantly negative for $PROCESS$ and negative but not significant for *R&D* and $PRODUCT$ in the specifications including all control variables. Furthermore, the impact of $THETA$ is significantly different for firms in group 1 compared to firms in group 2 and group 3. Hypothesis 4 is strongly confirmed since the impact of $THETA$ is never significantly negative for firms in group 2 and group 3. Moreover, in five out of six specifications, the effect of a more divergent demand lottery is significantly positive for firms in group 3. Furthermore, the impact is significantly larger for firms in group 3 than for firms in group 2 when process innovations are considered.

<Insert Table 4 about here>

4.3.3 Robustness checks

In this section, we investigate (i) the robustness of the results in Tables 2 and 3 by presenting specifications that include demand uncertainty, technical uncertainty or entry threat separately and specifications that combine demand uncertainty or technical uncertainty with entry threat, (ii) the robustness of the results in Table 4 by making a distinction between manufacturing and services and (iii) the effect of a more divergent demand and supply lottery on innovation expenditures.

Tables B.3 and B.4 in Appendix B present the robustness of the results in Tables 2 and 3 respectively. This allows us to assess whether multicollinearity between our main independent variables affects our results in Tables 2 and 3. It might be, for instance, that the perception of entry threat is correlated with the firm's past demand development. However, the results do not support this view. The significance as well as the magnitude of the estimated marginal effects (not shown in the tables) are very robust in both the full sample and the subsample. A weak exception is the effect of low and high threat on process innovation in the full sample: the marginal effects are significantly positive at the 10% level in specifications 5 and 6.

Table 5 illustrates the effect of an increase in θ and λ across groups of firms facing a different p_θ in manufacturing on the one hand and services on the other hand. Regarding the impact of an increase in θ, the derived hypotheses 3 and 4, i.e. a more divergent demand lottery does not increase (decrease) the likelihood of undertaking R&D in $G1$ ($G2$ and $G3$), are confirmed in both samples. However, there are some notable differences between manufacturing and services. While the effect of an increase in θ is significantly negative for group 1 in manufacturing across the different endogenous variables, this effect looses significance for group 1 in services. On the contrary, the effect of an increase in θ is significantly positive for group 3 in services across the different endogenous variables whereas this effect looses significance for group 3 in manufacturing in 2 out of 3 specifications. Hypothesis 2, postulating that the probability of undertaking R&D does not decrease with an increase in λ, is confirmed in manufacturing as well as in services. The marginal effects are significantly positive for $LAMBDA1$ and $LAMBDA2$ in manufacturing. In services, only $LAMBDA2$ has a significantly positive impact. The latter is estimated to be higher than in manufacturing.

<Insert Table 5 about here>

Finally, Table 6 investigates the impact of a more divergent demand and supply lottery on the amount of resources committed to the planned innovation project. More specifically, we measure this variable as the logarithm of planned expenditures for innovation activities in 2005 ($INEXP$). In general, our tobit results strongly confirm our probit findings (as illustrated in Tables 4 and 5). The amount of money spent for innovation projects is negatively affected by an increase in θ for group 1, whereas the opposite effect holds for group 3. Concerning an increase in λ, especially $LAMBDA2$ is strongly significantly positive across both manufacturing and services.

<Insert Table 6 about here>

5 Conclusion

This article contributes to the theoretical as well as the empirical literature on R&D decisions under uncertainty.

From a theoretical point of view, we study R&D decisions in the presence of entry threat under two types of uncertainty, demand uncertainty and technical uncertainty. We model a R&D project as a two-stage game with an abandonment option. Two types of uncertainty influence the decision to start R&D. Demand uncertainty is modelled as a lottery between a proportional increase and decrease in demand. Technical uncertainty is modelled as a lottery between a decrease and increase in the cost to continue R&D. A potential entrant is endowed with a superior technology and threatens to drive the incumbent out of the market. The incumbent has a time lead over the entrant and can obtain the same technology by completing the R&D project before the entrant can enter the market. Strategic interaction takes the form of preemptive behavior, where the investment of the incumbent discourages the entrant to enter the market. We derive under which lottery probabilities more divergent demand and supply lotteries positively or negatively affect the decision to start R&D. We find that a more divergent demand (supply) lottery cannot negatively affect the probability of starting R&D when the lotteries are such that an increase in demand or a decrease in the cost are more likely to prevail than a decrease in demand or an increase in the cost respectively. Under mild assumptions, relating (in the absence of technical uncertainty) the cost of starting R&D to the cost of continuing R&D and the total cost of the R&D project to the profit gain, we find that the more unfavorable the lotteries become, i.e. the more likely it is that the firm experiences a decrease in demand or an increase in the cost, the more divergent the demand (supply) lottery must be in order to positively affect the probability of starting R&D.

From an empirical point of view, we test the hypotheses derived from the theoretical model using data from the fourth Community Innovation Survey (CIS) data in Germany. The uniqueness of our data lies in the availability of proxies for demand and technical uncertainty as well as perceived entry threat for about 4000 firms to explain actual and planned R&D investments. Our main results, strongly confirming our model predictions, are that for firms facing lotteries where the good state is more likely to prevail (i) a 10% increase in the degree of divergence of the demand lottery increases the likelihood of undertaking R&D by 1.3% and (ii) a 10% increase in the degree of divergence of the supply lottery increases the likelihood of undertaking R&D by 1.5%. For firms facing a demand lottery where the bad state is more likely to prevail, a more divergent demand lottery decreases the probability of undertaking R&D significantly.

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Table 1
Descriptive Statistics - Full Sample

Variable	Unit	Mean	SD	Median	Skewness	Min	Max
Dependent variables							
R&D	[0/1]	0.484	0.500	0	–	0	1
PROCESS	[0/1]	0.460	0.498	0	–	0	1
PRODUCT	[0/1]	0.546	0.498	1	–	0	1
Independent variables							
Demand uncertainty							
THETA	%	0.141	0.152	0.094	2.777	0	1.242
G1	[0/1]	0.199	–	–	–	–	–
G2	[0/1]	0.412	–	–	–	–	–
G3	[0/1]	0.389	–	–	–	–	–
Technical uncertainty							
LAMBDA1	[0/1]	0.051	0.219	0	–	0	1
LAMBDA2	[0/1]	0.193	0.395	0	–	0	1
LAMBDA3	Mill. Euro	2.425	12.769	0.098	10.101	0	207.318
Additional control variables							
THREAT: no	[0/1]	0.093	0.291	0	–	0	1
THREAT: low	[0/1]	0.447	0.497	0	–	0	1
THREAT: medium	[0/1]	0.310	0.463	0	–	0	1
THREAT: high	[0/1]	0.150	0.357	0	–	0	1
SIZE	# Empl.	587.179	5495.988	45	27.059	1	232700
NUMCOMP: 0	[0/1]	0.021	0.142	0	–	0	1
NUMCOMP: 1-5	[0/1]	0.578	0.494	1	–	0	1
NUMCOMP: 6-15	[0/1]	0.211	0.408	0	–	0	1
NUMCOMP: >15	[0/1]	0.191	0.393	0	–	0	1
COMP: PRICE	[0/1]	0.527	0.499	1	–	0	1
COMP: QUAL	[0/1]	0.418	0.493	0	–	0	1
COMP: LEAD	[0/1]	0.110	0.313	0	–	0	1
COMP: VARIETY	[0/1]	0.052	0.222	0	–	0	1
COMP: DESIGN	[0/1]	0.033	0.180	0	–	0	1
DIVERS	[0-100]	71.312	23.427	75	-0.502	0.5	100
EXPORT	[0/1]	0.527	0.499	1	–	0	1
RATING	[1-6]	2.15	0.817	2.19	0.571	1	6
HIGHSKILLED	[0-100]	20.314	24.013	10	1.633	0	100
TRAINEXP	Mill. Euro	0.001	0.001	0	7.852	0	0.025
NOTRAIN	[0/1]	0.120	0.325	0	–	0	1
MVTRAIN	[0/1]	0.113	0.317	0	–	0	1
EAST	[0/1]	0.322	0.467	0	–	0	1
GROUP	[0/1]	0.579	0.494	1	–	0	1

Values for $LAMBDA3$, $SIZE$ and $TRAINEXP$ are not log-transformed. For estimation purposes, however, a log-transformation of these variables is used to take into account the skewness of the distribution.

Table 2
Effect of demand and technical uncertainty on innovation - Full Sample

Dep. variables	**R&D**		**PROCESS**		**PRODUCT**	
	(1)	(2)	(3)	(4)	(5)	(6)
Demand uncertainty						
THETA	0.229^{***}	0.134^{***}	0.123^{**}	0.108^{**}	0.120^{**}	0.057
	(0.047)	(0.046)	(0.055)	(0.055)	(0.052)	(0.051)
Technical uncertainty						
LAMBDA1	0.155^{***}	0.148^{***}	0.119^{***}	0.103^{**}	0.104^{***}	0.083^{**}
	(0.042)	(0.039)	(0.040)	(0.039)	(0.039)	(0.038)
LAMBDA2	0.255^{***}	0.203^{***}	0.193^{***}	0.171^{***}	0.241^{***}	0.200^{***}
	(0.020)	(0.021)	(0.023)	(0.023)	(0.021)	(0.021)
Additional control variables						
THREAT: low	0.009	0.006	0.044	0.038	0.010	-0.001
	(0.025)	(0.024)	(0.030)	(0.030)	(0.027)	(0.027)
THREAT: medium	-0.021	-0.006	0.059^{*}	0.063^{**}	-0.009	-0.001
	(0.026)	(0.025)	(0.031)	(0.031)	(0.029)	(0.028)
THREAT: high	-0.022	0.029	0.014	0.040	-0.037	0.003
	(0.029)	(0.028)	(0.035)	(0.035)	(0.032)	(0.031)
SIZE	0.061^{***}	0.051^{***}	0.069^{***}	0.061^{***}	0.048^{***}	0.035^{***}
	(0.004)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)
NUMCOMP: 0	–	0.017	–	-0.018	–	-0.070
		(0.049)		(0.063)		(0.059)
NUMCOMP: 1-5	–	0.039^{**}	–	-0.009	–	0.054^{***}
		(0.018)		(0.022)		(0.020)
NUMCOMP: 6-15	–	-0.009	–	-0.030	–	0.031
		(0.021)		(0.025)		(0.023)
COMP: PRICE	–	-0.055^{***}	–	-0.066^{***}	–	-0.041^{**}
		(0.016)		(0.019)		(0.017)
COMP: QUAL	–	-0.007	–	-0.005	–	0.011
		(0.015)		(0.018)		(0.017)
COMP: LEAD	–	0.134^{***}	–	0.000	–	0.112^{***}
		(0.023)		(0.027)		(0.026)
COMP: VARIETY	–	-0.041	–	-0.002	–	0.066^{**}
		(0.031)		(0.039)		(0.033)
COMP: DESIGN	–	-0.001	–	0.037	–	-0.024
		(0.037)		(0.045)		(0.039)
DIVERS	–	-0.115^{***}	–	-0.056	–	-0.149^{***}
		(0.029)		(0.035)		(0.033)
EXPORT	–	0.141^{***}	–	0.017	–	0.127^{***}
		(0.017)		(0.020)		(0.019)
RATING	–	0.003	–	-0.011	–	-0.001
		(0.009)		(0.011)		(0.009)
HIGHSKILLED	–	0.003^{***}	–	0.001	–	0.002^{***}
		(0.000)		(0.000)		(0.000)
TRAINEXP	–	0.049^{***}	–	0.043^{***}	–	0.039^{***}
		(0.007)		(0.008)		(0.007)
NOTRAIN	–	-0.439^{***}	–	-0.417^{***}	–	-0.423^{***}
		(0.031)		(0.034)		(0.044)
MVTRAIN	–	-0.342^{***}	–	-0.346^{***}	–	-0.271^{***}
		(0.031)		(0.037)		(0.051)
EAST	–	0.039^{***}	–	0.016	–	0.007
		(0.015)		(0.018)		(0.017)
GROUP	–	0.011	–	0.030^{*}	–	0.003
		(0.014)		(0.017)		(0.016)
$LogL$	-1944.3	-1765.4	-2062.0	-2007.1	-1877.8	-1748.7
R^2_{MF}	0.237	0.308	0.098	0.122	0.177	0.234
R^2_{MZ}	0.429	0.525	0.200	0.244	0.341	0.424
Count R^2	0.732	0.766	0.653	0.669	0.701	0.728
LM_{het} (p-value)	0.019	0.001	0.653	0.588	0.349	0.073
LM_{norm} (p-value)	0.421	0.864	0.329	0.563	0.187	0.579
# Obs.	3681	3681	3314	3314	3314	3314

Average marginal effects of the probit estimations are reported. Robust standard errors in parentheses.
***Significant at 1%; **Significant at 5%; *Significant at 10%. Industry dummies are included but not reported.
$LogL$: log likelihood value of the model with regressors. R^2_{MF} (likelihood ratio index): McFadden (1974) Pseudo R^2, comparing the likelihood of an intercept-only model to the likelihood of the model with regressors. R^2_{MZ}: McKelvey and Zavoina (1976) R^2, measuring the proportion of variance of the latent variable accounted for by the model. Count R^2: proportion of accurate predictions. LM_{het}: Davidson and MacKinnon (1984) test statistic for heteroskedasticity. LM_{norm}: Shapiro and Wilk (1965) test statistic for normality.

Table 3
Effect of demand and technical uncertainty on innovation - Subsample

Dep. variables	R&D		PROCESS		PRODUCT	
	(1)	(2)	(3)	(4)	(5)	(6)
Demand uncertainty						
THETA	0.275***	0.224**	0.216*	0.154	0.016	-0.039
	(0.107)	(0.101)	(0.129)	(0.122)	(0.109)	(0.103)
Technical uncertainty						
LAMBDA1	0.112*	–	0.126*	–	0.064	–
	(0.063)		(0.070)		(0.061)	
LAMBDA2	0.168***	–	0.138***	–	0.123***	–
	(0.035)		(0.043)		(0.037)	
LAMBDA3	–	0.017***	–	0.023***	–	0.018***
		(0.002)		(0.003)		(0.002)
Additional control variables						
THREAT: low	-0.025	0.001	0.001	0.018	0.024	0.039
	(0.039)	(0.039)	(0.054)	(0.054)	(0.043)	(0.041)
THREAT: medium	-0.016	-0.000	0.014	0.031	0.064	0.074*
	(0.041)	(0.040)	(0.056)	(0.055)	(0.043)	(0.040)
THREAT: high	-0.026	0.004	-0.067	-0.042	0.013	0.036
	(0.046)	(0.045)	(0.064)	(0.065)	(0.050)	(0.048)
SIZE	0.058***	0.042***	0.059***	0.037***	0.042***	0.025***
	(0.009)	(0.009)	(0.011)	(0.011)	(0.010)	(0.009)
NUMCOMP: 0	-0.079	-0.037	0.086	0.118	-0.065	-0.040
	(0.082)	(0.073)	(0.126)	(0.110)	(0.107)	(0.090)
NUMCOMP: 1-5	0.079***	0.052*	-0.020	-0.040	0.108***	0.085***
	(0.030)	(0.030)	(0.042)	(0.040)	(0.034)	(0.032)
NUMCOMP: 6-15	0.026	0.024	0.013	0.009	0.019	0.014
	(0.035)	(0.034)	(0.048)	(0.047)	(0.038)	(0.037)
COMP: PRICE	-0.100***	-0.089***	-0.089**	-0.079**	-0.049	-0.041
	(0.027)	(0.026)	(0.035)	(0.034)	(0.030)	(0.029)
COMP: QUAL	-0.048**	-0.061***	-0.012	-0.024	-0.007	-0.017
	(0.024)	(0.023)	(0.034)	(0.033)	(0.028)	(0.026)
COMP: LEAD	0.130***	0.096***	-0.001	-0.026	0.188***	0.150***
	(0.034)	(0.033)	(0.048)	(0.045)	(0.041)	(0.040)
COMP: VARIETY	-0.014	-0.052	-0.009	-0.061	-0.036	-0.066
	(0.051)	(0.046)	(0.076)	(0.074)	(0.059)	(0.054)
COMP: DESIGN	-0.063	-0.097*	0.128	0.092	-0.027	-0.062
	(0.061)	(0.055)	(0.092)	(0.085)	(0.076)	(0.068)
DIVERS	-0.050	-0.051	-0.022	-0.022	-0.136**	-0.140***
	(0.049)	(0.046)	(0.066)	(0.065)	(0.054)	(0.051)
EXPORT	0.162***	0.162***	-0.019	-0.031	0.164***	0.147***
	(0.032)	(0.032)	(0.041)	(0.040)	(0.037)	(0.035)
RATING	-0.002	-0.001	-0.015	-0.016	0.012	0.013
	(0.015)	(0.015)	(0.019)	(0.018)	(0.015)	(0.015)
HIGHSKILLED	0.002***	0.002***	0.001	0.001	0.002***	0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
TRAINEXP	0.068***	0.049***	0.049***	0.029*	0.041***	0.020*
	(0.012)	(0.012)	(0.015)	(0.015)	(0.012)	(0.012)
NOTRAIN	-0.546***	-0.476***	-0.421***	-0.302***	-0.448***	-0.259**
	(0.039)	(0.067)	(0.063)	(0.104)	(0.087)	(0.117)
MVTRAIN	-0.461***	-0.388***	-0.417***	-0.339***	-0.295***	-0.119
	(0.041))	(0.070)	(0.052)	(0.085)	(0.098)	(0.114)
EAST	0.010	0.002	-0.009	-0.027	0.026	0.014
	(0.025)	(0.024)	(0.036)	(0.034)	(0.028)	(0.028)
GROUP	0.016	-0.001	0.070**	0.056*	0.021	0.007
	(0.025)	(0.023)	(0.034)	(0.033)	(0.029)	(0.027)
$LogL$	-399.5	-376.8	-555.7	-531.9	-415.7	-389.0
R^2_{MF}	0.425	0.458	0.134	0.172	0.333	0.375
R^2_{MZ}	0.660	0.685	0.264	0.326	0.555	0.591
Count R^2	0.811	0.825	0.674	0.695	0.780	0.805
LM_{het} (p-value)	0.732	0.472	0.913	0.929	0.989	0.958
LM_{norm} (p-value)	0.240	0.679	0.265	0.417	0.008	0.018
# Obs.	1009	1009	929	929	929	929

Average marginal effects of the probit estimations are reported. Robust standard errors in parentheses. ***Significant at 1%; **Significant at 5%; *Significant at 10%. Industry dummies are included but not reported. For notes on goodness-of-fit and specification tests: see Table 2.

Table 4
Effect of demand uncertainty on innovation across groups of firms facing a different p_θ - Full Sample

Dep. variables	R&D		PROCESS		PRODUCT	
	(1)	(2)	(3)	(4)	(5)	(6)
Demand uncertainty						
THETA*G1	-0.188	-0.140	-0.395***	-0.308**	-0.290**	-0.214
	(0.122)	(0.118)	(0.142)	(0.143)	(0.131)	(0.131)
THETA*G2	0.208***	0.144**	0.010	0.021	0.084	0.050
	(0.061)	(0.059)	(0.072)	(0.072)	(0.065)	(0.064)
THETA*G3	0.290***	0.153***	0.280***	0.233***	0.201***	0.096
	(0.060)	(0.058)	(0.069)	(0.068)	(0.064)	(0.062)
$\alpha_{\theta*G1} >= \alpha_{\theta*G2}$ (p-value)	0.001	0.008	0.002	0.011	0.002	0.022
$\alpha_{\theta*G1} >= \alpha_{\theta*G3}$ (p-value)	0.000	0.007	0.000	0.000	0.000	0.010
$\alpha_{\theta*G2} >= \alpha_{\theta*G3}$ (p-value)	0.127	0.445	0.001	0.005	0.065	0.271
Technical uncertainty						
LAMBDA1	0.156***	0.148***	0.116***	0.102***	0.103***	0.082**
	(0.042)	(0.039)	(0.039)	(0.039)	(0.039)	(0.038)
LAMBDA2	0.252***	0.202***	0.190***	0.170***	0.238***	0.199***
	(0.020)	(0.021)	(0.022)	(0.023)	(0.021)	(0.021)
Entry threat						
THREAT: low	0.008	0.005	0.043	0.038	0.009	-0.001
	(0.025)	(0.024)	(0.030)	(0.030)	(0.027)	(0.026)
THREAT: medium	-0.023	-0.007	0.055*	0.060*	-0.011	-0.002
	(0.026)	(0.025)	(0.031)	(0.031)	(0.028)	(0.028)
THREAT: high	-0.018	0.032	0.019	0.044	-0.032	0.007
	(0.029)	(0.028)	(0.035)	(0.035)	(0.032)	(0.031)
$LogL$	-1935.7	-1761.7	-2048.1	-1998.3	-1870.2	-1745.6
R^2_{MF}	0.241	0.309	0.104	0.126	0.181	0.235
R^2_{MZ}	0.432	0.527	0.210	0.250	0.345	0.426
Count R^2	0.735	0.770	0.654	0.672	0.702	0.731
LM_{het} (p-value)	0.005	0.000	0.614	0.718	0.252	0.051
LM_{norm} (p-value)	0.220	0.783	0.236	0.952	0.241	0.480
# Obs.	3681	3681	3314	3314	3314	3314

Average marginal effects of the probit estimations are reported. Robust standard errors in parentheses. ***Significant at 1%; **Significant at 5%; *Significant at 10%. In columns (1), (3) and (5) $SIZE$ and industry dummies are included as control variables but not reported. In columns (2), (4) and (6) the full set of control variables including industry dummies is used but not reported (see Table 2). For notes on goodness-of-fit and specification tests: see Table 2.

Table 5
Effect of demand uncertainty on innovation across groups of firms facing a different p_θ - Manufacturing and Services

Sample	**Manufacturing**			**Services**		
Dep. variables	**R&D**	**PROCESS**	**PRODUCT**	**R&D**	**PROCESS**	**PRODUCT**
	(1)	(2)	(3)	(4)	(5)	(6)
Demand uncertainty						
THETA*G1	-0.387**	-0.668***	-0.326*	-0.008	-0.100	-0.168
	(0.178)	(0.231)	(0.174)	(0.149)	(0.176)	(0.182)
THETA*G2	0.041	-0.031	0.058	0.228***	0.062	0.058
	(0.083)	(0.107)	(0.091)	(0.081)	(0.099)	(0.094)
THETA*G3	0.021	0.273**	0.014	0.228***	0.188**	0.164**
	(0.085)	(0.112)	(0.096)	(0.076)	(0.086)	(0.079)
$\alpha_{\theta*G1} >= \alpha_{\theta*G2}$ (p-value)	0.008	0.003	0.015	0.059	0.181	0.108
$\alpha_{\theta*G1} >= \alpha_{\theta*G3}$ (p-value)	0.011	0.000	0.030	0.062	0.055	0.035
$\alpha_{\theta*G2} >= \alpha_{\theta*G3}$ (p-value)	0.579	0.007	0.655	0.499	0.130	0.156
Technical uncertainty						
LAMBDA1	0.176***	0.101**	0.109**	0.106	0.105	0.022
	(0.048)	(0.046)	(0.047)	(0.066)	(0.072)	(0.064)
LAMBDA2	0.182***	0.140***	0.170***	0.213***	0.222***	0.234***
	(0.024)	(0.028)	(0.025)	(0.036)	(0.038)	(0.038)
Entry threat						
THREAT: low	0.034	0.059	-0.018	-0.019	0.018	0.015
	(0.031)	(0.041)	(0.034)	(0.035)	(0.043)	(0.041)
THREAT: medium	0.017	0.082*	-0.019	-0.029	0.037	0.011
	(0.032)	(0.043)	(0.036)	(0.037)	(0.045)	(0.043)
THREAT: high	0.050	0.053	-0.006	0.014	0.034	0.015
	(0.036)	(0.048)	(0.041)	(0.042)	(0.051)	(0.049)
$LogL$	-890.4	-1089.4	-885.7	-851.3	-898.4	-845.1
R^2_{MF}	0.331	0.129	0.244	0.226	0.130	0.178
R^2_{MZ}	0.562	0.258	0.442	0.396	0.259	0.339
Count R^2	0.782	0.664	0.755	0.757	0.673	0.718
LM_{het} (p-value)	0.077	0.860	0.842	1.000	1.000	1.000
LM_{norm} (p-value)	0.801	0.022	0.264	0.175	0.174	0.634
# Obs.	1979	1806	1806	1702	1508	1508

Average marginal effects of the probit estimations are reported. Robust standard errors in parentheses.
***Significant at 1%; **Significant at 5%; *Significant at 10%. The full set of control variables including industry dummies is used but not reported (see Table 2). For notes on goodness-of-fit and specification tests: see Table 2.

Table 6
Effect of demand and technical uncertainty on innovation expenditure across groups of firms facing a different p_θ
Full Sample, Manufacturing and Services

Sample	**Full**	**Manuf.**	**Services**
Dep. variable	**INEXP**	**INEXP**	**INEXP**
Demand uncertainty			
THETA*G1	-3.164**	-5.289**	-1.841
	(1.265)	(2.104)	(1.511)
THETA*G2	-0.351	-1.345	0.238
	(0.657)	(1.031)	(0.824)
THETA*G3	2.173***	1.965*	1.908***
	(0.621)	(1.051)	(0.732)
$\alpha_{\theta*G1} >= \alpha_{\theta*G2}$ (p-value)	0.040	0.033	0.392
$\alpha_{\theta*G1} >= \alpha_{\theta*G3}$ (p-value)	0.000	0.002	0.007
$\alpha_{\theta*G2} >= \alpha_{\theta*G3}$ (p-value)	0.001	0.021	0.031
Technical uncertainty			
LAMBDA1	0.668*	0.458	1.060*
	(0.358)	(0.445)	(0.616)
LAMBDA2	2.200***	2.002***	2.470***
	(0.224)	(0.278)	(0.380)
Additional control variables			
THREAT: low	0.067	-0.323	0.421
	(0.275)	(0.400)	(0.372)
THREAT: medium	-0.036	-0.411	0.311
	(0.288)	(0.415)	(0.395)
THREAT: high	-0.015	-0.545	0.432
	(0.326)	(0.458)	(0.460)
SIZE	0.815***	0.891***	0.713***
	(0.056)	(0.081)	(0.076)
NUMCOMP: 0	-0.572	-0.188	-0.549
	(0.542)	(1.029)	(0.614)
NUMCOMP: 1-5	0.432**	0.101	0.577**
	(0.204)	(0.317)	(0.260)
NUMCOMP: 6-15	0.175	-0.234	0.472
	(0.246)	(0.356)	(0.340)
COMP: PRICE	-0.578***	-0.519**	-0.605**
	(0.176)	(0.249)	(0.243)
COMP: QUAL	-0.004	0.340	-0.267
	(0.170)	(0.241)	(0.235)
COMP: LEAD	0.987***	0.877***	1.228***
	(0.264)	(0.326)	(0.457)
COMP: VARIETY	0.188	-0.234	0.676
	(0.371)	(0.493)	(0.573)
COMP: DESIGN	0.268	-0.392	0.726
	(0.437)	(0.592)	(0.639)
DIVERS	-0.015***	-0.017***	-0.010**
	(0.003)	(0.005)	(0.005)
EXPORT	1.439***	1.890***	1.039***
	(0.186)	(0.262)	(0.261)
RATING	-0.001	-0.003*	0.001
	(0.001)	(0.001)	(0.001)
HIGHSKILLED	0.023***	0.040***	0.014***
	(0.004)	(0.007)	(0.005)
TRAINEXP	0.693***	0.689***	0.684***
	(0.074)	(0.107)	(0.102)
NOTRAIN	-5.644***	-6.251***	-5.063***
	(0.304)	(0.480)	(0.389)
MVTRAIN	-4.673***	-4.884***	-4.419***
	(0.339)	(0.578)	(0.388)
EAST	0.244	0.017	0.406*
	(0.173)	(0.248)	(0.236)
GROUP	0.436***	0.606***	0.337
	(0.162)	(0.234)	(0.219)
$LogL$	-7827.2	-4390.7	-3384.1
$LogL0$	-8612.2	-4854.0	-3664.9
R^2_{MF}	0.091	0.095	0.077

The change in the expected value of $INEXP$, given $INEXP$ is positive, is reported (at average values of the regressors). Average marginal effects of the tobit estimations are reported. Robust standard errors in parentheses. ***, **, *: Significant at 1%, 5% and 10% respectively. Industry dummies are included. R^2_{MF}: McFadden (1974) Pseudo R^2, comparing the likelihood of an intercept-only model ($LogL0$) to the likelihood of the model with regressors ($LogL$).

Appendix A: Proofs

Proof of Propositions 1 & 2: Consider the partial derivatives of the five arguments of Φ with respect to θ: $\frac{\partial \Delta_{NPV}^{(1,1,1,1)}}{\partial \theta} = (2p_\theta - 1)\Delta\pi$, $\frac{\partial \Delta_{NPV}^{(1,1,1,0)}}{\partial \theta} = (p_\theta - p_\lambda + p_\theta p_\lambda)\Delta\pi$, $\frac{\partial \Delta_{NPV}^{(1,1,0,0)}}{\partial \theta} = p_\theta \Delta\pi$, $\frac{\partial \Delta_{NPV}^{(1,0,1,0)}}{\partial \theta} = p_\lambda (2p_\theta - 1)\Delta\pi$, $\frac{\partial \Delta_{NPV}^{(1,0,0,0)}}{\partial \theta} = p_\theta p_\lambda \Delta\pi$. All five partial derivatives are either negative or equal to zero when $p_\theta = 0$ for all $p_\lambda \in [0,1]$. This is a sufficient condition to obtain Proposition 1. All five partial derivatives are either positive or equal to zero when $p_\theta \in [\frac{1}{2}, 1]$ for all $p_\lambda \in [0,1]$. This is a sufficient condition to obtain Proposition 2. ■

Proofs of Propositions 3a, 3b & 4: Before we prove Propositions 3a, 3b & 4 consequently, we introduce Lemma 1 and Lemma 1'. Lemma 1 identifies Φ for different ranges of the parameters θ and λ. Lemma 1 holds over the complete parameter space of (c, p_θ, p_λ).

Lemma 1:
(1) $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ *for all* $\theta \in [0, \frac{1}{2}]$ *and for all* $\lambda \in [0, \left(\frac{1}{2} - \theta\right)\Delta\pi]$.
(2) $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ *for all* $\theta \in [0, \frac{1}{2}]$ *and for all* $\lambda \in [\left(\frac{1}{2} - \theta\right)\Delta\pi, \lambda_{\max}]$.
(3) $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ *for all* $\theta \in [\frac{1}{2}, 1]$ *and for all* $\lambda \in [0, \left(\theta - \frac{1}{2}\right)\Delta\pi]$.
(4) $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ *for all* $\theta \in [\frac{1}{2}, 1]$ *and for all* $\lambda \in [\left(\theta - \frac{1}{2}\right)\Delta\pi, \lambda_{\max}]$.

Proof of Lemma 1: From Assumptions 1-2, it follows that $\lambda_{\max} = I = \frac{\Delta\pi}{2}$. Then, $\Delta_{NPV}^{GB} \geq 0$ when $\left(\frac{1}{2} + \theta\right)\Delta\pi \geq \lambda$. Therefore, $\Delta_{NPV}^{GB} \geq 0$ for all $\theta \in [0,1]$ and $\lambda \in [0, \lambda_{\max}]$. As a result, also $\Delta_{NPV}^{GG} \geq 0$ for all $\theta \in [0,1]$ and $\lambda \in [0, \lambda_{\max}]$ (cfr. section 2.3). Then, $\Delta_{NPV}^{BG} \geq 0$ when $\left(\theta - \frac{1}{2}\right)\Delta\pi \leq \lambda$. Therefore, $\Delta_{NPV}^{BG} \geq 0$ for all $\theta \in [0, \frac{1}{2}]$ and for all $\lambda \in [0, \lambda_{\max}]$, $\Delta_{NPV}^{BG} \leq 0$ for all $\theta \in [\frac{1}{2}, 1]$ and for all $\lambda \in [0, \left(\theta - \frac{1}{2}\right)\Delta\pi]$ and $\Delta_{NPV}^{BG} \geq 0$ for all $\theta \in [\frac{1}{2}, 1]$ and for all $\lambda \in [\left(\theta - \frac{1}{2}\right)\Delta\pi], \lambda_{\max}]$. Then, $\Delta_{NPV}^{BB} \geq 0$ when $\left(\frac{1}{2} - \theta\right)\Delta\pi \geq \lambda$. Therefore, $\Delta_{NPV}^{BB} \geq 0$ for all $\theta \in [0, \frac{1}{2}]$ and for all $\lambda \in [0, \left(\frac{1}{2} - \theta\right)\Delta\pi]$, $\Delta_{NPV}^{BB} \leq 0$ for all $\theta \in [0, \frac{1}{2}]$ and for all $\lambda \in [\left(\frac{1}{2} - \theta\right)\Delta\pi, \lambda_{\max}]$ and $\Delta_{NPV}^{BB} \leq 0$ for all $\theta \in [\frac{1}{2}, 1]$ and for all $\lambda \in [0, \lambda_{\max}]$. Lemma 1 follows from noting that $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ when $\Delta_{NPV}^{GB} \geq 0$, $\Delta_{NPV}^{BG} \geq 0$ and $\Delta_{NPV}^{BB} \geq 0$, that $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $\Delta_{NPV}^{GB} \geq 0$, $\Delta_{NPV}^{BG} \geq 0$ and $\Delta_{NPV}^{BB} \leq 0$ and that $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $\Delta_{NPV}^{GB} \geq 0$, $\Delta_{NPV}^{BG} \leq 0$ and $\Delta_{NPV}^{BB} \leq 0$. ■

We use Lemma 1, where λ is expressed as a function of θ, in the determination of x and y. For the determination of v and w, it is useful to rewrite Lemma 1 as Lemma 1' where we express θ as a function of λ. Again, Lemma 1' holds over the complete parameter space of (c, p_θ, p_λ).

Lemma 1':
(1) $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ *for all* $\lambda \in [0, \lambda_{\max}]$ *and for all* $\theta \in [0, \frac{\lambda_{\max}-\lambda}{\Delta\pi}]$.
(2) $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ *for all* $\lambda \in [0, \lambda_{\max}]$ *and for all* $\theta \in [\frac{\lambda_{\max}-\lambda}{\Delta\pi}, \frac{\lambda_{\max}+\lambda}{\Delta\pi}]$.
(3) $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ *for all* $\lambda \in [0, \lambda_{\max}]$ *and for all* $\theta \in [\frac{\lambda_{\max}+\lambda}{\Delta\pi}, 1]$.

Proof of Proposition 3a: We prove that the smallest p_θ for which a positive effect of an increase in θ on the decision to start R&D is found, equals $\frac{1}{4}$ by showing that $\Phi(\theta) = 0$ for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$, $\theta = 1$, $p_\theta = \frac{1}{4}$, $\lambda = \lambda_{\max}$ and $p_\lambda = 1$.

First, consider the partial derivatives of $\Delta_{NPV}^{(1,1,1,1)}$, $\Delta_{NPV}^{(1,1,1,0)}$ and $\Delta_{NPV}^{(1,1,0,0)}$ with respect to θ when $p_\theta \in [0, \frac{1}{2}]$. Note that $\frac{\partial \Delta_{NPV}^{(1,1,1,1)}}{\partial \theta} = (2p_\theta - 1)\,\Delta\pi \leq 0$, $\frac{\partial \Delta_{NPV}^{(1,1,1,0)}}{\partial \theta} = (p_\theta - p_\lambda + p_\theta p_\lambda)\,\Delta\pi \geq 0$ if and only if $p_\lambda \leq \frac{p_\theta}{1-p_\theta}$ and $\frac{\partial \Delta_{NPV}^{(1,1,0,0)}}{\partial \theta} = p_\theta \Delta\pi \geq 0$. A positive effect due to an increase in θ can only be found when $\frac{\partial \Phi(\theta)}{\partial \theta} \geq 0$ at some subdomain of θ.

Second, from the fact that $\Delta_{NPV}^{GG} \geq \Delta_{NPV}^{s} \geq \Delta_{NPV}^{BB}$ for $s \in \{GB, BG\}$ (cfr. section 2.3), it follows that $\frac{\partial \Delta_{NPV}^{(1,1,1,1)}}{\partial p_\theta} = p_\lambda \left(\Delta_{NPV}^{GG} - \Delta_{NPV}^{BG}\right) + (1-p_\lambda)\,(\Delta_{NPV}^{GB} - \Delta_{NPV}^{BB}) \geq 0$, $\frac{\partial \Delta_{NPV}^{(1,1,1,0)}}{\partial p_\theta} = p_\lambda\,(\Delta_{NPV}^{GG} - \Delta_{NPV}^{BG}) + (1-p_\lambda)\,\Delta_{NPV}^{GB} \geq 0$ and $\frac{\partial \Delta_{NPV}^{(1,1,0,0)}}{\partial p_\theta} = p_\lambda \Delta_{NPV}^{GG} + (1-p_\lambda)\,\Delta_{NPV}^{GB} \geq 0$. From these observations and the definition of x, it follows that when $p_\theta = x$, $\Phi(\theta) = 0$ when $\theta = 1$.

Third, from Lemma 1, $\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$. Solving $\Delta_{NPV}^{(1,1,0,0)}(1) = 0$ yields $p_\theta = \frac{\frac{1}{2}\Delta\pi}{\frac{3}{2}\Delta\pi + (2p_\lambda - 1)\lambda}$. We find x by solving $\min\limits_{\lambda, p_\lambda} p_\theta$. For $\lambda = \lambda_{\max}$ and $p_\lambda = 1$, $x = \frac{1}{4}$. ■

Proof of Proposition 3b: We prove that the smallest p_λ for which a positive effect of an increase in λ on the decision to start R&D is found, approximately equals 0.28 by showing that $\Phi(\lambda) = 0$ for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$, $\lambda = \lambda_{\max}$, $p_\lambda = 0.28$, $\theta = 1$, and $p_\theta = \frac{p_\lambda}{1-p_\lambda}$.

First, consider the partial derivatives of $\Delta_{NPV}^{(1,1,1,1)}$, $\Delta_{NPV}^{(1,1,1,0)}$ and $\Delta_{NPV}^{(1,1,0,0)}$ with respect to λ when $p_\lambda \in [0, \frac{1}{2}]$. Note that $\frac{\partial \Delta_{NPV}^{(1,1,1,1)}}{\partial \lambda} = (2p_\lambda - 1)\Delta\pi \leq 0$, $\frac{\partial \Delta_{NPV}^{(1,1,1,0)}}{\partial \lambda} = (-p_\theta + p_\lambda + p_\theta p_\lambda)\Delta\pi \geq 0$ if and only if $p_\theta \leq \frac{p_\lambda}{1-p_\lambda}$ and $\frac{\partial \Delta_{NPV}^{(1,1,0,0)}}{\partial \lambda} = p_\theta(2p_\lambda - 1)\Delta\pi \leq 0$ for all $p_\theta \in [0,1]$. A positive effect due to an increase in λ can only be found when $\frac{\partial \Phi(\lambda)}{\partial \lambda} \geq 0$ at some subdomain of λ.

Second, $\frac{\partial \Delta_{NPV}^{(1,1,1,1)}}{\partial p_\lambda} = p_\theta \left(\Delta_{NPV}^{GG} - \Delta_{NPV}^{GB}\right) + (1-p_\theta)\left(\Delta_{NPV}^{BG} - \Delta_{NPV}^{BB}\right) \geq 0$ and $\frac{\partial \Delta_{NPV}^{(1,1,0,0)}}{\partial p_\lambda} = p_\theta \left(\Delta_{NPV}^{GG} - \Delta_{NPV}^{GB}\right) \geq 0$. Also, $\frac{\partial \Delta_{NPV}^{(1,1,1,0)}}{\partial p_\lambda} = p_\theta \left(\Delta_{NPV}^{GG} - \Delta_{NPV}^{GB}\right) + (1-p_\theta)\,\Delta_{NPV}^{BG} \geq 0$ if and only if $\Delta_{NPV}^{BG} \geq 0$. This is the case when $\Phi = \Delta_{NPV}^{(1,1,1,0)}$. From these observations and the definition of v, it follows that when $p_\lambda = v$, $\Phi(\lambda) = 0$ when $\lambda = \lambda_{\max}$.

Third, from Lemma 1', $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $\theta \in [0,1]$. Solving $\Delta_{NPV}^{(1,1,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{\frac{1}{2}-p_\theta\theta}{1-\theta+p_\theta\theta}$. We find v by solving $\min\limits_{\theta,p_\theta} p_\lambda$ subject to $p_\theta \leq \frac{p_\lambda}{1-p_\lambda}$. For $\theta = 1$ and $p_\theta = \frac{p_\lambda}{1-p_\lambda}$, $v = 0.280776 \approx 0.28$. ■

Proof of Proposition 4: We first prove that the lowest p_θ for which no negative effect of an increase in θ on the decision to start R&D can be found, equals $\frac{1}{2}$ by showing that $\Phi(\theta) = 0$ for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$, $\theta = \frac{1}{2}$, $p_\theta = \frac{1}{2}$, $\lambda = 0$ and $p_\lambda = 1$.

First, a negative effect due to an increase in θ can only be found when $\frac{\partial\Phi(\theta)}{\partial\theta} \leq 0$ at some subdomain of θ. Hence, Φ has to be equal to $\Delta_{NPV}^{(1,1,1,1)}$or $\Delta_{NPV}^{(1,1,1,0)}$ when $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$ at some subdomain of θ.

Second, from the observation that $\frac{\partial\Delta_{NPV}^{(1,1,1,1)}}{\partial p_\theta} \geq 0, \frac{\partial\Delta_{NPV}^{(1,1,1,0)}}{\partial p_\theta} \geq 0$ and $\frac{\partial\Delta_{NPV}^{(1,1,0,0)}}{\partial p_\theta} \geq 0$ (cfr. proof of Proposition 3a) and from the definition of y, two possibilities arise. Either, $\Phi(\theta) = 0$ for $\theta = \frac{1}{2}$ and $p_\theta = y$, when (i) for $\theta \in [0, \frac{1}{2}]$, $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ or $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ and $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$ and for $\theta \in [\frac{1}{2}, 1]$, $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ or when (ii) for $\theta \in [0, \frac{1}{2}]$, $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ and for $\theta \in [\frac{1}{2}, 1]$, $\Phi = \Delta_{NPV}^{(1,1,0,0)}$or $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ and $p_\lambda \leq \frac{p_\theta}{1-p_\theta}$. Or $\Phi(\theta) = 0$ for $\theta = 1$ and $p_\theta = y$ when, for $\theta \in [0, \frac{1}{2}]$, $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ or $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ and $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$ and for $\theta \in [\frac{1}{2}, 1]$, $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ and $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$.

Third, from Lemma 1, $\Phi(\frac{1}{2}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ for all $\lambda \in [0, \lambda_{\max}]$ when $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$. Solving $\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2}) = 0$ yields $p_\theta = \frac{\frac{1}{2}\Delta\pi - p_\lambda\lambda}{\Delta\pi-(1-p_\lambda)\lambda}$. We find y by solving $\max\limits_{\lambda,p_\lambda} p_\theta$ subject to $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$. For $\lambda = 0$ and $p_\lambda = 1$, $y = \frac{1}{2}$. Since y cannot exceed $\frac{1}{2}$ (cfr. Proposition 2), the result follows.

We now prove that the lowest p_λ for which a no negative effect of an increase in λ on the decision to start R&D can be found, equals $\frac{1}{2}$ by showing that $\Phi(\lambda) = 0$ for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$, $\lambda = \lambda_{\max}$, $p_\lambda = \frac{1}{2}$, $\theta = 0$ and $p_\theta = 1$.

First, a negative effect due to an increase in λ can only be found when $\frac{\partial\Phi(\lambda)}{\partial\lambda} \leq 0$ at some subdomain of λ. Hence, in order to find a negative effect, Φ has to be equal to $\Delta_{NPV}^{(1,1,1,1)}$, $\Delta_{NPV}^{(1,1,0,0)}$ or $\Delta_{NPV}^{(1,1,1,0)}$ when $p_\theta \geq \frac{p_\lambda}{1-p_\lambda}$ at some subdomain of λ.

Second, from the observation that $\frac{\partial\Delta_{NPV}^{(1,1,1,1)}}{\partial p_\lambda} \geq 0, \frac{\partial\Delta_{NPV}^{(1,1,1,0)}}{\partial p_\lambda} \geq 0$ and $\frac{\partial\Delta_{NPV}^{(1,1,0,0)}}{\partial p_\lambda} \geq 0$ (cfr. proof of Proposition 3b) and from the definition of w, it follows that when $p_\lambda = w$, $\Phi(\lambda) = 0$ when $\lambda = \lambda_{\max}$.

Third, from Lemma 1', $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ for all $\theta \in [0,1]$ when $p_\theta \geq \frac{p_\lambda}{1-p_\lambda}$. Solving $\Delta_{NPV}^{(1,1,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{\frac{1}{2}-p_\theta\theta}{1-\theta+p_\theta\theta}$. We find w by solving $\max\limits_{\theta,p_\theta} p_\lambda$ subject to $p_\theta \geq \frac{p_\lambda}{1-p_\lambda}$. For $\theta = 0$ and $p_\theta = 1$, $w = \frac{1}{2}$. Since w cannot exceed $\frac{1}{2}$, the result follows. ■

Proof of sensitivity analysis (relaxing Assumption 1): Relaxing Assumption 1, setting $I_1 = aI_0$, where $a \in \mathbb{R}_{++}$, we respectively prove that (i) $x = \frac{1}{2(1+a)}$, (ii) $v = \min\{\frac{1}{1+a}, \frac{-3+\sqrt{9+8a}}{4a}\}$ and (iii) $y = w = \frac{1}{2}$.

From Assumption 2, it follows that $I_0 = \frac{\Delta\pi}{1+a}$ and $I_1 = \frac{a\Delta\pi}{1+a} = \lambda_{\max}$. Note that the signs of the partial derivatives of the different Δ_{NPV} with respect to θ, λ, p_θ and p_λ do not depend on the value of a. As a result, in (i) $\Phi(\theta) = 0$ for $\theta = 1$ and $p_\theta = x$ and in (ii) $\Phi(\lambda) = 0$ for $\lambda = \lambda_{\max}$ and $p_\lambda = v$.

(i). We show that for $\theta = 1$, $\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ for all $a \in \mathbb{R}_{++}$. First, $\Delta_{NPV}^{GG} \geq 0$ when $\frac{-1-\theta-a\theta}{1+a}\Delta\pi \leq \lambda$. Hence, $\Delta_{NPV}^{GG} \geq 0$ for all $\theta \in [0,1]$, all $a \in \mathbb{R}_{++}$ and $\lambda \in [0, \lambda_{\max}]$. Then $\Delta_{NPV}^{GB} \geq 0$ when $\frac{1+\theta+a\theta}{1+a}\Delta\pi \geq \lambda$. Hence, for $\theta = 1$, $\Delta_{NPV}^{GB} \geq 0$ for all $a \in \mathbb{R}_{++}$ and all $\lambda \in [0, \lambda_{\max}]$. Also $\Delta_{NPV}^{BG} \leq 0$ when $\frac{\theta+a\theta-1}{1+a}\Delta\pi \geq \lambda$. Hence, for $\theta = 1$, $\Delta_{NPV}^{BG} \leq 0$ for all $a \in \mathbb{R}_{++}$ and all $\lambda \in [0, \lambda_{\max}]$. From these observations, $\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$. Solving $\Delta_{NPV}^{(1,1,0,0)}(1) = 0$ when $\theta = 1$ yields $p_\theta = \frac{\frac{1}{1+a}\Delta\pi}{\frac{2+a}{1+a}\Delta\pi+(2p_\lambda-1)\lambda}$. We find x by solving $\min\limits_{\lambda,p_\lambda} p_\theta$. For $\lambda = \lambda_{\max}$ and $p_\lambda = 1$, $x = \frac{1}{2(1+a)}$.

(ii). We show that for $\lambda = \lambda_{\max}$, $\Phi(\lambda_{\max}) = 0$ holds for Φ equal to $\Delta_{NPV}^{(1,1,1,0)}$ or $\Delta_{NPV}^{(1,0,1,0)}$ for all $a \in \mathbb{R}_{++}$. Note that $\frac{\partial\Delta_{NPV}^{(1,0,1,0)}}{\partial\lambda} = p_\lambda\Delta\pi \geq 0$ and $\frac{\partial\Delta_{NPV}^{(1,0,1,0)}}{\partial p_\lambda} = p_\theta\Delta_{NPV}^{GG} + (1-p_\theta)\,\Delta_{NPV}^{BG} \geq 0$ when $\Phi = \Delta_{NPV}^{(1,0,1,0)}$. For $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{GB} \geq 0$ when $\theta \geq \frac{a-1}{a+1}$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{GB} \geq 0$ for all $a \in (0,1]$ and all $\theta \in [0,1]$, ${}_{NPV}^{GB} \geq 0$ for all $a \in [1,\infty)$ and for $\theta = [\frac{a-1}{a+1}, 1]$ and ${}_{NPV}^{GB} \leq 0$ for all $a \in [1,\infty)$ and for $\theta = [0, \frac{a-1}{a+1}]$. Also, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BG} \geq 0$ when $\theta \leq 1$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BG} \geq 0$ for all $a \in \mathbb{R}_{++}$ and all $\theta \in [0,1]$. Also, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BB} \geq 0$ when $\theta \leq \frac{1-a}{1+a}$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BB} \geq 0$ for all $a \in (0,1]$ and all $\theta \in [0, \frac{1-a}{1+a}]$, $\Delta_{NPV}^{BB} \leq 0$ for all $a \in (0,1]$ and all $\theta \in [\frac{1-a}{1+a}, 1]$ and $\Delta_{NPV}^{BB} \leq 0$ for all $a \in [1,\infty)$ and for $\theta \in [0,1]$. From these observations, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ for $a \in (0,1]$ and $\theta \in [\frac{1-a}{1+a}, 1]$. Solving $\Delta_{NPV}^{(1,1,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{1-p_\theta+ap_\theta-p_\theta\theta-ap_\theta\theta}{1+a-\theta-p_\theta+ap_\theta-a\theta+p_\theta\theta+ap_\theta\theta}$. We find the lowest p_λ by solving $\min\limits_{\theta,p_\theta} p_\lambda$ subject to $\theta \in [\frac{1-a}{1+a}, 1]$ and $p_\theta \leq \frac{p_\lambda}{1-p_\lambda}$. For $\theta = 1$ and $p_\theta = \frac{p_\lambda}{1-p_\lambda}$, solving the quadratic equation $2az^2 + 3z - 1 = 0$ yields $z = \frac{-3+\sqrt{9+8a}}{4a}$. Also, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ for $a \in [1,\infty)$ and $\theta \in [\frac{a-1}{a+1}, 1]$. We find z as shown above. Also $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,0,1,0)}$ for $a \in [1,\infty)$ and $\theta \in [0, \frac{a-1}{a+1}]$.. Solving $\Delta_{NPV}^{(1,0,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{1}{(1+a)((1+\theta)p_\theta+(1-\theta)(1-p_\theta))-2ap_\theta}$. We find the lowest p_λ by solving $\min\limits_{\theta,p_\theta} p_\lambda$ subject to $\theta \leq \frac{a-1}{a+1}$. For $\theta = 0$ and $p_\theta = 0$, $p_\lambda = \frac{1}{1+a}$. Concluding, $v = \min\{\frac{1}{1+a}, \frac{-3+\sqrt{9+8a}}{4a}\}$ for all $a \in \mathbb{R}_{++}$.

(iii). We only give the proof for y. The proof for w is analogous. We show that for $\theta = \frac{1}{2}$, $\Phi(\frac{1}{2}) = 0$ holds for Φ equal to $\Delta_{NPV}^{(1,1,1,1)}$ or $\Delta_{NPV}^{(1,1,1,0)}$ for all $a \in \mathbb{R}_{++}$.

For $\theta = \frac{1}{2}$, $\Delta_{NPV}^{BB} \geq 0$ when $\frac{1-a}{2(1+a)}\Delta\pi \geq \lambda$. Hence, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{BB} \geq 0$ for $a \leq \frac{1}{3}$ and $\lambda \in [0, \lambda_{\max}]$. Then, $\Phi(\frac{1}{2}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,1)}$. Setting $I_0 = \Delta\pi - I_1$, it is easy to check that $\frac{\partial\Delta_{NPV}^{(1,1,1,1)}(\frac{1}{2})}{\partial a} = \frac{\partial\Delta_{NPV}^{(1,1,1,1)}(\frac{1}{2})}{\partial I_1}\frac{\partial I_1}{\partial a} = 0$ since $\frac{\partial\Delta_{NPV}^{(1,1,1,1)}(\frac{1}{2})}{\partial I_1} = 0$. Solving $\Delta_{NPV}^{(1,1,1,1)}(\frac{1}{2}) = 0$ yields $p_\theta = \frac{\frac{\Delta\pi}{2} - \lambda - 2p_\lambda\lambda}{\Delta\pi}$. We find y by solving $\max\limits_{\lambda, p_\lambda} p_\theta$. For $\lambda = 0$ and $p_\lambda \in [0,1]$, $y = \frac{1}{2}$. Also, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{GB} \geq 0$ when $\frac{3+a}{2(1+a)}\Delta\pi \geq \lambda$. Hence, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{GB} \geq 0$ for $a \leq 3$ and $\lambda \in [0, \lambda_{\max}]$. Also, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{BG} \geq 0$ when $\frac{a-1}{2(a+1)}\Delta\pi \leq \lambda$. Hence, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{BG} \geq 0$ for $a \leq 1$ and $\lambda \in [0, \lambda_{\max}]$. Furthermore, for $\theta = \frac{1}{2}$, $\Delta_{NPV}^{BB} \geq 0$ for $a \geq \frac{1}{3}$ and $\lambda \leq \frac{1-a}{2(1+a)}\Delta\pi$, while $\Delta_{NPV}^{BB} \leq 0$ for $a \geq \frac{1}{3}$ and $\lambda \geq \frac{1-a}{2(1+a)}\Delta\pi$. From these observations, $\Phi(\frac{1}{2}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,1)}$ when $a \in [\frac{1}{3}, 1]$ and $\lambda \leq \frac{1-a}{2(1+a)}\Delta\pi$. Again, $\frac{\partial\Delta_{NPV}^{(1,1,1,1)}(\frac{1}{2})}{\partial I_1} = 0$ and the result follows as shown above. On the contrary, $\Phi(\frac{1}{2}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $a \in [\frac{1}{3}, 1]$ and $\lambda \geq \frac{1-a}{2(1+a)}\Delta\pi$. Then $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial I_1} = 1 - p_\theta - p_\lambda + p_\theta p_\lambda$. We obtain that $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial p_\lambda} = (1+p_\theta)\lambda + (1-p_\theta)\frac{\Delta\pi}{2} - (1-p_\theta)\frac{a\Delta\pi}{1+a}$. Hence, $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial p_\lambda} \geq 0$ for $a \in [\frac{1}{3}, 1]$ and $\lambda \geq \frac{1-a}{2(1+a)}\Delta\pi$. Setting $p_\lambda = 1$, $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial I_1} = 0$ and the result follows from the proof of Proposition 4. Similarly, $\Phi(\frac{1}{2}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $a \in [1,3]$ and $\lambda \geq \frac{a-1}{2(1+a)}\Delta\pi$ and when $a \geq 3$ and $\lambda \in [\frac{a-1}{2(1+a)}\Delta\pi, \frac{3+a}{2(1+a)}\Delta\pi]$. Again, $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial p_\lambda} \geq 0$ for $a \in [1,3]$ and $\lambda \geq \frac{a-1}{2(1+a)}\Delta\pi$ and for $a \geq 3$ and $\lambda \in [\frac{a-1}{2(1+a)}\Delta\pi, \frac{3+a}{2(1+a)}\Delta\pi]$. Setting $p_\lambda = 1$, $\frac{\partial\Delta_{NPV}^{(1,1,1,0)}(\frac{1}{2})}{\partial I_1} = 0$ and the result follows from the proof of Proposition 4. ■

Proof of sensitivity analysis (relaxing Assumption 2): Relaxing Assumption 2, setting $I_0 + I_1 = b\Delta\pi$, where $b \in \mathbb{R}_{++}$, we respectively prove that (i) $x = \min\{\frac{b}{4}, 1\}$, (ii) $v = \min\{\max\{\frac{-4+b+\sqrt{9b^2-8b+16}}{4b}, \frac{b}{4}\}, 1\}$, (iii) $y = \min\{\frac{b}{4(1-b)}, 1\}$ and (iv) $w = \frac{1}{2}$.

From Assumption 1, it follows that $I_0 = I_1 = \frac{b\Delta\pi}{2} = \lambda_{\max}$. Note that the signs of the partial derivatives of the different Δ_{NPV} with respect to θ, λ, p_θ and p_λ do not depend on the value of b. As a result, in (i) $\Phi(\theta) = 0$ for $\theta = 1$ and $p_\theta = x$ and in (ii) $\Phi(\lambda) = 0$ for $\lambda = \lambda_{\max}$ and $p_\lambda = v$.

(i). We show that for $\theta = 1$, $\Phi(1) = 0$ holds for Φ equal to $\Delta_{NPV}^{(1,1,0,0)}$ or $\Delta_{NPV}^{(1,0,0,0)}$ for $b \in [0,4]$. Note that $\frac{\partial\Delta_{NPV}^{(1,0,0,0)}}{\partial\theta} = p_\theta p_\lambda \Delta\pi \geq 0$ and $\frac{\partial\Delta_{NPV}^{(1,0,0,0)}}{\partial p_\theta} = p_\lambda \Delta_{NPV}^{GG} \geq 0$ when $\Phi = \Delta_{NPV}^{(1,0,0,0)}$. First, $\Delta_{NPV}^{GB} \geq 0$ when $\frac{2+2\theta-b}{2}\Delta\pi \geq \lambda$. Hence, for $\theta = 1$, $\Delta_{NPV}^{GB} \geq 0$ for all $b \in [0,2]$ and all $\lambda \in [0, \lambda_{\max}]$, $\Delta_{NPV}^{GB} \geq 0$ for all $b \in [2,4]$ and all $\lambda \in [0, (2-\frac{b}{2})\Delta\pi]$, $\Delta_{NPV}^{GB} \leq 0$ for all $b \in [2,4]$ and all $\lambda \in [(2-\frac{b}{2})\Delta\pi, \lambda_{\max}]$. Also $\Delta_{NPV}^{BG} \leq 0$ when $\frac{2\theta+b-2}{2}\Delta\pi \geq \lambda$. Hence, for $\theta = 1$, $\Delta_{NPV}^{BG} \leq 0$ for all $b \in \mathbb{R}_{++}$ and all $\lambda \in [0, \lambda_{\max}]$. From these observations,

$\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ when $b \in [0,2]$ and $\lambda \in [0, \lambda_{\max}]$. Solving $\Delta_{NPV}^{(1,1,0,0)}(1) = 0$ when $\theta = 1$ yields $p_\theta = \frac{b\Delta\pi}{4p_\lambda\lambda + 4\Delta\pi - b\Delta\pi - 2\lambda}$. We find the lowest p_θ by solving $\min\limits_{\lambda, p_\lambda} p_\theta$. For $\lambda = \lambda_{\max}$ and $p_\lambda = 1$, $p_\theta = \frac{b}{4}$. Also $\Delta_{NPV}^{GG} \geq 0$ when $\frac{b-2-2\theta}{2}\Delta\pi \geq \lambda$. Hence, for $\theta = 1$, $\Delta_{NPV}^{GG} \geq 0$ for all $b \in [0,4]$ and all $\lambda \in [0, \lambda_{\max}]$. From these observations, $\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,0,0)}$ when $b \in [2,4]$ and $\lambda \in [0, (2 - \frac{b}{2})\Delta\pi]$. As explained above, we find the lowest p_θ by solving $\min\limits_{\lambda, p_\lambda} p_\theta$ but now subject to $\lambda \leq (2 - \frac{b}{2})\Delta\pi$. For $\lambda = (2 - \frac{b}{2})\Delta\pi$ and $p_\lambda = 1$, $p_\theta = \frac{b}{8-2b}$. On the contrary, $\Phi(1) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,0,0,0)}$ when $b \in [2,4]$ and $\lambda \in [(2 - \frac{b}{2})\Delta\pi, \lambda_{\max}]$. Solving $\Delta_{NPV}^{(1,0,0,0)}(1) = 0$ when $\theta = 1$ yields $p_\theta = \frac{b\Delta\pi}{4p_\lambda\Delta\pi - bp_\lambda\Delta\pi + 2p_\lambda\lambda}$. We find the lowest p_θ by solving $\min\limits_{\lambda, p_\lambda} p_\theta$ subject to $\lambda \geq (2 - \frac{b}{2})\Delta\pi$. For $\lambda = \lambda_{\max}$ and $p_\lambda = 1$, $p_\theta = \frac{b}{4}$. The result follows from noting that $\frac{b}{4} \leq \frac{b}{8-2b}$ for all $b \in [2,4]$. Note that $p_\theta = 1$ for $b = 4$. Hence, we conclude that $x = \min\{\frac{b}{4}, 1\}$ for all $b \in \mathbb{R}_{++}$.

(ii). We show that for $\lambda = \lambda_{\max}$, $\Phi(\lambda_{\max}) = 0$ holds for Φ equal to $\Delta_{NPV}^{(1,1,1,0)}$ or $\Delta_{NPV}^{(1,0,1,0)}$ for $b \in [0,4]$. For $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{GB} \geq 0$ when $\theta \geq b-1$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{GB} \geq 0$ for all $b \in [0,1]$ and for all $\theta \in [0,1]$, $\Delta_{NPV}^{GB} \geq 0$ for all $b \in [1,2]$ and for all $\theta \in [b-1, 1]$, $\Delta_{NPV}^{GB} \leq 0$ for all $b \in [1,2]$ and for all $\theta \in [0, b-1]$ and $\Delta_{NPV}^{GB} \leq 0$ for all $b \in [2,4]$ and for all $\theta \in [0,1]$. Also, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BG} \geq 0$ when $(1-\theta)\Delta\pi \geq 0$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BG} \geq 0$ for all $b \in \mathbb{R}_{++}$ and for all $\theta \in [0,1]$. Also, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BB} \geq 0$ when $\theta \leq 1 - b$. Hence, for $\lambda = \lambda_{\max}$, $\Delta_{NPV}^{BB} \geq 0$ for all $b \in [0,1]$ and for all $\theta \in [0, 1-b]$, $\Delta_{NPV}^{BB} \leq 0$ for all $b \in [0,1]$ and for all $\theta \in [1-b, 1]$ and $\Delta_{NPV}^{BB} \leq 0$ for all $b \in [1,4]$ and for all $\theta \in [0,1]$. From these observations, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $b \in [0,1]$ and $\theta \in [1-b, 1]$. Solving $\Delta_{NPV}^{(1,1,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{\left(\frac{b}{2} - (1+\theta)p_\theta + bp_\theta\right)\Delta\pi}{bp_\theta\Delta\pi + (1-\theta)p_\theta(1-\Delta\pi)}$. We find the lowest p_λ by solving $\min\limits_{\theta, p_\theta} p_\lambda$ subject to $p_\theta \leq \frac{p_\lambda}{1-p_\lambda}$ and $\theta \geq 1-b$. For $p_\theta = \frac{p_\lambda}{1-p_\lambda}$ and $\theta = 1$, solving the quadratic equation $2bz^2 + (4-b)z - b = 0$ yields $z = \frac{-4+b+\sqrt{9b^2-8b+16}}{4b}$. Furthermore, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$ when $b \in [1,2]$ and $\theta \in [b-1, 1]$. The result for z follows as shown above. On the contrary, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,0,1,0)}$ when $b \in [1,2]$ and $\theta \in [0, b-1]$. Solving $\Delta_{NPV}^{(1,0,1,0)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{\frac{b}{2}}{p_\theta(1+\theta) + (1-p_\theta)(1-\theta)}$. We find the lowest p_λ by solving $\min\limits_{\theta, p_\theta} p_\lambda$ subject to $\theta \leq b-1$. For $\theta = b-1$ and $p_\theta = 1$, $p_\lambda = \frac{1}{2}$. The result follows from noting that $\frac{1}{2} \geq \frac{-4+b+\sqrt{9b^2-8b+16}}{4b}$ for all $b \in [1,2]$. Finally, $\Phi(\lambda_{\max}) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,0,1,0)}$ when $b \in [2,4]$ and $\theta \in [0,1]$. As before, we find the lowest p_λ by solving $\min\limits_{\theta, p_\theta} p_\lambda$. For $\theta = 1$ and $p_\theta = 1$, $p_\lambda = \frac{b}{4}$. Note that $\frac{b}{4} \geq \frac{-4+b+\sqrt{9b^2-8b+16}}{4b}$ for all $b \in [2,4]$ and that $p_\lambda = 1$ for $b = 4$. Hence, we conclude that $v = \min\{\max\{\frac{-4+b+\sqrt{9b^2-8b+16}}{4b}, \frac{b}{4}\}, 1\}$ for all $b \in \mathbb{R}_{++}$.

(iii). A negative effect due to an increase in θ can only be found when Φ equals

$\Delta_{NPV}^{(1,1,1,1)}$or $\Delta_{NPV}^{(1,1,1,0)}$ when $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$. We focus on $b \in (0,1]$. First, $\Delta_{NPV}^{GB} \geq 0$ when $\frac{2+2\theta-b}{2}\Delta\pi \geq \lambda$. Hence, $\Delta_{NPV}^{GB} \geq 0$ for all $b \in (0,1]$, all $\theta \in [0,1]$ and all $\lambda \in [0,\lambda_{\max}]$. Also $\Delta_{NPV}^{BG} \geq 0$ when $\frac{2\theta+b-2}{2}\Delta\pi \leq \lambda$. Hence, $\Delta_{NPV}^{BG} \geq 0$ for all $b \in (0,1]$, all $\theta \in [0,1-\frac{b}{2}]$ and all $\lambda \in [0,\lambda_{\max}]$, $\Delta_{NPV}^{BG} \geq 0$ for all $b \in (0,1]$, all $\theta \in [1-\frac{b}{2},1]$ and all $\lambda \in [\frac{2\theta+b-2}{2}\Delta\pi,\lambda_{\max}]$ and $\Delta_{NPV}^{BG} \leq 0$ for all $b \in (0,1]$, all $\theta \in [1-\frac{b}{2},1]$ and all $\lambda \in [0,\frac{2\theta+b-2}{2}\Delta\pi]$. Also $\Delta_{NPV}^{BB} \geq 0$ when $\frac{2-2\theta-b}{2}\Delta\pi \geq \lambda$. Hence, $\Delta_{NPV}^{BB} \geq 0$ for all $b \in (0,1]$, all $\theta \in [0,1-b]$ and all $\lambda \in [0,\lambda_{\max}]$, $\Delta_{NPV}^{BB} \geq 0$ for all $b \in (0,1]$, all $\theta \in [1-b,1-\frac{b}{2}]$ and all $\lambda \in [0,\frac{2-2\theta-b}{2}\Delta\pi]$, $\Delta_{NPV}^{BB} \leq 0$ for all $b \in (0,1]$, all $\theta \in [1-b,1-\frac{b}{2}]$ and all $\lambda \in [\frac{2-2\theta-b}{2}\Delta\pi,\lambda_{\max}]$ and $\Delta_{NPV}^{BB} \leq 0$ for all $b \in (0,1]$, all $\theta \in [1-\frac{b}{2},1]$ and all $\lambda \in [0,\lambda_{\max}]$. From these observations $\Phi(\theta) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,1)}$, $\theta = 1-b$, $b \in (0,1]$ and $\lambda \in [0,\lambda_{\max}]$. Solving $\Delta_{NPV}^{(1,1,1,1)}(1-b) = 0$ yields $p_\theta = \frac{\lambda(1-2p_\lambda)}{2(1-b)\Delta\pi}$. We find the highest p_θ by solving $\max\limits_{\lambda,p_\lambda} p_\theta$. For $\lambda = \lambda_{\max}$ and $p_\lambda = 0$, $p_\theta = \frac{b}{4(1-b)}$. Note that $p_\theta = \frac{1}{2}$ for $b = \frac{2}{3}$. Also $\Phi(\theta) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,1)}$, $\theta = 1-\frac{b}{2}$, $b \in (0,1]$ and $\lambda = 0$. Solving $\Delta_{NPV}^{(1,1,1,1)}(1-\frac{b}{2}) = 0$ yields $p_\theta = \frac{b}{2(2-b)}$. Note that $\frac{b}{4(1-b)} \geq \frac{b}{2(2-b)}$ for all $b \in (0,1]$. Also, $\Phi(\theta) = 0$ holds for $\Phi = \Delta_{NPV}^{(1,1,1,0)}$, $\theta = 1$, $b \in (0,1]$, $\lambda = \lambda_{\max}$ and $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$. Solving $\Delta_{NPV}^{(1,1,1,0)}(1) = 0$ when $\lambda = \lambda_{\max}$ yields $p_\theta = \frac{b}{2(2-b+p_\lambda b)}$. We find the highest p_θ by solving $\max\limits_{p_\lambda} p_\theta$ subject to $p_\lambda \geq \frac{p_\theta}{1-p_\theta}$. For $p_\lambda = \frac{p_\theta}{1-p_\theta}$, solving the quadratic equation $(4b-4)z^2 + (4-b)z - b = 0$ yields $z = \frac{-4+b+\sqrt{17b^2-24b+16}}{2(4b-4)}$. Note that $\frac{b}{4(1-b)} \geq \frac{-4+b+\sqrt{17b^2-24b+16}}{2(4b-4)}$ for all $b \in (0,1]$. Hence, we conclude that $y = \min\{\frac{b}{4(1-b)},1\}$ for all $b \in (0,1]$.

(iv). It is sufficient to show that $\Phi(\lambda) = 0$ for $\Phi = \Delta_{NPV}^{(1,1,1,1)}$, $\lambda = \lambda_{\max}$, $p_\lambda = \frac{1}{2}$, $\theta = 1-b$ and $p_\theta = 0$. From (iii), we know that $\Delta_{NPV}^{BB} \geq 0$ for all $b \in (0,1]$, all $\theta \in [0,1-b]$ and all $\lambda \in [0,\lambda_{\max}]$. Solving $\Delta_{NPV}^{(1,1,1,1)}(\lambda_{\max}) = 0$ yields $p_\lambda = \frac{\frac{3}{2}b-2p_\theta\theta+\theta-1}{b}$. We find w by solving $\max\limits_{\theta,p_\theta} p_\lambda$ subject to $\theta \in [0,1-b]$. For $\theta = 1-b$ and $p_\theta = 0$, $w = \frac{1}{2}$. Since w cannot exceed $\frac{1}{2}$, the result follows. ■

Appendix B: Statistical annex

Table B.1
Distribution of the Total Sample, Full Sample and Subsample

Distribution by:	Total Sample	Full Sample	Subsample
Industry			
Food/tobacco	3.16	3.10	2.68
Textiles	2.97	2.77	2.48
Paper/wood/print	6.7	6.82	6.05
Chemicals	4.1	4.13	4.96
Plastic/rubber	3.62	3.83	3.67
Glass/ceramics	2.14	2.25	2.97
Metal	8.35	8.53	10.31
Machinery	5.99	6.38	7.04
Electrical engineering	4.88	5.22	6.54
Medical, precision and optical instruments	4.92	5.51	6.64
Vehicles	2.66	2.53	2.58
Furniture	2.62	2.69	2.28
Wholesale	4.38	4.18	4.06
Retail	2.35	2.06	2.18
Transport/storage/post	8.46	8.10	5.55
Banks/insurances	5.05	4.48	3.87
Computer/telecommunication	4.59	4.75	4.66
Technical services	8.79	8.88	9.81
Consultancies	3.77	3.59	2.97
Other business related services	7.06	6.93	5.95
Real estate/renting	2.07	1.98	2.28
Media	1.38	1.28	0.50
Size (Number of employees)			
0-4	4.65	3.75	3.47
5-9	14.24	13.34	13.78
10-19	16.52	15.62	13.88
20-49	18.68	19.02	21.01
50-99	13.13	13.61	13.88
100-199	14.07	14.72	14.47
200-499	7.96	8.69	8.42
500-999	4.98	5.35	5.35
1000+	5.78	5.90	5.75
Region			
West Germany	66.86	67.81	64.42
East Germany	33.14	32.19	35.58
Innovation activities			
Non-innovators	36.12	33.14	31.71
Innovators[a]	63.88	66.86	68.29
# Obs.	4776	3681	1009

[a] Innovators are defined as firms having introduced product or process innovations in the period 2002-2004.

Table B.2
Variable definitions

Variable	Type	Definition
Dependent variables		
R&D	0/1	1 if the firm undertook R&D activities in the period 2002-2004.
PROCESS	0/1	1 if the firm planned to undertake process innovations in 2005.
PRODUCT	0/1	1 if the firm planned to undertake product innovations in 2005.
Independent variables		
Demand uncertainty		
THETA	c	Average of the absolute percentage change in sales over the last two years (2002/2003 and 2003/2004).
G1	0/1	1 if the firm experienced two negative demand shocks in the past two years, i.e. a decrease in sales in 2002/2003 as well as in 2003/2004.
G2	0/1	1 if the firm experienced a positive and a negative demand shock in the past, i.e. one decrease and one increase in sales within the last two years.
G3	0/1	1 if the firm experienced two positive demand shocks in the past two years, i.e. a positive growth in sales in 2002/2003 as well as in 2003/2004.
Technical uncertainty		
LAMBDA1	0/1	1 if the lack of technological information was of high-to medium-size importance and led to an extension of innovation projects in the period 2002-2004.
LAMBDA2	0/1	1 if high innovation costs were of high- to medium-size importance and led to an extension of innovation projects in the period 2002-2004.
LAMBDA3	c	Absolute deviation between in year 2003 expected R&D expenditure for 2004 and realized R&D expenditure in 2004, in log.
Additional control variables		
THREAT	0/1	3 dummy variables indicating whether the firm perceived a high/medium/low threat of its own market position due to the potential entry of new competitors (reference group: firms with no entry threat).
SIZE	c	Number of employees in 2003, in log.
NUMCOMP	0/1	3 dummy variables indicating the number of competitors: 0, 1-5 or 6-15 (reference group: more than 15 competitors).
COMP	0/1	5 dummy variables indicating the most important factors of competition: price, quality, technological lead, product variety or product design (multiple factors allowed).
DIVERS	0-100	Share of turnover of most important product in 2004.
EXPORT	0/1	1 if the firm sold its products to international markets in the period 2002-2004.
RATING	c	Credit rating index of the firm in year 2003, ranging between 1 (highest) and 6(worst creditworthiness).
HIGHSKILLED	0-100	Share of employees with a university or college degree in 2003.
NOTRAIN	0/1	1 if the firm did not invest in training in 2004.
TRAINEXP	c	Training expenditure per employee (in log.) if NOTRAIN=0, otherwise 0.
MVTRAIN	0/1	1 if the information on training expenditure is missing in the data.
EAST	0/1	1 if the firm is located in East Germany.
GROUP	0/1	1 if the firm belongs to a group.

$0/1$ indicates a binary variable, c a continuous variable and $0-100$ describes a continuous variable with range of 0 to 100.

Table B.3
Robustness checks - Full Sample

	R&D								
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
THETA	+++					+++			
LAMBDA1		+++		+++			+++		+++
LAMBDA2			+++	+++				+++	+++
THREAT: low					0	0	0	0	0
THREAT: medium					0	0	0	0	0
THREAT: high					0	0	0	0	0
	PROCESS								
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
THETA	++					++			
LAMBDA1		+++		++			+++		++
LAMBDA2			+++	+++				+++	+++
THREAT: low					+	+	0	0	0
THREAT: medium					++	++	++	++	++
THREAT: high					+	+	0	0	0
	PRODUCT								
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
THETA	++					++			
LAMBDA1		+++		++			+++		++
LAMBDA2			+++	+++				+++	+++
THREAT: low					0	0	0	0	0
THREAT: medium					0	0	0	0	0
THREAT: high					0	0	0	0	0

+++ Significantly positive at 1%; ++ Significantly positive at 5%; + Significantly positive at 10%; 0 not significant. Specifications (1) to (5) include only demand uncertainty, technical uncertainty or entry threat, respectively. Specification (6) combines demand uncertainty and entry threat whereas specifications (7) to (9) include only technical uncertainty and entry threat. In each regression, the full set of control variables including industry dummies is used but not reported.

Table B.4
Robustness checks - Subsample

	R&D										
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
THETA	++						++				
LAMBDA1		+ + +		0				+ + +		0	
LAMBDA2			+ + +	+ + +					+ + +	+ + +	
LAMBDA3					+ + +						+ + +
THREAT: low						0	0	0	0	0	0
THREAT: medium						0	0	0	0	0	0
THREAT: high						0	0	0	0	0	0
	PROCESS										
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
THETA					0		0				
LAMBDA1	+ + +		0					+ + +		0	
LAMBDA2		+ + +	+ + +						+ + +	+ + +	
LAMBDA3				+ + +							+ + +
THREAT: low						0	0	0	0	0	0
THREAT: medium						0	0	0	0	0	0
THREAT: high						0	0	0	0	0	0
	PRODUCT										
Specification	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
THETA	0						0				
LAMBDA1		++		0				++		0	
LAMBDA2			+ + +	+ + +					+ + +	+ + +	
LAMBDA3					+ + +						+ + +
THREAT: low						0	0	0	0	0	0
THREAT: medium						0	0	0	0	0	0
THREAT: high						0	0	0	0	0	0

+ + + Significantly positive at 1%; ++ Significantly positive at 5%; + Significantly positive at 10%; 0 not significant. Specifications (1) to (6) include only demand uncertainty, technical uncertainty or entry threat, respectively. Specification (7) combines demand uncertainty and entry threat whereas specifications (8) to (11) include only technical uncertainty and entry threat. In each regression, the full set of control variables including industry dummies is used but not reported.