# The Patent Quality Control Process: Can We Afford An (Rationally) Ignorant Patent Office?

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**Abstract**

This paper considers patent granting as a two-tiered process, which consists of patent office examination and court challenges. It argues that, when the patent-holder has private information about the patent validity, (i) a weak patent is more likely to be settled and thus escape court challenges than a strong patent; and (ii) a tighter examination by the patent office may strengthen private scrutiny over a weak patent. Both work against Lemley (2001)'s hypothesis of a "rationally ignorant" patent office. The paper also considers application fees and a pre-grant challenge procedure, and shows that the former, used as a tool to deter opportunistic patenting, may crowd out private enforcement but cannot replace public enforcement; while the usefulness of the latter is subject to several restrictions, including the private challenger's timing choice.

**Keywords**: Case Selection, Patent Quality, Public and Private Enforcement of Law.
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# 1 Introduction

The patent granting process is often described as a two-tiered system: Besides the inspection by patent office examiners (the public enforcement tier), private parties can also challenge the validity of issued patents in court or at the patent office (the private enforcement tier).[1] Indeed, private challengers are usually thought to have significant advantages over the public agency. They have more knowledge about which patents cover valuable inventions, so the granted monopoly entails serious consequences; they also closely follow technological developments and have more information about where to locate those prior arts useful in making patent granting decisions. Reflecting upon this view, Lemley (2001) advocates a "rationally ignorant" patent office, and argues that instead of carefully scrutinizing every patent application at the patent office, it would be more efficient to lower the examination standard and issue some patents with questionable quality, while letting private parties select which patents to dispute in court. A glance at the United States Patent Reform Act of 2007 also reveals this emphasis on the private sector to eliminate weak patents.

In this paper, we argue that this "rational ignorance" hypothesis ignores both private players' strategic behavior and how public efforts would affect private enforcement. Despite the advantages, private parties frequently settle cases, leaving the contested patents in force. Among those unsettled cases, the disputed patents may be systematically biased toward certain characteristics. This "case selection," as we will show in this paper, constrains the effectiveness of private force and needs to be taken into account in order to induce proper cooperation between private and public sectors in the patent quality control process.

We consider a situation where, before launching a validity challenge, the settlement bargaining between the patent-holder and a potential challenger is clouded with asymmetric information. That is, the patent-holder has some private information about the validity of the disputed patent. We use a simple two-type model where the patent-holder has either a strong or a weak patent (section 2), and the challenger optimally chooses his litigation efforts if bargaining breaks down. Fixing the litigation effort, a strong patent, assumed to be possessed by a true inventor, is more likely to withstand challenges. By contrast, a weak patent is more likely to be invalidated in court because, as an opportunistic player, its owner tried to patent an already existing technology.

[1] To challenge at the patent office, a private party can request patent reexamination in the United States, and patent opposition in the European Patent Office. Both occur at the post-grant stage.

We show in section 3 that bargaining breakdown is more likely to happen and a challenge ensue when the dispute involves a strong patent, for the patent-holder will be "tougher" at the bargaining table. Private force, then, may be exerted toward the wrong target, and the true inventor may face a higher litigation risk than the opportunistic player.

Even when the weak patent can be eliminated by private challenges, it doesn't necessarily imply that we can rely on private force to such an extent that the patent office should reduce or maintain low examination standards. In section 4, we show that a greater effort at the patent office may *increase* the chance to eliminate the weak patent through court challenges. There may be a positive relationship between public and private enforcement. Together with the case selection pattern, these results cast doubts on the "rational ignorance" hypothesis and call for reforms to improve patent office performance. In a sense, we provide a *raison-d'etre* for the patent office, and refute the idea of abolishing patent office examination and move toward a patent registration system.[2]

In section 5, we introduce two additional policy tools: application fees and a pre-grant challenge procedure. We show that in the two-type case a fee that fully deters the opportunistic player from filing a patent application will crowd out private enforcement, but can't substitute for public enforcement. Concerning a pre-grant challenge system, we point out some of its limitations, including the reversal of case selection pattern and the challenger's choice of timing to initiate a challenge. Section 6 concludes the paper and discusses future research. All proofs are relegated to Appendix A; and Appendix B extends our main results to alternative settings, especially the one where the patent-holder has continuous types.

□ **Related literature:** In law and economics, case selection has been extensively studied under two prominent approaches, that of "divergent expectations" and "asymmetric information".[3] Meurer (1989) provides an application of the asymmetric infor-

[2] See Merges (1999).

[3] A seminal paper using divergent expectations is Priest and Klein (1984). For the asymmetric information paradigm, the theoretical literature has been fairly well developed in several directions. Besides the screening model, where the uninformed party makes the offer (Bebchuk, 1984), there are also studies of: one-sided asymmetric information with the informed party makes the offer (the signaling case); two-sided asymmetric information; and the dynamic multiple-offer bargaining situation, *etc.*. Spier (2005) is a recent review of the literature. On the other hand, most empirical studies use the divergent expectations. But there is no definite evidence supporting either paradigm. Waldfogel (1998) favors the divergent expectations story, while Froeb (1993) supports the asymmetric information approach.

mation paradigm to the patent context.[4] We follow the same approach on the ground that the low patent quality problem can be alleviated through discouraging applications on technologies already in the public domain, a complaint widely shared, among others, in the software industry. A natural modeling strategy is to consider a situation where the patent applicant, but not other parties, is aware of this gaming behavior, and public policy should address this opportunism.

In the patent literature, recent concerns about the patent quality have attracted reform proposals from different sources, such as the United States Federal Trade Commission (FTC 2003), National Academies of Science (2004), as well as numerous law and economics scholars.[5] These reform proposals cover almost all aspects of patent life, from filing of applications, prosecution at the patent office, post-grant challenges, to patent litigation, but often lack sufficient formal analysis. One reason, perhaps, is that relative to the optimal policy design in terms of patent length, scope, and other instruments, very few theoretical efforts have been devoted to patent examination, or more generally the implementation of the patent system. A paper by a law scholar, Kesan (2005), describes how "bad," or weak patents can be settled in a symmetric information environment with legal expenses. On the other hand, two works by economists, Langinier and Marcoul (2003) and Caillaud and Duchêne (2005), elaborate on the patent application strategy and its relationship to patent office examination.

Langinier and Marcoul (2003) considers the patent applicant's search and disclosure of information to the patent office, while the later performs a complementary search and examination upon receiving the applicant's disclosure report. Caillaud and Duchêne (2005) considers multiple firms' R&D and patent filing strategies when the patent office faces the overload problem, that is, when the examination effort upon each application is decreasing due to application volume. For those firms pursuing opportunistic patenting, i.e., seeking patent protection on existing technologies, their applications' survival rate depends on others' strategy, and so multiple equilibria exist: if few file patent applications, then a high level of patent office scrutiny is received by each application; but as more firms "jointly attack" the patent office, an application receives a lower level of examination and a higher survival rate, as a consequence of the resource constraint of the patent office (the overload problem). Different from these papers, we emphasize the "second eye", that is, the role of the private sector in the

[4]But there is no litigation effort choice in his model. As we shall see, this is a crucial element for our results.

[5]Interested readers are referred to the special issue of *Berkeley Technology Law Journal*, 2004, 19 (3).

patent examination process, and consider the interaction between public and private sectors in improving patent quality.[6]

## 2 Model

There are three players: An inventor $A$ (she) seeks patent protection for her invention, which, if an application is filed, is examined by the patent office ($P$) and possibly by a private challenger ($B$, he) in court to verify whether the invention fulfills the patentability requirements specified in the patent law.

Suppose that, under perfect examination, $A$'s application will be rejected with a probability $\theta$. For instance, the patent examination body (say, the patent office) has full access to all relevant information, and with probability $\theta$ a piece of patent-defeating prior art exists which proves that $A$'s invention doesn't satisfy one or several of the patentability requirements. This probability is referred to as the "invalidity" of the patent (when issued). For simplicity, consider a two-type case $\theta \in \{\underline{\theta}, \bar{\theta}\}$, with $0 < \underline{\theta} < \bar{\theta} \leq 1$ (the case of $\underline{\theta} = 0$ will be treated in an example). An inventor with low invalidity $\underline{\theta}$, or high validity, is said to be a "true" inventor, or the "good" type: She spends considerable resources in R&D activities and brings about technological breakthrough. By contrast, an inventor with high invalidity $\bar{\theta}$ is called the "bad," or "opportunistic" type: She exploits the public domain and tries to patent an "old" technology. We also refer to a patent with high validity $\underline{\theta}$ as a "strong" patent, and one with $\bar{\theta}$ as a "weak" patent. Assume that $\theta$ is the inventor's private information, and other parties hold common initial belief that $Pr(\underline{\theta}) = \alpha$. Define $\theta^0 \equiv \alpha\underline{\theta} + (1-\alpha)\bar{\theta}$ as the *ex ante* average invalidity.

A positive probability to deny the true inventor patent protection, $\underline{\theta} > 0$, may come from a "type II" error in the patent examination process. Patentability standards may be inappropriately interpreted such that, for instance, once an invention is realized, others may perceive it as easier to achieve than it actually was. This "hindersight" bias may render an invention "obvious" or lacking an "inventive step," and so patent protection is denied. Alternatively, the patent authority may grant the monopoly rights to a good inventor only with some probability in order to reduce the deadweight loss

[6]This paper, in a broad sense, is therefore related to another research field in law and economics, namely, the cooperation of private and public sectors in law enforcement. Shavell (1993) discusses the costs and benefits of private enforcement, and the resulting optimal incorporation of private enforcement in different legal fields. This paper illustrates case selection bias as another limitation of private enforcement.

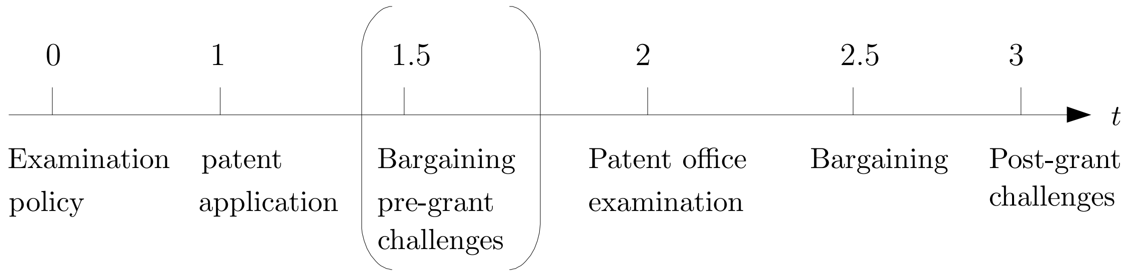


Figure 1: Timing

(Ayres and Klemperer, 1999).

We model patent examination as a "search and destroy" process: $P$ and $B$ can exert costly efforts $e_P$ and $e_B$, respectively, to search for the prior art, and the patent protection is denied if and only if the defeating prior art is found. Assume that, conditional on the existence of prior art, $P$'s and $B$'s search results are independent of each other. Given $\theta \in \{\underline{\theta}, \bar{\theta}\}$, the probability to eliminate $A$'s application by the patent office (the private challenger) is $\theta \cdot e_P$ ($\theta \cdot e_B$, respectively). The private party $B$'s search cost is $c(e_B)$, with $c(0) = c'(0) = 0$, $c(1) = c'(1) = \infty$, and $c'$ as well as $c'' > 0$. Later we will consider the patent office's cost, and assume that $P$ is less efficient than $B$. We call $e_P$ ($e_B$) public (private, respectively) enforcement efforts.

Concerning payoffs, regardless of her type, $A$ gets a monopoly profit $\pi > 0$ when receiving the patent protection, and $B$ gets a benefit $b \in (0, \pi)$ when the patent application is rejected. Otherwise the two receive no return. Except in section 5, private players are protected by limited liability. On the other hand, the patent office is concerned with the patent quality, which, in the two-type case, can be conveniently defined as the probability that the patent is issued to the true inventor. The patent office therefore aims to eliminate as much as possible the likelihood of granting patent rights to the opportunistic inventor, whether through private or public efforts.[7]

We first restrict the patent office's policy tool to examination efforts $e_P$. We then consider, separately, application fees and the possibility of mounting a private patent challenge at an alternative time, namely the pre-grant stage. We assume that the patent office can commit to its policy. FIGURE 1 illustrates the timing of the game: The patent office first announces its examination policy; and $A$ decides whether to file a patent application based on the policy. Under a post-grant challenge system, a

[7]For most part of the analysis, we ignore the impact of patent examination on the true inventor's returns from using the patent system and so her R&D incentives. See the concluding remark in section 4.

patent application first undergoes the patent office examination, and, upon issuance, encounters a private challenge by $B$. But the two parties bargain to settle the case before the court fight. On the other hand, under a pre-grant challenge the private enforcement and bargaining take place before the patent office examination. We assume that, when bargaining, $A$ makes a take-it-or-leave-it offer to $B$. (In Appendix B, we show that our main results are robust to the alternative distribution of bargaining power, i.e., when $B$ makes the offer, and a more general setting where $A$ has continuous types.)

# 3 The Limit of Private Enforcement

In this section we demonstrate that under a post-grant challenge system, a case involving a weak patent ($\bar{\theta}$) is more likely to be settled than that involving a strong patent ($\underline{\theta}$). This pattern of case selection points out the limit of private enforcement, and is key to subsequent analysis.

Suppose that $B$'s litigation effort $e_B$ is not contractible and so cannot be part of the settlement agreement.[8] A settlement offer is a transfer between $A$ and $B$. Let $\hat{\alpha} \in (0, 1)$ be the belief that $B$ faces a good inventor at the beginning of the bargaining subgame. This probability is affected by the patent office examination effort $e_P$ and can be seen as the quality of an issued patent. Define $\hat{\theta} \equiv \hat{\alpha}\underline{\theta} + (1 - \hat{\alpha})\bar{\theta}$ and the following terms: with $\theta \in \{\underline{\theta}, \bar{\theta}\}$,

$$e_B^*(\hat{\theta}) \equiv \arg\max_{e_B} \ \hat{\theta} e_B b - c(e_B),$$
$$u_A(\theta, e_B^*) = (1 - \theta e_B^*)\pi, \quad \text{and} \quad u_B(\hat{\theta}) = \hat{\theta} e_B^* b - c(e_B^*).$$

$e_B^*$ is $B$'s optimal litigation effort, and $u_A$ and $u_B$ are $A$'s and $B$'s expected payoffs in litigation, respectively. The optimal litigation effort is increasing in $\hat{\theta}$, and so decreasing in $\hat{\alpha}$. A lower probability to find the information and strike down the patent discourages $B$'s search activity. On the other hand, when engaging in a legal fight, $A$ always prefers a less intensive attack from $B$, i.e., a lower $e_B^*$, while $B$'s payoff is increasing in the probability of facing a weak patent $\hat{\theta}$.

Denote $\underline{e}_B \equiv e_B(\underline{\theta})$ and $\bar{e}_B \equiv e_B(\bar{\theta})$, and so $e_B^* \in [\underline{e}_B, \bar{e}_B]$. Note that $\underline{e}_B > 0$ for $\underline{\theta} > 0$. It is easy to check that $u_A(\underline{\theta}, e_B) > u_A(\bar{\theta}, e_B)$, $\forall e_B \in [\underline{e}_B, \bar{e}_B]$, and $u_A(\theta, e_B^*)$

[8] This effort may not be observable. Even if observable, the court may not enforce an agreed effort level to be exerted in litigation.

is increasing in $\hat{\alpha}$. That is, given the same private litigation effort, the true inventor's expected payoff from litigation is strictly higher than that of the opportunistic player; and through its effect on $e_B^*$ via $\hat{\theta}$, an inventor's litigation payoff is increasing in the belief $\hat{\alpha}$. Also note that by $b < \pi$, the case is always settled under symmetric information: $\pi - u_B(\underline{\theta}) > u_A(\underline{\theta}, \underline{e}_B)$ and $\pi - u_B(\bar{\theta}) > u_A(\bar{\theta}, \bar{e}_B)$.

Proposition 1. *(Case selection) After patent issuance, whether A or B makes a take-it-or-leave-it offer, there is no bargaining equilibrium in which only the true inventor settles.*

This result is fairly general and well-established in the literature of law and economics, regardless of the distribution of bargaining power. Intuitively, when one party holds private information about her case quality ($\theta$ here), a stronger case (lower $\theta$) makes a "tougher" player on the bargaining table, and so a settlement deal is harder to reach.

We now consider when private enforcement can be mounted against a weak patent. The weak patent is said to be fully (partially) exposed to private enforcement if the opportunistic $A$ engages in litigation for sure (with a probability, respectively). By Proposition 1, whenever the opportunistic $A$ litigates, so does the good $A$.

Proposition 2. *(Private enforcement) Suppose that A makes the settlement offer. The weak patent is subject to private enforcement when $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$. Suppose this is true.*

- *(Full exposure) When $u_A(\bar{\theta}, e_B^*(\hat{\theta})) \geq \pi - u_B(\bar{\theta})$, there is a Perfect Bayesian Equilibrium (henceforth, PBE) in which no settlement is reached at all, and B exerts litigation effort $e_B^*(\hat{\theta})$; and*
- *(partial exposure) if $u_A(\bar{\theta}, e_B^*(\hat{\theta})) < \pi - u_B(\bar{\theta}) < u_A(\bar{\theta}, \underline{e}_B)$, there is a PBE in which the opportunistic A litigates with probability $x^* \in (0,1)$, the good A always litigates, and B, with a belief $\alpha_x^*$ upon litigation, exerts an litigation effort $e_{B,x}^* < e_B^*(\hat{\theta})$, where $e_{B,x}^*$, $x^*$, and $\alpha_x^*$ are determined by*

$$u_A(\bar{\theta}, e_{B,x}^*) = \pi - u_B(\bar{\theta}), \ e_{B,x}^* = e_B^*\big(\alpha_x^*\underline{\theta} + (1-\alpha_x^*)\bar{\theta}\big), \text{ and } \alpha_x^* = \frac{\hat{\alpha}}{\hat{\alpha} + (1-\hat{\alpha})x^*}. \quad (1)$$

By this proposition, private enforcement can possibly eliminate the weak patent only when $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$. To understand this condition, note that $u_A(\bar{\theta}, \underline{e}_B)$ and $u_B(\bar{\theta})$ are the opportunistic $A$'s and $B$'s highest possible payoff in litigation, respectively, and so offering these amounts to corresponding players guarantees acceptance.

Suppose that $u_A(\bar{\theta}, \underline{e}_B) \leq \pi - u_B(\bar{\theta})$. When $A$ makes the offer, the opportunistic $A$'s highest possible litigation payoff is smaller than the lowest possible payoff from settlement, which is obtained by offering $B$'s highest litigation payoff to ensure settlement. She then has every incentive to settle.[9] In this case, private force is either exerted toward the wrong target (the strong patent), or simply not active; and patent quality cannot be improved by private enforcement.

COROLLARY 1. *When $u_A(\bar{\theta}, \underline{e}_B) \leq \pi - u_B(\bar{\theta})$, private enforcement doesn't improve the patent quality. It reduces the quality of issued patents when only the good patent-holder engages in litigation.*

REMARK 1. (EQUILIBRIUM REFINEMENT) In the proof we show that these equilibria survive the criterion $D1$ (Cho and Kreps, 1987). This criterion constrains the weight $B$ can put on the opportunistic type upon the off-path event of litigation. Roughly speaking, the good $A$ would have more to gain than the opportunistic $A$ in a legal fight, and so $D1$ requires the opportunistic $A$ be fully deleted from $B$'s off-path belief.

In the proof of PROPOSITION 2, we also consider other bargaining outcomes such as where both types of $A$ settle and there is no litigation, and where only the good $A$ litigates. However, no $PBE$ exists that fulfills the criterion $D1$ and implements the two outcomes.[10] ■

REMARK 2. ("HARASSING" THE TRUE INVENTOR) The case selection pattern also implies a higher litigation risk for the true inventor, which may translate into a higher probability to lose the patent protection. This happens when $B$ litigates only against the good $A$, while settling the case with the opportunistic $A$.[11] In other words, a true inventor may be "harassed" when trying to enforce her patent rights.[12] Private

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[9] In APPENDIX B we show that the same condition applies when $B$ makes the offer.

[10] "Divinity," though, retains these bargaining outcomes (Bank and Sobel, 1987). It is a weaker than $D1$ and only requires that $B$ believe the good $A$ plays the deviant move at least as often as the opportunistic $A$. The "passive belief," for example, is allowed under divinity but not under $D1$.

[11] When $u_A(\underline{\theta}, \underline{e}_B) \geq \pi - u_B(\bar{\theta}) \geq u_A(\bar{\theta}, \underline{e}_B)$, there is a $PBE$ where the good $A$ litigates for sure and the opportunistic $A$ settles for sure, with litigation efforts $\underline{e}_B$ (see proof of PROPOSITION 2). In this equilibrium, the probability that the opportunistic $A$ and good $A$ receive patent rights are $1-\bar{\theta}e_P$ and $(1-\underline{\theta}e_P)(1-\underline{\theta}\underline{e}_B)$, respectively. The opportunistic $A$ has higher probability to survive and receive patent protection than the good $A$ if

$$1 - \bar{\theta}e_P > (1 - \underline{\theta}e_P)(1 - \underline{\theta}\underline{e}_B) \Rightarrow \underline{\theta}\underline{e}_B(1 - \underline{\theta}e_P) > e_P(\bar{\theta} - \underline{\theta}).$$

It is more likely to be the case when $e_P$ is small.

[12] The harassment hypothesis usually refers to invalidation challenges facing a patent-holder from potential infringers or other stake-holders. One possible litigation shown in our model is exactly this invalidation suit.

enforcement, then, may reduce the true inventor's payoff and impair R&D incentives without offsetting gains to raise the patent quality. ■

Before proceeding to the relationship between public and private enforcement, let us consider two special cases of private bargaining.

Example 1. (An ironclad good patent) When the good patent can never be invalidated, $\underline{\theta} = 0$, the opportunistic $A$ can still be subject to private litigation. This is confirmed by that fact that, under this case, $u_A(\bar{\theta}, \underline{e}_B) = \pi > \pi - u_B(\bar{\theta})$.

However, without invalidation risk the true inventor will never pay $B$ to settle the case, there is no equilibrium in which private bargaining always reaches a deal, whoever makes the offer. Another equilibrium outcome ruled out by this assumption is one in which $B$ learns $A$'s true type and settles with the opportunistic player while litigating against the true inventor. By $\underline{\theta} = 0$ and so $\underline{e}_B = 0$, this equilibrium is busted by the opportunistic $A$'s attempt to mimic the good type (and engage in a "legal fight" with no litigation efforts from $B$). ■

Example 2. (Inelastic private enforcement capacity) Suppose that $\underline{\theta} > 0$ but $B$ has inelastic litigation capacity. For simplicity, let us consider the extreme case of fixed and costless $e_B > 0$.[13] After this modification, the weak patent is entirely exempted from private enforcement. A fixed $e_B$ renders $u_B(\bar{\theta}) = \bar{\theta} e_B b < \pi - u_A(\bar{\theta}, e_B) = \bar{\theta} e_B \pi$, which violates the necessary condition $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$.[14] This confirms that $B$'s litigation effort decision is a key ingredient in our analysis. ■

# 4 Public vs. Private Enforcement

The results we obtain in the previous analysis cast doubts on Lemley (2001)'s hypothesis of a "rationally ignorant patent office." Since private force cannot only be directed toward the "right target," that is, the weak patent, provoking private litigation at best improves the patent quality at the expense of the true inventor, who suffers from burdensome litigation and lower return from R&D.

Even if concerns about innovation incentives are not present, a closer look at Proposition 2 shows that a proposal to reduce or to maintain a low level of patent office examination may be detrimental to the overall patent quality control standard.

[13] With costly but fixed effort, we need only that $B$ has a credible threat to incur the cost in a legal fight, e.g., by assuming a cost smaller than $\underline{\theta} e_B b$.

[14] Introducing litigation cost only strengthens this result.

This section develops the relationship between public and private enforcement, through the former's effect on the patent quality $\hat{\alpha}$. For simplicity, suppose that the good inventor's R&D incentives are not too much damaged during the patent-granting process. For instance, we may assume that $\underline{\theta} > 0$ but low enough so that even if $e_P = e_B = 1$, the expected return from patenting, $(1-\underline{\theta})^2\pi$, covers her R&D expenditure.

Recall that $\theta^0 \equiv \alpha\underline{\theta} + (1-\alpha)\bar{\theta}$. When the patent office exerts an examination effort $e_P \geq 0$, the quality of an issued patent is

$$\begin{aligned} \hat{\alpha} &= \frac{\alpha(1-\underline{\theta}e_P)}{\alpha(1-\underline{\theta}e_P) + (1-\alpha)(1-\bar{\theta}e_P)} = \frac{\alpha(1-\underline{\theta}e_P)}{1-\theta^0 e_P} \\ \Rightarrow \qquad \frac{\partial\hat{\alpha}}{\partial e_P} &= \frac{\alpha(1-\alpha)(\bar{\theta}-\underline{\theta})}{\{1-\theta^0 e_P\}^2} > 0. \end{aligned} \tag{2}$$

A higher level of public enforcement raises the patent quality. Next, suppose that $\pi - u_B(\bar{\theta}) < u_A(\bar{\theta}, \underline{e}_B)$ and so the weak patent can be subject to private enforcement. We consider the full and partial exposure regime in turn, i.e., whether the opportunistic patent-holder litigates with probability equal to or less than one.

The full exposure regime requires patent quality $\hat{\alpha}$ be high enough, so that $\hat{\theta}$ and litigation effort $e_B^*$ low enough: $u_A(\bar{\theta}, e_B^*(\hat{\theta})) \geq \pi - u_B(\bar{\theta})$.[15] Intuitively, the opportunistic inventor is willing to mix with the good inventor and litigate only when she expects to encounter a low litigation effort. This is more likely to be the case when patent office exerts great examination effort $e_P$ and maintains high patent quality $\hat{\alpha}$. In addition, in this regime a marginal increase in public enforcement $e_P$ will *reduce* private enforcement effort $e_B$, for a higher patent quality $\hat{\alpha}$ weakens $B$'s search intensity. In other words, in this regime public enforcement crowds out private enforcement.

The partial exposure regime, on the other hand, happens for low $\hat{\alpha}$.[16] This regime exhibits an interesting relationship between public and private enforcement. By PROPOSITION 2 the opportunistic $A$'s litigation probability $x^* = [\hat{\alpha}(1-\alpha_x^*)]/[(1-\hat{\alpha})\alpha_x^*]$ is *increasing* in $\hat{\alpha}$. Together with the fact that the belief $\alpha_x^*$ and litigation effort $e_{B,x}^*$ are fixed in this case, the probability that the weak patent will be eliminated by private force, $x^* \cdot e_{B,x}^*$, is also *increasing* in $e_P$. Different from the full exposure regime, here public enforcement *crowds in* private enforcement.[17]

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[15] If $B$ makes the offer, by contrast, full expose happens only when $\hat{\alpha}$ is small enough (and $A$ accepts the offer upon indifference). Nevertheless, this is only the qualitative difference between the two distributions of bargaining power. See APPENDIX B.

[16] This is also true when $B$ makes the offer.

[17] The same holds true when $B$ makes the offer, provided that $\hat{\alpha}$ is low enough and $B$'s cost function is well-behaved.

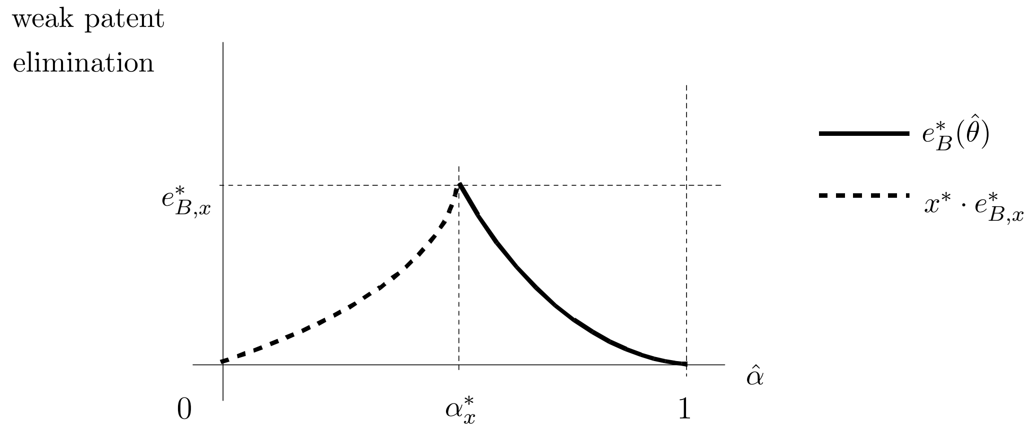


Figure 2: Patent quality and private enforcement

The reason is, referring to condition (1), under partial exposure the litigation effort $e^*_{B,x}$ is determined such that the opportunistic $A$ is indifferent between paying $u_B(\bar{\theta})$ to settle the case and facing a challenge with effort $e^*_{B,x}$. On the other hand, to have $e^*_{B,x}$ as the best response, $B$ should have a belief $\alpha^*_x$ when filing a challenge. And since a higher $e_P$ will raise the quality of an issued patent $\hat{\alpha}$, the opportunistic $A$ should litigate more (raise $x^*$) in order to fix $B$'s belief at $\alpha^*_x$.

PROPOSITION 3. *(Public and private enforcement) Assume $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$ so that the weak patent may be subject to private enforcement.*

- *(Full exposure) When $\hat{\alpha} \geq \alpha^*_x$, the weak patent is litigated for sure, and an higher level of public enforcement $e_P$ crowds out private litigation efforts $e^*_B(\hat{\theta})$.*
- *(Partial exposure) When $\hat{\alpha} < \alpha^*_x$, the weak patent is litigated with probability $x^*$, and the probability to eliminate a weak patent through private effort, $x^* \cdot e^*_{B,x}$ is increasing in $e_P$, even though $B$'s litigation efforts $e^*_{B,x}$ is not affected by $e_P$.*

FIGURE 2 summarizes the impact of patent quality $\hat{\alpha}$ on "weak patent elimination," which is defined as the probability that the weak patent will be eliminated in litigation. (Since $\hat{\alpha}$ is strictly increasing in $e_P$, it also depicts the effect of public enforcement on private enforcement.) When the patent quality increases, we move from the partial exposure (the dashed line) to the full exposure regime (the solid line). A marginal increase in the patent quality raises the probability of eliminating the weak patent in the former case, but not in the latter case. There is a non-monotonic relationship between weak patent elimination and the patent quality.

Notice the policy implication. A positive relationship between public enforcement and weak patent elimination occurs precisely under low patent quality. As previously discussed in the introduction, the current debate about patent quality is centered on the complaint that the patent office has issued too many unwarranted patents. To address this concern, we may want to improve the performance of the patent office not only to directly raise the patent quality, but also to enhance the involvement of private force in the quality control process.

□ **When to reduce public enforcement?** One might wonder, given a negative relationship between public and private enforcement at the full exposure regime, when the private challenger enjoys a cost advantage over the public agency, we should constrain the patent office examination and let a patent be scrutinized later through private litigation efforts. In other words, Lemley's "rational ignorance" hypothesis might be vindicated in this case.

To check this possibility, we assume that the patent office has a cost function $\gamma c(e_P)$, where $\gamma \geq 1$ and $c(\cdot)$ is $B$'s cost function. Define the total cost of patent examination as $C(e_P) \equiv \gamma c(e_P) + (1-\theta^0 e_P)c(e_B^*(\hat{\theta}))$. Also define the level of examination a patent application is expected to receive as $e_P + e_B$, for under this regime, a patent applicant with $\theta$ expects rejection with probability $1-(1-\theta e_P)(1-\theta e_B) = \theta(e_P+e_B) - \theta^2 e_P e_B \simeq \theta(e_P+e_B)$. We show when a marginal reduction in $e_P$ will reduce the total cost without deteriorating the examination standard.

A marginal change in $e_P$ causes a change in examination standards by

$$\frac{d[e_P + e_B^*(\hat{\theta})]}{de_P} = 1 + \frac{de_B^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P} = 1 - \frac{(\bar{\theta}-\underline{\theta})b}{c''(e_B)}\frac{\alpha(1-\alpha)(\bar{\theta}-\underline{\theta})}{(1-\theta^0 e_P)^2},$$

and a change in the total cost by

$$\frac{dC(e_P)}{de_P} = \gamma c'(e_P) - \theta^0 c(e_B^*(\hat{\theta})) + (1-\theta^0 e_P)c'(e_B^*(\hat{\theta}))\frac{de_B^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P}.$$

The following result is obtained from these two expressions in a straightforward manner.

Proposition 4. *(A rationally ignorant patent office under the full exposure regime) Under the full exposure regime, a marginal decrease in $e_P$ does not weaken the overall examination standard if and only if*

$$\frac{de_B^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P} \leq -1 \quad \Rightarrow \quad \frac{\alpha(1-\alpha)(\bar{\theta}-\underline{\theta})^2 b}{c''(e_B)(1-\theta^0 e_P)^2} \geq 1, \tag{3}$$

*and reduces the total examination cost if and only if*

$$\gamma > \frac{1}{c'(e_P)}\left[\theta^0 c(e_B^*(\hat{\theta})) - (1-\theta^0 e_P)c'(e_B^*(\hat{\theta}))\frac{de_B^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P}\right]. \tag{4}$$

*The rational ignorance hypothesis is supported when both conditions hold.*

Not surprisingly, the private sector's cost advantage $\gamma$ should be large enough to justify a not-so-excellent patent office. In the proof of this proposition we also obtain a sufficient condition for condition (3) to hold: $\forall e_B$, $\alpha(1-\alpha)(\bar{\theta}-\underline{\theta})^2 b \geq c''(e_B)$. This stems from the fact that the private sector's response should be large enough in order to compensate for a more lax public quality control. Among others, this requires a "less curved" cost function, i.e., $c''$ small enough, as $\partial e_B^*(\hat{\theta})/\partial\hat{\theta} = b/c''(e_B^*)$.

Remark. (R&D incentives) So far we've ignored the true inventor's R&D incentives. If this concern is introduced, to restrain the magnitude of type II error the patent office may want to constrain its examination effort $e_P$. However, previous analysis shows that a marginal reduction in $e_P$ may not always decrease the overall examination effort. This is indeed true in the partial exposure regime. In the full exposure regime, a reduction in $e_P$ causes $e_B$ to increase, and the general enforcement level decreases if and only if condition (3) fails. ■

# 5 Other Policy Choices

In this section we first erase the limited liability protection and allow negative returns for an inventor. This allows us to introduce applications fees as an additional policy tool. We then turn to an alternative timing to exert private efforts, i.e., a pre-grant challenge system.

□ **Application fees:** When the patent office can charge application fees, this may deter, ideally, the opportunistic inventor from seeking patent protection. In general, to achieve this goal, a more effective way is to condition the pecuniary punishment on the examination outcome, e.g., upon the rejection of a patent application or invalidation of an issued patent in court. However, a fine after invalidation is arguably under the discretion of the court, and an applicant, especially a "short-run player," might simply run away when her application is rejected by the patent office. Instead, we consider a uniform application fee $f$ for all patent applications. Nevertheless, our main result is robust to the exact shape of the pecuniary mechanism.

Suppose that an application fee $f$ fully deters the opportunistic inventor from applying for a patent, but not the true inventor. When this is true, at the bargaining stage $B$ holds belief that $\hat{\alpha} = 1$, and symmetric information prevents bargaining breakdown. In this two-type case, a fully deterrent application fee mutes entirely private enforcement. When $A$ holds the bargaining power,[18] it suffices to pay $u_B(\underline{\theta})$ to settle the case, and a deterrent fee $f$ should satisfy

$$(1 - \bar{\theta} e_P)\pi - u_B(\underline{\theta}) < f \leq (1 - \underline{\theta} e_P)\pi - u_B(\underline{\theta}).$$

Since this condition will not hold $e_P = 0$, a deterrent application fee cannot substitute for patent office examination. Furthermore, to preserve the good inventor's R&D incentives, the patent office should set $f$ as small as possible, without losing its deterrent power. Let $f^D = (1 - \bar{\theta} e_P)\pi - u_B(\underline{\theta}) + \epsilon$, with $\epsilon > 0$ but small. Since $f^D$ is decreasing in $e_P$, the good inventor's payoff, $(1 - \underline{\theta} e_P)\pi - u_B(\underline{\theta}) - f^D = (\bar{\theta} - \underline{\theta}) e_P \pi - \epsilon$, s increasing in $e_P$:

Proposition 5. *(Application fees) In the two-type case, an application fee that fully deters opportunistic patenting crowds out private enforcement but cannot substitute for public enforcement. A higher patent office examination level $e_P$ reduces the necessary fee. And when the application fee is set at the minimal necessary level $f^D$, the good inventor's payoff, and so the R&D incentive, is increasing in $e_P$.*

□ **Pre-grant challenges:** Lastly, let us consider a pre-grant challenge system. Suppose that after receiving a patent application but before starting its examination process (time 1.5 in Figure 1), the patent office publishes the application and allows third parties to challenge it (or submits information concerning its patentability).[19]

This alternative timing of the challenge allows the patent office to set different examination levels according to an application's history. Let $e_P^c$ be the examination effort exerted on an application that has survived private challenges, and $e_P^n$ on that which has not yet been challenged. Intuitively, the patent office should set $e_P^c \leq e_P^n$. In addition to the reason that private enforcement efforts perform as a "certificate"

[18]The distribution of bargaining power is not crucial to this result. It only changes the level of $f$ to deter opportunistic patenting, for the patent-holder's payoffs from fully settling the case depend on who makes the offer.

[19]Early publication of patent applications (18 months after filing) has been widely adopted in Japan and Europe; the U.S. has the same procedure but allows an applicant to opt out. About the pre-grant challenge, the 2007 Patent Reform Act in the U.S. introduces a procedure permitting third parties to submit relevant information before the issuance of a patent.

about the validity of an application, case selection (Proposition 1) provides further support of such a policy, because a weak patent (application) is less likely to receive private scrutiny.

However, under such a policy, an applicant may try to circumvent the high effort $e_P^n$ by arranging a "fake" challenge, in particular when the patent office is unable to verify the challenger's effort level, that is, whether the challenger only initiates a nominal challenge procedure without any serious effort to strike down the application. Besides, we show that (i) the "direction" of case selection may be reversed at the pre-grant challenge stage. That is, contrary to the previous result, there may exist an equilibrium where only the true inventor settles at the pre-challenge bargaining; and (ii) when $B$ does intend to initiate a challenge, and both pre- and post-grant challenges are available, he may want to wait and file a private challenge only after the failure of the patent office.[20]

For the first point, suppose that $B$ can only initiate a challenge at the pre-grant stage, and that $A$'s settlement payment comes from the monopoly rent and so is paid only when the patent is issued. (This is the case when $A$ is protected by limited liability.) Recall that $B$ cannot commit to $e_B$ in an agreement, and his initial belief of patent (application) quality is $\alpha$. We derive conditions under which there is a separating equilibrium where only the good inventor settles. A necessary condition is both $\underline{\theta}$ and $e_P^c > 0$. The former is simply due to the fact that a true inventor with $\theta = 0$ will never pay anything to settle. The latter can be justified in that the patent office doesn't "outsource" the examination task entirely to private parties.[21] Even if an application survives private challenges, the patent office still does its own work.

Intuitively, when the patent office sets different examination levels according to the challenge history, $A$ will take this into account when making settlement decisions. Consider if $e_P^n >> e_P^c$, that is, if an unchallenged application will receive a more detailed examination than an application surviving private challenges. This gives an applicant incentives not to settle with a private challenger in order to avoid stringent public scrutiny. But the magnitude of this effect depends on the true quality of the invention $\theta$. For instance, when $\underline{\theta}$ is very close to zero, even $e_P^n \simeq 1$ won't harm the true inventor too much. The case selection pattern at the pre-grant challenge stage may

[20] Of course, this is more likely the case when costs accrued to challengers are not so different for the post- and pre-grant challenge procedures.

[21] Or, equivalently, the patent office doesn't "rubber stamp" the issuance of a patent following private efforts.

be reversed. That is, only the good $A$ settles while the opportunistic $A$ experiences a private challenge. The following proposition confirms this possibility.

Proposition 6. *(Pre-grant challenges and reverse case selection) Suppose that $B$ can only file a challenge at the pre-grant stage. There is a PBE where only the opportunistic $A$ is challenged when*

$$\frac{(1-\bar{\theta}\bar{e}_B)(1-\bar{\theta}e_P^c)}{(1-\bar{\theta}e_P^n)} \geq \frac{\pi-\underline{s}}{\pi} \geq \frac{(1-\underline{\theta}\bar{e}_B)(1-\underline{\theta}e_P^c)}{(1-\underline{\theta}e_P^n)}, \tag{5}$$

*where* $\underline{s} = [u_B(\underline{\theta}) + (1-\underline{\theta}\underline{e}_B)\underline{\theta}e_P^c b]/(1-\underline{\theta}e_P^n)$.

First note that condition (5) won't hold when $e_P^c = 0$. In this case, a necessary condition of this equilibrium,

$$\frac{(1-\bar{\theta}\bar{e}_B)(1-\bar{\theta}e_P^c)}{(1-\bar{\theta}e_P^n)} \geq \frac{(1-\underline{\theta}\bar{e}_B)(1-\underline{\theta}e_P^c)}{(1-\underline{\theta}e_P^n)},$$

reduces to $e_P^n \geq \bar{e}_B$, contradictory with

$$\frac{1-\underline{\theta}\bar{e}_B}{1-\underline{\theta}e_P^n} \leq \frac{\pi-\underline{s}}{\pi} < 1.$$

In order to consider when it's more likely to have this equilibrium, let us fix $\bar{e}_B$, $\underline{\theta}$, and $e_P^c$ at strictly positive levels, but less than one. Suppose that $\underline{s}$ is small enough (due to, say, a small $b$) so that

$$\frac{\pi-\underline{s}}{\pi} \geq (1-\underline{\theta}\bar{e}_B)\frac{1-\underline{\theta}e_P^c}{1-\underline{\theta}} \geq (1-\underline{\theta}\bar{e}_B)\frac{1-\underline{\theta}e_P^c}{1-\underline{\theta}e_P^n}.$$

That is, the second inequality in condition (5) holds for all $e_P^n$. In this case, the separating equilibrium exists as long as

$$(1-\bar{\theta}\bar{e}_B)\frac{1-\bar{\theta}e_P^c}{1-\bar{\theta}e_P^n} \geq 1 \Rightarrow \frac{1-\bar{\theta}e_P^c}{1-\bar{\theta}e_P^n} \geq \frac{1}{1-\bar{\theta}\bar{e}_B}.$$

For all possible $\bar{\theta}$, it is more likely to hold as $e_P^n$ grows larger. In the extreme case of $\bar{\theta} = 1$, this condition is guaranteed when $e_P^n$ is large enough. This equilibrium exists exactly when the weak patent is of the worst kind, and the patent office exerts maximal efforts to eliminate it with the information provided by case selection!

Remark. (Can sequential private challenges reverse the pattern?) One might suspect that this reverse pattern of case selection is generated by sequential efforts to eliminate patent applications, and could happen as well under post-grant challenges and multiple potential challengers.

For simplicity, suppose there are two potential challengers $B_1$ and $B_2$, with identical cost $c(\cdot)$ and benefit $b$ . If $A$'s bargaining with $B_1$ results in the litigation of opportunistic $A$ and settlement of good $A$, then $B_1$ exerts litigation efforts $\bar{e}_B$. Denote the good $A$'s settlement offer as $s$. This separating equilibrium fully reveals $A$'s type, and so, knowing the litigation history, there will be no litigation between $B_2$ and $A$ (when the opportunistic $A$ survives $B_1$'s challenge). $B_2$ will settle with the good (opportunistic) $A$ with a payment $u_B(\underline{\theta})$ ($u_B(\bar{\theta})$, respectively). Since

$$\pi - s - u_B(\underline{\theta}) \geq (1 - \underline{\theta}\bar{e}_B)\pi - u_B(\bar{\theta}) > (1 - \bar{\theta}\bar{e}_B)\pi - u_B(\bar{\theta}),$$

the opportunistic $A$ will deviate to mimic the good $A$. The reverse pattern of case selection will not happen under sequential private challenges. ■

Now, consider a potential challenger's timing choice. Suppose that both pre- and post-grant challenges are available to $B$, but there is only one challenge opportunity. In the absence of a settlement agreement, with belief $\alpha$ and corresponding $\theta^0$,[22] $B$'s payoff from initiating a pre-grant challenge is $u_B(\theta^0)+[1-\theta^0 e_B^*(\theta^0)]e_P^c\theta^0 b$. If $B$ waits after the patent issuance, his expected payoff is $\theta^0 e_P^n b+(1-\theta^0 e_P^n)u_B(\hat{\theta})$, where $\hat{\theta} = \hat{\alpha}\underline{\theta}+(1-\hat{\alpha})\bar{\theta}$ and $\hat{\alpha}$ is determined according to condition (2), with $e_P = e_P^n$. Since $\hat{\alpha} > \alpha$ for all $e_P^n > 0$, $\hat{\theta} < \theta^0$, $e_B^*(\theta^0) > e_B^*(\hat{\theta})$, and $c(e_B^*(\theta^0)) > c(e_B^*(\hat{\theta}))$. We should expect more intensive private challenge efforts at the pre-grant stage than at the post-grant stage.

Since

$$\begin{aligned} u_B(\theta^0) + [1 - \theta^0 e_B^*(\theta^0)]e_P^c\theta^0 b &< \theta^0 e_B^*(\theta^0)b - (1 - \theta^0 e_P^n)c(e_B^*(\theta^0)) + [1 - \theta^0 e_B^*(\theta^0)]e_P^c\theta^0 b \\ &= -(1 - \theta^0 e_P^n)c(e_B^*(\theta^0)) + b\left[\theta^0 e_B^*(\theta^0) + (1 - \theta^0 e_B^*(\theta^0))\theta^0 e_P^c\right], \end{aligned}$$

and

$$\theta^0 e_P^n b + (1 - \theta^0 e_P^n)u_B(\hat{\theta}) = -(1 - \theta^0 e_P^n)c(e_B^*(\hat{\theta})) + b\left[\theta^0 e_P^n + (1 - \theta^0 e_P^n)\hat{\theta}e_B^*(\hat{\theta})\right],$$

a sufficient condition for $B$ to choose the post-grant procedure is

$$e_P^n - e_P^c > e_B^*(\theta^0)(1 - \theta^0 e_P^c). \tag{6}$$

It is more likely as $e_P^n$ gets larger and $e_P^c$ gets smaller. That is, $B$ will postpone and free ride on public efforts if the patent office targets and exert much higher efforts towards those applications not being protested by private players.

[22] This $\alpha$ may be the initial belief when there is no bargaining at all between $A$ and $B$, or the belief after the breakdown of a settlement negotiation.

Proposition 7. *(Choice of challenge timing) When condition (6) holds, a potential challenger prefers to challenge at the post-grant stage.*

# 6 Concluding Remarks

The limitation of private enforcement emphasized in this paper, namely the settlement bias toward weak patents, would persist despite the private challenger's information and cost advantages. These results highlight the importance of a patent office. Accordingly, future theoretical works and reform efforts should figure out how to improve the performance of the patent office in order to "get things right" in the first place. The agency problem and task allocations within the patent office are additional topics in our research agenda.[23]

In this aspect, our analysis sheds some lights on the design of incentive payments for patent examiners. One difficulty in the construction of such an incentive scheme is to find a proper index of examiners' efforts. A straightforward and somewhat "naive" application of incentive theory might suggest the use of court rulings as a measure of performance. A patent examiner would be punished if a patent issued by her is later invalidated in court. Several practical issues reduce the usefulness of this measure: the rare occurrence of patent disputes and the strong tendency toward settlement; upon dispute, the long delay from patent issuance to the final court judgment; and, at least in the United States, a significant portion of patent examiners who choose a career path in the private sector after a few years' experience in the patent office. Our analysis points out another restriction: the information content of a court ruling may be distorted by private bargaining. For instance, a positive relationship between public and private enforcement in the partial exposure regime suggests that a higher effort by the patent examiner may result in more patents being litigated and invalidated in court. It would then be undesirable to punish the examiner upon a successful post-grant court challenge.

# Appendix

(To be revised)

[23]Merges (1999) argues that the U.S. patent examiners are given incentives to approve, but not reject patent applications.

# A Proofs

### □ Proposition 1

*Proof.* Consider an equilibrium in which the good inventor settles (with some probability) but the opportunistic inventor always litigates. Let $s'$ be (one of) the good inventor's equilibrium settlement payment(s), which may be adopted for some probability, and $e'_B > 0$ be (one of) the litigation efforts facing the opportunistic inventor. When the good inventor prefers settlement and paying $s'$ than litigation against an effort $e'_B$, $\pi - s' \geq u_A(\underline{\theta}, e'_B) > u_A(\bar{\theta}, e'_B)$, the opportunistic inventor has incentives to deviate to $s'$ and settle. *Q.E.D.*

LEMMA 1. *(Off-path belief selection and full settlement) Consider a $PBE$ where no litigation occurs, and denote $s$ as the equilibrium settlement payment from $A$ to $B$. If this equilibrium fulfills the criterion $D1$ (divinity), it must be supported by off-path beliefs $\tilde{\alpha} = Pr(\underline{\theta}|\tilde{s})$ such that for $\tilde{s} < s$, $\tilde{\alpha} = 1$ ($\tilde{\alpha} \geq \hat{\alpha}$, respectively).*

*Proof.* To use $D1$ or divinity to eliminate or constrain the weight on the opportunistic type upon observing a deviation $\tilde{s} < s$, we show that whenever a (mixed strategy) best response of $B$ to $\tilde{s}$ makes the opportunistic $A$ (weakly) better off than under the equilibrium, the same response must give the good $A$ a strictly higher payoff than the equilibrium payoff.

Let $s$ be the equilibrium payment from $A$ to $B$ in a $PBE$ where no litigation ensues. Note that there can be only one such payment, otherwise the player making the offer will deviate to the payment that serves best his/her interests without intriguing law suits. $A$'s equilibrium payoff is $\pi - s$, regardless of her type. Consider $B$'s belief upon an off-path offer $\tilde{s} < s$.

When $A$ makes the offer, upon observing $\tilde{s} < s$, denote $B$'s mixed strategy best response as $(\tilde{\phi}, \tilde{e}_B)$ and belief as $\tilde{\alpha}$, where $\tilde{\phi}$ is the probability to accept the offer and $\tilde{e}_B = e^*_B(\tilde{\theta})$ the litigation effort when rejecting the offer, given $\tilde{\theta} = \tilde{\alpha}\underline{\theta} + (1 - \tilde{\alpha})\bar{\theta}$. $A$'s payoff from deviating to $\tilde{s}$ is therefore $\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_A(\theta, \tilde{e}_B)$, $\theta \in \{\underline{\theta}, \bar{\theta}\}$. By the shape of $c(\cdot)$, $B$ doesn't mix among different levels of $e_B$.

Since $\pi - \tilde{s} > \pi - s$, when $\tilde{\phi} = 1$ both types of $A$ strictly prefer the deviation. When $\tilde{\phi} = 0$, for any $\tilde{e}_B > 0$, $u_A(\underline{\theta}, \tilde{e}_B) > u_A(\bar{\theta}, \tilde{e}_B)$ and so whenever the opportunistic inventor is (weakly) better off by deviating to $\tilde{s}$, the good inventor strictly prefers doing so. The same holds when $\tilde{\phi} \in (0, 1)$.

When $B$ makes the offer, to support this equilibrium $A$ must reject $\tilde{s}$ and this deviant offer must lead to litigation. Previous argument guarantees that if the opportunistic inventor weakly prefers to deviate under some $\tilde{e}_B$, the good inventor must strictly prefer doing so. *Q.E.D.*

## □ Proposition 2

*Proof.* For similar reason in the previous proof, there can be at most one equilibrium litigation effort $e_B$.

⋄ Full exposure: Along the equilibrium path, both types of $A$ propose a settlement offer $s < u_B(\hat{\theta})$ and $B$ rejects this offer while maintaining belief at $\hat{\theta}$, with litigation effort $e_B^*(\hat{\theta})$. $A$'s equilibrium payoff is $u_A(\theta, e_B^*(\hat{\theta}))$, $\theta \in \{\underline{\theta}, \bar{\theta}\}$. To prevent deviation, (i) since $B$ will agree to settle with a payment $u_B(\bar{\theta})$, the opportunistic $A$ should prefer litigation to settlement for sure, $u_A(\bar{\theta}, e_B^*(\hat{\theta})) \geq \pi - u_B(\bar{\theta})$; and (ii) for other deviations $\tilde{s} < u_B(\bar{\theta})$, $B$ needs to reject $\tilde{s}$ and litigates with $\tilde{e}_B \geq e_B^*(\hat{\theta})$, to be supported by off-path belief $\tilde{\alpha} \leq \hat{\alpha}$.

⋄ Partial exposure: This is a semi-pooling equilibrium where the opportunistic $A$ mixes with the good $A$ and litigate with probability $x^* \in (0, 1)$. $B$'s equilibrium belief upon litigation therefore is $\alpha_x^*$ in condition (1), which in turn determines $e_{B,x}^*$. Since only the opportunistic $A$ settles, the settlement offer $s^* = u_B(\bar{\theta})$, and she is willing to play mixed strategy iff $\pi - u_B(\bar{\theta}) = u_A(\bar{\theta}, e_{B,x}^*)$. This guarantees that the good $A$ won't deviate to offer $s^*$. By $\alpha_x^* \in (\hat{\alpha}, 1)$ and so $e_{B,x}^* \in (\underline{e}_B, e_B^*(\hat{\theta}))$, we can find such $e_{B,x}^*$ iff $\pi - u_B(\bar{\theta}) \in (u_A(\bar{\theta}, e_B^*(\hat{\theta})), u_A(\bar{\theta}, \underline{e}_B))$. To support this equilibrium, $B$ should reject any deviant offer $\tilde{s} < u_B(\bar{\theta})$ and litigate with $\tilde{e}_B \geq e_{B,x}^*$. In other words, $B$ should put enough weight on the opportunistic $A$ upon receiving $\tilde{s} < u_B(\bar{\theta})$.

To show that both equilibria survive $D1$, it suffices to show that the opportunistic $A$ cannot be deleted in $B$'s off-path beliefs. Since $A$'s equilibrium payoff is $u_A(\theta, e_B)$, depending on $A$'s type and the prevailing $e_B$ for each equilibrium, upon a deviation offer, $B$'s response of rejection and litigation with the equilibrium efforts level makes both types of $A$ indifferent from deviation or not. And by $u_A(\bar{\theta}, e_B) < u_A(\underline{\theta}, e_B)$, whenever $B$'s acceptance of a deviant offer makes the good $A$ weakly better-off by deviating, the opportunistic $A$ strictly prefers that deviation. Hence $D1$ cannot rule out the opportunistic type.

For other bargaining outcomes:

⋄ No litigation: The minimal offer to settle with both types of $A$ is $u_B(\hat{\theta})$. Let it be an equilibrium payment. To support this equilibrium, let $B$ accept any deviant offers larger than $u_B(\hat{\theta})$ with, say, "passive belief" $\hat{\theta}$. When facing a smaller offer, $B$ should reject it and exert litigation effort $\tilde{e}_B$ such that $u_A(\underline{\theta}, \tilde{e}_B) \leq \pi - u_B(\hat{\theta})$. But by Lemma 1, $D1$ requires that $B$ believe that such an offer comes from the good type for sure, which in turn requires $B$ to accept any offer in $(u_B(\underline{\theta}), u_B(\hat{\theta}))$. Therefore no $PBE$ fulfilling $D1$ can implement this outcome. On the other hand, since the passive belief is allowed under divinity, and $u_A$ is decreasing in $e_B$, no litigation can be implemented by a $PBE$ satisfying divinity if $u_A(\underline{\theta}, e^*_B(\hat{\theta})) \leq \pi - u_B(\hat{\theta})$.

$\diamond$ Only the good $A$ litigates: First consider a full separating equilibrium such that the good $A$ always litigates while the opportunistic $A$ always settles. In this case, the opportunistic $A$'s equilibrium offer is $u_B(\bar{\theta})$, and the good $A$ litigates against an effort $\underline{e}_B$. Neither type will deviate to play the other's equilibrium strategy when $u_A(\underline{\theta}, \underline{e}_B) \geq \pi - u_B(\bar{\theta}) \geq u_B(\bar{\theta}, \underline{e}_B)$. No inventor would offer higher than $u_B(\bar{\theta})$ to settle the case. To support the equilibrium, $B$ has to reject a deviant offer $\tilde{s} < u_B(\bar{\theta})$ and litigating with $\tilde{e}_B \geq \underline{e}_B$. Since $A$ can be sure to face the minimal effort $\underline{e}_B$ by proposing the good $A$'s offer (it could be an empty offer), no patent-holder has incentives to deviate to any other offers strictly smaller than $u_B(\underline{\theta})$.

Consider a deviant offer $\tilde{s} \in [u_B(\underline{\theta}), u_B(\bar{\theta}))$. To reject this offer, $B$ should put enough weight on the opportunistic type, i.e., $\tilde{\theta}$ so high that $\tilde{s} < u_B(\tilde{\theta})$. We show that for $\tilde{s}$ small enough, $D1$ would require $Pr(\underline{\theta}|\tilde{s}) = 1$ and so this outcome cannot be supported as an equilibrium outcome. Relaxing the requirement to divinity, this outcome is possible only when $\hat{\alpha}$ small enough. Denote $(\tilde{\phi}, \tilde{e}_B)$ as $B$'s optimal response to $\tilde{s}$, which is rationalized by belief $\tilde{\alpha}$.

If $\tilde{s} \in [\pi - u_A(\underline{\theta}, \underline{e}_B), u_B(\bar{\theta}))$, $B$'s response $\tilde{\phi} = 1$ makes the opportunistic $A$ strictly better off but not the good $A$, relative to their equilibrium payoffs. $D1$ and divinity cannot constrain $\tilde{\theta}$. For $\tilde{s} \in [u_B(\underline{\theta}), \pi - u_A(\underline{\theta}, \underline{e}_B))$, (i) if $\tilde{\phi} = 1$, both types of $A$ strictly prefer $\tilde{s}$ than their equilibrium strategy; (ii) if $\tilde{\phi} = 0$ and $\pi - u_B(\bar{\theta}) > u_A(\bar{\theta}, \underline{e}_B)$, whatever $\tilde{e}_B$, this response cannot make the good (opportunistic) $A$ strictly (weakly, respectively) better off; and (iii) if $\tilde{\phi} \in (0, 1)$, then for $B$ to take mixed strategy response, $\tilde{s} = u_B(\tilde{\theta})$ and $\tilde{e}_B = e^*_B(\tilde{\theta})$. The opportunistic $A$ weakly prefers to deviate if

$$\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_A(\bar{\theta}, \tilde{e}_B) \geq \pi - u_B(\bar{\theta}) \Rightarrow \tilde{\phi} \geq \bar{\phi} \equiv \frac{\pi - u_B(\bar{\theta}) - u_A(\bar{\theta}, \tilde{e}_B)}{\pi - u_B(\tilde{\theta}) - u_A(\bar{\theta}, \tilde{e}_B)};$$

and the good $A$ strictly prefers to deviate if

$$\begin{aligned}
&\tilde{\phi}(\pi - \tilde{s}) + (1-\tilde{\phi})u_A(\underline{\theta}, \tilde{e}_B) > u_A(\underline{\theta}, \underline{e}_B) \\
\Rightarrow\quad &\pi - u_B(\tilde{\theta}) > u_A(\underline{\theta}, \tilde{e}_B) \ \text{ and } \ \tilde{\phi} > \underline{\phi} \equiv \frac{u_A(\underline{\theta}, \underline{e}_B) - u_A(\underline{\theta}, \tilde{e}_B)}{\pi - u_B(\tilde{\theta}) - u_A(\underline{\theta}, \tilde{e}_B)}.
\end{aligned}$$

$D1$ and divinity have no bite for those $\tilde{s}$ such that $\pi - u_B(\tilde{\theta}) \le u_A(\underline{\theta}, \tilde{e}_B)$. But this won't be the case for all $\tilde{\theta}$, for $\pi > u_A(\underline{\theta}, \underline{e}_B) + u_B(\underline{\theta})$ as $\tilde{\theta} \to \underline{\theta}$ (as $\tilde{s} \to u_B(\underline{\theta})$). Define $\tilde{S} \equiv \{\tilde{s} : u_A(\underline{\theta}, \tilde{e}_B) + u_B(\tilde{\theta}) < \pi, \bar{\phi} > \underline{\phi}\}$. $\tilde{S} \neq \emptyset$ since, as $\tilde{s} \to u_B(\underline{\theta})$,

$$\bar{\phi} \to \frac{\pi - u_B(\bar{\theta}) - u_A(\bar{\theta}, \underline{e}_B)}{\pi - u_B(\underline{\theta}) - u_A(\bar{\theta}, \underline{e}_B)} > 0, \quad \text{but} \quad \underline{\phi} \to \frac{u_A(\underline{\theta}, \underline{e}_B) - u_A(\underline{\theta}, \underline{e}_B)}{\pi - u_B(\underline{\theta}) - u_A(\underline{\theta}, \underline{e}_B)} = 0.$$

For all $\tilde{s} \in \tilde{S}$, the set of $B$'s strictly mixed strategy best responses that makes the good $A$ strictly prefer to deviate is strictly larger than the set that makes the opportunistic $A$ weakly prefer to deviate. Therefore, for any $s' \in S' \equiv \tilde{S} \cap [u_B(\underline{\theta}), \pi - u_A(\underline{\theta}, \underline{e}_B))$, $D1$ requires $B$ to hold belief $\theta' = \underline{\theta}$, and divinity requires a belief $\theta' \le \hat{\theta}$. Imposing $D1$ then eliminates this full separating equilibrium, as $B$ should accept the offer $u_B(\underline{\theta})$. And divinity will bust the equilibrium when $\hat{\alpha}$ is so large, and $\hat{\theta}$ so small that $u_B(\hat{\theta}) \le s'$ for some $s' \in S'$, since $B$ needs to reject $s'$ with some $\theta'$ such that $u_B(\theta') > s'$.

Lastly, suppose $\pi - u_B(\bar{\theta}) = u_A(\bar{\theta}, \underline{e}_B)$. In this case $D1$ and divinity have no bite for (i) when $\tilde{s} = u_B(\underline{\theta})$, $B$'s response $\tilde{\phi} = 0$ and $\tilde{e}_B = \underline{e}_B$ makes both types of $A$ indifferent between deviation or not; and (ii) when $\tilde{s} \in (u_B(\underline{\theta}), \pi - u_A(\underline{\theta}, \underline{e}_B))$,

$$\bar{\phi} = \frac{u_A(\bar{\theta}, \underline{e}_B) - u_A(\bar{\theta}, \tilde{e}_B)}{\pi - u_B(\hat{\theta}) - u_A(\bar{\theta}, \tilde{e}_B)} = \frac{\bar{\theta}(\tilde{e}_B - \underline{e}_B)\pi}{\bar{\theta}\tilde{e}_B\pi - u_B(\tilde{\theta})} < \underline{\phi} = \frac{\underline{\theta}(\tilde{e}_B - \underline{e}_B)\pi}{\underline{\theta}\tilde{e}_B\pi - u_B(\tilde{\theta})},$$

even when $\pi - u_B(\hat{\theta}) - u_A(\underline{\theta}, \tilde{e}_B) > 0$.

$\diamond$ The good $A$ plays mixed strategies: Lastly, if the good $A$ plays the mixed strategy, denote $y^*$ as her equilibrium probability to settle. $B$'s belief upon settlement then is $\alpha_y^*$, with $\theta_y^* = \alpha_y^* \underline{\theta} + (1 - \alpha_y^*)\bar{\theta}$, and the equilibrium settlement offer $s^* = u_B(\theta_y^*)$, such that

$$u_A(\underline{\theta}, \underline{e}_B) = \pi - u_B(\theta_y^*) \ \text{ and } \ \alpha_y^* = \frac{\hat{\alpha} y^*}{\hat{\alpha} y^* + 1 - \hat{\alpha}}.$$

Since only the good $A$ litigates, the equilibrium litigation effort is $\underline{e}_B$. The good $A$ is willing to play a mixed strategy iff $u_A(\underline{\theta}, \underline{e}_B) = \pi - u_B(\theta_y^*)$, which leaves the opportunistic $A$ no incentives to deviate and litigate. Since $\alpha_y^* \in (0, \hat{\alpha})$ and so $u_B(\theta_y^*) \in (u_B(\hat{\theta}), u_B(\bar{\theta}))$, this equilibrium requires $u_A(\underline{\theta}, \underline{e}_B) \in (\pi - u_B(\bar{\theta}), \pi - u_B(\hat{\theta}))$. Note that any deviant offer leading to litigation won't disturb this equilibrium, for the inventor's equilibrium payoff is $\pi - u_B(\theta_y^*) = u_A(\underline{\theta}, \underline{e}_B) > u_A(\bar{\theta}, \underline{e}_B)$. We then check whether there is belief satisfying divinity and inducing $B$'s rejection of a deviant offer $\tilde{s} \in [u_B(\bar{\theta}), u_B(\theta_y^*))$. Since $\alpha_y^* < \hat{\alpha}$ and so $u_B(\hat{\theta}) < u_B(\theta_y^*)$, (i) for $\tilde{s} \in [u_B(\underline{\theta}), u_B(\hat{\theta}))$, whether divinity can trim $B$'s off-path belief, upon deviation we can use the passive belief $\hat{\theta}$ to justify $B$'s rejection; and (ii) for $\tilde{s} \in [u_B(\hat{\theta}), u_B(\theta_y^*))$, it can be rejected only with belief $\tilde{\theta}$ such that $u_B(\tilde{\theta}) > \tilde{s} \geq u_B(\hat{\theta})$, and so to have $\tilde{\theta} > \hat{\theta}$ the weight on the opportunistic $A$ should not be constrained by divinity. $B$'s accepting $\tilde{s}$ makes both types of $A$ strictly better off; his rejection, together with litigation effort strictly higher than $\underline{e}_B$ makes $A$ worse off. But if $B$ plays a mixed strategy composed of $\tilde{\phi} \in (0,1)$ and $\tilde{e}_B$, since $A$'s equilibrium payoff doesn't not depend on her type, and

$$\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_A(\underline{\theta}, \tilde{e}_B) > \tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_A(\bar{\theta}, \tilde{e}_B),$$

whenever the opportunistic $A$ weakly prefers to deviate, the good $A$ strictly prefers to do so. For this range of $\tilde{s}$, divinity then requires off-path belief $\tilde{\theta} \leq \hat{\theta}$, and so this equilibrium cannot survive divinity. *Q.E.D.*

## □ Proposition 4

*Proof.* The necessary and sufficient conditions come directly from $d[e_P + e_B^*(\hat{\theta})]/de_P \leq 0$ and $dC(e_P)/de_P > 0$. The sufficient condition of no lower examination standard is obtained by setting $e_P = 0$ in condition (3), and the necessary condition of no larger cost is obtained by inserting $(de_B^*/d\hat{\alpha})(\partial\hat{\alpha}/\partial e_P) \leq -1$ into $dC(e_P)/de_P > 0$. *Q.E.D.*

## □ Proposition 6

*Proof.* In a separating equilibrium where only the good $A$ settles, along the equilibrium path the settlement payment $\underline{s}$ is determined by $B$'s indifference between accepting the offer or litigating against the good $A$. Note that upon settlement, $B$ receives $\underline{s}$ only when the application survives subsequent public enforcement $e_P^n$. And the opportunistic $A$ faces private challenge efforts $\bar{e}_B$, and public examination $e_P^c$ if survives the challenge. Condition (5) comes from that neither type of $A$ is willing to deviate to mimic the other type. That is, the good $A$ prefers paying $\underline{s}$ than encountering two stages of enforcement, $(1 - \underline{\theta}e_P^n)(\pi - \underline{s}) \geq (1 - \underline{\theta}\bar{e}_B)(1 - \underline{\theta}e_P^c)\pi$; and the opportunistic $A$ prefers examination than settlement, $(1 - \bar{\theta}\bar{e}_B)(1 - \bar{\theta}e_P^c)\pi \geq (1 - \bar{\theta}e_P^n)(\pi - \underline{s})$. To support this equilibrium, $B$ accepts any deviant offer $s' > \underline{s}$, and rejects any $s' < \underline{s}$ whiling litigating with efforts $\bar{e}_B$. *Q.E.D.*

# B Alternative settings

This appendix shows that the main results we obtained are robust to alternative settings where (i) the potential challenger $B$ makes the settlement offer; or (ii) $A$'s possible types are continuous.

□ **When $B$ makes the offer:** Assign the whole bargaining power to $B$ in the two-type case. Given belief $\hat{\alpha}$, and so average invalidity $\hat{\theta}$, if $B$ decides not to settle at all, his expected payoff from litigation is $u_B(\hat{\theta})$. If he wants to settle only with the opportunistic $A$, the settlement offer (the payoff he promises to $A$) is $u_A(\bar{\theta}, \underline{e}_B)$, and he will exert effort $\underline{e}_B$ against the good $A$ (recall that this effort cannot be part of the settlement agreement). His payoff under this "partial settlement" policy is $\hat{\alpha}u_B(\underline{\theta}) + (1-\hat{\alpha})[\pi - u_A(\bar{\theta}, \underline{e}_B)]$.

To fully settle the case $A$'s willingness to accept $B$'s offer depends on the $e_B$ at the off-path event of litigation, and a higher $e_B$ pushes down the settlement offer. But next proposition shows that only $\underline{e}_B$ fulfills the criterion $D1$.[24] By offering $u_A(\underline{\theta}, \underline{e}_B)$, $B$'s payoff from fully settlement is $\pi - u_A(\underline{\theta}, \underline{e}_B)$. Define the following terms:

$$\begin{aligned}
&\bar{\alpha}_1: \quad \pi - u_A(\underline{\theta}, \underline{e}_B) \equiv \bar{\alpha}_1 u_B(\underline{\theta}) + (1-\bar{\alpha}_1)[\pi - u_A(\bar{\theta}, \underline{e}_B)] \;\Rightarrow\; \bar{\alpha}_1 \equiv \frac{(\bar{\theta}-\underline{\theta})\underline{e}_B\pi}{\bar{\theta}\underline{e}_B\pi - u_B(\underline{\theta})},\\
&\bar{\alpha}_2: \quad u_B(\bar{\alpha}_2\underline{\theta} + (1-\bar{\alpha}_2)\bar{\theta}) \equiv \pi - u_A(\underline{\theta}, \underline{e}_B), \text{ and}\\
&\bar{\alpha}_3: \quad u_B(\bar{\alpha}_3\underline{\theta} + (1-\bar{\alpha}_3)\bar{\theta}) \equiv \bar{\alpha}_3 u_B(\underline{\theta}) + (1-\bar{\alpha}_3)[\pi - u_A(\bar{\theta}, \underline{e}_B)], \text{ s.t. } \bar{\alpha}_3 < 1.
\end{aligned}$$

$\bar{\alpha}_1$ is the cutoff level where $B$ is indifferent between full settlement and settling only with the opportunistic inventor (partial settlement). By the same token, $\bar{\alpha}_2$ is the cutoff where $B$ is indifferent between no settlement at all and full settlement; and $\bar{\alpha}_3$ the cutoff for indifference between no settlement and partial settlement. Note that $\bar{\alpha}_1 \in (0,1)$ is always well-defined, but there not may exist $\bar{\alpha}_2$ and $\bar{\alpha}_3$ in the open interval $(0,1)$.

Proposition 8. *(Bargaining equilibria when B makes the offer) Let B make the settlement offer. Suppose that A agrees to settle whenever she is indifferent between settlement or not, the offer to fully settle the case in a PBE surviving D1 is* $u_A(\underline{\theta}, \underline{e}_B)$. *In this case, the weak patent is fully exposed to private enforcement only when* $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$, *and (i)* $\hat{\alpha} < \bar{\alpha}_2$, *in the case of* $\bar{\alpha}_1 \leq \bar{\alpha}_2$; *or (ii)* $\hat{\alpha} < \bar{\alpha}_3$, *in the case of* $\bar{\alpha}_1 > \bar{\alpha}_2$. *Otherwise, either there is no litigation or only the good A litigates.*

[24] However, the general pattern of bargaining outcomes is not affected by this selection.

*Suppose that $A$ may also respond to $B$'s offer in mixed strategies, then $B$'s payoff is strictly higher when the weak patent is only partially exposed to private enforcement than when full exposure. When $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$ and $\hat{\alpha}$ small enough so that full litigation is optimal in the previous case, it is optimal for $B$ to make a settlement offer $u_A(\bar{\theta}, e_B^*(\theta_z))$ and exert litigation efforts $e_B^*(\theta_z)$ such that the opportunistic $A$ will litigate with probability $z \in (0,1)$ and the good $A$ will always litigate, where $\theta_z = \alpha_z \underline{\theta} + (1-\alpha_z)\bar{\theta}$ and $\alpha_z \equiv \hat{\alpha}/[\hat{\alpha} + (1-\hat{\alpha})z] \in (\hat{\alpha}, 1)$. $B$'s payoff is*

$$\max_{\alpha_z} U_z = \frac{\hat{\alpha}}{\alpha_z} u_B(\theta_z) + (1 - \frac{\hat{\alpha}}{\alpha_z})[\pi - u_A(\bar{\theta}, e_B^*(\theta_z))].$$

*Proof.* Suppose that $A$ will agree to settle upon indifference. To fully settle the case, $B$ needs to offer a payoff $u_A(\underline{\theta}, e)$, where $e \in [\underline{e}_B, \bar{e}_B]$ is determined by $B$'s off-path belief should $A$ reject the offer. The lowest offer, $u_A(\underline{\theta}, \bar{e}_B)$, is supported by the belief that the rejection must come from the opportunistic $A$. This, however, doesn't satisfy $D1$, according to Lemma 1. This lemma also shows that the only off-path belief surviving $D1$ is that such rejection must be from the good type; and so the offer could be supported by a $PBE$ with $D1$ is $u_A(\underline{\theta}, \underline{e}_B)$. By comparing $B$'s payoffs from different settlement policies, we get the range of $\hat{\alpha}$ such that $B$ will not settle at all.

Suppose that $A$ can respond to $B$'s offer with mixed strategies. First note that it won't be in $B$'s interests to let the good $A$ play a mixed strategy. In that case, $B$ offers a payoff $u_A(\underline{\theta}, \underline{e}_B)$ so that the good $A$ is indifferent between settlement and litigation; and since the opportunistic $A$ always settles, the litigation effort is $\underline{e}_B$. The good $A$'s probability of acceptance will only change the belief upon settlement, but neither the settlement offer nor the litigation effort. By $\pi - u_A(\underline{\theta}, \underline{e}_B) > u_B(\underline{\theta})$, $B$'s payoff is increasing in the probability of the good $A$'s settlement; $B$ can increase his offer by a very small amount to guarantee full settlement.

Now, suppose that opportunistic $A$ adopts mixed-strategy responses. Given $\hat{\alpha}$, if she litigates with probability $z \in (0,1)$ upon indifference, then $B$'s belief upon litigation becomes $\alpha_z \equiv \hat{\alpha}/[\hat{\alpha}+(1-\hat{\alpha})z] \in (\hat{\alpha}, 1)$, and litigation efforts $e_B^*(\theta_z) \in (\underline{e}_B, e_B^*(\hat{\theta}))$. As $z$ increases, $\alpha_z$ decreases and $e_B^*(\theta_z)$ increases. For the opportunistic $A$ to be indifferent,

$B$ offers a settlement payoff $u_A(\bar{\theta}, e_B^*(\theta_z))$. By doing so, $B$'s payoff is

$$\begin{aligned}
U_z &= \hat{\alpha}[\underline{\theta} e_B^*(\theta_z) b - c(e_B^*(\theta_z))] \\
&\quad + (1-\hat{\alpha})\left\{ z[\bar{\theta} e_B^*(\theta_z) b - c(e_B^*(\theta_z))] + (1-z)[\pi - u_A(\bar{\theta}, e_B^*(\theta_z))] \right\} \\
&= [\hat{\alpha} + (1-\hat{\alpha})z] u_B(\theta_z) + (1-\hat{\alpha})(1-z)[\pi - u_A(\bar{\theta}, e_B^*(\theta_z))] \\
&= \frac{\hat{\alpha}}{\alpha_z} u_B(\theta_z) + (1 - \frac{\hat{\alpha}}{\alpha_z})[\pi - u_A(\bar{\theta}, e_B^*(\theta_z))].
\end{aligned}$$

$B$ can obtain a payoff $U_z(\alpha_z)$, with any $\alpha_z \in (\hat{\alpha}, 1)$, when opportunistic $A$ sets $z = [\hat{\alpha}(1-\alpha_z)]/[(1-\hat{\alpha})\alpha_z]$.

Note that as $\alpha_z \to \hat{\alpha}$, $U_z \to u_B(\hat{\theta})$, $B$'s payoff under no settlement; and

$$\begin{aligned}
\left.\frac{du_B(\theta_z)}{d\alpha_z}\right|_{\alpha_z = \hat{\alpha}} &= \frac{1}{\hat{\alpha}}[\pi - u_A(\bar{\theta}, e_B^*(\hat{\theta})) - u_B(\hat{\theta})] + \frac{du_B(\hat{\theta})}{d\hat{\alpha}} + (1 - \frac{\hat{\alpha}}{\hat{\alpha}}) \frac{du_A(\bar{\theta}, e_B^*(\hat{\theta}))}{de_B} \frac{\partial e_B^*(\hat{\theta})}{\partial \hat{\alpha}} \\
&= \frac{1}{\hat{\alpha}}\left[\pi - u_A(\bar{\theta}, e_B^*(\hat{\theta})) - u_B(\hat{\theta}) - (\bar{\theta} - \underline{\theta}) e_B^*(\hat{\theta}) b\right] \\
&> \frac{1}{\hat{\alpha}} \bar{\theta}(\pi - b) e_B^*(\hat{\theta}).
\end{aligned}$$

Full litigation is strictly dominated when $A$ plays mixed strategies. This implies that, when $\hat{\alpha}$ is small enough so that $B$ doesn't want to settle at all in case where $A$ always settles upon indifference, it is optimal for $B$ to obtain a payoff $U_z$. On the other hand, when $\hat{\alpha} \to 1$, the feasible set of $\alpha_z$, $(\hat{\alpha}, 1)$ shrinks, and $U_z \to u_B(\underline{\theta})$, which is strictly smaller than $\pi - u_A(\underline{\theta}, \underline{e}_B)$, the payoff from full litigation. Therefore for $\hat{\alpha}$ large enough, it won't be optimal for $B$ to induce mixed-strategy response from $A$. *Q.E.D.*

Comparing this proposition with Proposition 2, the same condition, $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$, applies for the weak patent to be subject to private enforcement. However, since $u_B(\hat{\theta})$ is increasing in $\hat{\theta}$ and so decreasing in $\hat{\alpha}$, a higher patent quality makes settlement more attractive to $B$. Unlike the case where $A$ makes the offer, in this case the opportunistic $A$ is fully exposed to private enforcement only when the patent quality is low enough. This is the major difference between the two distributions of bargaining power.

But, in fact, in this case the full and partial exposure regimes take place for the same range of $\hat{\alpha}$. Different regimes ensue depending on whether $A$ is allowed to play mixed strategies, and $B$'s payoff improves when the opportunistic $A$ can be induced to play mixed strategies in a proper manner, and so only litigates with some probability.

Consider the impact of $e_P$ on different regimes. Under full exposure, there is no settlement, and $B$'s litigation effort is $e_B^*(\hat{\theta})$. The crowding out effect of public enforcement thus is robust to the distribution of bargaining power. And under partial exposure, we show in the following proposition that a positive relationship between public and private enforcement still obtains with some additional conditions.

Proposition 9. *(Partial exposure when $B$ makes the offer) When $B$ makes the offer, the weak patent may encounter a private challenge only when $u_A(\bar{\theta}, \underline{e}_B) > \pi - u_B(\bar{\theta})$, and at the full exposure regime a higher $e_P$ reduces $B$'s litigation efforts.*

*Under the partial exposure, if $B$'s cost function $c''' \geq 0$ and $\hat{\alpha}$ small enough, then $B$'s litigation efforts is independent of $e_P$ and the opportunistic $A$'s litigation probability is increasing in $e_P$.*

*Proof.* When $B$ makes the offer and the opportunistic $A$ litigates with probability $z \in (0,1)$ upon indifference, by the proof of Proposition 8 for $\hat{\alpha}$ smaller than $\bar{\alpha}_2$ or $\bar{\alpha}_3$, depending on $\bar{\alpha} \gtreqless \bar{\alpha}_2$, it is optimal for $B$ to induce the mixed-strategy response from the opportunistic $A$ and obtain a payoff $U_z$ for some $z$.

Given such $\hat{\alpha}$, denote $\alpha_z^* \in (\hat{\alpha}, 1)$ as the optimal belief upon litigation (derived from the optimal $z^*$), and $\theta_z^* = \alpha_z^* \underline{\theta} + (1-\alpha_z^*)\bar{\theta}$. $B$'s optimal payoff is

$$\begin{aligned} U_z(\theta_z^*) &= \frac{\hat{\alpha}}{\alpha_z^*} u_B(\theta_z^*) + (1 - \frac{\hat{\alpha}}{\alpha_z^*})[\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*))] \\ &= \pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - \frac{\hat{\alpha}}{\alpha_z^*}\left[\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)\right]. \end{aligned}$$

When $c''' \geq 0$, for all $\hat{\alpha}$, $U_z$ is strictly convex in $\alpha_z$:

$$\begin{aligned} FOC: \ \frac{\partial U_z}{\partial \alpha_z} &= \bar{\theta}\pi \frac{\partial e_B^*(\theta_z)}{\partial \alpha_z} + \frac{\hat{\alpha}}{\alpha_z^2}[\bar{\theta}\pi e_B^*(\theta_z) - u_B(\theta_z)] - \frac{\hat{\alpha}}{\alpha_z}[\bar{\theta}\pi \frac{\partial e_B^*(\theta_z)}{\partial \alpha_z} + (\bar{\theta} - \underline{\theta}) b e_B^*(\theta_z)], \\ SOC: \ \frac{\partial^2 U_z}{\partial \alpha_z^2} &= -\frac{2\hat{\alpha}}{\alpha_z^3}\left[\bar{\theta} e_B^*(\theta_z)(\pi - \alpha_z b) + c(e_B^*(\theta_z)) + (\bar{\theta} - \underline{\theta})\alpha_z b \frac{\bar{\theta}(\pi - \alpha_z b) + \underline{\theta}\alpha_z b}{c''(e_B^*(\theta_z))}\right] \\ &\quad + \bar{\theta}\pi(1 - \frac{\hat{\alpha}}{\alpha_z})\frac{\partial^2 e_B^*(\theta_z)}{\partial \alpha_z^2} < 0, \end{aligned}$$

where

$$\frac{\partial^2 e_B^*(\theta_z)}{\partial \alpha_z^2} = \frac{c'''}{(c'')^2}(\bar{\theta} - \underline{\theta}) b \frac{\partial e_B^*(\theta_z)}{\partial \alpha_z} \leq 0.$$

Together with $\partial U_z / \partial \alpha_z > 0$ as $\alpha_z \to \hat{\alpha}$ and $U_z \to \hat{\alpha} u_B(\underline{\theta}) + (1 - \hat{\alpha})[\pi - u_A(\bar{\theta}, \underline{e}_B)]$ as $\alpha_z \to 1$, the generalized program $\max_{\alpha_z} U_z$ has a unique solution over $\alpha_z \in (\hat{\alpha}, 1]$. If $\partial U_z / \partial \alpha_z < 0$ as $\alpha_z \to 1$, then the optimal $\alpha_z^* \in (\hat{\alpha}, 1)$; and if $\partial U_z / \partial \alpha_z \geq 0$ as $\alpha_z \to 1$, then we have a corner solution and $B$ should fully settle with the opportunistic $A$. In the former case, as $\alpha_z \to 1$, the first-order condition,

$$\frac{\partial U_z}{\partial \alpha_z}\bigg|_{\alpha_z\to 1} = \bar{\theta}\pi\frac{\partial e_B^*(\theta_z)}{\partial \alpha_z}\bigg|_{\alpha_z\to 1} + \hat{\alpha}\left[\bar{\theta}\pi\underline{e}_B - u_B(\underline{\theta}) - \bar{\theta}\pi\frac{\partial e_B^*(\theta_z)}{\partial \alpha_z}\bigg|_{\alpha_z\to 1} + (\bar{\theta}-\underline{\theta})be_B^*(\theta_z)\right],$$

becomes strictly negative for $\hat{\alpha}$ small enough, i.e., we must have an interior solution.

Suppose that $\hat{\alpha}$ is so small that the optimal $\alpha_z^* \in (\hat{\alpha}, 1)$. Considering a small increase in the patent quality $\hat{\alpha}' > \hat{\alpha}$, we show that the same $\alpha_z^*$ remains optimal when $\hat{\alpha}'$ is close enough to $\hat{\alpha}$. Let $\hat{\alpha}'$ be close enough to $\hat{\alpha}$ so that $\alpha_z^* \in (\hat{\alpha}', 1)$. We want to show that $\forall \alpha' \in (\hat{\alpha}', 1)$ and $\alpha' \neq \alpha_z^*$, with $\theta' = \alpha'\underline{\theta} + (1-\alpha')\bar{\theta}$,

$$\begin{aligned}
& \pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - \frac{\hat{\alpha}'}{\alpha_z^*}\left[\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)\right] \\
> \quad & \pi - u_A(\bar{\theta}, e_B^*(\theta')) - \frac{\hat{\alpha}'}{\alpha'}\left[\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')\right], \\
\Rightarrow \quad & u_A(\bar{\theta}, e_B^*(\theta')) - u_A(\bar{\theta}, e_B^*(\theta_z^*)) > \hat{\alpha}'\left\{\frac{\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)}{\alpha_z^*} - \frac{\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')}{\alpha'}\right\}.
\end{aligned}$$

By the definition and uniqueness of $\alpha_z^*$, since $\alpha'$ is also available under $\hat{\alpha}$ (for $(\hat{\alpha}', 1) \subset (\hat{\alpha}, 1)$),

$$\begin{aligned}
& \pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - \frac{\hat{\alpha}}{\alpha_z^*}\left[\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)\right] \\
> \quad & \pi - u_A(\bar{\theta}, e_B^*(\theta')) - \frac{\hat{\alpha}}{\alpha'}\left[\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')\right] \\
\Rightarrow \quad & u_A(\bar{\theta}, e_B^*(\theta')) - u_A(\bar{\theta}, e_B^*(\theta_z^*)) > \hat{\alpha}\left\{\frac{\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)}{\alpha_z^*} - \frac{\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')}{\alpha'}\right\}.
\end{aligned}$$

Therefore, if $\alpha' < \alpha_z^*$, then $e_B(\theta') > e_B(\theta_z^*)$ and so $u_A(\bar{\theta}, e_B^*(\theta')) < u_A(\bar{\theta}, e_B^*(\theta_z^*))$, any $\hat{\alpha}' > \hat{\alpha}$ will fulfill our objective. The same is true if $\alpha' > \alpha_z^*$ but

$$\frac{\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)}{\alpha_z^*} \leq \frac{\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')}{\alpha'}.$$

On the other hand, if $\alpha' > \alpha_z^*$ and

$$\frac{\pi - u_A(\bar{\theta}, e_B^*(\theta_z^*)) - u_B(\theta_z^*)}{\alpha_z^*} > \frac{\pi - u_A(\bar{\theta}, e_B^*(\theta')) - u_B(\theta')}{\alpha'},$$

a $\hat{\alpha}'$ close enough to $\hat{\alpha}$ guarantees the optimality of $\alpha_z^*$ under $\hat{\alpha}'$. *Q.E.D.*

□ **Continuous types:** Now, let $A$ keep the bargaining power, but assume continuous types $\theta \in [0, 1]$. Let *ex ante*, i.e., before the examination process begins, $CDF$ be

$F(\cdot)$ and *pdf* be $f(\cdot)$, with $f(\theta) > 0$ for all $\theta \in [0, 1]$. Again denote $\theta^0 \equiv \int_0^1 \theta dF$ as the *ex ante* expectation value of $\theta$. A higher $\theta^0$ implies a lower quality.

When all types of inventors file patent applications, under the post-grant challenge system and patent office efforts $e_P$, the probability to eliminate the application is $\int_0^1 \theta e_P dF = \theta^0 e_P$. Upon issuance, the distribution of $\theta$ is updated to

$$\hat{F}(\theta) \equiv \frac{1}{1-\theta^0 e_P} \int_0^\theta (1-\theta' e_P) dF \quad \text{and} \quad \hat{f}(\theta) \equiv \frac{1-\theta e_P}{1-\theta^0 e_P} f(\theta);$$

and the post-issuance expectation is

$$\hat{\theta} \equiv \int_0^1 \theta d\hat{F} = \frac{\theta^0 - e_P E(\theta^2)}{1 - e_P \theta^0}.$$

Intuitively, stronger public enforcement reduces $\hat{\theta}$:

$$\frac{\partial \hat{\theta}}{\partial e_P} = \frac{(\theta^0)^2 - E(\theta^2)}{(1-e_P\theta^0)^2} \leq 0,$$

by Jensen's inequality and the fact that $x^2$ is a convex function.

To facilitate the presentation, let us define the following terms: given $\tilde{\theta} \in (0, 1)$,

$$\hat{\theta}^+ \equiv E(\theta|\theta \geq \tilde{\theta}, e_P) = \frac{1}{1-\hat{F}(\tilde{\theta})} \int_{\tilde{\theta}}^1 \theta d\hat{F} \quad \text{and} \quad \theta^+ \equiv E(\theta|\theta \geq \tilde{\theta}, e_P = 0) = \frac{1}{1-F(\tilde{\theta})} \int_{\tilde{\theta}}^1 \theta dF.$$

$\hat{\theta}^+$ is the post-issuance expectation, conditional on $\theta$ greater than a threshold $\tilde{\theta}$; and $\theta^+$ is the conditional mean at the *ex ante* stage, or, equivalently, when $e_P = 0$. By the same token, we define $\hat{\theta}^-$ and $\theta^-$ as the conditional expectations when $\theta \leq \tilde{\theta}$:

$$\hat{\theta}^- \equiv E(\theta|\theta \leq \tilde{\theta}, e_P) = \frac{1}{\hat{F}(\tilde{\theta})} \int_0^{\tilde{\theta}} \theta d\hat{F} \quad \text{and} \quad \theta^- \equiv E(\theta|\theta \leq \tilde{\theta}, e_P = 0) = \frac{1}{F(\tilde{\theta})} \int_0^{\tilde{\theta}} \theta dF.$$

Maintain the assumption that $B$'s litigation effort $e_B$ cannot be part of the settlement agreement. Denote again $u_B(E(\theta|\mathcal{L}))$ as $B$'s expected payoff when challenging a patent with expected "case quality" $E(\theta|\mathcal{L})$. Upon bargaining breakdown, the optimal litigation effort $e_B^*$ also depends on $E(\theta|\mathcal{L})$, and is determined by the first-order condition $E(\theta|\mathcal{L})b \equiv c'(e_B^*)$. Given $e_B^*$, a patentee with of type $\theta$ has a expected payoff from litigation $(1-\theta e_B^*)\pi$. Since $\theta = 0$ is always one of the possible types, $f(0) > 0$, and cannot be eliminated by the patent office, under asymmetric information full settlement cannot be a bargaining outcome. As long as $Pr(\theta > 0) > 0$, $B$ will not accept an agreement under which $A$ keeps the whole monopoly profit $\pi$.

For simplicity, consider only pure strategies. The following proposition, In resemblance of Proposition 1, shows that a settled patent dispute involves weak patents, i.e., those with high values of $\theta$.

Proposition 10. *(Case selection under continuous types) Suppose that both private players use pure strategies. Whether $A$ or $B$ makes the settlement offer, there exists $\tilde{\theta} \in (0,1]$ such that a patent-holder litigates when having types $\theta' < \tilde{\theta}$, and settles when having types $\theta'' > \tilde{\theta}$.*

*Proof.* Since only pure strategies are allowed, there is only one equilibrium settlement payment $s$ (from $A$ to $B$). Without loss of generality, let $s = 0$ if no agreement is ever reached. A bargaining outcome consists of two elements: the equilibrium settlement offer $s$ and $B$'s litigation effort $e_B^*$ in case of bargaining breakdown. $A$'s payoffs from settlement and litigation are $\pi - s$ and $(1-\theta e_B^*)\pi$, respectively. The cut-off rule follows from the fact that the former is constant while the latter is decreasing in $\theta$. *Q.E.D.*

By this proposition, $B$'s equilibrium litigation effort is determined in accordance with the expectation $E(\theta|\mathcal{L}) = \hat{\theta}^-$. Let $\bar{\theta}_A$ be the equilibrium cutoffs. We first derive a sufficient condition under which $PBE$s exist, then consider the impact of a marginal change in $e_P$ and the possibility of a positive relationship between public and private enforcement.

Proposition 11. *(Bargaining equilibrium with continuous types) Consider continuous types and let $A$ make the settlement offer. If $u_B(1) < e_B^*(\hat{\theta})\pi$, there is no $PBE$ where no types of $A$ settle.*

*Any $\bar{\theta}_A \in (0,1)$ is an equilibrium cutoff of a $PBE$ if it satisfies*

$$\bar{\theta}_A e_B^*(\hat{\theta}^-)\pi \geq u_B(\hat{\theta}^+) \equiv \max_{e_B} \hat{\theta}^+ e_B b - c(e_B). \tag{7}$$

*A sufficient condition for the existence of an equilibrium cutoff $\bar{\theta}_A \in (0,1)$ is*

$$e_B^*\left(\frac{\theta^0 - E(\theta^2)}{1-\theta^0}\right)\pi > u_B(1) = \bar{e}_B b - c(\bar{e}_B), \tag{8}$$

*where $\bar{e}_B = e_B^*(1) \leq 1$ is the maximal possible litigation effort, and $E(\theta^2)$ is evaluated at the ex ante distribution.*

*Proof.* Firs, consider full litigation as the equilibrium outcome. The equilibrium litigation effort is $e_B^*(\hat{\theta})$, and equilibrium payoff for a patent-holder with type $\theta$ is $[1-\theta e_B^*(\hat{\theta})]\pi$, decreasing in $\theta$. To support this equilibrium, $B$ should reject any positive settlement offer with appropriate off-path beliefs. However, since $B$ will always agree to settle when offered a payment $u_B(1)$ (or plus a small amount in order to break the tie), the patentee with types close to $\theta = 1$ will find it profitable to deviate and settle when $\pi - u_B(1) > [1 - e_B^*(\hat{\theta})]\pi$.

Now, suppose that $\bar{\theta}_A \in (0,1)$ is an equilibrium cutoff, i.e., all $\theta' < \bar{\theta}_A$ litigate while all $\theta'' > \bar{\theta}_A$ settle. Let $\hat{\theta}^-$ and $\hat{\theta}^+$ be the conditional means corresponding to $\bar{\theta}_A$.

The type $\bar{\theta}_A$ must be indifferent between litigation (with a payoff $[1-\bar{\theta}_A e_B^*(\hat{\theta}^-)]\pi$) and settlement (with a payoff $\pi - s$), otherwise she and adjacent types will move toward the more profitable strategy and upset the equilibrium. The equilibrium settlement payment is $s = \bar{\theta}_A e_B^*(\hat{\theta}^-)\pi$. But this offer has to be no smaller than $B$'s expected payoff from litigating against $\hat{\theta}^+$ in order to accept the offer. Thus determines condition (7). This equilibrium can be supported by $B$'s off-path responses to accept any deviant offers greater than $\bar{\theta}_A e_B^*(\hat{\theta}^-)\pi$, and reject smaller deviant offers while litigate with efforts at least as strong as the equilibrium litigation level $e_B^*(\hat{\theta}^-)$.

For existence, note that as $\bar{\theta}_A \to 1$, $\hat{\theta}^- \to \hat{\theta}$ and $\hat{\theta}^+ \to 1$. The right-hand side of condition (7) is simply $B$'s maximal possible payoff from litigation: $\max_\theta u_B(\theta) = u_B(1) = \bar{e}_B b - c(\bar{e}_B)$. The left-hand side, as $\bar{\theta}_A \to 1$, approaches to $e_B^*(\hat{\theta})\pi$, where $\hat{\theta}$ is decreasing in $e_P$. To guarantee the existence for all $e_P$, condition (8) establishes the existence when $e_P \to 1$. *Q.E.D.*

Given an equilibrium cutoff $\bar{\theta}_A \in (0,1)$, the equilibrium settlement payment and litigation efforts are $\bar{\theta}_A e_B^*(\hat{\theta}^-)\pi$ and $e_B^*(\hat{\theta}^-)$, respectively.

Remark. (Equilibrium refinement) As in a typical signaling game, multiple equilibria may ensue.[25] The intuitive criterion has no bites here.[26] And, different from the two-type case, a more stringent criterion such as $D1$ will eliminate all the $PBE$s with positive probability of settlement. This is because, for all deviant offers $s' \neq s$, those types $\theta'' > \bar{\theta}_A$ will be eliminated under $D1$ by the type $\bar{\theta}_A$: With the same equilibrium payoff but lower probability to be invalidated for all $e_B > 0$, whenever a type $\theta''$ weakly prefers to deviate and offer $s'$, the type $\bar{\theta}_A$ must strictly prefer to do so. But

[25] Indeed, when $\pi >> b$ such that

$$\pi\left[e_B^*(\hat{\theta}^-) + \bar{\theta}_A \frac{\partial e_B^*}{\partial \theta}\bigg|_{\hat{\theta}^-} \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A}\right] > b e_B^*(\hat{\theta}^+)\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_A},$$

for any $\bar{\theta}_A$ satisfies condition (7), so does any $\theta > \bar{\theta}_A$.

[26] A $PBE$ here can be supported by off-path strategies such that $B$ accepts any deviant payment $s'$ higher than $s$, and rejects any smaller payment while exerting litigation efforts no smaller than $e_B^*$. Both responses can be justified by a belief that this offer comes from an inventor with an average type $\hat{\theta}^+$. Note that for $s' < s$, no type of $A$ can be eliminated by the intuitive criterion: Relative to their equilibrium payoffs, $B$'s acceptance of $s'$ is strictly preferred by those $\theta'' > \bar{\theta}$, and the rejection with a litigation effort higher than $e_B^*$ is strictly preferred by $\theta' \leq \bar{\theta}_A$. For the same reason, when $s' > s$, the intuitive criterion won't be able to eliminate a type $\theta' \leq \bar{\theta}_A$. So even if some types $\theta'' > \bar{\theta}_A$ can be deleted, a belief that a deviant offer comes from those types smaller than $\bar{\theta}_A$, with the resulting average quality $\hat{\theta}^-$, suffices to support $B$'s response.

this implies that the highest possible off-path belief is $\bar{\theta}_A$, which busts the equilibrium since $B$ has no reasonable off-path belief to reject a deviant offer $s'$ between $u_B(\bar{\theta}_A)$ and $u_B(\hat{\theta}^+)$. ■

We now proceed to consider the impact of public enforcement $e_P$. By $\hat{\theta}$ decreasing in $e_P$, a higher $e_P$ makes it easier to sustain an equilibrium with no settlement. This corresponds to the "full exposure" regime in the two-type case, and requires that the worst type $\theta = 1$ be willing to mix with all other types and fact an litigation effort $e_B^*(\hat{\theta})$ rather than offering $u_B(1)$ to guarantee settlement. This would happen when $e_P$ is high and so $e_B^*(\hat{\theta})$ is low enough.

Now, consider the effect of a marginal change in $e_P$. An increasing in $e_P$ changes the distribution function $\hat{F}$ at the private bargaining stage: $\forall \theta < 1$,

$$\frac{\partial \hat{F}(\theta)}{\partial e_P} = \frac{\theta^0 - E(\theta'|\theta' \leq \theta)}{(1 - \theta^0 e_P)^2} F(\theta) > 0.$$

A higher public enforcement effort shifts the distribution toward low values of $\theta$. Presumably, this change may simultaneously move the equilibrium cutoff $\bar{\theta}_A$ and effort $e_B^*$, with the latter both affected by the distribution and the equilibrium cutoff. This makes it difficult to define the extent of private enforcement. For simplicity, we restrict attention to a particular type of equilibrium adjustment. Similar to the partial exposure regime under the two-type case, we consider when an increase in $e_P$ will raise $\bar{\theta}_A$ but keep $e_B^*$ unchanged. If this holds, then a higher public effort enlarges the set of inventor types under private scrutiny without compromising challenge efforts.

We consider a pair of change $de_P$ and $d\bar{\theta}_A$ that keeps $\hat{\theta}^-$ unchanged, and so the equilibrium effort $e_B^*$ unchanged, and test when this pair of changes still satisfies condition (7). Formally, define $\Lambda \equiv \bar{\theta}_A e_B^* \pi - u_B(\hat{\theta}^+)$. In a $PBE$, $\Lambda \geq 0$. We consider $(de_P, d\bar{\theta}_A)$ such that

$$\frac{\partial \Lambda}{\partial e_P} de_P + \frac{\partial \Lambda}{\partial \bar{\theta}_A} d\bar{\theta}_A \geq 0 \ \ s.t. \ \ \frac{\partial \hat{\theta}^-}{\partial e_P} de_P + \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A} d\bar{\theta}_A = 0. \tag{9}$$

Proposition 12. *(Public and private enforcement under continuous types) In the continuous-type setting where $A$ makes the offer, a higher $e_P$ makes it more likely to have all types of $A$ involved in litigation. Full exposure occurs under high public enforcement.*

*In a $PBE$ with equilibrium cutoff $\bar{\theta}_A \in (0,1)$, a pair $(de_P, d\bar{\theta}_A)$ satisfies condition (9) if*

$$\frac{\partial \hat{\theta}^- / \partial e_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_A} \geq \frac{\partial \hat{\theta}^+ / \partial e_P}{\partial \hat{\theta}^+ / \partial \bar{\theta}_A}. \tag{10}$$

*Under ex ante uniform distribution $F(\theta) = \theta$, condition (10) is satisfied when $\bar{\theta}_A$ is small enough.*

*Proof.* Since $\hat{\theta}^-$ and so the equilibrium litigation effort $e_B^*$ are not affected by the changes of $e_P$ and $\bar{\theta}_A$, and by definition, $u_B(\hat{\theta}^+) = \hat{\theta}^+ e_B^*(\hat{\theta}^+)b - c(e_B^*(\hat{\theta}^+))$, we have

$$\frac{\partial \Lambda}{\partial e_P} = -e_B^*(\hat{\theta}^+)b\frac{\partial \hat{\theta}^+}{\partial e_P} \quad \text{and} \quad \frac{\partial \Lambda}{\partial \bar{\theta}_A} = e_B^*(\hat{\theta}^-)\pi - e_B^*(\hat{\theta}^+)b\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_A}.$$

By inserting the condition that keeps $\hat{\theta}^-$ intact,

$$d\bar{\theta}_A = -\frac{\partial \hat{\theta}^- / \partial e_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_A} de_P,$$

and after a few algebraic manipulation, we get

$$\frac{\partial \Lambda}{\partial e_P} de_P + \frac{\partial \Lambda}{\partial \bar{\theta}_A} d\bar{\theta}_A = \frac{de_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_A}\Big[ - e_B^*(\hat{\theta}^-)\pi \frac{\partial \hat{\theta}^-}{\partial e_P} + e_B^*(\hat{\theta}^+)b\big(\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_A}\frac{\partial \hat{\theta}^-}{\partial e_P} - \frac{\partial \hat{\theta}^+}{\partial e_P}\frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A}\big)\Big].$$

Since $\frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A} > 0 > \frac{\partial \hat{\theta}^-}{\partial e_P}$ (and so $d\bar{\theta}_A$ and $de_P$ should have the same sign), the whole term is guaranteed to be positive if

$$\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_A}\frac{\partial \hat{\theta}^-}{\partial e_P} - \frac{\partial \hat{\theta}^+}{\partial e_P}\frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A} \geq 0,$$

or, equivalently, if condition (10) holds.

With *ex ante* uniform distribution, $F(\theta) = \theta$, post-issuance $CDF$ and *pdf* are, respectively,

$$\hat{F}(\theta) = \frac{1}{1 - \theta^0 e_P}\int_0^\theta (1 - \theta' e_P)d\theta' = \frac{\theta(2 - \theta e_P)}{2 - e_P} \quad \text{and} \quad \hat{f}(\theta) = \frac{2 - 2\theta e_P}{2 - e_P}.$$

Given a cutoff $\bar{\theta}_A$, the conditional expectations are

$$\hat{\theta}^+ = \frac{2}{2 - (1 + \bar{\theta}_A)e_P}\big[\frac{1}{2}(1 + \bar{\theta}_A) - \frac{e_P}{3}(1 + \bar{\theta}_A + \bar{\theta}_A^2)\big] \quad \text{and} \quad \hat{\theta}^- = \frac{2\bar{\theta}_A}{2 - \bar{\theta}_A e_P}(\frac{1}{2} - \frac{e_P}{3}\bar{\theta}_A).$$

Therefore,

$$\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_A} = \frac{2(1 - \bar{\theta}_A e_P)[2(1 - e_P) + (1 - \bar{\theta}_A e_P)]}{3[2 - (1 + \bar{\theta}_A)e_P]^2}, \quad \frac{\partial \hat{\theta}^+}{\partial e_P} = -\frac{(1 - \bar{\theta}_A)^2}{3[2 - (1 + \bar{\theta}_A)e_P]^2},$$
$$\frac{\partial \hat{\theta}^-}{\partial e_P} = -\frac{\bar{\theta}_A^2}{3(2 - \bar{\theta}_A e_P)^2}, \quad \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_A} = \frac{2(3 - \bar{\theta}_A e_P)(1 - \bar{\theta}_A e_P)}{3(2 - \bar{\theta}_A e_P)^2},$$

and condition (10) requires:

$$\frac{\partial \hat{\theta}^- / \partial e_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_A} = -\frac{\bar{\theta}_A^2}{2(3 - \bar{\theta}_A e_P)(1 - \bar{\theta}_A e_P)} \geq \frac{\partial \hat{\theta}^+ / \partial e_P}{\partial \hat{\theta}^+ / \partial \bar{\theta}_A} = -\frac{(1 - \bar{\theta}_A)^2}{2(1 - \bar{\theta}_A e_P)[2(1 - e_P) + (1 - \bar{\theta}_A e_P)]},$$
$$\Rightarrow (\frac{1 - \bar{\theta}_A}{\bar{\theta}_A})^2 \geq \frac{3 - \bar{\theta}_A e_P - 2e_P}{3 - \bar{\theta}_A e_P}.$$

$\bar{\theta}_A$ has to be small enough. For instance, it is satisfied for all $\bar{\theta}_A \leq \frac{1}{2}$. *Q.E.D.*

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