# Above and Beyond the Inverted-U Relationship: Innovation and Product Market Competition in a Dynamic Duopoly

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**Comments welcome.**

**Abstract**

This paper revisits the theoretical grounds of Aghion, Bloom, Blundell, Griffith, and Howitt's (2005) seminal paper on competition and innovation. It uses a comparable framework but allows for simultaneous innovations, potentially free for the laggard. With quite a low probability of free catch-up by laggards, the inverted-U pattern between symmetric duopoly profits and innovation does not emerge. Besides, some "free" innovation is always beneficial to innovation. At last, the paper specifies a model of product market competition and simulates the dynamic output. This raises important interpretative issues. Overall, even inverted-U patterns between innovation and markups cannot be interpreted as a causal non-monotonic impact of "competition" on innovation.
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# Introduction

Given the complexity of the underlying concepts, the interplay between innovation and competition is very hard to analyse in general, either theoretically or empirically. Kamien and Schwartz (1972) as well as Cohen and Levin (1989) show that the impact of competition on innovation might be positive or negative, given the market characteristics (structure or technologies for instance) and the characteristics of the potential innovations.

When firms are symmetric and competing, the classical schumpeterian effect leads to maximum incentives for innovation when innovation is drastic and the innovator becomes a monopolist. However, when firms are not symmetric, for instance when a monopolist faces an entrant, two effects are classically competing. First, the monopolist has a low incentive to innovate as it would destroy his own previous innovation. This is the *replacement effect* (Arrow 1962). Second, if the entry is not drastic, the incentives of the entrants may be weak as he would have to share duopoly profits. This is the *efficiency effect* (Gilbert and Newbery 1982). Altogether, these effects are perfectly relevant and consistent. However, they provide competing results as regards to innovation. If the symmetric duopoly profit is very low, the schumpeterian effect is maximal, but there is not point entering the market or catching up for a firm with a lower technological level. If it is very high, this is the opposite. It is necessary to build a framework that encompasses all these effects in order to deal with the link between competition and innovation.[1]

Aghion, Bloom, Blundell, Griffith, and Howitt (2005) present an elegant dynamic framework to tackle the issue. They model a duopoly where two firms can differ by at most a technological level. The market can be *leveled*, the two symmetric (or *neck-to-neck*) firms getting symmetric duopoly profits. Alternatively, the market can be *unleveled*, one firm getting the leader's profit, and the second one lagging behind. Then, in each state, firms may invest in costly and uncertain innovation, in a continuous time framework. The comparative statics are straightforward. The leading firm cannot increase her advance and has no incentive to innovate. For given profits for the leader and for the laggards, the laggard's incentive is increasing in the

[1] In this paper, we implicitly focus on the direct link between profitability of innovation and incentives to innovate. However, a second strand of the literature emphasized that competition is a way for shareholders to discipline firms' managers (Aghion, Dewatripont, and Rey 1999a, Aghion, Dewatripont, and Rey 1999b).

symmetric duopoly profit. It is the opposite for neck-to-neck firms. This generates interesting dynamics. As a result of the previous incentives, the probability to be in leveled markets is larger when the symmetric duopoly profit is *ceteris paribus* larger. Quite naturally, this generates an inverted-U pattern between innovation and symmetric duopoly profits, the other profits given.

The theoretical results of the paper, and the related empirical test have been extensively quoted and commented. The main stylized result of an inverted-U relationship has, since its publication, been influential, either among economists and practitioners. However, two important issues arise.

First, timing and informational setups in investment games are crucial. The choice of simple continuous time models may have dramatic consequences and, in some cases, traditional continuous-time strategy spaces are not adequate to capture important heuristics of models (see Fudenberg and Tirole 1985). Here, the choice of the continuous time model rules out any static strategic interaction and, more generally, any effect related to simultaneous innovations. Nonetheless, it is by no mean obvious that potential simultaneous innovations play no role. This paper shows that they may play an important role. There exist devices meant to allow continuous-time models to more accurately represent discrete-time limits. Here, the process is directly modeled in a discrete-time framework as I do not view the process of research decisions as being short-time decisions. This paper shows that simultaneous innovation may play a role. The main result is that innovation by the laggard acts as a threat for leaders and increases the probability to be in leveled industries, which is the most innovative state. As a result, it is never optimal to totally protect the innovation of the leader. Conversely, it might be optimal to help laggards to catch-up.

Besides, the interpretation of the results in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) relies on comparative statics on the symmetric duopoly profits, the other profits given. However, this reduced form does not rely on strong theoretical arguments. On the contrary, several recent papers have shown that the link between "competition" and innovation depends on the nature of competition and innovation (Boone 2000, Bonanno and Haworth 1998, Cohen and Klepper 1996, Greenstein and Ramey 1998, Weiss 2003). This paper addresses this issue by a simple model of differentiation. In this model, when structural parameters change, all profits are modified. Besides, all parameters do not have the same influence on innovation, even though they are perfect candidates for proxies of competition (in the sense of Boone 2004). From a theoretical standpoint, I claim that the interplay between competition and innovation is complex, that competition is not a well defined normative concept and thus that the link between "competition" and innovation is not a well posed problem. this raises interpretative issues. From an empirical standpoint, this also raises important identification issues.

The paper is organized as follows. The first part presents the model and emphasizes the consequences of "free" innovation for the laggard. The second part focuses on linking the dynamics of the model with a simple model of product market competition. At stated before, this paper adds an important caveat to the previous interpretation of the so called "'inverted-U" relationship. Then, I conclude. At last, this paper includes several appendix. The first, most important one, focuses on a simple two-period model with static strategic interactions to understand how they might play a role in a more complex model. The following appendixes are technical. They include proofs of the propositions and additional figures.

# 1 Dynamics of Competition and Innovation

We consider an economy where two firms are competing in a final product market. Their profits depend of their relative technological levels. For simplicity, we only consider that firms can at most differ by one level of technology. They may either be neck-to-neck and both get profits $\Pi_0$. Alternatively, one firm may be in advance and get profit $\Pi_1$. Then, the laggard firm gets $\Pi_{-1}$. An example of profits will be developed in the last section but at this stage we only assume $\Pi_{-1} \leq \Pi_0 \leq \Pi_1$. As in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), both firms may invest in step by step innovation, without opportunity for leapfrogging. As firms can at most differ by one level of technology, when the firm in advance innovates, the laggard automatically catch up of one level. Their relative levels remain unchanged.

In order to allow for simultaneous innovations, we wish to consider a discrete time dynamic model. In general, this would generate a model that is not tractable (see appendix). It is then necessary to cancel static strategic interactions. It is however possible to keep the opportunity of simultaneous innovations for the laggard, which generates important and interesting effects for the static incentives as well as on the dynamics of the model. This model is close in spirit to this of Aghion, Bloom, Blundell, Griffith, and Howitt (2005), where continuous time also prevents static strategic interactions. The main difference is the possibility of "free" and simultaneous innovation by the laggard.[2] When there is no such potentially simultaneous innovation by the laggard, the model essentially replicates Aghion, Bloom, Blundell, Griffith, and

[2] There is some free innovation for the laggard in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), but it is not simultaneous.

Howitt (2005).

## 1.1 Model

The simplest way to cancel direct strategic interactions is to assume that, in each period, only one firm has an "idea" to implement. This is an important assumption of the model. Simultaneous innovation is only possible for the laggard and there exist no static strategic interaction. However, strategic interactions emerge from the dynamics of the model. The reaction of the other firm is not absent, it is simply delayed. This assumption is in line line with Aghion, Bloom, Blundell, Griffith, and Howitt (2005). The cancelation of static strategic interactions thus not only simplifies the resolution of the model, it also allows to compare the two equilibriums and thus the influence of simultaneous innovation by laggards. Static strategic interactions may play an additional role that would be hard to disentangle from the dynamics in a general model. Analyzing their potentiel influence is presented in the appendix in a two-period game.

This firm, and only this one, may invest some assets to innovate with a stochastic success. For simplicity, we also assume that, for each period, each firm ex-ante has the same probability ($\frac{1}{2}$) to have an idea. For each period, the schedule of the game is the following:

1. "Nature" decides which firm has an "idea" of innovation.
2. Given their technological levels, firms choose their prices or quantities. The output of the competition on the product market gives them profits $\Pi_{-1}$, $\Pi_0$ or $\Pi_1$.
3. The firm who has an idea may invest in innovation.
4. Her possible success determines the technological levels for the next period.

If a neck-to-neck firm or the leader in a leveled industry has an idea, she may invest $\frac{\gamma}{2}n^2$ for a probability of success of $n$. If the laggard firm has the opportunity, she may invest $\frac{\gamma}{2}n^2$ with a probability of success $n+\rho$. Without opportunity of investment in innovation, she still succeeds with probability $\rho$.[3]

When she has an idea, the leading firm maximizes:

$$\Pi_1 - \frac{\gamma}{2}n_1^2 + \delta\left(\rho(1-n_1)V_0 + (1-\rho(1-n_1))V_1\right)$$

[3] We thus keep the important element from the general model presented in appendix of potential simultaneous innovations by the leader and the laggard.

A laggard maximizes:

$$\Pi_{-1} - \frac{\gamma}{2} n_{-1}^2 + \delta\left((\rho + n_{-1})V_0 + (1 - (\rho + n_{-1}))V_{-1}\right)$$

In a leveled industry, the firm with an opportunity of innovation maximizes:

$$\Pi_0 - \frac{\gamma}{2} n_0^2 + \delta\left(n_0 V_1 + (1 - n_0)V_0\right)$$

We focus on Markov equilibrium. Then, stationary and symmetric equilibrium strategies are:

$$\begin{cases} \gamma n_{-1}^* &= \delta\left(V_0^* - V_{-1}^*\right) \\ \gamma n_1^* &= \delta\rho\left(V_1^* - V_0^*\right) \\ \gamma n_0^* &= \delta\left(V_1^* - V_0^*\right) \end{cases}$$

The equilibrium discounted profits are given by:

$$\begin{cases} (1-\delta)V_{-1}^* &= \Pi_{-1} - \frac{\gamma}{4} {n_{-1}^*}^2 + \frac{1}{2}\left\{\left((\rho + n_{-1}^*) + \rho(1 - n_1^*)\right)\left(V_0^* - V_{-1}^*\right)\right\} \\ (1-\delta)V_1^* &= \Pi_1 - \frac{\gamma}{4} n_1^{*2} - \frac{1}{2}\left\{\left((\rho + n_{-1}^*) + \rho(1 - n_1^*)\right)\left(V_1^* - V_0^*\right)\right\} \\ (1-\delta)V_0^* &= \Pi_0 - \frac{\gamma}{4} n_0^{*2} + \frac{1}{2}\left\{n_0^*(V_1^* - V_0^*) - n_0^*(V_0^* - V_1^*)\right\} \end{cases}$$

The first three equations are straightforward. As regards their incentives to innovate, neck-to-neck firms are solely concerned by the opportunity to outdistance their rivals. They are unable to counter an innovation by their rivals in this model. As a result, they are looking above only, but not below. The laggard is only concerned by the opportunity of catching-up. At last, the leader is also threatened by the laggard in this model, but only due to the potential "involuntary" innovation resulting from $\rho$. Hence, on the contrary to Aghion, Bloom, Blundell, Griffith, and Howitt (2005) leaders do innovate.

## 1.2 Comparative statics

This system leads to quite simple equations and can easily be solved.[4] In line with Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and for the sake of comparability, we present all comparative statics with fixed $\Pi_{-1}$ and $\Pi_1$.

**Proposition 1.1** *If* $\gamma \geq \frac{\delta(\Pi_1 - \Pi_{-1})}{(1-\rho)\left(1 - \frac{3}{4}(1-\rho)\right)}$, *there is a unique interior equilibrium. At the equilibrium, for given* $\Pi_{-1}$ *and* $\Pi_1$*:*

1. $n_0^*$ *is decreasing in* $\Pi_0$, $\gamma$, $\rho$ *and increasing in* $\delta$*;*
2. $n_{-1}^*$ *is decreasing in* $\gamma$, $\rho$, *and increasing in* $\Pi_0$, $\delta$*;*
3. $n_1^*$ *is decreasing in* $\Pi_0$, $\gamma$ *and increasing in* $\rho$, $\delta$.

[4] Proofs of all propositions are in the appendix.

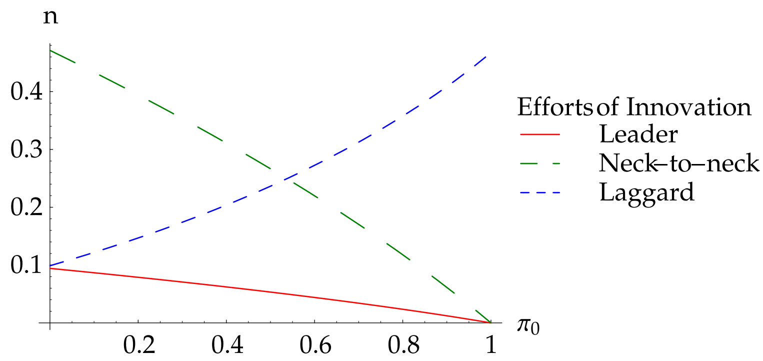
The results of comparative statics are intuitive: direct effects always dominate the indirect ones. The effects of $\delta$ and $\gamma$ are quite straightforward: if innovation is less costly or if the firms have a lower discount for the future, all firms will invest more.

The direct effect of an increase of $\Pi_0$ is to positively affect the value to be in a leveled industry and thus reduces the direct incentive to outdistance its rival. It also increases the potential gain of innovation for the laggard and the direct incentive to catch up. At last, it reduces the potential loss to be caught-up by the laggard for the leader. Given the complex forms of the results of this model, it is not possible to get formal results on the variations of the discounted values. However, discount values for the laggard and neck-to-neck firms should intuitively be increasing in $\Pi_0$, for given $\Pi_{-1}$ and $\Pi_1$. As the direct incentives of the laggard are also increasing, the value to be the leader might be slightly decreasing in $\Pi_0$ for some sets of parameters. If these intuitions cannot be proved in general, they are confirmed by all simulations. The values to be laggard or neck-to-neck firms are increasing in $\Pi_0$. This is also mostly the case for leaders, even though the value might be decreasing for smaller values of $\delta$ and smaller values of $\Pi_0$ (all values except this of neck-to-neck firms are then quite flat though). Very intuitively, the value in leveled markets is the most affected, as it is the most directly affected. Then, $V_1^* - V_0^*$ and the incentives of neck-to-neck firms are decreasing in $\Pi_0$. On the contrary $V_0^* - V_{-1}^*$ and the incentives of laggards are increasing in $\Pi_0$. These variations are proved in proposition 1.1. Besides, the effort of the laggard is never null, even when the direct effect is null ($\Pi_0 = \Pi_{-1}$). Catching the other firm up always have the option value of becoming a leader in the future. This indirect effect is even maximal when the direct effect is null.

$\rho$'s main effects are to threaten the leader and to level the playing field between the three types of firms. First, it directly increases the value to be a laggard. In the same time, the threat for the laggard increases. It then should invest more. As innovation is costly, it is unlikely to compensate its loss. Overall, as it is confirmed by all simulations, it thus reduces the value to be a leader. As a result, this also reduces the value to be a neck-to-neck firm (and at last to be a laggard). Neck-to-neck firms are less directly concerned by an increase in $\rho$. Then, $V_1^* - V_0^*$ and the incentives of neck-to-neck firms are decreasing in $\rho$. This is the same for $V_0^* - V_{-1}^*$ and thus the incentive of the laggard. At last, the leader is more threatened by the laggard as $\rho$ grows. Even if an increase in $\rho$ also decreases the relative advantage of being a leader, compared to neck-to-neck firms, the leader always react by investing more to protect its position. These variations are proved in proposition 1.1.

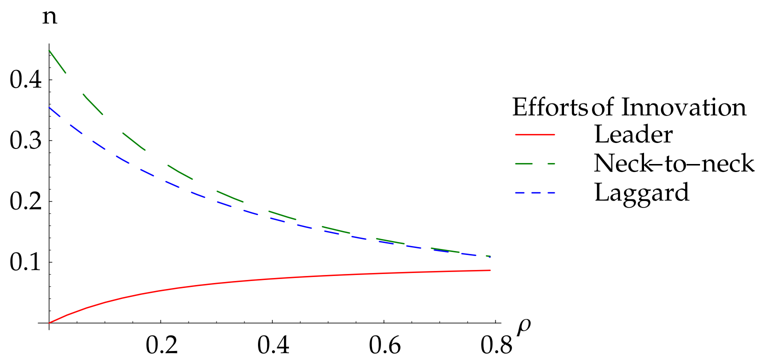
Figures 1 and 2 depict examples of variations of the innovation efforts in $\Pi_0$ and $\rho$ and illustrate the results of proposition 1.1. As expected, the pattern in $\Pi_0$ is close

Figure 1: Efforts of Innovation




**Note:** $\delta = 0.9$, $\gamma = 5$, $\rho = 0.2$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

Figure 2: Efforts of Innovation




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 = 0.5$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

to this in Aghion, Bloom, Blundell, Griffith, and Howitt (2005). The main difference is that the leader now innovates, all the more that $\rho$ is large.

## 1.3 Dynamics

### 1.3.1 Distribution of states

It is important to understand how composition effects affect the average rates of innovation outputs in a stationary equilibrium. The strategies in the previous game are stationary. As we considered a Markov equilibrium, transition probabilities only depend on the current state. Let $\mathbb{P}\{U \to L|U\}$ (resp. $\mathbb{P}\{L \to U|L\}$) be the probability of transition from unleveled to leveled state (resp. from leveled state to unleveled state). The probability to be in leveled state in time $t$ is:

$$\begin{aligned}\mathbb{P}_t\{L\} &= \mathbb{P}\{U\to L|U\}\,\mathbb{P}_{t-1}\{U\}+\mathbb{P}\{L\to L|L\}\,\mathbb{P}_{t-1}\{L\}\\ &= \mathbb{P}\{U\to L|U\}+(1-(\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}))\,\mathbb{P}_{t-1}\{L\}\\ &= \tfrac{\mathbb{P}\{U\to L|U\}}{\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}}\left(1-(1-(\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}))^k\right)\\ &\quad+(1-(\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}))^k\,\mathbb{P}_{t-k}\{L\}\end{aligned}$$

Thus, if $0<\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}<2$, there exists a steady state, in which the probabilities to be in any state are stationary:

$$\mathbb{P}\{L\} = \tfrac{\mathbb{P}\{U\to L|U\}}{\mathbb{P}\{U\to L|U\}+\mathbb{P}\{L\to U|L\}}$$

This result is intuitive: if the industry switches faster from unleveled to leveled states than the opposite, it is more often in a leveled state and *vice versa*. When the market is leveled, the ex-ante probability to switch to an unleveled industry is:

$$\mathbb{P}\{L\to U|L\} = n_0^*$$

Conversely, the probability to switch from unleveled to leveled is:

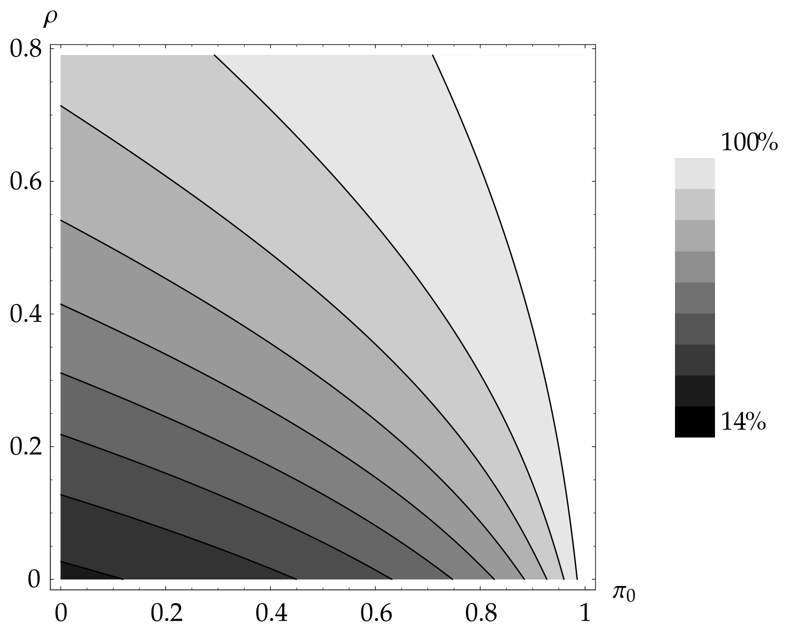
$$\mathbb{P}\{U\to L|U\} = \tfrac{1}{2}\left(\rho(1-n_1^*)+(\rho+n_{-1}^*)\right)$$

Here, if neck-to-neck firms innovate more, the time spent in leveled states is shorter. Conversely, if the laggard innovates more, or if the leader innovates less, then the time spent in unleveled states is shorter. Thus, $\mathbb{P}\{L\}|_{\{}n_{-1}^*, n_0^*, n_1^*, \rho\}$ is decreasing in $n_0^*$ and $n_1^*$, but increasing in $n_{-1}^*$ and $\rho$. As $n_{-1}^*$ is increasing in $\Pi_0$, but $n_0^*$ and $n_1^*$ have the opposite pattern, it is easy to conclude:

**Proposition 1.2** *The probability to be in the leveled state is increasing in* $\Pi_0$.

Besides, $\rho$ directly increases the probability to switch from unleveled to leveled state. It also indirectly decreases the effort of neck-to-neck firms, and hence the probability to switch from leveled to unleveled. However, it also increases the effort of the leader and decreases this of the laggard. Thus, the effect on the probability to switch from unleveled to leveled is ambiguous. The leading firm invests more in reaction to an increase of $\rho$ but, intuitively, she should not overreact in such a way that, overall, the probability to be caught up ($\rho(1-n_1^*)$) decreases. The intuition is the same for the laggard: she should not overreact to an increase of $\rho$ in such a way that, overall, its probability to catch up ($n_{-1}^*+\rho$) decreases. Then, the direct increase of the probability to switch from unleveled to leveled state should overwhelm the indirect effect due to the variations of the incentives of leaders and laggards. Overall, the

Figure 3: Probability to be in the leveled state




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 = 0.5$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

probability to be in leveled state would then be increasing in $\rho$. Given the complex formulas arising from the model, the formal result is hard to obtain. However, it is confirmed by many simulations. Figure 3 shows an example of a contour plot of the probability to be in leveled state for a set of parameter.[5] The influence of the other parameters is a priori ambiguous. $\delta$ raises all efforts of innovation, while $\gamma$ plays in the other direction.

### 1.3.2 Innovation

Aghion, Bloom, Blundell, Griffith, and Howitt (2005) focus on the average rate of innovation as the sole proxy for welfare. The average rate of innovation is an interesting characteristic of the equilibrium. Besides, it is easy to measure or proxy, for instance by patents. In both models, innovation is implicitly described as being vertical. New products are in all aspects better than the old ones. Then, innovations in themselves might not be relevant for welfare, eg. if innovations by laggards are sole (and most of the time costly) imitations. Then, this is mainly the innovations that are new *for the market* that are of importance. Moreover, it is this type of innovation that is related to the *pace of innovation.* If $k_t$ is the level of the more advanced of the two firms at

[5] Other examples of graphs are presented in appendix.

time $t$, let denote $\Delta = \mathbb{E}\{k_{t+1} - k_t|t, k_t\}$. $\Delta$ is the average incremental progress of the economy. This pace of innovation is thus more relevant for public policy.[6]

When the market is unleveled, there is one innovation with probability $\frac{1}{2}((\rho(1 - n_1^*) + (1-\rho)n_1^*) + (\rho + n_{-1}^*))$, there are two with probability $\frac{1}{2}(\rho n_1^*)$, hence none with probability $\frac{1}{2}((1-\rho)(1-n_1^*) + (1-(\rho+n_{-1}^*)))$. Thus, the average number of innovations in the unleveled state is:

$$\mathbb{E}\{I|U\} = \rho + \frac{n_1^* + n_{-1}^*}{2}$$

Only innovations by the leader are innovations for the market in this framework. Thus, the average number of innovations for the market is

$$\mathbb{E}\{Im|U\} = \mathbb{E}\{k_{t+1} - k_t|t, k_t, S_t = U\} = \frac{1}{2}(n_1^*) = \frac{1}{2}(\rho n_0^*)$$

Thus, catch-up mainly occur in this state. The overall innovation is increasing in $\Pi_0$, while the rate of innovations for the market is decreasing. The rate of innovation for the market is increasing in $\rho$. Relying on the intuition that $n_{-1}^* + \rho$ is also increasing in $\rho$, the overall average rate of innovations should also be increasing in $\rho$. This is confirmed by many simulations. Figure 26 in the appendix shows an example of a contour plot of average rates of innovation for a set of parameters.

When the market is leveled, an innovation occurs with probability $n_0^*$, this is always an innovation for the market. The average number of innovation, both relative and absolute is:

$$\mathbb{E}\{I|L\} = \mathbb{E}\{Im|L\} = \mathbb{E}\{k_{t+1} - k_t|t, k_t, S_t = L\} = n_0^*$$

Thus, the variations of $\mathbb{E}\{I|L\}$ and $\mathbb{E}\{Im|L\}$ are already well known (see figures 1 and 2). They are decreasing in $\Pi_0$ and in $\rho$. Besides, we have $n_1^* = \rho n_0^*$. Thus the average rate of innovation for the market is at least twice higher in the leveled state than in the unleveled.

Besides, at the stationary equilibrium:

$$\begin{aligned}
\mathbb{E}\{I\} &= \mathbb{P}\{L\}\mathbb{E}\{I|L\} + \mathbb{P}\{U\}\mathbb{E}\{I|U\} \\
\mathbb{E}\{Im\} &= \mathbb{P}\{L\}\mathbb{E}\{Im|L\} + \mathbb{P}\{U\}\mathbb{E}\{Im|U\} \\
\Delta &= \mathbb{E}\{Im\}
\end{aligned}$$

Simulations indicate unambiguous variations for the composition effects: as $\Pi_0$ and $\rho$ grow, the probability to be in leveled state is higher. However, the way these

[6]One could also think to focus on the duration of a cycle $T = \frac{1}{\mathbb{P}\{U \to L|U\}} + \frac{1}{\mathbb{P}\{L \to U|L\}}$. However, most qualitative conclusions are also valid for average rate of innovation and the duration of cycles.

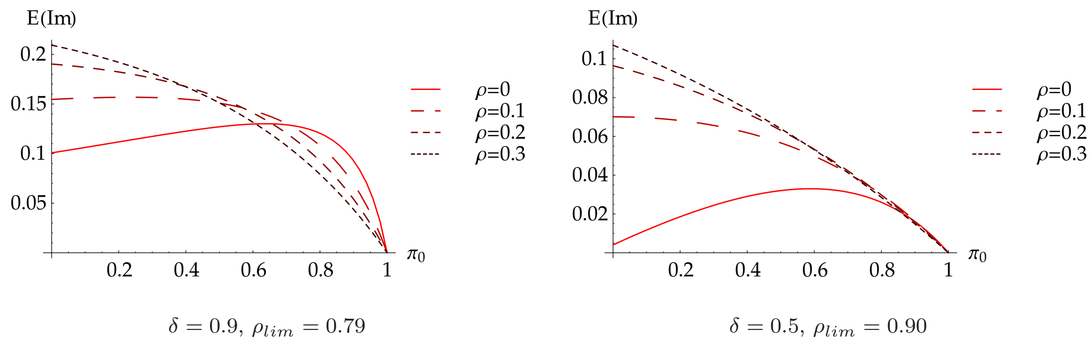
composition effects interact with the average rates of innovation in each state is ambiguous. If $\rho = 0$, there is no innovation for the market in the unleveled state. The rate of innovation in the leveled state is positive, but decreases as $\Pi_0$ grows. Overall, as $\Pi_0$ grows, the probability to be in the most innovative state increases, but the rate of innovation in this state decreases. This should generate an inverted-U pattern for the average rate of innovation for the market. The intuition is similar for overall innovations: for a small symmetric duopoly profit, leveled markets also are the most innovative. The rate of innovation in this state is decreasing in $\Pi_0$, while it is increasing in $\Pi_0$ in the unleveled markets. This should generate a comparable inverted-U pattern.

With a positive, and potentially "large" $\rho$, the situation is different. For a large $\rho$, the probability to be in the leveled state is always large, even for a small $\Pi_0$ (the probability to be in leveled state is increasing in $\rho$). Then the increase of the probability to be in the potentially most innovative state is always offset by the decrease of the innovation in this state. Besides, with a larger $\rho$, leveled states are less innovative, while unleveled ones are more innovative (either in absolute terms or for the market), which reinforces the previous effect. Then, for large $\rho$s the patterns for both types of innovations should be decreasing in $\Pi_0$.

As stated before, even though this model can be fully solved, the formulas of the equilibrium efforts of innovations are too complicated to allow to analyze the variations of average rates of innovation in general. However, patterns of innovation can be simulated and all simulations confirms the intuitions presented above. Figure 27 in the appendix shows two contour plots of the variations of the average rate of innovation for values of $\Pi_0$ and $\rho$ (and two values of $\delta$). Figure 28 in the appendix presents the similar graph for innovations for the market. $\gamma = 5$ in both examples and $\delta(\Pi_1 - \Pi_{-1})$ is equal to 0.9 (left graphs) or 0.5 (right graph). Then, the condition for $\rho = 0$ is $\gamma \leq 4\delta(\Pi_1 - \Pi_{-1})$ is satisfied (but $\gamma$ cannot be much smaller). Besides, in the left example $\rho_{lim} = 0.79$ while it is equal to 0.90 in the right one.

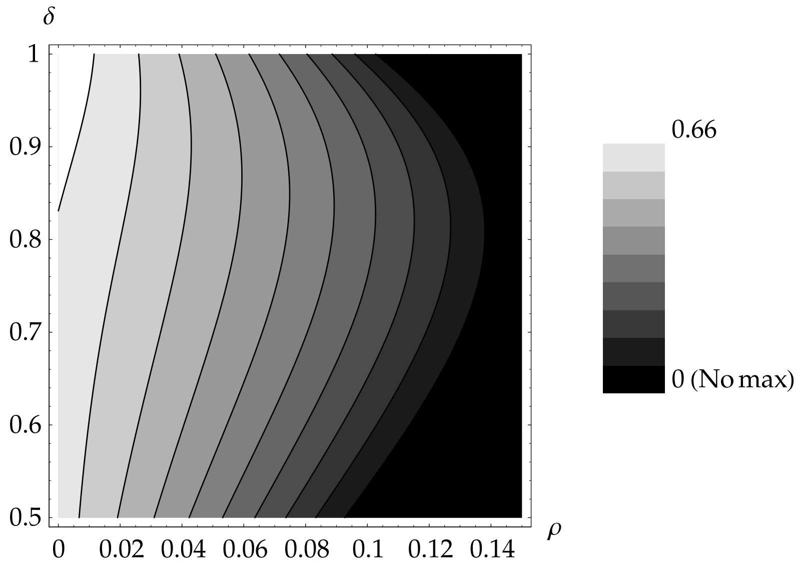
These two graphs confirm that, with a null $\rho$, innovation at the equilibrium behave quite similarly as in Aghion, Bloom, Blundell, Griffith, and Howitt (2005). From a qualitative standpoint, it is then nested within the model in this paper. If $\rho = 0$, innovation then have an inverted-U pattern in the symmetric duopoly profits, when the other profits are fixed. This pattern is robust to a small $\rho$, but not to a larger one. Then, simultaneous innovations do act as a threat for leaders and this effect cannot be captured by the continuous time model developed in Aghion, Bloom, Blundell, Griffith, and Howitt (2005). At last, the probability to be in leveled states is always high. Overall, innovative outputs are smaller when $\Pi_0$ *ceteris paribus* increases.

FIGURE 4: Pace of innovation




**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$, $\rho \in \{0, 0.1, 0.2, 0.3\}$.

FIGURE 5: Values of $\Pi_0$ maximizing $\Delta$




**Note:** $\delta \in [0.5, 1]$, $\gamma = 5$, $\rho \in [0, 0.2]$, $\Pi_{-1} = 0$, $\Pi_1$=1.

It is useful to go a step further. When the maximum exists, it is interesting to analyze for which values the rate of innovation is maximal. It is also interesting to analyze when such a maximum does not exist. For this analysis, we hereby focus on the pace of innovation, as it is the important output for public policies. We first plot two sets of average rate of innovation for the market for various values of $\rho$ and two values of $\delta$ (Figure 4). We then plot in figure 5 an example of the contour plot, as a function of $\rho$ and $\delta$, of the values of $\Pi_0$ that maximize $\Delta$, when such a maximum exists. In this example, $\gamma = 5$ and $\Pi_1 - \Pi_{-1} = 1$. For these values, the maximum only exists whenever $\rho$ is sufficiently small. As soon as $\rho$ is larger than about 0.14, then the average rage of innovation for the market, and hence the pace of innovation, is always decreasing in $\Pi_0$.

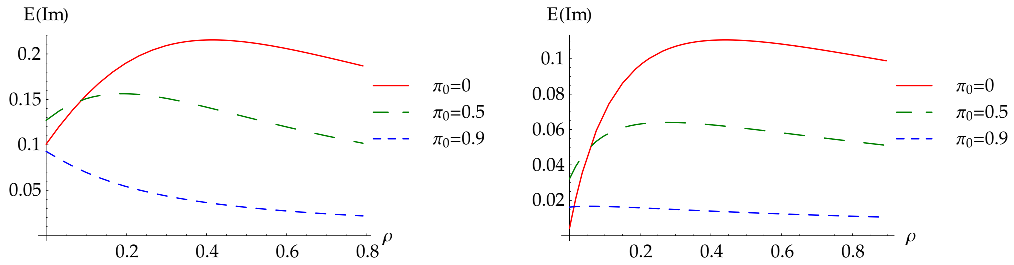
With a very low $\rho$, innovations for the market are due to neck-to-neck firms. As shown in appendix, with a large $\delta$, the probability to be in leveled state is very convex. Thus, it only weakly increases for small symmetric duopoly profits, but gets rapidly to one as $\Pi_0$ gets closer to $\Pi_1$. The effort in this state also decreases more rapidly as $\Pi_0$ gets closer to $\Pi_1$. Overall, the pattern of innovation for the market is only weakly increasing when $\Pi_0$ is small. However, when $\Pi_0$ gets closer to $\Pi_1$, it dramatically collapses.[7] From a qualitative standpoint, this collapse is the most significant stylized fact. On the contrary, for a smaller $\delta$, the probability to be in leveled state is quite low for small symmetric duopoly profits and both slopes of the probability to be in leveled states and the effort in this state are more steady. As result, the inverted-U pattern is more pronounced. At last, for a larger $\rho$ (the threshold being quite small), whatever the value of $\delta$, the pattern is unambiguously decreasing. These effects are illustrated in Figure 4.

Besides, one main result of this model is that some "free" and simultaneous innovation for laggards has a positive impact of the average pace of innovation. Figure 4 provides some insight on this, that is confirmed by the contour plots 27 and 28. An increase in $\rho$ increases the rate of innovation in unleveled markets (either in absolute terms or for the market). Moreover, a higher $\rho$ both decreases the effort in leveled industries, and increases the probability to be in this state. Thus, for a small $\rho$, the increase of the innovation in unleveled states and the composition effect offsets the negative consequences of this increase in terms of incentives in leveled states. For a larger $\rho$, this could be the contrary. Then, the pattern in $\rho$ should also be non monotonic. However, the increased rate of innovation in unleveled industries is mainly driven by the composition effects and the effort of the leader. For large symmetric duopoly profits, the leader is less threatened by the laggard and the probability to be in leveled state is already very large. Then, the benefits of a higher $\rho$ for the rate of innovation are small. Thus, in very uncompetitive markets (large $\Pi_0$), the pattern in $\rho$ could overall be decreasing. At the limit, when $\Pi_0 = \Pi_1$, the probability to be in leveled state is 1 and the effort in this state is null. These intuitions are confirmed by the contour plots in Figures 27 and 28.

As before, we plot two sets of average rate of innovation for the market as functions

[7] The pattern is quite similar for overall innovation. The same effects apply. Besides, for a large $\delta$, the laggard is weakly responsive to an increase of $\Pi_0$ as the opportunity to become a leader always have a strong option value, even when the direct incentive to invest is strong.

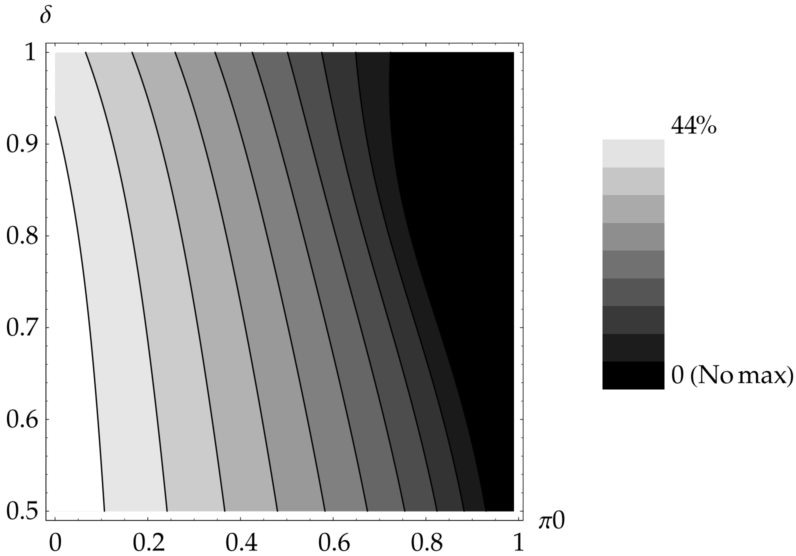
FIGURE 6: Pace of innovation




$\delta = 0.9$, $\rho_{lim} = 0.79$ $\delta = 0.5$, $\rho_{lim} = 0.90$

**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in \{0, 0.5, 0.9\}$, $\rho \in [0, \rho_{lim}]$.

FIGURE 7: Values of $\rho$ maximizing $\Delta$




**Note:** $\delta \in [0.5, 1]$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

of $\rho$ for various values of $\Pi_0$ and two values of $\delta$ (Figure 6, recall that for $\Pi_0 = \Pi_1$, there is no innovation). We also plot in figure 7 an example of the contour plot, as a function of $\Pi_0$ and $\delta$, of the values of $\rho$ that maximize $\Delta$, when such a maximum exists. In this example, $\gamma = 5$ and $\Pi_1 - \Pi_{-1} = 1$. It confirms that for large values of $\Pi_0$, no maximum exists (black zone). Besides, when this maximum exists, it is for rather small values of $\rho$, at most lower than a half.

Figures 4, 6 and 7 show that when $\Pi_0$ is very small, the benefits of an increased $\rho$ are very important when $\rho$ is very small. After that, the pattern is indeed decreasing, but only weakly. At the limit, a too large $\rho$ is preferable to a too small one as regards to the pace of innovation. This pattern is even more important when $\delta$ is smaller. $\delta$

may be smaller due to a larger discount rate for the future, but it also aggregates the probability of survival. When $\delta$ is small, innovation, which is an investment for the future, is naturally smaller. Then, an increased $\rho$ is very beneficial for the pace of innovation (this would be reinforced if laggards had a lower probability of survival). In these extreme cases, one would really be adverse on a too small $\rho$. When $\Pi_0$ is intermediate, the benefits of an increase of $\rho$ are smaller. However, an excessive $\rho$ does not make innovation collapse. Overall, it would rather be beneficial, in particular when $\delta$ is relatively small. At last, when $\Pi_0$ is very close to $\Pi_1$, the patterns are always decreasing in $\rho$. However, the pace of innovation is always very small then (there is no innovation when $\Pi_0 = \Pi_1$) and the small benefits of innovation for neck-to-neck firms should be the primordial matter of concerns for public policies.[8]

At last, it is interesting to investigate what is the optimal couple $(\rho^*, \Pi_0^*)$ for innovation, from a normative basis. Figure 30 in the appendix shows a plot of this couple for $\gamma = 5$, $\Pi_{-1} = 0$ and $\Pi_1 = 1$. It is interesting to note that the pace of innovation is always maximal for the lowest value of symmetric duopoly profits ($\Pi_0 = 0$) and for quite large values of $\rho$ (for various values of $\delta$, $\rho^*$ is between 0.45 and 0.5). Thus, the optimal policy would be to give the best direct incentives to neck-to-neck firms and to favor leveled markets, up to a certain limit, for instance by the use of limited patents. This result is quite intuitive in the light of the previous discussion.

## 1.4 Collusion

In this model, firms have at their disposal several control variable. They first control their prices (or quantities), which gives their current profits. They also control their investments in innovation. Until now, we solely considered non-cooperative equilibria. However, the model also provides some intuitions on the interplay between collusion and innovation. To simplify the analysis, we hereby consider an extreme version of the model of differentiation below. Demand is unitary, firms compete a la Bertrand with potentially different technologies and there exists a competitive fringe, lying two technological levels behind the most advanced firm. The value of the innovation is $a$. Then, the competitive fringe is such that customers have the reservation price $2a$ for the good sold by neck-to-neck firms or by the leader in unleveled market and of $a$ for the laggard.[9]

[8] A more extensive discussion on profits and competitiveness of markets is presented below.

[9] It is important to stress that in this framework, innovation is neutral as regards the aggregate profit of the industry in each period. Then, it is competition only, or imperfect coordination of the two firm, that makes then innovate. If innovation would increase the aggregate profit of the

The competitive benchmark is the following. In leveled markets, the competitive outcome is to sell at marginal cost (here null), and hence to make zero profits. In unleveled markets, the laggard prices zero, the leader just below $a$, gets all demand and makes profit $a$. Overall, $\Pi_{-1} = \Pi_0 = 0$ and $\Pi_1 = a$. For a small $\rho$, slight increases of $\Pi_0$ could be beneficial for innovation. For a larger $\rho$, competitive profits correspond to the maximum innovation.

The simplest form of collusion is the one presented in Aghion, Bloom, Blundell, Griffith, and Howitt (2005). Firms may collude in prices only, and only in leveled markets. Then, we would have $\Pi^m_{-1} = 0$ and $\Pi^m_0 = \Pi^m_1 = a$ (neck-to-neck firms both price just below $2a$, and share the demand). Implicitly, we assume that retaliation is triggered by a deviation on prices in leveled markets only, and consist to a return to Bertrand competition. A potential innovation would not trigger innovation, and laggards would not use grimmer strategies than normal asymmetric Bertrand competition. It can be shown that this is a sustainable equilibrium, for a threshold on $\delta$ that is actually quite close to this in a static collusive equilibrium.[10] Innovation does not significantly hinder collusion. Here, even though they do not collude on innovation, they still do not innovate. In this collusive equilibrium, collusive profits are such that each neck-to-neck firm do as well as the potential leader. This simplest mode of collusion (on prices only, and only in one state) is sufficient to prevent any type of innovation. In this case, collusion hinders innovation.

This type of equilibrium requires a commitment no to collude on prices in unleveled markets. In a certain sense, it might not be general. What could be static collusive profits in unleveled markets? Due to the competitive fringe, no firm can face positive demand and price above $2a$ (or $a$ for the laggard). If the difference between the price of the leader and the price of laggard is higher than $a$, the leader faces null demand. Conversely, if it is lower than $a$, the laggard faces null demand. If the leader faces all demand, it can at most price $2a$ and makes profit $2a$. Conversely, if the laggard faces all demand, it can at most price $a$ and makes profit $a$. If they share the demand (because their price difference is exactly $a$ and the leader does not price above $2a$), prices can be any pair $(p, p+a)$, with $p \in [0, a)$. Clearly, the laggard pricing just below $a$ is the optimal pricing strategy. Then, the laggard would price $a$, the leader would price $2a$. Their profits would be $\frac{a}{2}$ and $a$ respectively. Thus, the higher joined profit is generated by the leader selling just below $2a$ and the laggard just above $a$. The

industry, results would differ. However, the focus of the paper is on the link between innovation and competition. The analysis of coordination in a more general framework where firms innovate for other purposes than competition is beyond the scope of this paper.

[10] With $a = 1$,$\rho \in [0, 0.5]$ and $\gamma \in [4, 10]$, the threshold ranges between 0.50 (for $\rho = 0.5$, $\gamma = 10$) and 0.56 (for $\rho = 0$, $\gamma = 4$). It is only marginally affected by $\gamma$ or $\rho$.

laggard then makes null profit and the leader makes profit $2a$. The collusive joined profit in leveled industry is then the same as in unleveled.

If firms do not collude on innovation, it can be shown that the collusive equilibrium is sustainable[11]. However, this type of collusion generates some innovation, even though it does not generate any profit for the pair. In this sense, it is not optimal. It can be shown that in colluding in prices but also not to innovate in leveled industries is not harder to sustain and leads to higher profits for the firms.[12]

In this set-up, the size of the profit shared by the two firms is not affected by innovation (it is $2a$, at the most). Then, innovation is only triggered by competition. The first equilibrium shows that it is possible for firms not to innovate if they commit not to collude in unleveled markets. A direct collusion on investments in innovation would lead to the same result. In both situations, what is important is to penalize the innovator, and to reduce its expected profit. This is optimal and, on the contrary to some very grim trigger strategies, does not require losses by the laggard outside the equilibrium path. Then, collusion not only hinders innovation. The possibility to innovate does not significantly reduce the opportunities for collusion in prices either.

# 2 PRODUCT MARKET COMPETITION

In the previous part, all the comparative statics were presented with $\Pi_0$ moving between two fixed $\Pi_{-1}$ and $\Pi_1$. Aghion, Bloom, Blundell, Griffith, and Howitt (2005) interpret this reduces form profit as a direct proxy for competition in the sense of Boone (2004)and thus interpret all their results accordingly. Nonetheless, none of these profits is a structural parameter. Profits of firms might be higher due to horizontal or vertical differentiation, barriers to entry, collusion, etc. These characteristics of market equilibrium might have a differential impact on the profits of leaders, neck-to-neck firms and laggards. Moreover, it is unclear that symmetric duopoly profits can cover all the interval between the profits of laggards and leaders. For instance,

[11] The laggard and neck-to-neck firms have an opportunity to deviate. Both would gain $a$ from deviation. The incentives for neck-to-neck are slightly higher. For $\rho \in [0, 0.5]$ and $\gamma \in [4, 10]$, their threshold for instance range between 0.68 and 0.52, for respectively $(\gamma = 4, \rho = 0)$ and $(\gamma = 10, \rho = 0.5)$. Nonetheless, it is always the laggards who have the highest incentives to deviate. For $\rho \in [0, 0.5]$ and $\gamma \in [4, 10]$, the thresholds range between 0.83 and 0.60, for respectively $(\gamma = 10, \rho = 0)$ and $(\gamma = 10, \rho = 0.5)$. Thus, this type of collusion is generally harder to sustain.

[12] In a collusive path where firms are not expected to invest in innovation, neck-to-neck are not expected to innovate. Thus, an innovation by one firms is unambiguously detected as a deviation and might trigger retaliation. With $a = 1$,$\rho \in [0, 0.5]$ and $\gamma \in [4, 10]$, the threshold ranges between 0.51 (for $\rho = 0.5$, $\gamma = 10$) and 0.56 (for $\rho = 0$, $\gamma = 4$).

$\Pi_0$ close to $\Pi_1$ means that each neck-to-neck firm has a profit close to the one in advance. This would clearly be the case if innovations has no real market value. $\Pi_{-1}$ would then also be close to $\Pi_0$ and $\Pi_1$, which is an extreme case. It is not clear that it is possible in general for $\Pi_0$ to be close to $\Pi_1$ and significantly above $\Pi_{-1}$. The leading firms would then be a monopoly, with a superior product or technology. On the opposite, neck-to-neck firms would have to share a lower joined profit. In this situation, $\Pi_0$ could hardly be more than half of the profit of the leader.

Besides, profits are the result of the strategic interactions between players. The whole equilibrium depends on structural parameters, such as differentiation. Changes in these parameters are likely to affect all profits, with ambiguous consequences for innovation. The values of the profits in the various states has not yet been specified. To consistently interpret the previous results in terms of "competition, it is necessary to get some insight on the relative values of the profits given some structural parameters. Addressing these issues require to specify some example of product market competition.

## 2.1 A simple model of differentiation

In this part, we consider a very simple Hotelling model. The two firms ($A$ and $B$) are located at both extremities[13] and are marketing goods with technical levels $n/n$ or $n+1/n$ (without loss of generality, we can assume that the leader is always firm $A$ and located in 0). A technical level $n$ provides a utility $na$ to final customers, who have a transportation cost $t$. $a$ is thus a measure of *vertical differentiation*, while $t$ is a measure of *horizontal differentiation*. There exists a competitive fringe, in each extremity, marketing goods two technical level behind the most advanced of the two firms.[14] This means that the incumbents are also competing a la Bertrand with the competitive fringe. Then, prices are at most equal to their competitive advantage towards this competitive fringe ($2a$ the leader and for neck-to-neck firms, $a$ for the laggard).

Classically, in the leveled market, the customer who is indifferent between both

[13] For the sake of simplicity, we do not model the localization part of the model. We assume that, for some exogenous reason, the spatial differentiation is extreme.

[14] This mainly is a technical trick to avoid that consumers' valuations of the goods depend on the absolute technical levels. However, this is consistent with our choice to keep only two relative technological levels in the dynamic game. Besides, it is realistic: there should be some alternative to the incumbents apart from stone age technologies. Nonetheless, there is no reason to believe that consumers would not endure at least the same transportation costs. This is why these competitive fringes are also assumed to be located at each extremities.

manufacturers is located at $\frac{p_B - p_A + t}{2t}$. Both firms price $t$ and the equilibrium profits are:

$$\Pi_A = \Pi_B = \frac{t}{2}$$

Customers actually prefer neck-to-neck manufacturer to the competitive fringe iff.:

$$\forall x,\ (n-2)a - tx^2 \leq na - tx^2 - p \Leftrightarrow p = t \leq 2a$$

If their price is larger than $2a$, they face a null demand (the constraint does not depend on the location). The optimal strategy for the firms is then to price $2a$ and still get half of the demand. At last, we have:

$$\Pi_0 = \begin{cases} \frac{t}{2} \texttt{ if } t \leq 2a \\ a \texttt{ if } t \geq 2a \end{cases}$$

In the unleveled market, $A$ being the leader, the customer who is indifferent is located at $\frac{p_B - p_A + a + t}{2t}$. At the equilibrium, we have:

$$\begin{array}{lllllll} p_A & = & t + \frac{a}{3} & \quad & \Pi_A & = & \frac{t}{2}(1 + \frac{a}{3t})^2 \\ p_B & = & t - \frac{a}{3} & \quad & \Pi_B & = & \frac{t}{2}(1 - \frac{a}{3t})^2 \end{array}$$

This equilibrium requires all demands and prices to be positive. This is the case iff. $a \leq 3t$. Otherwise, if the value of innovation is very large, innovation is drastic. Then, there is no equilibrium where the two firms face a positive demand. The leader sets its price at the limit where demand for the laggard is nul, even if she gives its good for free. The binding condition is for the customer located in 1. As a result, we have $\Pi_A = a - t$ and $\Pi_B = 0$. Then, the profit of the leader is decreasing in $t$, which is mainly an artefact of the model.

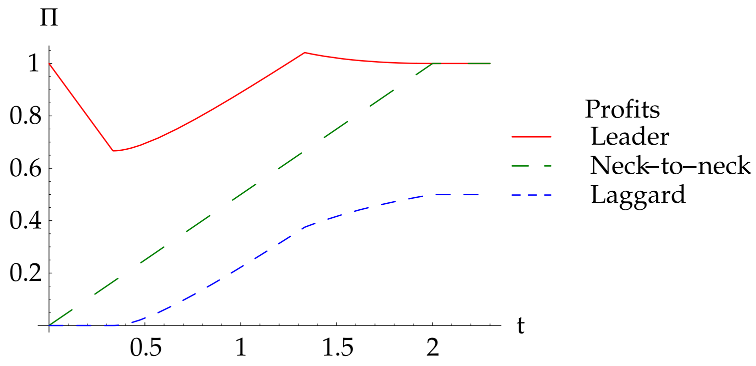
When the innovation is not drastic, firms may still meet the price cap that is the consequence of the existence of a competitive fringe. This would happen for $t \geq \frac{4a}{3}$ or $t \geq \frac{5a}{3}$ respectively for the laggard and for the leader. The laggard will then be the first to meet this constraint. It will set its price at its maximum ($p_A = a$), given the constraint of the competitive fringe (its demand is null for a higher price). The best answer by the leader will be $p_B = a + \frac{t}{2}$. Thus, he also meets its constraint if $t \geq 2a$.

At last, we have:

$$\Pi_1 = \begin{cases} a-t & \texttt{if} \quad t \leq \frac{a}{3} \\ \frac{t}{2}(1+\frac{a}{3t})^2 & \texttt{if} \quad \frac{a}{3} \leq t \leq \frac{4a}{3} \\ \frac{t}{2}(\frac{1}{2}+\frac{a}{t})^2 & \texttt{if} \quad \frac{4a}{3} \leq t \leq 2a \\ a & \texttt{if} \quad t \geq 2a \end{cases}$$

$$\Pi_{-1} = \begin{cases} 0 & \texttt{if} \quad t \leq \frac{a}{3} \\ \frac{t}{2}(1-\frac{a}{3t})^2 & \texttt{if} \quad \frac{a}{3} \leq t \leq \frac{4a}{3} \\ a(\frac{3t}{2}-a) & \texttt{if} \quad \frac{4a}{3} \leq t \leq 2a \\ \frac{a}{2} & \texttt{if} \quad t \geq 2a \end{cases}$$

FIGURE 8: Profits ($a = 1$)

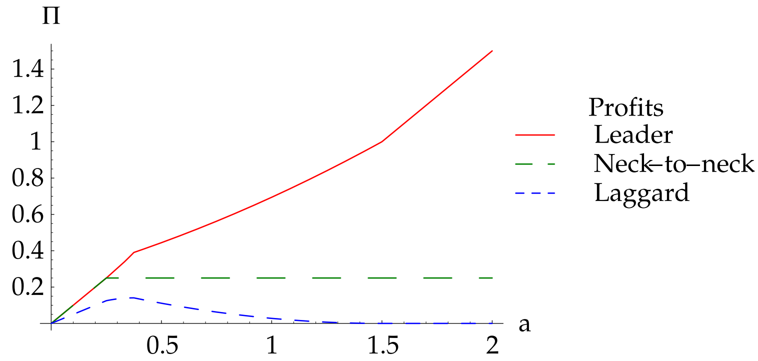



**Note:** $a = 1$, $t \in [0, 2]$.

For a given value of the innovation, it is drastic whenever transportation costs are small. The innovator is then a monopolist. As it cannot price discriminate, an increase of transportation costs induce a decrease of price, with a constant demand. The pattern is then decreasing for a small transportation cost. When the transportation cost is large enough, both firms are active in the market. Hence, both profits are increasing. For even larger transportation costs, the laggard is constrained by its competitive fringe. He is acting more competitively. As a reaction, the leader is also acting more competitively, but his price still increases in $t$. As a result, its demand is decreasing more rapidly in $t$. Overall, profits of the laggard significantly increase, while the profit of the leader is almost constant but slightly decreasing. Figure 8 shows an example of the three profits for $a = 1$.

For a given transportation cost, and a small value of innovation, incumbents in all situations are under the threat of their competitive fringes. All profits are driven by this value and all profits are thus increasing in $a$, with a slope of one for the leader and neck-to-neck firms and of a half for the laggard. When the value of the innovation

Figure 9: Profits ($t = \frac{1}{2}$)




**Note:** $a \in [0, 2]$, $t = \frac{1}{2}$.

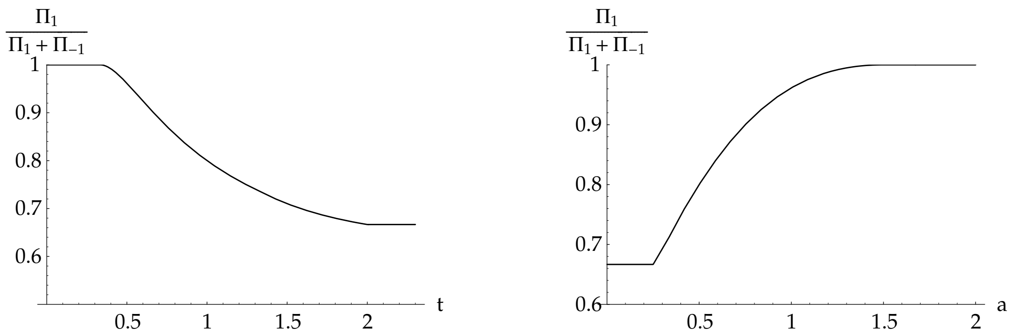
is large enough, the profit is constant for neck-to-neck (it is driven exclusively by transportation costs). Conversely, after the first threshold for the laggard, profits are quite intuitively increasing in $a$ for the leader, and decreasing for the laggard. Figure 9 shows an example of the three profits for $t = \frac{1}{2}$.

These graphs present some insight on the variations of profits. First, all profits are showing sharp patterns. The evolutions of the profits of laggards and leaders are by no mean of second order compared to this of neck-to-neck firms. Moreover, profits of neck-to-neck firms are always larger than profits of laggards, except when both are nul (either because $a$ or $t$ are null). At last, each neck-to-neck firm do as well as the leading firms only when both are constrained by their competitive fringe, i.e. when the value of the innovation is small compared to the transportation cost. Then, horizontal differentiation is qualitatively what matters for leaders and neck-to-neck firms and innovation plays a secondary role. For these values, the laggard firms also make their highest profits, which are solely related to $a$.

More generally, it is interesting to draw the share of the industry profit that is made by the leading firm (Figure 10). The parametrization of this Hotelling model was expected to provide proxies for "competition". Boone (2004) argues that a good proxy should be such that the share of the profit of the most competitive firm should be increasing in this proxy. In our example, both $t$ and $a$ meet this criteria: the share of profit of the leader in leveled markets is weakly increasing in $a$, and weakly decreasing in $t$ (see figure 10).[15] Overall, a larger $t$ would mean "less competition",

[15] Besides, the profits of the least advanced firms should be decreasing in this proxy. Here, the least advances firms are the competitive fringe and this criteria is not really relevant as they do not sell. However, if we consider the laggard as the least advanced firm in this model, $t$ is not a good proxy anymore. $a$ still is when the laggard is not constrained by its competitive fringe.

FIGURE 10: Share of industry profit of the leader in unleveled markets




$a = 1$, $t \in [0, 2]$ $a \in [0, 2]$, $t = \frac{1}{2}$

**Note:** The share of profits of the leader in leveled markets is $\frac{\Pi_1}{\Pi_1 + \Pi_{-1}}$.

while a larger $a$ would mean “more competition”. It is also interesting to focus on the set of parameter where innovation is not drastic, and where no firm is constrained by its competitive fringe. This will happen for $\frac{a}{3} \leq t \leq \frac{4a}{3}$. Then, the share of profits of the leading firm is strictly monotonic and range between 0.74 and 1. When both firms are constrained by their competitive fringes, it equals 0.67.

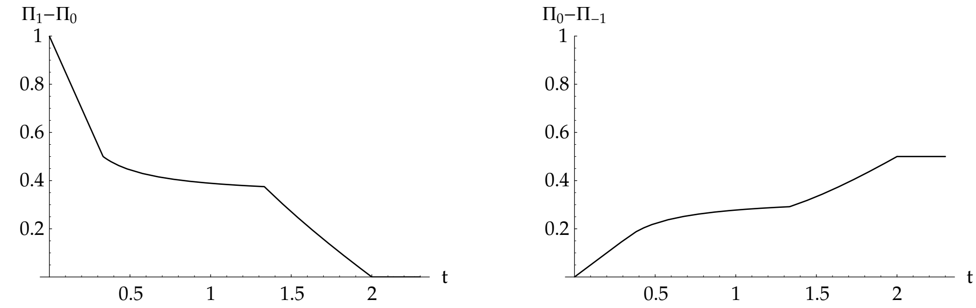
## 2.2 INNOVATION

Levels of profits are not directly relevant for innovation rates, nor are the shares of profits in a particular state. Efforts of innovation are directly driven by the variations of profits subsequent to a modification of relative technological levels. $n_1^*$ and $n_0^*$ are solely driven by $\Pi_1 - \Pi_0$. They are increasing as the direct benefits of innovation increase. For $n_{-1}^*$, direct and indirect effects are entangled. $\Pi_0 - \Pi_{-1}$ has a direct and positive influence on $n_{-1}^*$. However, it is also influenced by $n_0^*$, and thus by $\Pi_1 - \Pi_0$ in a non-monotonic way. The intuition of the previous part was that direct effects are stronger than indirect ones, but proving such a general result is quite difficult and this intuition might fail.

Understanding the way $a$ and $t$ influence the innovation at the equilibrium first requires to understand how the differences in profits change when $a$ or $t$ increase. We will first focus on variations of $t$ for a given $a$. The corresponding graphs can be found in figure 11. Both patterns are monotonic. The differences are very seldom null: it only happens when the value of the innovation is small compared to the transportation costs (then $\Pi_1 = \Pi_0$) or, conversely, when it is large (then $\Pi_0 = \Pi_{-1}$). If we focus, as before, on the set of parameters where innovation is not drastic, and where no firms

Figure 11: Variation of the relative profits ($a = 1$)

$\Pi_1 - \Pi_0$ $\Pi_0 - \Pi_{-1}$

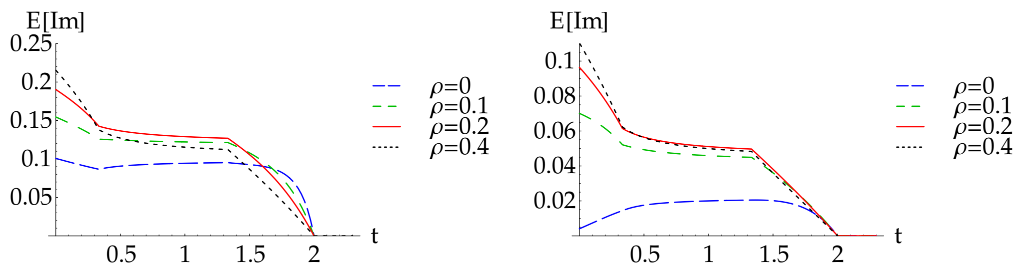



**Note:** $a = 1$, $t \in [0, 2]$.

is constrained by its competitive fringes, none of the differences is null. $\Pi_1 - \Pi_0$ then ranges between 0.17 and 0.29 and $\Pi_1 - \Pi_0$ between 0.38 and 0.51. When both firms are constrained by their competitive fringes, the differences respectively equal to zero and a half. At last, it is interesting to note that an increase of one difference due to variations of the "structural" parameters is, in general, at the detriment of the other one. On this issue, an increase of $t$ for a given $a$ is qualitatively close to an increase of $\Pi_0$, given $\Pi_{-1}$ and $\Pi_1$.

Figure 12: Pace of innovation ($a = 1$)

$\delta = 0.9$ $\delta = 0.5$




**Note:** $\gamma = 5$, $a = 1$, $t \in [0, 2]$.

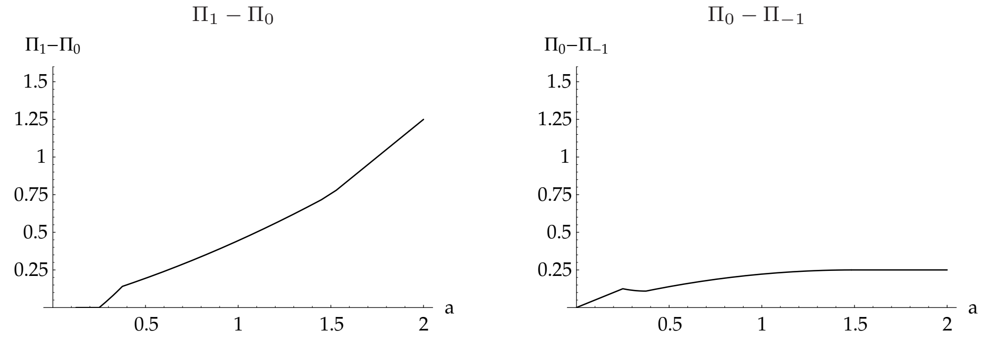
It is then possible to plot few examples of paces of innovation as functions of $t$ for a given $a$, $\gamma$ and $\delta$ (here, $a = 1$, $\gamma = 5$ and $\delta \in \{0.5, 0.9\}$). Such graphs are presented in Figure 12. The first part corresponds to sets where t is very small and innovation is drastic ($t \leq \frac{1}{3}$). Then, innovation rates all decrease, irrespective of $\rho$, except when *delta* and $\rho$ are small (in the example, $\delta = 0.5$ and $\rho = 0$). For higher values of $t$, the results are quite similar to these in the previous section. This was quite expected as an increase of $t$ qualitatively has the same effect of an increase of

$\Pi_0$, the other profits given: it increases the direct benefits to catch up, but decreases the benefits to outdistance its rival. Recall that here the laggard is constrained by its competitive fringe when $t \geq \frac{4}{3}$. From a qualitative standpoint, the decreasing part always occur after this value. When both firms are constrained, the directs incentives of the laggard firms are maximal, and neck-to-neck firms and leaders have no incentive. The probability to be in an unleveled market is null, and there is no effort is leveled markets. For the intermediate values where the laggard is constrained but not the leader, the incentives of the laggard are weakly increasing, while the incentives of leaders and neck-to-neck firms are collapsing (the profit of the leader is quite flat but the profit of neck-to-neck firms are strongly increasing). Overall, the average rate of innovation for the market is also collapsing. Otherwise, when innovation is not drastic and no firm is constrained, all patterns are quite flat and transportation costs play no role for innovation.

We then focus on variations of $a$ for a given $t$. Examples of the differences in profits can be found in Figure 13. Except for a small set of $a$ for a given $t = \frac{1}{2}$, both patterns are increasing. From a qualitative standpoint, the situation is very different to an increase of $t$ or to an increase of $\Pi_0$, the other profits given. For very small values of a, all firms are constrained by their competitive fringes. The set-up is such that leaders and neck-to-neck firms are making the same profits, and the laggard is making half of it. The probability to be in an unleveled state is then null, and so is the effort is the leveled state. For slightly larger values of $a$ ($a \geq \frac{1}{4}$), neck-by-neck firms are not constrained anymore, and their profits are solely driven by $t$. Then, as soon as $a \geq \frac{3}{8}$, the laggard is not constrained anymore, its profit is decreasing until it gets null when innovation is drastic. This is the opposite for the leader. On the contrary to the previous case, an increase of one difference does not occur at the detriment of the second one. An increase of $a$ generally increases the difference between the profit of the leader and the profit of neck to neck firms (that directly triggers innovation for both types of firms). However, it also increases the difference between the profits of neck-to-neck firms and laggards (that directly triggers innovation by the latter). Thus, the consequences of an increase of $a$ for a given $t$ are likely to be very different to those of an increase of $t$ for a given $a$ (or an increase of $\Pi_0$ for given $\Pi_{-1}$ and $\Pi_1$).

As before, we present examples of graphs of average innovation rates for the market as a function of $a$ in Figure 14. For very small values of a, all firms are constrained by their competitive fringes. Leaders and neck-to-neck firms are making the same profits. The probability to be in an unleveled state is then null, and so is the effort is the leveled state. When no firm is constrained, profits of neck-by-neck are constant (they are solely driven by $t$), while the profits of leaders and laggards are diverging. Thus,

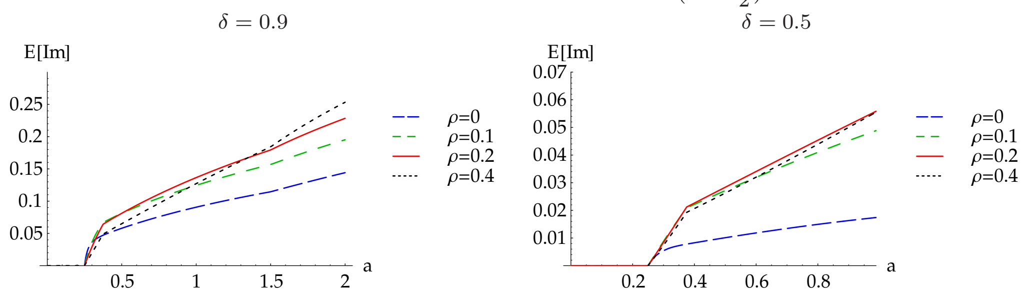
Figure 13: Variation of the relative profits ($t = \frac{1}{2}$)




**Note:** $a \in [0, 2]$, $t = \frac{1}{2}$.

both $\Pi_0 - \Pi_{-1}$ and $\Pi_1 - \Pi_0$ are increasing. As the latter is increasing more rapidly than the latter, average rates of innovation is increasing. For intermediate values of a ($\frac{1}{4} \leq a \leq \frac{3}{8}$), the first difference is decreasing while the second one is increasing. Then, average rates of innovation are also increasing. Overall, the pattern is very different to this presented in previous sections, even though $a$ also seems to be a relevant proxy for "competition": here "more competition" always mean "more innovation".[16] Besides, the previous conclusion that some 'free" innovation for laggards might be useful is also robust, at least to the specification of this model of market competition.

Figure 14: Pace of innovation ($t = \frac{1}{2}$)




**Note:** $\gamma = 5$, $a \in [0, 2]$, $t = \frac{1}{2}$.

The choice of this model is arbitrary. Other models might be more realistic. However, it shows that the link between "competition" and innovation in this type of framework is not obvious. For a given $a$, the results are very similar to these in the previous section. However, then symmetric duopoly profits might not take all the

[16] An increase of $a$ for a given $t$ reduces the share of the profit going to the less efficient firm. Besides, $a$ is a pure proxy for competition in Boone's sense when the laggard is constrained, but not the leader. However, the average rate of innovation for the market is increasing in $a$ for these values.

values between the profits of laggards and leaders. A significant pattern would emerge only in extreme parts of this model. The stylized fact remains that in this framework, patterns are either quite flat, or collapsing as markets get very uncompetitive. Relying on this parameter, the general conclusion would be that innovation is either neutral, or positive. On the contrary, given another proxy ($a$, for a given $t$), innovation would uniformly be increasing.

This raises important interpretative issues. First, in this context, it shows that the interplay between 'competition" and innovation is complex. $a$ is a measure of vertical differentiation and the model implicitly refers to vertical innovation. It is thus quite easy to understand that the influence of vertical differentiation on vertical innovation is very different than the impact of horizontal differentiation ($t$), even though they are intuitively perfect proxies for "competition". This conclusion has to be linked to the results in (Boone 2000) on competitive pressure, product and process innovation. Static profits, and hence the behavior of firms, are the consequences of differentiation, either vertical or horizontal. Innovation has to do with strategically acting on this differentiation. Causality is in both directions and it is hard to separate the two questions.

More fundamentally, the fact that various proxies of "competition" might not have the same impact on innovation indicates that "competition" is not a well defined concept. From a theoretical standpoint, either competition is perfect or there exist strategic interactions. In this latter case, firms' behaviors are influenced by the expected behaviors of the other firms and by exogenous parameters. When firms collude instead of competing, when firms exclude their rivals, one can say that, on this positive basis, a situation is more of less competitive. On the contrary, ranking these exogenous parameters on a normative basis as being more or less "competitive" is essentially artificial. Thus, the question of the link between "competition" and "innovation" is not a well posed theoretical problem; one should not refer to the reduced form concept of competition but on better defined concepts.

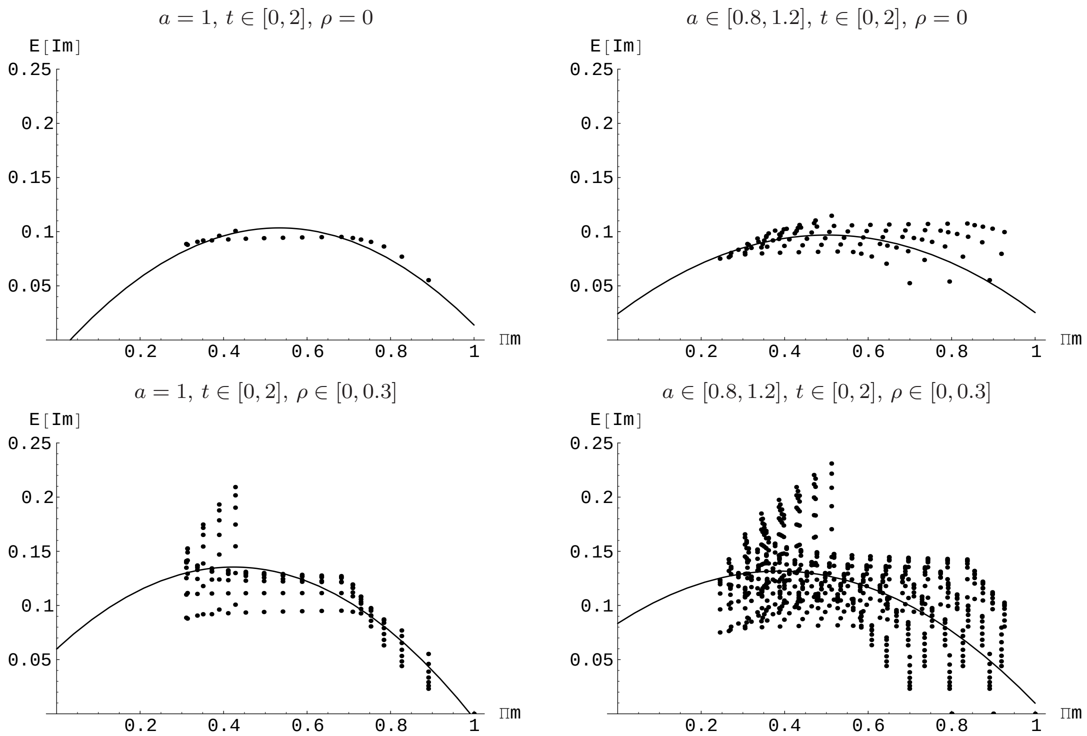
## 2.3 Empirical implications

One primordial step of the identification of the relationship between "competition" and innovation is empirical. The previous example of foundation of the various profits by a model of product market competition raises identification issues. One appealing empirical strategy using cross-section data might be to infer a potential relationship by plotting the average rate of innovation as a function of "profits". These profits will be hard to measure, and would probably be proxied by average markups in a market.

Here, this average markup would be the average profit:

$$\Pi_m \quad = \quad \mathbb{P}\{L\}\,\Pi_0 + \mathbb{P}\{U\}\,\frac{\Pi_{-1}+\Pi_1}{2}$$

Figure 15: Simulation of outputs and identification




**Note:** $\gamma = 5$, $\delta = 0.9$.

*A priori*, there is no reason to believe that all markets have the same values of innovation or the same transportation costs, or even the same $\rho$. Figure 15 depicts examples of simulated outputs for various simulations of outputs and of quadratic fits of the relation (quadratic fit is the simplest way to generate non monotonic estimates). The two graphs of the first row present the result of simulations using a null $\rho$ and $t$ uniformly distributed between 0 and 1. On the left, there is no heterogeneity of $a$, while in the second one, $a$ is uniformly distributed between 0.8 and 1.2. In both cases, the average markup never takes small values. The second row adds heterogeneity on $\rho$ to the two previous simulations ($\rho$ uniformly distributed on [0,0.3]). In all examples, quadratic fit shows a very pronounced inverted-U pattern. In these situations, there is however no general pattern of innovation as a function of all the structural parameters. There is thus a high risk of spurious identification when interpreting this fit as a causal non-monotonic impact of "competition" on innovation.

# CONCLUSION

Aghion et al. (2006) proposed an elegant dynamic framework to analyze the link between "competition" and innovation. It is one of the first paper to present the idea that the relation between "competition" and "innovation" is complex and might not be monotonic. This paper proposes to go a step further into the analysis.

First, some 'free" and simultaneous innovation for laggards might be useful. Thus, increasing the costs to catch-up for laggard, for instance by very restrictive patent policies, might not be optimal if they result in long periods in unleveled states. Alternatively, it might even be useful to help laggards to catch-up. This is all the more true that discount factors for the future are large, or that the probability to survive is low. It is never socially optimal either to provide laggards with a too large probability to catch up. This result is robust to the specification of product market competition.

Secondly, in the absence of simultaneous innovation by laggards, the dynamic output is very close to this of the model in Aghion et al. (2006). The latter is then qualitatively nested within the model in this paper. However, the inverted-U pattern is not robust to the introduction of automatic innovation by the laggard. From a qualitative standpoint, except when firms' discount for the future is very high, without "free" innovation for the laggard, innovation mainly collapse as symmetric duopoly profits get very close to the profit of the leading firm, either because of very high differentiation, or collusion. Otherwise, the pattern of innovation is quite flat compared to this collapse. Overall, the impact of a lower symmetric duopoly profit would either be strongly positive, or weakly negative. The optimal is a very low symmetric duopoly profit, associated with a positive simultaneous innovation by laggards.

Finally, the model gives very clear conclusions on the impact of collusion on innovation, if innovation is mainly due to competition. Potential innovation does not prevent firms to collude. On the contrary, collusion does hinder innovation. Antitrust authorities have all reasons to be concerned by collusion is potentially innovative industries.

In its second part, this paper proposes a foundation of duopoly profits, through the use of a simple model of differentiation with a competitive fringe. This foundation raises important interpretative issues. The interplay between innovation and "competition" is likely to be very complex. Firms innovate to increase their differentiation, either vertical or horizontal, or to reduce their costs. Competition is also related to substitutability, differentiation, barriers to entry, and other strategic behaviors. Disentangling innovation from competition as a normative concept is a tricky issue. From a theoretical point of view, this probably means that one should not first refer to the reduced form concept of innovation, but rather to more clearly defined concepts.

From an empirical standpoint, this means that the relationship between "competition" and innovation should not be sought to be identified using cross-section data, but on the longitudinal dimension and/or relying on structural models. This internal validity could come at the price of a reduced external validity. One would then for instance identify the impact of barriers to entry on innovation, instead of the impact of "competition" as a general concept.

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# A Static strategic interactions

## A.1 General model

We consider an economy where two firms are competing in a final product market. Their profits depend of their relative technological levels. For simplicity, we only consider that firms can at most differ by one level of technology. They may either be neck-to-neck and both get profits $\Pi_0$. Alternatively, one firm may be in advance and get profit $\Pi_1$. Then, the laggard firm gets $\Pi_{-1}$. As before, we only assume $\Pi_{-1} \leq \Pi_0 \leq \Pi_1$. As in Aghion et al (2006), both firms may invest in step by step innovation, without opportunity for leapfrogging. As firms can at most differ by one level of technology, when the firm in advance innovates, the laggard automatically catch up of one level. Their relative levels remain unchanged. In order to allow for strategic interaction and simultaneous innovations, we consider a discrete time dynamic model. For each period, the schedule of the game is the following:

1. Given their technological levels, firms choose their prices or quantities. The output of the competition on the product market gives them profits $\Pi_{-1}$, $\Pi_0$ or $\Pi_1$.

2. Firms may invest in innovation.

3. Their possible success determines their technological levels for the next period.

Neck-to-neck firms or the leader in an unleveled industry may invest $\frac{\gamma}{2}n^2$ and then attain the next step of technology with probability $n$. When the laggard invests $\frac{\gamma}{2}n^2$, she succeeds with probability $n + \rho$. The existence of the $\rho$ is a way to account for the fact that catching-up is less costly for the laggard and that it could occur for free with non null probability. This could for instance be a proxy for patent protection.[17]

In an unleveled industry, a leader $i$ faces a laggard $j$. Both firms choose their efforts of innovation, respectively $n_{1i}$ and $n_{-1j}$, in order to maximize their discounted profits, respectively $V_{1i}$ and $V_{-1j}$. We have:

$$\begin{aligned} V_{1i} &= \Pi_1 - \tfrac{\gamma}{2}n_{1i}^2 + \delta\left[(1-n_{1i})(n_{-1j}+\rho)V_{0i} + (1-(1-n_{1i})(n_{-1j}+\rho))V_{1i}\right] \\ V_{-1j} &= \Pi_{-1} - \tfrac{\gamma}{2}n_{-1j}^2 + \delta\left[(1-n_{1i})(n_{-1j}+\rho)V_{0j} + (1-(1-n_{1i})(n_{-1j}+\rho))V_{-1j}\right] \end{aligned}$$

Alternatively, in a leveled market, both firms $i$ and $j$ are neck-to-neck. They both choose their efforts $n_{0i}$ and $n_{0j}$ in order to maximize their discounted profits. For $i$,

[17] This paper shows that $\rho$ is often beneficial to innovation. This could speak in favor of subsidies for research and development specifically targeted at laggards.

we have ($j$'s discounted profit is symmetric):

$$\begin{aligned} V_{0i} &= \Pi_0 - \tfrac{\gamma}{2}n_{0i}^2 + \delta\left[(n_{0i}n_{0j} + (1-n_{0i})(1-n_{0j}))V_{0i}\right. \\ &\quad \left. +(1-n_{0j})n_{0i}V_{1i} + (1-n_{0i})n_{0j}V_{-1i}\right] \end{aligned}$$

We focus on Markov equilibrium. Then, stationary and symmetric equilibrium strategies would be:

$$\begin{cases} \gamma n_{-1}^* &= \delta(1-n_1^*)\left(V_0^* - V_{-1}^*\right) \\ \gamma n_1^* &= \delta(n_{-1}^* + \rho)\left(V_1^* - V_0^*\right) \\ \gamma n_0^* &= \delta\left((1-n_1^*)\left(V_1^* - V_0^*\right) + n_1^*\left(V_0^* - V_{-1}^*\right)\right) \end{cases}$$

The equilibrium discounted profits would be:

$$\begin{cases} (1-\delta)V_{-1}^* &= \Pi_{-1} - \tfrac{\gamma}{2}{n_{-1}^*}^2 + \delta(1-n_1^*)(n_{-1}^*+\rho)(V_0^* - V_{-1}^*) \\ (1-\delta)V_1^* &= \Pi_1 - \tfrac{\gamma}{2}n_1^{*2} - \delta(1-n_1^*)(n_{-1}^*+\rho)(V_1^* - V_0^*) \\ (1-\delta)V_0^* &= \Pi_0 - \tfrac{\gamma}{2}n_0^{*2} + \delta n_0^*(1-n_0^*)(V_1^* - V_0^* - (V_0^* - V_{-1}^*)) \end{cases}$$

The intuition behind the three first equations is quite straightforward: the laggard firm invests to catch-up with the leading firm. The latter invests to protect its technological advance. Neck-to-neck firms invest to outdistance their rivals, but also not to lag behind. The direct strategic interaction is the following: the incentive to catch up is smaller if the leading firm invests more, while the incentive to protect its leadership is larger when the risk to be caught up is high. At last, the incentive of neck-to-neck firms is a mix, and the relative importance of not being outdistanced is higher when the risk is higher.

However, solving this dynamic game leads to three nonlinear equations with no obvious solutions. Hence, it cannot be solved, in general. Besides, this model entangles direct strategic interactions (private efforts depend of the anticipated efforts of the rival) and dynamics. It is useful to understand the relative influence of both.

## A.2 Static strategic interactions

To get a flavor of the influence of static strategic interactions, it is useful to solve the game with two periods. Then, the three equations for control variables simply become:

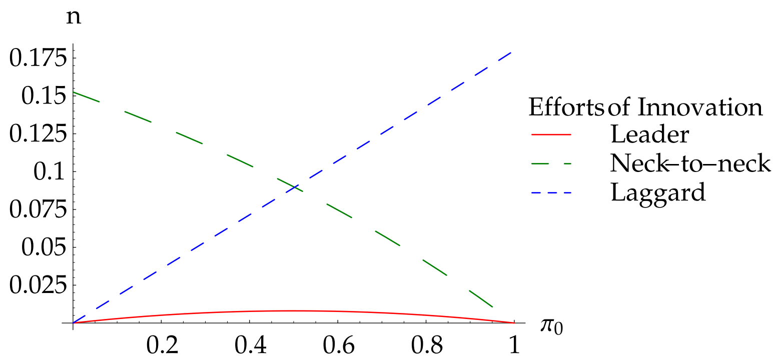
$$\begin{cases} \gamma n_{-1}^* &= \delta(1-n_1^*)\left(\Pi_0 - \Pi_{-1}\right) \\ \gamma n_1^* &= \delta(n_{-1}^* + \rho)\left(\Pi_1 - \Pi_0\right) \\ \gamma n_0^* &= \delta\left((1-n_0^*)\left(\Pi_1 - \Pi_0\right) + n_0^*\left(\Pi_0 - \Pi_{-1}\right)\right) \end{cases}$$

This model includes the same direct incentives for investing in innovation, apart from the indirect influence of dynamics. The leader is all the more threatened by the laggard that the advantage of being the leader and the risk to be caught up are high. Conversely, the laggard is all the more willing to invest as she will benefit to be in a leveled market and as the risk of a simultaneous innovation by the leader is small. The effort of the leader is a strategic substitute for the laggard but the effort of the laggard is a strategic complement for the leader. At last, neck-to-neck firms' incentives are mainly driven by the risk of being outdistanced when the effort of the other firm is high. On the contrary, it is mainly driven by the hope of outdistancing her rival when her effort is small. We show that there exist a unique interior equilibrium: [18]

**Proposition A.1** *If $\gamma(1-\rho) \geq \delta(\Pi_1 - \Pi_{-1})$, there is a unique interior equilibrium. At the equilibrium, for given $\Pi_{-1}$ and $\Pi_1$:*

1. *$n_0^*$ is decreasing in $\Pi_0$ and in $\gamma$, increasing in $\delta$, and independent of $\rho$;*
2. *$n_{-1}^*$ is decreasing in $\gamma$ and $\rho$, and increasing in $\Pi_0$ and $\delta$;*
3. *$n_1^*$ is decreasing in $\gamma$ and increasing in $\rho$ and $\delta$.*
    - *If $\rho \leq \rho_{lim}[\gamma, \delta, \Pi_1 - \Pi_{-1}]$, $n_1^*$ has an inverted-U shape in $\Pi_0$;*
    - *otherwise, it is decreasing in $\Pi_0$.*

Figure 16: Efforts of Innovation ($\rho = 0$)

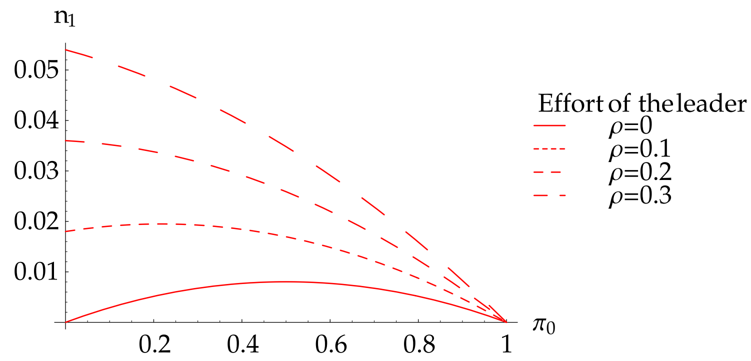



**Note:** $\delta = 0.9$, $\gamma = 5$, $\rho = 0$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

The impact of $\gamma$ and $\delta$ are quite straightforward. If innovation is less costly or if the firms have a lower discount for the future, all firms will invest more. The impact of $\rho$ is also intuitive: a higher probability of success for the laggard increases the effort of the leader. This also has the indirect effect of reducing the voluntary effort of the laggard. The direct effects of an increase of $\Pi_0$ is to increase the benefits to catch-up, while it reduces the benefits to outdistance its rivals and the loss to be caught up. For the laggard and neck-to-neck firms, this direct effect is always the stronger. For the

[18] The proofs of propositions are in appendix.

Figure 17: Efforts of Innovation ($\rho = 0.2$)




**Note:** $\delta = 0.9$, $\gamma = 5$, $\rho = 0.2$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

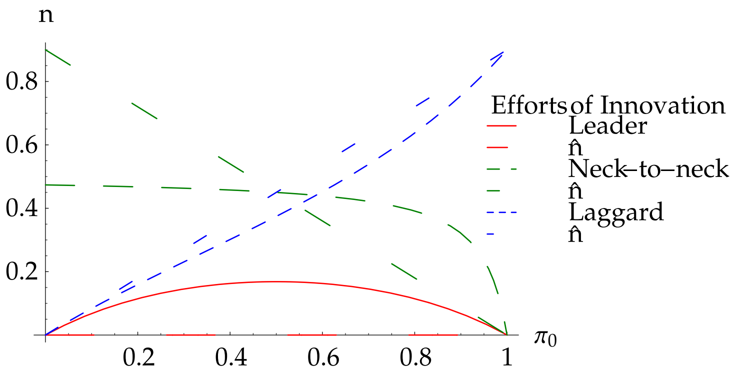
leader, an increase of the symmetric duopoly profits decreases the loss to be caught up but also increases the risk (as, overall, the effort of the laggard increases). The set-up is such that the stronger the risk, the smaller the loss. With a null $\rho$, it generates an inverted-U pattern for the effort of the leader: when the risk is limited and the loss is large, an increase of both induce a larger relative increase of the risk, and vice versa. When $\rho$ is larger, the risk to be caught-up is always strong, whatever value of $\Pi_0$. Then, the direct effect due to the reduction of the loss is always the stronger. Figure 16 show one examples of efforts of all firms as functions of $\Pi_0$ for $\rho = 0$. The three efforts are plot in the same figure. However, the efforts do not correspond to the same game: games in leveled and unleveled states are totally separated. For this value of $\rho$, the effort of the leader is non-monotonic. Figure 17 shows the effort of the leader as functions of $\Pi_0$ and $\rho$. It illustrate that for larger values of $\rho$, $n_1$ is monotonic.

In the previous examples, the slopes of the efforts of the laggard and of neck-to-neck firms were very close to the linear functions that would appear in the absence of static strategic interactions ($\widetilde{n}_{-1} = \frac{\delta(\Pi_0 - \Pi_{-1})}{\gamma}$ and $\widetilde{n}_0 = \frac{\delta(\Pi_1 - \Pi_0)}{\gamma}$ respectively). It is possible to analyze further what are the consequences of the static strategic interactions for these two types of firms. Here, we will limit our analysis to $\rho = 0$ and under the assumption of proposition A.1 ($\gamma \leq \delta(\Pi_0 - \Pi_{-1})$). Then, we have:

$$\begin{aligned}
\frac{n^*_{-1}}{\widetilde{n}_{-1}} &= \frac{\gamma^2}{\gamma^2 + \delta^2(\Pi_1 - \Pi_0)(\Pi_0 - \Pi_{-1})} &\in& \left[\tfrac{4}{5}, 1\right] \\
\frac{n^*_0}{\widetilde{n}_0} &= \frac{\gamma}{\gamma + \delta(\Pi_1 + \Pi_{-1} - 2\Pi_0)} &\in& \left[\frac{\gamma}{\gamma + \delta(\Pi_1 - \Pi_{-1})}, \frac{\gamma}{\gamma - \delta(\Pi_1 - \Pi_{-1})}\right]
\end{aligned}$$

The effort of the laggard is always very close to this without static strategic interactions. On the contrary, when the condition of proposition A.1 is weakly satisfied, strategic interactions have a dramatic impact on the effort of neck-to-neck firms. At the limit, when $\gamma = \delta(\Pi_0 - \Pi_{-1})$, we have $n^*_0 = \frac{1}{2}$. Static strategic interactions have the effect of smoothing the efforts of neck-to-neck firms. This effect is maximal when the value of the innovation $\delta(\Pi_0 - \Pi_{-1})$ is close to the cost of the innovation $\gamma$. It is very small otherwise (as in the previous examples where $\frac{\delta(\Pi_1-\Pi_{-1})}{\gamma} = 0.18$).

FIGURE 18: Efforts of the leader (limit case)




**Note:** $\delta = 0.9$, $\gamma = 1$, $\rho = 0$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

In Figure 16, the effort of the leader was quite small compared to this of the two other types of firms. For $\rho = 0$, $n_1^*$ is maximal for $\Pi_0 = \frac{\Pi_1+\Pi_{-1}}{2}$. For this value of $\Pi_0$, the ratio between $n_1^*$ and $n_{-1}^*$ is equal to $\frac{\delta(\Pi_1-\Pi_{-1})}{2\gamma}$. As previously, static strategic interactions have a significant effect for the effort of the leader only when the condition of proposition A.1 is weakly satisfied. Then, the effort of the leader is, at its maximum, equal to the half of this of the laggard. Otherwise, when the value of the innovation is smaller compared to its cost, the effort of the leader is negligible compared to this of the laggard. Figure 18 shows an example of evolutions of the efforts of the various types of firms in a limit case where $\frac{\delta(\Pi_1-\Pi_{-1})}{\gamma} = 0.9$. The effects of static strategic interactions are then maximal.

The conclusion as regards to static strategic interactions is twofold. When the value of the innovation ($\delta(\Pi_1 - \Pi_{-1})$) is relatively small compared to its cost ($\gamma$), static strategic interactions plays a qualitatively small role. $n_0^*$ and especially $n_{-1}^*$ are very close to their values without static strategic interactions. When $\rho$ is null, the effort of the leader presents a non-monotonic pattern, but it is overall negligible compared to this of the laggard. When $\rho$ is sufficiently positive (in the example before, this is always the case if it is larger than about 15%), the pattern of the effort of the leader is also very close to this without strategic interactions.[19]

On the contrary, when the incentives of innovation are maximal (at the limit of the

[19]The effort induced by $\rho$ is not *stricto sensu* a strategic interaction but is a consequence of the possibility of simultaneous innovations.

condition, some firms may invest in innovation with such an intensity that innovation occurs for sure), direct effects still dominate the indirect ones for the laggard and for neck-to neck firms. However, the pattern of the effort of the latter significantly differ from this without strategic interactions. As $\Pi_0$ increases, the advantage of outdistancing its rival decreases, but the loss of being outdistanced increases in the same time. The effort of both firms is a weighted average of the two incentives, the larger $n_0^*$, the heavier the weight on not being outdistanced. If the efforts of neck-to-neck firms are large enough, both effects might cancel each other out. It results a much flatter pattern, with a dramatic decrease as $\Pi_0$ gets very close to $\Pi_1$ (it is still null if $\Pi_0 = \Pi_1$, except at the frontier of condition in proposition A.1). Besides, the effort of the leader is still smaller than this of the laggard, but it is not negligible anymore. In the limit cases where $\delta(\Pi_1 - \Pi_{-1})$ is very close to $\gamma$, as condition in proposition A.1 has to hold, the pattern of $n_1^*$ is always non-monotonic. As $\gamma$ grows, there is a possibility for $\rho$ to be large enough so that $n_1^*$ is strictly decreasing in $\Pi_0$. For $\delta(\Pi_1 - \Pi_{-1}) = 1$, it will happen as soon as $\gamma \leq 1.75$ and then we have $\rho_{lim} = 0.43$. After this threshold in $\gamma$ (for a given $\delta(\Pi_1 - \Pi_{-1}) = 1$), $\rho_{lim}$ is decreasing in $\gamma$.

At last, it is important to note that the condition not to be in a corner solution in the dynamic game is more severe than in the two period game with strategic interactions. As firms account for a further future, the stakes of innovation are larger. This speaks in favor of larger efforts in the dynamic game and thus explains why the innovation has to be more costly to avoid corner solutions in the dynamic game than to avoid corner solutions in a two-period game.[20] For instance, for $\rho = 0$ and $\delta(\Pi_1 - \Pi_{-1}) = 1$, the condition in proposition A.1 is $\gamma \leq 1$ while it would be $\gamma \leq 4$ in proposition 1.1.[21] It has been shown in the previous section that in a two-period set-up, static strategic interactions have a significant impact only in limit cases where the condition in proposition A.1 is weakly satisfied. The severity of the condition in proposition 1.1 is such that with sets of parameters satisfying this conditions, static strategic interactions would be negligible in a two-period setup. Nonetheless, it is not sufficient to prove that they would not have any influence is a dynamic framework.[22]

[20] However, as efforts of innovation are mostly complementary (except for the laggard), the result was not obvious.

[21] More generally, the ration between limits in $\gamma$ in both setups is $\left(1 - \frac{3}{4}(1-\rho)\right)$ and both limits go to infinity as $\rho \to \infty$.

[22] In our example, $n_0^*$ could be as large as 0.5. With strategic interactions, neck-to-neck firms' incentives would then be the unweighted average between being outdistance and not being outdistanced. In the previous setup, this led to a very flat pattern.

## A.3 Influence of static strategic interactions: pseudo-dynamics

If it is not possible to fully solve the general dynamic model with static interactions, it is still possible to briefly analyse the pseudo dynamics that would be generated by the efforts in the simplified two period game. There, we would have:

$$\begin{aligned}\mathbb{P}\{U \to L|U\} &= (\rho + n^*_{-1})(1 - n^*_1)\\ \mathbb{P}\{L \to U|L\} &= 2n^*_0(1 - n^*_0)\end{aligned}$$

Besides, we will focus directly on the pace of innovation, which was previously equal to the innovation for the market. Here, we would have:

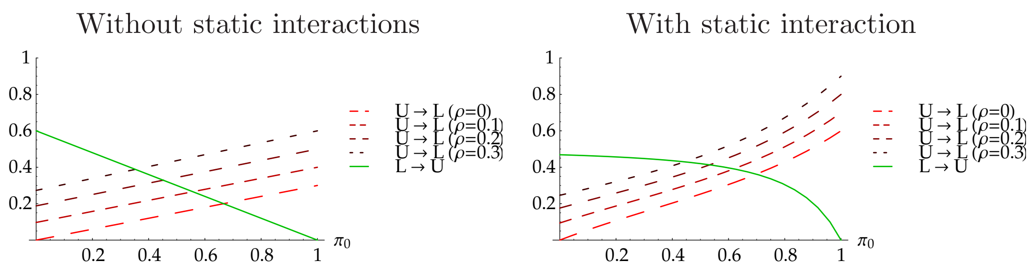
$$\begin{aligned}\mathbb{E}\{k_{t+1}|t, k_t, S_t = U\} - k_t &= n^*_1\\ \mathbb{E}\{k_{t+1}|t, k_t, S_t = L\} - k_t &= 1 - (1 - n^*_0)^2\end{aligned}$$

At last, understanding the influence of static strategic interactions cannot come from the comparison of the dynamics of the previous model with the pseudo-dynamics emerging from the two-period model. There are two main differences between the two models, and it is not possible to disentangle what difference between the two patterns is due to static strategic interaction and which is due to the dynamic interactions. Thus, we will compare the pseudo-dynamics generated by the simplified two-period game with this generated by the equivalent game without static interactions, namely the efforts $\widetilde{n}_{-1}$, $\widetilde{n}_0$ and $\widetilde{n}_1$ defined above. As seen previously, in the two-period model, strategic interactions have a qualitative influence only when the condition in proposition A.1 are weakly satisfied. Thus, we will focus on such sets of parameters.[23] For these values, the effort of the leading firm is always non-monotonic (see above). We then voluntarily focus on sets of parameters where an inverted-U pattern is the more likely to emerge for the average pace of innovation.

We first focus on the probabilities to switch form leveled to unleveled states and vice versa. Examples of plots are shown in figure 19 for two values of $\gamma$. $\rho$ has no influence in leveled industries. Two qualitative results emerge. First, the probability to switch from leveled to unleveled states is driven by the efforts of neck-to-neck firms. Static interactions have a similar effect on the earlier than on the latter. They flatten the pattern for small values of $\Pi_0$. The effort is then collapsing as $\Pi_0$ gets very close to $\Pi_1$, and so is the probability to switch. Static interactions also increase the convexity of the probability to switch from unleveled to leveled states. For small symmetric

[23] In the following examples, we use $\delta(\Pi_1 - \Pi_{-1}) = 0.9$ and $\gamma = 1.5$. Then, $\rho$ has to be smaller than 0.4 to avoid corner solutions.

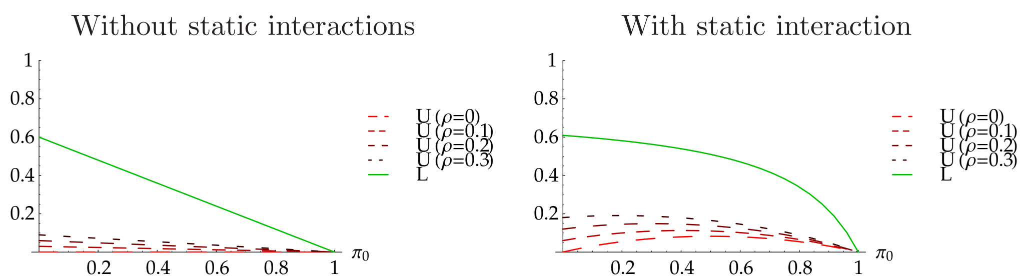
FIGURE 19: Probability to switch




**Note:** Pseudo-dynamics from the two-period simplified model. $\delta = 0.9$, $\rho \in \{0, 0.1, 0.2, 0.3\}$, $\Pi_{-1} = 0$, $\Pi_1{=}1$, $\Pi_0 \in [0, 1]$.

duopoly profits, the patterns with and without strategic interactions are quite close. As $\Pi_0$ grows, the pattern with static interactions increases much more rapidly, and to higher values. At last, the ranking for values of $\rho$ remains unchanged. Overall, the probability to be in the leveled state is always increasing in both $\Pi_0$ and $\rho$. The pattern in $\Pi_0$ is flatter for low values of $\Pi_0$ and increasing more rapidly as $\Pi_0$ gets closer to $\Pi_1$.

FIGURE 20: Pace of innovation

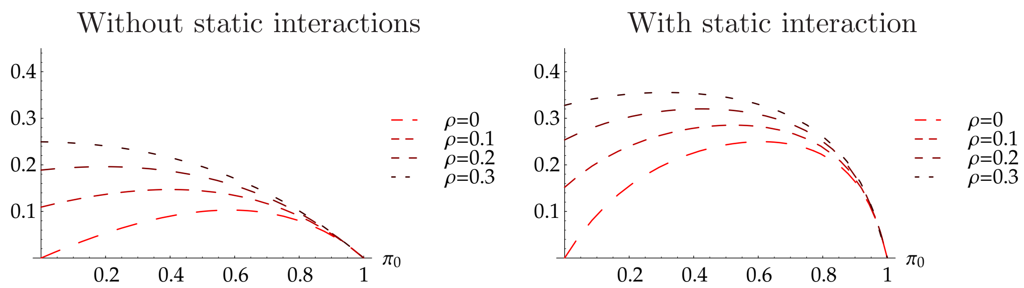



**Note:** Pseudo-dynamics from the two-period simplified model. "L" and "U" stand for leveled and unleveled states respectively. $\delta = 0.9$, $\rho \in \{0, 0.1, 0.2, 0.3\}$, $\Pi_{-1} = 0$, $\Pi_1{=}1$, $\Pi_0 \in [0, 1]$.

We then focus on the paces of innovation in each state. Examples of plots, with similar parameters as before, are presented in figure 20. As expected, with static interactions, the pattern in leveled industries is flatter and lower for small values of $\Pi_0$ and collapses more rapidly as it gets closer to $\Pi_1$. Besides, static interactions create a non-monotonic pattern for the pace in unleveled industries.

At last, we then focus on the average pace of innovation. An increase of $\Pi_0$ always increases the probability to be in the leveled state. It always decreases the pace of innovation in this state, which is always the most innovative. Without static

Figure 21: Pace of innovation




**Note:** Pseudo-dynamics from the two-period simplified model. $\delta = 0.9$, $\rho \in \{0, 0.1, 0.2, 0.3\}$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

interactions, it also decreases the pace in the unleveled state, while this pace has a non monotonic pattern with static interactions. Overall, it generates a non monotonic pattern for the average pace of innovation, either with and without static interactions. The pattern in the leveled state is flatter for low values of $\Pi_0$ with static interactions. Overall, as the pace in the unleveled state is then also itself non monotonic, the inverted-U pattern is more pronounced with static interactions.

With a higher $\rho$, the probability to be in the leveled state is always higher and never null, even when the symmetric duopoly profit equals the profit of the laggard. Besides, the pace of innovation in the unleveled state is higher for all value of $\Pi_0$. Overall, the non-monotonic pattern is less pronounced with static interactions.

This analysis enlightens the likely consequences of the absence of static strategic interactions in the dynamics of the model presented above. First, static interactions tend to bend the pattern of the average pace of innovation. Nonetheless, in the dynamic game, laggards always invest as catching up also has the option value of becoming a potential leader (this effect is maximal when the direct incentives are minimal). The effort of the leader would then be higher, as the leading firm reacts to the threat by increasing her own effort. Then, the pace with very low symmetric duopoly profits is dramatically underestimated by the two-period game. Even with a null $\rho$, the non-monotonic pattern in the unleveled state is likely to be less pronounced than in the two-period game (it would be closer to the pattern with a non null $\rho$). It remains that with very strong static interactions the pace in the leveled state is likely to be flatter for low symmetric duopoly profits and to collapse more rapidly as $\Pi_0$ gets very close to $\Pi_1$. As result, the non monotonic pattern might be more pronounced and arise for slightly larger values of $\rho$ but the inverted-U shall still not emerge for larger values of $\rho$, even in presence of static interactions.

Furthermore, in the simplified two-period game, an increase in $\rho$ is always beneficial to the pace of innovation. In the dynamic game, $\rho$ also influences the pace in leveled markets. As a result, a very large $\rho$ is still likely to be detrimental to the average pace of innovation. Nonetheless, the fact that some "free" and simultaneous innovation for laggards is beneficial to the pace of innovation shall remain unchanged. Overall, the two main conclusions of the previous section are likely to be very robust to the introduction of static interactions.

# B Proofs of Proposition 2.1

## B.1 Existence of an interior equilibrium

Equilibrium efforts of innovation are:

$$\begin{cases} n^*_{-1} &= \frac{\delta(\Pi_0-\Pi_{-1})(\gamma-\rho\delta(\Pi_1-\Pi_0))}{\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})} \\ n^*_1 &= \frac{\delta(\Pi_1-\Pi_0)(\gamma\rho+\delta(\Pi_0-\Pi_{-1}))}{\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})} \\ n^*_0 &= \frac{\delta(\Pi_1-\Pi_0)}{\gamma+\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1}))} \end{cases}$$

There exist an interior equilibrium iff $0 \leq n^*_0; n^*_1 \leq 1$ and $0 \leq n^*_{-1} + \rho \leq 1$. $n^*_0$ and $n^*_1$ are always positive, $n^*_{-1}$ is positive iff $\gamma \geq \rho\delta(\Pi_1 - \Pi_{-1})$ (1). Besides:

$$1 - n^*_1 = \frac{-\gamma\delta(\Pi_0 - \Pi_{-1}) + \delta^2(\Pi_0 - \Pi_{-1})(\Pi_1 - \Pi_0) + \gamma^2(1-\rho)}{\gamma^2 + \delta^2(\Pi_1 - \Pi_0)(\Pi_0 - \Pi_{-1})}$$

The numerator is maximum in $\Pi_{-1} + \frac{-\gamma+\delta(\Pi_{-1}-\Pi_0)}{2\delta} \leq \Pi_{-1}$ if $\gamma \geq \delta(\Pi_1 - \Pi_{-1})$ (2). Then, the numerator is always larger than its value for $\Pi_0 = \Pi_1$. It is then equal to $\frac{(1-\rho)\gamma-\rho\delta(\Pi_1-\Pi_{-1})}{\gamma}$. Thus, a sufficient condition is $(1-\rho)\gamma \geq \rho\delta(\Pi_1 - \Pi_{-1})$ (3). All three conditions (1), (2) and (3) are true if $(1-\rho)\gamma \geq \delta(\Pi_1 - \Pi_{-1})$ (C). We also have:

$$\begin{aligned} 1 - n^*_1 &= \frac{\gamma(\gamma-\delta\rho(\Pi_1-\Pi_0))}{\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})} \\ 1 - n^*_0 &= \frac{\gamma-\delta(\Pi_0-\Pi_{-1})}{\gamma+\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1}))} \end{aligned}$$

Thus, (C) is a sufficient condition for the existence of an internal equilibrium.

## B.2 Variations of $n^*_{-1}$

We have:

$$\frac{\partial n^*_{-1}}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} = \frac{\gamma\delta\left(\gamma^2+\delta^2(\Pi_0-\Pi_{-1})^2-\delta\gamma\rho(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1}))\right)}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}$$

The roots of the numerator are $\Pi_0 = \Pi_{-1} + \frac{+/-\sqrt{(\delta\gamma\rho)^2-\gamma\delta^2(\rho\delta(\gamma-(\Pi_1-\Pi_{-1})))}-\delta\gamma\rho}{\delta^2}$. Both roots are lower than $\Pi_{-1}$ if (C) is true. Thus, $n^*_{-1}$ is decreasing in $\Pi_1$.

Besides:

$$\begin{aligned}
\frac{\partial n_{-1}^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\gamma(\Pi_0-\Pi_{-1})(\gamma^2-\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})-2\gamma\delta\rho(\Pi_1-\Pi_0))}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}\\
\frac{\partial n_{-1}^*}{\partial \gamma}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\delta(\Pi_0-\Pi_{-1})(\gamma^2-\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})-2\gamma\delta\rho(\Pi_1-\Pi_0))}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}
\end{aligned}$$

$\gamma^2 - \delta^2(\Pi_1 - \Pi_0)(\Pi_0 - \Pi_{-1}) - 2\gamma\delta\rho(\Pi_1 - \Pi_0)$ has only one positive root:

$$\begin{aligned}
\tilde{\gamma} &= \underbrace{\delta\rho(\Pi_1 - \Pi_0)}_{\leq \delta\rho(\Pi_1-\Pi_{-1})} + \delta \underbrace{\sqrt{(\Pi_1 - \Pi_0)((\Pi_0 - \Pi_{-1}) + \rho^2(\Pi_1 - \Pi_0))}}_{\leq \frac{(\Pi_1-\Pi_{-1})}{2\sqrt{1-\rho^2}}}\\
&\leq \delta(\Pi_1 - \Pi_{-1})\left(\rho + \frac{1}{2\sqrt{1-\rho^2}}\right)\\
&\leq \frac{\delta(\Pi_1-\Pi_{-1})}{1-\rho}
\end{aligned}$$

Thus, under (C), $n_{-1}^*$ is decreasing in $\gamma$ and increasing in $\delta$.

At last:

$$\frac{\partial n_{-1}^*}{\partial \rho}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} = \frac{-\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}$$

Thus, $n_{-1}^*$ is decreasing in $\rho$.

## B.3 Variations of $n_0^*$

$$\begin{aligned}
\frac{\partial n_0^*}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\delta(\gamma-\delta(\Pi_1-\Pi_{-1}))}{(\gamma+\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1})))^2}\\
\frac{\partial n_0^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\gamma(\Pi_1-\Pi_0)}{(\gamma+\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1})))^2}\\
\frac{\partial n_0^*}{\partial \gamma}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\delta(\Pi_1-\Pi_0))}{(\gamma+\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1})))^2}\\
\frac{\partial n_0^*}{\partial \rho}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= 0
\end{aligned}$$

Thus, $n_0^*$ is independent of $\rho$, increasing in $\delta$ and decreasing in $\gamma$ and $\Pi_0$.

## B.4 Variations of $n_1^*$

We have:

$$\begin{aligned}
\frac{\partial n_1^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\gamma(\Pi_1-\Pi_0)(\gamma^2\rho-\delta^2\rho(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})+2\gamma\delta(\Pi_0-\Pi_{-1}))}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}\\
\frac{\partial n_1^*}{\partial \gamma}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\delta(\Pi_1-\Pi_0)(\gamma^2\rho-\delta^2\rho(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})+2\gamma\delta(\Pi_0-\Pi_{-1}))}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}
\end{aligned}$$

$\gamma^2\rho - \delta^2\rho(\Pi_1 - \Pi_0)(\Pi_0 - \Pi_{-1}) + 2\gamma\delta(\Pi_0 - \Pi_{-1})$ has only one positive root in $\delta$:

$$\begin{aligned}
\tilde{\delta} &= \frac{\gamma}{\rho}\frac{1+\sqrt{1+\rho^2\frac{\Pi_1-\Pi_0}{\Pi_0-\Pi_{-1}}}}{\Pi_1-\Pi_0}\\
&\geq \frac{\gamma}{\rho(\Pi_1-\Pi_0)}
\end{aligned}$$

Thus, under (C), $\delta \leq \tilde{\delta}$, and $n_1^*$ is increasing in $\delta$ and decreasing in $\gamma$.

Besides:

$$\frac{\partial n_1^*}{\partial \rho}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} = \frac{\gamma\delta(\Pi_1-\Pi_0)}{\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1})}$$

At last:

$$\frac{\partial n_1^*}{\partial \Pi_0}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} = \frac{\gamma\delta\left(\gamma\delta(\Pi_1-\Pi_0-(\Pi_0-\Pi_{-1}))+\rho(\gamma^2+\delta^2(\Pi_1-\Pi_0)^2)\right)}{(\gamma^2+\delta^2(\Pi_1-\Pi_0)(\Pi_0-\Pi_{-1}))^2}$$

The numerator has at most one root larger than $\Pi_{-1}$:

$$\Pi_{0lim} = \Pi_1 - \frac{\gamma}{\delta\rho}\left(1-\sqrt{1-\rho^2-\frac{\delta\rho}{\gamma}(\Pi_1-\Pi_{-1})}\right)$$

This root only exists if:

$$\begin{aligned} \rho \;&\leq\; \tilde{\rho_{lim}} = \frac{\delta(\Pi_1-\Pi_{-1})+\sqrt{4\gamma^2+\delta^2(\Pi_1-\Pi_{-1})^2}}{2\delta} \\ \text{and}\quad \rho \;&\leq\; \rho_{lim} = \frac{\gamma\delta(\Pi_1-\Pi_{-1})}{\gamma^2+\delta^2(\Pi_1-\Pi_{-1})} \end{aligned}$$

We have:

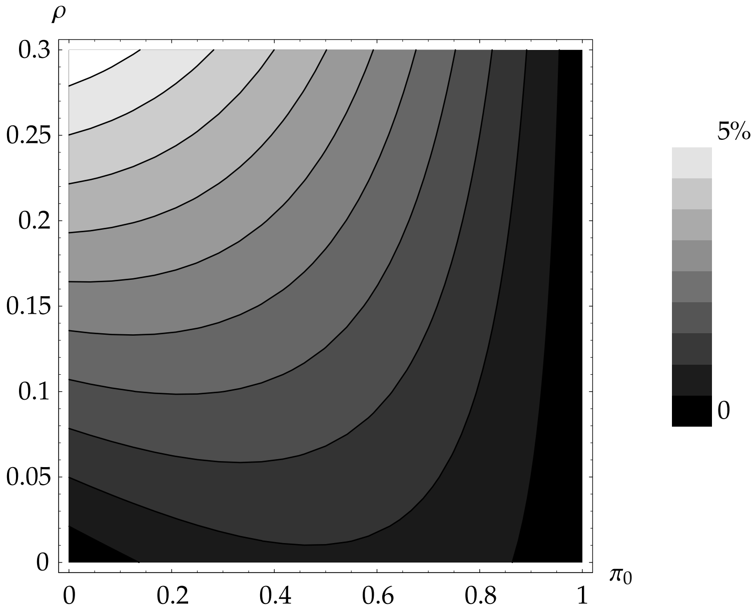
$$\tilde{\rho_{lim}} - \rho_{lim} = \frac{(\gamma^2+\delta^2(\Pi_1-\Pi_{-1}))\sqrt{4\gamma^2+\delta^2(\Pi_1-\Pi_{-1})^2}-(\Pi_1-\Pi_{-1})(3\gamma^2+\delta^2(\Pi_1-\Pi_{-1}))}{2\gamma(\gamma^2+\delta^2(\Pi_1-\Pi_{-1}))}$$

As:

$$(\gamma^2+\delta^2(\Pi_1-\Pi_{-1}))^2(4\gamma^2+\delta^2(\Pi_1-\Pi_{-1})^2)-(\Pi_1-\Pi_{-1})^2(3\gamma^2+\delta^2(\Pi_1-\Pi_{-1}))^2 = 4\gamma^6$$

$\rho_{lim}$ always bites before $\tilde{\rho_{lim}}$. Thus, for $\rho \leq \rho_{lim}$, $n_1^*$ has an inverted-U pattern, while it is always decreasing if $\rho$ is larger. $\rho_{lim}$ is increasing in $\delta(\Pi_1-\Pi_{-1})$ and decreasing in $\gamma$.

Figure 22: Efforts of the leader




**Note:** $\delta = 0.9$, $\gamma = 5$, $\rho \in [0, 0.3]$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$.

# C Proofs of Proposition 3.1

By replacing $n_1^*$ by $\rho n_0^*$, and by respectively subtracting $V_0^*$ to $V_1^*$ and $V_{-1}^*$ to $V_0^*$, we get the two following equations in $n_0^*$ and $n_{-1}^*$:

$$\begin{aligned} \delta\gamma(1-\rho^2){n_0^*}^2 + 4\gamma(1-\delta(1-\rho))n_0^* - 4\delta(\Pi_1^* - \Pi_0^*) &= 0 \\ \delta\gamma {n_{-1}^*}^2 + 2\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*))n_{-1}^* - \delta(\gamma {n_0^*}^2 + 4(\Pi_0 - \Pi_{-1})) &= 0 \end{aligned}$$

The first equation only has one positive root. Thus:

$$n_0^* = \frac{2(\sqrt{(\gamma(1-\delta(1-\rho)))^2+\gamma\delta^2(\Pi_1-\Pi_0)}-\gamma(1-\delta(1-\rho)))}{\gamma\delta(1-\rho^2)}$$

This is also the case of the second equation. Thus:

$$n_{-1}^* = \frac{\sqrt{(\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*)))^2+\gamma\delta^2(\gamma {n_0^*}^2+4(\Pi_0-\Pi_{-1}))}-\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*))}{\gamma\delta}$$

## C.1 interior equilibriums

Both solutions have been chosen to be positive. Hence, we have internal solutions if $n_1^* \leq n_0^* \leq 1$ and if $\rho + n_{-1}^* \leq 1$.

We have:

$$\frac{\partial n_0^*}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} = \frac{-\delta}{\sqrt{(\gamma(1-\delta(1-\rho)))^2+\gamma\delta^2(\Pi_1-\Pi_0)}}$$

It is decreasing in $\Pi_0^*$ and the constraint is harder to meet for $\Pi_0 = \Pi_{-1}$. Then:

$$1 - n_0^* \quad = \quad \frac{2\sqrt{(\gamma(1-\delta(1-\rho)))^2+\gamma\delta^2(\Pi_1-\Pi_{-1})}-\gamma(2-\delta(1-\rho)^2)}{\gamma\delta(1-\rho^2)}$$

This has only one root in $\gamma$ and is hence always positive if:

$$\gamma \quad \geq \quad \frac{4\delta(\Pi_1-\Pi_{-1})}{4-\delta(3-\rho)(1-\rho)}$$

Besides, using implicit functions derivation theorems, we have:

$$\begin{aligned}
\frac{\partial n_0^*}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\delta}{\gamma(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)} \\
\frac{\partial n_{-1}^*}{\partial n_0^*}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\delta(n_0^*-(1-\rho^2)n_{-1}^*)}{2-\delta(1-\rho)(2-(1+\rho)n_0^*)} \\
\frac{\partial n_{-1}^*}{\partial \Pi_0}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{2\delta}{\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*))}
\end{aligned}$$

We have:

$$\begin{aligned}
\frac{\partial n_{-1}^*}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\partial n_{-1}^*}{\partial \Pi_0}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} + \frac{\partial n_0^*}{\partial \Pi_0}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \frac{\partial n_{-1}^*}{\partial n_0^*}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \\
&= \frac{2\delta\big(2(1-\delta(1-\rho))+\delta(1-\rho^2)n_{-1}^*-\delta\rho^2 n_0^*\big)}{\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*))(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)}
\end{aligned}$$

The denominator is always positive. Besides, for $\gamma \geq \frac{4\delta(\Pi_1-\Pi_{-1})}{4-\delta(3-\rho)(1-\rho)}$:

$$\begin{aligned}
2(1-\delta(1-\rho)) + \delta(1-\rho^2)n_{-1}^* - \delta\rho^2 n_0^* &\geq 2(1-\delta(1-\rho)) - \delta\rho^2 \\
&\geq 2(1-\delta) + \delta(2\rho-\rho^2) \\
&\geq 0
\end{aligned}$$

Then, $n_{-1}^*$ is increasing in $\Pi_0$ and the condition in $n_{-1}^*$ is harder to meet in $\Pi_0 = \Pi_1$. Then, we have $n_0^* = 0$ and:

$$1 - \rho - n_{-1}^* \quad = \quad \frac{\gamma(2-\delta(1-\rho))-2\sqrt{(\gamma(2-\delta(1-\rho)))^2+\gamma\delta^2(\Pi_0-\Pi_{-1})}}{\gamma\delta}$$

This has always one root in $\gamma$ and is hence always positive if:

$$\gamma \quad \geq \quad \frac{\delta(\Pi_1-\Pi_{-1})}{(1-\rho)\big(1-\frac{3}{4}(1-\rho)\big)} \quad \left(\geq \frac{4\delta(\Pi_1-\Pi_{-1})}{4-\delta(3-\rho)(1-\rho)}\right)$$

We thus have a sufficient condition for an interior equilibrium.

## C.2 Variations of $n_0^*$

We have shown that $n_0^*$ is decreasing in $\Pi_0$. Using implicit functions derivation theorems, we have:

$$\begin{aligned}
\frac{\partial n_0^*}{\partial \rho}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\gamma\delta n_0^*(2-\rho n_0^*)}{\gamma(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)} \\
\frac{\partial n_0^*}{\partial \gamma}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-n_0^*(4-4\delta(1-\rho)+\delta(1-\rho^2)n_0^*)}{2\gamma(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)} \\
&= \frac{-2\delta(\Pi_1-\Pi_0)}{\gamma^2\gamma(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)} \\
\frac{\partial n_0^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-\gamma\delta^2(1-\rho^2){n_0^*}^2+4\gamma(1-\delta(2-\delta(1-\rho)))n_0^*+4\delta(\Pi_1^*-\Pi_0^*)}{2\gamma\delta(1-\delta)(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)} \\
&= \frac{2n_0^*}{\delta(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)}
\end{aligned}$$

Thus, $n_0^*$ is decreasing in $\rho$ and $\gamma$ and increasing in $\delta$.

## C.3 Variations of $n_1^*$

As $n_1^* = \rho n_0^*$, it is also decreasing in $\Pi_0$, $\gamma$ and increasing in $\delta$. Besides:

$$\begin{aligned}\frac{\partial n_1^*}{\partial \rho}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= n_0^* + \rho\frac{\partial n_0^*}{\partial \rho}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \\ &= \frac{n_0^*(2(1-\delta)+\delta n_0^*)}{2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*}\end{aligned}$$

Thus, $n_1^*$ is increasing in $\rho$.

## C.4 Variations of $n_{-1}^*$

We have already shown that $n_{-1}^*$ is increasing in $\Pi_0$. Similarly, using implicit functions derivation theorems, we have:

$$\begin{aligned}\frac{\partial n_{-1}^*}{\partial n_0^*}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\delta(n_0^*-(1-\rho^2)n_{-1}^*)}{2-\delta(1-\rho)(2-(1+\rho)n_0^*)} \\ \frac{\partial n_{-1}^*}{\partial \rho}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{-2\delta n_{-1}^*(1-\rho n_0^*)}{2-\delta(1-\rho)(2-(1+\rho)n_0^*)} \\ \frac{\partial n_{-1}^*}{\partial \gamma}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\delta({n_0^*}^2-{n_{-1}^*}^2)-n_{-1}^*\big(4-2\delta(1-\rho)(2-(1+\rho)n_0^*)\big)}{2\gamma(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\ &= \frac{-2\delta(\Pi_0-\Pi_{-1})}{\gamma^2(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\ \frac{\partial n_{-1}^*}{\partial \delta}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\delta^2({n_{-1}^*}^2+4(\Pi_0-\Pi_{-1})+\gamma n_{-1}^*\big(4-\delta(8+\delta(n_{-1}^*-2(1-\rho)(2-(1+\rho)n_0^*)))\big))}{2\gamma(1-\delta)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\ &= \frac{2n_{-1}^*}{2-\delta(1-\rho)(2-(1+\rho)n_0^*)}\end{aligned}$$

Then, direct effects of parameters are quite intuitive. Besides, we also have:

$$\begin{aligned}\frac{\partial n_{-1}^*}{\partial \rho}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\partial n_{-1}^*}{\partial \rho}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} + \frac{\partial n_0^*}{\partial \rho}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)}\frac{\partial n_{-1}^*}{\partial n_0^*}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \\ &= \frac{-\gamma\delta\big(\delta(2-\rho n_0^*)n_0^*+4(1-\delta(1-\rho))(1-\rho n_0^*)n_{-1}^*-\delta\rho(1-\rho^2)n_{-1}^*{n_0^*}^2\big)}{(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\ &\leq \frac{-\gamma\delta(4-\delta(4-\rho(3-\rho)))(1-\rho)n_{-1}^*}{(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\ &\leq 0\end{aligned}$$

Thus, $n_{-1}^*$ is decreasing in $\rho$.

We also have:

$$\begin{aligned}\frac{\partial n_{-1}^*}{\partial \gamma}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\partial n_{-1}^*}{\partial \gamma}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} + \frac{\partial n_0^*}{\partial \gamma}|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)}\frac{\partial n_{-1}^*}{\partial n_0^*}|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \\ &= \frac{1}{\gamma(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))}\Bigg\{ \\ &\quad \underbrace{-\gamma {n_0^*}^2(1-\delta(1-\rho))}_{\leq 0}\underbrace{-\delta {n_{-1}^*}^2(1-\delta(1-\rho)+\frac{1}{2}n_0^*\delta(1-\rho^2))}_{\leq 0} \\ &\quad \underbrace{-n_{-1}^*\left(4(1-\delta(1-\rho))^2+\frac{1}{2}{n_0^*}^2\delta^2(1-\rho^2)^2+2\delta n_0^*(1-\rho^2)(1-\delta(1-\rho))\right)}_{\leq 0}\Bigg\}\end{aligned}$$

$n_{-1}^*$ is hence decreasing in $\gamma$. At last, we have:

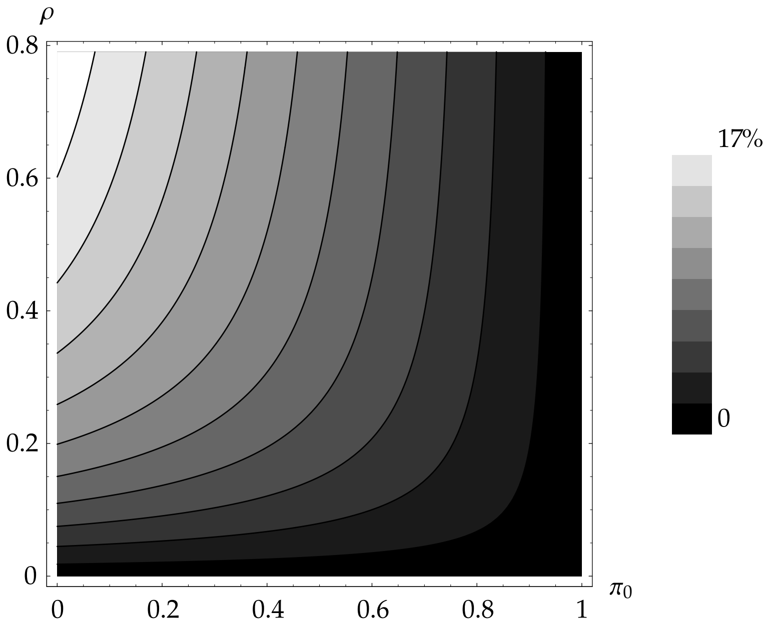
$$\begin{aligned}
\frac{\partial n_{-1}^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} &= \frac{\partial n_{-1}^*}{\partial \delta}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} + \frac{\partial n_0^*}{\partial \delta}\Big|_{(\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \frac{\partial n_{-1}^*}{\partial n_0^*}\Big|_{(n_0^*,\delta,\gamma,\rho,\Pi_{-1},\Pi_0,\Pi_1)} \\
&= \frac{2\gamma {n_0^*}^2 + 2\gamma n{-1^*}^2 \left(2(1-\delta(1-\rho)) - n_1^*(1-\delta)(1-\rho^2)\right)}{(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\
&\geq \frac{2\gamma {n_0^*}^2 + 2\gamma n{-1^*}^2 \left(1-\delta(1-\rho)^2+\rho^2\right)}{(2-2\delta(1-\rho)+\delta(1-\rho^2)n_0^*)(2-\delta(1-\rho)(2-(1+\rho)n_0^*))} \\
&\geq 0
\end{aligned}$$

Thus, $n_{-1}^*$ is also increasing in $\delta$.

# D Additional figures

## D.1 Efforts of innovation in the dynamic game

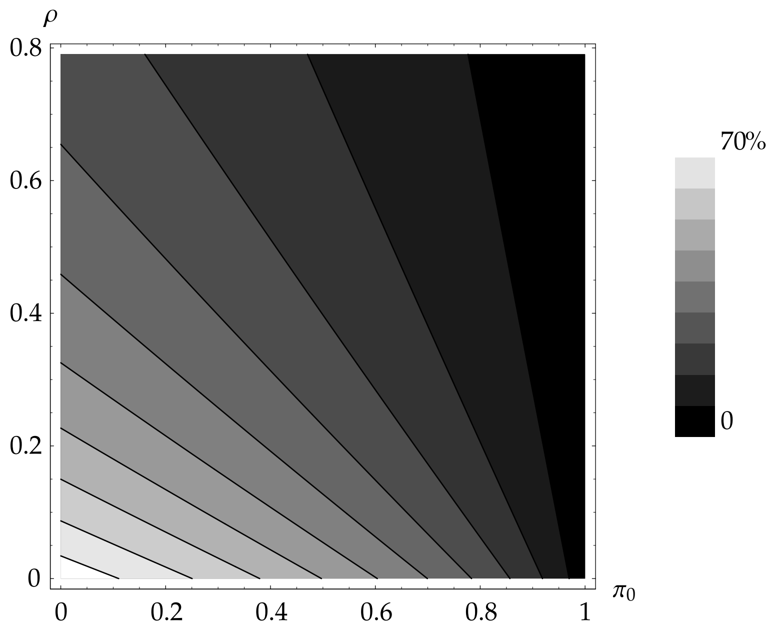
Figure 23: Efforts of the leader




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

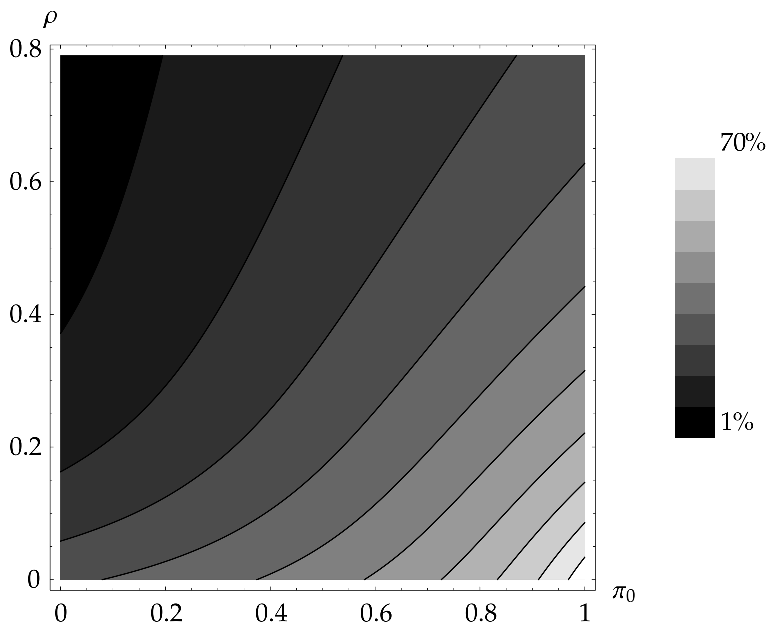
## D.2 Rates of innovation

FIGURE 24: Efforts of neck-to-neck firms




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

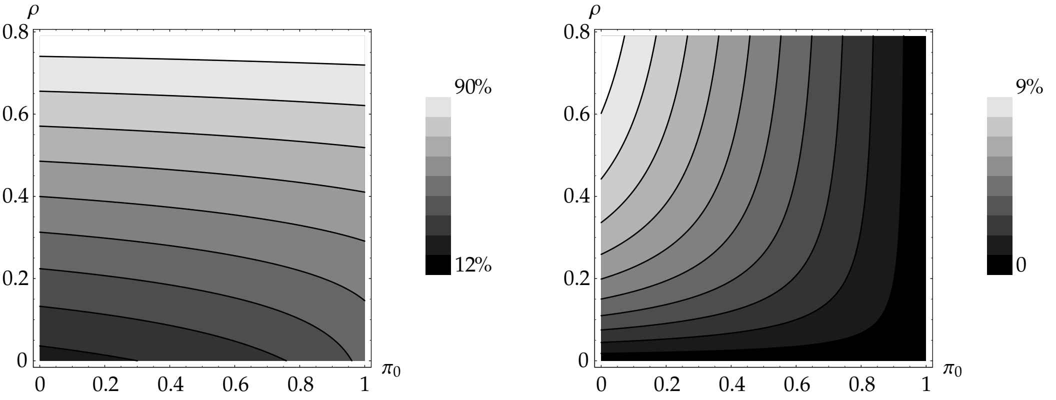
FIGURE 25: Efforts of the laggard




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

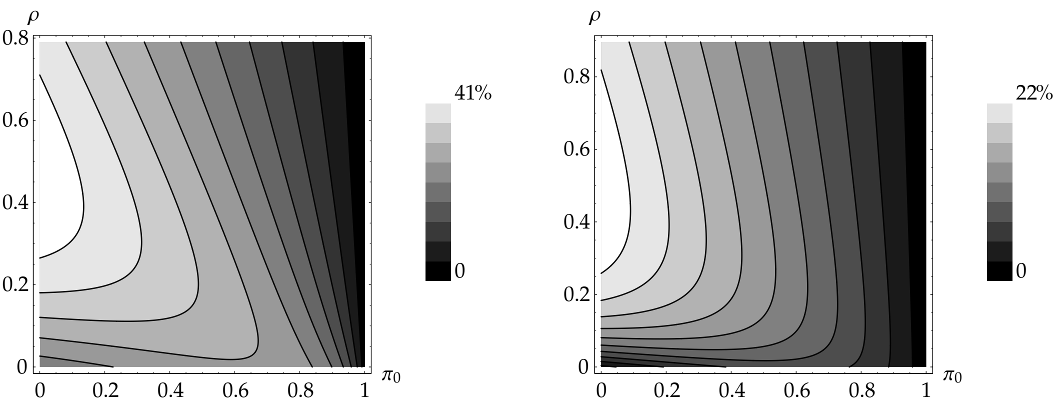
Figure 26: Average rate of innovation in the unleveled state

Overall innovations | Innovations for the market




**Note:** $\delta = 0.9$, $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 = 0.5$, $\rho \in [0, \rho_{lim}]$, $\rho_{lim} = 0.79$.

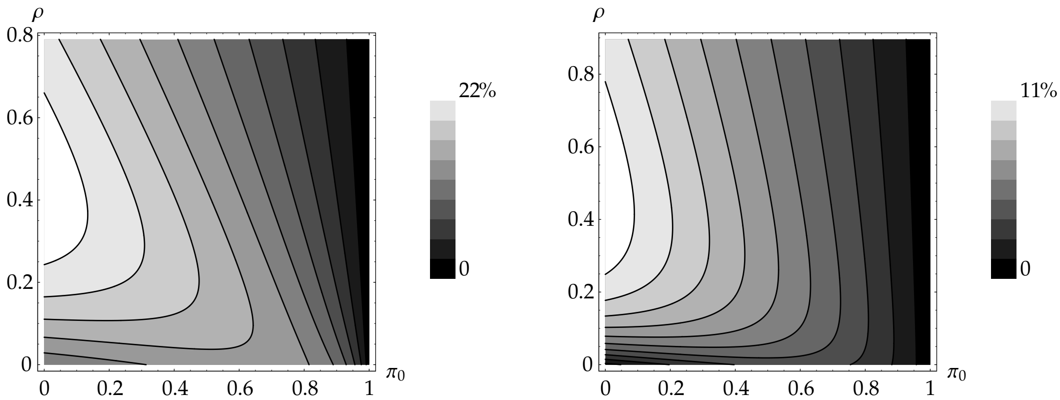
Figure 27: Average rate of innovations




$\delta = 0.9$, $\rho_{lim} = 0.79$ | $\delta = 0.5$, $\rho_{lim} = 0.90$

**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 = 0.5$, $\rho \in [0, \rho_{lim}]$.

Figure 28: Pace of innovation

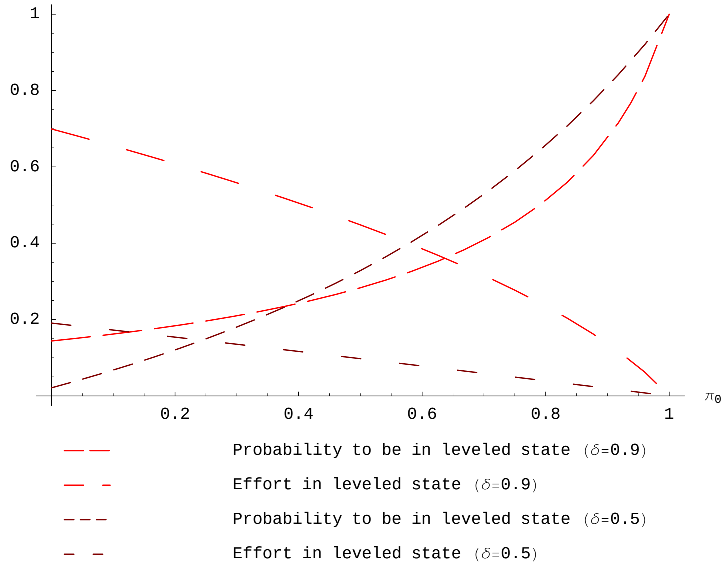



$\delta = 0.9$, $\rho_{lim} = 0.79$ $\delta = 0.5$, $\rho_{lim} = 0.90$

**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 = 0.5$, $\rho \in [0, \rho_{lim}]$.

## D.3 Explaining the patterns of $\Delta$ when $\rho = 0$

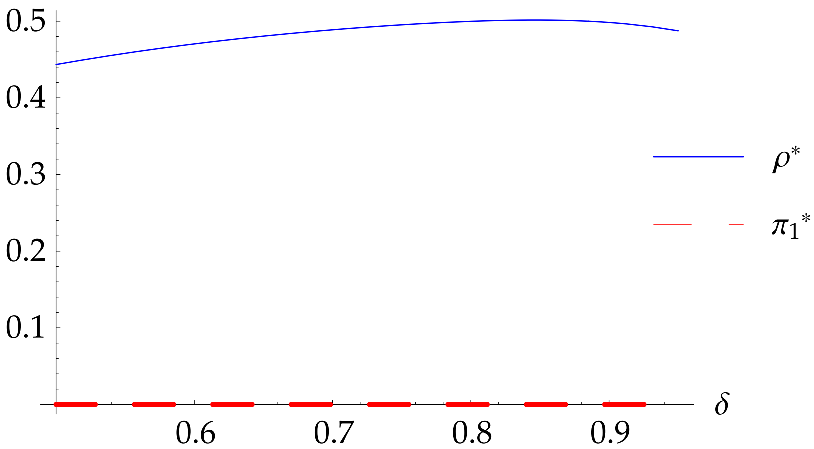
Figure 29: Probability to be in leveled state and effort in this state




**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1, $\Pi_0 \in [0, 1]$, $\rho = 0$.

## D.4 Optimal couple $(\rho^*, \Pi_0^*)$ for the pace of innovation

Figure 30: Optimal couple $(\rho^*, \Pi_0^*)$ for the pace of innovation




**Note:** $\gamma = 5$, $\Pi_{-1} = 0$, $\Pi_1$=1.