Abstract

The aim of this paper is twofold. First, I study how the proportion of fixed and variable-rate mortgages in an economy can affect the way shocks are propagated. Second, I analyze optimal implementable simple monetary policy rules and the welfare implications of this proportion. I develop and solve a New Keynesian dynamic stochastic general equilibrium model that features a housing market and a group of constrained individuals who need housing collateral to obtain loans. A given proportion of constrained households borrows at a variable rate, while the rest borrows at a fixed rate. The model predicts that in an economy with mostly variable-rate mortgages, an exogenous interest rate shock has larger effects on borrowers than in a fixed-rate economy. Aggregate effects are also larger for the variable-rate economy. For plausible parametrizations, differences are muted by wealth effects on labor supply and by the presence of savers. More persistent shocks, such as inflation target and technology shocks, cause larger aggregate differences. From a normative perspective I find that, in the presence of collateral constraints, the optimal Taylor rule is less aggressive against inflation than in the standard sticky-price model. Furthermore, for given monetary policy, a high proportion of fixed-rate mortgages is welfare enhancing.

Keywords: Fixed/Variable-rate mortgages, monetary policy, housing market, collateral constraint
"[...] the structure of mortgage contracts may matter for consumption behavior. In countries like the United Kingdom, for example, where most mortgages have adjustable rates, changes in short-term interest rates have an almost immediate effect on household cash flows. [...] In an economy where most mortgages carry fixed rates, such as the United States, that channel of effect may be more muted. I do not think we know at this point whether, in the case of households, these effects are quantitatively significant in the aggregate. Certainly, these issues seem worthy of further study". Ben Bernanke, June 15, 2007.

1 Introduction

Mortgage contracts in an economy can be fixed or variable rate. The proportion of variable-rate mortgages varies from country to country. In countries such as the United States, Germany and France, the majority of mortgages are fixed rate. However, the predominant type of mortgages in countries such as the United Kingdom, Australia and Spain is variable.

Mortgage rate changes affect the amount of mortgage interest payments, causing a direct cash-flow effect on consumption. Interest rate changes also affect housing demand and housing prices. If households are using housing as a collateral, the value of this collateral changes, inducing a wealth effect on household behavior and indirectly affecting consumption (ECB (2003), HM Treasury (2003)). Interest rate shocks affect mortgage rates differently depending on whether the mortgage is fixed or variable rate. Variable-rate mortgages are mortgage loans for which the interest rate is adjusted periodically, typically in line with some measured short-term interest rate. Hence, interest rate shocks directly affect variable rates. In contrast, fixed-rate mortgages are mortgage loans for which the interest rate remains constant through the term of the loan. The fixed interest rate is tied to a longer-term interest rate and is less sensitive to changes in the policy rate.

This raises important questions: How does the mortgage rate structure affect the way macroeconomic shocks are propagated? What are the implications in terms of monetary policy and welfare? These questions are of academic and policy interest. To give an illustrative example, the United Kingdom Treasury explicitly mentions the difference in mortgage structures as an important reason not to join the euro area. In the UK, the vast majority of borrowers have variable-rate mortgages, as opposed to the large countries of the euro area. According to the UK Treasury, British households are more exposed to monetary policy changes than, say, German households (HM Treasury (2003), Miles (2004)).
To address these questions, I build a New Keynesian dynamic stochastic general equilibrium model with housing and collateral constraints to explore how shocks are propagated in the presence of mortgage heterogeneity. I introduce fixed and variable-rate mortgages in the model. For the proportion of variable-rate mortgages to matter via the direct, cash-flow effect of mortgage interest payments on consumption, borrowers and savers are needed. Then, the effect of interest rate changes on borrowing does not cancel out by the presence of a representative consumer. For the indirect, wealth effect to appear, one needs non-durable consumption to be related to house prices. The introduction of collateral constraints tied to housing value for one type of consumers solves both problems since it motivates the presence of borrowers and savers and relates housing prices to consumption. In this model, monetary policy has real effects that are comparable with other sticky-price models. Furthermore, since the model is microfounded it allows me to study optimal monetary policy and welfare.\(^1\)

It is not the aim of this paper to explain how the decision between fixed and variable-rate mortgages is made.\(^2\) For simplicity, I hold the proportion of fixed and variable-rate borrowers constant and exogenous. Although this proportion can vary in reality, there is evidence that it fluctuates around a constant mean which is higher or lower depending on the country.\(^3\) We could think of these cross-country differences as due to institutional, historical or cultural factors, out of the scope of this model.\(^4\)

I use the model to compute impulse responses to interest rate, inflation target and technology shocks. I consider two extreme cases; one in which the economy is composed by variable-rate borrowers and one where the fixed rate is the predominant type of mortgage.

Results show that interest rate shocks affect more strongly those borrowers that have variable-rate mortgages. Given an increase in the interest rate set by the central bank, variable-rate borrowers reduce their consumption and housing demand by more than fixed-rate borrowers. The intuition is as follows: After a monetary policy shock (increase in the interest rate), fixed and variable-rate consumers differ in the real interest rate they face. Consider the most extreme case in which the variable rate changes one for one with the interest rate set by the central bank and the fixed rate is constant. After the shock, the nominal mortgage rate increases for the variable-rate individuals and inflation decreases. For the fixed-rate borrowers, the nominal mortgage interest rate does not react, but inflation is still decreasing.

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\(^1\)The analysis of optimal monetary policy is restricted to optimization over parameters of a simple implementable Taylor rule.

\(^2\)See Miles (2004) or Campbell and Cocco (2003) for studies that cover this from a microeconomic perspective.

\(^3\)See Appendix 1 for evidence for the UK and the US.

\(^4\)The European Mortgage Federation (EMF) highlights that cultural differences play an important role for the predominant type of mortgage contract in a country. They are linked to real estate law, borrowers’ risk aversion, funding system or frequency of house moves.
because the economy is contracting. As a result, real rates increase by more if the mortgage is variable rate. In real terms, payments are increasing by more for variable-rate consumers, and their consumption and housing decrease by more (this is a pure cash-flow effect). A second, wealth effect comes through the collateral constraint. Banks are willing to lend as long as debt repayments do not exceed a fixed proportion of the value of the house collateral. For borrowers with variable-rate mortgages the value of their collateral has been reduced by more since they are demanding less houses. These effects make consumption decrease more strongly for variable-rate borrowers.

Aggregate consumption also declines by more after a monetary policy shock when the economy is mainly borrowing at a variable rate. However, aggregate differences are more muted due to the behavior of savers. In equilibrium, borrowing and saving must be equal. If borrowing decreases, saving must also decrease. Savers are the owners of financial intermediaries in the model, so any loss for the borrowers is a gain for the savers. These manage to offset part of the decrease in consumption following a positive interest rate shock. Results for monetary policy shocks are very robust to different model specifications.

Some aggregate differences arise because the borrowers’ marginal propensity to consume is larger than the savers’. However, aggregate differences are not large because interest rate shocks are not very persistent. Also, income effects on labor supply are important in this model. With the type of preferences used in standard real business cycle models, labor effort is determined together with the intertemporal consumption choice. When consumption is reduced, individuals tend to work more to compensate and smooth consumption. Using preferences as in Greenwood, Hercowitz and Huffman (1988) (GHH henceforth), this effect is eliminated. In this case, the channels that are important for the purposes of this paper are emphasized and aggregate effects are larger.

In contrast, inflation target shocks generate larger aggregate differences between scenarios. In particular, when the inflation target increases, output responds by more when variable rates are predominant. Real interest rates fall persistently and house prices increase by less than with fixed mortgage rates. Variable-rate borrowers increase their nondurable consumption by more. Since house prices do not increase as much in the variable-rate case, also savers can consume more nondurables.

Finally, I consider technology shocks. A favorable technology shock increases output and lowers prices. Monetary policy responds in a persistent way and real rates increase. Variable-rate borrowers consume less because the real rate increase affects them and dampens the positive effects of the technology.

\footnote{In this model borrowers face collateral constraints and are more impatient than savers. This makes their consumption respond by more to changes in wealth.}
shock for them. The increase in real rates does not affect fixed-rate consumers as much and they can consume more. Output increases by more when fixed rates are predominant.

I also study welfare and optimal monetary policy in the context of fixed and variable rate mortgages. In particular, I search over parameters of a simple, implementable interest rate rule so that welfare is maximized. I find that, in the presence of collateral constraints, a social welfare maximizing central bank should respond to inflation less aggressively than in the absence of collateral constraints. Results also show that when the central bank focuses only on the savers’ welfare, thus ignoring the collateral constraint, the optimized inflation parameter in the Taylor rule is higher. However, when borrowers are taking into account, the central bank optimally responds less to inflation. The central bank faces a trade-off between the borrowers and savers’ welfare because on the one hand, low inflation corrects the sticky-price distortion but, on the other hand, inflation relaxes the collateral constraint and improves borrowers’ welfare. Comparing welfare across mortgage rate scenarios for given policy shows that this inflation channel is more effective the higher the proportion of fixed-rate mortgages in the economy. Therefore, borrowers are better off with fixed-rate mortgages although this comes at the cost of lower welfare for savers. For aggregate welfare, I find that predominantly fixed-rate contracts are welfare enhancing.

This paper relates to different strands of literature. First, it contributes to the literature on New Keynesian general equilibrium models with housing and collateral constraints such as Aoki et al. (2004) and Iacoviello (2005), who do not consider heterogeneous mortgage contracts. Second, it is also related to a literature that studies fixed and variable-rate mortgages. Campbell and Cocco (2003) and Miles (2004) study the fixed versus variable rate choice from a partial equilibrium perspective. Graham and Wright (2007) develop a model in which some households face binding credit constraints and debt contracts can be fixed or variable rate. However, they do not include a housing market and thus the constraint is not tied to housing stock and housing prices, eliminating the wealth channel. Calza et al. (2007) study how institutional factors, including mortgage contracts, can affect the monetary transmission mechanism. In my model, I focus on fixed versus variable rate mortgages. My results on monetary policy shocks are comparable to theirs under some parameter specifications. Relative to them, I do not only study the exogenous component of monetary policy but also the systematic response to other shocks. The existent literature is silent about how mortgage heterogeneity affects the way shocks such as changes in inflation target or technology are propagated. Finally the paper contributes to the literature on optimal monetary policy with heterogeneous consumers and collateral constraints. See for instance Monacelli
(2006) or Mendicino and Pescatori (2007). However, none of these papers studies optimal monetary policy in the context of different mortgage contracts.

Section II explains the basic model I build for the analysis. Section III shows the results and dynamics and business cycles of the model. Section IV analyzes optimal monetary policy. Section V presents the conclusions. Appendix 1 contains graphs and tables on the empirical evidence mentioned above. Appendix 2 shows model derivations.

2 The Baseline Model

I consider an infinite-horizon economy in which households consume, work and demand real estate. There is a representative financial intermediary that provides mortgages and accepts deposits from consumers. Firms set prices subject to Calvo (1983)-Yun (1996) nominal rigidity. The monetary authority sets interest rates endogenously, in response to inflation and output, following a Taylor rule.

2.1 The Consumer’s Problem

There are three types of consumers: unconstrained consumers, constrained consumers who borrow at a variable rate, and constrained consumers who borrow at a fixed rate. Constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments for both mortgages and loans cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. This assumption ensures that the borrowing constraint is binding, so that constrained individuals do not save and wait until they have the funds to self-finance their consumption. This generates an economy in which households divide into borrowers and savers. Furthermore, borrowers are divided into two groups, those who borrow at a fixed rate and those who borrow at a variable rate. The proportion of each type of borrower is fixed and exogenous. All households derive utility from consumption, housing services assumed proportional to the housing stock and leisure.6

6I do not allow for renting. This is needed to generate borrowers and savers in the economy. If renting were allowed, borrowers could use renting to save and the wealth effect would disappear. Furthermore, in the US, homeowneishments have been quite high in the last years (about 65 percent, according to the US Census Bureau).
2.1.1 Unconstrained Consumers (Savers)

Unconstrained consumers maximize:

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right),$$  \hspace{1cm} (1)$$

where, $E_0$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, and $C_t^u$, $H_t^u$ and $L_t^u$ are consumption at $t$, the stock of housing and hours worked, respectively; $1/(\eta - 1)$ is the labor supply elasticity, $\eta > 0$ and $j > 0$ represents the weight of housing in the utility function.

The budget constraint is:

$$C_t^u + q_t H_t^u + b_t^u \leq q_t H_{t-1}^u + w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t^u + S_t^u,$$ \hspace{1cm} (2)$$

where $q_t$ is the real housing price and $w_t^u$ is the real wage for unconstrained consumers. These can buy houses or sell them at the current price $q_t$. I assume zero housing depreciation for simplicity. As we will see, this group will choose not to borrow at all; they are the savers in this economy. $b_t^u$ is the amount they save. They receive interest $R_{t-1}$ for their savings. $\pi_t$ is inflation in period $t$. $S_t$ and $F_t$ are lump-sum profits received from the firms and the financial intermediary, respectively. We can think of these consumers as the wealthy agents in the economy, who own the firms and the financial intermediary.

The first-order conditions for this unconstrained group are:

$$\frac{1}{C_t^u} = \beta E_t \left( \frac{R_t}{\pi_t + 1 C_{t+1}^u} \right),$$ \hspace{1cm} (3)$$

$$w_t^u = (L_t^u)^{\eta - 1} C_t^u,$$ \hspace{1cm} (4)$$

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}.$$ \hspace{1cm} (5)$$

Equation (3) is the Euler equation for consumption, equation (4) is the labor-supply condition, and equation (5) is the Euler equation for housing. This states that the benefits from consuming housing must be equal to the costs at the margin.
2.1.2 Constrained Consumers (Borrowers)

Constrained consumers can be of two types: those who borrow at a variable rate and those who do it at a fixed rate. The proportion of variable-rate consumers is fixed and exogenous and equal to \( \alpha \in [0, 1] \).

Constrained and unconstrained consumers are different in the way they discount the future. Constrained consumers are more impatient than unconstrained ones. I assume that constrained consumers face a limit on the debt they can acquire. The maximum amount they can borrow is proportional to the value of their collateral, in this case the stock of housing. That is, the debt repayment next period cannot exceed a proportion of tomorrow’s value of today’s stock of housing:

\[
E_t \frac{R^c_i}{\pi_{t+1}} b^c_i t \leq k E_t q_{t+1} H^c_i t,
\]

where \( i = v \) if the constrained consumer borrows at a variable rate and \( i = f \) if he or she borrows at a fixed rate, and \( R^c_i = R_t \) if \( i = v \), \( R^f_i = \bar{R}_t \) if \( i = f \).

Constrained consumers maximize their lifetime utility function subject to the budget constraint and the collateral constraint:

\[
\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C^c_i t + j \ln H^c_i t - \frac{(L^c_i t) \eta}{\eta} \right),
\]

subject to:

\[
C^c_i t + q_t H^c_i t + \frac{R^c_i t - b^c_i t}{\pi_t} \leq q_{t-1} H^c_i t - 1 + w^c_i t L^c_i t + b^c_i t,
\]

and (6).

As noted above, constrained consumers are more impatient than unconstrained ones, so that \( \bar{\beta} < \beta \). This assumption is crucial for the borrowing constraint to be binding and therefore, for there to be both borrowers and savers in the economy.

The first-order conditions for constrained consumers are:

\[
\frac{1}{C^c_i t} = \bar{\beta} E_t \left( \frac{R^c_i t}{\pi_{t+1} C^c_{t+1}} \right) + \lambda_i^c R^c_i t,
\]

\[
w^c_i t = \left( L^c_i t \right)^{\eta-1} C^c_i t,
\]

\[\text{We will see from the firm’s problem that } w^v_i t = w^f_i t = w^c_i.\]
\[ \frac{j}{H_{ci}^t} = \frac{1}{C_{ci}^t} q_t - \tilde{\beta} E_t \frac{1}{C_{ci+1}^t} q_{t+1} - \lambda^c_i k E_t q_{t+1} \pi_{t+1}. \]  

(11)

These first-order conditions differ from those of the unconstrained individuals. In the case of constrained consumers, the Lagrange multiplier on the borrowing constraint \( \lambda^c_i \) appears in equations (9) and (11). From the Euler equations for consumption of unconstrained consumers, we know that \( R = 1/\beta \) in steady state. If we combine this result with the Euler equation for consumption of constrained individuals we have that \( \lambda^c_i = (\beta - \tilde{\beta}) / C^c_i > 0 \) in steady state. This means that the borrowing constraint holds with equality in steady state. Since we log-linearize the model around the steady state and assume that uncertainty is low, we can generalize this result to off-steady-state dynamics. Then, the borrowing constraint is always binding, so that constrained individuals are going to borrow the maximum amount they are allowed to and unconstrained consumers are never in debt.

Given the borrowing amount implied by (6) at equality, consumption for constrained individuals can be determined by their flow of funds:

\[ C_{ci}^t = w_{ci}^t L_{ci}^t + b_{ci}^t + q_t \left( H_{ci-1}^t - H_{ci}^t \right) - \frac{R_{ci-1}^t b_{ci-1}^t}{\pi_t}, \]

(12)

and the first-order condition for housing becomes:

\[ \frac{j}{H_{ci}^t} = \frac{1}{C_{ci}^t} \left( q_t - \frac{k E_t q_{t+1} \pi_{t+1}}{R_{ci}^t} \right) - \tilde{\beta} E_t \frac{1}{C_{ci+1}^t} (1 - k) q_{t+1}. \]

(13)

### 2.1.3 Aggregate Variables

Given the fraction \( \alpha \) of variable-rate borrowers, we can define aggregates across constrained consumers as \( C_i^c \equiv \alpha C_i^{cv} + (1 - \alpha) C_i^{cf}, L_i^c \equiv \alpha L_i^{cv} + (1 - \alpha) L_i^{cf}, H_i^c \equiv \alpha H_i^{cu} + (1 - \alpha) H_i^{cf}, b_i^c \equiv \alpha b_i^{cv} + (1 - \alpha) b_i^{cf}. \)

Therefore, economy-wide aggregates are: \( C_t \equiv C_u^c + C_i^c, L_t \equiv L_u^c + L_i^c, H_t \equiv H_u^c + H_i^c \). In this model, aggregate supply of housing is fixed, so that market clearing requires\(^8\): \( H_t = H_u^c + H_i^c = H \).

### 2.2 The Financial Intermediary

The financial intermediary accepts deposits from savers, and extends both fixed and variable-rate loans to borrowers. The profits of the financial intermediary are:

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\(^8\)This assumption provides an easy way to specify the supply of housing and have variable prices. A two-sector model with production of housing does not generate significantly different results (see Appendix 2).
\[ F_t = \alpha R_{t-1} b^{ct}_{t-1} + (1 - \alpha) \bar{R}_{t-1} b^{cf}_{t-1} - R_{t-1} b^u_{t-1}. \] (14)

To simplify, since the typical time horizon of a mortgage is large, I consider the maturity of mortgages to be infinite, although this assumption is not crucial for the dynamics of the problem.

In equilibrium, aggregate borrowing and saving must be equal, that is,

\[ b^C_t = b^u_t. \] (15)

Substituting (15) into (14), we obtain,

\[ F_t = (1 - \alpha) b^{cf}_{t-1} (\bar{R}_{t-1} - R_{t-1}). \] (16)

I assume that the financial intermediary operates under perfect competition. Therefore, the optimality condition for the financial intermediary implies that at each point in time \( \tau \), the intermediary is indifferent between lending at a variable or fixed rate. Hence, the expected discounted profits that the intermediary obtains by lending new debt in a given period at a fixed interest rate must be equal to the expected discounted profits the intermediary would obtain by lending it at variable rate:

\[
E\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i} R^*_{\tau} \left( b^f_{\tau} - b^f_{\tau-1} \right) = E\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i} R_{i-1} \left( b^f_{\tau} - b^f_{\tau-1} \right),
\] (17)

where \( \Lambda_{t,i} = \beta^{i-t} \left( \frac{C^{ct}_t}{C^{ct}_{t+i}} \right) \) is the unconstrained consumer relevant discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary’s problem.

We can obtain the optimal value of the fixed rate in period \( \tau \) from expression (17):

\[
\bar{R}^*_\tau = \frac{E\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i} R_{i-1}}{E\tau \sum_{i=\tau+1}^{\infty} \Lambda_{\tau,i}}.
\] (18)

Equation (18) states that, for every new debt issued at date \( \tau \), there is a different fixed interest rate that has to be equal to a discounted average of future variable interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period \( \tau \)
and the new amount of debt in period $\tau$ are fixed for all future periods. However, the fixed interest rate varies with the date the debt was issued, so that in every period there is a new fixed interest rate associated with new debt in this period. If we consider fixed-rate loans to be long-term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed interest rate that is the one the financial intermediary effectively charges every period. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period optimal fixed interest rate and new amount of debt. Therefore, the effective fixed interest rate that the financial intermediary charges for the stock of fixed-rate debt every period is:

$$R_t = \frac{R_{t-1}^{cf} + R_t^{*}}{b_t^{cf}} - b_{t-1}^{cf}.$$  (19)

Equation (19) states that the fixed interest rate that the financial intermediary is actually charging today is an average of what it charged last period for the previous stock of mortgages and what it charges this period for the new amount. Importantly, this assumption is not crucial for results. Both $R_t^{*}$ and $R_t$ are practically unaffected by interest rate shocks. This assumption is a way to reconcile the model with the fact that fixed-rate loans are not one-period assets but longer term ones.

As noted above, if any, profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and it does not make profits in absence of shocks, if there is a shock at a given point in time, the fact that only the variable interest rate is affected can generate non-zero profits.

### 2.3 Firms

#### 2.3.1 Final Goods Producers

There is a continuum of identical final goods producers that aggregate intermediate goods according to the production function

$$Y_t = \left[ \int_0^1 Y_t(z) \frac{z^{\varepsilon-1}}{z^{\varepsilon}} dz \right]^{\frac{1}{\varepsilon-1}}, \quad (20)$$

where $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The final good firm chooses $Y_t(z)$ to minimize its costs, resulting in demand of intermediate good $z$:
The price index is then given by:

\[ P_t = \left[ \int_0^1 P_t(z)^{1-\varepsilon} \, dz \right]^{\frac{1}{1-\varepsilon}}. \]  

(22)

Market clearing for the final good requires:

\[ Y_t = C_t = C^u_t + C^c_t. \]

2.3.2 Intermediate Goods Producers

The intermediate goods market is monopolistically competitive. Intermediate goods are produced according to the production function:

\[ Y_t(z) = A_t L^u_t(z)^{\gamma} L^c_t(z)^{(1-\gamma)}, \]  

(23)

where \( \gamma \in [0, 1] \) measures the relative size of each group in terms of labor. \(^9\) \( A_t \) represents technology and it follows the following autoregressive process:

\[ \log(A_t) = \rho_A \log(A_{t-1}) + u_{At}, \]  

(24)

where \( \rho_A \) is the autorregressive coefficient and \( u_{At} \) is a normally distributed shock to technology.

Labor demand is determined by:

\[ w^u_t = \frac{1}{X_t} \gamma \frac{Y_t}{L^u_t}, \]  

(25)

\[ w^c_t = \frac{1}{X_t} (1 - \gamma) \frac{Y_t}{L^c_t}, \]  

(26)

\(^9\)This Cobb-Douglas production function implies that labor efforts of constrained and unconstrained consumers are not perfect substitutes. This assumption can be justified by the fact that savers are the managers of the firms and their wage is not the same as the one of the borrowers. Experimenting with a production function in which hours are substitutes leads to very similar results (See Appendix 2). The Cobb-Douglas specification is analytically tractable and allows for closed form solutions for the steady state of the model.
where \( X_t \) is the markup, or the inverse of marginal cost.\(^{10}\)

The price-setting problem for the intermediate good producers is a standard Calvo-Yun setting. An intermediate good producer sells its good at price \( P_t(z) \), and \( 1 - \theta, \in [0, 1] \), is the probability of being able to change the sale price in every period. The optimal reset price \( P^*_t(z) \) solves:

\[
\sum_{k=0}^{\infty} (\theta \beta)^k E_t \left\{ \Lambda_{t,k} \left[ \frac{P^*_t(z)}{P_{t+k}} - \frac{\varepsilon}{\varepsilon - 1} X_{t+k} \right] Y^*_t(z) \right\} = 0. \tag{27}
\]

The aggregate price level is then given by:

\[
P_t = \left[ \theta P^c_{t-1} + (1 - \theta) (P^*_t)^{1-\varepsilon} \right]^{1/(1-\varepsilon)}. \tag{28}
\]

Using (27) and (28), and log-linearizing, we can obtain a standard forward-looking New Keynesian Phillips curve which is presented in the A.

### 2.4 Monetary Policy

The model is closed with a Taylor Rule with interest rate smoothing, to describe the conduct of monetary policy by the central bank:\(^{11}\)

\[
R_t = (R_{t-1})^\rho \left[ \left( \frac{\pi_t}{\pi^*_t} \right)^{(1+\phi_\pi)} R \right]^{1-\rho} \varepsilon_{Rt}, \tag{29}
\]

where \( 0 \leq \rho \leq 1 \) is the parameter associated with interest-rate inertia, and \( \phi_\pi > 0 \) measures the response of interest rates to current inflation. \( R \) is the steady-state values of the interest rate. \( \varepsilon_{Rt} \) is a white noise shock with zero mean and variance \( \sigma^2_{\varepsilon} \). \( \pi^*_t \) is the inflation target that evolves according to:

\[
\log (\pi^*_t) = \rho_n \log (\pi^*_{t-1}) + \varepsilon_{\pi t}, \tag{30}
\]

where \( \varepsilon_{\pi t} \) is normally distributed with variance \( \sigma^2_{\pi} \).

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\(^{10}\)Symmetry across firms allows us to write the demands without the index \( z \).

\(^{11}\)This is a realistic policy benchmark for most of the industrialized countries. A more realistic rule would also include output but it complicates building intuition about the workings of the model. Furthermore, estimations deliver a small response to the output gap in the last two decades (See Clarida, Gali and Gertler (2000)).
3 Shock Transmission and Business Cycles

I linearize the equilibrium equations around the steady state. Details are shown in Appendix 2. For calibration, I consider the following parameter values: The discount factor, $\beta$, is set to 0.99 so that the annual interest rate is 4% in the steady state. The discount factor for borrowers, $\tilde{\beta}$, is set to 0.98. Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency. Results are not sensitive to different values within this range. This value of $\tilde{\beta}$ is low enough to endogenously divide the economy into borrowers and savers. The weight of housing on the utility function, $j$, is set to 0.1 in order for the ratio of housing wealth to GDP in the steady state to be consistent with the data. This value of $j$ implies a ratio of approximately 1.40, in line with the Flow of Funds data.\footnote{See Table B.100. In this model, consumption is the only component of GDP. To make the ratio comparable with the data I multiply it by 0.6, which is approximately what nondurable consumption and services account for in the GDP, according to the data in the NIPA tables.} I set $\eta = 2$, implying a value of the labor supply elasticity of 1.\footnote{Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.} For the loan-to-value ratio, I pick $\kappa = 0.9$, consistent with the evidence that in the last years borrowing constrained consumers borrowed on average more than 90% of the value of their house.\footnote{We can identify constrained consumers with those that borrow more than 80% of their home. In the US, among those borrowers, the average LTV ratio exceeds 90% for the period 1973-2006. See the data from the Federal Housing Finance Board.} The labor income share of unconstrained consumers, $\gamma$, is set to 0.64, following the estimate in Iacoviello (2005). I pick a value of 6 for $\varepsilon$, the elasticity of substitution between intermediate goods. This value implies a steady state markup of 1.2. The probability of not changing prices, $\theta$, is set to 0.75, implying that prices change every four quarters. For the Taylor Rule parameters I use $\rho = 0.8$, $\phi_\pi = 0.5$. The first value reflects a realistic degree of interest-rate smoothing.\footnote{See McCallum (2001).} The second one, is consistent with the original parameter proposed by Taylor in 1993. For $\alpha$, I consider two polar cases for comparison. In the first case, the proportion of variable-rate mortgages in the economy is 0, that is, all constrained consumers in the economy borrow at a fixed rate. In the second case, the proportion of variable-rate mortgages is 1. Table 1 shows a summary of the parameter values.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>.99</td>
<td>Discount Factor for Savers</td>
</tr>
<tr>
<td>(\tilde{\beta})</td>
<td>.98</td>
<td>Discount Factor for Borrowers</td>
</tr>
<tr>
<td>(j)</td>
<td>.1</td>
<td>Weight of Housing in Utility Function</td>
</tr>
<tr>
<td>(\eta)</td>
<td>2</td>
<td>Parameter associated with labor elasticity</td>
</tr>
<tr>
<td>(k)</td>
<td>.9</td>
<td>Loan-to-value ratio</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>.64</td>
<td>Labor share for Savers</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0/1</td>
<td>Proportion of variable-rate borrowers</td>
</tr>
<tr>
<td>(X)</td>
<td>1.2</td>
<td>Steady-state markup</td>
</tr>
<tr>
<td>(\theta)</td>
<td>.75</td>
<td>Probability of not changing prices</td>
</tr>
<tr>
<td>(\rho_{\pi})</td>
<td>.975</td>
<td>Inflation target persistence</td>
</tr>
<tr>
<td>(\rho_A)</td>
<td>.9</td>
<td>Technology persistence</td>
</tr>
<tr>
<td>(\rho)</td>
<td>.8</td>
<td>Interest-Rate-Smoothing Parameter in Taylor Rule</td>
</tr>
<tr>
<td>(\phi_{\pi})</td>
<td>.5</td>
<td>Inflation Parameter in Taylor Rule</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

### 3.1 Impulse Responses

#### 3.1.1 Monetary Policy Shock

Impulse responses to a one standard deviation (0.29 percent) increase of the interest rate are presented in Figure 1.\(^{16}\) We can see that when the economy is mainly composed by individuals indebted at a variable-rate, the effects of monetary policy on consumption for the borrowers are stronger than in the fixed-rate case. Borrowers’ housing demand, initially, also decreases more strongly after a monetary policy shock if the predominant type of mortgages in the economy is variable rate. These findings show that the proportion of variable-rate mortgages matters for the monetary transmission mechanism. When the proportion of variable-rate borrowers is very high, a monetary policy shock affects more strongly those individuals who are constrained and need to borrow.

In the aggregate, output in the variable-rate economy also decreases more strongly (See Figure 2). There is a redistribution between borrowers and savers but we can still find aggregate differences because

\(^{16}\)Iacoviello (2005) estimates a Taylor Rule for the US economy and finds a 0.29 percent standard deviation on a quarterly basis. I use this number as an empirically plausible one-standard deviation increase in the interest rate.
borrowers are more sensitive to changes in wealth (they are more impatient and use housing wealth as collateral).

**Sensitivity Analysis** Differences between the two scenarios are not larger because monetary policy shocks are not very persistent and the share of borrowers in the economy is not very large. Results are sensitive to the wage share of unconstrained individuals in the economy. Figure 3 shows that by decreasing the size of the savers aggregate differences are amplified.

In this model income effects on the labor supply decision are important. In the baseline model preferences are separable in consumption and labor. In this case, the labor supply decision depends on the level of consumption. Given a negative shock to the economy, labor supply moves both in response to a substitution and an income effect. On the one hand, lower wages make consumers want to work less. On the other hand, lower consumption generates an income effect that makes consumers want to work more.

---

17This parameter represents the relative economic size of each group in the economy.
Income effects can partly offset aggregate differences. GHH preferences have the property of shutting down the income effect on the labor supply decision. In this preferences, labor and consumption are non-separable. This makes labor effort to be determined independently from the intertemporal consumption-savings choice.\footnote{See Appendix 2 for details on GHH preferences and derivations.} There an extensive literature that has also used these preferences to emphasize other channels that are partially offset by this income effects.\footnote{See for example Rafla (2006) and references therein.} Impulse responses, in line with other studies that use GHH preferences, show how consumption responses are stronger and aggregate differences are amplified (See Figure 4).

Results for monetary policy shocks with standard preferences are very robust to alternative model specifications. We can introduce capital in the basic model or assume nonseparability between housing and consumption in the utility function. The basic results for the variables of interest are maintained.\footnote{The details of the model are presented in Appendix 2. The parameter values used for the calibration are 0.025 for capital depreciation and 10 for capital adjustment costs. The elasticity of substitution between non-durable consumption goods and housing of 0.5. The rest of the parameter values are the same as in the baseline model.}

### 3.1.2 Inflation Target Shock

Instead of a shock to the interest rate, we can also consider a more persistent monetary policy disturbance such as a shock to the inflation target. Figure 5 shows the responses of the variables of interest to an
Aggregate differences are amplified with this type of shock. Output increases by more in the variable-rate case. Monetary policy responds systematically to the shock in a very persistent way. Real interest rates fall persistently and house prices increase by less in the variable-rate economy. Variable-rate borrowers increase by more their nondurable consumption because real rates fall. Since house prices do not increase that much in the variable-rate case, also savers can consume more nondurables.

3.1.3 Technology Shock

A shock to technology may also have different effects on the economy depending on whether individuals are mainly borrowing at variable or fixed rate. Impulse responses to a 1 percent positive shock to technology with 0.9 persistence are showed in Figure 6. We see that the economy responds more strongly after a technology shock when the majority of its borrowers have a fixed-rate mortgage. A technology shock increases output and lowers prices. As a reaction, real interest rates increase in a very persistent way. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. As

\[ \frac{\text{dev. steady state}}{\text{quarters}} \]
Figure 4: Aggregate Output Response to a Monetary Policy Shock. GHH Preferences.

As a result, output increases by more for fixed-rate consumers.

Monetary policy shocks or inflation target shocks cause the real interest rate to vary countercyclically, which is why flexible-rate mortgages amplify the effects those shocks. Technology shocks, by contrast, cause the real interest rate to vary procyclically: it rises when output rises, which is why flexible-rate mortgages dampen the effects of those shocks.

3.2 Second Moments

Table 2 shows the standard deviations of the main variable both from the model and the data. The model generates a standard deviation of GDP of 2.0127 for the variable-rate case and 2.126 for the fixed-rate economy. This is slightly smaller but close to the data (2.26), especially for the fixed-rate economy. The volatility of consumption and housing demand is always greater for those individuals that are constrained but smaller in the case of variable rates. The volatility of inflation and house prices is smaller in the model than in the data while the correlation between output and house prices is greater.

---

23 Theoretical moments calculated for technology shocks. Standard deviations from the data taken from Davis and Heathcote (2005).
24 Davis and Heathcote (2005) also find smaller output volatility.
Figure 5: Impulse Responses to an Inflation Target Shock. Baseline Specification.

<table>
<thead>
<tr>
<th>% SD Rel. to GDP</th>
<th>Data</th>
<th>Model (Fixed Rates)</th>
<th>Model (Variable Rates)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2.26</td>
<td>2.126</td>
<td>2.013</td>
</tr>
<tr>
<td>c^u</td>
<td>0.931</td>
<td>0.904</td>
<td></td>
</tr>
<tr>
<td>c^c</td>
<td>1.413</td>
<td>1.304</td>
<td></td>
</tr>
<tr>
<td>h^u</td>
<td>2.276</td>
<td>0.646</td>
<td></td>
</tr>
<tr>
<td>h^c</td>
<td>6.525</td>
<td>1.852</td>
<td></td>
</tr>
<tr>
<td>π</td>
<td>0.78</td>
<td>0.094</td>
<td>0.121</td>
</tr>
<tr>
<td>q</td>
<td>1.37</td>
<td>0.552</td>
<td>0.911</td>
</tr>
<tr>
<td>Correlations y, q</td>
<td>0.65</td>
<td>0.960</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Table 2: Business Cycle Properties.
4 Welfare and Optimal Monetary Policy

In this section, I compare different simple monetary policy rules based on welfare evaluations, both for the whole economy and for different types of consumers, in order to provide some normative assessment.

The individual welfare for savers and borrowers respectively is defined as follows:

\[ V_{u,t} = E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C_{t+m}^u + j \ln H_{t+m}^u - \frac{(L_{t+m}^u)^\eta}{\eta} \right), \]  \hspace{1cm} (31)

\[ V_{ci,t} = E_t \sum_{m=0}^{\infty} \tilde{\beta}^m \left( \ln C_{t+m}^{ci} + j \ln H_{t+m}^{ci} - \frac{(L_{t+m}^{ci})^\eta}{\eta} \right), \]  \hspace{1cm} (32)

Following Mendicino and Pescatori (2007), I define social welfare as a weighted sum of individual welfare for the different types of households:

\[ I \text{ numerically compute the second order approximation of the utility function as a measure of welfare.} \]
\[ V_t = (1 - \beta) V_{u,t} + \left( 1 - \frac{\beta}{\bar{\beta}} \right) \left[ \alpha V_{cv,t} + (1 - \alpha) V_{cf,t} \right]. \]  

(33)

Borrowers and savers’ welfare are weighted by \( \left( 1 - \frac{\beta}{\bar{\beta}} \right) \) and \((1 - \beta)\) respectively, so that the two groups receive the same level of utility from a constant consumption stream. As in Mendicino and Pescatori (2007), I take this approach to be able to evaluate the welfare of the three types of agents separately.\(^{26}\)

To begin, I evaluate the welfare achieved under the ad-hoc Taylor rule used in the baseline model. Results are presented in Table 3:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fixed Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Welfare</td>
<td>-6.0693</td>
</tr>
<tr>
<td>Savers Welfare</td>
<td>21.5748</td>
</tr>
<tr>
<td>Borrowers Welfare</td>
<td>-314.2514</td>
</tr>
<tr>
<td>(\sigma(\pi))</td>
<td>0.2436</td>
</tr>
</tbody>
</table>

Table 3: Welfare comparison. Ad-hoc Taylor Rule.

The economy with fixed-rate mortgages achieves a higher level of welfare than the variable-rate economy. Notice as well that there is a trade-off between savers and borrowers’ welfare: Although a larger fraction of fixed-rate borrowers raises aggregate welfare, this comes at the cost of lower welfare for savers.

Figure (7) shows how the welfare level varies with the proportion of variable rate mortgages in the economy.\(^{27}\) This figure clearly illustrates this trade-off. When mortgages are at a fixed rate, savers, who own the financial intermediary, bear all the risk associated with interest rate changes. Borrowers, are however insured against interest-rate risk and their collateral constraint is relaxed. If we look at the loglinearized collateral constraint (see equation (45) in Appendix 2), we can observe that, at a given level of inflation, in real terms, mortgage payments are lower, the lower the value of \(\alpha\) is. As a result of this trade-off between borrowers and savers, the economy achieves the maximum level of social welfare at around the value of \(\alpha = 0.3\), that is, when 70 percent of the mortgages are fixed rate.

Next, I study what is the monetary policy that maximizes welfare. The design of optimal monetary policy in the presence of collateral constraints is more complicated than in the standard sticky-price

\(^{26}\)See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.

\(^{27}\)Welfare is rescaled so that it appears in the positive axis. Additionally, borrowers and savers’ welfare is divided by 100.
setting. In this case, there are two types of distortions, price rigidities and credit frictions. On the one hand, the central bank should aim at lowering inflation volatility because, given sticky prices, inflation distorts production decisions. On the other hand, inflation relaxes the borrowing constraints and improves the borrowers’ welfare. And, as noticed above, this inflation channel is much more effective when fixed-rate mortgages are predominant. The loglinearized collateral constraint shows that mortgage payments decrease with inflation but increase with the interest rate. Inflation relaxes the collateral constraint for borrowers, as long as the interest rate does not react too much to it. Therefore, the inflation channel for borrower welfare is stronger the less the central bank responds to inflation but also the lower the value of $\alpha$. In the limit, an economy with just fixed-rate mortgages maximizes the favorable effects of inflation on the collateral constraint.

Given a grid of possible parameters for the Taylor rule, I perform a search that maximizes welfare, subject to determinacy requirements. For simplicity, I start by keeping the value of $\rho$ fixed to 0.8 and I search over different values of $\phi_\pi$, the response coefficient to inflation. In this way, I can build intuition about on much the central bank should respond to inflation in different cases for the same degree of interest-rate smoothing. Results are presented in Table 4:

![Figure 7: Welfare level for different values of $\alpha$. Ad-hoc Taylor rule.](image)
For the model with collateral constraints, I consider two cases: a central bank that is a social welfare maximizer and a central bank that neglects the borrowers’ welfare. Within each case, mortgage contracts are either fixed or variable rate. Then, I compare the results with a model without collateral constraints. As in Monacelli (2006) and Mendicino and Pescatori (2007), lenders prefer the central bank being aggressive against inflation. However, borrowers obtain welfare gains from a monetary policy that minimizes credit market inefficiencies. The central bank aggressively fights inflation if it considers only the welfare of those not facing credit constraints. However, economies with fixed-rate contracts achieve a higher welfare in all cases because they are less distorted by the collateral constraint. If we compare the results with a model without collateral constraints, we clearly see that the central bank should respond to inflation less aggressively than in the standard sticky-price model, without collateral constraints.
fact, for the model with collateral constraints the optimal value of $\phi_\pi$ corresponds to the minimum value allowed in the search while in the absence of collateral constraints it corresponds to the maximum one.

<table>
<thead>
<tr>
<th>Optimized Taylor Rule (Maximize Social Welfare)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Rate</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>$\rho^* = 0.9, \phi^*_\pi = 0.35$</td>
</tr>
<tr>
<td>Social Welfare</td>
</tr>
<tr>
<td>Savers Welfare</td>
</tr>
<tr>
<td>Borrowers Welfare</td>
</tr>
<tr>
<td>$\sigma (\pi)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optimized Taylor Rule (Maximize Savers Welfare)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Rate</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>$\rho^* = 0.1, \phi^*_\pi = 0.35$</td>
</tr>
<tr>
<td>Social Welfare</td>
</tr>
<tr>
<td>Savers Welfare</td>
</tr>
<tr>
<td>Borrowers Welfare</td>
</tr>
<tr>
<td>$\sigma (\pi)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optimized Taylor Rule (No Collateral Constraints)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^* = 0.1, \phi^*_\pi = 20$</td>
</tr>
<tr>
<td>Social Welfare</td>
</tr>
<tr>
<td>$\sigma (\pi)$</td>
</tr>
</tbody>
</table>

Table 5: Welfare Values for Optimized Taylor Rule

Table 5 shows results for an optimized Taylor rule in which I search for both the values of $\rho$ and $\phi_\pi$ so that welfare is maximized. Again in this case we can clearly see that the optimal response to inflation by the central bank is less aggressive in the presence of collateral constraints.

5 Conclusions

In this paper, I have developed a New Keynesian general equilibrium model with housing and collateral constraints to study first, how the proportion of variable-rate mortgages in the economy can affect the
transmission of shocks and then, what the welfare implications of mortgage contracts are. There are unconstrained and constrained individuals that correspond to the savers and borrowers of the economy. I explicitly introduce fixed and variable-rate mortgages, that is, constrained individuals can be of two types: those who borrow at a variable rate and those who borrow at a fixed rate.

Model responses are in line with the intuition. A monetary policy shock affects more strongly those individuals who are borrowing in economies in which the predominant type of mortgages is at variable rate. Consumption and housing demand decrease by more after an interest rate increase if constrained consumers are variable rate. In a general equilibrium framework, the partial equilibrium effects are maintained, but muted by a redistribution between borrowers and savers and strong wealth effects in labor supply decisions. GHH preferences generate larger aggregate differences between the two scenarios considered.

Monetary policy shocks are not persistent. More persistent shocks such as technology or inflation target shocks are able to generate much larger differences in the aggregate economy.\textsuperscript{28} Monetary policy responds to these shocks in a very persistent way causing large aggregate differences between the fixed and the variable-rate economy. Inflation target shocks have more effect on output in variable-rate economies. On the contrary, technology shocks increase output by more in those economies mainly borrowing at a fixed rate, due to the procyclicality of real interest rates in this case.

From a normative perspective, I find that the optimal interest-rate response to inflation by the Central Bank is weaker when a group of consumers need collateral to obtain loans, as compared to the standard sticky-price model. Inflation relaxes the collateral constraint and therefore reduces the distortions created by this extra friction. However, this channel is stronger the higher the proportion of fixed-rate mortgages in the economy. A high proportion of fixed-rate contracts is welfare enhancing.

The model presented here can set directions for future research. The proportion of fixed and variable-rate mortgages is kept constant. A natural extension would be to endogeneize it by modelling the mortgage choice. For instance, borrowers could be heterogeneous in their risk aversions or market-powered banks could price mortgages charging a spread on fixed-rate mortgages depending on economic conditions. Furthermore, this model is not able to keep track of the new fixed-rate mortgages issued every

\textsuperscript{28}This is also consistent with Krusell and Smith (1998) or Gourinchas (2001). They study the effects of the distribution of income and wealth and the implications of precautionary savings and life cycle for the macroeconomy in a general equilibrium framework with heterogeneous agents. Their results are not very different from what one would obtain in a representative agent model, behaviors of different agents practically offset each other in the aggregate when considering realistic parameter specification. They also find that permanent shocks would generate larger effects on the aggregate economy.
period. For tractability I assume that the financial intermediary charges an average of the new fixed interest rate and the old interest rate for fixed-rate mortgages every period. An overlapping generations version could solve this issue. It would also be interesting to study shock transmission and monetary policy in international versions of the model with heterogeneous mortgage structures across countries.
References


[16] HM Treasury (2003), EMU and the Monetary Transmission Mechanism

[17] HM Treasury (2003), Housing, Consumption and EMU


[21] Ireland, P., (2007), Changes in the Federal Reserve’s Inflation Target, mimeo


Appendix 1: Tables and Figures

![Proportion of Variable-Rate Mortgages](image)

Figure 8: Proportion of Variable-Rate Mortgages in the US and UK. Source: Federal Housing Finance Board and Council of Mortgage Lenders

<table>
<thead>
<tr>
<th>Residential Debt to GDP Ratio (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EU15</td>
</tr>
<tr>
<td>Austria</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>Greece</td>
</tr>
</tbody>
</table>

Table 6: Residential Debt to GDP Ratio. Source: European Mortgage Federation

<table>
<thead>
<tr>
<th>Predominant Type of Mortgage Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
</tr>
<tr>
<td>Austria</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>Greece</td>
</tr>
</tbody>
</table>

Table 7: Predominant Type of Mortgage Interest Rate. Source: ECB (2003), IMF
Appendix 2: Model Derivations and Alternative Specifications

Steady-State Relationships

Using (3) in the steady state we obtain $R = 1/\beta$. From (18) and (19) we have that $\overline{R} = \overline{R} = R = 1/\beta$.

From the first order conditions for housing we can obtain the steady-state consumption-to-housing ratio for both constrained and unconstrained consumers:

$$\frac{C^u}{qH^u} = \frac{1}{j} (1 - \beta), \quad (34)$$

$$\frac{C^c}{qH^c} = \frac{1}{j} \left(1 - \beta - k \left(\beta - \frac{1}{j}\right)\right) = \frac{q}{j} \Phi, \quad (35)$$

where $\Phi \equiv \left(1 - \beta - k \left(\beta - \frac{1}{j}\right)\right)$. From (12) and (26) we obtain the constrained and unconstrained consumption-to-output ratio in the steady state:

$$\frac{C^c}{Y} = 1 - \gamma \left(\frac{\Phi}{\Phi + jk (1 - \beta)}\right), \quad (36)$$

$$\frac{C^u}{Y} = 1 - \frac{C^c}{Y}, \quad (37)$$

where $X = \varepsilon / (\varepsilon - 1)$

The housing-to-output ratio for constrained and unconstrained consumers:

$$\frac{qH^c}{Y} = \frac{(1 - \gamma) j}{X} \left(\frac{1}{\Phi + jk (1 - \beta)}\right), \quad (38)$$

$$\frac{qH^u}{Y} = \frac{X j (\Phi + jk (1 - \beta)) - j (1 - \gamma) \Phi}{X (\Phi + jk (1 - \beta)) (1 - \beta)}. \quad (39)$$

Log-Linearized Model
The model can be reduced to the following linearized system in which all lower-case variables with a hat denote percent changes from the steady state and steady-state levels are denoted by dropping the time index:

**Financial Intermediary**

\[
\hat{r}_t^* = \frac{(1 - \beta)}{\beta} E_t \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \hat{r}_{t-1}, \quad (40)
\]

\[
\hat{r}_t = \hat{r}_{t-1} \Rightarrow \hat{r}_t = \hat{r} = 0.
\] (41)

Equation (40) is the log-linearized fixed interest rate in each period \(\tau\). Using this result we can obtain the log-linearized aggregate fixed interest rate, which is zero in deviations from the steady state (equation (41)), given the initial condition of being at the steady state in the absence of shocks.

**Aggregate Demand**

\[
\hat{y}_t = \frac{C^u}{Y} \hat{c}^u_t + \frac{C^c}{Y} \hat{c}^c_t, \quad (42)
\]

\[
\hat{c}^u_t = E_t \hat{c}^u_{t+1} - (\hat{r}_t - E_t \hat{r}_{t+1}), \quad (43)
\]

\[
\hat{c}^c_t = \left( \frac{\Phi + jk (1 - \beta)}{\Phi} \right) (\hat{y}_t - \hat{c}_t) - \frac{j}{\Phi} (\hat{h}^u_t - \hat{h}^u_{t-1}) + \frac{kj}{\Phi} (\beta \hat{b}^c_t - \hat{b}^c_{t-1}) - kj (\alpha \hat{r}_t - \hat{r}_t), \quad (44)
\]

\[
\hat{b}^c_t = E_t \hat{b}^c_{t+1} + \hat{h}^c_t - (\alpha \hat{r}_t - E_t \hat{r}_{t+1}). \quad (45)
\]

Equation (42) is the log-linearized goods market clearing condition. Equation (43) is the Euler equation for unconstrained consumption. Equation (44) is the budget constraint for constrained individuals, which determines constrained consumption. Equation (45) is the log-linearized collateral constraint.

**Housing Equations**

\[
\frac{H^u}{Y} \hat{h}^u_t + \frac{H^c}{Y} \hat{h}^c_t = 0, \quad (46)
\]
\[
\hat{h}_t^u = \frac{1}{1 - \beta} (\hat{c}_t^u - \hat{q}_t) - \frac{\beta}{1 - \beta} E_t (\hat{c}_{t+1}^u - \hat{q}_{t+1}) , \tag{47}
\]

\[
\hat{h}_t^c = \frac{1 - k\beta}{\Phi} \hat{c}_t^\gamma - \frac{1}{\Phi} \hat{q}_t - \frac{k\beta}{\Phi} (\alpha \hat{c}_t - E_t \hat{q}_{t+1}) + \frac{\overline{\beta}}{\Phi} \hat{q}_{t+1} + \frac{\overline{\beta} (1 - k)}{\Phi} E_t \hat{c}_{t+1}^c. \tag{48}
\]

Equation (46) is the log-linearized market clearing condition for housing. Equation (47) is the housing margin for unconstrained consumers. Equation (48) is the analogous expression for constrained consumers.

**Aggregate Supply**

\[
\hat{y}_t = - \frac{1}{\eta - 1} (\gamma \hat{c}_t^u + (1 - \gamma) \hat{c}_t^c + \tilde{x}_t) , \tag{49}
\]

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \tilde{k} \tilde{x}_t + u_{\pi t}. \tag{50}
\]

Equation (49) is the production function combined with labor market clearing. Equation (50) is the New Keynesian Phillips curve that relates inflation positively to future inflation and negatively to the markup \((\tilde{k} \equiv (1 - \theta) (1 - \beta \theta) / \theta)\). \(u_{\pi t}\) is a normally distributed cost-push shock.

**Monetary Policy**

\[
\hat{r}_t = \rho \hat{r}_{t-1} + (1 - \rho) [(1 + \phi_{\pi}) (\hat{\pi}_t - \hat{\pi}_t^*) + \phi_{y} \hat{y}_t] + e_t. \tag{51}
\]

**Alternative Model Specifications**

**The Model with GHH Preferences**

Under GHH preferences, savers maximize:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \ln \left( \frac{C_t^u - (L_t^u)^\eta}{\eta} \right) + j \ln H_t^u , \tag{52}
\]

The first-order conditions are:

\[
\frac{1}{C_t^u - (L_t^u)^\eta} = \frac{R_t}{\pi_{t+1}} \left( \frac{C_{t+1}^u - (L_{t+1}^u)^\eta}{\eta} \right) , \tag{53}
\]
\[ w_t^u = (L_t^u)^{\eta-1}, \]

\[ \frac{j}{H_t^u} = \frac{1}{C_t^u - (\frac{L_t^u}{\eta})^\eta} q_t - \beta E_t \frac{1}{C_{t+1}^u - (\frac{L_{t+1}^u}{\eta})^\eta} q_{t+1}. \]  

Note that consumption no longer appears in the labor-supply decision (equation (54)).

Similarly, we can obtain the first-order conditions for borrowers:

\[ \frac{1}{C_t^{ci} - (\frac{L_t^{ci}}{\eta})^\eta} = \beta E_t \left[ \sum_{t+1}^{\infty} \pi_t^{ci} \left( C_{t+1}^{ci} - (\frac{L_{t+1}^{ci}}{\eta})^\eta \right) \right] + \lambda_t^{ci} R_t^{ci}, \]

\[ w_t^{ci} = (L_t^{ci})^{\eta-1}, \]

\[ \frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci} - (\frac{L_t^{ci}}{\eta})^\eta} q_t - \beta E_t \frac{1}{C_{t+1}^{ci} - (\frac{L_{t+1}^{ci}}{\eta})^\eta} q_{t+1} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}. \]  

The Model with Capital

We can add capital to the model so that unconstrained consumers have more saving choices. Since borrowers would not hold capital, the only part of the model that changes is the one of the unconstrained consumers:

**Unconstrained consumers** maximize their expected lifetime utility function:

\[ \max \ E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \]

subject to the budget constraint which includes capital:

\[ C_t^u + q_t H_t^u + K_t - (1 - \delta) K_{t-1} + \phi \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^2 + R_{t-1} b_t^{b_{t-1}} \pi_t \leq q_t H_{t-1}^u \]

\[ + z_t K_{t-1} + w_t^u L_t^u + b_t^u + F_t^u + S_t^u, \]

So, in this case, savers can buy houses or sell them at the current price \( q_t \) and hold bonds. They can
also hold capital $K_t$, whose price is normalized to unity, which they rent to firms at rental price $z_t$. $\delta$ is the depreciation rate of capital. Consumers also have to pay quadratic adjustment costs for capital.

Maximizing (59) subject to (60), we obtain the first-order conditions:

$$\frac{1}{C_t^u} = \beta E_t \left( \frac{R_t}{\pi_{t+1} C_{t+1}^u} \right),$$

(61)

$$w_t^u = (L_t^u)^{\eta-1} C_t^u,$$

(62)

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} q_{t+1}.$$  

(63)

$$\frac{1}{C_t^u} \left[ 1 + \phi \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right) \right] = \beta E_t \frac{1}{C_{t+1}^u} \left( z_t + (1 - \delta) + \phi \frac{K_{t+1}}{K_t} \left( \frac{K_{t+1} - K_t}{K_t} \right) \right),$$

(64)

Now, we have a fourth first order condition, equation (64), which is the first order condition with respect to capital.

Unconstrained individuals are not going to hold capital in equilibrium so their problem remains unchanged.

Intermediate goods are going to be produced according to the following production function:

$$Y_t = (L_t^u)^{\mu\gamma} (L_t^c)^{\mu(1-\gamma)} K_{t-1}^{1-\mu},$$

(65)

where $\gamma$ measures the relative size of each group in terms of labor and $\mu$ is the labor share.

Firms choose employment and capital to

$$\min \ w_t^u L_t^u + w_c^c L_t^c + z_t K_{t-1},$$

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor and capital demand are the following:

$$w_t^u = \frac{1}{X_t} \gamma^\mu \frac{Y_t}{L_t^u},$$

(66)
Non Separability between Housing and Non-Durable Consumption in the Utility Function

Unconstrained consumers consume an index of non-durable goods and housing defined as:

$$I_t^u = \left[ (1 - \nu)^{\frac{1}{\mu}} \left( C_t^u \right)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} \left( H_t^u \right)^{\frac{-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \quad (69)$$

where $\nu$ is the share of housing in the composite consumption index and $\mu$ is the elasticity of substitution between non-durable consumption goods and housing.

Unconstrained consumers maximize an expected lifetime utility function with two arguments; the consumption index and labor/leisure.

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln I_t^u - \frac{(L_t^u)^{\eta}}{\eta} \right), \quad (70)$$

Subject to the budget constraint:

$$C_t^u + q_t H_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t H_{t-1}^u + w_t^u L_t^u + b_t^u + F_t^u + S_t^u, \quad (71)$$

Maximizing (70) subject to (71), we obtain the first-order conditions:

$$\left( \frac{(1 - \nu)^{\frac{1}{\mu}} \left( C_t^u \right)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} \left( H_t^u \right)^{\frac{-1}{\mu}}}{(1 - \nu)^{\frac{1}{\mu}} (C_t^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_t^u)^{\frac{-1}{\mu}}} \right) = \beta E_t \left[ \left( \frac{R_t}{\pi_{t+1}} \right) \left( \frac{(1 - \nu)^{\frac{1}{\mu}} \left( C_{t+1}^u \right)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} \left( H_{t+1}^u \right)^{\frac{-1}{\mu}}}{(1 - \nu)^{\frac{1}{\mu}} (C_{t+1}^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_{t+1}^u)^{\frac{-1}{\mu}}} \right) \right], \quad (72)$$

$$w_t^u = (1 - \nu)^{\frac{1}{\mu}} \left( C_t^u \right)^{\frac{-1}{\mu}} \left( \frac{(1 - \nu)^{\frac{1}{\mu}} (C_t^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_t^u)^{\frac{-1}{\mu}}}{(1 - \nu)^{\frac{1}{\mu}} (C_t^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_t^u)^{\frac{-1}{\mu}}} \right)^{\eta-1}, \quad (73)$$

$$\frac{\nu^{\frac{1}{\mu}} \left( H_t^u \right)^{\frac{-1}{\mu}}}{(1 - \nu)^{\frac{1}{\mu}} (C_t^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_t^u)^{\frac{-1}{\mu}}} = \frac{(1 - \nu)^{\frac{1}{\mu}} \left( C_t^u \right)^{\frac{-1}{\mu}}}{(1 - \nu)^{\frac{1}{\mu}} (C_t^u)^{\frac{-1}{\mu}} + \nu^{\frac{1}{\mu}} (H_t^u)^{\frac{-1}{\mu}}} q_t \quad (74)$$
In the same way, we have the problem of the constrained consumers.

They also consume a consumption index that aggregates non-durable goods and housing:

\[ I_t^{ci} = \left( 1 - \nu \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}}, \]  

(75)

Constrained consumers maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

\[
\begin{align*}
\max & \ E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln I_t^{ci} - \frac{(L_t^{ci})^\eta}{\eta} \right), \\
\text{subject to:} & \\
C_t^{ci} + q_t H_t^{ci} + \frac{R_{t-1}^{ci} b_{t-1}^{ci}}{\pi_t} & \leq q_t H_{t-1}^{ci} + w_t^{ci} L_t^{ci} + b_t^{ci}, \\
E_t \frac{R_t^{ci}}{\pi_{t+1}} - b_t^{ci} & \leq k E_t q_{t+1} H_t^{ci}.
\end{align*}
\]  

(76)

(77)

The first-order conditions for the consumers are:

\[
\begin{align*}
\frac{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}}} &= \bar{\beta} E_t \left[ \left( \frac{R_t^{ci}}{\pi_{t+1}} \right) \left( \frac{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci+1} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci+1} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci+1} \right)^{\frac{\mu-1}{\mu}}} \right) \right] + \lambda_t^{ci} R_t^{ci}, \\
\frac{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}}} &= (L_t^{ci})^{\eta-1}, \\
\frac{\nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}}} &= \frac{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci} \right)^{\frac{\mu-1}{\mu}}} q_t \\
-\bar{\beta} E_t \frac{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci+1} \right)^{\frac{\mu-1}{\mu}}}{(1 - \nu) \frac{1}{\mu} \left( C_t^{ci+1} \right)^{\frac{\mu-1}{\mu}} + \nu \frac{1}{\mu} \left( H_t^{ci+1} \right)^{\frac{\mu-1}{\mu}}} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}. 
\end{align*}
\]  

(79)

(80)

(81)

These first-order conditions differ from those of the unconstrained individuals. In the case of con-
strained consumers, the Lagrange multiplier on the borrowing constraint \( \lambda^{ci} \) appears in the equations. From the Euler equations for consumption of the unconstrained consumers, we know that \( R = 1/\beta \) in steady state. If we combine this result with the Euler equation for consumption for the constrained individual we have that \( \lambda^{ci} = \frac{(\beta - \tilde{\beta})(1 - \nu)\tilde{\beta}^\frac{1}{2}(C^{ci})^{\frac{1}{\nu}}}{(1 - \nu)^{\frac{1}{2}}(C^{ci})^{\frac{1}{\nu}} + \nu \tilde{\beta}^\frac{1}{2}(H^{ci})^{\frac{1}{\nu}}} > 0 \) in steady state, given that \( \tilde{\beta} < \beta \). This means that the borrowing constraint holds with equality in steady state. As in Iacoviello (2005), since we log-linearize around the steady state and assuming that uncertainty is low, we can generalize this steady-state result. Then, the borrowing constraint is always binding, so that constrained individuals are going to borrow the maximum amount they are allowed to and their savings are going to be zero:

\[
\begin{align*}
    b^{ci}_t &= \frac{kE_t q_{t+1} H^{ci}_t \pi_{t+1}}{R^{c}_t},
\end{align*}
\]

Therefore, consumption for constrained individuals is determined by their flow of funds:

\[
\begin{align*}
    C^{ci}_t &= w^{ci}_t L^{ci}_t + b^{ci}_t + q_t (H^{ci}_{t-1} - H^{ci}_t) - \frac{R^{c}_{t-1} b^{ci}_{t-1}}{\pi_t},
\end{align*}
\]

And the first-order condition for housing becomes:

\[
\begin{align*}
    \frac{\nu \tilde{\beta}^\frac{1}{2} (H^{ci}_t)^{\frac{1}{\nu}}}{(1 - \nu)^{\frac{1}{2}} (C^{ci}_t)^{\frac{1}{\nu}} + \nu \tilde{\beta}^\frac{1}{2} (H^{ci}_t)^{\frac{1}{\nu}}} &= \frac{(1 - \nu)^{\frac{1}{2}} (C^{ci}_t)^{\frac{1}{\nu}}}{(1 - \nu)^{\frac{1}{2}} (C^{ci}_t)^{\frac{1}{\nu}} + \nu \tilde{\beta}^\frac{1}{2} (H^{ci}_t)^{\frac{1}{\nu}}} \left( q_t - \frac{kE_t q_{t+1} \pi_{t+1}}{R^{c}_t} \right) \\
    -\tilde{\beta}E_t &= \frac{(1 - \nu)^{\frac{1}{2}} (C^{ci}_{t+1})^{\frac{1}{\nu}}}{(1 - \nu)^{\frac{1}{2}} (C^{ci}_{t+1})^{\frac{1}{\nu}} + \nu \tilde{\beta}^\frac{1}{2} (H^{ci}_{t+1})^{\frac{1}{\nu}}} (1 - k) q_{t+1}.
\end{align*}
\]

The problem of the financial intermediary, the firms and the monetary policy is identical to the baseline model.

### A Two-Sector Model

We can relax the assumption that the housing supply is fixed and consider a two-sector model in which consumers can supply labor to the housing sector and the consumption sector.

**Unconstrained consumers:**

\[
\max_{E_0} E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C^u_t + j \ln H^u_t - \left( (L^u_{ct})^{1-\nu} + (L^u_{ht})^{1-\nu} \right)^{\frac{1}{1-\nu}} \right),
\]

(85)
subject to the budget constraint:

\[ C_t^u + q_t H_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} \leq q_t (1 - \delta) H_{t-1}^u + w_{ct}^u L_{ct}^u + w_{ht}^u L_{ht}^u + b_t^u + F_t^u + S_t^u, \quad (86) \]

Maximizing (85) subject to (86), we obtain the first-order conditions:

\[ \frac{1}{C_t^u} = \beta E_t \left( \frac{R_t}{\pi_{t+1} C_{t+1}^u} \right), \quad (87) \]

\[ w_{ct}^u = (1 + \eta) \left( (L_{ct}^u)^{1-\nu} + (L_{ht}^u)^{1-\nu} \right) \frac{\nu + \eta}{\nu} (L_{ct}^u)^{-\nu} C_t^u, \quad (88) \]

\[ w_{ht}^u = (1 + \eta) \left( (L_{ct}^u)^{1-\nu} + (L_{ht}^u)^{1-\nu} \right) \frac{\nu + \eta}{\nu} (L_{ht}^u)^{-\nu} C_t^u, \quad (89) \]

\[ \frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta E_t \frac{1}{C_{t+1}^u} (1 - \delta) q_{t+1}. \quad (90) \]

Equations (87) is the consumption Euler equation. Equations (88) and (89) are the labor-supply condition for the consumption and the housing sector, respectively. Equation (90) is the Euler equation for housing and states that the benefits from consuming housing have to be equal to the costs.

**Constrained consumers** maximize the lifetime utility function subject to the budget constraint and the collateral constraint:

\[ \max \ E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left( \ln C_t^{ci} + j \ln H_t^{ci} - \left( (L_{ct}^{ci})^{1-\nu} + (L_{ht}^{ci})^{1-\nu} \right) \frac{1+\eta}{1+\nu} \right), \quad (91) \]

subject to:

\[ C_t^{ci} + q_t H_t^{ci} + \frac{R_{t-1} b_{t-1}^{ci}}{\pi_t} \leq q_t (1 - \delta) H_{t-1}^{ci} + w_{ct}^{ci} L_{ct}^{ci} + w_{ht}^{ci} L_{ht}^{ci} + b_t^{ci}, \quad (92) \]

\[ E_t \frac{R_t}{\pi_{t+1} b_t^{ci}} \leq k E_t q_{t+1} H_t^{ci}. \quad (93) \]

The first-order conditions for the consumers are:
\[
\frac{1}{C_t^{ci}} = \ddot{\beta} E_t \left( \frac{R_t^c}{\pi_{t+1} C_{t+1}^{ci}} \right) + \lambda_t^{ci} R_t^c,
\]
\[
w_t^{ci} = (1 + \eta) \left( (I_{ct}^{ci})^{1-\nu} + (L_{ht}^{ci})^{1-\nu} \right)^{\frac{\nu + \eta}{\nu}} (L_{ct}^{ci})^{-\nu} C_t^{ci},
\]
\[
w_t^{ci} = (1 + \eta) \left( (I_{ct}^{ci})^{1-\nu} + (L_{ht}^{ci})^{1-\nu} \right)^{\frac{\nu + \eta}{\nu}} (L_{ht}^{ci})^{-\nu} C_t^{ci},
\]
\[
\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} q_t - \ddot{\beta} E_t \frac{1}{C_t^{ci}} (1 - \delta) q_{t+1} - \lambda_t^{ci} k E_t q_{t+1} \pi_{t+1}.
\]

Using the fact that the collateral constraint is binding, consumption for constrained individuals is determined by their flow of funds:

\[
C_t^{ci} = w_t^{ci} I_{ct}^{ci} + w_t^{ci} L_{ht}^{ci} + b_t^{ci} + q_t (1 - \delta) H_{t-1}^{ci} - H_t^{ci} - \frac{R_t^c b_t^{ci}}{\pi_t},
\]

And the first-order condition for housing becomes:

\[
\frac{j}{H_t^{ci}} = \frac{1}{C_t^{ci}} q_t - \ddot{\beta} E_t \frac{1}{C_t^{ci}} (1 - \delta) q_{t+1}.
\]

The problem for the financial intermediary and the final good producer is identical to the baseline model. The problem for the intermediate good producer is slightly changed. Intermediate goods are produced according to the following production function:

\[
Y_{ct} = (L_{ct}^{u})^\gamma (L_{ct}^{c})^{(1-\gamma)},
\]

where \(\gamma\) measures the relative size of each group in terms of labor.

Analogously, the production function for the housing sector is the following:

\[
Y_{ht} = (L_{ht}^{u})^\gamma (L_{ht}^{c})^{(1-\gamma)},
\]

Firms choose employment to

\[
\min w_t^{cu} I_{ct}^{u} + w_t^{cu} L_{ct}^{c} + w_t^{cu} L_{ct}^{u} + w_t^{cu} L_{ht}^{c}.
\]
subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

\[ w^u_{ct} = \frac{1}{X_t} \gamma \frac{Y_{ct}}{L^u_{ct}}, \]  
(102)  

\[ w^c_{ct} = \frac{1}{X_t} (1 - \gamma) \frac{Y_{ct}}{L^c_{ct}}, \]  
(103)  

\[ w^u_{ht} = q_t \gamma \frac{Y_{ht}}{L^u_{ht}}, \]  
(104)  

\[ w^c_{ht} = q_t (1 - \gamma) \frac{Y_{ht}}{L^c_{ht}}, \]  
(105)  

Production function in which labor for savers and labor for borrowers are substitutes

The consumers’ and financial intermediary’s problem remains unchanged. However, the intermediate goods firm’s production function turns into the following one:

\[ Y_t = \omega L^u_t + (1 - \omega) \left[ \alpha L^c_t + (1 - \alpha) L^f_t \right] = \omega L^u_t + (1 - \omega) L^c_t, \]  
(106)  

where \( \omega \) is the size of the unconstrained group.

Firms choose employment to

\[ \min \omega w^u_t L^u_t + (1 - \omega) w^c_t L^c_t, \]

subject to the production function, demand and the constraint imposed by nominal rigidity.

The first-order conditions for labor demand are the following:

\[ w^u_t = w^c_t = \frac{1}{X_t}. \]  
(107)  

We see that in this case, the wage paid to each group is the same.

Aggregate variables are defined as follows:

\[ C_t = \omega C^u_t + (1 - \omega) C^c_t. \]  
(108)
\[ L_t \equiv \omega L_t^w + (1 - \omega) L_t^c. \quad (109) \]

\[ H_t \equiv \omega H_t^w + (1 - \omega) H_t^c. \quad (110) \]