# Fair and efficient taxation under partial control 

Erwin Ooghe \& Andreas Peichl

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- the more an outcome is determined by 'luck' (resp. 'effort')
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- under partial control [and lots of assumptions]


## Individual preferences / constraints

- Utility $U(c, \mathbf{x}, \mathbf{e})$ is a function of
- consumption $c$
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- A production function $f$ maps effort $\mathbf{e}$ into $(y, \mathbf{x})$
- Individuals solve

$$
\max _{\mathbf{e}} U(c, \mathbf{x}, \mathbf{e}) \text { s.t. } c \leq y-\tau(y, \mathbf{x}) \&(y, \mathbf{x})=f(\mathbf{e})
$$

## Simplifying assumptions

- quasi-linear \& additive structure on utility:
- $U(c, \mathbf{x}, \mathbf{e})=c+\sum_{j=1}^{J} \beta_{j} x_{j}-h(\mathbf{e})$, with
- $h(\mathbf{e})=\sum_{j=0}^{J} \frac{\delta_{j}}{\exp \gamma_{j}} \exp \left(\frac{e_{j}}{\delta_{j}}\right)$.


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- linear production function $f$ :
- $y=\alpha_{0} e_{0}+\left(1-\alpha_{0}\right) \theta_{0}$, and
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- $x_{j}=\alpha_{j} e_{j}+\left(1-\alpha_{j}\right) \theta_{j}, j=1,2, \ldots, J$.
- Unobserved abilities and tastes:
- $\boldsymbol{\theta}=\left(\theta_{0}, \theta_{1}, \ldots, \theta_{J}\right)$ is $N\left(\boldsymbol{\mu}^{\theta}, \boldsymbol{\Sigma}^{\theta}\right)$
- $\gamma=\left(\gamma_{0}, \gamma_{1}, \ldots, \gamma_{J}\right)$ is $N\left(\boldsymbol{\mu}^{\gamma}, \boldsymbol{\Sigma}^{\gamma}\right)$


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- The (per-capita) tax revenue is

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R(\tau)=\int_{\boldsymbol{\theta}} \int_{\gamma} \tau\left(y^{*}(\tau, \boldsymbol{\theta}, \gamma), \mathbf{x}^{*}(\tau, \boldsymbol{\theta}, \gamma)\right) d F(\boldsymbol{\theta}) d G(\gamma)
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- The planner solves

$$
\max _{\tau} W(\tau) \text { s.t. } R(\tau) \geq R_{0}
$$

with $R_{0}$ an exogenous (per-capita) revenue requirement.

## Simplifying assumptions

- Welfare is the 'average transformed well-being', i.e.,

$$
W(\tau)=\phi^{-1}\left[\int_{\boldsymbol{\theta}} \int_{\gamma} \phi(v(\tau, \boldsymbol{\theta}, \gamma)) d F(\boldsymbol{\theta}) d G(\boldsymbol{\gamma})\right]
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- Well-being $\widehat{v}=v(\tau, \boldsymbol{\theta}, \gamma)$ is a cardinalization of utility and implicitly defined as

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V(\tau, \boldsymbol{\theta}, \gamma)=V\left(R_{0},(\widehat{v}, \widehat{v}, \ldots, \widehat{v}), \gamma\right)
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- Taxation is linear, so

$$
\tau(y, \mathbf{x})=T+t_{0} y+\sum_{j=1}^{J} t_{j} x_{j} .
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## ... but 'defendable' assumptions

The choice of $\phi$ and $v$ guarantee that $W(\tau)$ satisfies

- Pareto: higher utilities are reflected in higher welfare
- Compensation (for abilities): a PD transfer between individuals with the same tastes improves social welfare
- Responsibility (for tastes): if all individuals have the same ability, then the laisser-faire should result $\left(\tau^{*}=R_{0}\right)$


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Tax up to the point where: marginal efficiency cost $=r \times$ marginal fairness gain,
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We focus on two special cases-income only \& adding a tag to income-before discussing the general case in more detail.

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- decreases with the degree of control $\alpha_{0}$.


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so, it should be higher

- the higher the direct effect $\beta_{1}$ of the tag on well-being
- the higher the signal $\sigma_{01}^{\theta}$. [+ other; see paper]


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- In particular, $\operatorname{cov}\left(x_{1}, y\right) / \operatorname{cov}\left(x_{1}, x_{1}\right)$ is an OLS estimate, so
- $\beta_{1}$ is the direct effect of the tag on well-being, and
- $\left(1-t_{0}^{*}\right) \times \operatorname{cov}\left(x_{1}, y\right) / \operatorname{cov}\left(x_{1}, x_{1}\right)$ is $E[$ indirect effect $]$.


## Testable conditions

- Consider income and non-income factors, partitioned into
- non-controllable non-income factors $N=\left\{j \mid \alpha_{j} \rightarrow 0\right\}$
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- Consider data collected in:
- a $n \times 1$ vector $y$ for gross incomes,
- a $n \times|N|$ matrix $\boldsymbol{X}_{N}$ for the non-controllable factors,
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- a $n \times|P|$ matrix $\boldsymbol{X}_{P}$ for the partially controllable factors.
- We obtain that the optimal tax rates $t_{j}^{*}$, for $j$ in $N$, are

$$
\boldsymbol{t}_{N}^{*}=\boldsymbol{\beta}_{N}+\left(1-t_{0}\right)\left(\boldsymbol{X}_{N}^{\prime} \boldsymbol{X}_{N}\right)^{-1} \boldsymbol{X}_{N}^{\prime} \boldsymbol{y}+\left(\boldsymbol{X}_{N}^{\prime} \boldsymbol{X}_{N}\right)^{-1} \boldsymbol{X}_{N}^{\prime} \boldsymbol{X}_{P}\left(\boldsymbol{\beta}_{P}-\boldsymbol{t}_{P}\right)
$$

[implementation + link with ${ }^{\text {E }}$ © ${ }^{\prime}$ '-literature]

