Social Security and the Interactions between Aggregate & Idiosyncratic Risk

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Motivation

- Question: Welfare effects of expanding PAYG system?
- Trade-off: Insurance vs. crowding out
- Social security as insurance against
  - Idiosyncratic risk (e. g., Imrohoroglu, et al. (1999))

    AND
  - Aggregate risk (e. g., Krueger & Kubler (2006))
- General conclusion: costs dominate

Interactions

1. Counter-cyclical variance of income risk (*CCV*)
   - Idiosyncratic risk higher in downturn than in boom
   - Mankiw(1986), Storesletten, et al. (2004)

2. Life-cycle interaction (*LCI*)
   - With independent risks
   - Value of savings at retirement depends on both shocks
   - First expose analytically

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Main Result

- Substantial welfare gains in general equilibrium despite crowding out (ex-ante perspective)
- Important quantitative role of risk interactions (LCI + CCV)

Two-Generations Model: Households

- Households live 2 periods, consume only when old
- Lifetime utility:

$$U_{i,t} = \beta \frac{1}{1-\theta} \, c_{i,2,t+1}^{1-\theta}$$

- Budget constraint:

$$c_{i,2,t+1} = a'_{i,1,t}(1 + r_{t+1}) + b_{t+1}$$
$$a'_{i,1,t} = (1-\tau)\eta_{i,1,t} w_t$$

Two-Generations Model: Endowments

- Partial equilibrium factor prices:

$$1 + r_t = \varrho_t \bar{R}$$

$$w_t = \zeta_t \bar{w}_t = \zeta_t \bar{w}_{t-1}(1 + \lambda)$$

- PAYG social security:

$$b_t = \tau w_t$$

- Distribution: jointly log-normal, mean one, independent

Two-Generations Model: Main Result

Proposition

A marginal introduction of social security increases $E_{t-1}U_t$ if

$$(1+\lambda)\cdot(1+V)^{\theta} > \bar{R},$$

where

$$\begin{aligned} V &\equiv var\left(\eta_{i,1,t}\zeta_t\varrho_{t+1}\right) \\ &= \underbrace{\sigma_\eta^2}_{IR} + \underbrace{\sigma_\zeta^2 + \sigma_\varrho^2 + \sigma_\zeta^2\sigma_\varrho^2}_{AR} + \underbrace{\sigma_\eta^2\left(\sigma_\zeta^2 + \sigma_\varrho^2 + \sigma_\zeta^2\sigma_\varrho^2\right)}_{LCI=IR\cdot AR}. \end{aligned}$$

Quantitative Model: Summary

1. Scale-up and extend simple model: ▸ Show details
   (a) 58 generations, 1-year periods
   (b) Population growth
   (c) Wage shocks ⇒ TFP shocks
   (d) Return shocks ⇒ depreciation shocks
   (e) (Auto-)correlation (TFP, depreciation) unrestricted
   (f) Idiosyncratic risk: autocorrelated, *CCV*
   (g) Deterministic age-productivity profile
   (h) Epstein-Zin preferences
2. Additional elements: ▸ Show details
   (a) Two assets: risk-free bond in addition to risky stock
   (b) Representative firm with exogenous capital structure
3. General equilibrium

Quantitative Model: Baseline Calibration

Parameter	*Target (Source)*	*Value*
Working age, retirement age, maximum age		21, 65, 78
Age productivity	earnings profiles (PSID)	$\{\epsilon_j\}_1^J$
Population growth, n	U.S. Social Sec. Adm. (SSA)	0.011
Technol. growth, λ	TFP growth (NIPA)	0.018
Capital share, α	wage share (NIPA)	0.32
Leverage ratio, $\overline{\kappa}_f$	U.S. capital structure (Croce (2010))	0.66
Autocorrelation of η	(Storesletten, et al. (2004))	0.952
CCV, $\sigma_{\nu,t}$	(Storesletten, et al. (2004))	$\{0.21, 0.13\}$
EIS, φ	exogenous (various)	0.5
CRRA, θ	exogenous in baseline	3.0
Discount factor, β	$K/Y = 2.65$ (NIPA)	0.981
Mean depreciation, δ_0	$E(r_f) = 2.3\%$ (Shiller)	0.10
Std. depreciation, $\bar{\delta}$	$\sigma(\frac{C_{t+1}}{C_t}) = 0.03$ (NIPA)	0.07
Std. TFP shocks, $\bar{\zeta}$	$\sigma(TFP) = 0.029$ (NIPA)	0.029
Prob$(\zeta' = \zeta_i \| \zeta = \zeta_i)$	$autoc(TFP) = 0.88$ (NIPA)	0.941
Prob$(\delta' = \delta_i \| \zeta' = \zeta_i)$	$cor(TFP, r) = 0.50$ (NIPA, Shiller)	0.885

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Results: General and Partial Equilibrium

- Experiment: $\tau = 0\% \rightarrow \tau = 2\%$, unanticipated
- Ex-ante expected CEV of a newborn, g_c
- GE vs. PE: "Small open economy"

	GE	PE	Crowd Out
g_c	+2.03%	+5.46%	-3.43%

▸ Show graphs

Results: Decomposition of Welfare Effects

- Decomposition procedure in PE ▸ Show details

Welfare effects in PE

g_c	$g_c(0)$	$dg_c(IR)$	$dg_c(AR)$	$dg_c(LCI)$	$dg_c(CCV)$
5.46% =	-0.62%	+0.84%	+2.00%	+1.66%	+1.57%

- $\frac{dg_c(LCI)+dg_c(CCV)}{g_c} = 0.59$

Results: Decomposition of Welfare Effects

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Results: Welfare in Different Economies

Consumption equivalent variation, g_c

Scenario	GE	PE	Crowd Out
AR-only	**-0.54%**	-0.38%	-0.16%
IR-only	**-1.91%**	0.27%	-2.18%
No-risk	**-1.13%**	-0.62%	-0.51%

- No interactions

Results: Welfare Across Calibrations

Consumption equivalent variation, g_c

Scenario	GE	PE	$\frac{dg_c(LCI)+dg_c(CCV)}{g_c}$
$IES = 0.5$			
Baseline	+2.03%	+5.46%	0.59
Sharpe ratio	+4.16%	+8.32%	0.65
Equity premium	+2.78%	+6.53%	0.67
$IES = 1.5$			
Baseline	+2.56%	+3.65%	0.60
Sharpe ratio	+5.08%	+8.09%	0.65
Equity premium	+4.32%	+7.65%	0.72

Conclusion

- Analysis of multiple risks
- Life-cycle structure: Interactions between risks
- Interactions between risks: Substantial welfare effects
- Introduction of social security: Robust welfare gains in GE

Outlook: Directions for Future Research

- Companion paper: Analytical GE extension
- Endogenous labor supply
- Guvenen et al. (2012): Left skewness
- Optimal size and/or structure of social security
- Government debt / buffer in pension system
- Interaction between social security and unemployment insurance

Two-Generations Model: Welfare Decomposition

Definition

- Consumption equivalent variation, $g_c(\cdot)$:

$$
\begin{aligned}
g_c(IR) &= g_c(0) + dg_c(IR) \\
g_c(AR) &= g_c(0) + dg_c(AR) \\
g_c(AR, IR) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI)
\end{aligned}
$$

- First-order Taylor series approximation of $g_c(AR, IR)$ gives:

$$
g_c(AR, IR) \approx \underbrace{\frac{1+g}{\bar{R}} - 1}_{g_c(0)} + \underbrace{\theta \frac{1+g}{\bar{R}} AR}_{dg_c(AR)} + \underbrace{\theta \frac{1+g}{\bar{R}} IR}_{dg_c(IR)} + \underbrace{\theta \frac{1+g}{\bar{R}} LCI}_{dg_c(LCI)}
$$

Quantitative Model: Preferences

- Epstein-Zin preferences:

$$U_{i,j,t} = \left[c_{i,j,t}^{\frac{1-\theta}{\gamma}} + \beta \left(\mathbb{E}\left[U_{i,j+1,t+1}^{1-\theta} \right] \right)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{1-\theta}}$$

- θ: Coefficient of relative risk-aversion
- φ: Elasticity of intertemporal substitution
- $\gamma = \frac{1-\theta}{1-\frac{1}{\varphi}}$

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Quantitative Model: Endowments

- Dynamic budget constraint:

$$a'_{i,j,t} + c_{i,j,t} = a_{i,j,t}(1 + r^f_t + \kappa_{i,j-1,t-1}(r_t - r^f_t)) + y_{i,j,t}$$
$$\text{with} \quad a'_{i,J,t} \geq 0$$

- Income:

$$y_{i,j,t} = \begin{cases} (1-\tau)\ \eta_{i,j,t}\ w_t\ \epsilon_j & \text{for } j < j_{ret} \\ b_t & \text{for } j \geq j_{ret} \end{cases}$$

- Idiosyncratic stochastic component:

$$\ln \eta_{i,j,t} = \rho \ln \eta_{i,j-1,t-1} + \nu_{i,t}, \qquad \sigma^2_\nu(contr) > \sigma^2_\nu(expans)$$

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Quantitative Model: Firms

- Neoclassical production:

$$Y_t = F(\zeta_t, K_t, L_t) = \zeta_t K_t^{\alpha} (\Upsilon_t L_t)^{1-\alpha}$$

- Wage rate:

$$w_t = \zeta_t (1-\alpha) k_t^{\alpha} (1+g) \Upsilon_{t-1}$$

- Net return on capital:

$$r_t^k = \zeta_t \alpha k_t^{\alpha-1} - \delta_t$$

- Leveraged stock return:

$$r_t = r_t^k (1 + \overline{\kappa}_f) - \overline{\kappa}_f r_t^f$$

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Quantitative Model: Governm & Social Security

- PAYG budget constraint:

$$\tau_t w_t L_t = Ret_t \int P_{i,j,t} d\Phi$$

- For today:
  - fixed contribution rate: $\tau_t = \tau$
  - lump-sum benefits: $P_{i,j,t} = P_t$
- Experiment: single, unanticipated increase in τ

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Quantitative Model: Transformations

- Rewrite in terms of cash at hand, x

$$x = a(1 + r^f + \kappa(r - r^f)) + y$$

- Denote measure over agents by $\Phi_t(j, x, \eta)$
- State space for each agent: $\mathcal{S} = (j, x, \eta, z, \Phi)$

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Stationary recursive competitive equilibrium

- Price functions $\{r(\Phi, z), r^f(\Phi, z), w(\Phi, z)\}$
- Policy functions $c(\mathcal{S})$, $a'(\mathcal{S})$, $\kappa(\mathcal{S})$ that maximize the household's utility for given $\{r, r^f, w, \tau, b\}$
- Firm choice k that maximizes profits for given $\{r, r^f, w\}$
- Govt policies $\tau(\Phi, z), b(\Phi, z)$ implying budget balance
- Market clearing, in particular:

$$k'(\Phi', z') = \int a'(\mathcal{S})\, d\Phi(j, x, \eta)$$

$$B'(\Phi', z') = \int (1 - \kappa(\mathcal{S}))\, a'(\mathcal{S})\, d\Phi(j, x, \eta)$$

- A law of motion $\Phi' = H(\Phi, z, z')$ consistent with policies

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Quantitative Model: Equilibrium and Solution

- Competitive recursive equilibrium: ▸ Show details

  competitive prices $\{r, r_f, w\}$, optimal household choices $\{c, a', \kappa\}$ and firm choices $\{K, L\}$, market clearing, soc. sec. budget balance $\{\tau, b\}$, law of motion

- Law of motion (Krusell & Smith (1997)):

  (i) capital stock, (ii) equity premium ▸ Show details

- Simulation periods > 80.000
- Regression fit: $R_k^2 = 0.999$, $R_\mu^2 = 0.977$
- Endogenous grid method (Carroll (2006)) ▸ Show details
- Parallel on 16 cores, computation time 20 - 60 hrs

Quantitative Model: Household problem

- First order conditions

$$\mathbb{E}\left[u(j+1,\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}}(c')^{\frac{1-\theta-\gamma}{\gamma}}\left(r'-r^{f'}\right)\right]=0$$

$$c=\left(\tilde{\beta}\frac{1+r^{f'}}{1+g}\left(\mathbb{E}\left[u(j+1,\cdot)^{1-\theta}\right]\right)^{\frac{1-\gamma}{\gamma}}\mathbb{E}\left[u(j+1,\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}}(c')^{\frac{1-\theta-\gamma}{\gamma}}\right]\right)^{\frac{\gamma}{1-\theta-\gamma}}$$

- Endogenous grid method (Carroll 2006) applied to portfolio choice
  - Avoid problem of jointly finding $\{a', \kappa\}$
  - Reduce 2-dimensional optimization to 2 sequential steps: first solve for κ, then for c

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Quantitative Model: Euler equation errors

- Relative Euler equation errors are computed as

$$e = 1 - \frac{\left(\tilde{\beta}\frac{1+r^{f\prime}}{1+g}\left(\mathbb{E}\left[u(j+1,\cdot)^{1-\theta}\right]\right)^{\frac{1-\gamma}{\gamma}}\mathbb{E}\left[u(j+1,\cdot)^{\frac{(1-\theta)(\gamma-1)}{\gamma}}(c')^{\frac{1-\theta-\gamma}{\gamma}}\right]\right)^{\frac{\gamma}{1-\theta-\gamma}}}{c}$$

- In baseline model
    - Average error: $e^{avg} = 0.001$

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Quantitative Model: Laws of motion

- Problem: agents need measure Φ to forecast prices
- Krusell and Smith (1998): approximate $\Phi' = H(\Phi, z, z')$ by low-dimensional object
- Our approximation is

$$(k', \mu) = \hat{H}(k, k^2, z)$$

  where $\mu = \mathbb{E}r' - r^{f\prime}$, the expected equity premium
- To find $\hat{H}$, need to simulate and update until convergence
- Regression fit: $R_k^2 = 0.999$, $R_\mu^2 = 0.977$

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Quantitative Model: Transition Matrix $\pi(z'|z)$

- $\pi^\zeta = \pi(\zeta' = 1 - \bar{\zeta} \mid \zeta = 1 - \bar{\zeta})$
- $\pi^\delta = \pi(\delta' = \delta_0 + \bar{\delta} \mid \zeta' = 1 - \bar{\zeta}) = \pi(\delta' = \delta_0 - \bar{\delta} \mid \zeta' = 1 + \bar{\zeta})$
- both symmetric

$$\pi^z = \begin{bmatrix} \pi^\zeta \cdot \pi^\delta & \pi^\zeta \cdot (1-\pi^\delta) & (1-\pi^\zeta) \cdot (1-\pi^\delta) & (1-\pi^\zeta) \cdot \pi^\delta \\ \pi^\zeta \cdot \pi^\delta & \pi^\zeta \cdot (1-\pi^\delta) & (1-\pi^\zeta) \cdot (1-\pi^\delta) & (1-\pi^\zeta) \cdot \pi^\delta \\ (1-\pi^\zeta) \cdot \pi^\delta & (1-\pi^\zeta) \cdot (1-\pi^\delta) & \pi^\zeta \cdot (1-\pi^\delta) & \pi^\zeta \cdot \pi^\delta \\ (1-\pi^\zeta) \cdot \pi^\delta & (1-\pi^\zeta) \cdot (1-\pi^\delta) & \pi^\zeta \cdot (1-\pi^\delta) & \pi^\zeta \cdot \pi^\delta \end{bmatrix}$$

- STY: $\pi^\delta = 1$
- GM: $\pi^\delta = 0.5$
- Our paper: $\pi^\delta = 0.885$

Results: General Equilibrium

- Experiment: $\tau = 0\% \rightarrow \tau = 2\%$, unanticipated
- g_C: ex-ante expected CEV of a newborn

	GE
g_C	+2.03%
$\Delta K/K$	-11.93%
Δr	+1.01%
Δr_f	+1.03%
$\Delta w/w$	-4.08%

Results: Decomposition Procedure

- Same PE experiment
- Sequentially "turn off" each risk
- Look at welfare change for each economy
- Recall decomposition of CEV:

$$\begin{aligned}
g_c(AR, IR, CCV) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI) + dg_c(CCV) \\
g_c(AR, IR) &= g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI) \\
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$$g_c(AR, IR) = g_c(0) + dg_c(AR) + dg_c(IR) + dg_c(LCI)$$

$$g_c(AR) = g_c(0) + dg_c(AR)$$

$$g_c(IR) = g_c(0) + dg_c(IR)$$

Results: Economy without Aggregate Risk

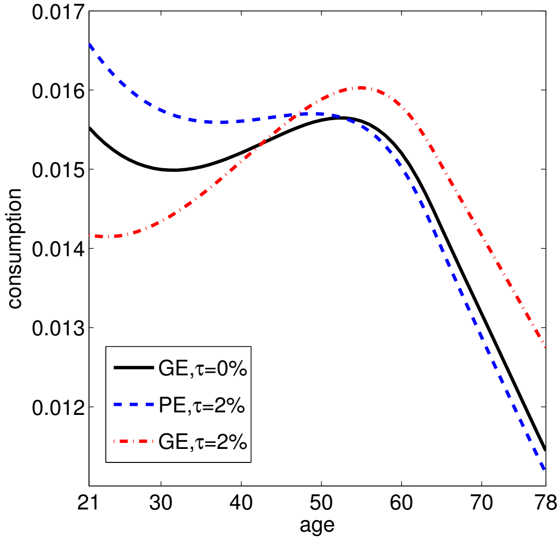
- Only one asset
- Empirical average asset return (Siegel (2002)): 4.2%
- Model average asset returns

Equity premium calibration

Median portfolio return	3.07%
E(mpk)	4.70%
Capital-structure weighted average of $E(r)$ and $E(r_f)$	5.24%

- Comparable and consistent results

Results: Life-Cycle Consumption



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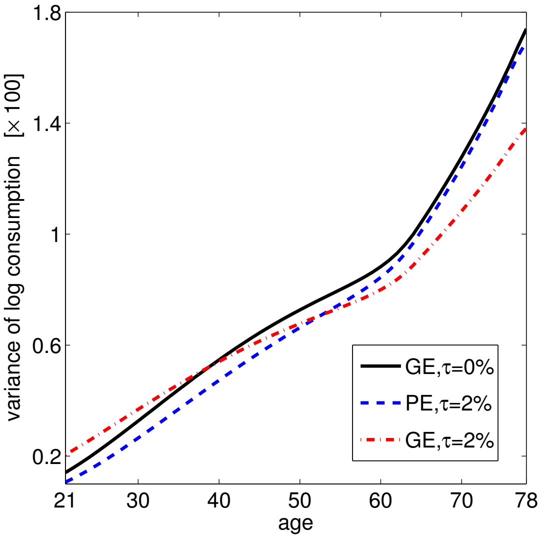
Results: Variance of Log-Consumption



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Results: The Role of Risks

Scenario	Calibrated parameters	Targets
AR-only	$\beta, \boldsymbol{\theta}, \delta_0, \boldsymbol{\bar{\delta}}, \pi^{\delta}$	$\frac{K}{Y}, \mathbb{E}(\boldsymbol{r_t - r_t^f}), \mathbb{E}(r_t^f), \boldsymbol{\sigma(r_t)}, \rho(\zeta, r_t)$
IR-only	$\beta, \bar{\delta}$	$\frac{K}{Y}, r^f$
No-risk	$\beta, \bar{\delta}$	$\frac{K}{Y}, r^f$

- Risk aversion in AR-economy: $\theta = 14.65$!

Results: Sensitivity Analysis

Scenario	Calibrated parameters	Targets
$IES = 0.5$		
Baseline	$\boldsymbol{\beta}, \delta_0, \bar{\delta}, \pi^{\delta}$	$\frac{\boldsymbol{K}}{\boldsymbol{Y}}, \mathbb{E}(r_t^f), \sigma(\frac{C_{t+1}}{C_t}), \rho(\zeta, r_t)$
Sharpe ratio	$\boldsymbol{\theta}, \delta_0, \bar{\delta}, \pi^{\delta}$	$\frac{\mathbb{E}(\boldsymbol{r_t - r_t^f})}{\boldsymbol{\sigma_{r_t}}}, \mathbb{E}(r_t^f), \sigma(\frac{C_{t+1}}{C_t}), \rho(\zeta, r_t)$
Equity premium	$\boldsymbol{\theta}, \delta_0, \boldsymbol{\bar{\delta}}, \pi^{\delta}$	$\mathbb{E}(\boldsymbol{r_t - r_t^f}), \mathbb{E}(r_t^f), \boldsymbol{\sigma(r_t)}, \rho(\zeta, r_t)$
$IES = 1.5$		
Baseline	as above	as above
Sharpe ratio	as above	as above
Equity premium	as above	as above

- Risk aversion in SR-economy ($IES = 0.5$): $\theta = 10.88$
- Risk aversion in EP-economy ($IES = 0.5$): $\theta = 5.51$