Search Costs and Risky Investment in Quality*

Arthur Fishman Nadav Levy

Bar Ilan University IDC Herzliya

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Abstract

One striking development associated with the explosion of e-commerce is the increased transparency of sellers' quality. In this paper we analyze how this affects firms' incentives to invest in quality when the outcome of investment is uncertain. We identify two conflicting effects. On the one hand, reducing the consumer's cost of search for quality exacerbates the negative effects of delivering poor quality. On the other hand, the fact that a firm, despite its best efforts, may fail to live up to consumers' more demanding expectations, makes investment less attractive. We show that reducing the search cost leads to higher quality if the initial level of the search cost is sufficiently high but may lead to lower quality if the initial level of the search cost is sufficiently low.

keywords: search, internet search, quality, risky investment

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Arthur Fishman, Department of Economics, Bar-Ilan University, Israel, afishman@mail.biu.ac.il; Nadav Levy, School of Economics, Interdisciplinary Center (IDC) Herzliya, Israel, nadavl@idc.ac.il.

1 Introduction

One striking development associated with the explosion of e-commerce is the increased transparency of sellers' quality. Sites like yelp.com, tripAdvisor.com and cnet.com. in which consumers and professionals rate the quality of a wide variety of products and services, facilitate comparison of competing vendors by new consumers. Indeed, consumers seem to have become increasingly dependent on such sources. For example, according to a survey by Forrester Research,[1] some 86% of respondents use ratings and reviews for online purchases and 44% go online before buying products in-store. Chevalier and Mayzlin (2006) find that an improvement in a book's reviews on Amazon.com significantly increase relative sales at that site.

In this paper we ask how such technological innovations, which lower consumers' search costs affect vendors' incentives to invest in quality improvements. As it becomes less costly for consumers to search for superior quality, the negative consequences for firms which fail to provide it become more severe. This would seem to suggest that lower search costs increase investment in quality.

However, lower search costs may also have a countervailing effect when the effect of investment on quality is uncertain. For example, a new restaurant may strive to maintain high standards of hygiene and buy the most expensive ingredients which nevertheless turn out to be contaminated and make its customers ill. Or a manufacturer may invest in a new model with novel features which fails to catch on with consumers. For example, although the Ford motor company invested heavily in launching the Edsel, the model was so unpopular that its name has become synonymous with failure. Similarly, Coca Cola's investment in 'new coke' was rejected by consumers. In these cases, where a firm may deliver poor quality in spite of its best efforts, lower search costs, by making consumers more demanding and less forgiving of less than perfect performance, may discourage investment in quality.

We explore this issue in a consumer search model in which firms choose whether or not to make investments which increase the likelihood of achieving high quality and consumers are able to search for quality.

Our main result is the following. If the initial level of the search cost is sufficiently high,

[1] ComputerWeekly.com, November 10, 2010.

reducing it increases the incentives to invest in quality. In this case, a higher proportion of firms invest in equilibrium, which leads to higher average product quality in the market. However, if the initial cost is sufficiently low, reducing it further can lower the incentive to invest, which lowers the proportion of firms which invest and leads to lower average product quality in the market.

This result raises the theoretical possibility that given the present state of internet technology, search costs may have already decreased sufficiently that further advances will lead to lower quality of some products. Ater and Orlov (2010) find that flight on-time performance is worse for flights originating in areas with greater internet access. Interpreting internet penetration as a proxy for the cost of search for flight on-time information (which is available from sites like flightstats.com) this finding is consistent with a negative correlation between quality and lower search costs.

The related literature includes, first of all, the vast consumer search literature. In the seminal papers (Diamond (1971), Burdett and Judd (1983)) products are homogenous and consumers search for the lowest price. In a second strand of this literature (Wolinsky (1986), Anderson and Renault (1999), Armstrong, Vickers and Zhou (2009)) firms' products are horizontally differentiated, and consumers search for both a lower price and a product they like. In all of these papers, product characteristics are exogenously determined.

In this paper, products are vertically differentiated by quality which is determined endogenously through investment.[2] All firms strive - but not all succeed - to achieve the highest possible quality. Consumers have identical preferences and search for the best price/quality combination.

Also related are papers (Horner (2002), Kranton (2003) and Bar-Isaac (2005)) which examine the effects of increasing competition on investment in quality. In those studies, consumers are costlessly informed about the qualities of all firms[3] whereas here, by contrast, consumers are uninformed and decide whether or not to become better informed.

Finally, the feature of our model that lower search costs can lead to lower quality also

[2] Bar-Isaac, Caruana and Cuñat (2009) develop a model in which products are horizontally differentiated amd firms chose product design.

[3] In Horner's model, consumers knows firms' customer base, which in equilibrium is equivalent to knowing its record.

connects to a literature showing that better information can lead to worse outcomes (e.g. Moav and Neeman (2010), Dranove, Kessler, McClellan and Satterthwaite (2003)).

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyses the equilibrium search behavior of consumers, the investment and pricing decisions of firms and considers how the firms' investments in quality depends on the level of the search cost. Section 4 concludes.

2 Model

There is a continuum of identical firms and the measure of consumers per firm is one. Each firm decides whether or not to invest a fixed amount, I, which probabilisticaly determines the quality of its product, as described in the next paragraph. A firm may produce at a unit cost of c which is independent of whether or not it invested.

We denote product quality by θ. There are N quality levels, denoted $\theta_1, ..., \theta_N$, $0 < \theta_1 < \theta_2 < ... < \theta_N$, where θ_N is the highest (most prized) quality, θ_{N-1} is the second highest (second most prized) quality level, etc. and θ_1 is the lowest (least desirable) quality. If a firm invests, it produces quality θ_r with probability α_r^I . If it does not invest, it produces θ_r with probability α_r^{NI}.

For notational simplicity, we shall henceforth refer to a firm with quality θ_r as an "r firm" and to its quality as r.

We assume that the distribution of qualities under investment satisfies the following assumption:

Assumption 1 (MLRP)

$$\frac{\alpha_N^I - \alpha_N^{NI}}{\alpha_N^I} > \frac{\alpha_{N-1}^I - \alpha_{N-1}^{NI}}{\alpha_{N-1}^I} > ... > \frac{\alpha_1^I - \alpha_1^{NI}}{\alpha_1^I}$$

This assumption means that investment has a greater effect on the probability of achieving a higher quality level than a lower one. Note that this assumption implies that the distribution of qualities of a firm that invests first-order stochastically dominates that of a firm that does not invest, i.e. $\sum_{r=1}^k \alpha_r^I < \sum_{r=1}^k \alpha_r^{NI}$ for all $k < N$. (FOSD).

Consumers have downward sloping demand functions. Specifically, a consumer's utility from a quantity Q of quality r is

$$\theta_r v(Q), \tag{1}$$

where $v(\cdot)$ is a concave function and $v'(0) = \infty$.

Let $Q_r(p) = \arg\max_Q [\theta_r v(Q) - pQ]$ denote the quantity demanded by a consumer from a *monopolist* firm with quality r when its price is p. $Q_r(p)$ is implicitly defined by the first-order condition

$$\theta_r v'(Q) - p = 0. \tag{2}$$

Let $S_r(p) = \theta_r v(Q_r(p)) - pQ_r(p)$ denote the consumer surplus from buying a quantity $Q_r(p)$ of quality r.[4] Let $\pi_r(p) = (p - c) Q_r(p)$ be the per-consumer profit of a monopolist firm with quality r and price p. We assume $\pi_r(p)$ is single-peaked, and denote $p_r^* = \arg\max \pi_r(p)$ as the unique monopoly price corresponding to a quality r and π_r^* as the monopoly profit. Finally, let p_r be the *equilibrium* price of firms with a quality of r.

It follows immediately that at monopoly prices, the consumer surplus is strictly increasing in quality. The proof and all other proofs that don't appear in the text are in the appendix.

Lemma 1 *Consider two qualities $r, \widehat{r}$ in $\{1, .., N\}$ where $r > \widehat{r}$. Then*

$$S_r(p_r^*) > S_{\widehat{r}}(p_{\widehat{r}}^*).$$

A consumer knows the distribution of prices and qualities in the economy, but not the price or quality of any particular firm. She randomly samples one firm's price and quality costlesstly, but must bear a search cost γ to learn the price and quality of each additional firm that she samples. She may sequentially sample an unlimited number of firms at a cost of γ per firm.

Remark 1 *If consumers had unit demands, as is often assumed in search models, the Diamond (1971) argument would imply that in equilibrium each quality is priced at its respective monopoly price, for any positive search cost. In that case, the firms' incentives to invest would be independent of the search cost, in contrast, to the results obtained below.*

[4] Since $v'(0) = \infty$ then $Q_r(p) > 0$ and $S_r(p) > 0$ for all p and r.

3 Analysis

Let $\mu \in [0,1]$ be the proportion of firms that invest and let

$$t_r = \mu\alpha_r^I + (1-\mu\ )\alpha_r^{NI} \tag{3}$$

be the proportion of firms with quality r. We first analyze equilibrium pricing and the consumers' optimal search strategy for a given μ. Then, in Section 3.3 we go back and determine the equilibrium value of μ that is consistent with these strategies.

3.1 Equilibrium pricing and search

As is well known (see e.g. Kohn and Shavell (1974)), under our assumptions, the consumers' optimal search rule is characterized by a constant reservation utility, S_γ, which is the consumer's expected surplus from the optimal search strategy.[5] That is, if a consumer is matched with a firm with quality r and price p, she optimally accepts p without further search if $S_r(p) \geq S_\gamma$ and otherwise rejects it to search.

The following lemma - which is reminiscent of Diamond (1971) - establishes that in equilibrium the price charged by a firm with the highest quality (N) equals the monopoly price, irrespective of the search cost.

Lemma 2 $p_N = p_N^*$ *.*

However, the prices of qualities below N are determined by the search cost and are generally below their monopoly price. The following proposition fully characterizes equilibrium prices.

Proposition 1 *Equilibrium prices are uniquely characterized by thresholds* $\underline{r}(\gamma)$ *and* $\overline{r}(\gamma)$*, where* $1 \leq \underline{r}(\gamma) \leq \overline{r}(\gamma) \leq N$ *such that*

1. *if* $r \geq \overline{r}(\gamma)$, $p_r = p_r^*$ *and* $S_r(p_r^*) \geq S_\gamma$.

2. *if* $\underline{r}(\gamma) \leq r < \overline{r}(\gamma)$, p_r *satisfies* $S_r(p_r) = S_\gamma$.

3. *if* $r < \underline{r}(\gamma)$, $p_r = c$ *and* $S_r(c) < S_\gamma$.

[5] It is well known that when the number of firms is infinite, as were assuming, S_γ does not depends on the currently offered quality\price and on whether the consumer can recall previously rejected prices.

where the thresholds $\underline{r}(\gamma)$ *and* $\overline{r}(\gamma)$ *are weakly decreasing step functions of* γ *and*

$$S_\gamma = \frac{\sum\limits_{r \geq \overline{r}(\gamma)} t_r S_r\left(p_r^*\right) - \gamma}{\sum\limits_{r \geq \overline{r}(\gamma)} t_r}. \tag{4}$$

Furthermore, p_r *is strictly increasing in* r *for* $r \geq \underline{r}(\gamma)$ *and is strictly increasing in* γ *for* $\underline{r}(\gamma) \leq r < \overline{r}(\gamma)$

In other words, only the highest qualities - above the upper threshold $\overline{r}$ - command monopoly prices. Intermediate qualities are constrained by the consumers' search option and are priced so that consumers are just indifferent between accepting them at the equilibrium price and searching for better quality. The lowest qualities - below the lower threshold $\underline{r}$ - are rejected by consumers and make no sales. Moreover, the thresholds themselves are determined by the level of the search cost: the lower the search cost, the higher the upper threshold - and the fewer qualities are priced at the monopoly level - and the higher the lower threshold - and the more qualities which are rejected by consumers.

The intuition is straightforward. Lowering the search cost increases the value to consumers of searching for superior quality, thus reducing their willingness to pay for every quality below the highest level. Thus qualities which may be priced at the monopoly level at a higher level of the search cost must now be priced more competitively (falling below the upper threshold), and qualities which are acceptable to consumers at a higher level of the search cost are now unacceptable (falling below the lower threshold).

The proposition is illustrated in Figure 1:

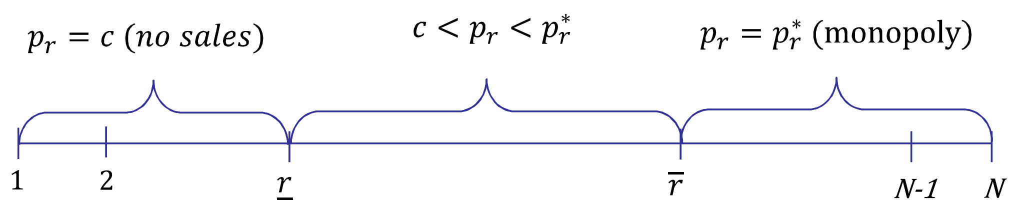

Figure 1: Pricing Equilibrium

3.2 The cost of search and the incentives to invest

Based on Proposition 1, we are now able to determine how changes in the search cost γ affect the incentives to invest. Denote the expected operating profit of a firm which invests as $\pi_I(\gamma)$, its expected profit if it does not invest as $\pi_{NI}(\gamma)$ and by $s_r \in [0,1]$ an r-firm's market share. Then

$$\pi_I(\gamma) = \sum_{r=1}^{N} \alpha_r^I s_r \pi_r \tag{5}$$

$$\pi_{NI}(\gamma) = \sum_{r=1}^{N} \alpha_r^{NI} s_r \pi_r \tag{6}$$

Firms with a quality worse than $\underline{r}(\gamma)$ make no sales and thus $s_r = 0$ for all $r < \underline{r}(\gamma)$. Since in equilibrium customers of firms with qualities $r \geq \underline{r}(\gamma)$ do not search, those firms have the same share, denoted $s(\gamma) = 1/\sum_{k \geq \underline{r}(\gamma)} t_k$, where $\sum_{k \geq \underline{r}(\gamma)} t_k$ is the proportion of firms that make positive sales.

Let $W(\gamma) = \pi_I(\gamma) - \pi_{NI}(\gamma)$ be the *return on investment* – the difference between the profits of a firm which invests and one which does not invest. Hence:

$$W(\gamma) = s(\gamma) \cdot \sum_{r=\underline{r}(\gamma)}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right) \pi_r(\gamma) \tag{7}$$

Thus the return on investment is the product of market share $s(\gamma)$ and the return on investment per customer $\sum_{r=\underline{r}(\gamma)}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right) \pi_r(\gamma)$.

The following proposition analyzes the effect of an increase in the search cost on the return on investment. First we observe that MLRP (Assumption 1) implies that there is a quality level $k_o > 1$ such that investment increases the likelihood of quality levels greater or equal to k_o (i.e. $\alpha_r^I - \alpha_r^{NI} > 0$ for $r \geq k_o$) and decreases the likelihood of quality levels below k_o (i.e. $\alpha_r^I - \alpha_r^{NI} < 0$ for $r < k_o$).

Proposition 2 *Consider an increase in the search cost from γ_1 to $\gamma_2 > \gamma_1$. The return on investment $W(\gamma)$: (i)* ***decreases*** *if $k_o \geq \overline{r}(\gamma_1)$ and (ii)* ***increases*** *if $k_o \leq \underline{r}(\gamma_2)$ and $s(\gamma_1) = s(\gamma_2)$.*

The intuition for part (i) of the proposition is the following. By (7), the return on investment is the product of market share and the return per customer. Recall that $r \geq \overline{r}$ earns monopoly profit. Since $\overline{r}(\gamma)$ is decreasing in γ, then if $k_o \geq \overline{r}(\gamma_1)$, all qualities above k_0 –

which are the qualities that are more likely outcomes if the firm invests than if doesn't – earn the same profit per customer whether the search cost is γ_2 or γ_1, while profits of qualities which are more likely if the firm doesn't invest ($r < k_0$) either increase or do not change. Hence the return on investment per customer decreases. Since more qualities may be viable at the higher search cost γ_2, $s(\gamma_2) \leq s(\gamma_1)$, and hence the total return on investment decreases.

Conversely, the intuition for part (ii) is the following. Recall that $r < \underline{r}$ earns zero profit. Since $\underline{r}(\gamma)$ is decreasing in γ, if $k_o \leq \underline{r}(\gamma_2)$, all qualities below k_0 - which are the qualities that are more likely outcomes if the firm doesn't invest – earn the same profit (zero) whether the search cost is γ_2 or γ_1, while profits of qualities which are more likely if the firm invests either increase or do not change. Hence the return on investment per customer decreases. If $s(\gamma_2) = s(\gamma_1)$ (which is the case if the change in the search cost is sufficiently small that $\underline{r}$ does not change), then by (2), the total return on investment increases.

In the appendix we provide further characterization for the intermediate case where $\underline{r}(\gamma_2) \leq k_o < \overline{r}(\gamma_1)$. Intuitively, both effects are then present and we derive a sufficient condition under which $W(\gamma)$ decreases.

An implication of Proposition 2 is the following:

Lemma 3 *If $k_o = N$ then $W(\gamma)$ is decreasing in γ, for all γ.*

Proof. If $k_o = N$, then for all γ, part (i) of Proposition 2 applies. Thus $W(\gamma)$ is decreasing in γ. ■

In other words, if investment increases only the probability of obtaining the highest quality (and decreases the probability of all other qualities), an increase in the search cost always lowers the incentives to invest. Two special cases in which this holds are the case of two qualities ($N = 2$) and the case in which investment is deterministic; that is, if a firm invests it produces the highest quality with probability 1.

Corollary 1

1. ***Two quality levels:*** *If $N = 2$ then $W(\gamma)$ is decreasing for all γ.*
2. ***Deterministic investment:*** *If $\alpha_N^I = 1$ then $W(\gamma)$ is decreasing for all γ.*

Thus a necessary condition for the return on investment $W(\gamma)$ to increase with the search cost γ is that $k_o < N$. That is, investment has to increase not only the probability of producing the highest quality, N, but also the probabilities of producing some of the intermediate qualities. To allow for richer effects we henceforth assume:

Assumption 2 $k_o < N$

Based on Proposition 2, we are now ready to state the main result of this section, which derive effects of changes in the search cost on the return on investment:

Proposition 3

1. *There exists* $\overline{\gamma}$ *such that for all* γ_1 *and* γ_2*,* $\gamma_2 \geq \gamma_1$ *in the interval* $[\overline{\gamma}, \infty)$*,* $W(\gamma_2) \leq W(\gamma_1)$.

2. *Suppose that*

$$S_{k_o-1}(c) < S_N(p_N^*). \tag{8}$$

Then there exists $\underline{\gamma} \geq 0$ *such that for all* γ_1 *and* γ_2*,* $\gamma_2 \geq \gamma_1$ *in the interval* $\left(0, \underline{\gamma}\right]$*,* $W(\gamma_2) \geq W(\gamma_1)$.

The explanation for part 1. of the proposition is as follows: Since $\overline{r}(\gamma)$ is decreasing in γ, if the search cost is sufficiently high, then $k_0 > \overline{r}(\gamma)$ and therefore part (i) of Proposition 2 applies.

The explanation for part 2. is as follows: Condition (8) implies that if the search cost is low enough, consumers reject qualities below k_0 to search for higher qualities, even if the former are priced at cost. Thus, if the initial cost is low enough, $\underline{r}(\gamma) > k_0$ and therefore part (ii) of Proposition 2 applies.[6]

[6] To investigate the plausibility of (8), consider a paramteric example with $v(Q) = Q^\beta/\beta$ for $0 < \beta < 1$. It is straightforward to show that $Q_r(p) = \left[\frac{p}{\theta_r}\right]^{\frac{1}{\beta-1}}$ and $S_r(p) = \frac{1-\beta}{\beta} \cdot \frac{\theta_r^{\frac{1}{1-\beta}}}{p^{\frac{\beta}{1-\beta}}}$. Substituting into (8) we obtain:

$$S_N(p_N^*) - S_{k_o-1}(c) = \frac{1-\beta}{\beta} \cdot \frac{1}{c^{\frac{\beta}{1-\beta}}} \cdot \left[\beta^{\frac{\beta}{1-\beta}} \theta_N^{\frac{\beta}{\beta-1}} - \theta_{k_0}^{\frac{1}{1-\beta}}\right]$$

Hence, Condition (8) obtains if and only if $\frac{\theta_N}{\theta_{k_0-1}} > \beta^{-\beta}$. Since $\lim_{\beta\to 0} \beta^{-\beta} = \lim_{\beta\to 1} \beta^{-\beta} = 1$, the condition is certainly satisfied if β is sufficiently high or sufficiently small. Furthermore, $\beta^{-\beta}$ is maximized at $\beta = 1/e$ giving $\beta^{-\beta} = 1.445$. Hence, if $\theta_N > 1.445 \cdot \theta_{k_0-1}$, Condition (8) holds for all β.

Note that the proposition implies that if the initial search cost is sufficiently high, reducing the search cost increases the return on investment and if the initial level of the search cost is sufficiently low, a further reduction of the search cost lowers the return on investment.

3.3 Equilibrium investment

The preceding analysis derives the equilibrium pricing and the return on investment for a given proportion μ of investing firms. We now complete the equilibrium analysis, by characterizing the value of μ which is consistent with equilibrium pricing and analyze how that value changes with the search cost.

If in equilibrium $\mu > 0$ it must be that $W \geq I$ (where recall that I is the cost of investment). Also, if $0 < \mu < 1$ then a firm must earn the same profit whether or not it invests, that is, $W = I$. Finally, an interior equilibrium is stable only if $dW/d\mu < 0$.

Lemma 4 *The return on investment W is decreasing as a function of μ except possibly for finite set of points where the number of viable qualities changes (i.e. $\underline{r}$ changes).*

Based on the preceding lemma, W is drawn in Figure 2 as a downward-sloping function of μ, for two different values of the search cost, γ_0 and γ_1, where the return on investment corresponding to γ_1 is greater than that corresponding to γ_0 and where $\underline{r}(\gamma_0) = \underline{r}(\gamma_1) = \underline{r}$ (that is the difference between γ_0 and γ_1 is sufficiently small not to affect $\underline{r}$).[7] The figure illustrates how the change in the search cost, via its effect on the return on investment, determines the number of firms that invest. Specifically, when the search cost is γ_0, the number of firms which invest is μ_0. A change in the search cost to γ_1 results in a new equilibrium $\mu_1 > \mu_0$. Thus if $\gamma_1 > \gamma_0$, corresponding for example to part 2. of Proposition 3, lowering the search cost from γ_1 to γ_0 reduces the number of firms which invest and hence the average quality in the market. Note that the lower is μ the greater is the proportion of firms whose realized quality is less than the threshold $\underline{r}$.[8] Thus, lowering μ increases the volume of

[7] In general, for a given I, multiple equilibria are possible, as the function W is discontinous at points in which $\underline{r}$ changes. In the case of multiple equilbria our above analysis applies to a local change in any such equilibrium, corresponding to a small change in γ.

[8] Since, by FOSD, $d[\sum_{r=1}^{\underline{r}} t_r]/d\mu = \sum_{r=1}^{\underline{r}} \left(\alpha_r^I - \alpha_r^{NI}\right) < 0$.

consumer search in equilibrium, since more consumers are initially matched with quality less than $\underline{r}$ and must search longer to find an acceptable quality.

Conversely, if $\gamma_1 < \gamma_0$, corresponding for example to part 1. of Proposition 3, lowering the search cost from γ_1 to γ_0 increases the number of firms which invest and hence the average quality in the market and reduces the volume of consumer search.

Thus, based on Proposition 3, we conclude that lowering the search cost leads to higher average quality if the initial search cost is sufficiently high, and may reduce average quality if the initial search cost is sufficiently low.

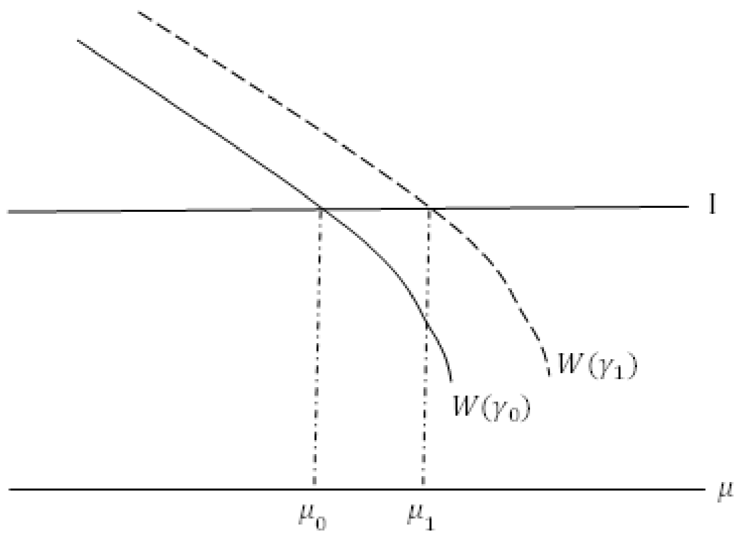

Figure 2: The effect of a change in the search cost on the number of investing firms

4 Conclusions

We have developed a model designed to address the question: How do technological developments which reduce consumer search costs affect product quality? In our model, firms can invest to (probablistically) improve product quality and consumers invest in search for higher quality products. In this setting, lower search costs incentivize consumers to search more intensively. Since even firms which invest may fail to deliver the highest quality, more intensive search reduces the prices of intermediate and low quality products, lowering the expected profits of investors and noninvestors alike. Thus the equilibrium effect of lower search costs on investment depends on whether the profit of investors declines by more or less than the profit of noninvestors. We have shown that when the initial search cost is sufficiently small, the profit from investing declines by more, leading to less investment and lower product quality.

A Proofs

Proof of Lemma 1. Clearly, given qualities $r, \widehat{r}$ such that $r > \widehat{r}$,

$$\begin{aligned} S_r(p) &= \theta_r v\left(Q_r(p)\right) - pQ_r(p) \\ &\geq \theta_r v\left(Q_{\widehat{r}}(p)\right) - pQ_{\widehat{r}}(p) > \theta_{\widehat{r}} v\left(Q_{\widehat{r}}(p)\right) - pQ_{\widehat{r}}(p) = S_{\widehat{r}}(p). \end{aligned}$$

Observe that $S_r(p_r^*) = \theta_r v\left(Q_r^*\right) - p_r^* Q_r^*$ and that $p_r^* = P_r\left(Q_r^*\right) = \theta_r v'\left(Q_r^*\right)$. Hence

$$S_r(p_r^*) = \theta_r\left(v\left(Q_r^*\right) - v'\left(Q_r^*\right) Q_r^*\right).$$

As $v\left(Q\right) - v'\left(Q\right) Q$ is increasing in Q (its derivative is $-v''\left(Q\right) > 0$), $S_r(p_r^*)$ is increasing in r, as Q_r^* is. This can be most readily be seen from the first-order condition for the monopoly problem, which sets the marginal revenue, $MR_r\left(Q\right) = dP_r\left(Q\right)/dQ$ equal to the marginal cost c. As $MR_r\left(Q\right) = \theta_r\left(v'\left(Q\right) + v''\left(Q\right) Q\right)$ is increasing in r and decreasing in Q (which follows from the second-order condition of the firm's maximization problem), Q_r^* is increasing in r. ■

Proof of Lemma 2. First note that for all r, $p_r \leq p_r^*$; otherwise an r firm could lower its price without losing customers and increase its profit. Suppose $p_N < p_N^*$. If $S_N(p_N) \geq S_r(p_r)$ for all $r < N$, then an N firm can slightly increase its price without inducing its customers to search, increasing thereby the profit per customer (by concavity of the profit function). Thus $S_N(p_N) \leq S_r(p_r)$ for at least one $r < N$ and let $k = \arg\max_{r<N} \{S_r(p_r)\}$. Then $p_k = p_k^*$; otherwise a k firm could increase its price slightly without losing customers and increase profit. But then, since $p_N < p_N^*$, $S_N(p_N) > S_N(p_N^*) > S_k(p_k^*) = S_k(p_k)$, a contradiction. This completes the proof. ■

Proof of Proposition 1. Given the consumer reservation utility S_γ,

1. If $S_r\left(p_r^*\right) \geq S_\gamma$ then $p_r = p_r^*$. Suppose to the contrary that $p_r < p_r^*$. Then $S_r\left(p_r\right) > S_r\left(p_r^*\right) \geq S_\gamma$, which implies that firm r can increase its price (and profits) without losing any customers.

2. If $S_r\left(p_r^*\right) < S_\gamma$ and $S_r\left(c\right) > S_\gamma$, then firm r can retain all costumers who samples it and make positive profits by charging a price above c. The maximal such price p_r the firm could charge satisfies $S_r(p_r) = S_\gamma$, where $p_r < p_r^*$.

3. If $S_r(c) \le S_\gamma$, then for any price $p > c$, $S_r(p) < S_\gamma$. Thus, such a firm can only sell by pricing below cost and optimally charges a price $\ge c$. Without loss of generality, $p_r = c$.

Define $\overline{r}(\gamma)$ as the lowest quality such that $S_r(p_r^*) \ge S_\gamma$. By Lemma 2, $S_N(p_N^*) \ge S_\gamma$ and thus $\overline{r}(\gamma) \le N$ exists. For $r < \overline{r}(\gamma)$, $S_r(p_r^*) < S_\gamma$ and therefore a firm with such a quality can only make sales if $p_r < p_r^*$. For $r \ge \overline{r}(\gamma)$, by Lemma 1, $S_r(p_r^*) \ge S_{\overline{r}(\gamma)}(p_{\overline{r}(\gamma)}^*) \ge S_\gamma$ and thus p_r^* maximizes the profits of such firms.

Next observe that a consumer will buy from a firm with a quality r and price p if and only if $S_r(p) \ge S_\gamma$. Let r' be defined as the highest value of r such that $S_r(c) \le S_\gamma$. If r' does not exist let $\underline{r}(\gamma) = 1$ and in that case any quality can earn positive profit by charging a price slightly above c. If $r' \ge 1$ does exist, then define $\underline{r}(\gamma) \equiv r' + 1$. From Lemma 1, for all $r < \underline{r}(\gamma)$, $S_r(c) \le S_{r'}(c) \le S_\gamma$ and hence a firm with this quality cannot earn positive profits. And for all $r \ge \underline{r}(\gamma)$, $S_r(c) \ge S_{\underline{r}(\gamma)}(c) > S_\gamma$ and thus a firm with this quality can earn positive profit by charging a price slightly above c.

Based on the above the consumer's reservation utility S_γ can be expressed as follows:

$$S_\gamma = \sum_{r<\overline{r}(\gamma)} t_r S_\gamma + \sum_{r\ge\overline{r}(\gamma)} t_r S_r(p_r^*) - \gamma$$

Rearranging, we obtain

$$S_\gamma = \frac{\sum_{r\ge\overline{r}(\gamma)} t_r S_r(p_r^*) - \gamma}{\sum_{r\ge\overline{r}(\gamma)} t_r}.$$

Since for $\underline{r}(\gamma) \le r < \overline{r}(\gamma)$, $S_r(p_r) = S_\gamma$ and since $S_r(p)$ is increasing in r, p_r is increasing in r in this range. For $r > \overline{r}(\gamma)$, $p_r = p_r^*$ which is increasing in r.

Next, we prove that a price equilibrium exists and is unique :

Uniqueness:

Uniqueness is proved as follows: for any $j \le N$, define

$$S_\gamma^{(j)} = \frac{\sum_{r=j}^{N} t_r S_r(p_r^*) - \gamma}{\sum_{r=j}^{N} t_r} \tag{9}$$

For future reference note that if $\overline{r} \le j$, then $S_\gamma^{(j)}$ is the expected consumer surplus from the following search strategy: search until a $r \ge j$ is found.

1. Suppose that an equilibrium with $\overline{r} = k$ exists. Then $S_\gamma^{(j)} < S_\gamma^{(k)}$ for every j such that $j < k$.

   Proof: Observe that we can express $S_\gamma^{(j)}$, using the following recursive formula.

$$\begin{aligned} S_\gamma^{(j)} &= \frac{t_j S_j\left(p_j^*\right) + \sum_{r=j+1}^{N} t_r S_r\left(p_r^*\right) - \gamma}{\sum_{r=j}^{N} t_r} \\ &= \frac{t_j}{\sum_{r=j}^{N} t_r} S_j\left(p_j^*\right) + \frac{\sum_{r=j+1}^{N} t_r}{\sum_{r=j}^{N} t_r} \cdot \left[\frac{\sum_{r=j+1}^{N} t_r S_r\left(p_r^*\right) - \gamma}{\sum_{r=j+1}^{N} t_r}\right] \end{aligned}$$

   and hence

$$S_\gamma^{(j)} = \frac{t_j}{\sum_{r=j}^{N} t_r} S_j\left(p_j^*\right) + \frac{\sum_{r=j+1}^{N} t_r}{\sum_{r=j}^{N} t_r} S_\gamma^{(j+1)} \tag{10}$$

   Applying (10) repeatedly we obtain for any $k > j$,

$$S_\gamma^{(j)} = \frac{\sum_{r=j}^{k-1} t_r S_r\left(p_r^*\right) + \sum_{r=k}^{N} t_r S_\gamma^{(k)}}{\sum_{r=j}^{N} t_r}.$$

   Since an equilibrium with $\overline{r} = k$ exists, it follows from the above analysis that for $r \leq k-1,\ \ S_r\left(p_r^*\right) < S_\gamma^{(k)}$. Thus

$$S_\gamma^{(j)} < \frac{\sum_{r=j}^{k-1} t_r S_\gamma^{(k)} + \sum_{r=k}^{N} t_r S_\gamma^{(k)}}{\sum_{r=j}^{N} t_r} = S_\gamma^{(k)}.$$

2. The equilibrium value of $\overline{r}$ is unique.

   Proof: Suppose there are two equilibria. One in which $\overline{r} = k$ and another where $\overline{r} = j < k$. Comparing (4) and (9) shows that in the latter equilibrium $S_\gamma = S_\gamma^{(j)}$. Consider the following strategy: search until a firm with quality $r \geq k$ is found (i.e. reject all qualities less than k). In either equilibrium, the expected surplus from following this strategy is $S_\gamma^{(k)}$. But by step 1, $S_\gamma^{(j)} < S_\gamma^{(k)}$, which means that the equilibrium search strategy in the $\overline{r} = j$ equilibrium is not optimal, a contradiction. By the same argument, an equilibrium with $\overline{r} > k$ cannot exist.

3. Equilibrium prices are uniquely determined.

   Proof: Given $\overline{r}$, $p_r = p_r^*$ for all $r \geq \overline{r}$, which, by step 2, is unique. Also given $\overline{r}$, S_γ is uniquely defined by (4) and thus, since $S_r(p)$ are monotonically decreasing in p, then p_r for $r < \overline{r}$, are uniquely determined as described above. This completes the proof that the equilibrium is unique.

Existence is proved by construction, using the following algorithm: Set $\overline{r} = N$ and calculate $S_\gamma^{(N)}$. If $S_{N-1}\left(p_{N-1}^*\right) < S_\gamma^{(N)}$, the unique equilibrium has $\overline{r} = N$ and prices are uniquely determined as described in the first part of the proof. Otherwise, set $\overline{r} = N-1$, calculate $S_\gamma^{(N-1)}$ and proceed as above. If the process reaches $\overline{r} = 2$ and $S_1\left(p_1^*\right) > S_\gamma^{(2)}$ then $p_k = p_k^*$ for all $k = 1, ..., N$.

Last, consider the effect of a change in the search cost γ on the price equilibrium and the thresholds $\overline{r}(\gamma)$ and $\underline{r}(\gamma)$. Note that $\overline{r}(\gamma)$ and $\underline{r}(\gamma)$ are defined on the integers and are thus step functions. Consider some γ_1, γ_2 such that $\gamma_1 < \gamma_2$. Then, as argued above, the value of search corresponding to γ_1 and γ_2 are respectively $S_{\gamma_1}^{(\overline{r}(\gamma_1))}$ and $S_{\gamma_2}^{(\overline{r}(\gamma_2))}$. Suppose that $\overline{r}(\gamma_2) > \overline{r}(\gamma_1)$. If $\gamma = \gamma_1$, consider the consumer search strategy: search until $r \geq \overline{r}(\gamma_2)$ is found. As argued above, the expected surplus from this strategy is $S_{\gamma_1}^{(\overline{r}(\gamma_2))}$. Since $S_{\gamma_1}^{(\overline{r}(\gamma_1))}$ is the surplus from the equilibrium search strategy, $S_{\gamma_1}^{(\overline{r}(\gamma_2))} \leq S_{\gamma_1}^{(\overline{r}(\gamma_1))}$. However, from the analysis above it follows that $S_{\gamma_1}^{(\overline{r}(\gamma_2))} > S_{\gamma_1}^{(\overline{r}(\gamma_1))}$ if $\overline{r}(\gamma_2) > \overline{r}(\gamma_1)$, a contradiction. This proves that $\overline{r}(\gamma)$ is weakly decreasing in γ.

Recall that $\underline{r}(\gamma)$ is the lowest value of r such that $S_r(c) > S_\gamma$. Thus if S_γ is decreasing in γ then $\underline{r}(\gamma)$ must be weakly decreasing. To prove that S_γ is decreasing in γ, recall from the uniqueness part above that $S_{\gamma_2} = S_{\gamma_2}^{(\overline{r}(\gamma_2))}$ and $S_{\gamma_1} = S_{\gamma_1}^{(\overline{r}(\gamma_1))}$. As $\overline{r}(\gamma_2) \leq \overline{r}(\gamma_1)$, $S_{\gamma_2}^{(\overline{r}(\gamma_2))} \leq S_{\gamma_2}^{(\overline{r}(\gamma_1))} < S_{\gamma_1}^{(\overline{r}(\gamma_1))}$, where the first inequality follows from the uniqueness part above and the second inequality follows directly from (9). Thus $S_{\gamma_2} < S_{\gamma_1}$ if $\gamma_2 > \gamma_1$.

Consider first an increase in γ that does not change $\underline{r}(\gamma)$ and $\overline{r}(\gamma)$. For $r < \underline{r}(\gamma)$ or $r \geq \overline{r}(\gamma)$, p_r does not change. For $r \in \{\underline{r}(\gamma), \ldots \overline{r}(\gamma) - 1\}$, $S_r(p_r) = S_\gamma$ and thus as S_γ is decreasing in γ, p_r is strictly increasing in γ. If only $\underline{r}(\gamma)$ decreases, S_γ does not change and so for all r for which previously $p_r > c$ there is no change in price while for all r which were previously less than $\underline{r}(\gamma)$, either p_r increases or does not change. Finally if $\overline{r}(\gamma)$ decreases, we argued above that S_γ decreases, and so all prices either decrease or stay the same. ■

It immediately follows from Proposition 1 that π_r is increasing in r for $r \geq \underline{r}(\gamma)$. For $r \geq \overline{r}(\gamma)$, $\pi_r = \pi_r^*$ is clearly increasing in r. For $\underline{r}(\gamma) \leq r < \overline{r}(\gamma)$, $0 < \pi_r < \pi_r^*$. In this range p_r is increasing in r and by the first-order condition (2), $Q_r(p)$ is strictly increasing in r. Thus π_r is increasing in r as well. Finally for $\underline{r}(\gamma) \leq r < \overline{r}(\gamma)$, π_r is increasing in γ, since p_r is increasing in γ and $p_r < p_r^*$, the monopoly price.

Proof of Proposition 2. Recall that the return on investment is defined by:

$$W(\gamma) = s(\gamma) \cdot \sum_{r=\underline{r}(\gamma)}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right) \pi_r(\gamma)$$

It is convenient to define the *per-customer return on investment* as

$$w(\gamma) = \sum_{r=\underline{r}(\gamma)}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right) \pi_r(\gamma)$$

and thus $W(\gamma) = s(\gamma) w(\gamma)$.

Recall from Proposition 1 that $\underline{r}(\gamma)$ and $\overline{r}(\gamma)$ are both decreasing in γ. Since for $r \geq \overline{r}(\gamma_1)$, $\pi_r(\gamma_1) = \pi_r(\gamma_2) = \pi_r^*$ and for $r \leq \underline{r}(\gamma_2)$, $\pi_r(\gamma_1) = \pi_r(\gamma_2) = 0$,

$$w(\gamma_2) - w(\gamma_1) = \sum_{r=\underline{r}(\gamma_2)}^{\overline{r}(\gamma_1)-1} \left(\alpha_r^I - \alpha_r^{NI}\right) \cdot \left(\pi_r(\gamma_2) - \pi_r(\gamma_1)\right)$$

As $\pi_r(\gamma_2) \geq \pi_r(\gamma_1)$ for all r it, then if $k_o \geq \overline{r}(\gamma_1)$, $\alpha_r^I - \alpha_r^{NI} < 0$ for all r in the summation term and thus $w(\gamma_2) - w(\gamma_1) \leq 0$. Conversely, if $k_o \leq \underline{r}(\gamma_2)$, $\alpha_r^I - \alpha_r^{NI} > 0$ for all r and thus $w(\gamma_2) \geq w(\gamma_1)$.

Now,

$$W(\gamma_2) - W(\gamma_1) = s(\gamma_2) w(\gamma_2) - s(\gamma_1) w(\gamma_1).$$

If $s(\gamma_1) = s(\gamma_2) = s$, then $W(\gamma_2) - W(\gamma_1) = s \cdot [w(\gamma_2) - w(\gamma_1)]$. In that case, the change in the total return on investment has the same sign as the change in the per-customer measure. Moreover if $s(\gamma_2) \leq s(\gamma_1)$ and if $w(\gamma_2) \leq w(\gamma_1)$ then $W(\gamma_2) - W(\gamma_1) \leq s(\gamma_1) \cdot [w(\gamma_2) - w(\gamma_1)] \leq 0$ and thus $W(\gamma_2) \leq W(\gamma_1)$. ■

For the intermediate case where $\underline{r}(\gamma_2) \leq k_o < \overline{r}(\gamma_1)$, we provide a partial characterization. Denote by $P_r(Q) = \theta_r v'(Q)$ the inverse demand function, by $\varepsilon_r(Q) = \frac{P_r(Q)}{P_r'(Q)Q}$ the elasticity of demand, and impose the following weak regularity condition

Assumption 3 *$\varepsilon_r(Q)$ is weakly increasing in Q, for all r.*

Under the assumption above we can prove the following:

Lemma 5 *Consider an increase in the search cost from γ_1 to $\gamma_2 > \gamma_1$. If $\underline{r}(\gamma_2) \leq k_o < \overline{r}(\gamma_1)$, the return on investment $W(\gamma)$ decreases provided $\sum_{r=\underline{r}(\gamma_2)}^{\overline{r}(\gamma_1)-1} \left(\alpha_r^I - \alpha_r^{NI}\right) \leq 0$.*

The next three auxiliary lemmas (A.1-A.3) are used in the proof of the lemma

Lemma A.1 *Suppose that there are* l, m *such that* $\underline{r} \leq l < m < \overline{r}$, *then* $Q_l = Q_l(p_l) > Q_m(p_m) = Q_m$.

Proof. As $\underline{r} \leq l < m < \overline{r}$, we have $S_l(p_l) = S_m(p_m) = S_\gamma$.Thus

$$S_l(p_l) = \theta_l v(Q_l) - p_l Q_l = \theta_m v(Q_m) - p_m Q_m = S_m(p_m)$$

In addition, from the first-order conditions to the consumer's problem $p_l = \theta_l v'(Q_l)$ and $p_m = \theta_m v'(Q_m)$. Hence

$$\theta_l \cdot [v(Q_l) - v'(Q_l) Q_l] = \theta_m \cdot [v(Q_m) - v'(Q_m) Q_m].$$

Because $\theta_m > \theta_l$, therefore $v(Q_l) - v'(Q_l) Q_l > v(Q_m) - v'(Q_m) Q_m$ and therefore $Q_l > Q_m$, as the function $v(Q) - v'(Q) Q$ is increasing in Q (its derivative is $-v''(Q) > 0$). ■

Lemma A.2 *For* r *in* $\{\underline{r}, \ldots \overline{r} - 1\}$, $\frac{p_r - c}{p_r} \cdot \varepsilon_r(Q_r)$ *is decreasing in* r

Proof. Note first that we can write

$$\varepsilon_r(Q) = \frac{P_r(Q)}{[dP_r(Q)/dQ] \cdot Q},$$

where $P_r(Q)$ is the inverse demand function given a quality r. Substituting $P_r(Q) = E[\theta|r] v'(Q)$, we obtain

$$\varepsilon_r(Q) = \frac{\theta_r v'(Q)}{\theta_r v''(Q) \cdot Q} = \frac{v'(Q)}{v''(Q) \cdot Q}.$$

Thus, $\varepsilon_r(Q)$ is invariant of the quality r and depends only on the quantity Q.

Now, let l, m be such that $\underline{r} \leq l < m < \overline{r}$. Hence

$$\frac{p_m - c}{p_m} \cdot \varepsilon_m(Q_m) < \frac{p_m - c}{p_m} \cdot \varepsilon_m(Q_l) = \frac{p_m - c}{p_m} \cdot \varepsilon_l(Q_l) < \frac{p_l - c}{p_l} \cdot \varepsilon_l(Q_l)$$

where the first inequality follows from Assumption 3, and the fact that $Q_l > Q_m$ (Lemma A.1) and the second inequality follows as $(p - c)/p$ is increasing in p, $p_l < p_m$ and $\varepsilon_l(Q_l) < 0$. ■

Lemma A.3 *Consider two levels of the search cost* $\gamma_2 > \gamma_1$. *Then* $\pi_r(\gamma_2) - \pi_r(\gamma_1)$ *is decreasing in* r.

Proof. For r in $\{\underline{r},\ldots\overline{r}-1\}$,

$$\frac{\partial \pi_r}{\partial \gamma}=\pi_r'\left(p_r\right)\cdot\frac{\partial p_r}{\partial \gamma}=\left[Q_r\left(p_r\right)+\left(p_r-c\right)Q_r'\left(p_r\right)\right]\frac{\partial p_r}{\partial \gamma}.$$

Recall that $\partial\pi_r/\partial\gamma>0$. Differentiating the equation $S_r(p_r)=S_\gamma$, which defines p_r implicitly, we obtain $\partial p_r/\partial\gamma=\left[\partial S_\gamma/\partial\gamma\right]/S_r'\left(p_r\right)$. Note that $S_r(p_r)=\max_Q\theta_r v\left(Q\right)-p_rQ$, and thus, by the envelope theorem, $S_r'(p_r)=-Q_r\left(p_r\right)$. Thus

$$\begin{aligned}\frac{\partial \pi_r}{\partial \gamma} &= -\frac{\partial S_\gamma}{\partial \gamma}\cdot\left[1+\left(p_r-c\right)\frac{Q_r'\left(p_r\right)}{Q_r\left(p_r\right)}\right]\\ &= -\frac{\partial S_\gamma}{\partial \gamma}\cdot\left[1+\frac{p_r-c}{p_r}\cdot\varepsilon_r\left(Q_r\right)\right]\end{aligned}$$

where $\varepsilon_r\left(Q_r\right)$ is the equilibrium price elasticity of the demand for an r firm. As $\partial S_\gamma/\partial\gamma$ <0, it follows from Lemma A.2 that $\partial\pi_r/\partial\gamma$ is decreasing in r. Finally, $\pi_r\left(\gamma_2\right)-\pi_r\left(\gamma_1\right)=\int_{\gamma_1}^{\gamma_2}\partial\pi_r/\partial\gamma$ is decreasing in r as well. ■

Based on these results, we can now prove:

Proof of Lemma 5. Provided $\underline{r}\left(\gamma_2\right)\leq k_o<\overline{r}\left(\gamma_1\right)$ one can write $r\left(\gamma_2\right)-r\left(\gamma_1\right)$ as

$$w\left(\gamma_2\right)-w\left(\gamma_1\right)=\sum_{r=\underline{r}(\gamma_2)}^{k_o-1}\left(\alpha_r^I-\alpha_r^{NI}\right)\left(\pi_r\left(\gamma_2\right)-\pi_r\left(\gamma_1\right)\right)+\sum_{r=k_o}^{\overline{r}(\gamma_1)-1}\left(\alpha_r^I-\alpha_r^{NI}\right)\left(\pi_r\left(\gamma_2\right)-\pi_r\left(\gamma_1\right)\right)$$

As $\pi_r\left(\gamma_2\right)-\pi_r\left(\gamma_1\right)$ is decreasing in r (as proved in Lemma A.3),

$$\begin{aligned}w\left(\gamma_2\right)-w\left(\gamma_1\right) &\leq \sum_{r=\underline{r}(\gamma_2)}^{k_o-1}\left(\alpha_r^I-\alpha_r^{NI}\right)\left(\pi_{k_o}\left(\gamma_2\right)-\pi_{k_o}\left(\gamma_1\right)\right)+\sum_{r=k_o}^{\overline{r}(\gamma_1)-1}\left(\alpha_r^I-\alpha_r^{NI}\right)\left(\pi_{k_o}\left(\gamma_2\right)-\pi_{k_o}\left(\gamma_1\right)\right)\\ &= \left(\pi_{k_o}\left(\gamma_2\right)-\pi_{k_o}\left(\gamma_1\right)\right)\cdot\sum_{r=\underline{r}(\gamma_2)}^{\overline{r}(\gamma_1)-1}\left(\alpha_r^I-\alpha_r^{NI}\right)\leq 0.\end{aligned}$$

Thus $w\left(\gamma_2\right)\leq w\left(\gamma_1\right)$. Since $s\left(\gamma_2\right)\leq s\left(\gamma_1\right)$, and as $W\left(\gamma\right)=s\left(\gamma\right)w\left(\gamma\right)$, then $W\left(\gamma_2\right)\leq W\left(\gamma_1\right)$. ■

Proof of Proposition 3.

1. Let $\overline{\gamma}$ be the smallest value of γ such that if $\gamma\geq\overline{\gamma}$, $\pi_r=\pi_r^*$ for all $r\geq k_o$ (i.e. $k_o\geq\overline{r}\left(\gamma\right)$), implying that part (i) of Proposition 2 applies. Thus, for any γ_1 and γ_2 such that $\gamma_2\geq\gamma_1$ in this interval $W\left(\gamma_2\right)\leq W\left(\gamma_1\right)$.[9]

[9] In fact, we can obtain an even lower threshold by defining $\overline{\gamma}$ to be the smallest value of γ such that if $\gamma\geq\overline{\gamma}$ either $\pi_r=\pi_r^*$ for all $r\geq k_o$ (as in done in the proof) or $\pi_r>0$ for every r (i.e. $\underline{r}\left(\gamma\right)=1$) . In the latter case, $\underline{r}\left(\gamma_2\right)<k_o$ and thus part (ii) of Proposition 2 does not apply. Moreover, it follows from FOSD that $\sum_{r=1}^{K}\left(\alpha_r^I-\alpha_r^{NI}\right)\leq 0$ for all K. Thus, either part (i) of Proposition 2, or the intermediate case described in the appendix apply, implying that $W\left(\gamma_2\right)\leq W\left(\gamma_1\right)$.

2. Note that, as γ goes to zero, $\overline{r}(\gamma) \to N$ and thus by (4), $S_\gamma \to S_N(p_N^*)$. Thus, if (8) obtains, there exists $\hat{\gamma}$ such that, for all $\gamma \leq \hat{\gamma}$, a consumer will rejects $r < k_o$ at any price greater or equal to c. Thus, for all $\gamma \leq \hat{\gamma}$, $\pi_r = 0$ for all $r < k_o$ (i.e. $k_o \leq \underline{r}(\gamma)$). It thus follows from part (ii) of Proposition 2 that $w(\gamma_2) \geq w(\gamma_1)$ for any γ_1 and γ_2 such that $\gamma_1 \leq \gamma_2 < \hat{\gamma}$. Finally, let $\underline{\gamma}$ be largest value of $\gamma \leq \hat{\gamma}$ such that $\underline{r}(\gamma)$ and thus $s(\gamma)$ are constant on $\left(0, \underline{\gamma}\right]$. For any γ_1 and γ_2 in $\left(0, \underline{\gamma}\right]$ such that $\gamma_2 \geq \gamma_1$, $W(\gamma_2) \geq W(\gamma_1)$.

■

Proof of Lemma 4. Consider first the effect of a small change in μ that does not affect the number of viable qualities (i.e. does not change $\underline{r}$). Recall that the return on investment can be expressed as a product of the market share s of a viable firm and w the return on investment per-customer, $W = s \cdot w$. It follows that

$$dW/d\mu = s \cdot dw/d\mu + ds/d\mu \cdot w,$$

where

$$dw/d\mu = \sum_{r=\underline{r}}^{N} \left[\left(\alpha_r^I - \alpha_r^{NI}\right) \cdot d\pi_r/d\mu\right].$$

To sign the last term note that, for $r \geq \underline{r}$, either $\pi_r = \pi_r^*$ in which case $d\pi_r/d\mu = 0$ or $\pi_r = \pi_r(p_r)$ where $S_r(p_r) = S_\gamma$. In the latter case $d\pi_r/d\mu = \pi_r' \cdot dp_r/d\mu$. Recall that S_γ is implicitly defined by the equation $S_\gamma = \sum_{r<\overline{r}} t_r S_\gamma + \sum_{r\geq\overline{r}} t_r S_r(p_r^*) - \gamma$ and it thus continuous in μ and differentiable at all points, expect to where $\overline{r}$ changes value. Wherever it is differentiable, $dS_\gamma/d\mu = \sum_{r=\overline{r}}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right)\left[S_r(p_r^*) - S_\gamma\right] \,/\, \sum_{r=\overline{r}}^{N} t_r$ (since by (3), $dt_r/d\mu = \alpha_r^I - \alpha_r^{NI}$). Since $S_r(p_r^*) > S_\gamma$ for $r > \overline{r}$, $dS_\gamma/d\mu$ is clearly positive if for all $r > \overline{r}$, $\alpha_r^I - \alpha_r^{NI} > 0$ (i.e. if $\overline{r} > k_o$). If $\overline{r} < k_o$:

$$\begin{aligned}\sum_{r=\overline{r}}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right)\left[S_r(p_r^*) - S_\gamma\right] &\geq \sum_{r=\overline{r}}^{k_o} \left(\alpha_r^I - \alpha_r^{NI}\right)\left[S_{k_0}\left(p_{k_0}^*\right) - S_\gamma\right] + \sum_{r=k_0}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right)\left[S_{k_0}\left(p_{k_0}^*\right) - S_\gamma\right] \\ &= \left[S_{k_0}\left(p_{k_0}^*\right) - S_\gamma\right] \sum_{r=\overline{r}}^{N} \left(\alpha_r^I - \alpha_r^{NI}\right) > 0\end{aligned}$$

Thus is either case, $dS_\gamma/d\mu \geq 0$. As $S_r'(p_r) < 0$ and $S_r(p_r) = S_\gamma$, $dp_r/d\mu < 0$. As $p_r < p_r^*$ it follows that $d\pi_r/d\mu < 0$ in this case. Thus $dw/d\mu \leq 0$.

Next, recall that the market share of viable firms is $s = (\sum_{k\geq\underline{r}}^{N} t_k)^{-1}$ and thus $ds/d\mu = -(\sum_{k\geq\underline{r}}^{N} t_k)^{-2} \cdot \sum_{k\geq\underline{r}}^{N} \left(\alpha_k^I - \alpha_k^{NI}\right) < 0$. Hence, for a small change in μ, $dW/d\mu < 0$. The return on investment W is decreasing in μ, except for a finite number of points where $\underline{r}$ (and thus s) jumps upwards. At these points W is jumping upwards as well. ■

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