To Sponsor or Not to Sponsor: Sponsored Search Auctions with Organic Links and Firm Dependent Click-Through Rates

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Abstract

In 2009, sponsored search advertisements generated over $11 billion in revenue for search engines in the US market and increased by 11% for the first half of 2010. Most of these advertisements were sold using an auction mechanism. Several existing papers analyze the auction mechanism currently used by search engines under the assumption that there are only sponsored links. We extend this literature to incorporate two important market features. In particular, we consider the impact of a second access channel, organic search listings which appear beneath the sponsored links, and we allow for the possibility that both relevance of the advertising firm as well as its position in the sponsored link listings impact the click-through-rate. Our results demonstrate that the existence of an outside alternative leads to less aggressive bidding behavior. The outside alternative also creates an important role for the minimum cost-per-click established by the search engine in maximizing auction revenue. In contrast to equilibrium results in the existing literature, the firm with the highest value per click does not necessarily win the first spot in the sponsored search listings. Moreover, under certain conditions, firms adopt a mixed strategy with regard to participation in the keyword auction, but have a pure bidding strategy when they do enter the auction.


1 Introduction

Consumers typically access firms in electronic markets through a commercial website where customers can find product information and make purchases. Key issues for firms with a website include visibility to the online audience of potential customers and the ability to convert online investments into revenue. Firms can use several strategies for this purpose. Search engine optimization (SEO) can increase the visibility of a website by improving its position among organic links on popular search engines. This strategy requires building a dense network of links and trackbacks through, for example, active participation in social networks or better internal organization of the website (e.g. cross linking, URL normalization). Yet, SEO has inherent limits because website designers are dependent on search engines regarding both the algorithm used to rank websites and the frequency of search engine (SE) updates.

Online advertising provides a more direct option for improving visibility.\(^1\) Since the end of the 1990s, the online advertising market has developed rapidly both in terms of technological possibilities (e.g. tracking opportunities) and business models (pay-per-click, pay-per-print, pay-per-sale).\(^2\) Sponsored search represents a significant segment of the online advertising market. Sponsored search enables firms to display sponsored ads alongside organic results produced by the SE. In 2010 sponsored search advertisements generated over $12 billion in revenue for search engines in the US market, which represented 46% of online advertising revenue (Source IAB). Many of these advertisements are sold through keyword auctions\(^3\). Sponsored search provides a balanced compromise between several concerns. First, sponsored links are displayed together with organic links. From a user’s perspective, they appear to be less intrusive than other types of ads such as pop-up windows or e-mail advertising. From the advertiser’s perspective, they provide the ability to better target customers based on a search query. This results in more qualified traffic viewing the sponsored ads. Finally, sponsored search typically entails a cost-per-click payment under which advertisers only incur a charge if the sponsored ad is sufficiently interesting to induce a consumer to click on the advertised

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\(^1\)See Evans(2007, 2011) for a survey on the economics of online advertising.

\(^2\)See Taylor(2010) for an economic rationale for current payment schemes in the online advertising industry. See also Moon and Kwon (2011) and Zhu and Wilbur (2010) for the advantages of CPC vs CPM schemes for the advertiser and the publisher.

\(^3\)See Edelman, Ostrovsky and Schwartz (2007) for a presentation of the features of search advertising industry and the evolution this industry over the last decade.
A growing body of economics and marketing literature is focused on keyword advertising and the response of firms and consumers to this advertising channel. The seminal papers of Varian (2007) and of Edelman et al. (2007) analyze optimal auction design to maximize SE revenue. Several subsequent studies have extended this analysis by considering more specific issues in the design of online auctions (Athey and Ellison, 2008; Animesh et al., 2010). Most of these extensions analyze keyword bidding strategies under the assumption that customers are only accessed through sponsored links. First, we consider the impact of a second access channel, organic search listings which appear beneath the sponsored links. Finally we allow for the possibility that both the relevance of the advertising firm as well as its position in the sponsored and organic link listings impact the return on keyword advertisements.

Our model enables us to examine how organic and sponsored links impact the keyword advertising strategies of individual firms and the reservation bids established by the search engines when websites differ in their relevance to consumers (reflected in the probability that a searching customer will click on a link to a given firm). Our analysis highlights two different effects of sponsored links. A location effect shifts clicks away from organic links to sponsored links because the sponsored link moves organic links further down the page. This may be exacerbated or offset by a crowding out/in effect which reflects the degree to which sponsored and organic links are substitutes or complements. Sponsored links may serve as a substitute for organic links regardless of location, or sponsored links may complement organic links and lead customers to click on both the sponsored and organic links. As a result, the existence of organic links leads to less aggressive bidding. In contrast to equilibrium results in much of the existing literature, the firm which is most relevant or has the highest value per click does not necessarily win the first spot in the sponsored search listings. The interplay between crowding out/in and location effects also creates an important role for the reservation price (minimum cost-per-click) established by the search engine that has not been considered in previous literature. In particular, the SE can minimize the effect of less competitive bidding by increasing the minimum cost per click. Under certain conditions, the optimal reservation price causes a less popular firm to use a sponsored link to increase its traffic while a more popular firm relies on its organic links to attract customers. In addition, for some parameterizations of our model, firms adopt mixed strategies. In contrast to previous
papers, the mixed strategies apply not to the bids submitted by each website but to the decision regarding whether or not to participate in the keyword auction.

The main interest of our paper is to provide an integrated game-theoretic model to understand the interplay between organic and sponsored links without stating restrictive assumptions on the characteristics of keywords and consumer preferences. Our setting enables to distinguish four categories of keywords that differs across two dimensions: the impact of sponsored link on the CTR of organic link (sponsored link as complements or substitutes for organic links) and the degree of effectiveness of sponsored link (relevance of sponsored link higher or lower than relevance of organic link). Classifying keywords in each of these categories is certainly a practical issue, but our model may help to understand how the nature of keywords affects keyword bidding strategies. Most of our assumptions about the perception of sponsored keywords are quite consistent with empirical studies. The latter underline how the effectiveness of a sponsored link depends on the nature of the keyword, its position on the search results page and the other links that are above and below. For example Ghose and Yang (2009) find that retailer-specific and brand-specific information in a sponsored link increases the efficiency of online advertising. Yang and Ghose (2010) also show that organic and sponsored links tend to be positively interdependant. In particular, total click-through rates, conversions rates, and revenues are significantly higher when both sponsored and organic links to the firm appear on the search results page. However, more recently Reiley, Li and Lewis (2010) find that sponsored links may substitute for organic links. Agarwal et. al. (2008) show that while the click-through-rate decreases with position, the conversion rate first increases and then decreases with position for longer keywords. They conclude that the top positions in sponsored search advertisements are not necessarily the optimal positions for advertisers. Complementary to these studies, Rutz and Bucklin (forth.) investigate the interactions between several types of keywords (generic versus branded keywords), and find that generic keywords may induce positive spillovers on the effectiveness (measured by click-through rate) of branded keywords. Similarly, Jeziorski and Segal (2009) and Chiou and Tucker (2010) show the prevalence of externalities across ads meaning that the click-through-rate on a given ad in a given position depends on which ads are shown in other positions and the words used in the text of these ads. Finally, Edelman and Gilchrist (2011) find that click-through-rates are influenced by the labeling of paid links (for example
replacing “sponsored link” with “ad” or “paid advertisement”).

The second contribution of our paper is to characterize bidding strategies in a "Generalized second-price" (GSP) auction system with positive reserve price when firms differ in relevance and value per consumer. This model extends the seminal papers of Varian (2007) and Edelman, Ostrovski & Schwartz (2007). The papers of Athey & Ellison (2008), Chen, De and Whinston (2009), Katona & Sarvary (2009) and Xu, Chen & Whinston (2009) are partially addressing some of these questions, but at the expense of restrictive assumptions on consumer perception of sponsored links. But our model allows for the click-through-rates on sponsored and organic links to depend on the number of effective sponsored links, the location and relevance of each link. Another distinction with the work by Xu et. al. (2009) and Katona and Sarvary (2009) is that in our model the firm’s decision to participate in the keyword auction is endogenous and depends upon the minimum cost per click (cpc) established by the SE.

Section 2 presents the model. Section 3 explains the nature of keyword auctions and analyzes equilibrium bidding strategies. Section 4 discusses and illustrates the results. Section 5 concludes.

Taylor (2009) considers a model in which consumers may choose between organic and sponsored links. With two competing search engines, he shows how the quality of organic results may cannibalize the revenues of the search engine. White (2009) focuses on the interplay between organic and sponsored results when sponsored links are sold at a fixed price and the SE can determine the quality of the ads it accepts. Higher quality ads reduce consumer search costs but may lead to increased competition in the final product market which ultimately reduces SE profit. However both papers don’t explicitly consider the auction process.

Xu et al. (2009) investigate a framework in which two firms compete in both the keyword (advertising) market and the product market. Firms sell a homogeneous product but are endowed with different marginal production costs. There are two types of consumers, shoppers who sample all firms, and non-shoppers who only sample the firm listed first in the search results. They find that the firm with the higher production cost always has an incentive to be ranked first while the firm with the lower production cost has an incentive to bid aggressively only when the advantage from being ranked first is significant. The effect of keyword advertising on the price of the product is ambiguous. See also Aggarwal et. al. (2006) for a model of keyword competition with uncertainty about product quality.

Katona and Sarvary (2009) show that a less popular firm can be ranked before a more popular firm in the list of sponsored links. They also extend these results by considering a dynamic setting to account for customer loyalty over time.
2 The Model

2.1 Consumer behavior on the search engine

We consider a duopoly market with sponsored search advertising with two firms $i = 1, 2$. We extend models of position auctions (Edelman et. al. (2007) and Varian (2007)) to incorporate organic search listings and to allow firm relevance to differ for organic and sponsored links. In particular, in our duopoly setting there are $n = 4$ possible positions on the search results page. Each position $k \in \{1, 2, 3, 4\}$ has a position specific parameter $x_k$ that measures the value or quality of this position, where $x_k \geq x_{k+1}$, and $1 \geq x_k > 0$ for all $k$. To incorporate differences in firm relevance, the position specific parameter is adjusted by a firm specific factor to determine the overall click-through-rate ($CTR$) for each firm/position specific combination. In particular, let $\beta_i$ denote the firm relevance factor for an organic link to firm $i$ when neither firm sponsors a link, where $1 \geq \beta_1 > \beta_2 > 0$. Because firm 1 is more relevant than firm 2, the organic (unsponsored) results produced by the search engine always list firm 1 before firm 2. When neither firm sponsors a link, the CTRs are $\beta_1 x_1$ and $\beta_2 x_2$ for firms 1 and 2, respectively.

In addition to providing organic search listings, the search engine conducts an auction to sell sponsored links which are listed in the highest positions. If only one of the two firms wins a sponsored link, then a link to that firm appears in the first position and the organic listings to firm 1 and then firm 2 appear in the second and third positions. Because the relevance of each firm may differ for sponsored and organic links, let $\delta_i$ denote the firm specific effect of a sponsored link to firm $i$ where $1 \geq \delta_1 \geq \delta_2 > 0$. If firm $i$ has a sponsored link in position $k \in \{1, 2\}$, then the CTR for that link is $\delta_i x_k$.

To allow for the possibility that sponsored links might be either complements or substitutes for organic links, let $\gamma_i$ be a firm specific adjustment to the relevance of the organic link to firm $i$ when that link appears after a sponsored link to firm $i$; the CTR for an organic link to firm $i$ in position $k$ following a sponsored link to firm $i$ is $\gamma_i \beta_i x_k$. If $\gamma_i < 1$, then the sponsored link serves as a substitute for the organic link in the sense that the presence

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7 We make no assumption about the relationship between $\delta_i$ and $\beta_i$. If firms employ effective targeted marketing by including specific phrases in the sponsored link related to the keyword search, then it is likely that $\delta_i > \beta_i$. However, if consumers have a distaste for sponsored links, then it is possible that $\delta_i < \beta_i$.


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of a sponsored link reduces the firm-specific relevance of the organic link. Similarly, the sponsored link complements the organic link if \( \gamma_i > 1 \). If both firms choose to advertise a sponsored link, then the SE must determine which sponsored link appears first. In this case, the sponsored links occupy the first two positions followed by the organic links to firm 1 and then firm 2 in the third and fourth positions.

Our assumptions about the structure of the search market generate click through rates which depend upon the number of sponsored links and the position of each firm in the sponsored links as depicted in Table 1. The number in parentheses denotes the firm located in the corresponding position. Figure 1 illustrates the case in which both firms have sponsored links and firm 2 is listed first. In this case the CTR for firm 1 is \( \delta_1 x_1 + \gamma_1 \beta_1 x_3 \) and for firm 2 is \( \delta_2 x_1 + \gamma_2 \beta_2 x_4 \).

<table>
<thead>
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<th>Position</th>
<th>None</th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Both, Firm 1 First</th>
<th>Both, Firm 2 First</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \beta_1 x_1 ) (1)</td>
<td>( \delta_1 x_1 ) (1)</td>
<td>( \delta_2 x_1 ) (2)</td>
<td>( \delta_1 x_1 ) (1)</td>
<td>( \delta_2 x_1 ) (2)</td>
</tr>
<tr>
<td>2</td>
<td>( \beta_2 x_2 ) (2)</td>
<td>( \beta_1 \gamma_1 x_2 ) (1)</td>
<td>( \beta_1 x_2 ) (1)</td>
<td>( \beta_2 x_2 ) (2)</td>
<td>( \delta_1 x_2 ) (1)</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>( \beta_2 x_3 ) (2)</td>
<td>( \beta_2 \gamma_2 x_3 ) (2)</td>
<td>( \beta_1 \gamma_1 x_3 ) (1)</td>
<td>( \beta_1 \gamma_1 x_3 ) (1)</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( \beta_2 \gamma_2 x_4 ) (2)</td>
<td>( \beta_2 \gamma_2 x_4 ) (2)</td>
</tr>
</tbody>
</table>

### 2.2 Firm profit

Firms are interested in maximizing profit generated by the search channel. Let \( p_{i,k} \) denote the cost-per-click (CPC) paid by firm \( i \) when firm \( i \) sponsors a link and that sponsored link is listed in position \( k \). Firms only incur the CPC if a customer reaches the firm through the sponsored link. Let \( \pi_{i}^{r,s} \) denote the profit of firm \( i \) when either none or one of the firms sponsors a link, firm 1 adopts a strategy \( r \), firm 2 adopts a strategy \( s \), and \( r, s \in \{N, A\} \), where \( A \) is a strategy of advertising a sponsored link and \( N \) is a strategy of not advertising. Finally, let \( \pi_{i}^{A,Ak} \) denote the profit for firm \( i \) when both firms have sponsored links and firm \( i \)'s sponsored link appears in position \( k \). The expected value to firm \( i \) of a customer who clicks on a link to firm \( i \) is \( v_i \).9

9This expected value is the product of the conversion rate (the probability the consumer makes a purchase after clicking on the sponsored link) and the average purchase amount. We make no assumptions on the
Figure 1: Example of search engine listing when both firms have sponsored links and firm 2’s paid link is listed first.

Using the above table, the expected profit for the firms under each possible strategy profile can be determined. If neither firm advertises, then

$$\pi_{1}^{NN} = \beta_1 x_1 v_1$$
$$\pi_{2}^{NN} = \beta_2 x_2 v_2.$$  

If only Firm 1 advertises, then

$$\pi_{1}^{AN} = \delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_2 \gamma_1 v_1$$
$$\pi_{2}^{AN} = \beta_2 x_3 v_2.$$  

If only Firm 2 advertises, then

$$\pi_{1}^{NA} = \beta_1 x_2 v_1$$
$$\pi_{2}^{NA} = \delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_3 \gamma_2 v_2.$$  

ordering of $v_1$ and $v_2$. Firm 2 is assumed less relevant ($\beta_2 < \beta_1$), but it may provide a more valuable product ($v_2 > v_1$).
If both firms advertise, then each firm’s profit depends upon the placement of its sponsored listing. If firm 1 is listed first, then

\[ \pi_{AA1}^1 = \delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_3 \gamma_1 v_1 \]
\[ \pi_{AA2}^2 = \delta_2 x_2 (v_2 - p_{2,2}) + \beta_2 x_4 \gamma_2 v_2. \]

If firm 2 is listed first when both firms advertise, then

\[ \pi_{AA2}^1 = \delta_1 x_2 (v_1 - p_{1,2}) + \beta_1 x_3 \gamma_1 v_1 \]
\[ \pi_{AA1}^2 = \delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2. \]

3 The Keyword Auction

Consider a position auction of the form analyzed by Edelman et. al. and Varian in which firms also have the option of not bidding and relying only on organic links as a source of customers and the search engine establishes a minimum \( CPC \) of \( c > 0.10 \). Each firm can submit a bid \( b_i \) which represents the maximum \( CPC \) that firm \( i \) can be assessed for a sponsored link (in either position 1 or position 2). We restrict attention to a generalized second price auction in which the \( CPC \) \( p_{i,k} \) paid by firm \( i \) for a sponsored link in position \( k \) is not a function of the bid submitted by firm \( i \). Furthermore, we assume firm \( i \) pays a \( CPC \) of \( c \) for its sponsored link if \( i \) is the only firm with a sponsored link or if both firms have sponsored links and \( i \) is listed second.

3.1 Optimal bidding strategies

The profit functions for each firm determine firm strategies for the game in which the firms simultaneously choose advertising strategies and bids. There will be an equilibrium in which neither firm chooses to advertise if \( \pi_{1}^{NN} > \pi_{1}^{AN} \), and \( \pi_{2}^{NN} > \pi_{2}^{NA} \). Noting that \( p_{i,1} = c \) if only one firm advertises, these restrictions require

\[ \beta_1 x_1 v_1 \geq \delta_1 x_1 (v_1 - c) + \beta_1 x_2 \gamma_1 v_1, \text{ and} \]
\[ \beta_2 x_2 v_2 \geq \delta_2 x_1 (v_2 - c) + \beta_2 x_3 \gamma_2 v_2. \]

\(^{10}\)Estimates provided by the Google AdWords keyword tool suggest a minimum \( CPC \) of at least \$.05 for any keyword on Google..
which imply that firm 1 will prefer not to advertise conditional on firm 2 not advertising if
\[ c \geq v_1 \left( 1 - \beta_1/\delta_1 + \beta_1 \gamma_1 x_2/\delta_1 x_1 \right) \equiv c_1, \]
and firm 2 will prefer not to advertise conditional on firm 1 not advertising if
\[ c \geq v_2 \left( 1 - \beta_2 x_2/\delta_2 x_1 + \beta_2 \gamma_2 x_3/\delta_2 x_1 \right) \equiv c_2. \]

**Proposition 1** If \( c > \max \{c_1, c_2\} \), then there is an equilibrium in which neither firm bids on a sponsored link.

If \( c < \max \{c_1, c_2\} \), then at least one firm will advertise with strictly positive probability. In addition, while a high minimum CPC of \( c > \max \{c_1, c_2\} \) ensures existence of an equilibrium in which neither firm chooses to sponsor a link, as we will show below, it does not rule out the possibility of an equilibrium in which both firms sponsor a link.

To determine optimal strategies for each firm, several conditions must be considered. The first case 1.1 corresponds directly to the envy-free equilibrium bidding condition in Edelman et. al. and Varian for the case of two firms with \( c > 0 \). The second and third conditions can be thought of as no-regret conditions which ensure that advertising is preferred to the alternative of encountering customers through the organic channel alone.

- **Condition 1.1.** Firm 1 prefers to advertise and be listed first over advertising and being listed second (given firm 2 does advertise) if \( \pi_{AA_1}^1 > \pi_{AA_2}^1 \) which implies

\[
\delta_1 x_1 \left( v_1 - p_{1,1} \right) + \beta_1 x_3 \gamma_1 v_1 > \delta_1 x_2 \left( v_1 - c \right) + \beta_1 x_3 \gamma_1 v_1
\]

or

\[
p_{1,1} < v_1 \left( 1 - x_2/\delta_1 \right) + c x_2/\delta_1 \equiv \tilde{p}_1. \tag{11}
\]

Note that \( \tilde{p}_1 \) is increasing in \( c \) and \( \tilde{p}_1 < v_1 \) if and only if \( c < v_1 \).

- **Condition 1.2.** Firm 1 prefers to advertise and be listed first over not advertising given firm 2 does advertise if \( \pi_{AA_1}^1 > \pi_{NA}^1 \) which implies

\[
\delta_1 x_1 \left( v_1 - p_{1,1} \right) + \beta_1 x_3 \gamma_1 v_1 > \beta_1 x_2 v_1
\]

or

\[
p_{1,1} < v_1 \left( 1 - \beta_1 x_2/\delta_1 \right) + \beta_1 \gamma_1 x_3/\delta_1 x_1 \equiv \tilde{p}_1.
\]

Note that \( \tilde{p}_1 \leq v_1 \) if and only if \( \gamma_1 \leq x_2/\delta_1 \).

\[ \text{For } c = 0, \text{ this expression is equivalent to the bounds established in equations (10) and (11) in Varian – these bounds are equal for the case of two bidders.} \]
• Condition 1.3. Firm 1 prefers to advertise and be listed second over not advertising given firm 2 advertises if \( \pi_1^{AA} > \pi_1^{NA} \) which implies

\[
\delta_1 x_2 (v_1 - c) + \beta_1 x_3 \gamma_1 v_1 > \beta_1 x_2 v_1
\]

or

\[
c < v_1 \left( 1 - \beta_1 / \delta_1 + \beta_1 \gamma_1 x_3 / \delta_1 x_2 \right) = \tilde{c}_1.
\]

Note that \( \tilde{p}_1 = \hat{p}_1 \) when \( c = \tilde{c}_1 \). If \( c > \tilde{c}_1 \), then \( \hat{p}_1 > \tilde{p}_1 \), and if \( c < \tilde{c}_1 \), then \( \hat{p}_1 < \tilde{p}_1 \). In addition, \( \tilde{c}_1 \leq v_1 \) if and only if \( \gamma_1 \leq x_2 / x_3 \). If there are sufficiently strong complementarities between firm 1’s sponsored and organic links so that \( \gamma_1 > x_2 / x_3 > 1 \), then \( \tilde{c}_1 > v_1 \). This implies that conditional on firm 2 advertising, if \( \gamma_1 > x_2 / x_3 \), then firm 1 may choose to advertise even if \( c > v_1 \). In particular, firm 1 will be willing to incur a CPC of \( c > v_1 \) if \( \gamma_1 > x_2 / x_3 \) and \( c < \tilde{c}_1 \).

Similar conditions also apply to firm 2.

• Condition 2.1. Firm 2 prefers to advertise and be listed first over advertising and being listed second (given firm 1 advertises) if \( \pi_2^{AA} > \pi_2^{A2} \) which implies

\[
\delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2 > \delta_2 x_2 (v_2 - c) + \beta_2 x_4 \gamma_2 v_2
\]

or

\[
p_{2,1} < v_2 \left( 1 - x_2 / x_1 \right) + cx_2 / x_1 = \tilde{p}_2.
\]

Note \( \tilde{p}_2 \) is increasing in \( c \) and that \( \tilde{p}_2 < v_2 \) if and only if \( c < v_2 \) which holds by assumption.

• Condition 2.2. Firm 2 prefers to advertise and be listed first over not advertising given firm 1 advertises if \( \pi_2^{AA} > \pi_2^{AN} \) which implies

\[
\delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2 > \beta_2 x_3 v_2
\]

or

\[
p_{2,1} < v_2 \left( 1 - \beta_2 x_3 / \delta_2 x_1 + \beta_2 \gamma_2 x_4 / \delta_2 x_1 \right) = \hat{p}_2.
\]

Note that \( \hat{p}_2 \leq v_2 \) if and only if \( \gamma_2 \leq x_3 / x_4 \).
• Condition 2.3. Firm 2 prefers to advertise and be listed second over not advertising given firm 1 advertises if $\pi^{AA}_2 > \pi^{AN}_2$ which implies

$$\delta_2x_2(v_2 - c) + \beta_2x_4\gamma_2v_2 > \beta_2x_3v_2$$

or

$$c < v_2(1 - \beta_2x_3/\delta_2x_2 + \beta_2\gamma_2x_4/\delta_2x_2) \equiv \tilde{c}_2.$$ 

Note that $\tilde{p}_2 > \hat{p}_2$ if and only if $c > \tilde{c}_2$ and that $\tilde{c}_2 \leq v_2$ if and only if $\gamma_2 \leq x_3/x_4$. As was true for firm 1, if there are strong complementarities between firm 2’s sponsored and organic links so that $\gamma_2 > x_3/x_4$, then conditional on firm 1 advertising, firm 2 will always advertise as well.

The above conditions enable us to specify the optimal bidding behavior of a given firm $i$ as a function of the minimum cost per click and the parameter $\gamma_i$ which determines the extent to which sponsored links are complements to or substitutes for organic links. Conditions i.1 imply that each firm prefers to be listed first in the sponsored links over being listed second for any $CPC$ up to $\hat{p}_i$. However, conditions i.2 imply that firm $i$ is better off not sponsoring than sponsoring an ad in the first position (assuming firm $j$ sponsors an ad in the second position) if the $CPC$ exceeds $\hat{p}_i$, and conditions i.3 imply that firm $i$ is better off not sponsoring an ad than sponsoring an ad in the second position if the $CPC$ exceeds $\tilde{c}_i$. These Note that because the $CPC$ for the second sponsored link is determined by the search engine, not by the firms competing for sponsored links, standard arguments imply that bidding the maximum willingness to pay per click (e.g., $\tilde{p}_i$ or $\hat{p}_i$) is a weakly dominant strategy for each firm in the generalized section price auction we consider.

If $\gamma_i < x_{i+1}/x_{i+2}$, then $\tilde{c}_i < \hat{p}_i < v_i$, and if $\gamma_i > x_{i+1}/x_{i+2}$, then $v_i < \hat{p}_i < \tilde{c}_i$. Furthermore, $\hat{p}_i \leq v_i$ for $c \geq v_i$, and $\hat{p}_i \geq \tilde{c}_i$ for $c \geq \tilde{c}_i$. Thus, assuming that both firms bid on sponsored links, if $\gamma_i < x_{i+1}/x_{i+2}$, then $\hat{p}_i \geq \tilde{c}_i$ so conditions i.1, i.2 and i.3 imply firm $i$ bids $\hat{p}_i$ if $c \leq \tilde{c}_i$. The firm prefers to win a sponsored link in the first position, but also is willing to sponsor a link in the second position. If $\tilde{c}_i \leq c \leq \hat{p}_i$, then conditions i.1 and i.2 imply firm $i$ is willing to pay a $CPC$ of $\hat{p}_i$ to sponsor a link in the first position, but condition i.3 implies firm $i$ is not willing to sponsor a link in the second position. If $c > \hat{p}_i$, then firm $i$ is not willing to sponsor a link in either position. The optimal bid for firm $i$ given firm $j$ sponsors a link and $\gamma_i < x_{i+1}/x_{i+2}$ is depicted in figure 2. The dashed section of the bid function for values of
that for values of $c$ in this range firm $i$ is only willing to bid $\hat{p}_i$ if it is certain to win the first position. The optimal bid for firm $i$ if $\gamma_i > x_{i+1}/x_{i+2}$ is slightly more complicated and is depicted in figure 3. In this case firm $i$ bids $\hat{p}_i$ if $c \leq v_i$. If $\hat{p}_i \geq c > v_i$, then $c > \hat{p}_i$, so firm $i$ prefers to be listed second instead of first and should bid $c$. However, if both firms submit a bid if $c$, then firm $i$ may be randomly awarded the first sponsored link. This is acceptable as long as $c \leq \hat{p}_i$ (condition i.2 implies firm $i$ prefers to be listed first over not advertising as long as $p_{i,1} < \hat{p}_i$). Thus, for $v_i < c \leq \hat{p}_i$, firm $i$ will submit a bid of $c$ and hope to be listed second instead of first, but will accept whichever sponsored position it wins in the auction process. If $c > v_i$, then the benefit of the sponsored link accrues only through the complement effect of increasing clicks on the organic link. The firm prefers the second sponsored link position in order to realize the complementarity while minimizing the number of clicks on the sponsored link. However, firm $i$ is still better off sponsoring a link than not sponsoring a link with a $CPC$ of $c \leq \hat{p}_i$ even if it wins the first sponsored link position. If $\hat{p}_i < c \leq \tilde{c}_i$, then firm $i$ is only willing to sponsor a link if it is in the second sponsored position with a $CPC$ of $c$. If firm $j$ sponsors a link in the first position, then firm $i$ is better off sponsoring a link in the second position than not. However, if both firms bid $c$ and firm $j$ loses the auction and firm $i$ is awarded a sponsored link in the first position, then firm $i$ prefers not to sponsor a link. In figure 3 the dashed bid function for values of $c$ satisfying $\hat{p}_i < c \leq \tilde{c}_i$ indicates that firm $i$ is only willing to bid for values of $c$ in this range if it is guaranteed to win the second position. As discussed in the appendix, if firm parameters are such that $c$ lies in this range for both firms and $c < c_i$ for at least one firm, then the firms must adopt mixed advertising strategies in equilibrium. Finally, if $c > \tilde{c}_i$, then firm $i$ is always better off not sponsoring a link if firm $j$ sponsors.

The discussion of optimal bidding strategies for firm $i$ in the previous paragraph is summarized in Lemmas 2 through 4. Note that these results are contingent upon the assumption that the competing firm $j$ sponsors a link. If firm $j$ does not sponsor a link, then firm $i$ will sponsor a link if and only if $c < c_i$. In section 4 below, we consider conditions under which firms choose to bid on sponsored links or rely only on organic links in equilibrium.

**Lemma 2** If $\gamma_i < x_{i+1}/x_{i+2}$, then $v_i > \hat{p}_i > \tilde{c}_i$. If $\gamma_i > x_{i+1}/x_{i+2}$, then $\tilde{c}_i > \hat{p}_i > v_i$.

**Lemma 3** Assume $\gamma_i \leq x_{i+1}/x_{i+2}$ and that firm $j$ participates in the auction for sponsored links. If $c \leq \tilde{c}_i$, then firm $i$ bids $\hat{p}_i$. If $\hat{p}_i \geq c > \tilde{c}_i$, then firm $i$ participates and bids $\hat{p}_i$ if and
Figure 2: Relationship between the minimum CPC and the optimal bids for firm $i$ only if firm $i$ will secure the first position in the sponsored links with a bid of $\hat{p}_i$. If $c > \hat{p}_i$, then firm $i$ will not sponsor a link.

**Lemma 4** Assume $\gamma_i > x_{i+1}/x_{i+2}$ and that firm $j$ participates in the auction for sponsored links. If $c \leq v_i$, then firm $i$ bids $\tilde{p}_i$. If $v_i < c \leq \hat{p}_i$, then firm $i$ bids $c$. If $\hat{p}_i < c \leq \tilde{c}_i$, then firm $i$ participates and bids $c$ if and only if firm $i$ will secure the second position in the sponsored links with a bid of $c$. If $c > \tilde{c}_i$, then firm $i$ will not sponsor a link.

Note that $\min(\tilde{p}_i, \hat{p}_i) = \tilde{p}_i$ if $c \leq \tilde{c}_i$, and $\min(\tilde{p}_i, \hat{p}_i) = \hat{p}_i$ if $c > \tilde{c}_i$. Thus, lemmas 3 and 4 imply the following proposition.

**Proposition 5** If $c > \max\{\tilde{p}_i, \hat{p}_j\}$, then there is no equilibrium in which both firms bid on sponsored links.

### 3.2 Search Engine Allocation of Sponsored Links

The search engine must determine which firm to list first when both firms submit bids exceeding the reservation price $c$. The expected revenue to the search engine from listing firm 1 first is

$$\delta_1 x_1 p_{1,1} + \delta_2 x_2 p_{2,2}$$
and from listing firm 2 first is  
\[ \delta_2 x_1 p_{2,1} + \delta_1 x_2 p_{1,2}. \]

The price paid by any firm cannot exceed that firm’s bid, so  \( p_{i,k} \leq b_i \), and the CPC for the firm listed second is  \( p_{i,2} = c \) for  \( i = 1, 2 \). Thus, the search engine will list firm 1 first if  
\[ \delta_1 x_1 b_1 + \delta_2 x_2 c \geq \delta_2 x_1 b_2 + \delta_1 x_2 c \]
or
\[ b_1 \geq \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1} \tag{1} \]
and will list firm 2 first otherwise. While equation (1) determines the optimal position assignments based on bids submitted, it does not provide a general rule for determining the price paid by the firm listed in the first sponsored link position. Note that if  \( c = 0 \), then a rule of ranking firms by their quality adjusted bids  \( \delta_ib_i \) is optimal and the second price auction rule charges the firm  \( i \) listed first a CPC equal to the quality adjusted bid  \( b_j \delta_i / \delta_j \) of the firm listed second. It is easily shown that such an ordering rule generalizes to the case of  \( N \) bidders with  \( K \) sponsored link positions provided either  \( c = 0 \), or  \( c > 0 \) and  \( N > K \) and  \( \delta_{g(K+1)} b_{g(K+1)} > \delta_{g(K)} c \), where  \( g(k) \) is the identity of the firm awarded a sponsored link in position  \( k < K \) and  \( g(K+1) \) is the identity of the highest ranked firm not awarded a sponsored link. (In this case the firm awarded the last sponsored link position pays a CPC of  \( \delta_{g(K+1)} b_{g(K+1)}/\delta_{g(K)} \)). However, if the firm awarded the last sponsored link position pays  \( c \), either because  \( N < K \) or because  \( N > K \) and  \( \delta_{g(K+1)} b_{g(K+1)} < \delta_{g(K)} c \), then the search engine should award higher sponsored link positions based on quality adjusted bids plus a premium (or discount if the less relevant firm is ranked first) which depends upon the minimum CPC,  \( c \). Casual observations from Google searches indicate that for many keywords some of the sponsored link positions often go unsold, so understanding the implications of a binding minimum cost-per-click,  \( c > 0 \) is important.

Assuming it is optimal for the search engine to award firm 1 the first sponsored link position, any price  \( p_{1,1} \) satisfying  \( b_1 \geq p_{1,1} \geq \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1} \) would be possible. However, because we are considering a second price auction rule and because setting  \( p_{1,1} \) strictly greater than  \( \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1} \) would result in a suboptimal assignment of firms to sponsored link positions by the search engine if  \( b_1 \in \left( \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}, p_{1,1} \right) \), we assume the auction rule establishes a price  \( p_{1,1} \) for the first sponsored link position that is just sufficient to
ensure positions are awarded based on the condition (1). In particular, if bids are such that equation (1) is satisfied, then firm 1 is awarded the first sponsored position and pays a cost-per-click (CPC) of

$$p_{1,1} = \max \left( \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}, c \right) < b_2,$$

where the inequality follows from the assumptions that $b_2 > c$, $x_1 \geq x_2$, and that the firm in the second sponsored position pays a CPC of $c$. If bids are such that equation (1) is violated, then firm 2 is awarded the first sponsored position with a CPC of

$$p_{2,1} = \frac{\delta_1}{\delta_2} b_1 - \frac{x_2 c (\delta_1 - \delta_2)}{\delta_2 x_1} > b_1.$$

Because $b_1 > c$ by assumption, firm 2 incurs a CPC greater than $c$ if it is awarded the first sponsored link position. Given the assumptions that $\delta_1 > \delta_2$ and $x_1 \geq x_2$, firm 1 may be listed first even if it bids less than firm 2, and firm 2 must bid strictly more than firm 1 in order to be listed first.

The premium $(p_{2,1} - b_1)$ that firm 2 must pay in order to be listed first is increasing in $\delta_1$ (the relevance of the sponsored link to firm 1), and decreasing in $\delta_2$ (the relevance of the sponsored link to firm 2). Similarly, the discount $(b_2 - p_{1,1})$ for firm 1 is decreasing in $\delta_1$ and increasing in $\delta_2$. In the limiting case in which $\delta_1 \rightarrow \delta_2$, so that firms are equally relevant to consumers who utilize sponsored links, the search engine will simply rank the firms according to their bids, and the firm submitting the highest bid pays a CPC equal to the second highest bid. The premium paid by firm 2 and the discount to firm 1 are both decreasing in the ratio $x_2 / x_1$. As this ratio increases, the incremental gain in the click-through-rate from being in the first as opposed to the second sponsored position decreases. This lessens the search engine incentive to provide a discount to firm 1 and lessens its ability to extract a premium from firm 2 for the first sponsored position. Similar logic explains why both the premium to firm 2 and discount to firm 1 are decreasing in $c$.

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12 Establishing a lower minimum price would clearly not maximize search engine expected revenue. Establishing a higher minimum price would create incentives for advertisers to repeatedly lower their bids in an attempt to learn the lowest price the search engine would accept. As Edelman et. al., note, similar behavior under first-price auction rules in early sponsored search markets was a significant reason for the transition to the use of a generalized second price auction.
4 Equilibrium Outcomes

We now present equilibrium results for ranges of $c$ which generate relatively straightforward equilibrium strategies.

**Proposition 6** Suppose $\gamma_i \leq x_{i+1}/x_{i+2}$ for $i = 1, 2$.

1. If $c \leq \min(c_1, \tilde{c}_1, c_2, \tilde{c}_2)$, then there is a unique equilibrium in which each firm submits a bid of $\tilde{p}_i$ for sponsored links.

2. If $\max(c_1, c_2) < c < \min(\tilde{c}_1, \tilde{c}_2)$, then there are two pure strategy equilibria; in one equilibrium, neither firm bids on sponsored links, and in the other equilibrium each firm submits a bid of $\tilde{p}_i$.

3. If $\max(c_i, \tilde{c}_i) < c < c_j$, then there is a unique equilibrium in which only firm $j$ sponsors a link, and firm $j$ bids $\tilde{p}_j$.

4. If $c > \max(c_1, \tilde{c}_1, c_2, \tilde{c}_2)$, then neither firm bids on sponsored links.

**Proof.** The first and third and fourth results follow directly from comparison of firm profit from advertising versus not advertising for $c$ in the specified range. The second result follows from Proposition 1 if $c > \max(c_1, c_2)$, and from lemma 3 and the fact that if firm $j$ advertises, then cases i.1, i.2, and i.3 imply that firm $i$ should advertise and bid $\tilde{p}_i$ if $c < \min(\tilde{c}_1, \tilde{c}_2)$.

Cases (1), (3) and (4) in the above proposition generate straightforward equilibrium outcomes. Under the conditions of case (2) firms engage in a coordination game in which equilibrium entails matching the strategy adopted by the competing firm. More generally, the equilibrium bidder response to the minimum $CPC$, $c$, established by the search engine depends on the ordering of the critical values $c_i$ and $\tilde{c}_i$, $\tilde{p}_i$ and $\hat{p}_i$. Because the ordering of these critical values depends upon the differences in position specific click through rates $x_k$ as well as the degree of substitutability (or complementarity) $\gamma_i$ between sponsored and organic links, several different possibilities for the relationship between the critical values and the actual minimum $CPC$, $c$ established by the search engine must be considered. As lemma 2 demonstrates, the magnitude of $\gamma_i$ determines the ordering of $\tilde{c}_i$ and $\tilde{p}_i$. This analysis is straightforward, but cumbersome, so a discussion of is left to the appendix.
Proposition 7 If \( \gamma_i < (\beta_i x_i - \delta_i x_i) / \beta_i x_{i+1} \) for \( i = 1, 2 \), then for any \( c \geq 0 \) there is an equilibrium in which neither firm bids on sponsored links.

Proof. This follows from part 2 of proposition 6 and the fact that \( c_1 < 0 \) and \( c_2 < 0 \) given the values of \( \delta_1, \delta_2, \gamma_1, \) and \( \gamma_2 \) in the statement of the proposition. \( \blacksquare \)

Note that because \( \gamma_i > 0 \), the conditions of proposition 7 require \( \delta_1 < \beta_1 \) and \( \delta_2 < \beta_2 x_2 / x_1 \). These conditions imply that consumers are averse to sponsored links – they are more likely to click on the organic link to firm \( i \) when presented with both a sponsored and an organic link to the firm. Thus, proposition 7 implies that if consumers are sufficiently averse to sponsored links and sponsored links are not strong complements to organic links, then firms may choose not to participate in keyword auctions even if the search engine offers to provide sponsored links at no cost to the advertising firm. Note that this result can apply even if sponsored links are strict complements to organic links so that \( \gamma_i > 1 \). For example, if \( \delta_1 < \beta_1 (x_1 - x_2) / x_1 \), then \( (\beta_1 - \delta_1) x_1 / \beta_1 x_2 > 1 \), so \( c_1 < 0 \) will hold for some \( \gamma_1 > 1 \).

Under the conditions of Proposition 7, an equilibrium in which neither firm bids on sponsored links exists for any \( c > 0 \), but this equilibrium may not be unique. In particular, it is possible that \( \tilde{c}_1 > 0 \) and \( \tilde{c}_2 > 0 \), and a second equilibrium in which both firms participate in the auction with bids of \( \tilde{p}_i \) would exist if \( c < \min \{ \tilde{c}_1, \tilde{c}_2 \} \). However, if \( \delta_1, \delta_2, \gamma_1, \) and \( \gamma_2 \) are all sufficiently small, then there is a unique equilibrium in which neither firm participates in the keyword auction for any \( c \geq 0 \).

Alternatively, as demonstrated in proposition 8 below, sufficiently strong complementarities between sponsored and organic links give the search engine significant market power regardless of the relationship between the relevance \( \beta_i \) of organic links and \( \delta_i \) of sponsored links.

Proposition 8 If \( \gamma_i > x_{i+1} / x_{i+2} \) for \( i = 1, 2 \), then for any \( c \leq \min \{ \tilde{p}_1, \tilde{p}_2 \} \) there is an equilibrium in which both firms participate in the keyword auction and submit bids of \( \max \{ \tilde{p}_i, c \} \).

In addition, if \( \gamma_i > (x_{i+1} / x_{i+2}) \) for \( i = 1, 2 \), then the equilibrium is unique.

\(^{13}\)More generally, for any \( c \geq 0 \), \( c_1 < c \) and \( c_2 < c \) will both hold, so there is an equilibrium in which neither firm participates in the keyword auction, if \( \gamma_1 < (\beta_1 x_1 + \delta_1 x_1 (c - v_1) / v_1) / \beta_1 x_2 \) and \( \gamma_2 < (\beta_2 x_2 + \delta_2 x_2 (c - v_2) / v_2) / \beta_2 x_3 \).

\(^{14}\)In particular, \( \max (c_1, \tilde{c}_1, c_2, \tilde{c}_2) < 0 \) is a sufficient condition for a unique equilibrium in which neither firm participates for any \( c \geq 0 \). If \( \delta_1 < \beta_1 \) and \( \gamma_1 < \min \{ (\beta_1 - \delta_1) x_2 / \beta_1 x_3, (\beta_1 - \delta_1) x_1 / \beta_1 x_2 \} \) then both \( c_1 < 0 \) and \( \tilde{c}_1 < 0 \). Similarly, if \( \delta_2 < \min \{ \beta_2 x_2 / x_1, \beta_2 x_3 / x_2 \} \) and \( \gamma_2 < \min \{ (\beta_2 x_2 - \delta_2 x_1) / \beta_2 x_3, (\beta_2 x_3 - \delta_2 x_2) / \beta_2 x_4 \} \), then both \( c_2 < 0 \) and \( \tilde{c}_2 < 0 \).
Proof. This follows directly from lemmas 2 through 4 and the fact that $c_i > \hat{p}_i$ if $\gamma_i > \bar{\gamma}$.

Proposition 8 implies that if complementarities between sponsored and organic links are sufficiently strong for both firms, then the search engine can extract all of the surplus directly generated by the sponsored link from the firm with the lower expected value $v_i$ of a click-through by setting $c = v_i$. In fact, because both firms participate in the auction for any $c \leq \min\{\hat{p}_1, \hat{p}_2\}$ and because $\gamma_i > x_{i+1}/x_{i+2}$ implies $\hat{p}_i > v_i$, the search engine can even extract a premium in excess of $v_i$. This result holds even if the probability $\delta_i, x_k$ that a consumer clicks on the sponsored link to firm $i$ in position $k$ is very small. Proposition 8 indicates that when there are sufficiently strong complementarities between sponsored and organic links, the driving force in a firm’s decision to participate in a keyword auction is not the net profit generated by clicks on the sponsored link, but rather the increased traffic that the sponsored link generates on the firm’s organic link for which the firm pays nothing to the search engine. The positive externality accrued through the complementarity justifies purchase of a sponsored link even at a $CPC$ of $c > v_i$. Furthermore, because this result depends only on the extent of the complementarity $\gamma_i$ and the ratio $x_k/x_{k+1}$ of the position specific click-through rates of adjacent search listings, even a small complementarity ($\gamma_i$ greater than but close to 1) can convey substantial market power to the search engine if the difference in click-through-rates for adjacent positions is small so that $x_k/x_{k+1} \approx 1$. However, under currently utilized pricing practices the search engine’s ability to exert this market power is limited to the minimum $CPC$ $c$. In particular, the search engine is unable to extract rents generated by the positive externality the sponsored link creates by increasing traffic to the organic link.\footnote{This suggests that the search engine may be able to increase profits by implementing alternative pricing schemes such as a two part tariff.}

Propositions 7 and 8 demonstrate the important role that organic links as well as the extent to which sponsored links complement or substitute for organic links play in determining equilibrium outcomes in keyword auction markets. At one end of the spectrum, the presence of organic links combined with consumer resistance to sponsored links (low $\delta_i$) and crowding out effects (low $\gamma_i$) can lead to equilibria in which firms are unwilling to utilize sponsored links even if they are available at no charge. At the other end of the spectrum, even small complementarities between sponsored and organic links can induce firms to participate in a
keyword auction even if the search engine establishes a minimum CPC equal to the expected value \( v_i \) of a customer. Other implications of organic links and the parameter \( \gamma_i \) for keyword auction markets are illustrated in two simple examples.

**Example 9** \( \gamma_1 = \gamma_2 = \gamma, \ v_1 = v_2 = v, \ \frac{x_{i+1}}{x_i} = \alpha, \) and \( \frac{\beta_1}{\delta_1} = \frac{\beta_2}{\delta_2}. \)

The parameter assumptions imply that the two firms are identical with the exception that firm 1 is more relevant than firm 2. In addition, the fraction \( x_{k+1} \) of customers who consider a link in location \( k + 1 \) on the search results page (independent of any firm specific relevance effects) is a fixed proportion \( \alpha \) of the fraction \( x_k \) of customers who consider the next highest location. The parameter assumptions imply \( c_1 = \tilde{c}_1 = v(1 - \frac{\beta_1}{\delta_1}(1 - \gamma \alpha)) \) and \( c_2 = \tilde{c}_2 = v(1 - \frac{\beta_2}{\delta_2} \alpha(1 - \gamma)), \) and \( \alpha \leq 1 \) implies \( c_2 = \tilde{c}_2 \geq c_1 = \tilde{c}_1. \)

If \( \gamma > \frac{1}{\alpha} \) (the sponsored link strongly complements the organic link), then \( v < \min \{c_1, c_2\}. \)

In this case, equilibrium bidding strategies are straightforward: both firms participate and bid \( (\tilde{p}_1, \tilde{p}_2) \) with \( \tilde{p}_1 = \tilde{p}_2 = v(1 - \alpha) + c\alpha \) as long as \( c < v. \) In this case, both firms bid exactly the same amount, but the more relevant firm (firm 1) is always listed first.\(^{16}\)

If \( \gamma < \frac{1}{\alpha}, \) then three situations are possible:

- If \( c < \min \{c_1, c_2\}, \) then both firms bid \( \tilde{p}_1 = \tilde{p}_2 = v(1 - \alpha) + c\alpha \) and firm 1 is listed first.

- If \( c > c_2 = \tilde{c}_2, \) then neither firm participates in the keyword auction because \( c_2 > c_1 = \tilde{c}_1. \)

- If \( c_1 < c < c_2, \) then firm 2 bids \( \tilde{p}_2 = v(1 - \alpha) + c\alpha. \) In addition, firm 1 will bid \( \hat{p}_1 = v(1 - \frac{\beta_1}{\delta_1} \alpha(1 - \alpha \gamma)) \) only if this bid results in firm 1 being listed first in the sponsored listings. This requires
  
  \[ \hat{p}_1 > \frac{\delta_2}{\delta_1} \tilde{p}_2 + \frac{c}{\delta_1} \frac{(\delta_1 - \delta_2)}{\delta_1} \alpha. \]

  Substituting expressions for \( \hat{p}_1 \) and \( \tilde{p}_2 \) and simplifying yields
  
  \[ c < \frac{v(1 - \frac{\beta_1}{\delta_1} \alpha(1 - \alpha \gamma) - \frac{\delta_2}{\delta_1} (1 - \alpha))}{\alpha} \equiv \tilde{c}. \]  

\(^{16}\)Condition (1) always strictly holds because

\[ \tilde{p}_1 > \frac{\delta_2}{\delta_1} \tilde{p}_2 + \frac{c}{\delta_1} \frac{(\delta_1 - \delta_2)}{\delta_1} \frac{x_2}{x_1} = \frac{\delta_2}{\delta_1} v(1 - \alpha) + c\alpha. \]
If \( c < \bar{c} \), then firm 1 announces a lower bid than firm 2 \((\hat{p}_1 < \hat{p}_2)\), but is listed first and pays a \(CPC\) equal to \(\frac{\delta_2}{\delta_1}v(1-\alpha) + c\alpha\). This \(CPC\) increases in the value \(v\) of the products and in the minimum \(CPC\) set by the search engine. If condition (2) does not hold, then firm 1 does not participate in the keyword auction and firm 2 bids \(\hat{p}_2\) and pays a \(CPC\) of \(c\). Recall that \(\alpha\) indicates how consumer interest in a link depreciates for positions sequentially lower on the search engine results page. As \(\alpha\) increases, the four top positions are considered more similar (ignoring any differences in firm specific relevance) by consumers. As \(\alpha \to 0\) (the first position provides a kind of winner-take-all advantage), \(c_1 \to v(1 - \frac{\delta_1}{\delta_1})\) and \(c_2 \to v\). Condition 2 becomes \(v(1 - \frac{\delta_2}{\delta_1}) > 0\) which is always satisfied. This implies that if the location advantage is extreme for the first position and the minimum \(CPC\) has intermediate value, then both firms bid on keywords and firm 1 secures the first position even though it bids less aggressively than firm 2. As \(\alpha \to 1\), (there is no location advantage), \(c_1 \to c_2\) and the likelihood for firm 1 to bid \(\hat{p}_1\) decreases. However, firm 1 is more likely to participate if organic and sponsored links are not strong substitutes (\(\gamma\) remains close to 1) and the sponsored link of firm 1 is highly relevant (\(\delta_1\) close to 1). More generally if the minimum \(CPC\) increases in the interval \([c_1, c_2]\), then firm 1 is less likely to bid.

To summarize, the less relevant firm 2 (that is always listed second in the organic links) has a stronger incentive to participate in the keyword auction than firm 1, even if firm 2 is less likely to obtain the first position in sponsored links. Sponsored links can largely rebalance positions between competing firms on the search engine if location doesn’t provide a real advantage, but also when organic links and sponsored links are strong substitutes or the minimum cost per click is quite high.

In the context of this example, it also is straightforward to consider the optimal \(CPC\) established by the search engine. If \(\gamma > \frac{1}{\alpha}\), then the strategy of the search engine is trivial. If \(c\) is set at the highest possible level, \(c = v\), then both firms participate in the auction with bids of \(\hat{p}_1 = \hat{p}_2 = c\) and the search engine extracts the maximum possible surplus. More interestingly when \(\gamma < \frac{1}{\alpha}\), as long as \(c \leq c_1\), both firms participate with bids of \(\hat{p}_i\), and the the \(CPC\) paid by each firm increases with \(c\). This implies the search engine will never set \(c < c_1\). However, if \(c\) is between \(c_1\) and \(c_2\), increasing \(c\) can have two offsetting effects: an increase in \(c\) increases the \(CPC\) paid by firm 2 (through a higher minimum \(CPC\)), but it
reduces the likelihood that firm 1 participates in the keyword auction. Thus, it might be optimal for the search engine to set \( c \) between \( c_1 \) and \( c_2 \), but strictly lower than \( c_2 \), in order to encourage firm 1 to participate.

5 Managerial implications and concluding remarks

This paper investigates strategic behavior of firms that utilize sponsored search as a means of attracting potential customers. We develop a model which accounts for the existence of asymmetry between these firms. First, some firms may be more relevant for customers than others (i.e. consumers are more likely to follow a link to one firm than another). Second, firms may differ in the value generated by a consumer that visits their website – some firms may offer product that is more valuable to consumers than another firm even though consumers reach both firms by initially searching the same keyword. Our model also integrates several empirical features regarding consumers behavior on search engines. Our model explicitly integrates the possibility that some consumers may consider sponsored links as substitutes or complements to organic links, or may be ad-lover or ad-adverse.

Our framework allows us to determine what parameters are important for bidding strategies and how they affect the decision to bid and the amount of bidding. Our results highlight two kinds of equilibrium outcomes. In the first equilibrium outcome both firms bid on sponsored links. Under such an equilibrium, either firm can be listed first in the sponsored links. In general, the more relevant firm, firm 1, is more likely to be listed first as the difference \( \delta_1 - \delta_2 \) in the relevance of each firm in the sponsored links increases, because this implies consumers are more likely to click on firm 1’s sponsored link, so the SE prefers to list firm 1 first. In addition, as \( \beta_1 \) decreases (\( \beta_2 \) increases), firm 1’s bid becomes more competitive relative to firm 2, which also makes it more likely that firm 1 will be listed first.

A second type of equilibrium outcome involves only one of the two firms bidding in the keyword auction. This equilibrium occurs when there is a significantly large difference in the willingness to pay between the two firms and the search engine sets the minimum \( CPC \) sufficiently high. For example, if firm 1 has a relatively high click through rate \( \beta_1 \) on its organic links and firm 2 does not, then it may be optimal for the SE to set \( c \) high enough so that firm 1 chooses not to bid in the keyword auction. Similarly, if the value \( v_2 \) to firm 2 of attracting customers is much higher than \( v_1 \), then the SE will optimally set a high value of
which extracts surplus from firm 2 while excluding firm 1. The analysis also demonstrates how the SE’s ability to extract surplus in this manner is constrained by each firm’s ability to attract customers through its organic links.17

The results and specific examples presented above demonstrate that organic listings can have a significant impact on equilibrium in sponsored search auctions – both in terms of the firm’s decision to participate in the auction, the optimal bidding strategy when the firm does participate, and the optimal reservation price established by the SE. In particular, the results are consistent with outcomes in sponsored search auctions in which less relevant firms are often listed ahead of more relevant firms in the sponsored links. The analysis also predicts that under certain conditions, highly relevant firms will not appear at all in the sponsored links, while less relevant firms will bid and have a sponsored link.

The model can be extended in several directions. Initial analysis extending the model to include more than two firms suggests that it is possible that a Vickrey auction will generate more revenue for the SE than a generalized second price auction under certain conditions. This merits further investigation. Another possible extension would endogenize parameters γ and δi as parts of the search engine strategy. Indeed, the SE has the possibility to improve the visibility of the first sponsored link (in contrast to the other sponsored and organic links). For instance, it may choose to display only one sponsored link at the top of a result page, all the other sponsored links being displayed at the left-hand-side of the page which is a less favorable location. In that case, the advantage given to the first sponsored links is significantly increased. Secondly, one may empirically observe that for valuable or popular keywords, the ranking and the identity of sponsored links may change for two identical and successive requests. The ads displayed in the sponsored links section are the results of a random selection process of ranking. Yet, this creates some opportunity for the SE to “manipulate” the probability of clicking on a sponsored link (δi in our model) so as to extract more revenues from keywords advertising.

17 In a third type of equilibrium (not presented explicitly) the firms adopt mixed strategies to determine whether to participate in the auction process. This equilibrium requires that the SE set c sufficiently large that one of the two firms, say firm 1, is better off not advertising if firm 2 also chooses not to advertise, but firm 2 is better off advertising and paying c if firm 1 does not advertise, and firm 1 is better off advertising and being listed first if firm 2 does advertise.
6 Appendix

Lemmas 3 and 4 characterize optimal bidding strategies when both firms bid on sponsored links. If $c$ falls in a range in which both firms bid on sponsored links, then these lemmas determine the bid submitted by each firm. However, if $c$ is such that only one firm chooses to bid conditional on the other firm bidding, then lemmas 3 and 4 do not apply. The nature of the equilibrium strategies then depends upon $c_1$ and $c_2$. For example, if $\hat{p}_i > c > \tilde{c}_i$ for both firms, then each firm only wants to bid if it will be listed first, and each firm will bid $\hat{p}_i$ if that bid will win the first sponsored link position. However, unless $\hat{p}_1 = \hat{p}_2$, one of the two firms will win the first position with probability 1 and the other firm will choose not to bid. Set $i$ so that $\hat{p}_i > \hat{p}_j$. If $c < c_i$, then firm $i$ will bid $\hat{p}_i$ and firm $j$ will not bid. Because $c < c_i$, firm $i$ will sponsor a link regardless of whether or not firm $j$ sponsors a link. Therefore, firm $j$ knows that it will be in the second sponsored position if it chooses to bid, so it will not bid. However, if $c > c_i$, then if firm $j$ decides not to bid because $\hat{p}_i > \hat{p}_j$, then firm $i$ is better off not sponsoring a link and also will choose not to bid. If $c > c_j$, then the fact that firm $i$ chooses not to bid will not change firm $j$’s decision – the conditions of part 4 of proposition 6 are satisfied. However, if $c_i < c < c_j$, then a decision by firm $i$ to not bid knowing that firm $j$ will not bid makes bidding $\hat{p}_j$ a best response for firm $j$. But if firm $j$ bids $\hat{p}_j$, then it is optimal for firm $i$ to bid $\hat{p}_i$ and win the first sponsored link position. Thus, in this case there is no equilibrium in pure strategies. However, there will be an equilibrium in mixed strategies in which each firm advertises with probability $q_i$ such that given the probability that the other firm advertises with probability $q_j$, firm $i$’s expected profit from advertising with a bid of $\hat{p}_i$ is equal to firm $i$’s expected profit from not advertising. In particular, if $\hat{p}_i > \hat{p}_j$, then $q_j$ and $q_i$ satisfy

$$
q_j \pi_i^{AA1} + (1 - q_j) \pi_i^{AN} = q_j \pi_i^{NA} + (1 - q_j) \pi_i^{NN},
$$

and

$$
q_i \pi_j^{AA2} + (1 - q_i) \pi_j^{NA} = q_i \pi_j^{AN} + (1 - q_i) \pi_j^{NN}.
$$

More generally, a mixed strategy equilibrium will exist whenever conditions are such that one firm $i$ will bid on a sponsored link conditional on firm $j$ bidding on a sponsored link, firm $j$ will not bid on a sponsored link conditional on firm $i$ bidding, and $c_j > c > c_i$. The following lemmas establish conditions determining the relationship between $c_i$ and $\tilde{c}_i$. 

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Lemma 10 \( c_1 \geq \tilde{c}_1 \) if and only if \( x_2^2 \geq x_1 x_3 \).

**Proof.** Follows directly from the definitions of \( c_1 \) and \( \tilde{c}_1 \). ■

Lemma 11  Fix \( x_2^2 > x_1 x_3 \). If \( \gamma_1 < (x_1 - x_2) / (x_2 - x_3) \), then \( \hat{p}_1 > c_1 > \tilde{c}_1 \). If \( x_2/x_3 > \gamma_1 > (x_1 - x_2) / (x_2 - x_3) \), then \( c_1 > \hat{p}_1 > \tilde{c}_1 \). If \( \gamma_1 > x_2/x_3 \), then \( c_1 > \hat{p}_1 > \tilde{c}_1 \).

Fix \( x_2^2 < x_1 x_3 \). If \( \gamma_1 < x_2/x_3 \), then \( \tilde{c}_1 > c_1 > \hat{p}_1 \). If \( (x_1 - x_2) / (x_2 - x_3) > \gamma_1 > x_2/x_3 \), then \( \hat{c}_1 > \hat{p}_1 > c_1 \). If \( \gamma_1 > (x_1 - x_2) / (x_2 - x_3) \), then \( \hat{p}_1 > \tilde{c}_1 > c_1 \).

**Proof.** This follows directly from the definitions of \( c_1 \), \( \tilde{c}_1 \), and \( \hat{p}_1 \). ■

Lemma 12 \( c_2 \geq \tilde{c}_2 \) if and only if \( x_2^2 - x_1 x_3 + \gamma_2 (x_1 x_4 - x_2 x_3) < 0 \).

**Proof.** Follows directly from the definitions of \( c_2 \) and \( \tilde{c}_2 \). ■

Lemma 13  Fix \( x_2^2 > x_2 x_4 \). If \( \gamma_2 > x_3/x_4 \), then \( \hat{p}_2 < \min(c_2, \tilde{c}_2) \). If \( x_3/x_4 > \gamma_2 > (x_2 - x_3) / (x_3 - x_4) \), then \( c_2 > \hat{p}_2 > \tilde{c}_2 \). If \( \gamma_2 < (x_2 - x_3) / (x_3 - x_4) \), then \( \hat{p}_2 \not\in (c_2, \tilde{c}_2) \).

Fix \( x_2^2 < x_2 x_4 \). If \( \gamma_2 > (x_2 - x_3) / (x_3 - x_4) \), then \( \hat{p}_2 < \min(c_2, \tilde{c}_2) \). If \( (x_2 - x_3) / (x_3 - x_4) > \gamma_2 > x_3/x_4 \), then \( \hat{c}_2 > \hat{p}_2 > c_2 \). If \( \gamma_2 < x_3/x_4 \), then \( \hat{p}_2 \not\in (c_2, \tilde{c}_2) \).

**Proof.** This follows directly from the definitions of \( c_2 \), \( \tilde{c}_2 \), and \( \hat{p}_2 \). ■
7 Illustration

Let consider a numerical simulation with $\gamma_1 = \gamma_2 = \gamma = 0.8$ or 1.2, $v_1 = v_2 = v = 10$, $\delta_2 = 0.8$, $\frac{x_{i+1}}{x_i} = \alpha$, and $\delta_1 = \delta_2 = \phi = 0.8$ or 1.2.

<table>
<thead>
<tr>
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<th>Ad-lover $\phi \equiv 0.8$</th>
<th>Ad-adverse $\phi \equiv 1.2$</th>
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</thead>
<tbody>
<tr>
<td>Substitutes $\gamma = 0.8$</td>
<td>Case 1</td>
<td>Case 3</td>
</tr>
<tr>
<td>Complements $\gamma = 1.2$</td>
<td>Case 2</td>
<td>Case 4</td>
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For each case, we characterize the different equilibria that depends on the minimum cost per click and the value per consumer.

Four equilibrium outcomes:

Area I $\Rightarrow (\hat{p}_1, \hat{p}_2)$
Area II $\Rightarrow (\hat{p}_1, \hat{p}_2)$
Area III $\Rightarrow (\emptyset, \hat{p}_2) \Rightarrow$ only firm 2 bids
Area IV $\Rightarrow (\emptyset, \emptyset) \Rightarrow$ no firm bids

Case 1: sponsored links as substitute for organic links, but less effective than organic links to induce click throughs
Figure 3: Case 2: sponsored links as complement for organic links, but less effective than organic links to induce click throughs

Case 3: sponsored links as substitute for organic links, and more effective than organic links to induce click throughs

Case 4: sponsored links as complement for organic links, and more effective than organic links to induce click throughs

In case 3, only areas I, II and IV exist. In case 4, the four areas are present. The main
findings are:

- Both firms are more likely to bid when consumers are ad-lovers
- The more relevant firm (firm 1) is more likely to bid when sponsored links complement organic links
- The less relevant firm (firm 2) is more likely to be the only firm to bid when sponsored and organic links are substitute.

References


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