Anti-competitive compatibility: the effects of the elasticity of consumers' expectations and the shape of the network effects function*

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Abstract

I find that interconnection – standardization or compatibility – causes the market to be less competitive, and leads to an increase in the price firms charge for their product. Absent interconnection, firms compete for a consumer for two reasons. The first reason is to obtain revenue from selling the product to a consumer (as in the case without network effects). The second reason is that by expanding the network by one more consumer, the product becomes more attractive to all other consumers. Interconnection eliminates the second reason – when firms interconnect, they are no longer concerned with consumers following the crowd. I show that consumers and society might be worse off from interconnection, and that firms might interconnect when it is not socially optimal. Factors that make the [post-compatibility] price increase bigger are as follows: high product differentiation, consumer expectations which are highly sensitive to prices, and consumers putting a high value on small increases in network size at the equilibrium market shares. The results hold with asymmetric quality, only a few of the firms interconnecting, and when it is possible to expand the market through interconnection.

1 Introduction

A product has a positive network effect when a consumer purchasing it benefits from others purchasing the same product. Products which have positive network effects range from phones

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to game consoles.[1] Interconnection (or compatibility) between product A and product B means that a consumer who has purchased good A also enjoys the additional benefit when someone buys good B.

For example, some car companies share platforms – consumers buying a Nissan sedan enjoy the indirect benefit of having all the Renault service facilities at their disposal.[2] Instead of interconnection the reader can think of interoperability or setting a common standard.

Ever since Katz and Shapiro (1985) the prevalent opinion among economists has been that firms do not interconnect enough from the perspectives of consumer and social welfare.[3] As a result, most of the public policy in this field is aimed at encouraging firms to interconnect (see Gandal (2002) for a summary), with network effects-related issues becoming one of the prime examples of market failure.[4] The legal literature, too, encourages formation of standard setting organizations, basing its reasoning on the literature in economics – see Gates (1998) and Curran (2003).[5]

I show throughout the paper that demand becomes less elastic after interconnection, allowing firms to compete less aggressively, and to charge more. This means that prices might increase more than the benefits from a larger network, and that interconnection might be bad for consumers and for society. I show how three factors affect the magnitude of the price increase and the consumer welfare change: product differentiation, shape of the network effects function, and the sensitivity of consumers' expectations of how a price change affects firms' network sizes.

The contribution to the literature is two-fold: I examine the effects of the sensitivity of consumer expectations and the shape of the network effects function; and I show that the anti-competitive compatibility appears in other contexts than a Hotelling interval with symmetric firms. Besides the network interconnection setting, the results with consumer expectations and the shape of the externality function should hold up in other contexts, for example competition

[1] See Gandal (2008) for empirical studies of network effects. In case of phones or email consumers get direct externality of more people being on the network being better. In case of the game consoles consumers derive benefits from other people having the same console indirectly by getting more software in a bigger market. Also there is the direct benefit of being able to interact with other users online, and the more users there are, the better off each user is. See Clements (2004) for when direct and indirect network effects produce different qualitative results.

[2] Another example is video game platforms. Even though the hardware is incompatible (one cannot play a Sony PS3 disk on a Microsoft Xbox), Corts and Lederman (2009) show that availability of non-exclusive software acts as partial interconnection – users of a given game console derive more utility even if someone buys another game console. For some of the examples generally given in the network externalities literature, a model with two-sided platforms might be a better fit. I consider network effects to be somewhat of a reduced approach to two-sided markets, which lets the researcher focus on particular details, and this paper could have been presented within the two-sided setup. For recent papers examining compatibility in a two-sided setting, see Casadesus-Masanell and Ruiz-Aliseda (2009) who show compatible platforms deriving higher profits, and Miao (2009), in which the author examines the incentives of a monopolist to provide compatibility in a complementary market.

[3] Several authors has shown that interconnection in different contexts might be anti-competitive. See literature review in the end of this section for comparisons with the mix-and-match and telecommunications literatures.

[4] Some of the heavily cited example are Windows v. Apple OS, QWERTY v. Dvorak, and Beta v VHS. See Liebowitz and Margolis (1994) for one of the few articles holding the opposite view.

[5] A more complete study of standard setting would include bargaining and side payments between the parties, however this is outside of the scope of this paper. I conjecture that the qualitative results still hold even in a richer model with side payments.

with aftermarkets.[6]

The results of an after interconnection price increase and consumer welfare decrease hold if the products are sufficiently differentiated after interconnection. Low product differentiation after interconnection leads to the familiar Katz and Shapiro (1985) results: increase in consumer welfare after interconnection and firms not interconnecting enough from the perspective of social welfare.[7]

Literature on network effects has focused on the settings where the only differentiation between the products in the market is the number of consumers in each network. However, in the majority of the competitive network effects settings, there is a brand differentiation component as well. My goal in this paper is to point out the difference in the intuition and the qualitative results between differentiated and not differentiated network interconnection, and show how the aforementioned three factors affect the magnitude of the changes in welfare and in prices after the products become compatible.

Two of the most common settings for interconnection are electronics and the internet. Electronic products have natural lifecycles where prices decrease, and these effects might dominate any post interconnection price increase. Many internet services which have experienced interconnection recently, like instant messengers, are free for consumers and are either tied with some other product or are supported by advertising. Thus, keeping everything else equal, it is difficult to document a price increase or decrease post interconnection.[8]

Another highly visible sector where we see interconnection is the airline industry. Interconnection, or belonging to one alliance, is a major strategic decision for the airlines – recently, Delta was an making offer of up to a billion dollars to align with Japan Airlines. Meanwhile, American Airlines were making a similar offer to keep their alliance with Japan Airlines intact (New York Times 2009). Park et. al. (2001) find that prices increase after airlines enter into an alliance, when airlines share routes.[9]

Without interconnection, firms compete for a consumer for two reasons. The first is to increase revenue, as without network effects. The second is that capturing this consumer makes the product more attractive to all the other consumers. Interconnection eliminates the second reason – when firms interconnect, they are no longer concerned with consumers following the crowd. This allows firms to charge higher prices, and to not fear competitors' price cuts. The firms interconnect too much (from social and consumer welfare perspectives) when the marginal network effects are sufficiently high. This is not intuitive since steep network effects imply the social need of having one big network.

[6]See Cabral (2009) for a recent article on aftermarkets with externalities.

[7]Katz and Shapiro (1985) assume that the products are not differentiated.

[8]Brynjolfsson and Kemerer (1996) provide some indirect evidence. They examine pricing of spreadsheet software from 1987 to 1991, and find that 'a product [compatible with Lotus 1-2-3, the dominant software at the time] commands a price approximately 46% greater, all else being equal'. See Gandal (1995) for similar findings. Gandal (1995) also includes a model (in the appendix) of two firms competing, with one of them compatible with the dominant firm. That model describes the reality of having one player with a large installed base. The problem is that it does not allow for the leader to act strategically, and it does not allow for two firms compatible with the leader to compete on prices between themselves and a third incompatible firm.

[9]The same article goes deeper into what benefits consumers derive from being in a bigger alliance.

Sensitivity of consumers' expectations of the networks' sizes to a price change has a similar effect as the steepness of network effects. Absent interconnection, firms compete more aggressively if after a price decrease consumers expect that a large number of other consumers will switch networks. With interconnection, the firms do not have this incentive to compete anymore.[10]

In Katz and Shapiro's model consumers always benefit from interconnection.[11] The authors based their conclusion on the assumption that products are (post interconnection) homogeneous. In my model, when firms interconnect, consumers that already belong to the network derive the same additional utility when additional consumers purchase any product (join any network), but product differentiation remains after interconnection.[12] I impose assumptions on the model that do not let one of the firms take over the whole market – I am interested in examining the incentives to interconnect, and interconnection becomes unnecessary once there is just one firm left.[13]

Another difference between my model and Katz and Shapiro (1985) is that in the base case I assume a fully covered market. With this assumption, I can concentrate on the effects of interconnection on the current consumers. In one of the later sections, I show that the same conclusions are achieved in a not fully covered market. The ability to expand the market after interconnection mitigates the strategic effect: firms are less aggressive toward each other because they want to capture new consumers, and thus might end up charging lower prices.

I also examine asymmetric cases. If one of the firms has an early lead with an installed base of consumers, then the firms are more likely to interconnect. The leader doesn't want to have a price competition with other firms in the market, and interconnection results in higher prices. The follower benefits from having a larger network to offer its consumers, and from the leader not competing aggressively.

On the other hand, if one of the firms has a higher quality product, then the firms are not as eager to interconnect. The firms are already differentiated in two dimensions in this case, and thus interconnection is not as valuable as it would have been otherwise.

I start out with a non-localized random utility model of competition (Perloff and Salop (1985)), and derive the main results. I assume that consumers' utility is additive in network benefits, price, and random brand utility, and that consumers' network-size expectations increase if a firm decreases its price.

While the Perloff and Salop (1985) model is general on the demand side, a big drawback is that it is hard to deal with asymmetric firms or with an outside option. I deal with these issues in later sections by employing the Hotelling (1929) model. I use the Hotelling model for the case of asymmetric installed base or quality; and Hotelling with hinterlands to incorporate

[10]This seems to be the least testable result of the model, however there is a growing literature on measuring expectations, see Manski (2004), and following papers.

[11]I refer to firms interconnecting their products as simply firms interconnecting from now on.

[12]See conclusion for more on this assumption. I am looking at products where even after interconnection consumers perceive the products to be sufficiently differentiated on other dimensions, for example all the cameras using the SD-type memory card.

[13]The assumption is sufficiently high product differentiation, relative to the strength of network effects.

market expansion and outside option effects (the possibility of bringing in more consumers to the market mitigates the effect of the existing consumers becoming less elastic). The main results hold up under all of the specifications above. In the appendix, I use Salop (1979) circle to examine only some of the firms interconnecting, finding similar qualitative results.

The literature is generally concerned with two ways in which firms can interconnect. The first way is mutually agreeing on a common standard, with each firm paying some fixed cost to interconnect. The second way is a firm providing an adapter to let consumers (possibly imperfectly) enjoy the benefits of having the other network around, with each firm deciding how much interconnectivity to provide. I examine the first case of mutual agreement on a standard in this paper.[14]

Shy (2001) examines a duopoly model with network effects with two equally sized consumer segments, where each segment prefers one of the firms, and network effects are linear. The setup is similar to a switching cost model with network effects and to a Hotelling-type setup, so the result is a price increase with interconnection. However, the covered two-segment market with linear network effects assumption, and not explicitly accounting for expectation formation implies that his model is the polar opposite of Katz and Shapiro – consumers are always worse off after interconnection. The point that interconnection might hurt consumers in a Hotelling setup was made by Spulber in his (2008a) paper without a model, and in his (2008b) paper (see footnotes 79-82 in the paper for the model and solution). In the base model I extend the model to include N firms in the random utility Perloff and Salop (1985) setup, I explicitly deal with expectations formation, and different shapes of the network effect functions, and describe the intuition of the result. Then I extend the model to include vertical differentiation, installed bases, partial interconnection, and not fully covered market.

Katz and Shapiro (1986) show how interconnection might hurt consumers in a two-period model with a not differentiated Bertrand competition for homogeneous customers and inelastic demand (in each period one of the firms gets the whole market).[15] Compatibility reduces price competition in that model too, however only if the firms can do side payments, only in the first period, and for different reasons – the need of firm B to establish itself in period 1, so that the other firm does not have too much of an advantage in period 2 and compatibility breaking the

[14] Farrell and Saloner (1992) use the spatial setup to study a market where firms can produce converters for one-way compatibility with the other firm. They find that the availability of converters might be bad for society. While this also sounds like a result of too much interconnection being bad for consumers, this is not actually the case in their setup. The availability of converters is bad for the society only when without the converters consumers would have all joined the same network and achieved perfect standardization – i.e. there is still not enough interconnection in equilibrium, even with adapters. Garcia and Vergari (2008) also examine adapter interconnection with differentiation (vertical in their case). The authors find that with weak network effects market may achieve full compatibility (although with stronger network effects compatibility is underprovided). Moreover they find that a bigger firm might want to interconnect just as much as (or more than) a smaller firm, something that I show as well. Including adapters in my model would probably lead to something akin to endogenous travel cost models like Hendel and Neiva de Figueiredo (1997), at least in the linear version of my model.

[15] Assume one of the firms (say, A) has lower cost in the first period, and the other (say, B) in the second. Then if the products are incompatible, B might price below marginal cost in period 1, to get everyone in period 2 (since A cannot commit to pricing below cost in the final period). If the products are compatible, then B does not have the incentive to do that, and so first period consumers might suffer.

link between the two periods.

The mix-and-match literature assumes that there are no explicit network effects, but products ('systems') are made up of components, and the consumer has to buy all the components to get the utility from the system.[16] Matutes and Regibeau (1988) and Economides (1989) find that compatibility lets the firms charge higher prices in the mix-and-match literature. Chou and Shy (1996) use a similar model, with an endogenously determined amount of supporting services. They endogenously derive the effect of the number of other consumers buying a product on a given consumer, and find that compatibility makes the effect more negative. While mix-and-match literature is close to the network effects literature, the intuition for this result is different in their setup. In those papers (except for Chou and Shy), without compatibility, a price cut of a given size leads to an increased purchase of the whole system sold by the firm as opposed to a given component with compatibility, therefore price cuts are much more effective without compatibility.[17]

In the context of telecommunications, interconnection refers to firms allowing customers receive calls from the competitors' customers. The key difference between that setup and mine is that customers in my model simply enjoy having more customers buy a similar product, either directly or indirectly. There are no costs or foregone opportunities after two firms interconnect, and, even more importantly, no access charges. In the context of telecommunications, Armstrong (1998) and Laffont et.al.(1998) both show that firms can increase prices because of interconnection. However, the price increase is due solely to a reciprocal access fee between the firms – by raising the fee, the firms ensure that they charge high prices to customers. See Vogelsang (2003) for a review of the literature on access pricing in telecommunications.

There are several related works in which interconnection is not the main topic of the paper. Suleymanova and Wey (2009) examine Bertrand competition in a market with network effects and heterogeneous switching costs. For a range of parameters they find that social welfare might be lower with compatibility, however this is due to more consumers incurring switching costs. Grilo et.al. (2001) examine horizontally differentiated competition with network effects, and derive several benchmark results both for positive and negative network effects. Griva and Vettas (2004) examine both horizontally and vertically differentiated market with two firms, noting that, "generally, the presence of network effects may lead firms to compete more intensively and their profits to be lower," which is a claim of this paper as well, and I show it in more detail in one of the subsections below.

[16]See Economides (1996) for more on mix-and-match models and comparison with the network externality approach.

[17]Knittel and Stango (2009) examine the mix and match theory using a bank ATM dataset, and find that prices actually increase with incompatibility in that setting.

2 Base Model and Welfare Implications

2.1 Base Model

I adopt the setup of Perloff and Salop (1985), in which I include network effects as well as the option to interconnect.[18] In this section I consider symmetric equilibria only. I make the assumption that in a symmetric equilibrium no firm has an incentive to charge a price that is low enough to cause all consumers to switch to their product (a sufficient amount of product differentiation, relative to the strength of network effects, implies that this assumption is satisfied).

There are N symmetric firms in the market, with marginal costs normalized to 0. The firms can either all interconnect or not before consumers make their purchase decisions.[19] Each firm has to pay F to interconnect. After the interconnection decision, the firms simultaneously set prices.

There is a mass 1 of consumers. Each consumer has firm-specific preference, θ_i for firm i. For each consumer, the θs are independently and identically distributed, according to a p.d.f. $g(\bullet)$. If the firms are not interconnected, then the consumer expects to get a benefit of $v(E[Q_i])$, where $E[Q_i]$ is the expected number of consumers buying the same product, and $v(\bullet)$ is the network effect function. If the firms interconnect, then the consumers expect everyone to sign up, resulting in an externality benefit of $v(1)$. The externality benefit function, $v(\bullet)$, is increasing and differentiable. Thus a given consumer's utility from buying from firm i is:

$$u_i = \begin{cases} \beta\theta_i - p_i + v(E[Q_i]) & if\ the\ networks\ don't\ interconnect \\ \beta\theta_i - p_i + v(1) & if\ the\ networks\ interconnect, \end{cases} \tag{1}$$

where β is the strength of preferences and is similar to the familiar Hotelling's travel cost. Consumers decide on their expectations after the firms set prices. If a firm lowers its price, then the consumers expect a bigger network for that firm, and smaller networks for the rivals, or $\frac{\partial E[Q_i]}{\partial p_i} < 0$ and $\frac{\partial E[Q_j]}{\partial p_i} > 0$.

I assume a fulfilled expectations equilibrium is played, $E[Q_i] = Q_i$, and that the beliefs change symmetrically, in other words, if all the other firms are charging the same price p_j, then $\frac{\partial E[Q_i]}{\partial p_i} = -(N-1)\frac{\partial E[Q_j]}{\partial p_i}$ – if a firm lowers its price, all the symmetric competitors lose the same amount of consumers in expectation.[20] In the base model, there is no outside option – each consumer has to buy a product; I include the outside option in a later section.

Proposition 1 *If the networks do not interconnect, then the symmetric equilibrium market price is*

$$p^* = \frac{\beta}{M(N)} \frac{1}{1 - v'\left(\frac{1}{N}\right) \frac{N}{N-1} \frac{\partial(E[Q_i])}{\partial p_i}}, \tag{2}$$

[18] I could have used the Salop (1979) circle instead, or a Hotelling line with two firms, to get similar results.

[19] See the appendix examining only some firms interconnecting.

[20] I do not impose further assumptions on the rationality of consumer expectations with respect to quantity. For example, I do not require $\frac{\partial E[Q_i]}{\partial p_i}$ to be equal to $\frac{\partial Q_i^*}{\partial p_i}$. My results go through as long as consumers expect that an (own) price decrease results in an (own) larger network.

where $M(N) = N(N-1)\int G^{N-2}(\theta)g^2(\theta)d\theta$.

Proof. A consumer buys brand i if and only if $u_i > u_j$ for all $j \neq i$. Thus

$$\beta\theta_i + v(E(Q_i)) - p_i > \beta\theta_j + v(E(Q_j)) - p_j \tag{3a}$$

$$\theta_j < \theta_i + \frac{p_j - p_i + v(E(Q_i)) - v(E(Q_j))}{\beta}, \tag{3b}$$

for all $j \neq i$. Since θs are i.i.d.,

$$Q_i = \int \left[\prod_{j \neq i} G(\theta_i + \frac{p_j - p_i + v(E(Q_i)) - v(E(Q_j))}{\beta}) \right] g(\theta_i)d\theta_i. \tag{4}$$

The firm's profit is

$$\Pi_i = p_i Q_i, \tag{5}$$

and assuming that all firms $j \neq i$ choose the same price $\overline{p}$, the FOC is

$$Q_i - p_i \frac{N-1}{\beta}\left(1 - \frac{\partial\left(v(E(Q_i)) - v(E(\overline{Q}))\right)}{\partial p_i}\right) \times$$
$$\int G^{N-2}\left(\theta_i + \frac{\overline{p} - p_i + v(E(Q_i)) - v(E(\overline{Q}))}{\beta}\right) g\left(\theta_i + \frac{\overline{p} - p_i + v(E(Q_i)) - v(E(\overline{Q}))}{\beta}\right) g(\theta_i)d\theta_i = 0,$$

Since in a symmetric equilibrium $Q_i = \frac{1}{N}$, we get the result in the proposition. The second order conditions come from Perloff and Salop (1985), endnote 6. ■

Term M(N) is the same as in Perloff and Salop (1985) and represents a combination of the particular functional form of the probability distribution function of consumer valuations, and the number of firms in the market. If $\overline{G} = 1 - G$ is log-concave, then M(N) increases in N, and if there are more firms in the market, then the equilibrium price is lower.[21]

The price above differs from the standard Perloff and Salop price by the second term, which without network effects ($v' = 0$) reduces to 1. Since $\frac{\partial(E[Q_i])}{\partial p_i} < 0$, the price is lower than in the absence of network effects.

If the firms interconnect, then consumers derive the same network effect benefits ($v(1)$) regardless of which product others purchase. Instead of (3b), we get

$$\theta_j < \theta_i + \frac{p_j - p_i}{\beta}, \tag{6}$$

which is the same condition as in Perloff and Salop (1985).

Corollary 1 *In a symmetric equilibrium with interconnection firms charge*

$$p^* = \frac{\beta}{M(N)}. \tag{7}$$

[21] See Weyl (2009) and Gabaix et. al. (2009) for more detailed results about M(N) in Perloff and Salop (1985).

As discussed above, this price is strictly higher than the one absent interconnection. The difference between the two expressions is a product of the steepness of network effects ($v'\left(\frac{1}{N}\right)$) and the steepness of change in expected network sizes due to a price change ($\frac{\partial(E[Q_i])}{\partial p_i}$).

Either all firms interconnect or none do. Firms interconnect if, and only if, the additional profit brought about by interconnection is greater than the sunk cost (F) firms pay to interconnect. Recall that in this model I consider a symmetric equilibrium, and that no outside option is available. Therefore at issue is whether prices increase by a sufficient amount. For price increase to be relatively large, firms must be sufficiently differentiated. If firms are not sufficiently differentiated, the price increase is not large enough to cover the cost of interconnection, and the consumers are the main beneficiaries of interconnection.

Corollary 2 *Firms interconnect if and only if*

$$\frac{1}{N}\frac{\beta}{M(N)}\left(1-\frac{1}{1-v'\left(\frac{1}{N}\right)\frac{N}{N-1}\frac{\partial(E[Q_i])}{\partial p_i}}\right) > F. \tag{8}$$

When marginal network effects ($v'(\bullet)$) are high at the equilibrium market share, firms are able to gain more from interconnection. With interconnection, consumers' marginal benefit from one more consumer in the network is irrelevant (in a covered market). Absent interconnection, a firm's incentive to undercut its rivals depends on consumers' valuation of the benefit an additional consumer in the network provides. If consumers highly value each additional consumer who joins their network, then a firm has a strong incentive to undercut their rivals. The opposite is the case if consumers do not put a high value on an additional consumer joining the network.

Another benefit of interconnection for firms is that consumers' expectations of other consumers' behavior is irrelevant, eliminating any incentives that firms might have to undercut its competitors for this reason. Absent interconnection, a firm's incentive to undercut its rivals depends on consumers' anticipation of other consumers' behavior. If consumers believe that many other consumers will switch to a different product because of a price decrease, then a firm has strong incentives to undercut their rivals. The opposite is the case if consumers do not anticipate other consumers to switch to a different product due to a price decrease.

2.2 Welfare Implications

Interconnection affects consumers in two different ways - the price they pay for a product and the size of the network they are a part of. While price increases (hurting consumers), the network size grows (adding to consumers' utility). Specifically, the network effect increases by $v(1)-v\left(\frac{1}{N}\right)$, the difference between the full network and one of the N networks.

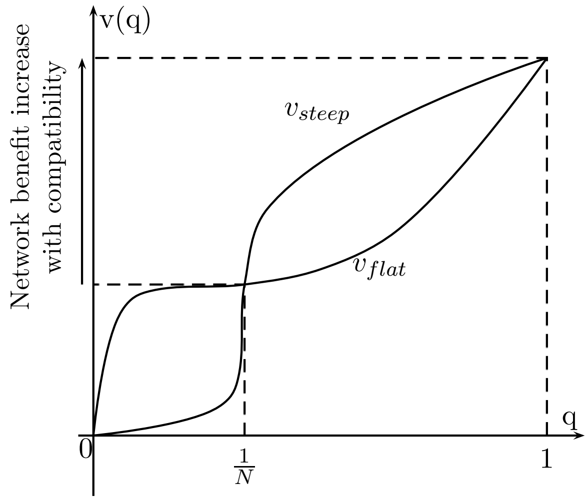

Figure 1: Switch to compatibility with v_{steep} increases prices by more than with v_{flat}.

Corollary 3 *With interconnection, consumer welfare decreases if and only if*

$$v(1) - v\left(\frac{1}{N}\right) < \frac{\beta}{M(N)}\left(1 - \frac{1}{1 - v'\left(\frac{1}{N}\right)\frac{N}{N-1}\frac{\partial(E[Q_i])}{\partial p_i}}\right). \tag{9}$$

The above inequality can be satisfied since on the right hand side we simply have a v' (and not a v). From the right hand side we can see that firms interconnect when consumers would prefer them not to exactly when consumers' put a high value on an additional consumer joining the network. Sensitive consumer expectations with respect to prices also lead to a decrease in consumer welfare.

Consider Figure 1. Two network effect functions, labeled as v_{steep} and v_{flat}, give consumers the same benefits with and without compatibility. Recall from above that the increase in consumer network benefits is $v(1) - v\left(\frac{1}{N}\right)$. Let us examine what happens if firms decide to make their products compatible. On one hand, in looking at v_{steep} at the equilibrium market share $\left(\frac{1}{N}\right)$, the steepness of the curve indicates that the price in the market increases sharply. That is, firms compete aggressively for consumers without compatibility because the benefit from each additional consumer who joins the network is very large. On the other hand, in looking at v_{flat}, the flatness of the curve indicates that the price in the market remains almost the same. Firms barely compete for consumers without compatibility because the benefit from an additional consumer to the entire network utility is minimal. From these two observations, firms would prefer to make their products compatible when the network effects look like v_{steep}, but not when they look like v_{flat}. There are two effects on consumer welfare, a positive and a negative. The network becomes bigger (by $v(1) - v\left(\frac{1}{N}\right)$). But the price increases as well. In this example, the utility from a bigger network is the same from both functions, but the price increase is much higher with v_{steep}.

Some products' network effects function is increasing sharply (in the number of consumers) up to some point, and then starts to flatten out, or even decrease in some cases. This turning point can either come before $\frac{1}{N}$ (the number of users in one of the N networks) or after. If it comes before, then firms don't have a good incentive to interconnect, since the network effects are flat. If this point comes after, then the slope at the equilibrium absent interconnection $\left(v'\left(\frac{1}{N}\right)\right)$ is relatively large, so that the firms interconnect, however the difference in consumer utility between the big network and one of the smaller networks $\left(v(1) - v\left(\frac{1}{N}\right)\right)$ is much smaller since the whole network is an overkill as far as the network size is concerned.

The sensitivity of consumers' expectations of the networks' sizes after a price change, has a similar effect on price as the steepness of network effects. If by lowering price a firm convinces consumers that a large number of other consumers will switch networks, then absent interconnection firms compete more aggressively. With interconnection, the firms do not have this incentive to compete anymore. If the consumers hold beliefs that no matter what prices are charged, all the firms will end up with the same market shares, then the firms have exactly the same incentives for lowering prices absent interconnection as they do with interconnection, and thus the price does not increase with compatibility.

The social welfare effect of interconnection is a combination of the increase in the network externality benefit to consumer and the sunk cost paid by the firms. The difference in price (absent interconnection and with interconnection) does not influence social welfare since this is simply a transfer from consumers to firms. Nevertheless, it is important to note that firms interconnect only if the price difference is big enough to cover the sunk cost of interconnection. In certain cases, however, even if the price difference is large enough to compensate for the (sunk) cost, the network externalities might be too small to justify interconnection from the social perspective. In this case, consumers are also worse off since they are paying for interconnection costs.

In Katz and Shapiro (1985) interconnection always increases consumer welfare. The result of consumer welfare possibly falling because of interconnection immediately leads to reexamination of Katz and Shapiro's central result that firms do not interconnect enough from the society's point of view.[22]

Corollary 4 *Firms interconnect when it is not socially optimal if and only if*

$$\frac{\beta}{M(N)}\left(1-\frac{1}{1-v'\left(\frac{1}{N}\right)\frac{N}{N-1}\frac{\partial(E[Q_i])}{\partial p_i}}\right) > NF > v(1)-v\left(\frac{1}{N}\right). \tag{10}$$

Consumer welfare always decreases in this case.

The condition above is similar to the one under which consumers suffer from firms interconnecting. An additional constraint that I add is that the sunk cost of interconnection must

[22] $\Delta SW = \Delta CW + \Delta\Pi$. In Katz and Shapiro $\Delta CW > 0$, and therefore $\Delta\Pi < \Delta SW$, and in particular it is possible that $\Delta\Pi < F < \Delta SW$, where F is the cost of interconnection, and in this case it it socially optimal to interconnect, but the firms do not want to.

be higher than the difference in the network effects. Increasing the number of firms does not necessarily solve the problem. When N is larger, the inequality on the left side becomes more difficult to satisfy.[23] For this reason, firms do not interconnect as much. The right side of the inequality, however, becomes easier to satisfy and shows that when firms do interconnect, it is more likely that consumers will be worse off.

Higher intensity of consumer preferences (or product differentiation), β, is always beneficial to the firms. With a sufficiently low preference intensity, we get Katz and Shapiro (1985) results – consumer welfare always increases after interconnection and sometimes firms do not interconnect when it is socially optimal to. Higher preference intensity increases the post and pre interconnection profit difference, and encourages firms to interconnect more, to the point where consumer welfare decreases after interconnection and firms interconnect when it is not socially optimal to.

I have talked about the base case to make the arguments easier. It is possible to get similar results in all the other cases covered below. The same logic goes through – interconnection can hurt consumers and the society overall. Below I only present proofs for price increases; all the consumer welfare decrease proofs are in the appendix. I also do not examine the effects of the shape of network effects function and the sensitivity of consumer expectations in the sections below. I present the extensions for asymmetric qualities, asymmetric installed bases, and an uncovered market below, and I present the extension to three firms with only two firms interconnecting at a time in the appendix.

3 Effects of Asymmetry on Interconnection Decision

3.1 Different Quality - Vertical Differentiation Together With Horizontal

The ongoing assumption in the paper so far had been that the products are just horizontally differentiated. Another issue to examine is what happens if there are *also* quality differences between the products. For most of the markets there is also a vertical component of differentiation, a consumer who gets the same random shock for two products with equal prices would still prefer one of them.

There are two firms, located at 0 and 1 of the Hotelling interval [0,1], with consumers distributed uniformly with mass 1. Assume that both the travel cost and the network externality functions are linear, with coefficients of t and j, and $t > j$.

I assume consumers form their network size expectations by finding the marginal consumer, just like in Hotelling. Suppose adjacent networks 1 and 2 are not interconnected, and there is distance 1 between them. Then the marginal consumer between the two networks is located at the distance x from firm 1, such that

$$jx - p_1 - tx = j(1-x) - p_2 - t(1-x), \tag{11}$$

[23] With reasonable assumptions on the p.d.f.: $1 - G$ must be log-concave.

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Also, without loss of generality, assume that all the consumers derive an additional utility of $Q > 0$ from consuming firm 0's product. High Q means that firm 0 is much better than firm 1.

Proposition 2 *With quality differences, both firms are less willing to interconnect. If the firms do interconnect, prices increase and market concentration decreases.*

Proof. The indifferent consumer is at x which is defined by

$$2x = \frac{p_2 - p_1}{t - j} + \frac{Q}{t - j} + 1, \tag{12}$$

with subscript 2 denoting the firm at 1, subscript 1 denoting the firm at 0 (the one with the superior product). The only difference from the familiar formula is the fraction with Q, and as one would expect, setting $Q = 0$ gives us back the standard formula. From 12 and the first order conditions for each firm we can get the following system describing the optimal prices:

$$p_1 = \frac{p_2}{2} + \frac{Q}{2} + (t - j)\frac{1}{2} \tag{13a}$$

$$p_2 = \frac{p_1}{2} - \frac{Q}{2} + (t - j)\frac{1}{2} \tag{13b}$$

The difference is the vertical Q term, which makes the firms asymmetrical, as expected. Solving the system, we get

$$p_1^* = (t - j) + \frac{Q}{3} \tag{14a}$$

$$p_2^* = (t - j) - \frac{Q}{3}, \tag{14b}$$

and from there, $x^* = \frac{1}{2} + \frac{Q}{6(t-j)}$. The prices and the indifferent consumer above give us the profits of the firms before interconnection. Firm 1's profit is xp_1 and firm 2's profit is $(1 - x)p_2$. After interconnection the profits are the before interconnection profits with $j = 0$. The difference between pre- and post interconnection profits for *both* firms is

$$\Pi_\Delta = \frac{j}{2} - \frac{j}{18t(t - j)} \times Q^2, \tag{15}$$

note that Q enters with a negative sign - for both firms the higher the quality difference is, the less they are willing to spend on interconnection. ■

Quality differences introduce another dimension of differentiation into the setup, allowing the firms to derive higher profits than otherwise. Interconnection still means that firms do not compete for consumers as aggressively as before, however this is not as valuable since the firms are already differentiated in two dimensions, as firms do not compete on prices as hard to begin with. Both firms are less willing to interconnect in this setup. For both of them the extra dimension of differentiation brings in more profits, and the interconnection is not as enticing.

The result is likely to generalize to more dimensions than two - the more dimensions the firms are differentiated on, the less incentive they have to interconnect.

3.2 Previous Installed Base – Competing With Incumbent

We have looked at the quality asymmetry in the previous subsection. Now, assume that the quality of the products is the same, but one of the firms is a new entrant, with the other already having some consumers captured.

There are two firms, located at 0 and 1 of the Hotelling interval $[0, 1]$, with consumers distributed uniformly. As in the previous section, assume that both the travel cost and the network externality functions are linear, with coefficients of t and j, and $t > 2j$. Consumers form their expectations as in (11). Also, without loss of generality, assume that the firm at 0 already has A consumers captured from the previous periods. Suppose that these A consumers have infinitely high switching costs, and the firm cannot charge different prices to the old and to the new consumers.[24]

Proposition 3 *The bigger the difference in installed base is, the more the firms want to interconnect. If the firms do interconnect, prices increase and market concentration decreases.*

Proof. The proof is similar to the one for the previous proposition, and therefore relegated to the appendix. ■

Here we have the opposite result from the quality competition. The reason is that while the firms were differentiated in quality in the example before, there were no captured consumers. In this example, there is a captured installed base at the incumbent's disposal. This base is not under a threat from the entrant, and therefore the bigger the installed base is, the more the incumbent is interested in higher prices which come after interconnection. The entrant is not going to get the installed base one way or another, but with the incumbent becoming softer as the installed base grows, the entrant has more market share and a higher price charged to gain as the two firms interconnect.

Chen et. al. (2009) examine whether firms still want to stay interconnected after some random shocks leave one of the firms with a larger installed base. In my setting, the previous proposition shows that this is indeed the case here, as usual subject to the qualification that the bigger firm will not be able to takeover the whole market. However, the previous subsection showed that if the firms might get some random quality (or at least perceived quality) shocks, that might change their decision, especially if some of the interconnection cost is not sunk.

4 Not Covered Market

In format (or standard) wars many consumers sit out and wait for one of the standards to emerge as the winner, or for the firms to agree on a common standard. For example, some

[24] If the captured consumers are just there, and do not have to pay anything, but contribute to network effects anyway, then we are back in the previous subsection. If consumers have finite switching costs, the comparative statics of the section might change. However, see Suleymanova and Wey (2008).

of the consumers would have been in the market for a high definition DVD equipment much sooner had there been an established standard from the beginning. Likewise, one of the reasons that the consumer welfare always increases in Katz and Shapiro (1985) is that total demand is greater after interconnection, which is not the case in any of the previous sections.

I tweak the model from above to address this issue. I use the Hotelling with hinterlands model.[25] Two firms, 1 and 2, are located at -1 and 1. Consumers are distributed uniformly, with mass 1 along the an infinite line. As in the previous sections, assume that both the travel cost and the network externality functions are linear, with coefficients of t and j, and $t > j$. Consumers form their expectations as in (11). Also, consumers derive a fixed benefit of R from buying a product.

This results in three submarkets. One of the submarkets is the standard Hotelling competition between firm 1 and firm 2, and this submarket consists of all the consumers in $[-1, 1]$. I call this submarket 12 from now on. The other two submarkets are standard Hotelling monopoly setups, and consist of the consumers with $x < -1$ for firm 1's monopoly submarket (call this submarket 10); and of the $x > 1$ consumers for firm 2's monopoly submarket (call this submarket 20).

I chose to work with an infinite line instead of a bigger interval for the overall market because I am not interested now in what happens if the market is covered, since I have examined this case in detail in the sections above. With an infinite line there is always additional demand if the firms interconnect. Note that the line instead of an interval assumption stacks the model in favor of the result of always too little interconnection. The firms have a much lower incentive of increasing prices, since there is an incentive of lowering prices to attract more consumers from the monopoly submarkets. Moreover, with interconnection the firms play a non-cooperative game in the small submarket 12 competing for consumers, however they are interested in cooperating on the rest of the line, since more consumers for my rival means that the value of my product increases for my monopoly market and I can get more revenue, all of which would imply strong incentives to keep prices down post interconnection.

Proposition 4 *With sufficiently high product differentiation or sufficiently low network effects, the prices increase with interconnection and consumer welfare decreases, resulting in the possibility of too much interconnection.*

Proof. See appendix. ■

Even with the model stacked against a decrease in consumer welfare, I still get the result of too much interconnection with sufficiently high product differentiation. The possibility of market expansion mitigates the results of an automatic post interconnection price increase, but high enough product differentiation still delivers a result similar to the ones from previous sections.

[25] In what follows, I could instead use a slightly modified version of the spokes model of Chen and Riordan (2007) with two firms and three spokes. That setup could be extended easier to more firms, and is non-localized.

5 Conclusion

This paper is a complement to Katz and Shapiro (1985). They examine homogeneous products. I examine differentiated products. In their case, consumer welfare always increases, and firms do not interconnect enough. Katz and Shapiro argue that consumer welfare always increases, and that firms should interconnect more. I, on the contrary, reach the conclusion that consumer welfare might decrease, and firms should interconnect less. The intuition behind my result is that interconnection eliminates the extra incentive of firms to compete for consumers to make their network larger.

Sufficient product differentiation is key for this result – it ensures that with interconnection firms charge higher prices. The possibility of market expansion mitigates the effects of product differentiation, but does not qualitative results still hold. It is up to the decision makers and empirical researchers to figure out which intuition and which model – with homogeneous or with sufficiently differentiated products – better suits a given industry.

The established view on interconnection is that it benefits consumers, but does not necessarily benefits firms; the market expands, yet the mark-ups diminish. One way to explain that was through the intuition of Katz and Shapiro (1985); the other way is through thinking about standardization rather than interconnection. With standardization, products not only become compatible, but also start sharing some characteristics, and therefore become less differentiated:

> while a more extensive standard leads to [...] stronger network externalities, it also can reduce the ability of each supplier to differentiate its products, thereby intensifying price competition.[26]

If firms' decision is not just about compatibility, but about sharing some product characteristics as well, then product differentiation indeed decreases after interconnection. This view can be reconciled with mine. The vast majority of product markets has several dimensions of differentiation. Standardization brings the products closer together on some of the dimensions, by setting common standards. However, it will also make the firms less aggressive on price competition, since bringing in one more consumer does not mean anything for the other consumers. The question for the firms is which effect dominates.

Firms interconnect if prices increase sufficiently after interconnection. The factors that make the price increase bigger are as follows: high product differentiation, consumer expectations which are highly sensitive to prices, consumers putting a high value on small increases in network sizes at the current market shares, installed base differences, and the market being close to full coverage. Consumers are worse off if the price increase is not compensated for by the benefits provided by a larger network. This is a more serious problem for consumers buying products whose network effects have some sort of a saturation point after which additional members in the network are not nearly as valuable, and absent interconnection the networks are close in size to this saturation point.

[26] Shapiro and Varian (1999), p.231.

My findings suggest that we should reevaluate our suggestions on interconnection to both managers and policy makers. For the managers, if the products are still differentiated after interconnection, or one of the firms is enjoying an installed base advantage, or consumers highly value the next person joining the network, or if the consumers' market share expectations are particularly sensitive to prices, then it makes sense to interconnect even without market expansion effects. For the policy makers, standardization is not always welfare improving, especially when the network effects are particularly strong only up to some level of enrollment or consumers' expectations are really sensitive to price changes.

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Appendix

Partial and Sequential Interconnection

Some industries are fully interconnected, some are fully not interconnected and in some industries one company is interconnected with another, but not with a third one. An example would be the airline miles points networks – United is interconnected with Lufthansa, but not with American Airlines. Another one would be instant messengers, where Yahoo! Instant Messenger became interconnected with Google's after a recent deal, while not interconnecting with others. To focus on the interesting issues, and to keep the model as tractable as possible, there are three firms in the market. The market is a Salop's circle, where unit demand customers are distributed uniformly with mass 1, and have linear transportation cost t, and linear network benefit j. I assume that $t > 2j$ and that the market is covered (each consumer buys a unit). The three firms are located at equal distances from each other. A consumer located d_k from firm k derives the following utility from buying that firm's product:

$$u_i = jE(Q_K) - p_k - td_k, \tag{16}$$

where $E(Q_K)$ is the consumer's expectation about the firm's network size. I assume consumers form their network size expectations by finding the marginal consumer, just like in Hotelling. Suppose adjacent networks 1 and 2 are not interconnected, there is distance M between them. Then the marginal consumer between the two networks is located at the distance x from firm 1, such that

$$j[(E(Q_1^{other}) + x] - p_1 - tx = j[(E(Q_2^{other}) + M - x] - p_2 - t(M - x), \tag{17}$$

where the other expectations are what consumers expect to happen in the other submarket of the firm or with any firms that are interconnected with the firm. This is the standard Hotelling way of determining the market share, with the network effects thrown in.
I assume that only two out of the three firms are interconnected. The complication is that each of the interconnected firms has two different submarkets. In one of the submarkets the firm is competing with another firm it is interconnected with. In the other the firm is competing with the outside firm. The competition is much more intense on the side of the outside firm, however the firm cannot charge different prices.

Proposition 5 *The interconnected firms charge*

$$p_{interconnected} = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} t(5t - 6j), \tag{18}$$

and the stand-alone firm charges

$$p_{alone} = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} \left(5t^2 - 10tj + 3j^2\right). \tag{19}$$

Proof. Two of the firms are interconnected. I drop the subscript for one of the interconnected firms, denote the other by subscript *i2*, and the stand-alone firm by *alone*. Since the two interconnected firms are symmetric, the proof of the following proposition only involves solving a system of two equations with two variables[27].

Fix an interconnected firm (The Firm from now on - the one without the subscript). The Firm charges one price, but has two different submarkets, on either side of The Firm. The Firm and i2 are interconnected, but they still fight for consumers between them – with x consumers in the arc between the two interconnected firms buying from The Firm, and $\frac{1}{3} - x$ buying from the other firm. Similarly, x_{a1} consumers buy from The Firm in the arc between The Firm and the stand-alone firm; and x_{a2} consumers buying from the interconnected firm.

The demand in the first submarket, where the competition is the other interconnected firm, is the standard Salop/Hotelling demand, since from the previous sections we know that interconnected firms compete like standard product differentiated firms.

$$x = \frac{1}{6} + \frac{p_{i2} - p}{2t}, \tag{20}$$

where p_{i2} is the price of the other interconnected firm, and $\frac{1}{6}$ comes from the fact that there are three submarkets overall (each of size $\frac{1}{3}$), and if the firms are exactly the same, they split the submarket equally. The Firm competes with the stand-alone firm in the other submarket, and here we need to consider that the firms might potentially have different sized networks, and add the size of the other interconnected firm's network. Therefore, with utility from buying The Firm on the left hand side, and the utility from buying from the stand-alone firm on the right hand side, we get:

$$-p - tx_{a1} + j(x_{a1} + \frac{1}{3} + x_{a2}) = -p_{alone} - t(\frac{1}{3} - x_{a1}) + j(\frac{2}{3} - x_{a1} - x_{a2}). \tag{21}$$

The term by j on the left hand side is the size of the interconnected network - what The Firm takes from the stand-alone firm, the arc between the interconnected firms, and what the other interconnected firm takes from the stand-alone firm. The similar expression on the other side is just the remainder (1 minus the expression on the left hand side). From this equation we get

$$x_{a1} = \frac{1}{6} + \frac{p_{alone} - p}{2(t-j)} + \frac{jx_{a2}}{t-j}. \tag{22}$$

The demands for The Firm and the stand-alone firm respectively are $D = x + x_{a1}$ and $D_{alone} = \frac{2}{3} - x_{a1} - x_{a2}$. Differentiating profit of The Firm with respect to own price, and substituting from previous equations we get

$$\frac{\partial \Pi}{\partial p} = \frac{1}{3} + \frac{p_{i2} - 2p}{2t} + \frac{p_{alone} - 2p}{2(t-j)} + \frac{jx_{a2}}{t-j}. \tag{23}$$

[27] Equations are the first order conditions of an interconnected firm and the stand alone firm, and the variables are the respective prices.

It is clear that the second order conditions are satisfied. We do not know x_{a2}, however in equilibrium the two interconnected firms are symmetric, and therefore $x_{a2} = x_{a1}$, and $p_{i2} = p$. Substituting that into 22 we get

$$x_{a2} = \frac{p_{alone} - p}{2(t-j)} + \frac{t-j}{6(t-2j)}. \tag{24}$$

Then the first order condition for The Firm (plugging the right hand side above into 23 and making it equal to 0) gives us

$$t(t-j)(2t-3j) - 3p(3t^2 - 6jt + 2j^2) + 3p_{alone}t(t-j) = 0. \tag{25}$$

The FOC from the stand-alone firm gives

$$p_{alone} = \frac{t-j}{3(2t-3j)}(t - 3j + 3p). \tag{26}$$

This with equation 25 gives a system of two equations with two variables. The answers are in the proposition statement. ■
Since $t > j$, a simple corollary follows.

Corollary 5 *In equilibrium, the stand-alone firm*

- *charges less than the interconnected firms* ($p_{alone} < p_{interconnected}$),
- *sells less quantity than either of the interconnected firms* ($D_{alone} < \frac{1}{3} < D_{interconnected}$),
- *derives less profits than either of the interconnected firms* ($\Pi_{alone} < \Pi_{interconnected}$).

In the standard Cournot model cooperation between two of the many firms leads to reduced output on the part of the cooperating firms, and therefore more profits for the not cooperating firms. The similar exercise is much harder to reproduce in a localized competition model, since one needs to take into account the fact that all the other firms are not symmetric any more – the firms closer to the cooperating firms are different than the firms farther away[28].
The results above show that even partial network interconnection makes the competition softer. While it is not a merger, it is a form of cooperation, and one would expect cooperating firms to benefit at the expense of the other firm[29].

Corollary 6 *If all the firms interconnect, prices are even higher than with only two firms interconnecting.*

[28] See Giraud-Heraud (2003) for an elegant solution of the case where cooperating firms are neighbors, with a system of N equations. Another way to model this would be to use the spokes model of Chen and Riordan (2007), as it would cut down on the number of equations in the system, since the interconnected firms would be symmetric, and so would be the not interconnected firms.

[29] Again, because we are dealing with the Hotelling setup here. In Cournot mergers the firms that do not merge benefit more.

Proof. From the previous proposition we have to show that $p_{interconnected} < \frac{t}{3}$, where the right hand side of the inequality is the familiar Salop price with three firms. The price of the two interconnected firms already contains $\frac{t}{3}$, and one can see that the prices with all the firms interconnecting are higher as long as $2t > 3j$, which is satisfied because of the $t > 2j$ assumption. ■

As expected, the prices in the market increase even more if the third firm joins the network. From the previous corollary one can see that if this happens, the original members of the wide network lose market share. Therefore, for the first two firms to let the third firm join in, the price increase must be dramatic enough to cover not only for the costs of interconnection, but also for the market share decrease. Sequential (versus simultaneous) nature of interconnection does not influence the main result – firms charge higher prices with interconnection.

Other Proofs

Proof of Proposition 3 **Proof.** I denote the firm with an installed base by subscript a, and the other firm by subscript b. As before, we have to see where is the indifferent consumer located, and she is located at x, where x satisfies the following equation

$$2x = \frac{p_b - p_a}{t - j} + \frac{Aj}{t - j} + 1. \tag{27}$$

Therefore, bigger firm's demand is $A + x$, and the smaller firm's demand is $1 - x$, where x is defined above. The profits are then, respectively, $(A + x)p_a$ and $(1 - x)p_b$. From this we get the following first order conditions:

$$2(A + x) - \frac{p_a}{t - j} = 0, \tag{28a}$$

$$2(1 - x) - \frac{p_b}{t - j} = 0. \tag{28b}$$

Since $t > 2j$, the second order conditions are satisfied for both firms, and we just have to solve a system of three equations and three variables, with the third one being 27. Skipping (quite a few) steps, we have the following results:

$$p_a^* = \frac{4}{3}At + (t - j) - Aj, \tag{29a}$$

$$p_b^* = \frac{2}{3}At + (t - j) - Aj, \tag{29b}$$

$$x^* = \frac{1}{2} - \frac{A}{3} + \frac{Aj}{6(t - j)}. \tag{29c}$$

Since x^* increases monotonically with j, and both prices decrease monotonically with j, we get the second part of the proposition (as before, the prices and market shares are going to be determined by the same equations with $j = 0$). The less trivial part are the incentives to interconnect. After plugging in profits after interconnection and subtracting the profits from before, we get the following differences in profits:

$$\Pi_a^{\Delta} = \Pi_b^{\Delta} = Aj + \frac{j}{2} + \frac{jA^2}{2} - \frac{tjA^2}{18(t-j)}. \tag{30}$$

Since $t > 2j$, the expression above increases in A - the bigger the installed base is, the more the firms want to interconnect, giving us the first part of the proposition. ■

Proof of the Not Fully Covered Market Proposition (Proposition 4) **Proof.** While the logic of the proofs stays roughly the same, the algebra becomes much more involved. First, I find the equilibrium prices without interconnection. For that I need to find demand of the firms in each of the submarkets. The competitive submarket (12) can be characterized by the following, where x_{12} is how far away from firm 1 the consumer indifferent between buying from firm 1 and from firm 2 is located.

$$-p_1 - tx_{12} + j(x_{12} + x_{10}) = -p_2 - t(2 - x_{12}) + j(2 - x_{12} + x_{20}), \tag{31}$$

where p's are the prices, x_{10} and x_{20} are consumers on the line who, respectively, are indifferent between buying firm 1's product and not buying at all and buying firm 2's product and not buying at all. We have to divide the demand by 2 for network effects, since only half of the consumers before (for example) x_{12} are in the competitive submarket, the rest are in one of the monopoly submarkets. Similarly, we can characterize the other two x's:

$$R - p_1 - tx_{10} + j(x_{12} + x_{10}) = 0, \tag{32a}$$

$$R - p_2 - tx_{20} + j(2 - x_{12} + x_{20}) = 0. \tag{32b}$$

From the definitions of x's ((31),(32a), and (32b)), we have a system of three equations with three variables (the x's). Solving the system, we get:

$$x_{12} = 1 + \frac{p_2 - p_1}{2(t-2j)}, \tag{33a}$$

$$x_{10} = \frac{R+j}{t-j} + \frac{3jp_1 + jp_2 - 2tp_1}{2(t-2j)(t-j)}. \tag{33b}$$

The profit function of firm 1 is

$$\Pi_1 = p_1(x_{10} + x_{12}), \tag{34}$$

differentiating with respect to p_1, we get:

$$\frac{\partial \Pi_1}{\partial p_1} = 1 + \frac{p_2 - 2p_1}{2(t-2j)} + \frac{R+j}{t-j} + \frac{6jp_1 + jp_2 - 4tp_1}{2(t-j)(t-2j)}, \tag{35}$$

The second order condition holds, so setting the expression above to zero and invoking the symmetry conditions we get

$$p^* = \frac{2(t-2j)(R+t)}{5t-8j}. \tag{36}$$

Now we need to find the equilibrium prices with interconnection. It used to be that the intensity of network effects would not matter with interconnection. This is not the case now. While it will not matter (directly) for the competitive submarket, j directly effects the monopoly submarkets, since the higher the strength of the network effects, and the bigger the network, the more consumers prefer to purchase from their monopolist than not to purchase at all. Therefore the new equations which define the x's are going to be the same for the monopoly submarkets (x_{10} and x_{20}), and the previous equation (31) with j set to 0 for the competitive submarket (x_{12}). Solving this new system we get:

$$x_{12} = 1 + \frac{p_2 - p_1}{2t}, \tag{37a}$$

$$x_{10} = \frac{R + 2j}{t - 2j} - \frac{p_1}{t - 2j} + \frac{p_1 - p_2}{2t(t - 2j)}. \tag{37b}$$

Given the new values, and differentiating the same profit function, we get

$$\frac{\partial \Pi_1}{\partial p_1} = 1 + \frac{p_2 - 2p_1}{2t} + \frac{R + 2j}{t - 2j} - \frac{2p_1}{t - 2j} + \frac{2p_1 - p_2}{2t(t - 2j)}, \tag{38a}$$

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} = -\frac{1}{t} - \frac{2}{t - 2j} + \frac{1}{t(t - 2j)}. \tag{38b}$$

The second order condition is satisfied iff $3t > 2j+1$ and I assume it for the rest of the discussion. Involving the usual symmetry conditions the FOC gives us

$$p^* = \frac{2t(R + t)}{5t - 2j - 1}. \tag{39}$$

Prices after interconnection are higher for sufficiently high product differentiation (t). ■

Proofs of the possibility of consumer welfare decreasing with compatibility under different scenarios

1. Partial interconnection This case is probably the most tedious one. I will show that the change in profits of two interconnecting firms might be bigger than the change in social welfare. The change is from none of the firms being interconnected to two out of three being interconnected.

From the proof of the proposition 5:

$$p_{interconnected} = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} t(5t - 6j), \tag{40}$$

and the stand-alone firm charges

$$p_{alone} = \frac{t - j}{3(5t^2 - 9tj + 3j^2)} \left(5t^2 - 10tj + 3j^2\right). \tag{41}$$

The tedious part is that there are consumers who had switched from one firm to another. There are

$$K = \frac{p_{alone} - p_{intercon}}{t-j} + \frac{t-j}{3(t-2j)} - \frac{1}{3} = j\frac{t^2 - tj + 3j^2}{3(5t^2 - 9tj + 3j^2)(t-2j)} \tag{42}$$

such consumers. K is positive.

The consumers who were with the two now interconnected firms now have bigger networks. The networks are bigger by $\frac{1}{3} + K$ – the other firm's initial third of consumers joined, and so did the K consumers who left the other network. The K consumers who left their network experience the same change – all the networks started out being symmetric. Therefore there are $\frac{2}{3} + K$ consumers whose networks got bigger by $\frac{1}{3} + K$. The consumers who are left with the not-interconnected firms suffered because of the interconnection of the other two firms, at least in network size. There are $\frac{1}{3} - K$ of them left, and their network went down by K.

Another thing that has happened is that consumers who had switched now have to travel to another firm. Their additional travel cost (over all consumers) is $2\int_0^{\frac{K}{2}} 2xdx$. The total welfare change is then (not accounting for the fixed costs of interconnection):

$$\Delta SW = j\left((\frac{2}{3} + K)(\frac{1}{3} + K) - K(\frac{1}{3} - K)\right) - 2\int_0^{\frac{K}{2}} 2xdx. \tag{43}$$

The profit difference for the two now interconnected firms is a bit easier to compute. The consumers who were already with the firms (two thirds of the market) pay $p_{intercon} - \frac{t-j}{3}$ more, where the second term is the equilibrium price in a Salop circle with three not interconnected firms. The K new consumers pay the full price more. Thus, the difference in profits not accounting for the fixed costs of interconnection is:

$$\Delta\Pi = \frac{2}{3}\frac{j(t-j)^2}{5t^2 - 10tj + 3j^2} + K\frac{t-j}{3(5t^2 - 9tj + 3j^2)}t(5t - 6j). \tag{44}$$

It is possible, but tedious, to verify that for a set of parameters $\Pi > 0$, $\Delta\Pi > \Delta SW$, and $t > 2j$, for example t=.15 and j=.05.

2. Asymmetric quality or installed bases I use M to look at asymmetric quality, but if one substitutes (A+1) for M, the same result comes about for asymmetric installed bases.

The prices increase by Mj for every consumer. The network increases by less than M (the whole market) for every consumer, since each of the firms had a positive market share. Thus the prices increase by more than the network effects.

3. Uncovered market The change in consumer welfare (consumer welfare with interconnection less the consumer welfare without interconnection) is (with superscript i denoting interconnection, and superscript n denoting no interconnection):

$$CW^i - CW^n = \left[p^i - p^n + 2j(x^i_{10} - x^n_{10})\right](1 + 2x^n_{10}) + (x^i_{10} - x^n_{10})\left[R + j(1 + 2x^i_{10}) - p^i\right] - 2t\int_{x^n_{10}}^{x^i_{10}} zdz, \tag{45}$$

where all the x's and p's are the equilibrium values. The first term is how the welfare of existing consumers changed. The second term is the welfare of the new consumers, who did not buy before interconnection. The third term is the travel cost that the new consumers have to pay. The sum of the last two terms is positive, otherwise the new consumers would not buy. The first term is negative as long as the prices increase with interconnection. Overall, apriori it is not clear what happens. Note however that all the positive terms decrease with t, and the travel cost increases with t. Since all of our restrictions on the shapes of the travel cost and network effect functions involved t being sufficiently high, this means that a sufficiently large t still satisfies those restrictions, and makes the expression above negative. By previous arguments this implies that firms might interconnect too much.