Operating system adoption decisions in the presence of indirect network externalities*

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Abstract

The paper studies operating system adoption decisions of competing sellers of hardware products such as smart-phones, computers, or videogame consoles. Each seller can either develop a new operating system, which gives the seller control over the price at which the system's code is licensed to application developers, or buy the operating system from an independent platform. Adoption decisions affect the equilibrium number of applications written for each system, and the degree of differentiation between sellers. We show that even if sellers are symmetric ex ante, in equilibrium one seller may develop its own OS while its rival adopts the independent system. The independent platform may pay the sellers to adopt its operating system in order to make more revenues from developers. We also find that if users have a high value for applications, it is optimal for the platform to commit to distribute its code to developers free of charge in order to make more profits from sellers.

Keywords: indirect network effects; operating system; industry structure; free software; product differentiation; two-sided markets

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1 Introduction

The decision which operating system (OS) to adopt is of utmost importance for producers of smart-phones, computers, or video game consoles. The benefits consumers can derive from the hardware and the profits that producers can reap depend on the functionality of the operating system as well as the number and nature of applications available for the OS. The producers normally face the problem to either buy an existing OS or to costly develop an one own.

In the smart-phone industry, several major handset manufacturers use operating systems developed by independent software companies. For example, Nokia and Samsung use mainly the Symbian OS,[1] or HTC Corporation and Asus, two leading smart-phone producers in Asia, adopted Windows Mobile OS. However, other producers developed their own OS although they also had the possibility to adopt existing ones. Examples are RIM who developed the BlackBerry OS, or recently Apple Inc. who developed the iPhone OS. Operating system adoption decisions thus differ between companies, although one may argue that some of the smart-phone producers initial market positions and adoption incentives were quite similar.

A prevalent feature in these industries, is that for a smart-phone, video game console, or PC producer to be successful, it needs to attract "two sides", namely users and developers. The latter write applications (games in the case of video game consoles) for the adopted OS. Each group exerts positive indirect network externalities on the other. If a producer attracts many users, application developers face a larger market base, which makes the OS that runs on the producer's hardware more attractive for developers. Vice versa, the more applications are available for an OS, the more attractive is any hardware using this OS for users. As a result, if a producer develops its own OS, it acts as a two-sided platform who controls the pricing decision on the user and the developer side. If a company chooses the buy the operating system, on the other hand, then it sets the price for users but the independent platform sets the fees to developers, i.e. royalties and the fee to license its code. At the same time, if several hardware producers decide to adopt the same OS, then they all benefit from the fact that the OS's larger market base is likely to induce more applications.

Several questions arise. First, when is it optimal for a hardware producer to buy the OS from an independent platform instead of developing its own OS? In particular, how does competition between producers affect their OS adoption decision? Second, what is the optimal pricing strategy of an independent OS developer?

The existing literature on indirect network externalities, which we will summarize

[1] In 2008, Nokia acquired Symbian Ltd. and transferred it into the Symbian Foundation, an organization that is nevertheless still independent of Nokia.

below, only considers the case in which platforms control both user and developer prices. It has therefore neglected the case that the two sides of the market, application developers and users, do not necessarily buy from the same firm, as is the case in the industries described above. Instead, the OS owner, who sets licensing fees for application developers, may be distinct from the firm that sells to users.

In this paper we develop a model that accounts for this possibility, allows such a market structure to arise endogenously and provides conditions under which different market structures occur. In addition, we show how the independent platform can influence the chosen structure by its pricing policy. We thereby provide an answer to the questions proposed above. In particular, the framework is chosen so as to capture the key features of the smart-phone industry and other markets with similar characteristics. We consider a model with two producers who independently choose between developing an OS or buying an independent OS. The utility of final users is increasing in the number of available applications, and application developers earn more if they can sell to a larger number of users. Developing an OS is costly, but allows a producer to differentiate from its rival, and to internalize indirect network externalities by adjusting prices on both sides of the market. If both producers adopt the independent OS, this (endogenously) leads to more applications being developed for the OS because the user base for this OS is larger. We also study the implications of freeware, whereby the independent OS owner commits not to charge application developers, as is commonly observed.[2]

We show that the producers' equilibrium OS adoption decisions depend on their OS development costs and the degree of differentiation if the producers adopt different OS. The independent platform attracts both producers whenever the degree of differentiation between different operating systems is small, even if the development costs are negligible. The effect of higher indirect network externalities if both producers adopt the same OS is crucial in this case, and dominates the gains from coordinated pricing decisions and from differentiation that a producer could achieve by developing its own OS. This may contribute to the explanation why in the market for smart-phones the independent OS of Symbian has by far the largest market share globally.[3]

The outcome is different if the degree of differentiation between the OS of a producer and the independent platform is relatively large. Three different scenarios can occur in equilibrium, depending on the size of the development costs. If development

[2]For example, Windows Mobile does not charge developers for getting access to the development tools but only a certification fee of $99 to sell their applications. Recently Niklas Savander, Executive Vice President Services at Nokia said: "Our goal is ... to make it effortless for our partners to create highly appealing, context-relevant applications." This also includes to charge developers no or only small fees for access to the software.

[3]For example, in the second quarter of 2009 this market share was 50.3% followed by RIM's BlackBerry OS with 20.9% and Apples's iPhone OS with 13.9%.

costs are small, both producers develop their own OS. If costs are intermediate, an asymmetric market structure arises, in which one producer creates its own OS while the other buys the existing one. If costs are high, both producers buy the independent OS. The asymmetric market structure is the most interesting. A producer who creates its own OS induces differentiation from its rival and gets revenues from both users and developers but has to incur development costs. By contrast, the producer who buys the existing OS saves the development cost and free-rides on the differentiation effect. Although this producer earns revenue only from users, its profit is often larger than that of its rival who developed its own OS. Intuitively, this producer obtains differentiation for free. Each producer therefore wants to create a new OS to differentiate only if its rival does not do so. Thus, there are two asymmetric equilibria dependent on which producer develops its own OS and which one buys from the independent platform.

Turning to the pricing policy of the independent platform, we show that it can be optimal for the platform to pay producers to adopt its OS. The reason is that the platform can recoup losses on the producer side from application developers. Such negative fees are optimal if one or both producers are sufficiently close to being indifferent between adopting the OS from the platform or developing their own. A slight change in the parameters may induce a large reaction in the platforms pricing policy. For example, if development costs for producers increase slightly, the platform may find optimal to attract both producers instead of just one, and will do so by setting large negative fees. If the independent platform aims to attract only one producer, on the other hand, it will set a positive fee.

We also analyze under which conditions the independent platform can gain from committing to let application developers access its code free of charge. We find that such a strategy is profitable if the indirect network externality from developers to users is large, that is, if users value applications highly. In this case commitment to zero licensing fees allows the platform to attract one or both producers for a larger range of parameters. Additionally, the commitment to freeware increases the independent platform's profits even if the market structure remains unchanged. The intuition is twofold. First, more developers create applications for the independent OS if it can commit to freeware. Therefore, producers receive higher revenues from users and are willing to pay more for the OS. The second effect is more subtle. Without freeware, if a producer attracts more users by lowering its price, developers gain as well. But the independent platform has an incentive to keep part of this gain by charging higher licensing fees to developers, which deters producers from reducing their prices. This lowers producers' profits. By committing to freeware the independent platform commits to refrain from such behavior, and can therefore charge higher fees to producers in the first place. Overall, the analysis shows that the possibility to commit to free software can reverse the revenue source of the independent OS. Without commitment, the independent platform may pay producers to adopt its software and make money from developers only. With commitment, the platform foregoes any profits from developers, but can charge large producer fees.

The paper contributes to the literature on competition between two-sided platforms, initiated by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006) and Armstrong (2006). Among many other things, this literature points out that the price set to a group of customers crucially depends on whether this group can join only one platform (so-called single-homing) or several platforms (multi-homing). The single-homing side is treated favorable because platforms need to compete fiercely for these buyers while the multi-homing side is usually exploited. In our model end users single-home, which is a realistic assumption when considering smart-phones or game consoles; our modelling of the developers side permits both a single-homing and a multi-homing interpretation. The main difference to our model is that we allow for the case that the OS is different from the hardware producer which implies that the company that prices to users is different from the one that prices to developers. In addition, the existing papers deal with the case in which the hardware of each producer uses a different OS, which implies that the applications that run on the OS of one producer are incompatible with that of any other producer. In our framework, both producers may decide to license the same OS from an independent platform. We thus allow for compatibility between applications to arise endogenously.

Casadesus-Masanell and Ruiz-Aliseda (2009) examine compatibility issues in a model of platform competition with indirect network externalities. They compare the case in which the applications developed for one platform are compatible with the other platform to the case in which they are incompatible.[4] They find, for example, that under incompatibility, but not under compatibility, an asymmetric equilibrium in which only one platform is active can arise. As Casadesus-Masanell and Ruiz-Aliseda (2009), we allow applications to be compatible. In contrast to their paper, however, compatibility arises only if both producers use the same independent OS, instead of investing into development. The equilibrium market structure is therefore endogenously determined by the producers' decisions, whereas it is exogenous in Casadesus-Masanell and Ruiz-Aliseda (2009). In addition, we extend the analysis to incorporate commitment to freeware.[5]

[4]The question about compatibility and competition was pioneered by Katz and Shapiro (1985) who consider direct network effects. For further discussions on this issues see, among other papers, Katz and Shapiro (1994), Crémer, Rey and Tirole (2000) or Doganoglu and Wright (2006).

[5]The developer side in Casadesus-Masanell and Ruiz-Aliseda (2009) is modelled in a more elaborate way than in our model. In particular they allow the developer price to depend on the platforms' developer fees. As will become clear later, we abstract from this by supposing that the optimal price a developer charges is independent of the platform fees. We make this sacrifice to focus on our main

Hagiu (2006a) considers a situation where developers enter the market before users, and analyzes whether the platform can gain from committing to a user price at the time developers enter. He finds that platforms benefit from this commitment possibility if developers can multi-home, while under single-homing the option to commitment leaves profits unaffected. Our analysis complements the study by Hagiu (2006a) in two respects. First, we consider the reverse sequence of events, namely users entering before developers, a more realistic assumption in the smart-phone industry. Second, when analyzing the case of freeware, we allow the independent platform to commit to zero prices. Thus, is our case commitment is possible before the market structure is determined. As in the papers above, the market structure is exogenous and there is no independent platform in Hagiu (2006a).

Our analysis of commitment to freeware is also related to studies of mixed oligopoly in which one firm distributes its product for free while another charges a positive price. The most prominent analyses concern competition between an open source and a proprietary platform, see e.g. Economides and Katsamamkas (2006), or Casadesus-Masanell and Ghemawat (2006).[6] The proprietary platform follows a profit-maximizing strategy while the open-source platform charges zero prices both to users and developers. Economides and Katsamamkas (2006, Section 6) provide conditions under which industry profits in case of a proprietary system are larger than in case of an open-source system. Casadesus-Masanell and Ghemawat (2006) consider a dynamic model of competition between a proprietary and an open-source platform. They find that although demand-side learning is larger for the open-source platform, both platforms obtain positive market shares in the long-run. In contrast to these papers, we do not consider open-source but (partially) free software: although platform charges a zero fee to developers, it still makes revenues from producers. Moreover, in the above mentioned papers the industry structure is exogenously given, while we provide conditions under which it is indeed optimal to commit to free software.

The rest of the paper is organized as follows. The next section sets out the basic model. Section 3 derives the equilibrium and analyzes under which conditions producer fees are negative. Section 4 considers the case of commitment to free software and Section 5 concludes. All proofs are relegated to the Appendix

questions.

[6]For further studies, see also Hagiu (2006b) and Ormen (2008).

2 The Model

Industry Structure There are two ex ante symmetric hardware (e.g., smart-phone) producers, denoted by $i \in \{1, 2\}$, that sell directly to final users.[7] Prior to selling, each producer installs an OS on its hardware, which allows end users to fully exploit the hardware and to add applications later on. There are two possible sources of OS for each producer: either the producer develops its own OS, or it uses the OS of an independent platform denoted by I. We exclude the possibility that several producers share an OS developed by one of them. All large smart-phone producers indeed either use their own OS (Apple uses MAC OS X , RIM its own Blackberry OS) or buy from an independent company like Symbian, which is used by several producers like Nokia, Sony-Ericsson and Samsung. There are applications developers who write applications that each run on a specific OS. Applications developers spend time and money on developing software for these applications and sell them to end users.

Timing In the first stage, the independent platform I sets a price h_I for its OS that can either be positive or negative. The producers then simultaneously decide whether to adopt the OS from I, or to incur cost $C > 0$ to develop a new OS. The OS choice of $i = 1, 2$ is denoted by $s_i \in \{I, o\}$, where o stands for *own* OS. In the second stage, each producer sets the price p_i it charges to final users for its "system", the bundle of its hardware with the adopted OS. Each potential user decides which system to buy if any. We denote the number of users who choose to buy the system from producer i by q_i. In the third stage, OS owners (I and potentially 1 and/or 2) set fees charged to developers. We denote by f_I the fee set by I, and by f_i the fee set by i if it has its own OS. For simplicity, we suppose that this fee to developers is a fixed charge. At the beginning of Section 3 we show that our analysis remains unchanged if this fee is instead a royalty—that is, OS owners charge a price for each application that runs on their OS—because profits of OS owners and developers are the same in both scenarios.[8] Finally, in the fourth stage developers learn their development costs for all potential applications, decide which applications to write for which operating system(s), and set the price at which they sell their applications to end users. End users decide which applications to buy. All past decisions are public information. Table 1 gives a graphical representation of the time line:

[7] In some countries, such as the US, the majority of consumers buy their cell phones from their service provider bundled with a contract rather than directly from the producer. We abstract from this additional station in the supply channel to focus on the OS adoption decision.

[8] As will become evident there, even if both fees are possible, the profits an therefore the results are the same.

Stage 1	Stage 2	Stage 3	Stage 4
I sets h_I	Producers set p_i	OS owners	Dev. join OS
Producers set $s_i \in \{I, o\}$	Users decide to buy	set f_I (and f_i)	and set prices

Table 1

Two remarks on our time structure are in order: First, we suppose that the independent platform has already developed an OS and therefore does not have to incur costs to do so. This is a realistic description in many industries. Consider for example the smart-phone industry. In this industry, Symbian OS was a successor of the OS EPOC that was developed in the 1980's by Psion Plc. for electronic organizers. Only in 1998 Nokia, Motorala and Ericsson adopted this OS under the name Symbian for use in their smartphones. To the contrary, RIM developed its BlackBerry OS only in the late 1990's. This feature that an independent firm has developed an OS some time in advance—perhaps for some other purpose—is also present in other industries. Second, we suppose that users decide which hardware to buy in advance of developers' decisions for which OS to write applications. This assumption can be observed in the smart-phone industry, where the development of an application does not take much time and so developers can switch between systems at relatively little costs. Therefore, developers can wait to observe users' choices before investing in application development, while users buy their phones also to enjoy basic services, namely making phone calls or sending text messages.

Applications market We assume that a user's willingness to pay for an application that runs on his system is equal to $v_H > 1$ with probability ρ and equal to 1 with probability $1 - \rho$, where $1 > \rho v_H$. This reflects the idea that most users have a relatively low valuation for many applications but each user has a high reservation price for the few applications that interest him most. Total demand at any price up to 1 is therefore q_i if an application runs only on i's system, and $q_1 + q_2$ if an application runs on both systems, where q_i denotes the number of users who previously purchased system i.

The cost of developing any given application is a random draw from the uniform distribution with support $[0, \bar{c}]$. For any available OS, there is a continuum of potential applications of measure 1. Once developed, the marginal cost of producing an application is zero. The intellectual property rights of any potential application belong to a single developer, so that successful developers can charge the profit-maximizing price, which is 1 (recall that $1 > \rho v_H$). The expected gain for an end user from every application that runs on the system he buys is then equal to

$$x \equiv \rho(v_H - 1). \tag{1}$$

Given all applications are priced at 1, it is profitable for a developer with cost realization c to write an application for an OS *only* whenever

$$c \leq q_j - f_j,$$

with $j \in \{I, 1, 2\}$. In particular, if the two producers both adopt OS I, then all applications with costs c below $(q_1 + q_2) - f_I$ are profitable to develop, and the number of applications for the OS of I is

$$N_i = \frac{(q_1 + q_2) - f_I}{\bar{c}}. \tag{2}$$

On the other hand, if producer i develops its own OS, the number of applications for this OS is

$$N_i = \frac{q_i - f_i}{\bar{c}}, \tag{3}$$

independent of the rival's decision to develop an own OS or not. Finally, if producer i adopts I's OS, while the rival creates an own one, the number of applications for the OS adopted by i can be written as

$$N_i = \frac{q_i - f_I}{\bar{c}}. \tag{4}$$

To avoid having to deal with corner solutions, we assume in the remainder of the analysis that $\bar{c}$ is so high that it is always unprofitable to develop applications at this cost.

In both (2) and (3) the number of profitable applications for an OS increases with the number of users of the OS. In other words, positive *indirect network externalities* from end users to application developers affect the supply of applications. This suggests that an important advantage of both producers' adopting the same OS is that the resulting scale will lead to a larger number of applications, which in turn renders the hardware plus OS bundles more attractive for consumers.

Systems market Producers sell hardware plus OS bundles, also called systems, to end users. A key feature of the market for systems is that positive *indirect network externalities* arise from application developers on end users. A system becomes more attractive the more applications users anticipate will run on the system, because users derive an expected value of x as defined in (1) from every application. At the time users buy systems, they cannot yet observe the numbers of applications. Strictly speaking, their demands hence depend on anticipated numbers of applications. Since users' expectations are accurate in equilibrium, however, we take a short-cut by directly writing demands as functions of the numbers of applications.

If $s_1 = s_2 = I$, the inverse demand for system i, $i \neq -i \in \{1,2\}$, is

$$p_i^{II} = K + xN_i - q_i - \delta q_{-i}, \quad i \in \{1,2\}, \tag{5}$$

where (p_1, p_2) are the prices of the two systems.[9] $K > 0$ is the gross utility users derive from the system without any applications. The parameter $\delta < 1$ measures the substitutability between two systems that employ the same OS. Since both producers adopt the same OS, users know that $N_1 = N_2$. Inverting the demand system (5), using $N_1 = N_2$ and solving for the direct demand of firm $i \neq -i \in \{1,2\}$ gives

$$q_i^{II}(p_i, p_{-i}, N_i) = \frac{(K + xN_i)(1-\delta) - p_i + \delta p_{-i}}{1-\delta^2}, \quad i \in \{1,2\}. \tag{6}$$

If $s_i \neq s_{-i}$, i.e. producers do not adopt the same system, the inverse demand function is given by

$$p_i^{s_i s_{-i}} = K + xN_i - q_i - \gamma q_{-i}, \quad i \in \{1,2\}.$$

Inverting this demand system and recognizing that the number of applications for each of the two systems may differ since $s_i \neq s_{-i}$ yields a direct demand system of

$$q_i^{s_i s_{-i}}(p_i, p_{-i}, N_i, N_{-i}) = \frac{K(1-\gamma) + x(N_i - \gamma N_{-i}) - p_i + \gamma p_{-i}}{1-\gamma^2}, \quad i \in \{1,2\}. \tag{7}$$

It follows that in this case the demand of a producer increases in the number of applications for its OS but falls in the number of applications for the rival's OS.

An important assumption is that

$$\gamma < \delta.$$

This assumption captures the fact that OS typically differ along dimensions other than the available applications. Some consumers may simply prefer one OS's interface over that of another, or there exist different applications for for the two systems. Systems that run on different operating systems are therefore less good substitutes than two sys-

[9] This demand can be derived for example by a population of users each with unit demand who differ in their willingness to pay θ for the hardware component of each producer where θ uniformly distributed on $[0, K]$. It can also be generated by a representative consumer with utility function.

$$U = (K + xN_i)q_i + (K + xN_{-i})q_{-i} - \frac{1}{2}(q_i^2 + q_{-i}^2 + 2\beta q_i q_{-i}) - p_i q_i - p_{-i} q_{-i} + M,$$

where M is the utility from income. Differentiating this utility function with respect to q_i and q_{-i} again yields the inverse demand system above.

tems that both employ the same OS, even if the two distinct OS offer the same number of applications. Note that for simplicity we assume that the degree of differentiation is γ whenever the two systems use different OS, irrespectively of whether one of the OS is I or not.

The producers' marginal costs of producing hardware are assumed to be constant and normalized to zero.

To guarantee interior solutions, we assume that

$$8\bar{c}(\bar{c} - x - \bar{c}\delta^2) + 2x^2 - x - \bar{c} > 0.$$

This assumption guarantees that the indirect network externality from developers on users is not too large (x not too close to 1), and that the two producers are not too close rivals (δ not too close to 1); otherwise, one producer would corner the market in equilibrium and the other would be inactive.

Finally, we assume that C is small relative to the profit producers earn in the market. This is just a simplification to avoid that producers are deterred from developing their own OS because they would earn negative profits by doing so. As will become clear later, all our results would be qualitatively the same without this assumption but the analysis is more cumbersome since there are more cases to differentiate.

Equilibrium concept We analyze the subgame-perfect equilibria of the sequential game described above, but exclude any equilibria that rely on coordination failure. In particular, this rules out situations in which both producers adopt the OS of I although they would both be better off in another equilibrium where each develops its own OS, or vice versa.

3 Equilibrium Operating System Adoption Decisions

We solve the game backwards. In stage four developers decide how many applications to write for each system. We determined the outcome of this stage in the last section, i.e. dependent on the adoption decisions of producers the number of applications for a system is given by (2), (3) and (4), respectively. We can therefore move directly to the determination of the developer licensing fees and the user prices.

User prices and application developer licensing fees In stages two and three, when systems first set end user prices and OS owners then set licensing fees, we have to distinguish between three different cases: either both producers adopt OS I, one producer developed its own OS while the other adopted OS I, or both producers developed their own OS.

(i) Both producers adopt the OS I

If all systems adopted OS I, the number of applications as a function of how many users the systems have, is given in (2). The profit I earns from licensing its code to application developers at a fee of f_I is

$$\left(\frac{q_1 + q_2 - f_I}{\bar{c}}\right) f_I.$$

Maximizing this profit with respect to f_i yields

$$f_I = \frac{1}{2}(q_1 + q_2) \quad \text{and} \quad N_i = \frac{1}{2\bar{c}}(q_1 + q_2) \text{ for both } i, \tag{8}$$

which gives a profit to I of $\Pi_I = (q_1 + q_2)^2/(4\bar{c})$.

We can easily check that if the platform could charge a royalties in addition to the fixed fee, it would get the same profit. To see this we can first write the platform's profit as $\Pi_I = ((q_1 + q_2)(1 - r_I) - f_I)\,(f_I + (q_1 + q_2)r_I)/\bar{c}$. Maximizing with respect to f_I and r_I yields that $f_I = 1/2(q_1 + q_2) - (q_1 + q_2)r_I$ while r_I is left undefined.[10] Independently of the exact amount of f_I, I's profit is $\Pi_I = (q_1 + q_2)^2/(4\bar{c})$, which is the same as without royalties.

Now we turn to the third stage when producers set user prices. If both producers use OS I, the demand functions are given by $q_i^{II}(p_i, p_{-i}, N_i, N_{-i})$ in (6) while N_i, $i \in \{1, 2\}$ is derived in (8). Inserting (8) into (6) and solving the system of equations for q_i yields

$$q_i = \frac{2K\bar{c}(1-\delta) - p_i(2\bar{c} - x) + p_{-i}(2\delta\bar{c} - x)}{2(\bar{c}(1+\delta) - x)(1-\delta)}, \quad i \in \{1, 2\}.$$

Since production costs are zero, the profit producer i earns from selling to end users is simply $p_i q_i^{II}(p_i, p_{-i}, N_i, N_{-i})$. Maximizing profits with respect to p_i to derive reaction functions and solving for the equilibrium prices yields

$$p_i = \frac{2K\bar{c}(1-\delta)}{4\bar{c} - 2\bar{c}\delta - x}$$

and equilibrium profits of

$$\Pi^{II} = \frac{2K^2\bar{c}(1-\delta)(2\bar{c} - x)}{(4\bar{c} - 2\bar{c}\delta - x)^2(1 + \delta - x)}.$$

One can easily check that Π^{gg} is increasing in x and K and decreasing in δ. The corre-

[10] Armstrong (2006) obtains a similar result in a model of two-sided platform competition when considering membership fees in addition to per-transaction fees.

sponding profit of platform g from licensing its code to developers is

$$\Pi_I^{II} = \frac{K^2\bar{c}(2\bar{c}-x)^2}{(4\bar{c}-2\bar{c}\delta-x)^2(\bar{c}(1+\delta)-x)^2}. \tag{9}$$

(ii) Producer i develops its own OS, producer $-i$ buys from I

The number of developers system i attracts is now given by (3) while the number of developers that system $-i$ attracts is given by (4). In the same way as in case (i) we can calculate the profit-maximizing licensing fees, which yields $f_I = q_{-i}/2$ and $f_i = q_i/2$. The corresponding numbers of applications are $N_i = q_i/(2\bar{c})$ and $N_{-i} = q_{-i}/(2\bar{c})$. The resulting profits of OS owners are $(q_i)^2/(4\bar{c})$ for producer i and $(q_{-i})^2/(4\bar{c})$ for OS I.[11]

Inserting $N_i = q_i/(2\bar{c})$ and $N_I = q_{-i}/(2\bar{c})$ in the demand functions in (7) gives

$$q_i = \frac{2\bar{c}(K(2\bar{c}(1-\gamma)-x)-p_i(2\bar{c}-x)+2\gamma\bar{c}p_{-i})}{(2\bar{c}(1+\gamma)-x)(2\bar{c}(1-\gamma)-x)}, \quad i \in \{1,2\}.$$

The continuation profit of firm i in stage 3 includes the future revenue from licensing to application developers, and is hence equal to $p_iq_i + q_i^2/(4\bar{c})$, while $-i$'s profit is simply its immediate revenue $p_{-i}q_{-i}$. Solving for the equilibrium prices yields

$$p_i = \frac{K(2\bar{c}+2\gamma\bar{c}-x)(2\bar{c}-2\gamma\bar{c}-x)(4\bar{c}^2(1-\gamma^2)+x(1+x)-2\bar{c}(1+2x))}{\sigma}$$

and

$$p_{-i} = \frac{K(8(\bar{c}-x)-4\bar{c}^2\gamma(1+\gamma)+2(x^2-\bar{c}+\gamma\bar{c}x))(2\bar{c}+2\gamma\bar{c}-x)(2\bar{c}-2\gamma\bar{c}-x)}{2\sigma},$$

where

$$\sigma = 8\bar{c}^4(4+\gamma^4-5\gamma^2)-4\bar{c}^3(2+16x-\gamma^2-10x\gamma^2)+2\bar{c}^2x(6+24x-\gamma^2-5x\gamma^2)-2\bar{c}x^2(3+8x)+x^3(2x+1).$$

Inserting prices back into profits yields the following profit for producer i who develops its own OS:

$$\Pi^{oI} = \frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x+\bar{c}\gamma^2)+2(x^2-\bar{c})+x)(2\bar{c}+2\gamma\bar{c}-x)^2(2\bar{c}-2\gamma\bar{c}-x)^2}{\rho^2}.$$

Producer $-i$'s profit can be written as

$$\Pi^{Io} = \frac{K^2\bar{c}(2\bar{c}-x)(8(\bar{c}-x)-4\bar{c}^2\gamma(1+\gamma)+2(x^2-\bar{c}+\gamma\bar{c}x))^2(2\bar{c}+2\gamma\bar{c}-x)(2\bar{c}-2\gamma\bar{c}-x)}{2\rho^2}.$$

[11] As in case (i) if the firms could charge royalties in addition to developers fees, this would not change profits.

The profit of OS g from licensing its code to application developers is given by

$$\Pi_I^{Io} = \frac{K^2\bar{c}(2\bar{c}-x)^2(8(\bar{c}-x)-4\bar{c}^2\gamma(1+\gamma)+2(x^2-\bar{c}+\gamma\bar{c}x))^2(2\bar{c}+2\gamma\bar{c}-x)^2}{4\rho^2}. \quad (10)$$

(iii) Both producers develop their own OS

If both producers develop their own software, then by the same logic as above we get that $f_i = q_i/2$ and $N_i = q_i/(2\bar{c})$, $i \in \{1,2\}$. User demand function are the same as in case (ii). When setting user prices, both producers' profit functions include future licensing revenues equal to $(q_i/2)^2/\bar{c}$ in addition to revenues from selling systems to end users. Calculating the equilibrium prices in the third stage in the same way as above yields

$$p_i = \frac{K(2\bar{c}^2+x^2-3x\bar{c}-4\gamma^2\bar{c}^2)}{(6\bar{c}^2+2x^2-7x\bar{c}-2\gamma x\bar{c}+4\gamma\bar{c}^2(1-\gamma))},$$

which yields an equilibrium profit for each firm of

$$\Pi^{oo} = \frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x-\bar{c}\gamma^2)+x(2x-1)-2\bar{c})}{(8\bar{c}(\bar{c}-x)+x(2x+1)+2\bar{c}\gamma(2\bar{c}(1-\gamma)-\gamma x)-2\bar{c})^2}.$$

The profit of I is zero since no producer adopted its OS.

OS adoption decisions In stage one, each producer decides if it wants to build its own OS or accept the offer from the independent OS I. Clearly, if a producer in the first stage accepted to use the OS of I, it does not develop it own OS because this costs $C > 0$ but the producer cannot use it due to the contracting requirement. The profit of the producer who accepts is therefore $\Pi^{II} - h_I$ if the rival also uses OS I and $\Pi^{Io} - h_I$ if the rival develops its own OS. If producer i refuses to buy the OS of I in the first stage, its profit from developing its own OS also depends on the decision of its rival $-i$. In particular, if the rival develops its own OS, then the profit of producer i is $\Pi^{oo} - C$ while if $-i$ uses the OS from I, then i's profit $\Pi_i^{oI} - C$. By our assumption that market profits are large relative to C, these profits are positive. This gives the following pay-off matrix in stage 2:

1\2	o	I
o	$(\Pi^{oo}-C, \Pi^{oo}-C)$	$(\Pi^{oI}-C, \Pi^{Io}-h_I)$
I	$(\Pi^{Io}-h_I, \Pi^{oo}-C)$	$(\Pi^{II}-h_I, \Pi^{II}-h_I)$

Pay-off matrix in stage 2

The subgame has an equilibrium in which both producers adopt the OS of I if and only if no producer has an incentive to unilaterally deviate to developing its own OS:

$$\Pi^{II} - h_I \geq \Pi^{oI} - C. \quad (11)$$

The subgame has an equilibrium in which each producer develops its own OS if and only if

$$\Pi^{oo} - C \geq \Pi^{Io} - h_I. \tag{12}$$

Finally, an equilibrium of the subgame in which the two producers make different choices exists if and only if the following two conditions hold:

$$\begin{aligned} \Pi^{oI} - C &> \Pi^{II} - h_I \\ \Pi^{Io} - h_I &\geq \Pi^{oo} - C \end{aligned}$$

or

$$h_I > \Pi^{II} - \Pi^{oI} + C \tag{13}$$

$$h_I \leq \Pi^{Io} - \Pi^{oo} + C \tag{14}$$

are both satisfied. For (13) and (14) to be satisfied simultaneously for some h_I, it is necessary that

$$\Pi^{oI} - \Pi^{II} \geq \Pi^{oo} - \Pi^{Io}. \tag{15}$$

In words, a producer's change in profit from developing an own OS instead of adopting the one from I must be higher if the rival adopts I's OS than if the rival also develops its own. If (15) holds, then there exist two asymmetric equilibria for the range of offers h_I that fulfill $\Pi^{II} - Pi^{oI} + C < h_I \leq \Pi^{Io} - \Pi^{oo} + C$, and no symmetric equilibria. On the other hand, if (15) is violated, any equilibrium must be symmetric.

The proof of Proposition 1 shows that (15) is violated if δ is close to γ but holds if δ is sufficiently above γ. In other words, an asymmetric equilibrium can exist only if the adoption of different operating systems increases system differentiation sufficiently.

If (15) is violated, then conditions (11) and (12) are satisfied simultaneously for some h_I. For this range of h_I, it is optimal for a producer to develop its own OS if and only if the rival is also doing so. Without further equilibrium selection, both symmetric equilibria co-exist in this case. As mentioned, we use a simple and natural selection criterion that producers can coordinate on the equilibrium that yields larger profits (to each of them). As we show in the proof of Proposition 1, even at the highest price that g can charge such that (11) still holds the producers earn more if they both adopt OS I than if both develop their own OS. We will therefore assume that both producers accept I's offer in this case.

We can now determine the optimal fee that I can charge to induce one or the other equilibrium. The highest fee h_I to induce an equilibrium in which both producers adopt OS I is such that (11) hold with equality. In this case the overall profit of I is

$$2(\Pi^{II} - \Pi^{oI} + C) + \Pi_I^{II},$$

where Π_I^{II} is the future profit from licensing to application developers, as given in (9).

Similarly, the highest price that I can charge in an asymmetric equilibrium in which only one producer adopts the OS of I is such that (14) holds with equality, and I's overall profits in this case is

$$\Pi^{Io} - \Pi^{oo} + C + \Pi_I^{Io},$$

where Π_I^{Io} is as given in (10).

The equilibrium outcome depends on the interplay between the developing costs and the degree of differentiation. To simplify the exposition we define

$$y \equiv \delta/\gamma,$$

with $1 \leq y \leq 1/\gamma$. A higher y means that there is a larger difference, in terms of the degree of substitutability, between the situation where both producers use the same OS and situation where they use different operating systems. The following Proposition summarizes the equilibrium adoption decision as a function of y and C.

Proposition 1

There exists a unique y^* such that the following holds:
For all $y \leq y^*$ both producers adopt the OS of I if $C \geq \hat{C}(y)$, and no producer adopts the OS of I if $C < \hat{C}(y)$. $\hat{C}(y) = 0$ for $y \leq \underline{y}$ and is strictly increasing in y for $y \geq \underline{y}$.
For all $y > y^*$ both producers adopt the OS of I if $C \geq \check{C}(y)$, only one producer adopts the OS of I if $\check{C}(y) \leq C < \tilde{C}(y)$, and no producer adopts the OS of I if $C < \tilde{C}(y)$. Finally, $\check{C}(y)$ is strictly increasing in y while $\tilde{C}(y)$ is constant in y.

Figure 1 displays equilibrium adoption decisions.

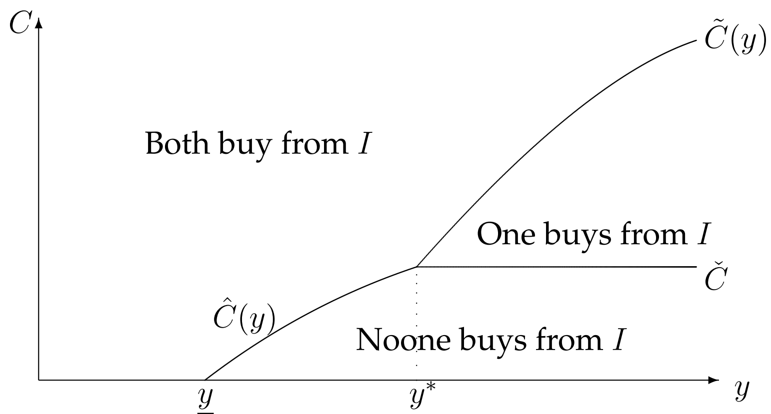

Figure 1: Equilibrium Configuration

The equilibrium configuration has two interesting features. First, even for C close to zero both producers adopt I's OS if the degree of differentiation between different operating systems is relatively small. This result is driven by the indirect network effect from developers on users, which are larger if both producers use the same OS. If the incentive to differentiate is weak (δ is close to γ), platform I optimally attracts both producers by charging negative fees. It recoups the resulting losses later on, since developers are willing to pay higher licensing fees for an OS that is used by both producers and, hence, will attract many end users.

Second, if the difference between δ and γ is large enough and the costs C are intermediate, an asymmetric equilibrium emerges in which one producer develops its own OS while the other buys the OS from I. The benefit from developing an OS depends on the rival's adoption decision. If the rival uses the OS of I, the decision to develop a new OS induces differentiation and thereby reduces competitive pressure. If the rival develops its own OS, the two producers are already differentiated, so this effect vanishes. In this case it is therefore optimal to use the independent platform's OS if development costs are sizable. There are parameter constellations such that the producer that buys from I earns more than its rival that develops its own OS, i.e. the first producer free-rides on the differentiation induced by its rival's investment decision. The producers thus face a coordination problem: out of the two asymmetric equilibria that exist, each producer prefers the one in which its rival develops its own OS.

Finally, if development costs C are sufficiently low, it is optimal for both producers to develop their own OS so as to control pricing not only on the user but also on the developer side.

The exact location of the curves in the y-C-diagram depends on the parameters of the model, i.e. x—the measure of the positive externality from application developers on users defined in (1)—$\bar{c}$ and K. When x rises, the curves for $\hat{C}$ and $\tilde{C}$ shift downwards while the $\check{C}$-curve shifts upwards. Hence, the outcome that both producers adopt I's OS becomes more likely, while the asymmetric equilibrium occurs only for a smaller set of parameters. This is intuitive: A higher x reflects the case where the indirect network externality from application developers on users is larger. Thus, I can attract both producers more easily since the number of applications is larger if both producers adopt the same OS.

An interesting result that we already alluded to is that the independent platform may find it profitable to pay producers to use its OS. If I attracts more producers, it can make higher profits from developers. The following proposition gives a more systematic statement on when h_I will be negative:

Proposition 2

(i) In any equilibrium in which just one producer adopts the OS from I and C is close

to $\check{C}$, we have $h_I < 0$.
(ii) In any equilibrium in which both producers adopt the OS from I and C is close to $\hat{C}$ or $\tilde{C}$, we have $h_I < 0$.
(iii) In any equilibrium in which both producers adopt the OS from I there exists a unique threshold value for y, so that $h_I < 0$ for all y larger than this threshold.

The fee h_I is negative at the lower borders of each equilibrium region in Figure 1. This is the case because, say, at $\tilde{C}$ both producers are indifferent between accepting the offer of I or creating an own OS. Thus, the platform must pay a producer to adopt its OS. The results in Proposition 2 also imply that there is a downward jump of the platform fee at $C = \tilde{C}$. If C is slightly above $\tilde{C}$, the platform finds it optimal to attract both producers. But to do so the fee must be negative. However if C is slightly below $\tilde{C}$, the platform earns a larger profit by attracting only one producer. In this case, the platform fee is likely to be positive. Thus, a small increase in C induces the platform to lower its fee discretely to attract both producers. Nevertheless, the platform's profit is continuously increasing in C but displays a kink at $C = \tilde{C}$.

4 Commitment to Freeware

We now turn to the case where the independent platform I can choose to commit to distribute its code to developers free of charge. In terms of our model, this means that at the beginning of the game I can commit to freeware, i.e., to set $f_I = 0$, later on.[12] We are especially interested in comparing the case of freeware with the previous analysis where I could not commit on f_I.

Suppose I indeed commits to set $f_I = 0$ in stage three. The model is the same as described in Section 2 otherwise. To guarantee interior solutions we have to modify the assumption on x and δ by assuming that $(1+\delta)\bar{c} - 2x > 0$ and $4\bar{c}^2(1-\gamma^2) + x(2+x) - 6\bar{c}(x+1) > 0$. This again guarantees that in equilibrium we do not have a corner solution in which only one firm is active.[13]

We start by examining the difference with respect to the analysis in the last section in the third and fourth stage. Let us first consider the case in which both producers adopt the OS from I. Since $f_I = 0$ we get that in the fourth stage $N_I = (q_1 + q_2)/\bar{c}$. Calculating the equilibrium price that the producers set in stage three in the same way

[12] This modelling structure is similar to Church and Gandal (1993) who analyze a model with two incompatible platforms that can only charge users. In contrast to our model, Church and Gandal (1993), as the previous literature, do not consider the case of an independent platform that can sell its OS to producers.

[13] The conditions are slightly tighter than the one in the last section. The reason is that the externality is larger in this case because via committing to $f_I = 0$ more developers are attracted.

as above gives

$$p_i = \frac{K\bar{c}(1-\delta)}{((2-\delta)\bar{c}-x)}, \quad i \in \{1,2\},$$

and equilibrium profits of

$$\widetilde{\Pi}^{II} = \frac{K^2\bar{c}^2(1-\delta)(\bar{c}-x)}{((2-\delta)\bar{c}-x)^2((1+\delta)\bar{c}-2x)}.$$

In the asymmetric case in which firm i is independent while firm $-i$ buys the OS from I we now get $N_i = q_i/(2\bar{c})$ while $N_{-i} = q_{-i}/\bar{c}$. The equilibrium prices in this case are

$$p_i = \frac{K\left(2\bar{c}^2(1-\gamma^2)-\bar{c}(1+3x)+x(1+x)\right)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)}{2\xi}.$$

and

$$p_{-i} = \frac{K\left(2\bar{c}^2(1-\gamma^2)-3\bar{c}x)+x^2\right)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}(6x+1-2x\gamma)+2x^2+x\right)}{2\xi},$$

with

$$\xi = \Big(2\bar{c}^4(4+\gamma^4-5\gamma^2)-\bar{c}^3(2-\gamma^2(1+15x)+24x)+$$
$$+\bar{c}^2x(5+26x-\gamma^2(1+5x))-4\bar{c}x^2(1+3x)+x^3(1+2x)\Big).$$

The equilibrium profits are then given by

$$\widetilde{\Pi}^{oI} = \frac{K^2\bar{c}(2\bar{c}-x)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)^2\left(4\bar{c}(1-\gamma^2)-\bar{c}(6x+1)+x(2x+1)\right)}{4\xi^2}$$

for firm i and by

$$\widetilde{\Pi}^{Io} = \frac{K^2\bar{c}(\bar{c}-x)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)^2\left(4\bar{c}(1-\gamma^2)-\bar{c}(6x+1)+x(2x+1)\right)}{4\xi^2}$$

for firm $-i$.

Clearly, the profits in the case in which both producers develop their own OS are the same as in the last section. For all cases, $\widetilde{\Pi}_I = 0$ since the independent platform does not receive revenues from the developer side.

The analysis of the first stage is the same as the one in the last section but with the adjusted profit functions. In the same way as in the proof of Proposition 1 we can show that the equilibrium with commitment looks qualitatively similar to the one without commitment. This means if y is below a certain threshold both producer use the OS of I if C is large but abstain if C is small, while if y is above this threshold, the three different regions as in the last section emerge.

We can now determine how the boundaries between the different regions differ in the case with commitment compared to the case without. Here we obtain the following result:

Proposition 3
There exists a unique x denoted by $\hat{x}^*$ such that $\hat{C}$ in case of commitment is above $\hat{C}$ in case without commitment for all $x < \hat{x}^*$ and below for all $x > \hat{x}^*$.
Also, there exists a unique x denoted by $\check{x}^*$ (respectively $\tilde{x}^*$) such that $\check{C}$ (resp. $\tilde{C}$) in case of commitment is above $\check{C}$ (resp. $\tilde{C}$) in case without commitment for all $x < \check{x}^*$ (resp. $x < \tilde{x}^*$) and below for all $x > \check{x}^*$ (resp. $x > \tilde{x}^*$).

Proposition 3 shows that if the independent platform commits to freeware, it depends crucially on x—the strength of the indirect network externality—how the equilibrium regions change. If x is relatively large, the boundaries for all equilibrium regions shift down which implies that I can now attract producers for a larger range of parameters. Specifically, there exist parameter combinations in which no producer buys the OS from I in case commitment to $f_I = 0$ is impossible, but in which one or both producers (dependent on y being above or below y^*) buy from I if such a commitment is possible. This obviously implies that in this region the platform now makes positive profits while the profit was zero without commitment. In addition, there also exists a parameter range in which the platform attracts both producers with commitment but just one without commitment.

The intuition behind the result is the following: Via committing to freeware, the platform attracts more developers for its OS. Due to the positive externality from developers on users, this in turn attracts more users and, thus, allows producers to reap larger profits on the user market. However, the independent platform foregoes any profits it can get from developers and, therefore, it can no longer afford to subsidize producers. Which of these two effects dominates depends on the strength of the indirect network externality x. If x is large, a producer gains a lot by the larger number of developers since many more users are attracted. Thus, the producer is willing to pay a much higher fee to the platform. In addition, in case of no commitment the platform charges larger developer fees, the more users are attracted by producers. Thus, the platform curbs producers to lower their user prices because it keeps part of this revenue via higher developer fees. The platform can avoid this by committing to freeware. As a consequence, the equilibrium region for which the producers adopt the OS of I gets broader if x is large. Conversely, if x is small, producers are not willing to pay a much larger fee to g because they earn only little profits from users. Thus, the platform cannot recoup its foregone revenues from developers, and the region in which one or both producers adopt I's OS shrinks.

So far we analyzed how the equilibrium regions change with freeware. However, we have not yet looked at the change in the profit of the independent platform. It is evident that if x is large enough, its profit rises for the parameter range in which the platform can attract one or both producers with freeware but none without freeware. The following proposition answers the question how its profit changes even if the equilibrium region is unchanged by the use of freeware.

Proposition 4
There exists a unique x denoted by x^*_{II} (resp. x^*_{Io}) such that Π_I^{II} (resp. Π_I^{Io}) in case of commitment is higher than in case without commitment for all $x > x^*_{II}$ (resp. $x > x^*_{Io}$) and lower for all $x < x^*_{II}$ (resp. $x < x^*_{Io}$).

The result shows that if x is large, not only the equilibrium regions in which the platform can attract producers gets larger if the platform commits to freeware, but also the platform's profit inside a region rise. The intuition is similar to the one for the last proposition. If x is large enough, producers are willing to pay a higher access fee for the platform's OS. This dominates any foregone profits of the platform on the developer's side and yields higher profits. The converse holds true for small values of x.

The section shows that the possibility to commit to freeware can completely reverse the revenue source of an independent platform. In case of no commitment the platform at times subsidize producers to make revenue from developers. By contrast, in case of commitment the producer fees are the only source of revenue for the platform and they are therefore relatively high. Nevertheless, producers pay this higher fees because they in turn make larger revenues from users.

5 Conclusion

We examined hardware producers' operating system adoption decisions in a setting with indirect network effects where an independent platform sells access to its OS but producers can also develop their own OS. By developing its own OS a producer can differentiate itself from its rival, and, in addition, it gains control over setting licensing fees to application developers. However, it also has to bear the OS development costs and foregoes the larger network effect that arises if both producers use the same OS. We show that this network effect can be so large that both producers adopt the OS of the independent platform even if development costs are negligible. If adopting different operating systems increases differentiation substantially, an asymmetric equilibrium can occur in which one producer develops its own OS while the rival buys from the independent platform. In this case, the rival free-rides on the differentiation created by the producer who develops its own OS. This may explain why some smart-phone producers develop operating systems used exclusively for their own phones, while others adopt independently developed operating systems used by several producers.

For the independent platform it is sometimes optimal to pay producers for using its OS, because this allows it to make more money from licenses sold to application developers. If it has the possibility to commit to distribute its software to developers free of charge, however, the independent platform finds it optimal to do so as long as users value applications sufficiently. In this case, the independent platform foregoes all profits from developers in order to charge high access fees to producers.

The analysis restricted attention to the case of two producers, but the qualitative insights remain valid in an oligopoly. With three producers, for instance, there can be an asymmetric equilibrium in which exactly two producers adopt the OS sold by independent platform. The strategic trade-off between softer competition via differentiation and larger network effects is unchanged. The incentive for the independent platform to commit to freeware is qualitatively similar as well.

The assumption that users decide which hardware to buy before developers write applications matches the smart-phone industry, where application development takes relatively little time. Developers can therefore afford to wait and observe users' choices before investing in application development. In other markets decisions are closer to being simultaneous or even in reverse order. For example, as Hagiu (2006a) argues, in the market for video games the development of an application is a long and costly process that can take more than one year. Developers therefore have to make their choices before users do. Analyzing the implications of this could be an interesting topic for future research. We expect the main trade-offs of the present paper to persist, but the analysis could yield additional insights concerning the relative strengths of these forces in industries characterized by different technologies.

Another direction for further research would be to consider sequential adoption decision of producers. Apple, for instance, decided to develop its iPhone OS in the knowledge that other manufacturers like Nokia or Samsung were using Symbian. An open question is whether sequential adoption decision would lead to more or less differentiation than simultaneous decisions.

6 Appendix

Proof of Proposition 1

From the analysis of stages 2 and 3 we know that three possible outcomes can emerge in equilibrium and we determined the respective profits of the two producers and of firm I in all of these three outcomes. In stage 1 we determined the conditions on h_I such that either of the three outcomes arises.

We can now check under which conditions the inequality (15), $\Pi^{oI} - \Pi^{II} > \Pi^{oo} - \Pi^{Io}$, which is the necessary and sufficient condition for an asymmetric equilibrium to emerge, is fulfilled. In the following we denote the asymmetric equilibrium by (o, I). To do so we first insert the values for the respective profits obtained in stages 2 and 3 into $\Pi^{oI} - \Pi^{II} - (\Pi^{oo} - \Pi^{Io})$. Since we know that $\delta \geq \gamma$ we can substitute $\delta = y\gamma$ with $1 < y \leq 1/\gamma$. It is then easy to show that $\Pi^{oI} - \Pi^{II} - (\Pi^{oo} - \Pi^{Io})$ is strictly negative in the limit as $y \to 1$. However, it is strictly positive at the maximum value of y, that is given by $y^{max} = \left(\sqrt{2(2\bar{c} - x)(1 + 2x - 4\bar{c})}\right) / (4\gamma\bar{c}) > y$,[14] or, if this value does not exist because $\bar{c}$ is large, for any sufficiently high y.[15] In addition, differentiating $\Pi^{oI} - \Pi^{II} - (\Pi^{oo} - \Pi^{Io})$ with respect to y yields

$$\frac{2K^2\gamma\bar{c}(2\bar{c} - x)(4\bar{c}(1 - y\gamma) - x(2 - x) + 2y\gamma\bar{c}(2y\gamma - x))}{(1 + y\gamma - x)^2(2(2\bar{c} - y\gamma) - x)^3},$$

which is strictly positive because $y\gamma$ is between 0 and $\left(\sqrt{2(2\bar{c} - x)(1 + 2x - 4\bar{c})}\right) / (4\bar{c})$. Thus, there exists a unique threshold of y below which only the symmetric equilibria can exist while above which all three equilibria can exist.

We first look at the case in which y is below this threshold. We know from (11) that if firm I sets $h_I \leq \Pi^{II} - \Pi^{oI} + C$, there exists a continuation equilibrium in stage 2 such that both producers the OS from I. As mentioned, it is optimal for firm I to set $h_I = \Pi^{II} - \Pi^{oI} + C$ to extract the highest profit from producers. In this case the profit of firm I is $2(\Pi^{II} - \Pi^{oI} + C) + \Pi_I^{II} \equiv \Pi_I^2$ which is positive only if $C \geq -\Pi^{II} + \Pi^{oI} - \Pi_I^{II}/2$. Inserting the respective values for $-\Pi^{II} + \Pi^{oI} - \Pi_I^{II}/2$ and differentiating this expression with respect to y gives

$$\frac{K^2\bar{c}\gamma(2\bar{c} - x)(12\bar{c}^2(\bar{c} - x) + x^2(5\bar{c} - 3x) - \gamma y\bar{c}(8\bar{c}^2 - 8\gamma^2y^2\bar{c}^2 - 4x\bar{c} - 6x^2 + 12\gamma yx\bar{c}))}{(\bar{c} + \gamma y\bar{c} - x)^3(4\bar{c} - 2\gamma y\bar{c} - x)^3}$$

which is strictly positive for the range of x that fulfill $8\bar{c}(\bar{c} - x - c\delta^2) + x(2x - 1) - 2\bar{c} > 0$. Therefore, the critical $C(y)$ such that $\Pi_I^2 \geq 0$ is increasing in y. We denote this $C(y)$ by $\hat{C}(y)$. Routine calculations show that $\lim_{y \to 1} \hat{C}(y) < 0$. Thus, if $\delta \to \gamma$, the profit of

[14] The maximum value of y stems from the condition $8\bar{c}(\bar{c} - x - c(y\gamma)^2) + x(2x - 1) - 2\bar{c} > 0$.

[15] We do not show the full expressions since they are rather complicated. However, determining their sign is an easy task and this is all that is needed for our purposes.

platform I from attracting both producers is positive even for $C = 0$.

However, it is possible that at $h_I = \Pi^{II} - \Pi^{oI} + C$ also condition $h_I > \Pi^{Io} - \Pi^{oo} + C$ is satisfied. This implies that in the first stage also the continuation Nash equilibrium in which both producers develop their own OS, denoted by (o, o)-equilibrium, can exist. If this is the case, our equilibrium selection criterion is that producers coordinate on the equilibrium that gives them the highest profit. We now show that the profits in the equilibrium in which both producers adopt the OS from g, denoted by (I, I)-equilibrium, are always higher than in the (o, o)-equilibrium. In the (I, I)-equilibrium a producer gets $\Pi^{II} - h_I = \Pi^{oI} - C$ while in the (o, o)-equilibrium a producer receives $\Pi^{oo} - C$. Subtracting Π^{oo} from Π^{oI} yields that the sign of this difference is given by the sign of

$$\big(8\bar{c}(\bar{c} - x - c\delta^2) + x(2x - 1) - 2\bar{c}\big) \times$$

$$\Big(16\bar{c}^4(2 + \gamma)(1 + \gamma)(2 - \gamma)(1 - \gamma) - 4\bar{c}^3(4 + 2x(17 - 10\gamma^2) - \gamma(1 + 2\gamma))+$$

$$+4\bar{c}^2 x(6 - x(24 - 5\gamma^2) + \gamma(1 + \gamma)) - \bar{c}x^2(32x + 12 - \gamma)2x^3(2x + 1)\Big).$$

We know from the assumption in Section 2 that the first term, $8\bar{c}(\bar{c}-x-c\delta^2)+x(2x-1)-2\bar{c}$, is positive. It is easy to check that the second term is decreasing in x for any γ. Now inserting the largest possible value of x which is $x = 2\bar{c} - 1/4\left(1 + \sqrt{1 + 64(\bar{c}y\gamma)^2}\right)$, in this second term yields

$$\Big(16\bar{c}^4\gamma^4(1 + 4y^4 - 5y^2) + \bar{c}^2\gamma^2(2y^2 + 4\bar{c}\gamma y^2 - 3/2) + 1/8\gamma\bar{c}+$$

$$+1/8\sqrt{1 + 64\bar{c}^2y^2\gamma^2}\bar{c}\gamma(16\bar{c}y^2\gamma + 1 - 12\bar{c}^2\gamma)\Big).$$

This expression is 0 for $\gamma = 0$ but positive for all $\gamma > 0$. Therefore, the second term is positive which implies that $\Pi^{oI} > \Pi^{II}$. As a consequence, the (I, I)-equilibrium gives producers larger profits than the (o, o)-equilibrium and, thus, they coordinate on the former.

Now let us turn to the case in which y is above the threshold and so (15) is satisfied. In this case, via setting h_I firm I in the first stage can induce different equilibria in the continuation game at stage 2. If $h_I \le \Pi^{II} - \Pi^{oI} + C$, the (I, I)-equilibrium emerges, if $\Pi^{II} - \Pi^{oI} + C < h_I \le \Pi^{Io} - \Pi^{oo} + C$, the (o, I)-equilibrium emerges and if $h_I > \Pi^{Io} - \Pi^{oo} + C$, the (o, o)-equilibrium emerges. To extract most profits it is optimal for I to set $h_I = \Pi^{II} - \Pi^{oI} + C$ in the first case and $h_I = \Pi^{Io} - \Pi^{oo} + C$ in the second case. Therefore, in the first case firm I gets a profit of Π_I^2, in the second case it gets $\Pi^{Io} - \Pi^{oo} + C + \Pi_I^{Io} \equiv \Pi_I^1$ and in the third case it gets zero. We can now compare these three profits with each other.

Above, we already determined that $\Pi_I^2 \ge 0$ if and only if $C \ge \hat{C}(y)$. Turning to Π_I^1

we obtain that $\Pi_I^1 \geq 0$ if $C \geq \Pi^{oo} - \Pi^{Io} - \Pi_I^{Io}$. We denote this critical C by $\check{C}$. Since none of the terms in $\Pi^{oo} - \Pi^{Io} - \Pi_I^{Io}$ depends on δ and therefore on y, $\check{C}$ is constant for all y. In addition, it is easy to check that $\check{C} > 0$ for all admissible x-γ-$\bar{c}$-combinations.

We can now compare Π_I^1 and Π_I^2. Doing so reveals that $\Pi_I^2 \geq \Pi_I^1$ if $C \geq 2(\Pi^{oI} - \Pi^{II}) - (\Pi^{oo} - \Pi^{Io}) - \Pi_I^{II} + \Pi_I^{Io}$. This inequality defines a third critical $C(y)$ that we denote by $\tilde{C}(y)$. Above $\tilde{C}(y)$ the profit of I from attracting both producers is larger than from attracting just one. Inserting the respective profits for $2(\Pi^{oI} - \Pi^{II}) - (\Pi^{oo} - \Pi^{Io}) - \Pi_I^{II} + \Pi_I^{Io}$ and differentiating this expression with respect to y yields now

$$2\frac{K^2\bar{c}\gamma(2\bar{c}-x)(12\bar{c}^2(\bar{c}-x)+x^2(5\bar{c}-3x)-\gamma y\bar{c}(8\bar{c}^2-8\gamma^2y^2\bar{c}^2-4x\bar{c}-6x^2+12\gamma yx\bar{c}))}{(\bar{c}+\gamma y\bar{c}-x)^3(4\bar{c}-2\gamma y\bar{c}-x)^3}$$

which is exactly two times the slope of $\hat{C}(y)$.

From the above we know that $\hat{C}(y)$ is increasing in y while $\check{C}$ is constant. In addition, at $y = 1$ we have $\check{C} > \hat{C}(y)$. One can easily check, that if y is large enough we obtain $\check{C} < \hat{C}(y)$. But since $\hat{C}(y)$ is strictly increasing in y, we know that there exists a unique y such that $\check{C} = \hat{C}(y)$. In the following, we denote this y by y^*. We know that at $\Pi_I^1 = 0$ and $\Pi_I^2 = 0$ at this y^*. Since $\tilde{C}(y)$ is defined as the C at which $\Pi_I^1 = \Pi_I^2$, we have that $\check{C} = \hat{C}(y^*) = \tilde{C}(y^*)$.

Both $\hat{C}(y)$ and $\tilde{C}(y)$ are increasing in y but the slope of $\tilde{C}(y)$ is steeper than the one of $\hat{C}(y)$. Since all three critical values are the same at $y = y^*$, it follows that $\tilde{C}(y^*) < \hat{C}(y^*) < \check{C}$ for all $y < y^*$ and $\tilde{C}(y^*) > \hat{C}(y^*) > \check{C}$ for all $y > y^*$. This gives Figure 2.

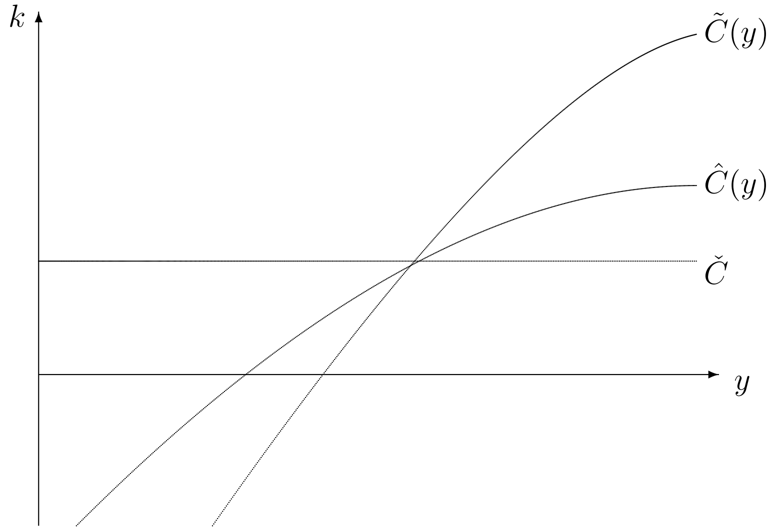

Figure 2: Display of $\hat{C}(y)$, $\check{C}$ and $\tilde{C}(y)$

We know that it is optimal for I to attract both producers if $C \geq \max\{\hat{C}(y), \tilde{C}(y)\}$. It therefore follows that both producers buy from I if $C \geq \hat{C}(y)\ \forall\ y \leq y^*$ and if $C \geq \tilde{C}(y)$ $\forall\ y > y^*$. Furthermore, it is optimal for I to attract only one producer if $\check{C} \leq C < \tilde{C}(y)$.

We know that such a region only exists for $y > y^*$. Thus, the result follows. Finally, it does not pay off for I to attract any producer if $C < \min\{\hat{C}(y), \check{C}\}$. This proves the result for the different adoption regions.

The results concerning the slopes of the functions has been established above. ■

Proof of Proposition 2

(i) If just one producer adopts the OS from I, we have $h_I = \Pi^{Io} - \Pi^{oo} + C$. We know that $\check{C}$ is defined as $\check{C} \equiv \Pi^{oo} - \Pi^{Io} - \Pi_I^{Io}$. As a consequence, if $C = \check{C}$ we have $h_I = -\Pi_I^{Io} < 0$. Therefore, also for C that are slightly above $\check{C}$, h_I is strictly negative.

(ii) If both producers adopt the OS from I, we have $h_I = \Pi^{II} - \Pi^{oI} + C$. We know that $\tilde{C}$ is defined as $\check{C} \equiv 2(\Pi^{oI} - \Pi^{II}) - (\Pi^{oo} - \Pi^{Io}) - \Pi_I^{II} + \Pi_I^{Io}$. Thus, at $C = \tilde{C}$ we have $h_I = (\Pi^{oI} - \Pi^{II}) - (\Pi^{oo} - \Pi^{Io}) - \Pi_I^{II} + \Pi_I^{Io}$. From the proof of Proposition 1 we know that there exists a C close to $\tilde{C}$ for which the (I, I)-equilibrium occurs only if $y \geq y^*$. So let us first determine h_I at y^*. At y^* we have that $\Pi_I^1 = \Pi_I^2 = 0$ which implies that $\Pi^{Io} - \Pi^{oo} + C + \Pi_I^{Io} = 0$ and $\Pi^{II} - \Pi^{oI} + C + \Pi_I^{II} = 0$. But from that it follows that $-\Pi^{oo} + \Pi^{Io} + \Pi_I^{Io} = -C$ and $\Pi^{oI} - \Pi^{II} - \Pi_I^{II} = C$. This implies that $h_g = 0$ at $y = y^*$. Differentiating $h_I = (\Pi^{oI} - \Pi^{II}) - (\Pi^{oo} - \Pi^{Io}) - \Pi_I^{II} + \Pi_I^{Io}$ with respect to y yields

$$-2K^2\bar{c}\gamma\Big(\frac{\bar{c}(2\bar{c}-x)^2(2\bar{c}(1-2y\gamma)-x)}{(\bar{c}(1+y\gamma)-x)^3(2\bar{c}(2-y\gamma)-x)^3}+$$

$$+\frac{(2\bar{c}-x)(4\bar{c}(1-y\gamma)+x(x-2)+2\bar{c}y\gamma(2y\gamma-x))}{(1-x+y\gamma)^2(2\bar{c}(2-y\gamma)-x)^3}\Big).$$

One can easily check that this expression is negative for all admissible combinations of $\bar{c}$, x and $y\gamma$. It follows that $h_I < 0$ for all $y > y^*$ if C is slightly above $\tilde{C}$.

Now suppose that C is close to $\hat{C}$ which is defined as $\hat{C} \equiv \Pi^{oI} - \Pi^{II} - \Pi_I^{II}/2$. Since $h_I = \Pi^{II} - \Pi^{oI} + C$ in the case in which both producer use the OS from I, we have that at $C = \hat{C}$, $h_I = -\Pi_I^{II}/2 < 0$. Therefore, h_I is strictly negative also for C that are slightly above $\hat{C}$.

(iii) Since both producers adopt the OS from I, we have $h_I = \Pi^{II} - \Pi^{oI} + C$. Differentiating the right-hand side with respect to y yields

$$-2K^2\bar{c}\gamma\frac{\bar{c}(2\bar{c}-x)^2(2\bar{c}(1-2y\gamma)-x)}{(\bar{c}(1+y\gamma)-x)^3(2\bar{c}(2-y\gamma)-x)^3} < 0.$$

We also $\Pi^{II} - \Pi^{oI}$ at $y = 1$ can either be positive or negative while $\Pi^{II} < \Pi^{oI}$ for large y. It follows that if C is close to zero, $h_I < 0$ if $\Pi^{oI} > \Pi^{II}$. However, if C is large and/or $\Pi^{II} > \Pi^{oI}$, we have that there is a unique threshold of y such that for all y above this threshold $h_I > 0$, since, by our assumption, the profits from the market Π^{oI} and Π^{II} are larger than the development costs C. ■

Proof of Proposition 3

Inserting the respective profits in the equation that determines $\tilde{C}$ yields in the case of no commitment

$$\tilde{C}_{nc} = 2\Big(\frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x+\bar{c}\gamma^2)+2(x^2-\bar{c})+x)(2\bar{c}+2\gamma\bar{c}-x)^2(2\bar{c}-2\gamma\bar{c}-x)^2}{\rho^2}-$$

$$-\frac{2K^2\bar{c}(1-\delta)(2\bar{c}-x)}{(4\bar{c}-2\bar{c}\delta-x)^2(1+\delta-x)}-\frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x-\bar{c}\gamma^2)+x(2x-1)-2\bar{c})}{(8\bar{c}(\bar{c}-x)+x(2x+1)+2\bar{c}\gamma(2\bar{c}(1-\gamma)-\gamma x)-2\bar{c})^2}+$$

$$+\frac{K^2\bar{c}(2\bar{c}-x)^2(8(\bar{c}-x)-4\bar{c}^2\gamma(1+\gamma)+2(x^2-\bar{c}+\gamma\bar{c}x))^2(2\bar{c}+2\gamma\bar{c}-x)^2}{4\rho^2}\Big)-$$

$$-\frac{K^2\bar{c}(2\bar{c}-x)^2}{(4\bar{c}-2\bar{c}\delta-x)^2(\bar{c}(1+\delta)-x)^2}+$$

$$+\frac{K^2\bar{c}(2\bar{c}-x)^2(8(\bar{c}-x)-4\bar{c}^2\gamma(1+\gamma)+2(x^2-\bar{c}+\gamma\bar{c}x))^2(2\bar{c}+2\gamma\bar{c}-x)^2}{4\rho^2},$$

while for the case of commitment we get

$$\tilde{C}_c = 2\Big(\frac{K^2\bar{c}(2\bar{c}-x)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)^2\left(4\bar{c}(1-\gamma^2)-\bar{c}(6x+1)+x(2x+1)\right)}{4\xi^2}-$$

$$\frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x-\bar{c}\gamma^2)+x(2x-1)-2\bar{c})}{(8\bar{c}(\bar{c}-x)+x(2x+1)+2\bar{c}\gamma(2\bar{c}(1-\gamma)-\gamma x)-2\bar{c})^2}-\frac{K^2\bar{c}^2(1-\delta)(\bar{c}-x)}{((2-\delta)\bar{c}-x)^2((1+\delta)\bar{c}-2x)}+$$

$$+\frac{K^2\bar{c}(\bar{c}-x)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)^2\left(4\bar{c}(1-\gamma^2)-\bar{c}(6x+1)+x(2x+1)\right)}{4\xi^2}\Big).$$

We then calculate $\tilde{C}_{nc}-\tilde{C}_c$. First, setting $x=0$ we get

$$\left(\tilde{C}_{nc}-\tilde{C}_c\right)_{x=0} = -\frac{K^2}{4\bar{c}(1+\delta)^2(2-\delta)^2(8\bar{c}-2-\gamma^2-2\bar{c}\gamma^2(5-\gamma^2))^2}\times$$

$$\times(2\bar{c}(2+\gamma)(1-\gamma)(2+2\gamma(1-\gamma)+\delta(1-\delta))-2(1-\gamma^2)+\delta(1-\delta))\times$$

$$\times(2\bar{c}(2+\gamma)(1-\gamma)(6+2\gamma(1-\gamma)+\delta(1-\delta))+2(3-\gamma^2)+\delta(1-\delta)).$$

One can easily check that this expression is negative in the admissible range of γ and δ.

On the other hand, the largest admissible value of x is given by $x=(1+\delta)\bar{c}/2$. Inserting $x=(1+\delta)\bar{c}/2$ into $\check{C}_{nc}-\check{C}_c$ yields that the sign of this difference is given by the sign of

$$\text{sign}\left\{\left(\tilde{C}_{nc}-\tilde{C}_c\right)_{x=(1+\delta)\bar{c}/2}\right\} =$$

$$\text{sign}\Big\{-(1+\delta)^2(1-\delta)^2(7-5\delta)^2\Big(\bar{c}\delta^4-\delta^3(12\bar{c}-1)+\delta^2(54\bar{c}+9\bar{c}\gamma^2-20)+$$

$$+\delta(120\bar{c}\gamma^2+27-8\gamma^2+108\bar{c})+\bar{c}(81-180\gamma+64\gamma^4)-27+24\gamma^2)\Big)^2\Big(\bar{c}\delta^4-\delta^3(8\bar{c}-1)+\delta^2(22\bar{c}-10\bar{c}\gamma^2-5)+$$

$$+\delta(40\bar{c}\gamma^2+7-4\gamma^2-24\bar{c})+\bar{c}(9+16\gamma-3064\gamma^4)-3+4\gamma^2)\Big)^2\Big\},$$

which is clearly negative.

Finally, tedious but routine calculations show that $\check{C}_{nc}-\check{C}_c$ is strictly decreasing in x which proves the result. The proof for $\hat{C}$ and $\tilde{C}$ proceeds in exactly the same way and is therefore omitted. ■

Proof of Proposition 4

We first calculate Π_I^2 for the case of no commitment and for the case of commitment. For the case of no commitment we get $\Pi_I^2 = 2(\Pi^{II} - \Pi^{oI} + C) + \Pi_I^{II}$. Inserting the respective expression derived in Section 3 we get

$$\Pi_I^2 = 2\Big(\frac{2K^2\bar{c}(1-\delta)(2\bar{c}-x)}{(4\bar{c}-2\bar{c}\delta-x)^2(1+\delta-x)}- \tag{16}$$

$$-\frac{K^2\bar{c}(2\bar{c}-x)(8\bar{c}(\bar{c}-x+\bar{c}\gamma^2)+2(x^2-\bar{c})+x)(2\bar{c}+2\gamma\bar{c}-x)^2(2\bar{c}-2\gamma\bar{c}-x)^2}{\rho^2}+C\Big)+$$

$$+\frac{K^2\bar{c}(2\bar{c}-x)^2}{(4\bar{c}-2\bar{c}\delta-x)^2(\bar{c}(1+\delta)-x)^2}.$$

For the case of commitment we get $\Pi_I^2 = 2(\Pi^{II} - \Pi^{oI} + C)$. Inserting the respective expression derived in Section 4 we get

$$\Pi_I^2 = 2\Big(\frac{K^2\bar{c}^2(1-\delta)(\bar{c}-x)}{((2-\delta)\bar{c}-x)^2((1+\delta)\bar{c}-2x)}- \tag{17}$$

$$-\frac{K^2\bar{c}(2\bar{c}-x)\left(2\bar{c}^2(2-\gamma-\gamma^2)-\bar{c}x(6-\gamma)+2x^2\right)^2\left(4\bar{c}(1-\gamma^2)-\bar{c}(6x+1)+x(2x+1)\right)}{4\xi^2}+C\Big).$$

Subtracting the right-hand side of (17) from the right-hand side of (16) and setting $x=0$ we get

$$\left(\Pi_I^2(nc)-\Pi_g^2(c)\right)_{x=0} = \frac{K^2}{\bar{c}(1+\delta)^2(2-\delta)^2} > 0.$$

Taking the other extreme, i.e. setting $x=(1+\delta)/2$ yields that the sign of $\Pi_I^2(nc) - \Pi_I^2(c)$ is given by the sign of

$$\text{sign}\left\{\left(\Pi_I^2(nc)-\Pi_I^2(c)\right)_{x=(1+\delta)/2}\right\} =$$

$$\text{sign}\Big\{-(1+\delta)^2(1-\delta)^2(7-5\delta)^2\Big(\bar{c}\delta^4-\delta^3(12\bar{c}-1)+\delta^2(54\bar{c}+9\bar{c}\gamma^2-20)+$$

$$+\delta(120\bar{c}\gamma^2+27-8\gamma^2+108\bar{c})+\bar{c}(81-180\gamma+64\gamma^4)-27+24\gamma^2)\Big)^2\Big(\bar{c}\delta^4-\delta^3(8\bar{c}-1)+\delta^2(22\bar{c}-10\bar{c}\gamma^2-5)+$$

$$+\delta(40\bar{c}\gamma^2 + 7 - 4\gamma^2 - 24\bar{c}) + \bar{c}(9 + 16\gamma - 3064\gamma^4) - 3 + 4\gamma^2)\Big)^2\Big\},$$

which is negative.

As in the proof of Proposition 2 routine calculations show that $\Pi_I^2(nc) - \Pi_I^2(c)$ is strictly increasing in x which proves the result. The proof for $\Pi_I^1(nc) - \Pi_I^1(c)$ proceeds in exactly the same way and is therefore omitted. ■

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