

Interim report on multi-country Extension of the Baseline Model. Revised version

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Revised version.

Author: Francesco Bianchi, Diego Comin, Thilo Kind, and Howard Kung

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Coordinator: Dr. Georg Licht, ZEW

Email: licht@zew.de





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Project Information Summary

Table 1: Project Information Summary

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	Diego Comin (Dartmouth, CEPR, NBER),
	Thilo Kind (LBS), Howard Kung (LBS, CEPR)
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Executive Summary

We develop a multi-country DSGE model to study the effect of fiscal policies in the presence of asymmetric business cycle shocks. Our model features endogenous technology adoption and a rich fiscal side as well as international interactions through trade and portfolio composition. These features allow us to provide a realistic characterization of the effect of fiscal policies on the supply side and of the international propagation of shocks and policy interventions. We use our model to study the effects of different fiscal policies in Spain and Germany in the context of the global financial crisis.

The calibrated model is able to provide a realistic account of the fluctuations in sovereign credit spreads, business cycle fluctuations, and growth rates. We perform counterfactual experiments to decompose the effects of large severe liquidity shocks on the eurozone. Overall, we show the the technology diffusion margin is important for propagating the adverse fiscal shocks from one country to the rest of the eurozone.



1 Introduction

The great recession has staged the tensions between two views on fiscal policy. The traditional Keynesian view advocated for expansionary fiscal policy to make up for the decline in aggregate demands. An opposite view advocated for contractionary fiscal policy in response to the increase in government deficits and sovereign bond spreads (See Figures 1 and 2). Proponents of fiscal austerity argued that the low productivity problems were as important for the downturn as aggregate demand and that structural reforms that alleviated these supply side problems could restore economic activity.

These divergent views on fiscal policy were reflected by the implementation of different fiscal policies across countries. As illustrated in Figure 3, countries in the euro area adopted a more austere fiscal policy than the US. Some have argued that this difference in fiscal policy explains the faster recovery experienced by the US economy.

Any exercise that aims at accurately evaluate alternative fiscal policies needs to rely on a rich enough model that captures two elements that we consider key. First, the interconnections between national economies which are both deep and widespread in the euro area. Second, the inter-relation between business cycle conditions and the current and future productive capacity of the economy.

The goal of this paper is to develop a setting that captures these key components and to use it to evaluate the impact of a rich array of fiscal policies in the euro area during the great recession. Our framework contains two open economies that interact through international trade and capital flows derived from the transaction of sovereign bonds. Financial rigidities are introduced through a stochastic cost to the acquisition of government bonds which results in time-varying bond spreads across countries. The world technology frontier grows at a stochastic rate, but domestic productivity is determined by the endogenous range of technologies adopted by producers. The adoption margin connects local supply conditions to local and foreign economic and financial conditions. A rich set of fiscal details that include explicit fiscal policy rules for tax rates, government expenditures and transfers, and distortionary taxation of labor income, capital, consumption, as well as subsidies to the adoption of new technologies.

We plan on estimating the model using Bayesian methods using as home and foreign countries Spain and Germany, respectively. For the time being, we have explored the model insights through a calibration exercise.

Our key findings are as follows. We compare the effects of robust and non-robust fiscal policies to a domestic financial shock. The robust fiscal policy rule reacts aggressively to an increase debt-to-GDP ratio, and as a result the government's increase in taxes is more pronounced. As a result of the higher taxes, the adoption rate declines more under the robust rule leading to a significantly larger decline in GDP over the medium term.

We also compare the effects of using different distortionary tax instruments after a domestic liquidity demand shock. In particular, we consider taxes to capital, consumption and labor that yield similar levels of debt-to-GDP ratio after 25 years. We find that labor income tax leads to the most severe recessions, while capital taxes lead to the least severe recessions.

Finally, we show that a domestic recession caused by a liquidity demand shock also affects negatively the foreign country. In particular, we observe a drop in foreign consumption and investment s well as in the rate of technology adoption.

The rest of the paper is organized as follows. Section 1.1 presents the related literature. Section 2 develops the model. Section 3 conducts a quantitative evaluation of the fiscal policy responses to liquidity demand shocks and section 4 concludes.



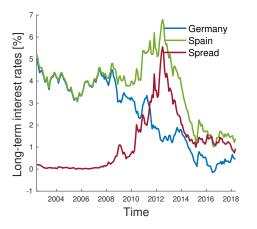


Figure 1: Long-term interest rates of Germany and Spain, as well as their difference in annualized percentage points.

Figure 2: The ratio of government debt to the gross domestic product.

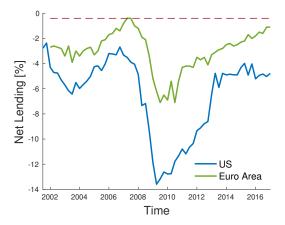


Figure 3: The government's net lending as a fraction of the gross domestic product.

1.1 Related Literature

[INSERT]

2 A Two Country Model

The model consists of two open economies which the variables are denoted as (X_t, X_t^*) . The final goods are produced competitively and is tradable between both countries. The differentiated intermediate goods are produced by a monopolist and non-tradable, while all prices are denoted in the same currency. A representative agent consumes a consumption index, consisting of the final goods of each country. The household owns the domestic capital, all domestic firms,



and holds a portfolio of domestic and foreign government bonds. Besides issuing bonds, the government finances expenditures through distortionary taxes to labor income, capital income, and on consumption goods, in addition to lump-sum transfers. Further, there is an exogenously evolving world technology frontier. Firms of each country independently invest to adopt technology using the final goods such that the intermediate goods variety for producing the final good expands. Furthermore, markets are incomplete and only non-state-contingent government bonds are traded. We describe the set of equations for the home country, however, the foreign country's problem is characterized in the same way.

2.1 Representative Household

The household's preferences over the consumption goods index C_t , and total labor supply L_t are given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi_0 Z_t^{1-\sigma} \frac{L_t^{1+\varphi}}{1+\varphi} \right) \right] \tag{1}$$

(2)

$$C_{t} = \left((1 - v)^{\frac{1}{\eta}} \left(C_{Ht} \right)^{\frac{\eta - 1}{\eta}} + v^{\frac{1}{\eta}} \left(C_{Ft} \right)^{\frac{\eta - 1}{\eta}} \right)^{\frac{\eta}{\eta - 1}}$$
(3)

$$\mathcal{P}_{t} = \left((1 - v) (P_{Ht})^{1-\eta} + v (P_{Ft})^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{4}$$

where we scale the disutility of labor by the level of the world technological frontier Z_t^1 . Here, the representative household accumulates the capital stock K_t according to:

$$K_{t+1} = (1 - \delta) K_t + I_t \left[1 - \Psi \left(\iota_t \frac{I_t}{I_{t-1}} \right) \right]$$

$$(5)$$

$$\log \iota_t = \rho_\iota \log \iota_{t-1} + \sigma_\iota \varepsilon_t^\iota, \tag{6}$$

where the convex function Ψ has the property that $\Psi(\cdot)_{|ss} = \Psi'(\cdot)_{|ss} = 0$ and ι_t is a mean one shock to the marginal efficiency to investment. Then, the household's budget constraint reads

$$(1 + \tau_t^c) \mathcal{P}_t \mathcal{C}_t = \left(1 - \tau_t^l\right) W_t L_t + \Omega_t + \left(1 - \tau_t^k\right) R_{kt} K_t - P_{Ht} I_t + T_t \tag{7}$$

$$-\mathcal{P}_t\left(\left(\kappa_{Ht} + \eta_{Ht}B_{Ht}\right)B_{Ht} - R_{B,t-1}^r B_{Ht-1}\right) \tag{8}$$

$$-\mathcal{P}_{t}^{*}\left(\left(\kappa_{Ft} + \eta_{Ft}B_{Ft}\right)B_{Ft} - R_{B,t-1}^{r*}B_{Ft-1}\right),\tag{9}$$

where R_{Bt}^r is the return on a government bond and R_{kt} is the return on physical capital. Ω_t represents the total domestic profits² and T_t are the lump-sum transfers from the government. τ_t^l and τ_t^k are tax rates on labor income and capital income, whereas τ_t^c is the domestic tax rate on consumption goods of domestic and foreign goods. In addition, the household pays a

¹The corresponding demand functions for the domestic and foreign final good are $C_{Ht} = (1 - v) (P_{Ht}/\mathcal{P}_t)^{-\eta} \mathcal{C}_t$ and $C_{Ft} = v (P_{Ft}/\mathcal{P}_t)^{-\eta} \mathcal{C}_t$.

 $^{{}^2\}Omega_t = \Omega_{At} + \Omega_{Bt}$ where Ω_{At} are firms' profits. Implicitly, Ω_{At} in the consumer's budget constraint accounts for when a new technology is developed; consumers must pay J to get the option to adopt it. Then, after successfully investing in adoption, the technology's value becomes V. Ω_{Bt} are the total trading fees on the domestic government bond payed by the domestic and foreign household.



fee $\kappa_{Ht} + \eta_{Ht}B_{Ht} - 1$ per unit of bond traded³. The corresponding first order conditions with respect to domestic and foreign bonds are

$$1 + \underbrace{2\frac{\eta_{Ht}}{\kappa_{Ht}}}_{\equiv (\tilde{\eta}_{Ht} - \bar{\eta}_H)/Z_t} B_{Ht} = \frac{R_{B,t}^r}{\kappa_{Ht}} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \right]$$

$$(10)$$

$$1 + \frac{(\tilde{\eta}_{Ft} - \bar{\eta}_F)}{Z_t} B_{Ft} = \frac{R_{B,t}^{r*}}{\kappa_{Ft}} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left(\frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} \right) \right]$$
(11)

where $Q_t = \frac{\mathcal{P}_t}{\mathcal{P}_t^*}$ is the real exchange rate between the two countries and $\Lambda_{t,t+1} = \beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\sigma}$ is the real stochastic discount factor. We model the trading costs, κ_t and $\tilde{\eta}_t$, in reduced-form according to

$$\log \kappa_t = (1 - \rho_\kappa) \,\bar{\kappa} + \rho_\kappa \log \kappa_{t-1} + \sigma_\kappa \varepsilon_t^\kappa \tag{12}$$

$$\log \tilde{\eta}_t = (1 - \rho_{\tilde{\eta}}) \, \bar{\tilde{\eta}} + \rho_{\tilde{\eta}} \log \tilde{\eta}_{t-1} + \sigma_{\tilde{\eta}} \varepsilon_t^{\tilde{\eta}}. \tag{13}$$

The spread between the two countries is given by

$$R_{B,t}^{r} - R_{B,t}^{r*} = \frac{\kappa_{Ht} + \kappa_{Ht} \left(\tilde{\eta}_{Ht} - \bar{\eta}_{H} \right) B_{Ht}}{\mathbb{E}_{t} \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_{t}^{c}}{1 + \tau_{t+1}^{c}} \right) \right]} - \frac{\kappa_{Ft} + \kappa_{Ft} \left(\tilde{\eta}_{Ft} - \bar{\eta}_{F} \right) B_{Ft}}{\mathbb{E}_{t} \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_{t}^{c}}{1 + \tau_{t+1}^{c}} \right) \left(\frac{\mathcal{Q}_{t}}{\mathcal{Q}_{t+1}} \right) \right]}$$
(14)

Other optimality conditions for labor supply, investment, and the return on capital are

$$Z_t^{1-\sigma} \varphi_0 \mathcal{C}_t^{\sigma} L_t^{\varphi} = \left(\frac{1-\tau_t^l}{1+\tau_t^c}\right) \frac{W_t}{\mathcal{P}_t} \tag{15}$$

$$\frac{P_{Ht}}{\mathcal{P}_t} \frac{C_t^{-\sigma}}{(1+\tau_t^c)} = \mu_t \left(1 - \iota_t \frac{I_t}{I_{t-1}} \Psi'(\cdot) - \Psi\left(\frac{I_t}{I_{t-1}}\right) \right) + \beta \mathbb{E}_t \left[\mu_{t+1} \iota_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 \Psi'(\cdot) \right]$$
(16)

$$\mu_{t} = \beta \mathbb{E}_{t} \left[\mu_{t+1} \left(1 - \delta \right) + \frac{C_{t}^{-\sigma}}{\mathcal{P}_{t+1}} \left(\frac{1 - \tau_{t+1}^{k}}{1 + \tau_{t+1}^{c}} \right) R_{kt+1} \right]$$
(17)

where μ_t is the Lagrange multiplier on the capital accumulation equation.

2.2 Final Good

The competitive final good producer faces a constant elasticity production function which nests all differentiated domestic goods

$$Y_{Ht} = \left(\int_0^{A_t} (Y_t(j))^{\frac{\varepsilon - 1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$
(18)

$$P_{Ht} = \left(\int_0^{A_t} (P_t(j))^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$
(19)

³For constant coefficients, this specification can be mapped to Benigno (2009) or Ghironi, Lee, Rebucci (2006). There, a quadratic cost in changes in the real asset position when trading in the foreign bond market is incurred. The cost of moving the holdings of foreign assets serves for the purpose of determining the steady-state value of the foreign-asset position



and the corresponding demand function reads

$$Y_t(j) = \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\varepsilon} Y_{Ht} \tag{20}$$

2.3 Intermediate Goods

Each differentiated domestic good $j \in [0, A_t]$ is produced by a monopolist that uses labor and capital as factor inputs subject to the demand function of the final good producer in each country. The production technology of monopolist j is characterized by

$$Y_t(j) = \xi_t (K_t(j))^{\alpha} (L_t(j))^{1-\alpha}$$
(21)

$$\log \xi_t = \rho_{\xi} \log \xi_{t-1} + \sigma_{\xi} \varepsilon_t^{\xi} \tag{22}$$

where ξ_t is the stationary productivity process for the intermediate production technology. Using profit maximization, the optimality conditions read

$$W_{t} = \frac{\epsilon - 1}{\epsilon} P_{t}(j) (1 - \alpha) \frac{Y_{t}(j)}{L_{t}(j)}$$

$$(23)$$

$$R_{kt} = \frac{\epsilon - 1}{\epsilon} P_t(j) \alpha \frac{Y_t(j)}{K_t(j)}$$
(24)

$$\pi_t(j) = \frac{1}{\epsilon} P_t(j) Y_t(j) \tag{25}$$

2.4 Technology

As in Comin et al. (2009), the world technological frontier, Z_t , evolves exogenously and according to

$$Z_{t+1} = \left(\bar{\chi}\chi_t^{\zeta} + \phi\right) Z_t \tag{26}$$

$$\log\left(\chi_{t}\right) = \rho_{\gamma}\log\left(\chi_{t-1}\right) + \sigma_{\gamma}\varepsilon_{t}^{\chi} \tag{27}$$

An adoption firm k tries to make a technology $k \in Z_t \cap A_t$ usable by investing the final domestic good. The firm is owned by the domestic representative household and can sell the adopted technology on the open market. Let $J_t(k)$ be the value of an unadopted innovation. Then, the optimization problem and the corresponding optimality conditions read

$$J_{t}(k) = \max_{H_{t}(k)} \mathbb{E}_{t} \left\{ -P_{Ht}H_{t}(k) + \phi \Lambda_{t,t+1} \frac{\mathcal{P}_{t}}{\mathcal{P}_{t+1}} \left[\lambda_{t}(k) V_{t+1}(k) + (1 - \lambda_{t}(k)) J_{t+1}(k) \right] \right\}$$
(28)

$$P_{Ht} = \rho_{\lambda} \frac{\lambda_{t}(k)}{H_{t}(k)} \phi \mathbb{E}_{t} \left\{ \Lambda_{t,t+1} \frac{\mathcal{P}_{t}}{\mathcal{P}_{t+1}} \left(V_{t+1}(k) - J_{t+1}(k) \right) \right\}$$

$$(29)$$

$$V_{t}(k) = \pi_{t}(k) + \phi \mathbb{E}_{t} \left\{ \Lambda_{t,t+1} \frac{\mathcal{P}_{t}}{\mathcal{P}_{t+1}} V_{t+1}(k) \right\}$$

$$(30)$$

$$\lambda_t(k) = \bar{\lambda} \left(F_t H_t(k) \right)^{\rho_{\lambda}} \tag{31}$$

$$F_t = \frac{Z_t}{K_t}. (32)$$

where $\Lambda_{t,t+1}$ is the real stochastic discount factor. Given that the solution is symmetric, the total cost of adoption and the time evolution of adopted technologies is given by

$$H_t^T = H_t \left(Z_t - A_t \right) \tag{33}$$

$$A_{t+1} = \lambda_t \phi \left[Z_t - A_t \right] + \phi A_t \tag{34}$$



2.5 Government

The government finances expenditures, G_t , by issuing real bonds (to domestic and foreign investors), lump-sum transfers, and taxes on consumption, labor, and capital income:

$$\frac{P_{Ht}}{P_t}G_t = B_t - R_{Bt-1}^r B_{t-1} + \tau_t^l W_t^r L_t - T_t^r + \tau_t^c C_t + \tau_t^k R_{kt}^r K_t,$$
(35)

where R_{Bt}^r is the real interest rate on real government bond holdings. We model the fiscal policy rules for expenditures, tax rates, and transfers as in Leeper et al. (2010). Their log-deviations from the steady state follow

$$\hat{u}_t^x = \rho_x \hat{u}_{t-1}^x + \sigma_x \varepsilon_t^x, \quad x \in \{g, l, c, k, T\}$$
(36)

$$\hat{G}_t = -\varphi_g \hat{Y}_{Ht}^{HW} - \zeta_g \hat{b}_{t-1} + \hat{u}_t^g \tag{37}$$

$$\hat{T}_{t} = -\varphi_{T} \hat{Y}_{Ht}^{HW} - \zeta_{T} \hat{b}_{t-1} + \hat{u}_{t}^{T}$$
(38)

$$\hat{\tau}_t^k = \varphi_k \hat{Y}_{Ht}^{HW} + \zeta_k \hat{b}_{t-1} + \varphi_{kl} \hat{u}_t^l + \varphi_{kc} \hat{u}_t^c + \hat{u}_t^k$$
(39)

$$\hat{\tau}_t^l = \varphi_l \hat{Y}_{Ht}^{HW} + \zeta_l \hat{b}_{t-1} + \varphi_{lk} \hat{u}_t^k + \varphi_{lc} \hat{u}_t^c + \hat{u}_t^l$$

$$\tag{40}$$

$$\hat{\tau}_t^c = \varphi_c \hat{Y}_{Ht}^{HW} + \zeta_c \hat{b}_{t-1} + \varphi_{ck} \hat{u}_t^k + \varphi_{cl} \hat{u}_t^l + \hat{u}_t^c. \tag{41}$$

The government reacts to the log-output gap which is smoothed by the Holt-Winters filter and to the previous deviation from the non-stochastic steady state of the log Debt-to-GDP ratio \hat{b}_{t-1} :

$$\log Y_{Ht}^{HW} = \tau_{HW}^g \log Y_{Ht-1}^{HW} + (1 - \tau_{HW}^g) \left(\log Y_{Ht} - \log Y_{Hss}\right) \tag{42}$$

$$\hat{B}_{t-1} = \log\left(\frac{P_{t-1}B_{t-1}}{P_{Ht-1}Y_{Ht-1}}\right) - \log\left(\frac{P_{ss}B_{ss}}{P_{Hss}Y_{Hss}}\right)$$
(43)

2.6 Resource Constraints

$$\underbrace{P_{Ft}C_{Ft} - P_{Ht}C_{Ft}^*}_{-NX_t} = \mathcal{P}_t \left(B_{Ft}^* - R_{Bt-1}^r B_{Ft-1}^* \right) - \mathcal{P}_t^* \left(B_{Ft} - R_{B,t-1}^{r*} B_{Ft-1} \right)$$
(44)

$$-\mathcal{P}_{t}^{*}\left(\left(\kappa_{Ft} + \eta_{Ft}B_{Ft} - 1\right)B_{Ft} - \frac{\mathcal{P}_{t}}{\mathcal{P}_{t}^{*}}\left(\kappa_{Ft}^{*} + \eta_{Ft}^{*}B_{Ft}^{*} - 1\right)B_{Ft}^{*}\right)$$
(45)

$$Y_{Ht} = C_{Ht} + C_{Ft}^* + H_t^T + I_t + G_t (46)$$

$$B_t = B_{Ht} + B_{Ft}^* \tag{47}$$

$$L_t = \int_0^{A_t} L_t(j) \, dj \tag{48}$$

$$K_t = \int_0^{A_t} K_t(j) \, dj \tag{49}$$

2.7 International Identities

The terms-of-trade is defined as the price ratio of imported to exported goods of the home country. The real exchange rate is defined as the price ratio of the two price indices.





$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}} \tag{50}$$

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}}$$

$$Q_t = \frac{P_t}{P_t^*}$$
(50)

Competition in the final sector insures that the law of one price holds for the tradable goods, i.e.

$$P_{Ft} = P_{Ht}^* \tag{52}$$

$$P_{Ft} = P_{Ht}^*$$
 (52)
 $P_{Ht} = P_{Ft}^*$ (53)



3 Results

Parameter	Value	Parameter	Value
β	0.98	ζ_g	0.00
σ	1.00	ζ_l	2.80
v	0.40	ζ_k	3.50
arphi	3.00	ζ_c	1.70
α	0.60	$arphi_g$	0.00
ϵ	3.50	$ar{\chi}$	0.06
δ	0.02	$ ho_\chi$	0.84
η	2.00	$ ho_{\lambda}$	0.90
ϕ	0.95	$ar{\lambda}_1$	0.63
ζ	0.80	$ar{\lambda}_2$	0.68

Table 4: Parameter values of the model, calibrated at quarterly frequencies

	Home country		Foreign country		
Variable	mean	ac1	mean	ac1	
A. Macro Variables					
Consumption index growth	1.62%	0.54	1.54%	0.58	
Investment growth	6.54%	0.69	6.07%	0.75	
GDP growth	2.68%	0.31	2.49%	0.37	
B. Financial Variables					
Debt-to-GDP	42.5%	0.69	30.2%	0.74	
Return on bonds	5.67%	0.72	3.41%	0.75	
Return on capital	7.06%	0.59	6.82%	0.63	

Table 5: Annualized macroeconomic and financial moments

Consumption correlation between the two countries: 0.35

3.1 Technology Adoption

We show the transmission of a liquidity shock to bonds issued by the home country over a horizon of 25 years. Figure 4 plots the impulse response functions for government Debt-to-GDP, the interest rate spread between the two countries, adoption probability, physical investment, GDP, and consumption of the home country. The IRFs compare an economy with and without



adoption, holding everything else equal. While the effect on fiscal distress is small, the endogenous adoption margin amplifies the dynamics of the macroeconomic variables, particularly at medium-term frequencies. Adoption-related investment activities decrease with respect to a liquidity shock which ultimately leads to a deeper and longer recession. Capital investment, GDP, and consumption all decline more strongly and persistently with endogenous technology adoption.

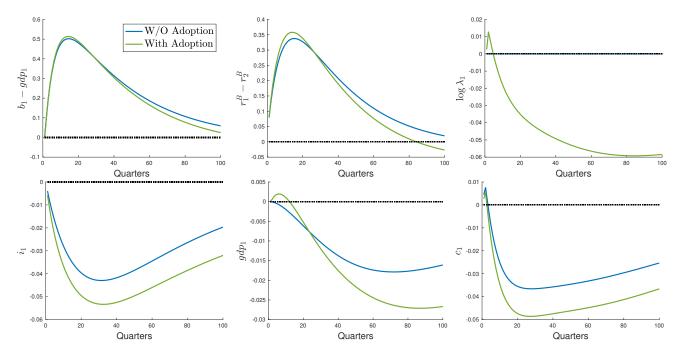


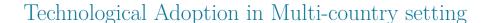
Figure 4: Home country's impulse response functions to a liquidity shock to bonds issued by the home country: Debt-to-GDP, Spread, Adoption, Investment, GDP, and Consumption.

3.2 Fiscal Robustness

Figure 5 compares two different government responses to a liquidity shock to the home country. The robust fiscal policy rule reacts aggressively to an increase in the Debt-to-GDP ratio. The government's increase in taxes is more pronounced in the robust case. In contrast, the non robust fiscal policy government increases taxes only slightly. Crucially, figure 5 highlights the potential cost of a tight fiscal policy in the light of endogenous technology adoption. A strong fiscal reaction to fiscal distress leads to a strong and persistent decline in GDP.

3.3 Taxation

We compare different government tax instruments. Figure 6 shows the home country's impulse response functions to a liquidity shock to bonds issued by the home country for various tax policies. We compare three different regimes in which the government stabilizes debt by either raising capital tax, consumption tax, or labor tax. Each of those instruments distorts the optimal equilibrium outcome. However, labor tax has the strongest effect and leads to the most severe recession as it distorts the household's optimal consumption leisure decision. Capital tax on the other hand, which distorts the optimal investment decision of the household, has the most moderate effect on macroeconomic dynamics. Further, figure 7 shows that the foreign country is





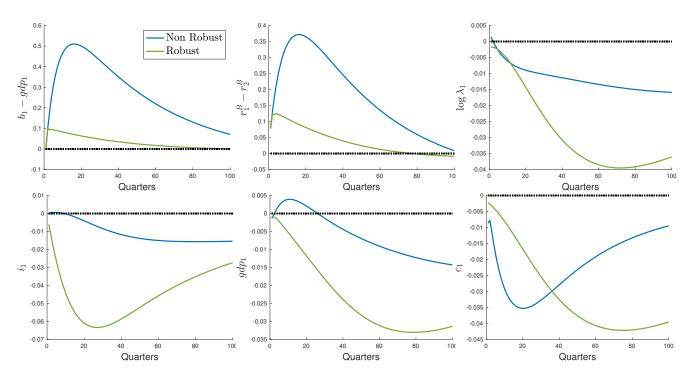


Figure 5: Home country's impulse response functions to a liquidity shock to bonds issued by the home country: Debt-to-GDP, Spread, Adoption, Investment, GDP, and Consumption.

adversely affected by a liquidity shock to the home country. Similarly, the two key ingredients, adoption in conjunction with different fiscal policy instruments, have strong effects on growth. It indicates that not only the response to fiscal stress in the first country is important. In perspective, Germany's fiscal choice to a reaction to financial distress in Spain has also strong implications on macroeconomic fluctuations in Germany.

4 Conclusion

We develop a multi-country DSGE model to study the effects of fiscal policies in the presence of asymmetric business cycle shocks. Our model features endogenous technology adoption and a rich fiscal side as well as international interactions through trade and portfolio composition. These features allow us to provide a realistic characterization of the effects of fiscal policies on the supply side and of the international propagation of shocks and policy interventions. We use our model to study the effects of different fiscal policies in Spain and Germany in the context of the global financial crisis. The calibrated model is able to provide a realistic account of the fluctuations in sovereign credit spreads, business cycle fluctuations, and growth rates. We perform counterfactual experiments to decompose the effects of large severe liquidity shocks on the eurozone. Overall, we show the the technology diffusion margin is important for propagating the adverse liquidity shocks from one country to the rest of the eurozone.



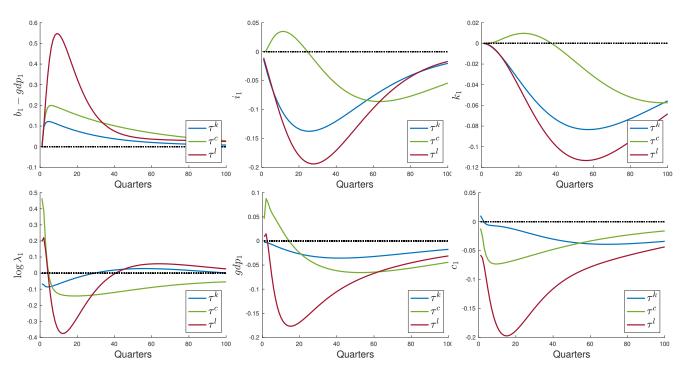


Figure 6: Home country's impulse response functions to a liquidity shock to bonds issued by the home country: Debt-to-GDP, Spread, Adoption, Investment, GDP, and Consumption.

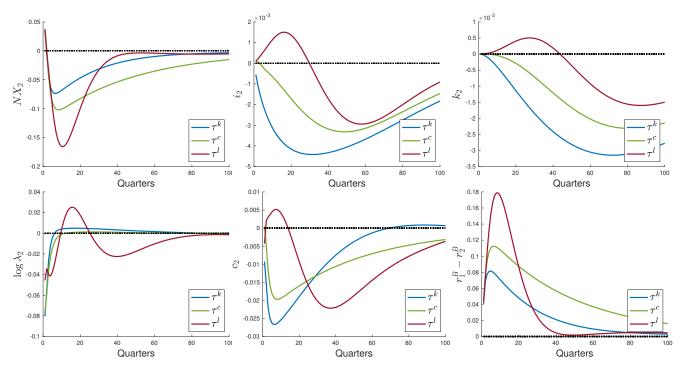


Figure 7: Foreign country's impulse response functions to a liquidity shock to bonds issued by the home country: Net-Exports, Investment, Capital, Adoption, Consumption, and the spread.



A Real Symmetric Conditions

• Exogenous processes

$$Z_{t+1} = \left(\bar{\chi}\chi_t^{\zeta} + \phi\right) Z_t \tag{54}$$

$$\log\left(\chi_{t}\right) = \rho_{\chi}\log\left(\chi_{t-1}\right) + \sigma_{\chi}\varepsilon_{t}^{\chi} \tag{55}$$

$$\log \kappa_t = (1 - \rho_\kappa) \,\bar{\kappa} + \rho_\kappa \log \kappa_{t-1} + \sigma_\kappa \varepsilon_t^\kappa \tag{56}$$

$$\log \tilde{\eta}_t = (1 - \rho_{\tilde{\eta}}) \, \bar{\tilde{\eta}} + \rho_{\tilde{\eta}} \log \tilde{\eta}_{t-1} + \sigma_{\tilde{\eta}} \varepsilon_t^{\tilde{\eta}} \tag{57}$$

$$\log \xi_t = \rho_{\xi} \log \xi_{t-1} + \sigma_{\xi} \varepsilon_t^{\xi} \tag{58}$$

$$\log \iota_t = \rho_\iota \log \iota_{t-1} + \sigma_\iota \varepsilon_t^\iota \tag{59}$$

$$\varsigma_t = \varsigma_0 + \varsigma_1 \left(\hat{b}_t - \bar{\hat{b}} \right) + \sigma_\varsigma \varepsilon_t^\varsigma \tag{60}$$

$$\hat{u}_t^x = \rho_x \hat{u}_{t-1}^x + \sigma_x \varepsilon_t^x, \quad x \in \{g, l, c, k, T\}$$

$$\tag{61}$$

• Price relations

$$\frac{\mathcal{P}_t}{P_{Ht}} = \left((1 - v) + v \left(\mathcal{S}_t \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} \equiv h \left(\mathcal{S}_t \right)$$
(62)

$$\frac{\mathcal{P}_t}{P_{Ft}} = \frac{h\left(\mathcal{S}_t\right)}{\mathcal{S}_t} \tag{63}$$

$$Q_t = \frac{\mathcal{P}_t}{\mathcal{P}_t^*} = \frac{\mathcal{P}_t}{P_{Ht}} \frac{P_{Ht}}{\mathcal{P}_t^*} = \frac{\mathcal{P}_t}{P_{Ht}} \frac{P_{Ht}}{P_{Ft}} \frac{P_{Ft}}{\mathcal{P}_t^*} = \frac{1}{\mathcal{S}_t} \frac{h(\mathcal{S}_t)}{f(\mathcal{S}_t)}$$
(64)

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}} \tag{65}$$

$$P_{Ft} = P_{Ht}^* \tag{66}$$

$$P_{Ht} = P_{Ft}^* \tag{67}$$

$$\frac{P_{Ht}}{P_t} = A_t^{\frac{1}{1-\varepsilon}} \tag{68}$$

• Household

$$C_{t} = \left((1 - v)^{\frac{1}{\eta}} \left(C_{Ht} \right)^{\frac{\eta - 1}{\eta}} + v^{\frac{1}{\eta}} \left(C_{Ft} \right)^{\frac{\eta - 1}{\eta}} \right)^{\frac{\eta}{\eta - 1}}$$
(69)

$$C_{Ht} = (1 - v) \left(\frac{P_{Ht}}{\mathcal{P}_t}\right)^{-\eta} \mathcal{C}_t \tag{70}$$

$$C_{Ft} = \upsilon \left(\frac{P_{Ft}}{\mathcal{P}_t}\right)^{-\eta} \mathcal{C}_t \tag{71}$$

$$K_{t+1} = (1 - \delta) K_t + I_t \left[1 - \Psi \left(\iota_t \frac{I_t}{I_{t-1}} \right) \right]$$
 (72)

$$Z_t^{1-\sigma}\varphi_0 \mathcal{C}_t^{\sigma} L_t^{\varphi} = \left(\frac{1-\tau_t^l}{1+\tau_t^c}\right) \frac{W_t}{\mathcal{P}_t}$$

$$\tag{73}$$

$$\frac{P_{Ht}}{\mathcal{P}_t} \frac{\mathcal{C}_t^{-\sigma}}{(1+\tau_t^c)} = \mu_t \left(1 - \iota_t \frac{I_t}{I_{t-1}} \Psi'(\cdot) - \Psi\left(\iota_t \frac{I_t}{I_{t-1}}\right) \right) + \beta \mathbb{E}_t \left[\mu_{t+1} \iota_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \Psi'(\cdot) \right]$$
(74)



• Returns

$$\mu_{t} = \beta \mathbb{E}_{t} \left[\mu_{t+1} \left(1 - \delta \right) + \frac{C_{t}^{-\sigma}}{\mathcal{P}_{t+1}} \left(\frac{1 - \tau_{t+1}^{k}}{1 + \tau_{t+1}^{c}} \right) R_{kt+1} \right]$$
 (75)

$$1 + \frac{(\tilde{\eta}_{Ht} - \bar{\eta}_H)}{Z_t} B_{Ht} = \frac{R_{B,t}^r}{\kappa_{Ht}} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) (1 - \mathbb{I}_{Dt+1} \gamma_{t+1}) \right]$$
(76)

$$1 + \frac{(\tilde{\eta}_{Ft} - \bar{\eta}_F)}{Z_t} B_{Ft} = \frac{R_{B,t}^{r*}}{\kappa_{Ft}} \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left(\frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} \right) \left(1 - \mathbb{I}_{Dt+1}^* \gamma_{t+1}^* \right) \right]$$
(77)

$$p_{Dt} = \frac{1}{1 + e^{-\varsigma_t}} \tag{78}$$

$$\Lambda_{t,t+1} = \beta \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t}\right)^{-\sigma} \tag{79}$$

• Final good

$$Y_{Ht} = A_t^{\frac{\varepsilon}{\varepsilon - 1}} Y_t \tag{80}$$

$$Y_t = \left(\frac{P_t}{P_{Ht}}\right)^{-\epsilon} Y_{Ht} \tag{81}$$

• Intermediate goods

$$Y_t = \frac{1}{A_t} \xi_t K_t^{\alpha} L_t^{1-\alpha} \tag{82}$$

$$W_t^r = \frac{\epsilon - 1}{\epsilon} \frac{P_t}{P_t} (1 - \alpha) A_t \frac{Y_t}{L_t}$$
(83)

$$R_{kt}^{r} = \frac{\epsilon - 1}{\epsilon} \frac{P_{t}}{\mathcal{P}_{t}} \alpha A_{t} \frac{Y_{t}}{K_{t}}$$
 (84)

$$\pi_t^r = \frac{1}{\epsilon} \frac{P_t}{P_t} Y_t \tag{85}$$

• Technology

$$J_{t}^{r} = \max_{H_{t}} \mathbb{E}_{t} \left\{ -\frac{P_{Ht}}{P_{t}} H_{t} + \phi \Lambda_{t,t+1} \left[\lambda_{t} V_{t+1}^{r} + (1 - \lambda_{t}) J_{t+1}^{r} \right] \right\}$$
(86)

$$\frac{P_{Ht}}{\mathcal{P}_{t}} = \rho_{\lambda} \frac{\lambda_{t}}{H_{t}} \phi \mathbb{E}_{t} \left\{ \Lambda_{t,t+1} \left(V_{t+1}^{r} - J_{t+1}^{r} \right) \right\}$$
(87)

$$V_t^r = \pi_t^r + \phi \mathbb{E}_t \left\{ \Lambda_{t,t+1} V_{t+1}^r \right\}$$
(88)

$$\lambda_t = \bar{\lambda} \left(F_t H_t \right)^{\rho_{\lambda}} \tag{89}$$

$$F_t = \frac{Z_t}{K_t} \tag{90}$$

$$H_t^T = H_t \left(Z_t - A_t \right) \tag{91}$$

$$A_{t+1} = \lambda_t \phi \left[Z_t - A_t \right] + \phi A_t \tag{92}$$



Technological Adoption in Multi-country setting

• Government

$$\frac{P_{Ht}}{P_t}G_t = B_t - R_{Bt-1}^r \left(1 - \mathbb{I}_{Dt}\gamma_t\right) B_{t-1} + \tau_t^l W_t^r L_t - T_t^r + \tau_t^c C_t + \tau_t^k R_{kt}^r K_t, \tag{93}$$

$$GDP_t = P_{Ht}Y_{Ht} \tag{94}$$

$$\hat{G}_t = -\varphi_g \hat{Y}_{Ht}^{HW} - \zeta_g \hat{b}_{t-1} + \hat{u}_t^g \tag{95}$$

$$\hat{T}_t = -\varphi_T \hat{Y}_{Ht}^{HW} - \zeta_T \hat{b}_{t-1} + \hat{u}_t^T \tag{96}$$

$$\hat{\tau}_t^k = \varphi_k \hat{Y}_{Ht}^{HW} + \zeta_k \hat{b}_{t-1} + \varphi_{kl} \hat{u}_t^l + \varphi_{kc} \hat{u}_t^c + \hat{u}_t^k \tag{97}$$

$$\hat{\tau}_t^l = \varphi_l \hat{Y}_{Ht}^{HW} + \zeta_l \hat{b}_{t-1} + \varphi_{lk} \hat{u}_t^k + \varphi_{lc} \hat{u}_t^c + \hat{u}_t^l$$

$$\tag{98}$$

$$\hat{\tau}_t^c = \varphi_{ck} \hat{u}_t^k + \varphi_{cl} \hat{u}_t^l + \hat{u}_t^c \tag{99}$$

• Budget and resource constraints

$$\frac{P_{Ft}}{\mathcal{P}_t}C_{Ft} - \frac{P_{Ht}}{\mathcal{P}_t}C_{Ft}^* = \left(B_{Ft}^* - R_{Bt-1}^r \left(1 - \mathbb{I}_{Dt}\gamma_t\right)B_{Ft-1}^*\right) - \frac{1}{\mathcal{Q}_t}\left(B_{Ft} - R_{B,t-1}^{r*} \left(1 - \mathbb{I}_{Dt}^*\gamma_t^*\right)B_{Ft-1}\right)$$
(100)

$$-\frac{1}{Q_t} \left((\kappa_{Ft} + \eta_{Ft} B_{Ft} - 1) B_{Ft} - Q_t \left(\kappa_{Ft}^* + \eta_{Ft}^* B_{Ft}^* - 1 \right) B_{Ft}^* \right)$$
 (101)

$$Y_{Ht} = C_{Ht} + C_{Ft}^* + H_t^T + I_t + G_t + \vartheta_t \mathbb{I}_{\mathbf{Dt}}$$

$$\tag{102}$$

$$B_t = B_{Ht} + B_{Ft}^* (103)$$



B Stationary Representation

Non-stationary variables are scaled by Z_t , i.e. $\tilde{X}_t = \frac{X_t}{Z_t}$ with the corresponding rate $\Gamma_{t+1} = \frac{Z_{t+1}}{Z_t}$. Stationarity requires

$$Y_{Ht} = A_t^{\frac{\varepsilon}{\varepsilon - 1}} \frac{\xi_t}{A_t} K_t^{\alpha} L_t^{1 - \alpha} \sim A_t^{\frac{\varepsilon}{\varepsilon - 1} - 1 + \alpha}$$
(104)

$$\Rightarrow 2 = \frac{\epsilon}{\epsilon - 1} + \alpha \tag{105}$$

• Exogenous processes

$$\Gamma_{t+1} = \left(\bar{\chi}\chi_t^{\zeta} + \phi\right) \tag{106}$$

$$\log\left(\chi_{t}\right) = \rho_{\chi}\log\left(\chi_{t-1}\right) + \sigma_{\chi}\varepsilon_{t}^{\chi} \tag{107}$$

$$\log \kappa_t = (1 - \rho_\kappa) \,\bar{\kappa} + \rho_\kappa \log \kappa_{t-1} + \sigma_\kappa \varepsilon_t^\kappa \tag{108}$$

$$\log \tilde{\eta}_t = (1 - \rho_{\tilde{\eta}}) \, \bar{\tilde{\eta}} + \rho_{\tilde{\eta}} \log \tilde{\eta}_{t-1} + \sigma_{\tilde{\eta}} \varepsilon_t^{\tilde{\eta}} \tag{109}$$

$$2\frac{\eta_t}{\kappa_t} = \frac{\tilde{\eta}_t - \bar{\eta}}{Z_t} \tag{110}$$

$$\log \xi_t = \rho_{\xi} \log \xi_{t-1} + \sigma_{\xi} \varepsilon_t^{\xi} \tag{111}$$

$$\varsigma_t = \varsigma_0 + \varsigma_1 \left(\hat{b}_t - \bar{\hat{b}} \right) + \sigma_\varsigma \varepsilon_t^\varsigma \tag{112}$$

$$\hat{u}_t^x = \rho_x \hat{u}_{t-1}^x + \sigma_x \varepsilon_t^x, \quad x \in \{g, l, c, k, T\}$$
(113)

• Price relations

$$\frac{\mathcal{P}_t}{P_{Ht}} = \left((1 - v) + v \left(\mathcal{S}_t \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} \equiv h \left(\mathcal{S}_t \right)$$
(114)

$$\mathcal{P}_{t}^{*} = \left((1 - v) \left(P_{Ht}^{*} \right)^{1 - \eta} + v \left(P_{Ft}^{*} \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} = P_{Ft} \left((1 - v) + v \left(\frac{1}{\mathcal{S}_{t}} \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} \equiv P_{Ft} f \left(\mathcal{S}_{t} \right)$$
(115)

 $\frac{\mathcal{P}_t}{P_t} = \frac{\mathcal{P}_t}{P_{Ht}} \frac{P_{Ht}}{P_t} = h\left(\mathcal{S}_t\right) A_t^{\frac{1}{1-\varepsilon}} \tag{116}$

$$\frac{\mathcal{P}_t}{P_{Ft}} = \frac{h\left(\mathcal{S}_t\right)}{\mathcal{S}_t} \tag{117}$$

$$Q_t = \frac{\mathcal{P}_t}{\mathcal{P}_t^*} = \frac{\mathcal{P}_t}{P_{Ht}} \frac{P_{Ht}}{\mathcal{P}_t^*} = \frac{\mathcal{P}_t}{P_{Ht}} \frac{P_{Ht}}{P_{Ft}} \frac{P_{Ft}}{\mathcal{P}_t^*} = \frac{1}{\mathcal{S}_t} \frac{h\left(\mathcal{S}_t\right)}{f\left(\mathcal{S}_t\right)}$$
(118)

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}} \tag{119}$$

$$P_{Ft} = P_{Ht}^* \tag{120}$$

$$P_{Ht} = P_{Ft}^* \tag{121}$$

$$\frac{P_{Ht}}{P_t} = A_t^{\frac{1}{1-\varepsilon}} \tag{122}$$



• Household

$$\tilde{C}_{H} = (1 - v) \left(h \left(\mathcal{S}_{t} \right) \right)^{\eta} \tilde{\mathcal{C}}_{t}$$
(123)

$$\tilde{C}_F = \upsilon \left(\frac{h\left(\mathcal{S}_t\right)}{\mathcal{S}_t}\right)^{\eta} \tilde{\mathcal{C}}_t \tag{124}$$

$$\Gamma_{t+1}\tilde{K}_{t+1} = (1-\delta)\,\tilde{K}_t + \tilde{I}_t \left[1 - \Psi\left(\Gamma_t \iota_t \frac{\tilde{I}_t}{\tilde{I}_{t-1}}\right) \right] \tag{125}$$

$$\varphi_0 \tilde{\mathcal{C}}_t^{\sigma} L_t^{\varphi} = \left(\frac{1 - \tau_t^l}{1 + \tau_t^c}\right) \tilde{W}_t^r \tag{126}$$

$$\frac{1}{h\left(\mathcal{S}_{t}\right)} \frac{\tilde{\mathcal{C}}_{t}^{-\sigma}}{\left(1 + \tau_{t}^{c}\right)} = \frac{\mu_{t}}{Z_{t}^{-\sigma}} \left(1 - \Gamma_{t} \iota_{t} \frac{\tilde{I}_{t}}{\tilde{I}_{t-1}} \Psi'\left(\Gamma_{t} \iota_{t} \frac{\tilde{I}_{t}}{\tilde{I}_{t-1}}\right) - \Psi\left(\Gamma_{t} \iota_{t} \frac{\tilde{I}_{t}}{\tilde{I}_{t-1}}\right)\right) + \beta \mathbb{E}_{t} \left[\Gamma_{t+1}^{-\sigma} \frac{\mu_{t+1}}{Z_{t+1}^{-\sigma}} \iota_{t+1} \left(\Gamma_{t+1} \frac{\tilde{I}_{t+1}}{\tilde{I}_{t}}\right)^{2}\right] \tag{127}$$

• Returns

$$\frac{\mu_t}{Z_t^{-\sigma}} = \beta \mathbb{E}_t \left[\Gamma_{t+1}^{-\sigma} \frac{\mu_{t+1}}{Z_{t+1}^{-\sigma}} (1 - \delta) + \Gamma_{t+1}^{-\sigma} \tilde{\mathcal{C}}_{t+1}^{-\sigma} \left(\frac{1 - \tau_{t+1}^k}{1 + \tau_{t+1}^c} \right) R_{kt+1}^r \right]$$
(128)

$$1 + (\tilde{\eta}_{Ht} - \bar{\eta}_H) \, \tilde{B}_{Ht} = \frac{R_{B,t}^r}{\kappa_{Ht}} \mathbb{E}_t \left[\beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_t} \right)^{-\sigma} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) (1 - \mathbb{I}_{Dt+1} \gamma_{t+1}) \right]$$
(129)

$$1 + (\tilde{\eta}_{Ft} - \bar{\eta}_F) \, \tilde{B}_{Ft} = \frac{R_{B,t}^{r*}}{\kappa_{Ft}} \mathbb{E}_t \left[\beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_t} \right)^{-\sigma} \left(\frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right) \left(\frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} \right) (1 - \mathbb{I}_{Dt+1}^* \gamma_{t+1}^*) \right]$$
(130)

$$p_{Dt} = \frac{1}{1 + e^{-\varsigma_t}} \tag{131}$$

$$\Lambda_{t,t+1} = \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_t} \right)^{-\sigma} \tag{132}$$

• Goods

$$\tilde{Y}_{Ht} = \tilde{A}_t^{\frac{1}{\varepsilon - 1}} \xi_t \tilde{K}_t^{\alpha} L_t^{1 - \alpha} \tag{133}$$

$$\tilde{W}_{t}^{r} = \frac{\epsilon - 1}{\epsilon} \frac{1}{h(\mathcal{S}_{t})} \tilde{A}_{t}^{\frac{1}{\varepsilon - 1}} (1 - \alpha) \xi_{t} \left(\frac{\tilde{K}_{t}}{L_{t}}\right)^{\alpha}$$
(134)

$$R_{kt}^{r} = \frac{\epsilon - 1}{\epsilon} \frac{1}{h(\mathcal{S}_{t})} \tilde{A}_{t}^{\frac{1}{\varepsilon - 1}} \alpha \xi_{t} \left(\frac{L_{t}}{\tilde{K}_{t}}\right)^{1 - \alpha}$$
(135)

$$\pi_t^r = \frac{1}{\epsilon} \frac{1}{h(\mathcal{S}_t)} \tilde{A}_t^{\frac{1}{\varepsilon - 1} - 1} \xi_t \tilde{K}_t^{\alpha} L_t^{1 - \alpha}$$
(136)



• Technology

$$J_{t}^{r} = \max_{H_{t}} \mathbb{E}_{t} \left\{ -\frac{1}{h\left(\mathcal{S}_{t}\right)} H_{t} + \phi \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_{t}} \right)^{-\sigma} \left[\lambda_{t} V_{t+1}^{r} + \left(1 - \lambda_{t} \right) J_{t+1}^{r} \right] \right\}$$
(137)

$$\frac{1}{h\left(\mathcal{S}_{t}\right)} = \rho_{\lambda} \frac{\lambda_{t}}{H_{t}} \phi \mathbb{E}_{t} \left\{ \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_{t}} \right)^{-\sigma} \left(V_{t+1}^{r} - J_{t+1}^{r} \right) \right\}$$
(138)

$$V_t^r = \pi_t^r + \phi \mathbb{E}_t \left\{ \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}}{\tilde{\mathcal{C}}_t} \right)^{-\sigma} V_{t+1}^r \right\}$$
 (139)

$$\lambda_t = \bar{\lambda} \left(F_t H_t \right)^{\rho_{\lambda}} \tag{140}$$

$$F_t = \frac{1}{\tilde{K}_t} \tag{141}$$

$$\tilde{H}_t^T = H_t \left(1 - \tilde{A}_t \right) \tag{142}$$

$$\Gamma_{t+1}\tilde{A}_{t+1} = \lambda_t \phi \left[1 - \tilde{A}_t \right] + \phi \tilde{A}_t \tag{143}$$

• Government

$$\frac{1}{h\left(\mathcal{S}_{t}\right)}\tilde{G}_{t} = \tilde{B}_{t} - R_{Bt-1}^{r}\left(1 - \mathbb{I}_{Dt}\gamma_{t}\right)\frac{\tilde{B}_{t-1}}{\Gamma_{t}} + \tau_{t}^{l}\tilde{W}_{t}^{r}L_{t} - \tilde{T}_{t}^{r} + \tau_{t}^{c}\tilde{\mathcal{C}}_{t} + \tau_{t}^{k}R_{kt}^{r}\tilde{K}_{t}, \tag{144}$$

$$GDP_t^r = \frac{1}{h(\mathcal{S}_t)} Y_{Ht} \tag{145}$$

$$NX_t^r = \frac{1}{h(\mathcal{S}_t)} C_{Ft}^* - \frac{\mathcal{S}_t}{h(\mathcal{S}_t)} C_{Ft}$$
(146)

$$\hat{G}_t = -\varphi_g \hat{Y}_{Ht}^{HW} - \zeta_g \hat{b}_{t-1} + \hat{u}_t^g \tag{147}$$

$$\hat{T}_t = -\varphi_T \hat{Y}_{Ht}^{HW} - \zeta_T \hat{b}_{t-1} + \hat{u}_t^T \tag{148}$$

$$\hat{\tau}_t^k = \varphi_k \hat{Y}_{Ht}^{HW} + \zeta_k \hat{b}_{t-1} + \varphi_{kl} \hat{u}_t^l + \varphi_{kc} \hat{u}_t^c + \hat{u}_t^k$$
(149)

$$\hat{\tau}_t^l = \varphi_l \hat{Y}_{Ht}^{HW} + \zeta_l \hat{b}_{t-1} + \varphi_{lk} \hat{u}_t^k + \varphi_{lc} \hat{u}_t^c + \hat{u}_t^l$$

$$\tag{150}$$

$$\hat{\tau}_t^c = \varphi_{ck}\hat{u}_t^k + \varphi_{cl}\hat{u}_t^l + \hat{u}_t^c \tag{151}$$



• Budget and resource constraints

$$\frac{S_{t}}{h\left(S_{t}\right)}\tilde{C}_{Ft} - \frac{1}{h\left(S_{t}\right)}\tilde{C}_{Ft}^{*} = \left(\tilde{B}_{Ft}^{*} - \frac{R_{Bt-1}^{r}}{\Gamma_{t}}\left(1 - \mathbb{I}_{Dt}\gamma_{t}\right)\tilde{B}_{Ft-1}^{*}\right) - \frac{1}{Q_{t}}\left(\tilde{B}_{Ft} - \frac{R_{B,t-1}^{r*}}{\Gamma_{t}}\left(1 - \mathbb{I}_{Dt}^{*}\gamma_{t}^{*}\right)\tilde{B}_{Ft-1}\right) \\
- \frac{1}{Q_{t}}\left(\left(\left(1 + \frac{\tilde{\eta}_{Ft} - \bar{\eta}_{F}}{2}\right)\kappa_{Ft}\tilde{B}_{Ft} - 1\right)\tilde{B}_{Ft} - Q_{t}\left(\left(1 + \frac{\tilde{\eta}_{Ft}^{*} - \bar{\eta}_{F}^{*}}{2}\right)\kappa_{Ft}^{*}\tilde{B}_{Ft}^{*} - 1\right) \\
\eta_{Ht} = \frac{\tilde{\eta}_{Ht} - \bar{\eta}_{H}}{Z_{t}}\frac{\kappa_{Ht}}{2} \tag{154}$$

$$\eta_{Ft} = \frac{\tilde{\eta}_{Ft} - \bar{\eta}_F}{Z_t} \frac{\kappa_{Ft}}{2} \tag{155}$$

$$\tilde{Y}_{Ht} = \tilde{C}_{Ht} + \tilde{C}_{Ft}^* + \tilde{H}_t^T + \tilde{I}_t + \tilde{G}_t + \tilde{\vartheta}_t \mathbb{I}_{\mathbf{Dt}}$$
(156)

$$\tilde{B}_t = \tilde{B}_{Ht} + \tilde{B}_{Ft}^* \tag{157}$$

B.1 Foreign Country

B.1.1 Real Symmetric Conditions

Most equilibrium conditions are identical with a few exceptions with respect to the price-ratios.

• Exogenous processes

$$Z_{t+1} = \left(\bar{\chi}\chi_t^{\zeta} + \phi\right) Z_t \tag{158}$$

$$\log\left(\chi_{t}\right) = \rho_{\chi}\log\left(\chi_{t-1}\right) + \sigma_{\chi}\varepsilon_{t}^{\chi} \tag{159}$$

$$\log \kappa_t^* = (1 - \rho_\kappa) \,\bar{\kappa}^* + \rho_\kappa \log \kappa_{t-1}^* + \sigma_\kappa \varepsilon_t^{\kappa^*} \tag{160}$$

$$\log \tilde{\eta}_t^* = (1 - \rho_{\tilde{\eta}}) \,\bar{\tilde{\eta}}^* + \rho_{\tilde{\eta}} \log \tilde{\eta}_{t-1}^* + \sigma_{\tilde{\eta}} \varepsilon_t^{\tilde{\eta}^*} \tag{161}$$

$$\log \xi_t^* = \rho_\xi \log \xi_{t-1}^* + \sigma_\xi \varepsilon_t^{\xi^*} \tag{162}$$

$$\log \iota_t^* = \rho_\iota \log \iota_{t-1}^* + \sigma_\iota \varepsilon_t^{\iota *} \tag{163}$$

$$\varsigma_t^* = \varsigma_0 + \varsigma_1 \left(\hat{b}_t^* - \bar{\hat{b}}^* \right) + \sigma_\varsigma \varepsilon_t^{\varsigma_*} \tag{164}$$

$$\hat{u}_t^{x*} = \rho_x \hat{u}_{t-1}^{x*} + \sigma_x \varepsilon_t^{x*}, \quad x \in \{g, l, c, k, T\}$$
(165)



• Price relations

$$\frac{\mathcal{P}_t^*}{P_{Ht}^*} = \left((1 - \upsilon) + \upsilon \left(\frac{1}{\mathcal{S}_t} \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} = f\left(\mathcal{S}_t \right)$$
 (166)

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}} \tag{167}$$

$$\frac{\mathcal{P}_t^*}{P_t^*} = f\left(\mathcal{S}_t\right) A_t^{*\frac{1}{1-\varepsilon}} \tag{168}$$

$$\frac{P_{Ht}^*}{P_t^*} = A_t^{*\frac{1}{1-\varepsilon}} \tag{169}$$

$$\frac{P_{Ft}^*}{P_{Ht}^*} = \frac{P_{Ht}}{P_{Ft}} = \frac{1}{S_t} \tag{170}$$

$$\frac{\mathcal{P}_{t}^{*}}{P_{Et}^{*}} = \mathcal{S}_{t} f\left(\mathcal{S}_{t}\right) \tag{171}$$

$$\frac{\mathcal{P}_{t}^{*}}{P_{Ft}^{*}} = \frac{h\left(\mathcal{S}_{t}\right)}{\mathcal{S}_{t}} \tag{172}$$

$$Q_t = \frac{1}{S_t} \frac{h(S_t)}{f(S_t)} \tag{173}$$

• Household

$$C_{Ht}^* = (1 - \upsilon) \left(f \left(\mathcal{S}_t \right) \right)^{\eta} \mathcal{C}_t^* \tag{174}$$

$$C_{Ft}^* = \upsilon \left(f \left(\mathcal{S}_t \right) \mathcal{S}_t \right)^{\eta} \mathcal{C}_t^* \tag{175}$$

$$K_{t+1}^* = (1 - \delta) K_t^* + I_t^* \left[1 - \Psi \left(\iota_t \frac{I_t^*}{I_{t-1}^*} \right) \right]$$
 (176)

$$Z_t^{*1-\sigma} \varphi_0^* \mathcal{C}_t^{*\sigma} L_t^{*\varphi} = \left(\frac{1-\tau_t^{l*}}{1+\tau_t^{c*}}\right) W_t^{*r} \tag{177}$$

$$\frac{1}{f(S_t)} \frac{C_t^{-\sigma*}}{(1 + \tau_t^{c*})} = \mu_t^* \left(1 - \iota_t \frac{I_t^*}{I_{t-1}^*} \Psi'(\cdot) - \Psi\left(\iota_t \frac{I_t^*}{I_{t-1}^*}\right) \right) + \beta \mathbb{E}_t \left[\mu_{t+1}^* \iota_{t+1} \left(\frac{I_{t+1}^*}{I_t^*} \right)^2 \Psi'(\cdot) \right]$$
(178)

Returns

$$\mu_t^* = \beta \mathbb{E}_t \left[\mu_{t+1}^* \left(1 - \delta \right) + C_t^{*-\sigma} \left(\frac{1 - \tau_{t+1}^{k*}}{1 + \tau_{t+1}^{c*}} \right) R_{kt+1}^{r*} \right]$$
 (179)

$$1 + \frac{(\tilde{\eta}_{Ht}^* - \bar{\eta}_{H}^*)}{Z_t} B_{Ht}^* = \frac{R_{B,t}^{r*}}{\kappa_{Ht}^*} \mathbb{E}_t \left[\Lambda_{t,t+1}^* \left(\frac{1 + \tau_t^{c*}}{1 + \tau_{t+1}^{c*}} \right) \left(1 - \mathbb{I}_{Dt+1}^* \gamma_{t+1}^* \right) \right]$$
(180)

$$1 + \frac{(\tilde{\eta}_{Ft}^* - \bar{\eta}_F^*)}{Z_t} B_{Ft}^* = \frac{R_{B,t}^r}{\kappa_{Ft}^*} \mathbb{E}_t \left[\Lambda_{t,t+1}^* \left(\frac{1 + \tau_t^{c*}}{1 + \tau_{t+1}^{c*}} \right) \left(\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \right) (1 - \mathbb{I}_{Dt+1} \gamma_{t+1}) \right]$$
(181)

$$p_{Dt}^* = \frac{1}{1 + e^{-\varsigma_t^*}} \tag{182}$$

$$\Lambda_{t,t+1}^* = \beta \left(\frac{\mathcal{C}_{t+1}^*}{\mathcal{C}_t^*}\right)^{-\sigma} \tag{183}$$



• Final good

$$Y_{Ht}^* = A_t^* \frac{\epsilon}{\epsilon - 1} Y_t^* \tag{184}$$

• Intermediate goods

$$Y_t^* = \frac{\xi_t^*}{A_t^*} K_t^{*\alpha} L_t^{*1-\alpha} \tag{185}$$

$$W_t^{*r} = \frac{\epsilon - 1}{\epsilon} \frac{A_t^{*\frac{1}{\varepsilon - 1}}}{f(\mathcal{S}_t)} (1 - \alpha) A_t^* \frac{Y_t^*}{L_t^*}$$
(186)

$$R_{kt}^{*r} = \frac{\epsilon - 1}{\epsilon} \frac{A_t^{*\frac{1}{\epsilon - 1}}}{f(\mathcal{S}_t)} \alpha A_t^{*} \frac{Y_t^{*}}{K_t^{*}}$$

$$\tag{187}$$

$$\pi_t^{*r} = \frac{1}{\epsilon} \frac{A_t^{*\frac{1}{\varepsilon - 1}}}{f(\mathcal{S}_t)} Y_t^* \tag{188}$$

• Technology

$$J_t^{*r} = \mathbb{E}_t \left\{ -\frac{H_t^*}{f(S_t)} + \phi \Lambda_{t,t+1}^* \left[\lambda_t^* V_{t+1}^{*r} + (1 - \lambda_t^*) J_{t+1}^{*r} \right] \right\}$$
 (189)

$$\frac{1}{f\left(\mathcal{S}_{t}\right)} = \rho_{\lambda} \frac{\lambda_{t}^{*}}{H_{t}^{*}} \phi \mathbb{E}_{t} \left\{ \Lambda_{t,t+1}^{*} \left(V_{t+1}^{*r} - J_{t+1}^{*r} \right) \right\}$$

$$\tag{190}$$

$$V_t^{r*} = \pi_t^{r*} + \phi \mathbb{E}_t \left\{ \Lambda_{t,t+1}^* V_{t+1}^{*r} \right\}$$
 (191)

$$\lambda_t^* = \bar{\lambda}^* \left(\frac{Z_t}{K_t^*} H_t^* \right)^{\rho_{\lambda}} \tag{192}$$

$$H_t^{*T} = H_t^* \left(Z_t - A_t^* \right) \tag{193}$$

$$A_{t+1}^* = \lambda_t^* \phi \left[Z_t - A_t^* \right] + \phi A_t^* \tag{194}$$

• Government

$$\frac{G_t^*}{f(S_t)} = B_t^* - R_{Bt-1}^{r*} \left(1 - \mathbb{I}_{Dt}^* \gamma_t^*\right) B_{t-1}^* + \tau_t^{l*} W_t^{r*} L_t^* - T_t^{r*} + \tau_t^{c*} C_t^* + \tau_t^{k*} R_{kt}^{r*} K_t^*$$
 (195)

$$GDP_t^{r*} = \frac{Y_{Ht}^*}{f(\mathcal{S}_t)} + NX_t^{r*} \tag{196}$$

$$\hat{G}_t^* = -\varphi_g \hat{Y}_{Ht}^{HW*} - \zeta_g \hat{b}_{t-1}^* + \hat{u}_t^{g*}$$
(197)

$$\hat{T}_{t}^{*} = -\varphi_{T}\hat{Y}_{Ht}^{HW*} - \zeta_{T}\hat{b}_{t-1}^{*} + \hat{u}_{t}^{T*}$$
(198)

$$\hat{\tau}_t^{k*} = \varphi_k \hat{Y}_{Ht}^{HW*} + \zeta_k \hat{b}_{t-1}^* + \varphi_{kl} \hat{u}_t^{l*} + \varphi_{kc} \hat{u}_t^{c*} + \hat{u}_t^{k*}$$
(199)

$$\hat{\tau}_t^{l*} = \varphi_l \hat{Y}_{Ht}^{HW*} + \zeta_l \hat{b}_{t-1}^* + \varphi_{lk} \hat{u}_t^{k*} + \varphi_{lc} \hat{u}_t^{c*} + \hat{u}_t^{l*}$$
(200)

$$\hat{\tau}_t^{c*} = \varphi_{ck} \hat{u}_t^{k*} + \varphi_{cl} \hat{u}_t^{l*} + \hat{u}_t^{c*} \tag{201}$$

• Resource constraint

$$Y_{Ht}^* = C_{Ht}^* + C_{Ft} + H_t^{T*} + I_t^* + G_t^* + \vartheta_t^* \mathbb{I}_{Dt}^*$$
 (202)

$$B_t^* = B_{Ht}^* + B_{Ft} \tag{203}$$



B.1.2 Stationary Representation

• Exogenous processes

$$Z_{t+1} = \left(\bar{\chi}\chi_t^{\zeta} + \phi\right) Z_t \tag{204}$$

$$\log(\chi_t) = \rho_{\chi} \log(\chi_{t-1}) + \sigma_{\chi} \varepsilon_t^{\chi}$$
(205)

$$\log \kappa_t^* = (1 - \rho_\kappa) \,\bar{\kappa}^* + \rho_\kappa \log \kappa_{t-1}^* + \sigma_\kappa \varepsilon_t^{\kappa^*} \tag{206}$$

$$\log \tilde{\eta}_t^* = (1 - \rho_{\tilde{\eta}}) \,\bar{\tilde{\eta}}^* + \rho_{\tilde{\eta}} \log \tilde{\eta}_{t-1}^* + \sigma_{\tilde{\eta}} \varepsilon_t^{\tilde{\eta}^*} \tag{207}$$

$$\log \xi_t^* = \rho_{\xi} \log \xi_{t-1}^* + \sigma_{\xi} \varepsilon_t^{\xi^*} \tag{208}$$

$$\log \iota_t^* = \rho_\iota \log \iota_{t-1}^* + \sigma_\iota \varepsilon_t^{\iota *} \tag{209}$$

$$\varsigma_t^* = \varsigma_0 + \varsigma_1 \left(\hat{b}_t^* - \bar{\hat{b}}^* \right) + \sigma_\varsigma \varepsilon_t^{\varsigma*} \tag{210}$$

$$\hat{u}_t^{x*} = \rho_x \hat{u}_{t-1}^{x*} + \sigma_x \varepsilon_t^{x*}, \quad x \in \{g, l, c, k, T\}$$
(211)

• Price relations

$$\frac{\mathcal{P}_t^*}{P_{Ht}^*} = \left((1 - \upsilon) + \upsilon \left(\frac{1}{\mathcal{S}_t} \right)^{1 - \eta} \right)^{\frac{1}{1 - \eta}} = f\left(\mathcal{S}_t \right)$$
 (212)

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{P_{Ht}^*}{P_{Ht}} \tag{213}$$

$$\frac{\mathcal{P}_t^*}{P_t^*} = f\left(\mathcal{S}_t\right) A_t^{*\frac{1}{1-\varepsilon}} \tag{214}$$

$$\frac{P_{Ht}^*}{P_t^*} = A_t^{*\frac{1}{1-\varepsilon}} \tag{215}$$

$$\frac{P_{Ft}^*}{P_{Ht}^*} = \frac{P_{Ht}}{P_{Ft}} = \frac{1}{S_t} \tag{216}$$

$$\frac{P_t^*}{P_{Ft}^*} = \mathcal{S}_t f\left(\mathcal{S}_t\right) \tag{217}$$

$$\frac{\mathcal{P}_t^*}{P_{Ft}^*} = \frac{h\left(\mathcal{S}_t\right)}{\mathcal{S}_t} \tag{218}$$

$$Q_{t} = \frac{1}{S_{t}} \frac{h\left(S_{t}\right)}{f\left(S_{t}\right)} \tag{219}$$



• Household

$$\tilde{C}_{Ht}^* = (1 - v) \left(f\left(\mathcal{S}_t\right) \right)^{\eta} \tilde{\mathcal{C}}_t^* \tag{220}$$

$$\tilde{C}_{Ft}^* = \upsilon \left(f \left(\mathcal{S}_t \right) \mathcal{S}_t \right)^{\eta} \tilde{\mathcal{C}}_t^* \tag{221}$$

$$\Gamma_{t+1}\tilde{K}_{t+1}^* = (1 - \delta)\,\tilde{K}_t^* + \tilde{I}_t^* \left[1 - \Psi \left(\Gamma_t \iota_t^* \frac{\tilde{I}_t^*}{\tilde{I}_{t-1}^*} \right) \right]$$
(222)

$$\varphi_0^* \tilde{\mathcal{C}}_t^{*\sigma} L_t^{*\varphi} = \left(\frac{1 - \tau_t^{l^*}}{1 + \tau_t^{c^*}}\right) \tilde{W}_t^{*r} \tag{223}$$

$$\frac{1}{f\left(\mathcal{S}_{t}\right)} \frac{\tilde{\mathcal{C}}_{t}^{-\sigma*}}{(1+\tau_{t}^{c*})} = \frac{\mu_{t}^{*}}{Z_{t}^{-\sigma}} \left(1 - \Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}} \Psi'\left(\Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}}\right) - \Psi\left(\Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}}\right)\right) + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{I}_{t+1}^{*}}{\tilde{I}_{t}^{*}}\right)^{2} \Psi'\left(\Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}}\right)\right] + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{I}_{t+1}^{*}}{\tilde{I}_{t}^{*}}\right)^{2} \Psi'\left(\Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}}\right)\right] + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{I}_{t+1}^{*}}{\tilde{I}_{t}^{*}}\right)^{2} \Psi'\left(\Gamma_{t} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t-1}^{*}}\right)\right] + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{I}_{t+1}^{*}}{\tilde{I}_{t}^{*}}\right)\right] + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{I}_{t}^{*}}{\tilde{I}_{t}^{*}}\right)\right] + \beta \mathbb{E}_{t} \left[\mu_{t+1}^{*} \left(\Gamma_{t+1} \iota_{t}^{*} \frac{\tilde{$$

• Returns

$$\frac{\mu_t^*}{Z_t^{-\sigma}} = \beta \mathbb{E}_t \left[\frac{\mu_{t+1}^*}{Z_{t+1}^{-\sigma}} (1 - \delta) + \tilde{C}_t^{*-\sigma} \left(\frac{1 - \tau_{t+1}^{k*}}{1 + \tau_{t+1}^{c*}} \right) R_{kt+1}^{r*} \right]$$
(225)

$$1 + (\tilde{\eta}_{Ht}^* - \bar{\eta}_H^*) \, \tilde{B}_{Ht}^* = \frac{R_{B,t}^{r*}}{\kappa_{Ht}^*} \mathbb{E}_t \left[\beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^*}{\tilde{\mathcal{C}}_t^*} \right)^{-\sigma} \left(\frac{1 + \tau_t^{c*}}{1 + \tau_{t+1}^{c*}} \right) \left(1 - \mathbb{I}_{Dt+1}^* \gamma_{t+1}^* \right) \right]$$
(226)

$$1 + (\tilde{\eta}_{Ft}^* - \bar{\eta}_F^*) \, \tilde{B}_{Ft}^* = \frac{R_{B,t}^r}{\kappa_{Ft}^*} \mathbb{E}_t \left[\beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^*}{\tilde{\mathcal{C}}_t^*} \right)^{-\sigma} \left(\frac{1 + \tau_t^{c*}}{1 + \tau_{t+1}^{c*}} \right) \left(\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \right) (1 - \mathbb{I}_{Dt+1} \gamma_{t+1}) \right]$$
(227)

$$p_{Dt}^* = \frac{1}{1 + e^{-\varsigma_t^*}} \tag{228}$$

$$\Lambda_{t,t+1}^* = \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^*}{\tilde{\mathcal{C}}_t^*} \right)^{-\sigma} \tag{229}$$

• Final good

$$Y_{Ht}^* = A_t^* \frac{\epsilon}{\epsilon - 1} Y_t^* \tag{230}$$

• Intermediate goods

$$\tilde{Y}_{Ht}^* = \tilde{A}_t^{*\frac{1}{\varepsilon - 1}} \xi_t^* \tilde{K}_t^{*\alpha} L_t^{*1 - \alpha} \tag{231}$$

$$W_t^{*r} = \frac{\epsilon - 1}{\epsilon} \frac{1}{f(S_t)} (1 - \alpha) \xi_t^* \left(\frac{\tilde{K}_t^*}{L_t^*} \right)^{\alpha}$$
 (232)

$$R_{kt}^{*r} = \frac{\epsilon - 1}{\epsilon} \frac{1}{f(\mathcal{S}_t)} \alpha \tilde{A}_t^{*\frac{1}{\epsilon - 1}} \xi_t^* \left(\frac{L_t^*}{\tilde{K}_t^*}\right)^{1 - \alpha}$$
(233)

$$\pi_t^{*r} = \frac{1}{\epsilon} \frac{\tilde{A}_t^{*\frac{1}{\varepsilon - 1} - 1}}{f(\mathcal{S}_t)} \xi_t^* \tilde{K}_t^{*\alpha} L_t^{*1 - \alpha}$$
(234)



• Technology

$$J_{t}^{*r} = \mathbb{E}_{t} \left\{ -\frac{H_{t}^{*}}{f(\mathcal{S}_{t})} + \phi \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^{*}}{\tilde{\mathcal{C}}_{t}^{*}} \right)^{-\sigma} \left[\lambda_{t}^{*} V_{t+1}^{*r} + (1 - \lambda_{t}^{*}) J_{t+1}^{*r} \right] \right\}$$
(235)

$$\frac{1}{f\left(\mathcal{S}_{t}\right)} = \rho_{\lambda} \frac{\lambda_{t}^{*}}{H_{t}^{*}} \phi \mathbb{E}_{t} \left\{ \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^{*}}{\tilde{\mathcal{C}}_{t}^{*}} \right)^{-\sigma} \left(V_{t+1}^{*r} - J_{t+1}^{*r} \right) \right\}$$
(236)

$$V_t^{r*} = \pi_t^{r*} + \phi \mathbb{E}_t \left\{ \beta \left(\Gamma_{t+1} \frac{\tilde{\mathcal{C}}_{t+1}^*}{\tilde{\mathcal{C}}_t^*} \right)^{-\sigma} V_{t+1}^{*r} \right\}$$
 (237)

$$\lambda_t^* = \bar{\lambda}^* \left(\frac{1}{\tilde{K}_t^*} H_t^* \right)^{\rho_{\lambda}} \tag{238}$$

$$\tilde{H}_t^{*T} = H_t^* \left(1 - \tilde{A}_t^* \right) \tag{239}$$

$$\Gamma_{t+1}\tilde{A}_{t+1}^* = \lambda_t^* \phi \left[1 - \tilde{A}_t^* \right] + \phi \tilde{A}_t^* \tag{240}$$

• Government

$$\frac{\tilde{G}_{t}^{*}}{f(\mathcal{S}_{t})} = \tilde{B}_{t}^{*} - R_{Bt-1}^{r*} \left(1 - \mathbb{I}_{Dt}^{*} \gamma_{t}^{*}\right) \frac{\tilde{B}_{t-1}^{*}}{\Gamma_{t}} + \tau_{t}^{l*} \tilde{W}_{t}^{r*} L_{t}^{*} - \tilde{T}_{t}^{r*} + \tau_{t}^{c*} \tilde{\mathcal{C}}_{t}^{*} + \tau_{t}^{k*} R_{kt}^{r*} \tilde{K}_{t}^{*}$$
(241)

$$GDP_t^{r*} = \frac{Y_{Ht}^*}{f(\mathcal{S}_t)} \tag{242}$$

$$\hat{G}_t^* = -\varphi_g \hat{Y}_{Ht}^{HW*} - \zeta_g \hat{b}_{t-1}^* + \hat{u}_t^{g*}$$
(243)

$$\hat{T}_{t}^{*} = -\varphi_{T} \hat{Y}_{Ht}^{HW*} - \zeta_{T} \hat{b}_{t-1}^{*} + \hat{u}_{t}^{T*}$$
(244)

$$\hat{\tau}_t^{k*} = \varphi_k \hat{Y}_{Ht}^{HW*} + \zeta_k \hat{b}_{t-1}^* + \varphi_{kl} \hat{u}_t^{l*} + \varphi_{kc} \hat{u}_t^{c*} + \hat{u}_t^{k*}$$
(245)

$$\hat{\tau}_t^{l*} = \varphi_l \hat{Y}_{Ht}^{HW*} + \zeta_l \hat{b}_{t-1}^* + \varphi_{lk} \hat{u}_t^{k*} + \varphi_{lc} \hat{u}_t^{c*} + \hat{u}_t^{l*}$$
(246)

$$\hat{\tau}_t^{c*} = \varphi_{ck}\hat{u}_t^{k*} + \varphi_{cl}\hat{u}_t^{l*} + \hat{u}_t^{c*} \tag{247}$$

• Resource constraint

$$\tilde{Y}_{Ht}^{*} = \tilde{C}_{Ht}^{*} + \tilde{C}_{Ft} + \tilde{H}_{t}^{T*} + \tilde{I}_{t}^{*} + \tilde{G}_{t}^{*} + \tilde{\vartheta}_{\mathbf{t}}^{*} \mathbb{I}_{\mathbf{Dt}}^{*}$$
(248)

$$\tilde{B}_t^* = \tilde{B}_{Ht}^* + \tilde{B}_{Ft} \tag{249}$$





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