D3.1: Final report on endogenous human capital extension of the baseline model

Deliverable D3.1: Final report on endogenous human capital extension of the baseline model. **Revised.**

Author: Diego Comin and Mario Giarda

Version: 1.0

Quality review: All Partners

Date: March 31st, 2018

Grant Agreement number: 727073

Starting Date: 01/04/2017

Duration: 24 months

Coordinator: Dr. Georg Licht, ZEW

Email: licht@zew.de

Contents

Project Information Summary

Table 1: Project Information Summary

Project Acronym	FRAME
Project Full Title	Framework for the Analysis of Research and Adoption Activities and their Macroeconomic Effects
Grant Agreement	727073
Call Identifier	H2020-SC6-CO-CREATION–2016 -1
Topic	CO-CREATION-08-2016/2017: Better integration of evidence on the impact of research and innovation in policy making
Funding Scheme	Medium-scaled focused research project
Project Duration	1st April 2017 – 31st March 2019 (24 months)
Project Officer(s)	Hinano SPREAFICO (Research Executive Agency)
	Roberto MARTINO (DG Research and Innovation)
Co-ordinator	Dr. Georg Licht, Zentrum für Europäische Wirtschaftsforschung GmbH
Consortium Partners	Centre for Economic Policy Research
	Lunds Universitet
	Università Luigi Bocconi
	Universitat Pompeu Fabra
	London Business School
Website	http://www.h2020frame.eu/frame/home.html

Deliverable Documentation Sheet

Table 2: Deliverable Documentation Sheet

Number	**D3.1**
Title	Interim report on endogenous human capital extension of the baseline model with revisions
Related WP	WP3
Lead Beneficiary	UPF
Author(s)	Diego Comin (CEPR), Mario Giarda (UPF)
Contributor(s)	
Reviewer(s)	All partners
Nature	R (Report)
Dissemination level	PU (Public)
Due Date	31.03.2018
Submission Date	
Status	

Quality Control Assessment Sheet

Table 3: Quality Control Assessment Sheet

Issue	Date	Comment	Author
V0.1	31.01.2018	First draft	Diego Comin, Mario Giarda
V0.1	31.03.2018	Second deliverable	Diego Comin, Mario Giarda
V1.0	24.07.2018	Second deliverable with corrections	Diego Comin, Mario Giarda

1 Executive Summary

In work-package 3, we include endogenous human capital accumulation to the model of endogenous growth developed in the WP1. This extension is useful since it considers two important features. 1. Human capital is a state variable. 2. Agents react to economic conditions and policies when deciding how much to invest in human capital. Moreover, making human capital endogenous is important to obtain an accurate understanding of the effects of innovation policies for several reasons. First, human capital is an input in production and impacts the productivity of the other inputs of the economy. Therefore, policies that affect the stock of human capital must affect aggregate productivity. Second, human capital is an important input in R&D activities and in efforts to adopt new technologies. Therefore, policies that affect the human capital stock may also affect the cost of R&D and adoption services. Finally, it is important to remark that since human capital is a state variable, by including it in our analysis, the effect of policies in the economy may not only increase but also may become more long-lasting than if we ignored the endogeneity of human capital.

We conducted several policy exercises, extending the ones developed in the Baseline model. First, we consider in our model an explicit measure of educational quality that we model as the rate of transformation of hours of education into human capital. This allows us to explore the macroeconomic impact of improving educational quality. Second, we include spending in education. As educational expenditures are not negligible in a macroeconomic perspective (it is 5% of the GDP in the OECD), studying the effect of increasing subsidies to education is crucial.

We show that both educational policies interact significantly with our features of endogenous growth. The intuition is simple: for example, after an increase to education subsidies, investment in human capital increases immediately and human capital reacts in the same direction. This has spillover effects on all the sectors of production side the economy. Now, there is more effective labor available to work at the same wage. This generates an increase in all the forms of labor (devoted to R&D, diffusion, and production), that pushes the economy upwards.

We also show that innovation policies interact with human capital by distorting the labor markets and the return to education. In our baseline calibration, public investment in adoption has a negative impact in the short run. This because it stimulates wages in the medium-run and, due to income effects the supply of labor falls, inducing a recession. While the opposite happens after an R&D investment shock. We showed that this depends on the calibration of human capital. When depreciation is larger, innovation policies have the expected effect (just as WP1).

2   Introduction

In the baseline model of FRAME, a fixed number of workers (normalized to one) decides the number of hours of skilled and unskilled labor services they supply in the market. This set up abstracts from two important features. 1. Human capital is a state variable. 2. Agents react to economic conditions and policies when deciding how much to invest in human capital. In work-package 3, we take on the task of modeling the process of human capital accumulation in the context of FRAME. Making human capital endogenous is important to obtain an accurate understanding of the effects of innovation policies for several reasons. First, human capital is an input in production. Therefore, policies that affect the stock of human capital must affect aggregate productivity. Second, human capital is an important input in R&D activities and in efforts to adopt new technologies. Therefore, policies that affect the human capital stock may also affect the cost of R&D and adoption services. Finally, it is important to remark that since human capital is a state variable, by including it in our analysis, the effect of policies in the economy may not only increase but also may become more long-lasting than if we ignored the endogeneity of human capital. To explore the role of the accumulation of human capital we first strip down the baseline model from several bells and whistles that are not essential for this work package. In particular, we eliminate the nominal rigidities in price setting and wages. Modeling human capital accumulation requires taking steps in four basic dimensions.

1. How does human capital enter the production function

2. How is human capital accumulated

3. How does time spent in school enter in the utility function

4. How is education financed

The array of policy interventions that we consider in this work package includes R&D subsidies, education subsidies, and education policies that affect the efficiency in the accumulation of human capital.

The rest of the report is organized as follows: section 3 describes the model; section; and section 4 concludes.

3   Model

We take the real economy block of Anzoategui et al. (2015) maintaining the features that help to match the data: habit formation in consumption, flow investment adjustment costs, and variable capital utilization. In addition, we include educational choice and human capital accumulation.

As in the first work package, the fundamental non-standard feature is that total factor productivity depends on two endogenous variables: the creation of new technologies via R&D and the speed of adoption of these new technologies. Skilled labor is used as an input for the R&D and the adoption processes, as well as the production of intermediate

goods. Furthermore, as the technological process demands human capital and educated labor, the decision process of skill level and education impacts the levels of R&D and diffusion inducing dependence between technology and households decisions.

For this reason, we include educational and skill level decisions explicitly. Our main assumption is that every period the worker chooses its investment on education, which determines the level of worker's human capital. Then, these decisions spillover the supply side of the economy as higher effective labor, which affects investment in technology as well as input on production.

In order to study the impact of public policies in this environment, we include an active government that is, in the one hand, able to directly invest in both education and technological activities. And in the other, to deliver subsidies to education and to R&D and adoption.

3.1 Households

The representative household consumes and saves in the form of capital and riskless bonds which are in zero net supply. It rents capital to intermediate goods firms. As in the standard DSGE model, there is habit formation in consumption.

The household supplies only one type of labor, effective skilled labor H_t, which is used on the production of the intermediate good, R&D, or adoption. Effective labor is given by $H_t = h_t L_t$ which is productivity of labor times hours worked. Productivity h_t is also determined by the household, that is accumulated through education e_t. Education is defined as the amount of time the household devotes to educational activities. Hence, our household derives utility from consumption and leisure, the latter given by $(1 - L_t - e_t)$.

Let C_t be consumption, B_t holdings of the riskless bond, Π_t profits from ownership of monopolistically competitive firms, K_t capital, Q_t the price of capital, R_{kt} the rate of return, and D_t the rental rate of capital. Then the households' decision problem is given by

$$\max_{C_t, B_{t+1}, L_t, e_t, h_{t+1}, K_{t+1}} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ \log(C_{t+\tau} - bC_{t+\tau-1}) + \frac{\nu(1 - L_{t+\tau} - e_{t+\tau})^{1-\kappa}}{1-\kappa} \right\} \quad (1)$$

subject to

$$\frac{P_t^C}{P_t} C_t = (1 - \tau_t^l) w_t L_t h_t + \Pi_t + R_{kt} Q_{t-1} K_t - Q_t K_{t+1} + R_t B_t - B_{t+1} + T_t - \frac{P_t^e}{P_t} (1 - \tau_t^e) e_t \quad (2)$$

and

$$h_{t+1} = \Lambda_t e_t + (1 - \delta_h(A_t)) h_t. \quad (3)$$

with

$$R_{kt} = \frac{D_t + Q_t}{Q_{t-1}} \quad (4)$$

This problem departs from the standard one in several ways. First, it considers an explicit disutility of education. Education and working hours contest for a scarce resource, time.

Second, education has a cost, that is the after subsidies price of education $(1 - \tau_t^e)P_t^e$. Third, the benefit of education is that it allows the agent to accumulate human capital. Hence, it is the flow that feeds human capital accumulation. The return to education is given by Λ_t; the higher is this value, the higher is the impact of education on human capital.

Also, the agent must decide its optimal human capital levels for tomorrow given its level today, which makes this problem dynamic. In this setup, human capital is a stock variable that evolves with education. As equation (2) shows, higher human capital levels imply higher labor income, so the agent will derive utility from lower working hours needed to get the same labor income or from higher consumption when h_t increases. Human capital depreciates at a rate δ_h to consider that capital becomes obsolete.[1] We denote with μ_t to be the Lagrange multiplier on (2) and with $\mu_t\gamma_t$ the one on restriction (3). After optimization, we get the following first order conditions for the labor problem:

$$h_{t+1}: \qquad \gamma_t = E_t\left\{M_{t,t+1}\left((1-\tau_{t+1}^l)w_{t+1}L_{t+1} + \gamma_{t+1}(1-\delta_h(A_{t+1}))\right)\right\}, \qquad (5)$$

$$e_t: \qquad \frac{\nu(1-L_t-e_t)^{-\kappa}}{\mu_t} + p_t^e(1-\tau_t^e) = \gamma_t\Lambda_t, \qquad (6)$$

$$L_t: \qquad \nu(1-L_t-e_t)^{-\kappa} = \mu_t(1-\tau_t^l)w_th_t, \qquad (7)$$

Equation (5) is the law of motion of the marginal value of increasing human capital stock by one unit γ_t, that is determined by the flow value of income per unit of human capital after income taxes plus the expected value of future benefits of an additional unit of human capital today. It implies that human capital decision is dynamic and makes the agent look forward. The essential parameter for the evolution of the value of human capital is its depreciation rate $\delta_h(A_t)$, which is crucial to determine the optimal level of h_t, as well as in the transmission of shocks and policies. The depreciation rate depends on the level of technology A_t. We include this to capture the fact that depreciation of human capital is increasing in technology[2]. Hence it also becomes obsolete. Moreover, this assumption assures the existence of a balanced growth path.

Second, equation (6) is the first order condition on education, e_t. The household has to equate the cost in consumption units μ_t of investing one unit of education with its expected value, where the left-hand side is the cost: the utility loss of additional hours of education plus the monetary cost of an additional unit of education. The right-hand side is the benefit, and is given by the utility improvement derived from the increase in human capital, which depends on λ_t, the productivity of education. This variable is key in the optimal value of education and human capital. We consider Λ_t as a policy variable because it is a proxy of educational quality (the efficiency of hours of education); hence, it can be affected by policies. Also, we include subsidies to education τ_t^e, that make education cheaper.

Recall that $H_t = h_tL_t$. Therefore, the relevant labor supplied is effective labor H_t that is what the production side of the economy demands. This will be reflected in the labor

---

[1]In further analysis, we will consider more complex processes for Λ_t that could depend on a policy variable or technology.

[2]We assume an exponential function of A_t with elasticity 0.03. We claim this is small, hence we need a small reaction of depreciation to assure a balanced growth path for reasonable calibration of δ_h.

market clearing. Education increases the supply of effective hours H_t, so all the sectors that demand H_t are benefited by a more slack labor market. Therefore, they produce more technology and goods. In a linearized economy, an increase in the supply of either L_t or h_t is distributed among the different sectors depending on its shares in steady state.

Additionally, we denote with $M_{t,t+1}$ the household's stochastic discount factor, that is given by

$$M_{t,t+1} \equiv \beta \mu_{t+1}/\mu_t \equiv \beta u'(C_{t+1})/u'(C_t) \tag{8}$$

where $u'(C_t) = 1/(C_t - bC_{t-1}) - b/(C_{t+1} - bC_t)$.

Finally, we can express the first order necessary conditions for capital as:

$$1 = E_t\{M_{t,t+1}R_{kt+1}\} \tag{9}$$

3.2 Production Sector and Endogenous TFP: Preliminaries

In this section we describe the production sector and sketch how endogenous productivity enters the model. In a subsequent section we present the firm optimization problems.

We consider two types of firms in this economy (i) final goods producers and (ii) intermediate goods producers. There exists a continuum of measure A_t of monopolistically competitive intermediate goods firms that each make a differentiated product. The endogenous predetermined variable A_t is the stock of types of intermediate goods adopted in production, i.e., the stock of adopted technologies. Intermediate goods firm j produces output Y_t^j. The final good is an intermediate goods CES composite:

$$Y_t = \left(\int_0^{A_t} (Y_t^j)^{\frac{1}{\vartheta}} dj \right)^{\vartheta} \tag{10}$$

with $\vartheta > 1$.

Let K_t^j be the stock of capital firm j employs, U_t^j be how intensely this capital is used, and H_{yt}^j the effective labor employed in production. Then firm j uses capital services $U_t^j K_t^j$ and effective labor $H_{yt}^j = h_t L_{yt}^j$ as inputs to produce output Y_t^j according to the following Cobb-Douglas technology:

$$Y_t^j = \theta_t \left(U_t^j K_t^j \right)^{\alpha} \left(H_{yt}^j \right)^{1-\alpha} \tag{11}$$

where θ_t is an exogenous random disturbance. As we will make clear shortly, θ_t is the exogenous component of total factor productivity. Finally, we suppose that intermediate goods firms set prices each period.

Finally, given a symmetric equilibrium for intermediate goods (recall prices are flexible in this sector) it follows from equation (10) that we can express the aggregate production function for the finally good composite Y_t as

$$Y_t = \left[A_t^{\vartheta-1} \theta_t \right] \cdot (U_t K_t)^{\alpha} H_{yt}^{1-\alpha} \tag{12}$$

where the term in brackets is total factor productivity, which is the product of a term that reflects endogenous variation, $A_t^{\vartheta-1}$, and one that reflects exogenous variation θ_t. In this model we can also write equation (12) in the following way

$$Y_t = \left[A_t^{\vartheta-1} h_t^{1-\alpha} \theta_t \right] \cdot (U_t K_t)^{\alpha} L_{yt}^{1-\alpha}, \tag{13}$$

where the "TFP" is now a compound of three variables: the two described before and human capital, $h_t^{1-\alpha}$. Therefore, there is have a role for education and human capital accumulation not only as an input but as a part of technology in this economy. The more advanced is human capital, the more productive are capital and hours. But also, and more importantly, the higher is the impact of innovation in this economy. With human capital, the effect of increases in the adoption of technologies depends on the state of human capital as well.

In sum, endogenous productivity effects enter through the expansion in the variety of adopted intermediate goods, measured by A_t that is reinforced by the state of human capital, h_t. We next describe the mechanisms through which new intermediate goods are created and adopted.

3.3 R&D and Adoption

The processes for creating and adopting new technologies are based on Comin and Gertler (2006). Let Z_t denote the stock of technologies, while as before A_t is the stock of adopted technologies (intermediate goods). In turn, the difference $Z_t - A_t$ is the stock of unadopted technologies. R&D expenditures increase Z_t while adoption expenditure increase A_t. We distinguish between creation and adoption because we wish to allow for realistic lags in the adoption of new technologies. We first characterize the R&D process and then turn to adoption.

3.3.1 R&D: Creation of Z_t

There are a continuum of measure unity of innovators that use effective labor to create new intermediate goods. Let H_{rt}^p be skilled labor employed in R&D by innovator p and let φ_t be the number of new technologies at time $t+1$ that each unit of skilled labor at t can create. We assume φ_t is given by

$$\varphi_t = \chi_t Z_t H_{rt}^{\rho_z - 1} H_{purt}^{\gamma_z} \tag{14}$$

where χ_t is an exogenous disturbance to the R&D technology H_{purt} is the number of public R&D labor, and H_{rt} is the aggregate amount of skilled labor working on R&D, which an individual innovator takes as given. Following Romer (1990), the presence of Z_t, which the innovator also takes as given, reflects public learning-by-doing in the R&D process. We assume $\rho_z < 1$ which implies that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. We introduce this congestion externality so that we can have constant returns to scale in the creation of new technologies at the individual innovator level, which simplifies aggregation, but diminishing returns at the aggregate level. Our assumption of diminishing returns is consistent with the empirical evidence (see Griliches (1990)); further, with our specification the elasticity of creation of new technologies with respect to R&D becomes a parameter we can estimate, as we make clear shortly.[3]

---

[3]An added benefit from having diminishing returns to R&D spending is that, given our parameter estimates, steady state growth is relatively insensitive to tax policies that might affect incentives for R&D. Given the weak link between tax rates and long run growth, this feature is desirable.

Let J_t be the value of an unadopted technology, $M_{t,t+1}$ the representative household's stochastic discount factor and w_{st} the real wage for a unit of skilled labor. We can then express innovator p's decision problem as choosing H_{rt}^p to solve

$$\max_{H_{rt}^p} E_t\{M_{t,t+1}J_{t+1}\varphi_t H_{rt}^p\} - (1-\tau_t^r)w_t H_{rt}^p \tag{15}$$

where τ_{rt}^s is a R&D subsidy. The optimality condition for R&D is then given by

$$E_t\{M_{t,t+1}J_{t+1}\varphi_t\} - (1-\tau_t^r)w_t = 0$$

which implies

$$E_t\{M_{t,t+1}J_{t+1}\chi_t Z_t H_{rt}^{\rho_z-1} H_{purt}^{\gamma_z}\} = (1-\tau_t^r)w_t \tag{16}$$

The left side of equation (16) is the discounted marginal benefit from an additional unit of skilled labor, while the right side is the marginal cost.

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will be also be pro-cyclical. This consideration, in conjunction with some stickiness in the wages of skilled labor which we introduce later, will give rise to pro-cyclical movements in H_{rt}.[4]

Finally, we allow for obsolescence of technologies.[5] Let ϕ be the survival rate for any given technology. Then, we can express the evolution of technologies as:

$$Z_{t+1} = \varphi_t H_{rt} + \phi Z_t \tag{17}$$

where the term $\varphi_t H_{rt}$ reflects the creation of new technologies. Combining equations (17) and (14) yields the following expression for the growth of new technologies:

$$\frac{Z_{t+1}}{Z_t} = \chi_t H_{rt}^{\rho_z} H_{purt}^{\gamma_z} + \phi \tag{18}$$

where ρ_z is the elasticity of the growth rate of technologies with respect to R&D, a parameter that we estimate.

3.3.2 Adoption: From Z_t to A_t

We next describe how newly created intermediate goods are adopted, i.e. the process of converting Z_t to A_t. Here we capture the fact that technology adoption takes time on average, but the adoption rate can vary pro-cyclically, consistent with evidence in Comin (2009). In addition, we would like to characterize the diffusion process in a way that minimizes the complications from aggregation. In particular, we would like to avoid having to keep track, for every available technology, of the fraction of firms that have and have not adopted it.

Accordingly, we proceed as follows. We suppose there are a competitive group of "adopters" who convert unadopted technologies into ones that can be used in production.

---

[4]Other approaches to motivating procyclical R&D, include introducing financial frictions Aghion et al. (2010), short term biases of innovators Barlevy (2007), or capital services in the R&D technology function Comin and Gertler (2006).

[5]We introduce obsolescence to permit the steady state share of spending on R&D to match the data.

They buy the rights to the technology from the innovator, at the competitive price J_t, which is the value of an adopted technology. They then convert the technology into use by employing skilled labor as input. This process takes time on average, and the conversion rate may vary endogenously.

In particular, the pace of adoption depends positively on the level of adoption expenditures in the following simple way: an adopter succeeds in making a product usable in any given period with probability λ_t, which is an increasing and concave function of the amount of skilled labor employed, H_{at}, and H_{puat} is the amount of skilled public R&D workers used in the adoption of the technology:

$$\lambda_t = \bar{\lambda}_0 * (Z_t H_{at})^{\rho_\lambda} * \left(1 + \bar{\lambda}_{pu} * (Z_t H_{puat})^{\rho_{\lambda pu}}\right) \tag{19}$$

with ρ_λ, $\rho_{\lambda pu} \in (0,1)$ and $\omega > 0$. We augment H_{at} by a spillover effect from the total stock of technologies Z_t - think of the adoption process as becoming more efficient as the technological state of the economy improves. The practical need for this spillover is that it ensures a balanced growth path: as technologies grow, the number of new goods requiring adoption increases, but the supply of labor remains unchanged. Hence, the adoption process must become more efficient as the number of technologies expands. Unlike the specification used for R&D, there is no separate shock to the productivity of adoption activities in (19). We are constrained to introduce this asymmetry because we do not have a direct observable to measure adoption labor or λ_t. The identified series of adoption hours, L_{sat}, can be interpreted as the effective number of adoption hours.

Our adoption process implies that technology diffusion takes time on average, consistent with the evidence. If λ is the steady state value of λ_t, then the average time it takes for a new technology be adopted is $1/\lambda$. Away from the steady state, the pace of adoption will vary with skilled input H_{at}. We turn next to how H_{at} is determined.

Once in usable form, the adopter sells the rights to the technology to a monopolistically competitive intermediate goods producer that makes the new product using the production function described by equation (12). Let Π_{mt} be the profits that the intermediate goods firm makes from producing the good, which arise from monopolistically competitive pricing. The adopter sells the new technology at the competitive price V_t, which is the present discounted value of profits from producing the good, given by

$$V_t = \Pi_{mt} + \phi E_t\{M_{t,t+1} V_{t+1}\} \tag{20}$$

Then we may express the adopter's maximization problem as choosing L_{sat} to maximize the value J_t of an unadopted technology, given by

$$J_t = \max_{H_{at}} E_t\{-(1 - \tau_t^a) w_t H_{sat} + \phi M_{t,t+1}[\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}\} \tag{21}$$

subject to equation (19). The first term in the Bellman equation reflects total adoption expenditures that considers a subsidy to technological adoption τ_t^a, while the second is the discounted benefit: the probability weighted sum of the values of adopted and unadopted technologies.

The first order condition for H_{at} is

$$\lambda_t' \cdot \phi E_t\{M_{t,t+1}[V_{t+1} - J_{t+1}]\} = (1 - \tau_t^a)w_t \tag{22}$$

The term on the left is the marginal gain from adoption expenditures: the increase in the adoption probability λ_t times the discounted difference between an adopted versus unadopted technology. The right side is the marginal cost.

The term $V_t - J_t$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones. Given this consideration and the stickiness in w_t which we alluded to earlier, H_{at} varies pro-cyclically. The net implication is that the pace of adoption, given by λ_t, will also vary pro-cyclically.

Given that λ_t does not depend on adopter-specific characteristics, we can sum across adopters to obtain the following relation for the evolution of adopted technologies

$$A_{t+1} = \lambda_t \phi[Z_t - A_t] + \phi A_t \tag{23}$$

where $Z_t - A_t$ is the stock of unadopted technologies.

3.4 Firms

3.4.1 Intermediate goods firms: factor demands

Given the CES function for the intermediate good composite (10), in the symmetric equilibrium each of the monopolistically competitive intermediate goods firms charges the markup ϑ. Let p_{mt} be the relative price of the intermediate goods composite. Then from (10) and the production function (11), cost minimization by each intermediate goods producer yields the following standard first order conditions for capital, capital utilization, and unskilled labor:

$$\alpha \frac{p_t Y_t}{K_t} = \vartheta[D_t + \delta(U_t)Q_t] \tag{24}$$

$$\alpha \frac{p_t Y_t}{U_t} = \vartheta \delta'(U) Q_t K_t \tag{25}$$

$$(1 - \alpha) \frac{p_t Y_t}{H_{yt}} = \vartheta w_t \tag{26}$$

3.4.2 Capital producers: investment

Competitive capital producers use final output to make new capital goods, which they sell to households, who in turn rent the capital to firms. Let I_t be new capital produced and p_{kt} the relative price of converting a unit of investment expenditures into new capital (the replacement price of capital), and γ_y the steady state growth in I_t. In addition, following Christiano et al. (2005), we assume flow adjustment costs of investment. The capital producers' decision problem is to choose I_t to solve

$$\max_{I_t} E_t \sum_{\tau=0}^{\infty} M_{t,t+\tau} \left\{ Q_{t+\tau} I_{t+\tau} - p_{kt+\tau} \left[1 + f\left(\frac{I_{t+\tau}}{(1+\gamma_y)I_{t+\tau-1}}\right) \right] I_{t+\tau} \right\} \tag{27}$$

where the adjustment cost function is increasing and concave, with $f(1) = f'(1) = 0$ and $f''(1) > 0$. We assume that p_{kt} follows an exogenous stochastic process.

The first order condition for I_t the relates the ratio of the market value of capital to the replacement price (i.e. "Tobin's Q") to investment, as follows:

$$
\frac{Q_t}{p_{kt}} = 1 + f\left(\frac{I_t}{(1+\gamma_y)I_{t-1}}\right) + \frac{I_t}{(1+\gamma_y)I_{t-1}}f'\left(\frac{I_t}{(1+\gamma_y)I_{t-1}}\right)
$$
$$
- E_t M_{t,t+1}\left(\frac{I_{t+1}}{(1+\gamma_y)I_t}\right)^2 f'\left(\frac{I_{t+1}}{(1+\gamma_y)I_t}\right)
\tag{28}
$$

3.4.3 Fiscal policy

We take two approaches when including government, lump-sum or distortionary taxes. If we assume that government activities G_t, H_{purt}, H_{puat}, τ_t^r, τ_t^a, and τ_t^e are financed with lump sum taxes T_t, government's budget constraint is

$$
G_t + w_t(H_{purt} + H_{puat}) + w_t(\tau_t^r H_{rt} + \tau_t^a H_{at}) + \tau_t^e P_t^e e_t = T_t
\tag{29}
$$

while with distortionary taxes it writes

$$
G_t + w_t(H_{purt} + H_{puat}) + w_t(\tau_t^r H_{rt} + \tau_t^a H_{at}) + \tau_t^e P_t^e e_t = \tau_t^l w_t H_t
\tag{30}
$$

Further, the (log) deviation of G_t, H_{purt}, H_{puat}, τ_t^r, τ_t^a, τ_t^e, and Λ_t from the deterministic trend of the economy follows AR(1) processes. Formally, for each $\mathcal{X}_t \in \{G_t, H_{purt}, H_{puat}, \tau_t^r, \tau_t^a, \tau_t^e, \Lambda_t\}$, we have

$$
\log(\mathcal{X}_t/(1+\gamma_y)^t) = (1 - \rho_{\mathcal{X}})\bar{\mathcal{X}} + \rho_{\mathcal{X}}\log(\mathcal{X}_{t-1}/(1+\gamma_y)^{t-1}) + \epsilon_t^{\mathcal{X}}
\tag{31}
$$

3.5 Resource constraints and equilibrium

The resource constraint is given by

$$
Y_t = C_t + p_{kt}\left[1 + f\left(\frac{I_{t+\tau}}{(1+\gamma_y)I_{t+\tau-1}}\right)\right]I_t + G_t + p_t^e e_t
\tag{32}
$$

Capital evolves according to

$$
K_{t+1} = I_t + (1 - \delta(U_t))K_t
\tag{33}
$$

The market for effective labor must clear:

$$
H_t = (Z_t - A_t) * (H_{at} + H_{puat}) + H_{rt} + H_{purt} + H_{yt}
\tag{34}
$$

Finally, the market for risk-free bonds must clear, which implies that in equilibrium, risk-free bonds are in zero net supply

$$
B_t = 0
$$

This completes the description of the model.

3.6 Calibration

In this section we concentrate in the calibration of the labor-education block of our model, while the rest of the calibration follows WP1. This requires to set steady state values of hours worked, hours of education, aggregate expenditure in education, and the depreciation of human capital.

Time spent in education and labor activities are obtained from the OECD. To calibrate hours worked in steady state we use the mean of hours worked. For a representative worker this represents 0.42 of her available time.[6] To compute the time the economy spends working we must consider the proportion of the population that actually works, which is 0.44. Therefore, we obtain $L = 0.186$. Then we conduct a similar exercise to compute hours of education in steady state. A representative student spends 0.192 of her time in educating. Considering that about the 35% of the population study full-time (people 0 to 25 years old represent 40% of the population, so we consider this number an upper bound), and considering that about 10% of the population over 25 years also study (we also consider this number as an upper bound), we set $e = 0.087$. Robustness analyses are going to be conducted to check the impact of these assumptions on the results. Human capital depreciation is set to 1.6%, which we consider an upper bound per quarter (6.4% yearly). The estimates od depreciation of human capital are quite diverse. Heckman (1976) finds an upper bound of 4% annually. Also, Groot (1998) finds numbers between 11% and 17% annually for Europe, estimates that have increased in time due to technological progress. In our case, we take a stance on a low depreciation in the body of the text but compare it with a high depreciation (18% annually) in the appendix.

Finally, we calibrate expenditure in education as a share of GDP. For this, we also follow OECD data. According to the report "Education at a Glance", the OECD average for total expenditure in education is 5.2% of GDP. We have to split this into public and private. Public expenditure in education is 4.4% of GDP while private is 0.8%. In this way, we set $s_e = 0.8\%$.

We use the estimates of WP6 to calibrate the parameters in φ_t. The private technological parameter ρ_z, is set to 0.38 and the public technological elasticity to 0.29.

4 Conclusion

In work-package 3, we include endogenous human capital accumulation to the model of endogenous growth developed in the WP1. This extension is useful since it considers two important features. 1. Human capital is a state variable. 2. Agents react to economic conditions and policies when deciding how much to invest in human capital. Moreover, making human capital endogenous is important to obtain an accurate understanding of the effects of innovation policies for several reasons. First, human capital is an input in production and impacts the productivity of the other inputs of the economy. Therefore, policies that affect the stock of human capital must affect aggregate productivity. Sec-

---

[6]We define available time as the time an average worker/student as sixteen hours each day of five days per week.

ond, human capital is an important input in R&D activities and in efforts to adopt new technologies. Therefore, policies that affect the human capital stock may also affect the cost of R&D and adoption services. Finally, it is important to remark that since human capital is a state variable, by including it in our analysis, the effect of policies in the economy may not only increase but also may become more long-lasting than if we ignored the endogeneity of human capital.

We conduct several policy exercises, extending the ones developed in the Baseline model. First, we consider in our model an explicit measure of educational quality that we model as the rate of transformation of hours of education into human capital. This allows us to explore the macroeconomic impact of improving educational quality. Second, we include spending in education. As educational expenditures are not negligible in a macroeconomic perspective (it is 5% of the GDP in the OECD), studying the effect of increasing subsidies to education is crucial.

References

Aghion, P., Angeletos, G.-M., Banerjee, A., and Manova, K. (2010). Volatility and growth: Credit constraints and the composition of investment. *Journal of Monetary Economics*, 57(3):246 – 265.

Anzoategui, D., Comin, D., Gentler, M., and Martinez, J. (2015). Endogenous technology adoption and r&d as sources of business cycle persistence. *Dartmouth mimeo.*

Barlevy, G. (2007). On the cyclicality of research and development. *American Economic Review*, 97(4):1131–1164.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.

Comin, D. (2009). On the integration of growth and business cycles. *Empirica*, 36:165–176.

Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.

Griliches, Z. (1990). Patent statistics as economic indicators: A survey. *Journal of Economic Literature*, 28:1661–1707.

Groot, W. (1998). Empirical estimates of the rate of depreciation of education. *Applied Economics Letters*, 5(8):535–538.

Heckman, J. J. (1976). A life-cycle model of earnings, learning, and consumption. *Journal of Political Economy*, 84(4, Part 2):S9–S44.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.

A   Calibration

Parameter	Description	Value
α	Capital share	1/3
δ	Capital depreciation	0.02
β	Discount factor	0.995
φ	Inv. Frisch elasticity	3.381
$\frac{G}{Y}$	SS govt. consumption/output	0.2
γ_y	SS output growth	1.87%
ϑ	Intermediate goods mark-up	1.35
$1 - \phi$	Obsolescence rate	0.08/4
$\overline{\lambda}$	SS adoption lag	0.15/4
ρ_λ	Private adoption elasticity	0.95
ρ_z	Private R&D elasticity	0.38
f''	Investment adj. cost	1.386
$\frac{\delta'(U)}{\delta}$	Capital utiliz. Elast.	3.868
b	Consumption habit	0.389

Table 4: Calibration.

Parameter	Description	Value
ρ_θ	TFP	0.91
ρ_{pk}	Investment	0.87
ρ_ϱ	Liquidity demand	0.91
ρ_g	Govt. expenditures	0.99
ρ_χ	R&D	0.84
σ_θ	TFP	0.51
σ_{pk}	Investment	0.74
σ_ϱ	Liquidity demand	0.23
σ_g	Govt. expenditures	2.87
σ_χ	R&D	2.13

Table 5: Calibration.

Parameter	Description	Value
$\overline{\lambda}_{pu}$	SS public adoption lag	0.2
$\rho_{\lambda_{pu}}$	Public adoption elasticity	0.7
γ_z	Public R&D elasticity	0.29
ρ_{lr}	Persistence in Pub Inv in Adoption	0.9
ρ_{la}	Persistence in Pub Inv in R&D	0.9
ρ_{κ_r}	Persistence Pub subsidies in Adoption	0.9
ρ_{κ_a}	Persistence Pub subsidies in R&D	0.9
σ_{lr}	Pub Inv in Adoption	0.01
σ_{la}	Pub Inv in R&D	0.01
σ_{κ_r}	Pub subsidies in Adoption	0.01
σ_{κ_a}	Pub subsidies in R&D	0.01
$G/Earnings$	SS gov spending share	0.8
$L_{spua}/Earnings$	SS adoption inv share	0.05
$L_{spur}/Earnings$	SS R&D inv share	0.05
$\kappa_a/Earnings$	SS adoption subsidy share	0.05
$\kappa_r/Earnings$	SS R&D subsidy share	0.05

Table 6: Calibration.