Can Aggregation across Goods be Achieved by Neglecting the Problem?

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Abstract. This paper studies the problem of composite commodity in two different frameworks. In one case, the aggregation across goods is analyzed for elementary goods that satisfy an optimality condition. The unrestricted case is also examined. I formalize the notion of an approximate aggregate representation and show that it is always possible. Can thereby aggregation issues simply be neglected in economic contributions? I show that the standard economic properties of initial functions are not necessarily inherited by the approximate aggregate. However, the conditions which guarantee the inheritance are shown to be weaker that those required in the case of exact aggregation.

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1. Introduction

The microeconomic theory of producer and consumer behavior derives structural relationships on the basis of elementary goods and prices. The concept of elementary good is, however, difficult to handle both because of the huge amount of such goods and because of the difficulty in defining what an elementary good even represents (in the Arrow-Debreu framework, time and localization lead to distinguish physically identical goods). Thus, composite commodities are considered instead in empirical analyses. A direct consequence is the loss of some information relative to the original problem. A more worrying consequence is that the initial structural framework must be adapted to cope with composite commodities and to retain coherency with the original theory referring to elementary goods. Two principal approaches have been proposed.

Exact aggregation across goods, developed by e.g. Leontief [1947], Sono [1961], Blackorby, Primont and Russell [1978] requires that the aggregate representation holds for any value of the elementary variables. The exact aggregation approach states which conditions on the preferences and the technologies of the individual units have to hold for exact aggregation to be possible. The conclusions of this approach are rather pessimistic since the required conditions seem rather implausible and are most often rejected in empirical investigations (see e.g. Blackorby, Schworm and Fisher [1986] and Diewert and Wales [1995]). Of course, the generality of the requirement of exact aggregation is directly at the origin of the restrictive conditions under which an aggregate description of the initial relationships exists.

In order to avoid these implausible restrictions on admissible preferences (technologies), some authors have followed a practice suggested by Hicks [1936] and Leontief [1936], which consists in restricting the *distribution* of the elementary prices. Often, the strict proportionality of all elementary prices of a given subset is assumed for achieving aggregation. This alternative approach is also problematic for two main reasons: first, when decision units behave rationally, there exist links between restrictions on the preferences and restrictions on the distribution of elementary goods and prices as underlined in Koebel [1998]; second, strict proportionality of prices is usually rejected when tested. Lewbel [1993b, 1996] weakens this assumption and shows that aggregation across goods is achieved when prices are approximately proportional, an assumption which still appears as quite restrictive.

The purpose of this paper is to show that an aggregate representation can be achieved under broader conditions than those presented in this stream of the literature. Whereas the exact aggregation approach raises doubts on the very existence of an aggregate representation, we show that it is always possible, by simple transformations of the microeconomic model, to obtain relationships depending on aggregate variables and on the way the elementary variables are distributed. We also present the necessary and sufficient conditions for modelling economic relationships on the basis of aggregate goods and prices only: it is shown that these conditions are weaker than those required by exact aggregation and approximate price proportionality. Of course, these conditions are still restrictive and may not hold in general. Neglecting the distribution of elementary variables when modelling individual relationships then has the same implication as omitting some relevant explanatory variables, as already underlined by Theil [1954]. In this case, we show that no microeconomic regularity relationships necessarily hold at the aggregate level (across goods and prices). However, the conditions which guarantee the inheritance of these properties are shown to be weaker that those required in the case of exact aggregation.

In the second section, the framework and the notations are outlined. The relations between exact aggregated models and those obtained when aggregation is neglected are presented in the third section, in a context where optimality relationships are not considered. In the fourth section, I study the implication of neglecting aggregation when optimality conditions drive the allocation of some goods. In the fifth section, the properties of optimized and indirect objective functions are presented. An empirical investigation is presented in the sixth section.

2. From elementary to aggregate goods

The definition of aggregation theory involves several concepts: elementary and aggregate goods, microeconomic and aggregate theory. The meaning given to these concepts varies across contributions, so that it appears useful to state some generally accepted points before describing these distinct, alternative approaches.

2.1 The microeconomic framework

Italic letters are chosen to denote elementary goods and microeconomic relationships, while bold letters are used for aggregate goods and relationships. Let $x \in \mathbb{R}^{S_x}$ and $z \in \mathbb{R}^{S_z}$ be the vectors of elementary goods, where S_x and S_z denote the dimensions of the vectors x and z. By convention, any positive components of x and z correspond to a net supply and negative ones to a net demand. Furthermore, x is a vector of choice variables and z a vector of fixed goods.

Let $f : \mathbb{R}^{S_x} \times \mathbb{R}^{S_z} \to \mathbb{R}$ be a continuously differentiable transformation (or utility) function of an optimizing agent. In accordance with all these conventions, the transformation (or utility) function f must be strictly decreasing in x and z to match the standard economic theory. Besides, f is also strictly quasi-concave in x. In the present case, the functional form of f is assumed to be known and parameterized by the vector of technological characteristics $\alpha \in \mathbb{R}^{S_\alpha}$, parameters whose estimation and interpretation represent the purposes of empirical studies. In this paper, aggregation of the vector of elementary parameters α will be considered along with the aggregation of elementary goods.

Contrary to the objective function f, the precise functional form of the profit constraint $\pi(p, x) \geq b$ is known and given by $p'x \geq b$, where p denotes the exogenous price vector and $b \in \mathbb{R}$ corresponds to the exogenous part of the profit (budget). We also assume that a continuously differentiable solution to the optimization problem

$$\max_x \{f(x, z; \alpha) : p'x \geq b\} \tag{1}$$

exists. The microeconomic optimality conditions (for an interior solution) are then given by $\partial f/\partial x = -\lambda_f p$, and by the binding profit constraint $p'x = b$; the parameter $\lambda_f \geq 0$ denotes the Lagrange multiplier. We will also consider the dual problem obtained by inverting the role of the former objective and constraint:

$$\max_x \{p'x : f(x, z; \alpha) \geq 0\}. \quad (2)$$

This last optimization problem is more usual in production economics, whereas the former is common in consumer analysis.

2.2 Elementary goods and aggregates

For simplification, the aggregation approach considered in this paper is based on real valued prices and quantity aggregates for describing a set of quantities and prices. Let $\mathcal{J} = \{\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_J\}$ and $\mathcal{K} = \{\mathcal{K}_1, \mathcal{K}_2, \ldots, \mathcal{K}_K\}$ denote a partition of the set of integers indexing the elementary goods into respectively J and K subsets. According to this partition, we decompose the vectors of goods x and z as follows

$$\begin{aligned} x &= \left(x_1', \ldots, x_j', \ldots, x_J'\right)' = \left(x_{11}, \ldots, x_{1S_{x_1}}, \ldots, x_{J1}, \ldots, x_{JS_{x_J}}\right)', \\ z &= \left(z_1', \ldots, z_k', \ldots, z_K'\right)' = \left(z_{11}, \ldots, z_{1S_{z_1}}, \ldots, z_{K1}, \ldots, z_{KS_{z_K}}\right)', \end{aligned}$$

where S_{x_j} and S_{z_k} denote the dimensions of the vectors x_j and z_k. For each subset, the aggregates $\mathbf{x}_j$ and $\mathbf{z}_k$ are defined as $\mathbf{x}_j = a_{x_j}\left(x_j; \gamma_{x_j}\right)$, for $j = 1, \ldots, J$ and by $\mathbf{z}_k = a_{z_k}\left(z_k, \gamma_{z_k}\right)$, for $k = 1, \ldots, K$; the aggregate prices for the goods belonging to $\mathcal{J}_j$ are given by $\mathbf{p}_j = a_{p_j}\left(p_j; \gamma_{p_j}\right)$. The aggregator functions a_{x_j}, a_{p_j} and a_{z_k} are real valued functions, increasing in x_j, p_j and z_k respectively, and twice continuously differentiable. As we will see below, the specification of the aggregators and the parameters γ_{x_j} and γ_{z_k} differs in the literature.

In empirical work, the aggregates correspond frequently to sums, i.e. $\mathbf{x}_j \equiv \sum_{h \in \mathcal{J}_j} x_{jh}$, or to weighted sums i.e. $\mathbf{x}_j \equiv \sum_{h \in \mathcal{J}_j} k_{jh} x_{jh}$ with $k_{jh} = p_{jh}/\mathbf{p}_j$ or $k_{jh} = p_{jh}^0 / \left(\sum_{h \in \mathcal{J}_j} p_{jh}^0 x_{jh}^0\right)$, and where $\mathbf{p}_j$ is an aggregate price for group $\mathcal{J}_j$. When aggregate prices $\mathbf{p}_j$ are not available at the level of individual units, they are often replaced by a price level $\mathbf{P}_j$ aggregated across several decision units. Vectors of aggregates are denoted by $\mathbf{x}$, $\mathbf{p}$ and $\mathbf{z}$.

2.3 The aggregation problems

Two kind of aggregation problems are studied in this paper. The first one studies the conditions under which $f(x, z; \alpha)$ and $p'x$ can be described on the basis of aggregate goods and prices $\mathbf{x}$, $\mathbf{z}$ and $\mathbf{p}$, but without considering that some goods may be optimally allocated. The second kind of problem considered, studies whether the elementary optimization problem can be equivalently expressed in terms of aggregates.

Out of the optimum, the aggregation across elementary goods usually requires that the following criterion be fulfilled:

$$f(x, z; \alpha) = \mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha}), \quad (3)$$

and that

$$p'x = \mathbf{p}'\mathbf{x}. \quad (4)$$

Since the right hand side of the equalities (3) and (4) depend on aggregates, the left hand side must also be so. This can only be achieved if the elementary variables x, z and p depend on the aggregates. This dependance must not be strictly deterministic but may be stochastic. The available tools for achieving the aggregate representations are (i) the specification of the microeconomic and aggregate functions f and $\mathbf{f}$ and (ii) the definition of the aggregate variables and parameters $\boldsymbol{\alpha}$ for given microeconomic relationships. In this paper, the form of the aggregate goods are mainly given by those usually computed by statistical offices.

The second kind of aggregation problems considered here studies whether the elementary optimization problem can be equivalently expressed in terms of aggregates, i.e., if the following aggregation criterion is considered:

$$\begin{aligned} & \max_{\mathbf{x}} \{\mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha}) : \mathbf{p}'\mathbf{x} \geq b\} = \max_{x} \{f(x, z; \alpha) : p'x \geq b\} \\ \Leftrightarrow \quad & \mathbf{f}^*(\mathbf{p}, \mathbf{z}; \boldsymbol{\alpha}, b) = f^*(p, z; \alpha, b) . \end{aligned} \tag{5}$$

The microeconomic optimality conditions are then given by $\partial f / \partial x = -\lambda_f p$, and by the binding profit constraint $p'x = b$, where the parameter $\lambda_f \geq 0$ denotes the Lagrange multiplier. It is also interesting to consider aggregation within the dual problem:

$$\begin{aligned} & \max_{\mathbf{x}} \{\mathbf{p}'\mathbf{x} : \mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha}) \geq 0\} = \max_{x} \{p'x : f(x, z; \alpha) \geq 0\} \\ \Leftrightarrow \quad & \boldsymbol{\pi}^*(\mathbf{p}, \mathbf{z}; \boldsymbol{\alpha}) = \pi^*(p, z; \alpha) . \end{aligned} \tag{6}$$

Again, the specification of the microeconomic relationships f and the definition of the parameters $\boldsymbol{\alpha}$ are the available tools for achieving the above aggregation criteria.

The exact aggregation approach to solve these problems principally studies which conditions on the microeconomic relationship f make the above representations possible (see Blackorby, Primont and Russell [1978]). When aggregation is neglected, the analysis begins with a function $\mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha})$ and assumes that the different aggregation criteria above are fulfilled. For given microeconomic relationships f and given aggregates $\mathbf{p}$, $\mathbf{x}$ and $\mathbf{z}$, the only remaining possibility for fulfilling (3) to (6), is to allow the aggregate parameters $\boldsymbol{\alpha}$ to become functions of the elementary goods x and z.

3. Neglecting aggregation in f and π out of optimality

Up to this point, the aggregate parameter vector $\boldsymbol{\alpha}$ has not been specified. Therefore, it can take any elementary goods as components and an aggregate representation of the problem is always possible through a convenient reparameterization of the initial problem. By using, for example, a transformation suggested by Lewbel [1992], we can rewrite:[1]

$$f(x, z; \alpha) = f\left(x_1 \frac{\mathbf{x}_1}{\mathbf{x}_1}, \ldots, x_J \frac{\mathbf{x}_J}{\mathbf{x}_J}, z_1 \frac{\mathbf{z}_1}{\mathbf{z}_1}, \ldots, z_K \frac{\mathbf{z}_K}{\mathbf{z}_K}; \alpha\right).$$

Let us define vectors of the shares of the elementary goods in the aggregate as $\rho_{x_j} = (x_{jh}/\mathbf{x}_j)'$, $\rho_{z_k} = (z_{kh}/\mathbf{z}_k)'$ and $\rho_{p_j} = (p_{jh}/\mathbf{p}_j)'$, and denote with ρ_x, ρ_z and ρ_p the corresponding vectors with respectively the ρ_{x_j}, ρ_{z_k} and ρ_{p_j} as subvectors. Further, form the

[1] In fact, Lewbel considers aggregation across decision units.

vectors $\widetilde{\mathbf{x}}$, $\widetilde{\mathbf{z}}$ and $\widetilde{\mathbf{p}}$ according to the following pattern for $\widetilde{\mathbf{x}}$

$$\widetilde{\mathbf{x}} = (\underbrace{\mathbf{x}_1, \ldots, \mathbf{x}_1}_{S_{x_1} \text{ terms}}, \mathbf{x}_2, \ldots, \underbrace{\mathbf{x}_j, \ldots, \mathbf{x}_j}_{S_{x_j} \text{ terms}}, \ldots, \mathbf{x}_J)'.$$

Then, the aggregate relations can be written on the basis of available information on the aggregates:

$$f(x, z; \alpha) = f(\widetilde{\mathbf{x}} * \rho_x, \widetilde{\mathbf{z}} * \rho_z; \alpha) = \mathbf{f}^a(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z), \tag{7}$$

where the sign $*$ represents the Hadamard product.[2] The superscript a of $\mathbf{f}^a(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z)$ is introduced for denoting the aggregated transformation (or utility) function, which is parameterized by the technological parameters α, and by the distributions ρ_x and ρ_z of elementary goods. When only data on $\mathbf{x}$ and $\mathbf{z}$ are used for the estimation of $\mathbf{f}^a(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z)$, the aggregate parameters $\boldsymbol{\alpha}_j$ are implicitly defined as a function of α, ρ_x and ρ_z:

$$f(x, z; \alpha) = f(\mathbf{x}, \mathbf{z}; a_\alpha(\alpha; \rho_x, \rho_z)) = \mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha}). \tag{8}$$

Remark that the above representation, although based on no restrictions on the microeconomic function f is also exact, that is to say, is true for any value of the elementary goods x and z. However, is the aggregate parameter vector $\boldsymbol{\alpha}$ taken to be constant (the aim of many empirical methods), then the representation will only be *approximate*.[3]

Thus, we see from (8) that approaches neglecting aggregation and starting directly with a function $\mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha})$ also admit some foundations in the theory of aggregation. The aggregation criterion (8) appears as antipodal to the weak separability approach: now aggregate goods depend solely on elementary goods but the aggregate parameters $\boldsymbol{\alpha}_\ell$ are complex functions of the elementary parameters and of the shares of elementary goods.

Such a transformation is clearly possible for any functional form f. In order to better underline the implications of such a transformation, let us consider a Cobb-Douglas production function for example, and rewrite:

$$\begin{aligned}
f(x, z; \alpha) \geq 0 &\Leftrightarrow \prod_{k=1}^{K} \prod_{h=1}^{S_{z_k}} (-z_{kh})^{\alpha_{kh}} - x \geq 0 \\
&\Leftrightarrow \prod_{k=1}^{K} \prod_{h=1}^{S_{z_k}} \left(-\frac{z_{kh}}{\mathbf{z}_k} \mathbf{z}_k \right)^{\alpha_{kh}} - \mathbf{x} \geq 0 \\
&\Leftrightarrow \prod_{k=1}^{K} (-\mathbf{z}_k)^{\boldsymbol{\alpha}_k} \prod_{h=1}^{S_{z_k}} \left(\frac{z_{kh}}{\mathbf{z}_k} \right)^{\alpha_{kh}} - \mathbf{x} \geq 0 \\
&\Leftrightarrow \boldsymbol{\alpha}_0 \prod_{k=1}^{K} (-\mathbf{z}_k)^{\boldsymbol{\alpha}_k} - \mathbf{x} \geq 0 \\
&\Leftrightarrow \mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha}) \geq 0,
\end{aligned}$$

where $z_{kh} \leq 0$ are the inputs and $x \geq 0$ is a scalar output. Note that in this example, we have assumed that the elementary commodity x coincides with the aggregate $\mathbf{x}$.

[2] The Hadamard product between two vectors u and v of the same dimension $u * v$ gives a vector of the same dimension again and with $u_i v_i$ as components.

[3] Therefore, the parameter determination will in last instance proceed to aggregation by expressing the initial information in a more condensed manner.

The elementary good x could also be grouped along with some goods z_k of z. Then, however, the aggregate function becomes nonlinear in all variables. In this example, the aggregate parameters are defined as $\boldsymbol{\alpha}_0 = \prod_h (z_{kh}/\mathbf{z}_k)^{\alpha_{kh}}$, $\boldsymbol{\alpha}_k = \sum_h \alpha_{kh}$, for $k \neq 0$ and $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_K)$. Aggregate parameters have different meanings from disaggregate ones. Even when each production unit possesses the same technology (i.e., α_{kh} are identical for each production unit, $\forall k, h$), the aggregate parameter $\boldsymbol{\alpha}_0$ can differ across production units because of different shares of elementary goods used. Then, differences in $\boldsymbol{\alpha}_0$ do not convey differences in technologies alone. Studies aimed at measuring technical inefficiencies by analyzing how $\boldsymbol{\alpha}_0$ varies across production units are also implicitly testing the significance of aggregation over goods (see e.g. Cornwell and Schmidt [1996] for a recent survey of this literature). Under Lewbel's [1992] mean scaling assumption, $\boldsymbol{\alpha}_0$ is independent of $\mathbf{z}$. This assumption can in some cases be tested without any data on elementary goods: with panel data, a Hausman type test could be used for this aim. If the independence assumption is not satisfied, the choice of an adequate method for the estimation of $\boldsymbol{\alpha}$ is required. Thus, econometric tools can be useful to handle some aggregation problems.

In general, there is an identification problem precluding the interpretation of the parameters $\boldsymbol{\alpha}$ as reflecting strictly individual behavior. Thus, a similar difficulty to the one arising in the context of aggregation across agents (see e.g. Stoker [1984]) also characterizes aggregation across goods. The marginal impact of the aggregate variable can be decomposed into a direct and an indirect shift which cannot be identified independently when the function $\mathbf{f}(\mathbf{x}, \mathbf{z}; \boldsymbol{\alpha})$ is retained. Indeed, since the aggregate representation only makes sense when elementary goods depend on the aggregate, ρ_x will also be shifted when $\mathbf{x}_j$ vary (hence the notation $\overline{\rho}_x$):

$$\frac{d\mathbf{f}^a}{d\mathbf{x}_j} = \frac{\partial \mathbf{f}^a}{\partial \mathbf{x}_j} + \frac{\partial \mathbf{f}^a}{\partial \rho_x'} \frac{\partial \overline{\rho}_x}{\partial \mathbf{x}_j},$$

where the last term represents the shift in the distribution of the elementary goods $\partial \overline{\rho}_x / \partial \mathbf{x}_j$ consecutively to a variation in the aggregate. In two extreme cases, the distributional impact vanishes. The assumption of weak separability would imply that $\partial \mathbf{f}^a / \partial \rho_x = 0$.[4] Strict (or approximate) proportionality of all elementary goods of a given partition with the corresponding aggregate would imply $\partial \overline{\rho}_x / \partial \mathbf{x}_j = 0$. These conditions appear however more restrictive than necessary since the indirect impact $\partial \mathbf{f}^a / \partial \rho_x' \left[\partial \overline{\rho}_x / \partial \mathbf{x}_j \right]$ vanishes under the weaker condition of orthogonality between both vectors. Thus, restrictions on both functional form and distribution of the elementary goods may appear helpful for avoiding distributional impacts in the first derivative.

A further alternative is to impose restrictions, not on the components of the model (functional forms, distribution of elementary goods), but on the interpretation of the estimates. Indeed, if only the total impact of a shift in the aggregate $\mathbf{x}_j$ can be estimated but not the impact of $\mathbf{x}_j$ for a given distribution of x, it suffices to content ourselves with what is available: $d\mathbf{f}^a / d\mathbf{x}_j$.

[4] Under weak separability, $f(x, z; \alpha)$ can be written as $\mathbf{f}^{ws}(\mathbf{x}, \mathbf{z}; \overline{\alpha})$ where $\overline{\alpha}$ is a subvector of α. This aggregate function $\mathbf{f}^{ws}(\mathbf{x}, \mathbf{z}; \overline{\alpha})$ is not parameterized by the distribution of elementary goods in the aggregate.

A similar transformation can also be applied to aggregate profits, so that we can rewrite $p'x$ as

$$p'x = \pi(p,x) = \left(\widetilde{\mathbf{p}} * \rho_p\right)' \left(\widetilde{\mathbf{x}} * \rho_x\right) = \boldsymbol{\pi}^a\left(\mathbf{p},\mathbf{x};\rho_x,\rho_p\right). \tag{9}$$

A more stringent aggregative criterion than (9) is often required: the product of the aggregate price and quantity should equal the costs (or revenues) of all elementary goods, i.e., $p_j'x_j = \mathbf{p}_j\mathbf{x}_j$. If we allow that price aggregators for the subset $\mathcal{J}_j$ may depend on elementary quantities of this subset (in which case, a_{p_j} has also x_j among its components), this last equality can always be achieved by defining $\mathbf{p}_j$ as the *unit value* of the goods x_j, i.e., $\mathbf{p}_j^u = \sum_h p_{jh}x_{jh}/\mathbf{x}_j$.[5] Thus, profits can also be represented in an aggregate way, but will be subject to similar inconveniences than those occurring in the transformation function. Indeed, the marginal profit of good $\mathbf{x}_j$ is not equal to its unit price in general. When $\boldsymbol{\pi}^a = \mathbf{p}^{u\prime}\mathbf{x}$ for example,

$$\frac{\partial\boldsymbol{\pi}^a}{\partial\mathbf{x}_j} = \mathbf{p}_j^u + \frac{\partial\mathbf{p}^{u\prime}}{\partial\mathbf{x}_j}\mathbf{x}.$$

To summarize, in this section we have seen that an aggregate representation of the initial relationships which depend on elementary goods and prices is possible without restricting microeconomic behavior, by adequately defining aggregate quantities, prices and parameters.

4. First order optimality conditions when aggregation is neglected

Both aggregate functions $\mathbf{f}^a$ and $\boldsymbol{\pi}^a$ depend on the distribution of elementary goods and prices. In such a context, it seems interesting to see under which conditions, the regularity properties verified by the microeconomic functions are inherited in the aggregate. Can the microeconomic optimization framework be adopted for aggregate functions? Which properties do the aggregate optimized relationships verify? In the present section, we show that when aggregation is neglected and when a transformation (or utility) function $\mathbf{f}(\mathbf{x},\mathbf{z};\boldsymbol{\alpha})$ is considered without further justification in aggregation theory, the usual first order optimality conditions hold for the model relying on $\mathbf{f}(\mathbf{x},\mathbf{z};\boldsymbol{\alpha})$. For this aim, the preceding model is adapted to a context where some goods are optimally allocated. Besides, we will see how aggregate prices should be defined to be coherent with the theory.

Using the transformations (7) and (9), the initial maximization problem can be expressed as

$$\begin{aligned} & f^*(p,z,b;\alpha) = \max_x \{f(x,z;\alpha) : p'x \geq b\} \\ = \; & \max_{\overline{\mathbf{x}}}\left\{\max_{\rho_x}\left\{f\left(\widetilde{\mathbf{x}} * \rho_x, z;\alpha\right) : p'\left(\widetilde{\mathbf{x}} * \rho_x\right) \geq b \wedge a_{x_j}(x_j) = \overline{\mathbf{x}}_j, j=1,\ldots,J\right\}\right\} \quad (10) \\ = \; & \max_{\overline{\mathbf{x}}}\left\{\max_{\rho_x}\left\{\mathbf{f}^a\left(\overline{\mathbf{x}},\mathbf{z};\alpha,\rho_x,\rho_z\right) : \boldsymbol{\pi}^a\left(\mathbf{p},\overline{\mathbf{x}};\rho_x,\rho_p\right) \geq b\right\}\right\}. \end{aligned}$$

Thus, the initial optimization problem can be written equivalently in the form of a two stage optimization problem. In the second stage, the optimal level of the elementary goods

[5] Or reciprocally, we could define aggregate quantities as $\mathbf{x}_j = \sum_h p_{jh}x_{jh}/\mathbf{p}_j$.

are chosen for the given value $\overline{\mathbf{x}}_j$ of the aggregates; in the first stage, the optimal levels of the aggregate goods are chosen given the aggregate prices and the optimal levels of the elementary goods chosen in the second stage.[6] The solution of the second stage gives the optimal choices of the shares of elementary goods in the aggregates as $\overline{\rho}_x\left(p, z, \overline{\mathbf{x}}, b; \alpha\right)$, and depends explicitly on the level $\overline{\mathbf{x}}$ of the aggregate goods. Thus, changes in the aggregate $\overline{\mathbf{x}}_j$ have a direct impact on the aggregate objective and an indirect impact through changes in the way elementary goods and prices are distributed:

$$\frac{\partial \mathbf{f}^a}{\partial \mathbf{x}'}\frac{\partial \overline{\mathbf{x}}}{\partial \overline{\mathbf{x}}_j} + \frac{\partial \mathbf{f}^a}{\partial \rho_x'}\frac{\partial \overline{\rho}_x}{\partial \overline{\mathbf{x}}_j} = \lambda_f \frac{\partial \boldsymbol{\pi}^a}{\partial \mathbf{x}'}\frac{\partial \overline{\mathbf{x}}}{\partial \overline{\mathbf{x}}_j} + \lambda_f \frac{\partial \boldsymbol{\pi}^a}{\partial \rho_x'}\frac{\partial \overline{\rho}_x}{\partial \overline{\mathbf{x}}_j} + \lambda_f \frac{\partial P}{\partial \overline{\mathbf{x}}_j}, \tag{11}$$

where λ_f denotes the Lagrange multiplier and $\partial P/\partial \overline{\mathbf{x}}_j$ represents the change in the price and in the distribution of relative prices implied by a change in the aggregate quantity:

$$\frac{\partial P}{\partial \overline{\mathbf{x}}_j} = \frac{\partial \boldsymbol{\pi}^a}{\partial \mathbf{p}'}\frac{\partial \mathbf{p}}{\partial \overline{\mathbf{x}}_j} + \frac{\partial \boldsymbol{\pi}^a}{\partial \rho_p'}\frac{\partial \overline{\rho}_p}{\partial \overline{\mathbf{x}}_j}.$$

When the price aggregator is perfect (i.e. depends solely on elementary prices and not on elementary quantities or their distribution), the impact of the change of an aggregate quantity on prices and price distribution vanishes in (11): $\partial P/\partial \overline{\mathbf{x}}_j = 0$. It is interesting to note that even when aggregators are imperfect, $\partial P/\partial \overline{\mathbf{x}}_j = 0$ holds under rather weaker conditions.

Proposition 1. *$\partial P/\partial \overline{\mathbf{x}}_j = 0$ if and only if $\left(\partial p/\partial \overline{\mathbf{x}}_j\right)'\left(\widetilde{\mathbf{x}} * \rho_x\right) = 0$.*

Proof. Since by (9), profits $p'x$ can be written as $p'\left(\widetilde{\mathbf{x}} * \rho_x\right)$, it follows that $\partial P/\partial \overline{\mathbf{x}}_j$ corresponds to $\partial P/\partial \overline{\mathbf{x}}_j = \left(\partial p/\partial \overline{\mathbf{x}}_j\right)'\left(\widetilde{\mathbf{x}} * \rho_x\right)$.

The condition stated in Proposition 1 is an orthogonality condition between the elementary quantities x and the shift in elementary prices induced by the change in the aggregate quantity $\overline{\mathbf{x}}_j$. This requirement is verified when perfect competition occurs, for example. The main interest of this result is that it permits to relax the assumption of perfect price aggregators without complicating the problem. In the following we assume that $\partial P/\partial \overline{\mathbf{x}}_j = 0$ and focus only on the remaining distributional effects occurring in equation (11).

When only aggregate data on expenditures, prices and goods are available, it seems impossible to model the distribution effects occurring in (11) explicitly. Exact aggregation provides conditions under which these effects do not emerge. Indeed, when the objective f is weakly separable in $\mathcal{J}$, ρ_x does not appear as an argument of the aggregate transformation function. Under weak homogeneous separability of f in $\mathcal{J}$, the term $\partial \overline{\rho}_x/\partial \mathbf{x}_j = 0$ (see e.g. Koebel [1998]) and (11), the first order condition for optimality, becomes

$$\frac{\partial \mathbf{f}^{whs}\left(\mathbf{x}, z; \boldsymbol{\alpha}\right)}{\partial \overline{\mathbf{x}}_j} = \lambda_f \sum_{h \in \mathcal{J}_j} p_{jh} \frac{x_{jh}}{\overline{\mathbf{x}}_j} \equiv \mathbf{p}_j^u,$$

where the superscript whs denotes a function being weakly homogeneous separable in $\mathcal{J}$. In the present case, the way elementary goods x_{jh} are chosen affects the first order conditions only through the aggregate quantity $\mathbf{x}_j$ and price $\mathbf{p}_j^u$ which can then be seen

[6] See Koebel [1998] for a formulation in the exact aggregation framework

as sufficient statistics for modeling aggregate first order conditions.

However, nothing ensures that the transformation function verifies such restrictions and that the first order condition can be simplified as described above. How do the first order conditions look like when the weak homogeneous separability restriction does not hold? How should the aggregate prices arising in the definition of aggregate profits (9) be specified? In order to find the aggregate price corresponding to the right hand side of (11), let us write this equation in terms of elementary goods and prices by using (7) and (9) and the fact that $\partial P/\partial \overline{\mathbf{x}}_j = 0$:

$$\begin{aligned} \frac{\partial f(x,z;\alpha)}{\partial x'}\left(\frac{\partial \widetilde{\mathbf{x}}}{\partial \overline{\mathbf{x}}_j} * \overline{\rho}_x + \widetilde{\mathbf{x}} * \frac{\partial \overline{\rho}_x}{\partial \overline{\mathbf{x}}_j}\right) &= \lambda_f \left(\widetilde{\mathbf{p}} * \rho_p\right)'\left(\frac{\partial \widetilde{\mathbf{x}}}{\partial \overline{\mathbf{x}}_j} * \overline{\rho}_x + \widetilde{\mathbf{x}} * \frac{\partial \overline{\rho}_x}{\partial \overline{\mathbf{x}}_j}\right) \\ &= \lambda_f \left[\sum_{h \in \mathcal{J}_j} \frac{p_{jh} x_{jh}}{\overline{\mathbf{x}}_j} + p'\left(\widetilde{\mathbf{x}} * \frac{\partial \overline{\rho}_x}{\partial \overline{\mathbf{x}}_j}\right)\right]. \end{aligned} \quad (12)$$

The impact of a variation in aggregate quantity $\overline{\mathbf{x}}_j$ is decomposed into two effects in (12). A first effect conveys the impact of a marginal change in the aggregates, the distribution of the elementary goods remaining unchanged, and a second effect is implied by the shifts in the distribution of elementary goods following a change in the aggregate $\overline{\mathbf{x}}_j$. The right hand side of (12) represents the marginal relative price of a change in the aggregate $\overline{\mathbf{x}}_j$. Using the equality

$$p'\left(\widetilde{\mathbf{x}} * \partial \overline{\rho}_x / \partial \overline{\mathbf{x}}_j\right) = p' \partial \overline{x}^p / \partial \overline{\mathbf{x}}_j - \sum_{h \in \mathcal{J}_j} p_{jh} \overline{x}^p_{jh} / \overline{\mathbf{x}}_j,$$

the right hand side of (12) can be rewritten $\lambda_f p' \partial \overline{x}^p / \partial \overline{\mathbf{x}}_j$. This result tells us that without weak separability and additive aggregation assumptions, the marginal value of the good $\overline{\mathbf{x}}_j$ corresponds to the weighted sum of elementary marginal values, with $\partial \overline{x}^p_j / \partial \overline{\mathbf{x}}_j$ as weights. Thus, the aggregate price conforming with economic theory should be defined as $p' \partial \overline{x}^p / \partial \overline{\mathbf{x}}_j$, which is not commonly available from statistical offices.[7] Notice that when the weak homogeneous separability is verified, $p' \partial \overline{x}^p / \partial \overline{\mathbf{x}}_j = p'_j \overline{x}^p_j / \overline{\mathbf{x}}_j$ (see Koebel [1998]) and unit values $p'_j \overline{x}^p_j / \overline{\mathbf{x}}_j$ can be retained for modeling first order conditions (in which case these unit values are also perfect aggregators).

Is there no other possibility for representing aggregate first order conditions than to make strong restrictions on $f(x,z;\alpha)$ or to retain 'monsters' as aggregate prices? By using the optimality conditions for the elementary goods, we see that the impact of the change in the distribution of elementary goods related to a change in $\mathbf{x}_j$ can be netted out from the left and right side of (12), which can be written as

$$\sum_{h \in \mathcal{J}_j} \frac{\partial f(x,z;\alpha)}{\partial x_{jh}} \frac{x_{jh}}{\overline{\mathbf{x}}_j} = \lambda_f \sum_{h \in \mathcal{J}_j} p_{jh} \frac{x_{jh}}{\overline{\mathbf{x}}_j}. \quad (13)$$

Thus, defining unit values as $\mathbf{p}^u_j \equiv p'_j x_j / \overline{\mathbf{x}}_j$, the equations (13) can be used for estimating the parameters of the objective function $\mathbf{f}^a(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z)$. Note that unit values can be retained for the estimation, without requiring weak homogeneous separability (or additive price aggregation) is not required. Briefly, we have shown that the aggregate marginal

[7] For the aggregate price $\mathbf{p}^v_j = p' \partial \overline{x}^p / \partial \mathbf{x}_j$, the product $\mathbf{p}^v_j \mathbf{x}_j$ will generally be different from the profit $p'_j x_j$ of commodities in group j.

conditions (11) can be modelled indirectly by (13), as it is usually done in studies neglecting completely the problem of aggregation. However, the function $\mathbf{f}^a$ will not necessarily fulfill the second order conditions for a maximum with respect to $\mathbf{x}$. Indeed, the matrix of second derivative of $\mathbf{f}^a$ with respect to $\mathbf{x}$ need not to be negative definite on the hyperplane $(\partial \mathbf{f}^a/\partial \mathbf{x})^{\perp}$:

$$\begin{aligned} \frac{d^2\mathbf{f}^a}{d\mathbf{x}d\mathbf{x}'} &= \frac{d}{d\mathbf{x}}\left(\frac{\partial \mathbf{f}^a}{\partial \mathbf{x}'} + \frac{\partial \overline{\rho}_x'}{\partial \mathbf{x}}\frac{\partial \mathbf{f}^a}{\partial \rho_x}\right) \\ &= \frac{\partial^2 \mathbf{f}^a}{\partial \mathbf{x}\partial \mathbf{x}'} + 2\frac{\partial^2 \mathbf{f}^a}{\partial \mathbf{x}\partial \rho_x'}\frac{\partial \overline{\rho}_x}{\partial \mathbf{x}'} + \frac{\partial \overline{\rho}_x'}{\partial \mathbf{x}}\frac{\partial^2 \mathbf{f}^a}{\partial \rho_x'\partial \rho_x}\frac{\partial \overline{\rho}_x}{\partial \mathbf{x}'} + \frac{\partial^2 \overline{\rho}_x}{\partial \mathbf{x}\partial \mathbf{x}'}\frac{\partial \mathbf{f}^a}{\partial \rho_x}. \end{aligned}$$

In fact, only the negative definiteness of the matrix $[\partial^2\mathbf{f}^a/\partial \mathbf{x}\partial \mathbf{x}']$ on the hyperplane $(\partial \mathbf{f}^a/\partial \mathbf{x})^{\perp}$ can be inferred from the quasi-concavity of f in x, but the remaining matrixes may offset this effect if the impact of a change in the aggregate commodity on ρ_x is important enough. These results are summarized in the following proposition.

Proposition 2. *(i) The first order optimality conditions of problem (10) can be represented by* $\partial \mathbf{f}^a/\partial \mathbf{x}_j = \lambda_f \mathbf{p}_j^u$ *where* $\mathbf{p}_j^u = p_j' x_j/\mathbf{x}_j$ *and* $\mathbf{p}^{u\prime}\mathbf{x} = b$.

(ii) The second order conditions for a maximum in $\mathbf{x}$ *are not necessarily verified by* $\mathbf{f}^a(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z)$.

Thus, neither separability nor additive price aggregation, nor the requirement of perfect aggregators, nor the assumption that the distributions of elementary goods be independent from the aggregate ones, is required in order to model implicitly the first order optimality conditions of $\mathbf{f}^a$. The first part of this result can be useful for several purposes. Solving the aggregate first order conditions in $\mathbf{x}_j$ permits us to derive the optimal solution $\mathbf{x}_j^a(\mathbf{p}^u, \mathbf{z}, b; \alpha, \rho_x, \rho_z)$ of the first stage of the optimization problem, which can be modelled with aggregate prices, goods and profit b as explanatory variables. Further, the result also tells us that the aggregate marginal productivity (utility) $\partial \mathbf{f}^a/\partial \mathbf{x}_j$ is a weighted average of the elementary marginal productivities (utilities) of the goods within the subset $\mathcal{J}_j$. Thus, when the aggregate price is defined as a unit value, the Hotelling-Wold identity holds in the aggregate, i.e.,

$$\frac{\mathbf{p}_j^u}{b} = \frac{\partial \mathbf{f}^a/\partial \mathbf{x}_j}{\sum_{k=1}^J \mathbf{x}_j \partial \mathbf{f}^a/\partial \mathbf{x}_j}.$$

Indeed $\mathbf{p}_j^u = \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}/\mathbf{x}_j$, $\sum_{j=1}^J \mathbf{p}_j \mathbf{x}_j = b$ and $\partial \mathbf{f}^a/\partial \mathbf{x}_j = \lambda_f \mathbf{p}_j^u$.

The result (ii) of Proposition 2 may appear of pessimistic content. Note however that the assumption required for strict quasi-concavity to hold in the aggregate are weaker than the conditions under which an exact aggregate representation would be possible.

All this discussion of the impact of a change in $\mathbf{x}_j$ on the distribution of elementary goods could, in fact, be spared by immediately remarking that the way the distributions of elementary goods and prices are affected by a change in $\mathbf{x}_j$ is irrelevant for the solution $\mathbf{x}_j^{a*}$ of the problem.

5. Neglecting aggregation in optimized relationships

We consider aggregation within both the primal and the dual optimization frameworks, characterized by (1) and (2). We will show, that the usual regularity conditions of elementary optimal transformation and profit functions $f^*(p,z,b;\alpha)$ and $\pi^*(p,z;\alpha)$ may not be transmitted to $\mathbf{f}^*(\mathbf{p},\mathbf{z};b,\boldsymbol{\alpha})$ and $\boldsymbol{\pi}^*(\mathbf{p},\mathbf{z};\boldsymbol{\alpha})$ when aggregation is neglected. Another objective of this section is to determine how aggregate prices and quantities should be specified in order to be coherent with the theory.

5.1 The profit function

Under optimality conditions, similar results to those derived above can be obtained for the dual optimization problem (2) and the corresponding first order conditions. Here, we directly consider the aggregate profit function which is related to the elementary profit function $\pi^*(p,z;\alpha)$ by the following transformations:

$$\begin{aligned}
&\pi^*(p,z;\alpha) = \max_x \{p'x : f(x,z;\alpha) \geq 0\} \\
= \ & \max_{\overline{\mathbf{x}}} \left\{ \max_{\rho_x} \left\{ (\widetilde{\mathbf{p}} * \rho_p)' (\widetilde{\mathbf{x}} * \rho_x) : f(\widetilde{\mathbf{x}} * \rho_x, z; \alpha) \geq 0 \wedge a_{x_j}(x_j) = \overline{\mathbf{x}}_j, j = 1, \ldots, J \right\} \right\} \\
= \ & \max_{\overline{\mathbf{x}}} \left\{ (\widetilde{\mathbf{p}} * \rho_p)' \left(\widetilde{\overline{\mathbf{x}}} * \rho_x^d \right) : \mathbf{f}^a \left(\overline{\mathbf{x}}, \mathbf{z}; \alpha, \rho_x^d, \rho_z \right) \geq 0 \right\} \\
= \ & \boldsymbol{\pi}^{*a} \left(\mathbf{p}, \mathbf{z}; \alpha, \rho_z, \rho_p \right). \qquad (14)
\end{aligned}$$

Now, the solution of the second stage of this optimization problem is given by $\rho_x^d\left(\mathbf{p},\overline{\mathbf{x}},\mathbf{z};\alpha,\rho_z,\rho_p\right)$. Notice that the aggregate profit function is not parameterized by the same parameters as $\mathbf{f}^a$: ρ_x no longer appears in the expression of $\boldsymbol{\pi}^{*a}$, which depends instead on the distribution of the elementary price ratios ρ_p. The above representation makes only sense when elementary prices and fixed good goods depend on the aggregates: then $\rho_z = \overline{\rho}_z(\mathbf{z},.)$ and $\rho_p = \overline{\rho}_p(\mathbf{p},.)$.

The microeconomic profit function $\pi^*(p,z;\alpha)$ is linearly homogeneous and convex in prices. Since $\pi^*(p,z;\alpha) = \boldsymbol{\pi}^{*a}\left(\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right)$, the last function will also be linearly homogeneous and convex in p. But which properties does $\boldsymbol{\pi}^{*a}$ verify with respect to aggregate prices $\mathbf{p}$? Using (7) and (9), we see that *for given relative prices*, $\boldsymbol{\pi}^{*a}$ will also be linearly homogeneous and convex in $\mathbf{p}$. Linear homogeneity in $\mathbf{p}$ results from the fact that:

$$\begin{aligned}
\boldsymbol{\pi}^{*a}\left(\kappa\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right) &= \max_{\mathbf{x}} \left\{ \left(\kappa\widetilde{\mathbf{p}}*\rho_p\right)' (\widetilde{\mathbf{x}}*\rho_x) : \mathbf{f}^a(\mathbf{x},\mathbf{z};\alpha,\rho_x,\rho_z) \geq 0 \right\} \\
&= \kappa \max_{\mathbf{x}} \left\{ \left(\widetilde{\mathbf{p}}*\rho_p\right)' (\widetilde{\mathbf{x}}*\rho_x) : \mathbf{f}^a(\mathbf{x},\mathbf{z};\alpha,\rho_x,\rho_z) \geq 0 \right\} \\
&= \kappa \boldsymbol{\pi}^{*a}\left(\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right).
\end{aligned}$$

Convexity of $\boldsymbol{\pi}^{*a}$ in $\mathbf{p}$ (for given ρ_p) is obtained in a similar manner as convexity of $\pi^*(p, z;\alpha)$ in p (see e.g. Diewert [1982]): take two price vectors $\mathbf{p}^0 > 0$, and $\mathbf{p}^1 > 0$ and define

$$\begin{aligned}
\boldsymbol{\pi}^{*a\ell} \equiv \boldsymbol{\pi}^{*a}\left(\mathbf{p}^\ell,\mathbf{z};\alpha,\rho_z,\rho_p\right) &= \max_{\overline{\mathbf{x}}} \left\{ \left(\widetilde{\mathbf{p}}^\ell*\rho_p\right)' (\widetilde{\mathbf{x}}*\rho_x) : \mathbf{f}^a(\mathbf{x},\mathbf{z};\alpha,\rho_x,\rho_z) \geq 0 \right\} \\
&= \left(\widetilde{\mathbf{p}}^\ell*\rho_p\right)' \left(\widetilde{\mathbf{x}}^{a\ell}*\rho_x^\ell\right), \quad \ell = 0,1.
\end{aligned}$$

For $0 \leq \kappa \leq 1$, define $\mathbf{p}^{\kappa} = \kappa \mathbf{p}^{0} + (1-\kappa)\, \mathbf{p}^{1}$. Then

$$\begin{aligned}
\boldsymbol{\pi}^{*a}\left(\mathbf{p}^{\kappa}, \mathbf{z}; \alpha, \rho_z, \rho_p\right) &= \max_{\widetilde{\mathbf{x}}} \left\{ \left(\widetilde{\mathbf{p}}^{\kappa} * \rho_p\right)' (\widetilde{\mathbf{x}} * \rho_x) : \mathbf{f}^{a}\left(\mathbf{x}, \mathbf{z}; \alpha, \rho_x, \rho_z\right) \geq 0 \right\} \\
&= \left(\widetilde{\mathbf{p}}^{\kappa} * \rho_p\right)' \left(\widetilde{\mathbf{x}}^{a\kappa} * \rho_x^{\kappa}\right) \\
&= \kappa \left(\widetilde{\mathbf{p}}^{0} * \rho_p\right)' \left(\widetilde{\mathbf{x}}^{a\kappa} * \rho_x^{\kappa}\right) + (1-\kappa) \left(\widetilde{\mathbf{p}}^{0} * \rho_p\right)' \left(\widetilde{\mathbf{x}}^{a\kappa} * \rho_x^{\kappa}\right) \\
&\leq \kappa \left(\widetilde{\mathbf{p}}^{0} * \rho_p\right)' \left(\widetilde{\mathbf{x}}^{a0} * \rho_x^{0}\right) + (1-\kappa) \left(\widetilde{\mathbf{p}}^{1} * \rho_p\right)' \left(\widetilde{\mathbf{x}}^{a1} * \rho_x^{1}\right) \\
&= \kappa \boldsymbol{\pi}^{*a0} + (1-\kappa)\, \boldsymbol{\pi}^{*a1}.
\end{aligned}$$

However, nothing ensures a priori that the distribution of elementary prices is stable and does not shift when $\mathbf{p}$ and $\mathbf{z}$ vary. Therefore, the linear homogeneity and convexity in $\mathbf{p}$ may, in fact, not be verified. This leads to the following proposition.

Proposition 3. *The aggregate profit function $\boldsymbol{\pi}^{*a}$ is not necessarily linearly homogeneous and convex in the aggregate prices $\mathbf{p}$.*

We know that $\boldsymbol{\pi}^{*a}\left(\mathbf{p}, \mathbf{z}; \alpha, \rho_z, \rho_p\right)$ is linearly homogeneous in p: i.e. when p is multiplied by a scalar κ, the profits are multiplied by κ. However, this is not sufficient to imply that $\boldsymbol{\pi}^{*a}\left(\mathbf{p}, \mathbf{z}; \alpha, \rho_z, \rho_p\right)$ is linearly homogeneous in $\mathbf{p}$. Indeed, when $\mathbf{p}$ is multiplied by the scalar κ it does not generally imply that all elementary prices are also multiplied by κ (even in the cases where $\mathbf{p}$ in linearly homogeneous in p). Similarly, the convexity in aggregate prices is not ensured, since the second order total differentiation yields

$$\begin{aligned}
\frac{d^2 \boldsymbol{\pi}^{*a}}{d\mathbf{p} d\mathbf{p}'} &= \frac{d}{d\mathbf{p}} \left(\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \mathbf{p}'} + \frac{\partial \overline{\rho}_p'}{\partial \mathbf{p}} \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p} \right) \\
&= \frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \mathbf{p} \partial \mathbf{p}'} + 2 \frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \mathbf{p} \partial \rho_p'} \frac{\partial \overline{\rho}_p}{\partial \mathbf{p}'} + \frac{\partial \overline{\rho}_p'}{\partial \mathbf{p}} \frac{\partial^2 \boldsymbol{\pi}^{*a}}{\partial \rho_p' \partial \rho_p} \frac{\partial \overline{\rho}_p}{\partial \mathbf{p}'} + \frac{\partial^2 \overline{\rho}_p'}{\partial \mathbf{p} \partial \mathbf{p}'} \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p},
\end{aligned} \tag{15}$$

where only the matrix $[\partial^2 \boldsymbol{\pi}^{*a} / \partial \mathbf{p} \partial \mathbf{p}']$ is ensured to be positive semi-definite. A marginal variation in the aggregate fixed goods $\mathbf{z}_k$ also has a distributional effect in general, indeed

$$\frac{d\boldsymbol{\pi}^{*a}\left(\mathbf{p}, \mathbf{z}; \alpha, \rho_z, \rho_p\right)}{d\mathbf{z}_k} = \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \mathbf{z}_k} + \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_z'} \frac{\partial \overline{\rho}_z}{\partial \mathbf{z}_k} + \frac{\partial \boldsymbol{\pi}^{*a}}{\partial \rho_p'} \frac{\partial \overline{\rho}_p}{\partial \mathbf{z}_k}. \tag{16}$$

In summary, up to now we have found that neither linear homogeneity in prices nor convexity is ensured to hold in the aggregate. Of course, restrictions like those made by exact aggregation may ensure some regularities. Alternatively, following Lewbel [1993b, 1996], and assuming that the shift in the distribution of prices is zero also permits us to retrieve the linear homogeneity and the convexity of $\boldsymbol{\pi}^{*a}$ in $\mathbf{p}$. It seems interesting, however, to note that convexity in $\mathbf{p}$ can be verified under milder conditions. Indeed, it suffices for this that the impacts of ρ_p on profits be small enough for not destroying the positive semi-definiteness of the first matrix $\partial^2 \boldsymbol{\pi}^{*a} / \partial \mathbf{p} \partial \mathbf{p}'$.

Are really all properties of the profit function lost? We show that Hotelling's lemma can still be used for deriving the optimal aggregate supply and demand functions.

Proposition 4. *The aggregate profit function $\boldsymbol{\pi}^{*a}$ verifies Hotelling's lemma in the sense that:*

$$\frac{\partial \boldsymbol{\pi}^{*a}}{\partial \mathbf{p}_j} = \sum_{h \in \mathcal{J}_j} \frac{p_{jh} x_{jh}^{d}}{\mathbf{p}_j}, \tag{17}$$

where x_{jh}^{d} denotes the elementary solution of the dual optimization problem. By defining aggregate prices as $\mathbf{p}_j^u \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}^d/\mathbf{x}_j^a$, it follows that $\boldsymbol{\pi}^{*a} = \mathbf{p}_j^u\mathbf{x}_j^a$ and $\partial\boldsymbol{\pi}^{*a}/\partial\mathbf{p}_j^u = \mathbf{x}_j^a$.

Proof. By definition,

$$\boldsymbol{\pi}^{*a}\left(\mathbf{p}^0,\mathbf{z};\alpha,\rho_z,\rho_p^0\right) = \max_{\mathbf{x}}\left\{\left(\widetilde{\mathbf{p}}^0*\rho_p^0\right)'\left(\widetilde{\mathbf{x}}*\rho_x\right) : \mathbf{f}^a\left(\mathbf{x},\mathbf{z};\alpha,\rho_x,\rho_z\right)\geq 0\right\}.$$

Let $\mathbf{x}^{a0} \equiv \mathbf{x}^a\left(\mathbf{p}^0,\mathbf{z};\alpha,\rho_z,\rho_p^0\right)$ be the solution of this optimization program. For arbitrary elementary prices p and the corresponding aggregate price $\mathbf{p}$, we have

$$\boldsymbol{\pi}^{*a}\left(\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right) \geq \left(\widetilde{\mathbf{p}}*\rho_p\right)'\left(\widetilde{\mathbf{x}}^{a0}*\rho_x^0\right),$$

where $\rho_x^0 = \left(x_1^{d0}/\mathbf{x}_1^{a0},\ldots,x_J^{d0}/\mathbf{x}_J^{a0}\right)$ and $x_j^{d0} \equiv x_j^d(\mathbf{p}^0,\mathbf{x}^{a0},\mathbf{z};\alpha,\rho_z,\rho_p^0)$. Thus, the function defined as

$$\boldsymbol{\pi}^{*a}\left(\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right) - \left(\widetilde{\mathbf{p}}*\rho_p\right)'\left(\widetilde{\mathbf{x}}^{a0}*\rho_x^0\right)$$

is positive and minimized at $\mathbf{p} = \mathbf{p}^0$; the corresponding first order conditions are given by:

$$\frac{\partial\boldsymbol{\pi}^{*a}\left(\mathbf{p},\mathbf{z};\alpha,\rho_z,\rho_p\right)}{\partial\mathbf{p}_j} - \sum_{h\in\mathcal{J}_j}\frac{p_{jh}x_{jh}^d}{\mathbf{p}_j} = 0, \quad j = 1,\ldots,J,$$

which yields (17). By specifying $\mathbf{p}_j$ as $\mathbf{p}_j^u \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}^d/\mathbf{x}_j^a$, it follows that the optimal profit function is:

$$\boldsymbol{\pi}^{*a}\left(\mathbf{p}^u,\mathbf{z};\alpha,\rho_z,\rho_p\right) = \max_{\overline{\mathbf{x}}}\left\{p'\overline{x}^d\left(\mathbf{p},\overline{\mathbf{x}},\mathbf{z};\alpha,\rho_z,\rho_p\right)\right\} = p'x^d = \mathbf{p}^{u\prime}\mathbf{x}^a.$$

Q.E.D.

This proposition tells us that Hotelling's lemma holds with respect to aggregate prices $\mathbf{p}_j$. The equality (17) also dictates how aggregate quantities must be defined in order to find Hotelling's lemma in the aggregate: either $\mathbf{x}_j \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}/\mathbf{p}_j$, for any specification of the aggregate price $\mathbf{p}_j$, or $\mathbf{p}_j = \mathbf{p}_j^u \equiv \sum_{h\in\mathcal{J}_j} p_{jh}x_{jh}/\mathbf{x}_j$, for any specification of the aggregate quantity $\mathbf{x}_j$. Then, $\partial\boldsymbol{\pi}^{*a}/\partial\mathbf{p}_j^u = \mathbf{x}_j^a$ is retrieved at the aggregate level. However, this is not sufficient to imply that $\boldsymbol{\pi}^{*a}\left(\mathbf{p}^u,\mathbf{z};\alpha,\rho_z,\rho_p\right)$ is linearly homogeneous in $\mathbf{p}^u$. Although by Proposition 4, $\mathbf{p}^{u\prime}\partial\boldsymbol{\pi}^{*a}/\partial\mathbf{p}^u = \boldsymbol{\pi}^{*a}$, the total differentiation of $\boldsymbol{\pi}^{*a}$ yields $\mathbf{p}^{u\prime}d\boldsymbol{\pi}^{*a}/d\mathbf{p}^u = \boldsymbol{\pi}^{*a} + \left(\partial\boldsymbol{\pi}^{*a}/\partial\rho_p'\right)\partial\rho_p/\partial\mathbf{p}^u$, which may differ from $\boldsymbol{\pi}^{*a}$. The usefulness of this aggregate Hotelling's lemma for empirical analysis is not obvious; indeed, the observation of the complete disaggregate information on ρ_z and ρ_p is required for its application. When not available, the modeler may be led to retain a profit function specified as $\boldsymbol{\pi}\left(\mathbf{p}^u,\mathbf{z};\boldsymbol{\alpha}\right)$ for estimating a parameter vector $\boldsymbol{\alpha}$ which is constant over the sample. Then, the validity of Hotelling's lemma is called into doubt as soon as ρ_z or ρ_p are not stable.

Briefly, we have shown that important properties of profit functions, linear homogeneity and convexity in aggregate prices $\mathbf{p}^u$ are lost when goods and prices are aggregated. These properties, which are usually used for comparative static analyses and for specifying systems of demands and supplies in empirical studies, should, therefore, not be imposed without precaution.

5.2 The indirect objective function

The aggregate indirect objective function can be obtained from its microeconomic counterpart as

$$\begin{aligned} f^*(p,z;\alpha,b) &= \max_x \{f(x,z;\alpha): p'x \geq b\} \\ &= \max_{\overline{\mathbf{x}}} \left\{ \max_{\rho_x} \left\{ f(\widetilde{\mathbf{x}} * \rho_x, z; \alpha) : (\widetilde{\mathbf{p}} * \rho_p)' (\widetilde{\mathbf{x}} * \rho_x) \geq b \wedge a_{x_j}(x_j) = \overline{\mathbf{x}}_j, j = 1, \ldots, J \right\} \right\} \\ &= \mathbf{f}^{*a}(\mathbf{p}, \mathbf{z}; \alpha, b, \rho_z, \rho_p). \end{aligned}$$

It is easy to see that when the ratios of elementary to aggregate prices stay constant, the usual properties are verified for $\mathbf{f}^{*a}(\mathbf{p}, \mathbf{z}; \alpha, b, \rho_z, \rho_p)$ with respect to $\mathbf{p}$ and b. In general however, for varying ρ_z and ρ_p, these properties may vanish. The proposition and its proof are left to the interested reader.

6. An empirical investigation: heterogeneous skills and aggregate labor demand

Input price changes, wage variations, growth and technical change have rather dissimilar impacts on the demands for different skills of labor.[8] Therefore, when only aggregate information on the aggregate quantity of labor and aggregate wage is available, some interesting information on the demand for different labor skills is lost. A further consequence of neglecting disaggregate information may be bias in the estimates relying on aggregate data. In this section, we study the importance and the impact of this bias. More precisely, we study aggregation of labor and the consequences of representing the categories of labor h_t, s_t and u_t, which denote respectively 'high-skilled', 'semi-skilled' and 'unskilled' labor, by a scalar quantity of labor $\mathbf{x}_{\ell t}$. The subscript t is introduced for characterizing time. The other arguments of the transformation function are materials m_t, capital k_t, which are assumed to be flexible inputs whereas output y_t and time t are exogenous; therefore $x_t \equiv (h_t, k_t, m_t, s_t, u_t)$ and $z_t \equiv (y_t, t)$ in the expression of $f(x_t, z_t; \alpha) \geq 0$. The dataset consists of five aggregate sectors of economic activity in West Germany: manufacturing; energy, water & mining; construction; wholesale & retail trade; banking & insurance. All data are available semi-annually from 1977 to 1994. In 1994, approximately 60 percent of all workers paying social security contributions were employed in these five sectors. These data are fully described in Falk and Koebel [1997].

The aggregate labor input is defined as the sum of elementary labor inputs, as commonly defined by statistical offices and retained in empirical studies: $\mathbf{x}_{\ell t} = h_t + s_t + u_t$. Aggregate wages are given by the unit wage $\mathbf{p}_{\ell t} = (p_{ht} h_t + p_{st} s_t + p_{ut} u_t) / \mathbf{x}_{\ell t}$. For the use of such aggregates to be possible, the 'exact aggregation' approach would require that an aggregate function $\mathbf{f}^s$ exists such that:

$$f(x_t, z_t; \alpha) = \mathbf{f}^s(k_t, m_t, h_t + s_t + u_t, y_t, t; \alpha).$$

[8] See for example Falk and Koebel [1999] for such empirical findings with the same dataset as the one used in this section.

In this case however, a rational production unit would use only one labor input: the cheapest one. This last situation is clearly not met with our sectorial data and therefore exact aggregation cannot be achieved. This short reasoning leads to reject the strategy relying on exact aggregation theory for modelling aggregate labor demand. Is therefore any empirical study useless? Not necessarily, because the production process can be represented approximately by $\mathbf{f}^a\left(k_t, m_t, \mathbf{x}_{\ell t}, z_t; \alpha_t, \rho_{xt}\right)$, with $\rho_{xt} = \left(h_t/\mathbf{x}_{\ell t}, s_t/\mathbf{x}_{\ell t}, u_t/\mathbf{x}_{\ell t}\right)'$.

6.1 Empirical investigation of price proportionality

Since in applied production analysis, the determinants of input demand functions are often of interest, we begin with a disaggregate system $x^*\left(p_t, z_t; \alpha\right)$, from which the aggregate relationships $\mathbf{x}^a\left(\mathbf{p}_t, y_t, t; \alpha, \rho_{pt}\right)$ with $\mathbf{p}_t \equiv \left(p_{et}, p_{kt}, p_{mt}, \mathbf{p}_{\ell t}\right)'$ are obtained by using the above reparameterization. When not observable, ρ_{pt} is neglected in the empirical estimation. In this case, the underlying cost and demand functions noted $\mathbf{c}\left(\mathbf{p}_{\ell t}, p_{mt}, p_{kt}, y_t, t; \boldsymbol{\alpha}\right)$ and $\mathbf{x}\left(\mathbf{p}_{\ell t}, p_{mt}, p_{kt}, y_t, t; \boldsymbol{\alpha}\right)$ will not necessarily verify the microeconomic regularity properties. The conditions under which $\mathbf{c}$ and $\mathbf{x}$ verify the microeconomic properties are given in section 3 and 4. Stochastic proportionality of prices is a sufficient condition for this; in which case,

$$\rho_{p_{jt}} \equiv \frac{p_{jt}}{\mathbf{p}_{\ell t}} = \widetilde{\gamma}_{j0} + \widetilde{\omega}_{jt}, \quad j = h, s, u, \tag{18}$$

where $\widetilde{\gamma}_{j0}$ is a constant parameter and $\widetilde{\omega}_{jt}$ a random term with zero mean and constant variance. The validity of this assumption is tested by Lewbel [1996], who investigates the stationarity and cointegration properties of relative prices in the consumer context. Here, being interested in the properties of $\mathbf{x}\left(\mathbf{p}_t, y_t, t; \boldsymbol{\alpha}\right)$, we test the absence of correlation between ρ_{p_t} and other explanatory variables $\mathbf{p}_t$, y_t and t of the aggregate demand functions. In the case where ρ_{p_t} is correlated with $\mathbf{p}_{\ell t}$, the sufficient condition for the fulfillment of microeconomic regularities in the aggregate is rejected. The procedure therefore suggest to compare the specification (18) with a more general one, having $\mathbf{p}_t$, y_t and t as arguments and chosen as

$$\rho_{p_t} \equiv \frac{p_{jt}}{\mathbf{p}_{\ell t}} = \gamma_{j0} + \gamma_{j\ell}\mathbf{p}_{\ell t} + \gamma_{jm}p_{mt} + \gamma_{jk}p_{kt} + \gamma_{jy}y_t + \gamma_{jt}t + \omega_{jt}, \tag{19}$$

where $j = h$, s, u and ω_{jt} is a residual term with zero mean conditional on regressors and constant variance. This last relationships are very similar to the auxiliary equations introduced by Theil [1954, 1971] in the analysis of aggregation, for studying the relationship between the estimates for $\boldsymbol{\alpha}$ and α.

The estimates of the coefficients of (19) are listed in Table 6. A Fisher test for the joint nullity of all slopes rejects in all cases (for all qualifications and sectors) the assumption of stochastic proportionality of the prices for different qualifications of labor. This rejection may appear surprising in the light of the very high correlations between the three elementary prices of labor: over all five sectors retained, the lowest coefficient of correlation observed was 0.986. Despite this fact, the Fisher test unambiguously rejects the null hypothesis of stable relative prices $p_{jt}/\mathbf{p}_{\ell t}$ over the period. Note that the rejection may be artificial: by using an instrumental variable regression, the precision of the estimates may fall and the F-test statistic decrease.

Table 1: Testing the constancy of relative prices for different skills

Sector	explained variable	Coefficient estimates (and t-values in parenthesis)													
		γ_{j0}		$\gamma_{j\ell}$		γ_{jm}		γ_{jk}		γ_{jy}		γ_{jt}		R^2	F-Test(1)
	$p_{ht}/\mathbf{p}_{\ell t}$	1.21	(24.8)	-0.30	(-6.9)	0.00	(0.0)	0.04	(1.5)	-0.00	(-5.4)	0.01	(10.1)	0.98	267.0
Manufacturing	$p_{st}/\mathbf{p}_{\ell t}$	0.98	(77.1)	0.05	(4.2)	-0.01	(-1.4)	0.00	(0.1)	0.00	(0.8)	-0.00	(-5.6)	0.93	77.2
	$p_{ut}/\mathbf{p}_{\ell t}$	0.99	(36.5)	-0.05	(-2.0)	0.04	(2.4)	-0.02	(-1.2)	0.00	(2.1)	0.00	(2.0)	0.63	10.2
Water,	$p_{ht}/\mathbf{p}_{\ell t}$	1.27	(16.1)	-0.36	(-9.5)	0.04	(2.1)	-0.03	(-1.6)	-0.00	(-0.9)	0.01	(7.3)	0.87	38.5
Energy	$p_{st}/\mathbf{p}_{\ell t}$	0.88	(26.9)	0.13	(8.3)	-0.01	(-1.4)	-0.00	(-0.4)	0.00	(1.9)	-0.00	(-7.1)	0.82	26.6
and Mining	$p_{ut}/\mathbf{p}_{\ell t}$	1.34	(11.8)	-0.35	(-6.4)	0.02	(0.5)	0.03	(1.1)	-0.00	(-1.9)	0.01	(5.7)	0.72	15.2
	$p_{ht}/\mathbf{p}_{\ell t}$	1.31	(9.1)	-0.65	(-8.7)	0.07	(0.3)	-0.08	(-0.6)	0.00	(1.1)	0.01	(7.7)	0.76	19.5
Construction	$p_{st}/\mathbf{p}_{\ell t}$	0.89	(34.2)	0.05	(3.8)	0.03	(0.7)	0.02	(1.0)	0.00	(3.1)	-0.00	(-5.9)	0.71	14.6
	$p_{ut}/\mathbf{p}_{\ell t}$	1.24	(17.7)	-0.05	(-1.4)	-0.06	(-0.5)	-0.06	(-1.0)	-0.00	(-3.2)	0.00	(3.4)	0.59	8.6
Wholesale	$p_{ht}/\mathbf{p}_{\ell t}$	1.28	(12.8)	-0.16	(-5.4)	0.02	(0.5)	-0.00	(-0.1)	-0.00	(-3.0)	0.01	(4.8)	0.76	19.1
and Retail	$p_{st}/\mathbf{p}_{\ell t}$	1.01	(16.1)	-0.04	(-2.1)	0.01	(0.4)	-0.01	(-0.9)	0.00	(0.2)	0.00	(0.8)	0.40	3.9
Trade	$p_{ut}/\mathbf{p}_{\ell t}$	0.81	(3.9)	0.20	(3.1)	-0.01	(-0.2)	0.01	(0.4)	0.00	(0.5)	-0.00	(-1.6)	0.58	8.4
Banking	$p_{ht}/\mathbf{p}_{\ell t}$	1.13	(39.7)	-0.07	(-2.6)	-0.08	(-2.2)	0.01	(1.2)	-0.00	(-4.6)	0.01	(13.4)	0.98	279.9
and	$p_{st}/\mathbf{p}_{\ell t}$	1.01	(82.1)	-0.00	(-0.4)	-0.02	(-1.1)	0.01	(3.3)	0.00	(0.9)	-0.00	(-1.9)	0.82	27.3
Insurances	$p_{ut}/\mathbf{p}_{\ell t}$	0.91	(20.1)	0.06	(1.4)	0.09	(1.5)	-0.04	(-3.4)	-0.00	(-0.3)	-0.00	(-1.2)	0.40	4.0

(1) F-Test for the hypothesis that the five parameters $\gamma_{j\ell}$, γ_{jm}, γ_{jk}, γ_{jy} and γ_{jt} are zero for $j = h$, s, u. The statistic F follows the Fisher-Snedecor distribution with 5 and 30 degrees of freedom. The critical value at the 1% signifiance level is 3.7

Thus, when data on elementary labor qualifications and prices are not available, the fulfillment of microeconomic properties is not ensured in the aggregate. From Table 6, we can see that in all sectors $p_{ht}/\mathbf{p}_{\ell t}$ decreases with $\mathbf{p}_{\ell t}$ and increases with t. Therefore, any change in the explanatory variables of $\mathbf{x}\left(\mathbf{p}_t, y_t, t; \boldsymbol{\alpha}\right)$ cannot be interpreted ceteris paribus, since they also measure changes in the distribution of the omitted elementary prices. This is the reason why microeconomic properties need not to hold for $\mathbf{x}\left(\mathbf{p}_t, y_t, t; \boldsymbol{\alpha}\right)$ even when they are verified with respect to $x^*\left(p_t, y_t, t; \alpha\right)$.

Note that the rejection of the *sufficient* condition for the microeconomic condition to hold in the aggregate does not imply their rejection: the different distributional effects might compensate each other and vanish in the aggregate. From Table 6 one can see that within each industry, no parameter estimate has a constant sign for the different labor qualifications. Further empirical investigations are needed for studying if the *necessary* conditions for the microeconomic properties are also rejected. However, the rejection of stochastic proportionality already suffices to cast some doubts on the validity of these restrictions.

6.2 The aggregation biases

7. Conclusion

In this paper, we show that some studies neglecting aggregation issues may admit foundation in aggregation theory: it is not pure nonsense to consider a relationship having some aggregates as arguments. Besides, an aggregate representation of the initial problem is possible without restrictions on the admissible microeconomic functional forms. Such a representation exists whether goods are optimally allocated or not. These findings contrast with the pessimistic conclusions of the exact aggregation approach for which an aggregate representation is only possible under rather implausible restrictions on microeconomic relationships.

However, we also show that the aggregate parameters do not in general reflect only economic behavior but also shifts in the distribution of elementary goods and prices. Such a feature was underlined by e.g. Stoker [1984] in the context of aggregation across individuals. Therefore, several regularity properties of individual behavior may be lost at the aggregate level: linear homogeneity and convexity in the aggregate prices are not necessarily verified by profit functions, for example. Therefore the imposition of microeconomic regularities at the aggregate level (across goods) may lead to estimation bias.

When the conditions for an exact aggregation are not verified however, the fact that the parameters to be estimated are 'complicated combinations of technological characteristics and the distribution of inputs' (Blackorby, Schworm and Fisher [1986]) does not only imply a loss of some regularity properties in the aggregate. Another consequence is that the interpretation of the aggregate estimations must be enlarged: the marginal impact of aggregate variables must be thought of as being composed of a direct and an indirect effect on the distribution of elementary components. Besides, this dependence between aggregate parameters and the elementary characteristics of the model also demands further econometric considerations since this represents a departure from the usual assumptions required for consistent and efficient parameter estimation.

Three main recommendations for economic research can be draw from our analysis. First, any restrictions at the aggregate level which is not motivated by empirical purposes (as identification, collinearity, parsimony of parameterization etc.) should be avoided, or at least tested. Second, as already underlined by Stoker [1984], the interpretation of aggregate estimates in terms of individual behavior should be given up. Third, since in general aggregate parameters are complex functions of elementary components of the model, their stability over the sample should be investigated. These suggestions are not new: they can be found in several textbooks and are seen as both reasonable and valuable by most economists. Surprisingly, these issues are rarely considered in empirical studies. Although presented in textbooks (e.g. Berndt [1990]), tests of the validity of microeconomic restrictions or of the stability of parameters are not often carried out in economic analysis.

An appealing extension of the model presented would consist to study whether regularities are more likely to be verified in the aggregate (across goods) than in the original framework. Such a modelling approach is advocated by Hildenbrand [1994] in the context of aggregation across agents.

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