

# **PANEL DATA METHODS\***

By

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## **I. Introduction**

Panel data refers to data sets consisting of multiple observations on each sampling unit. This could be generated by pooling time-series observations across a variety of cross-sectional units including countries, states, regions, firms, or randomly sampled individuals or households. Two well known examples in the U.S. are the Panel Study of Income Dynamics (PSID) and the National Longitudinal Survey (NLS). The PSID began in 1968 with 4802 families, including an over-sampling of poor households. Annual interviews were conducted and socioeconomic characteristics of each of the families and of roughly 31000 individuals who have been in these or derivative families were recorded. The list of variables collected is over 5000. The NLS, followed five distinct segments of the labor force. The original samples include 5020 older men, 5225 young men, 5083 mature women, 5159 young women and 12686 youths. There was an oversampling of blacks, hispanics, poor whites and military in the youths survey. The list of variables collected runs into the thousands. Panel data sets have also been constructed from the U.S. Current Population Survey (CPS) which is a monthly national household survey conducted by the Census Bureau. The CPS generates the unemployment rate and other labor force statistics. Compared with the NLS and PSID data sets, the CPS contains fewer variables, spans a shorter period and does not follow movers. However, it covers a much larger sample and is representative of all demographic groups. European panel data sets include, the German Social Economic Panel, the Swedish study of household market and nonmarket activities and the Intomart Dutch panel of households.

Some of the benefits and limitations of using panel data sets are listed in Hsiao (1986). Obvious benefits are a much larger data set with more variability and less collinearity among the variables than is typical of cross-section or time-series data. With additional, more informative data, one can get more reliable estimates and test more sophisticated behavioral models with less

restrictive assumptions. Another advantage of panel data sets are their ability to control for individual heterogeneity. Not controlling for these unobserved individual specific effects leads to bias in the resulting estimates. Panel data sets are also better able to identify and estimate effects that are simply not detectable in pure cross-sections or pure time-series data. In particular, panel data sets are better able to study complex issues of dynamic behavior. For example, with cross-section data set one can estimate the rate of unemployment at a particular point in time. Repeated cross-sections can show how this proportion changes over time. Only panel data sets can estimate what proportion of those who are unemployed in one period remain unemployed in another period.

Limitations of panel data sets include the following: problems in the design, data collection and data management of panel surveys, see Kasprzyk, Duncan, Kalton and Singh (1989). These include the problems of coverage (incomplete account of the population of interest), non-response (due to lack of cooperation of the respondent or because of interviewer error), recall (respondent not remembering correctly), frequency of interviewing, interview spacing, reference period, the use of bounding to prevent the shifting of events from outside the recall period into the recall period and time in sample bias. Another limitation of panel data sets are the distortions due to measurement errors. Measurement errors may arise because of faulty response due to unclear questions, memory errors, deliberate distortion of responses (e.g., prestige bias), inappropriate informants, missrecording of responses and interviewer effects. Although these problems can occur in cross-section studies, they are aggravated in panel data studies. Duncan and Hill (1985) in a validation study on the PSID data set compare the records of a large firm with the response of its employees and find the ratio of measurement error variance to true variance to be of the order of 184% for average hourly earnings. These figures are for a one-year recall,

i.e., 1983 for 1982, and are more than doubled with two years' recall. Panel data sets may also exhibit bias due to sample selection problems. For the initial wave of the panel, respondents may refuse to participate or the interviewer may not find anybody at home. This may cause some bias in the inference drawn from this sample. While this non-response can also occur in cross-section data sets, it is more serious with panels because subsequent waves of the panel are still subject to non-response. Respondents may die, or move, or find that the cost of responding is high. The rate of attrition differs across panels and usually increases from one wave to the next, but the rate of increase declines over time. Beckett, Gould, Lillard and Welch (1988) studied the representativeness of the PSID, 14 years after it started. They find that only 40% of those originally in the sample in 1968 remained in the sample in 1981. Typical panels involve annual data covering a short span of time for each individual. This means that asymptotic arguments rely crucially on the number of individuals in the panel tending to infinity. Increasing the time span of the panel is not without cost either. In fact, this increases the chances of attrition with every new wave and increases the degree of computational difficulty in the estimation of qualitative limited dependent variable panel data models, see Baltagi (1995b).

## II. The Error Components Regression Model

Although, random coefficient regressions can be used in the estimation and specification of panel data models, see Swamy (1971), Hsiao (1986) and Dielman (1989), most panel data applications have been limited to a simple regression with error components disturbances

$$y_{it} = x'_{it}\beta + \mu_i + \lambda_t + v_{it} \quad i=1, \dots, N ; t=1, \dots, T \quad (1)$$

where  $i$  denotes individuals and  $t$  denotes time.  $x_{it}$  is a vector of observations on  $k$  explanatory variables,  $\beta$  is a  $k$  vector of unknown coefficients,  $\mu_i$  is an unobserved individual specific effect,  $\lambda_t$  is an unobserved time specific effect and  $v_{it}$  is a zero mean random disturbance with variance

$\sigma_v^2$ . The error components disturbances follow a two-way analysis of variance (ANOVA). If  $\mu_i$  and  $\lambda_t$  denote fixed parameters to be estimated, this model is known as the fixed effects (FE) model. The  $x_{it}$ 's are assumed independent of the  $v_{it}$ 's for all  $i$  and  $t$ . Inference in this case is conditional on the particular  $N$  individuals and over the specific time-periods observed. Estimation in this case amounts to including  $(N-1)$  individual dummies and  $(T-1)$  time dummies to estimate these time invariant and individual invariant effects. This leads to an enormous loss in degrees of freedom. In addition, this attenuates the problem of multicollinearity among the regressors. Furthermore, this may not be computationally feasible for large  $N$  and/or  $T$ . In this case, one can eliminate the  $\mu_i$ 's and  $\lambda_t$ 's and estimate  $\beta$  by running least squares of  $\tilde{y}_{it} = y_{it} - \bar{y}_i - \bar{y}_{.t} + \bar{y}_{..}$  on the  $\tilde{x}_{it}$ 's similarly defined, where the dot indicates summation over that index and the bar denotes averaging. This transformation is known as the within transformation and the corresponding estimator of  $\beta$  is called the within estimator or the FE estimator. Note that the FE estimator cannot estimate the effect of any time invariant variable like sex, race, religion, or union participation. Nor can it estimate the effect of any individual invariant variable like price, interest rate, etc., that vary only with time. These variables are wiped out by the within transformation.

If  $\mu_i$  and  $\lambda_t$  are random variables with zero means and constant variances  $\sigma_\mu^2$  and  $\sigma_\lambda^2$ , this model is known as the random effects model. The preceding moments are conditional on the  $x_{it}$ 's. In addition,  $\mu_i$ ,  $\lambda_t$  and  $v_{it}$  are assumed to be conditionally independent. The random effects (RE) model can be estimated by GLS which can be obtained using a least squares regression of  $y_{it}^* = y_{it} - \theta_1 \bar{y}_i - \theta_2 \bar{y}_{.t} + \theta_3 \bar{y}_{..}$  on  $x_{it}^*$  similarly defined, where  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  are simple functions of the variance components  $\sigma_\mu^2$ ,  $\sigma_\lambda^2$  and  $\sigma_v^2$ , see Fuller and Battese (1974). The corresponding GLS estimator of  $\beta$  is known as the RE estimator. Note that for this RE model, one can estimate the

effects of time invariant and individual invariant variables. The Best Quadratic Unbiased (BQU) estimators of the variance components are ANOVA type estimators based on the true disturbances and these are Minimum Variance Unbiased (MVU) under normality of the disturbances. One can obtain feasible estimates of the variance components by replacing the true disturbances by OLS residuals, see Wallace and Hussain (1969). Alternatively, one could substitute the fixed effects residuals as proposed by Amemiya (1971). In fact, Amemiya (1971) shows that the Wallace and Hussain (1969) estimates of the variance components have a different asymptotic distribution from that knowing the true disturbances, while the Amemiya (1971) estimates of the variance components have the same asymptotic distribution as that knowing the true disturbances. Other estimators of the variance components were proposed by Swamy and Arora (1972) and Fuller and Battese (1974). Maximum Likelihood Estimation (MLE) under the normality of the disturbances is derived by Amemiya (1971). The first-order conditions are non-linear, but can be solved using an iterative GLS scheme, see Breusch (1987). Finally one can apply Rao's (1972) Minimum Norm Quadratic Unbiased Estimation (MINQUE) methods. These methods are surveyed in Baltagi (1995b). Wallace and Hussain (1969) compare the RE and FE estimators of  $\beta$  in the case of nonstochastic (repetitive)  $x_{it}$ 's and find that both are (i) asymptotically normal (ii) consistent and unbiased and that (iii)  $\hat{\beta}_{RE}$  has a smaller generalized variance (i.e., more efficient) in finite samples. In the case of nonstochastic (nonrepetitive)  $x_{it}$ 's they find that both  $\hat{\beta}_{RE}$  and  $\hat{\beta}_{FE}$  are consistent, asymptotically unbiased and have equivalent asymptotic variance-covariance matrices, as both  $N$  and  $T \rightarrow \infty$ . Under the random effects model, GLS based on the true variance components is BLUE, and all the feasible GLS estimators considered are asymptotically efficient as  $N$  and  $T \rightarrow \infty$ . Maddala and Mount (1973) compared OLS, FE, RE and MLE methods using Monte Carlo experiments. They found little to choose among the various feasible GLS estimators in small

samples and argued in favor of methods that were easier to compute. MINQUE was dismissed as more difficult to compute and the applied researcher given one shot at the data was warned to compute at least two methods of estimation. If these methods give different results, the authors diagnose misspecification. Taylor (1980) derived exact finite sample results for the one-way error component model ignoring the time-effects. He found the following important results. (1) Feasible GLS is more efficient than the FE estimator for all but the fewest degrees of freedom. (2) The variance of feasible GLS is never more than 17% above the Cramer-Rao lower bound. (3) More efficient estimators of the variance components do not necessarily yield more efficient feasible GLS estimators. These finite sample results are confirmed by the Monte Carlo experiments carried out by Baltagi (1981a).

One test for the usefulness of panel data models is their ability to predict. For the RE model, the Best Linear Unbiased Predictor (BLUP) was derived by Wansbeek and Kapteyn (1978) and Taub (1979). This derivation was generalized by Baltagi and Li (1992) to the RE model with serially correlated remainder disturbances. More recently, Baillie and Baltagi (1995) derived the asymptotic mean square prediction error for the FE and RE predictors as well as two other misspecified predictors and compared their performance using Monte Carlo experiments.

### **III. Test of Hypotheses**

Fixed versus random effects has generated a lively debate in the biometrics literature. In econometrics, see Mundlak (1978). The random and fixed effects models yield different estimation results, especially if  $T$  is small and  $N$  is large. A specification test based on the difference between these estimates is given by Hausman (1978). The null hypothesis is that the individual and time-effects are not correlated with the  $x_{it}$ 's. The basic idea behind this test is that the fixed effects estimator  $\tilde{\beta}_{FE}$  is consistent whether the effects are or are not correlated with the



$x_{it}$ 's. This is true because the fixed effects transformation described by  $\tilde{Y}_{it}$  wipes out the  $\mu_i$  and  $\lambda_t$  effects from the model. However, if the null hypothesis is true, the fixed effects estimator is not efficient under the random effects specification, because it relies only on the within variation in the data. On the other hand, the random effects estimator  $\hat{\beta}_{RE}$  is efficient under the null hypothesis but is biased and inconsistent when the effects are correlated with the  $x_{it}$ 's. The difference between these estimators  $\hat{q} = \tilde{\beta}_{FE} - \hat{\beta}_{RE}$  tend to zero in probability limits under the null hypothesis and is non-zero under the alternative. The variance of this difference is equal to the difference in variances,  $\text{var}(\hat{q}) = \text{var}(\tilde{\beta}_{FE}) - \text{var}(\hat{\beta}_{RE})$ , since  $\text{cov}(\hat{q}, \hat{\beta}_{RE}) = 0$  under the null hypothesis. Hausman's test statistic is based upon  $m = \hat{q}'[\text{var}(\hat{q})]^{-1}\hat{q}$  and is asymptotically distributed a  $\chi^2$  with  $k$  degrees of freedom under the null hypothesis.<sup>1</sup> The Hausman test can also be computed as a variable addition test by running  $y^*$  on the regressor matrices  $X^*$  and  $\tilde{X}$  testing that the coefficients of  $\tilde{X}$  are zero using the usual F-test. This test was generalized by Arellano (1993) to make it robust to heteroskedasticity and autocorrelation of arbitrary forms. In fact, if either heteroskedasticity or serial correlation is present, the variances of the FE and RE estimators are not valid and the corresponding Hausman test statistic is inappropriate. Ahn and Low (1995) show that the Hausman test statistic can be obtained as  $(NT)R^2$  from the regression of GLS residuals on  $\tilde{X}$  and  $\bar{X}$  where the latter denotes the matrix of regressors averaged over time. Also, an alternative Generalized Method of Moments (GMM) test is recommended for testing the joint null hypothesis of exogeneity of the regressors *and* the stability of regression parameters over time. If the regression parameters are nonstationary over time, then both  $\hat{\beta}_{RE}$  and  $\tilde{\beta}_{FE}$  are

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<sup>1</sup>For the one-way error components model with individual effects only, Hausman and Taylor (1981) show that Hausman's specification test can also be based on two other contrasts that yield numerically identical results. Kang (1985) extends this analysis to the two-way error components model.

inconsistent even though the regressors may be exogenous. Ahn and Low perform Monte Carlo experiments which show that both the Hausman and the alternative GMM test have good power in detecting endogeneity of the regressors. However, the alternative GMM test dominates if the coefficients of the regressors are nonstationary. Li and Stengos (1992) propose a Hausman specification test based on  $\sqrt{N}$ -consistent semiparametric estimators. They apply it in the context of a dynamic panel data model of the form

$$y_{it} = \delta y_{i,t-1} + g(x_{it}) + u_{it} \quad i=1, \dots, N; \quad t=1, \dots, T \quad (2)$$

where the function  $g(\cdot)$  is unknown, but satisfies certain moment and differentiability conditions. The  $x_{it}$  observations are IID with finite fourth moments and the disturbances  $u_{it}$  are IID( $0, \sigma^2$ ) under the null hypothesis. Under the alternative, the disturbances  $u_{it}$  are IID in the  $i$  subscript but are serially correlated in the  $t$  subscript. Li and Stengos base the Hausman test for  $H_0: E(u_{it}|y_{i,t-1}) = 0$  on the difference between two  $\sqrt{N}$ -consistent instrumental variables estimators for  $\delta$ , under the null and the alternative respectively.

For panels with large  $N$  and small  $T$ , testing for poolability of the data amount to testing the stability of the cross-section regression across time. In practice, the Chow (1960) test for the equality of the regression coefficients is popular. This is proper only under the spherical disturbances assumption. It leads to improper inference under the random effects specification. In fact, Baltagi (1981a) shows that in this case, the Chow-test leads to rejection of the null too often when in fact it is true. However, applying the F-test for the equality of slopes accounting for the random effects variance-covariance matrix performs well in Monte Carlo experiments. Recently, Baltagi, Hidalgo and Li (1995) derive a non-parametric test for poolability which is robust to functional form misspecification. In particular, they consider the following nonparametric panel data model

$$y_{it} = g_t(x_{it}) + \epsilon_{it} \quad (i= 1, \dots, N ; t= 1, \dots, T) \quad (3)$$

where  $g_t(\cdot)$  is an unspecified functional form that may vary over time.  $x_{it}$  is a  $k \times 1$  column vector of predetermined explanatory variables with  $(p \geq 1)$  variables being continuous and  $k-p (\geq 0)$ . Poolability of the data over time is equivalent to testing that  $g_t(x) = g_s(x)$  almost everywhere for all  $t$  and  $s= 1, 2, \dots, T$ ; versus  $g_t(x) \neq g_s(x)$  for some  $t \neq s$  with probability greater than zero. The test statistic is shown to be consistent and asymptotically normal and is applied to a panel data set on earnings.

In choosing between pooled homogeneous parameter estimators versus non-pooled heterogeneous parameter estimators, some Mean-Square Error (MSE) criteria can be used as described in Wallace (1972) to capture the tradeoff between bias and variance. Bias is introduced when the poolability restriction is not true. However, the variance is reduced by imposing the poolability restriction. Hence, the MSE criteria may choose the pooled estimator despite the fact that the poolability restriction is not true. Ziemer and Wetzstein (1983) suggest comparing pooled and non-pooled estimators according to their forecast risk performance. They show for a wilderness recreation demand model that a Stein-rule estimator gives better forecast risk performance than the pooled or individual cross-section estimates. More recently, the fundamental assumption underlying pooled homogeneous parameters models has been called into question. For example, Robertson and Symons (1992) warned about the bias from pooled estimators when the estimated model is dynamic and homogeneous when in fact the true model is static and heterogeneous. Pesaran and Smith (1995) argued in favor of heterogeneous estimators rather than pooled estimators for panels with large  $N$  and  $T$ . They showed that when the true model is dynamic and heterogeneous, the pooled estimators are inconsistent whereas an average estimator of heterogeneous parameters can lead to consistent estimates as long as both  $N$  and  $T$  tend to

infinity. Using a different approach, Maddala, Srivastava and Li (1994) argued that shrinkage estimators are superior to either heterogeneous or homogeneous parameter estimates especially for prediction purposes. In this case, one shrinks the individual heterogeneous estimates toward the pooled estimate using weights depending on their corresponding variance-covariance matrices. Baltagi and Griffin (1995) compare the short-run and long-run forecast performance of the pooled homogeneous, individual heterogeneous and shrinkage estimators for a dynamic demand for the gasoline across eighteen OECD countries. Based on one, five and ten year forecasts, the results support the case for pooling. Alternative tests for structural change in panel data include Han and Park (1989) who used the cumulative sum and cusum of squares to test for structural change based on recursive residuals. They find no structural break over the period 1958-1976 in U.S. foreign trade of manufacturing goods.

Testing for random individual effects is of utmost importance in panel data applications. Ignoring these effects lead to huge bias in estimation, see Moulton (1986). A popular Lagrange Multiplier test for the significance of the random effects  $H_0: \sigma_\mu^2 = 0$  was derived by Breusch and Pagan (1980). This test statistic can be easily computed using least squares residuals. This assumes that the alternative hypothesis is two-sided when we know that the variance components are non-negative. A one-sided version of this test is given by Honda (1985). This is shown to be a *uniformly most powerful* and robust to non-normality. However, Moulton and Randolph (1989) showed that the asymptotic  $N(0, 1)$  approximation for this one-sided LM statistic can be poor even in large samples. They suggest an alternative Standardized Lagrange Multiplier (SLM) test whose asymptotic critical values are generally closer to the exact critical values than those of the LM test. This SLM test statistic centers and scales the one-sided LM statistic so that its mean is zero and its variance is one.

For  $H_0^b$ ;  $\sigma_\mu^2 = \sigma_\lambda^2 = 0$ , the two-sided LM test is given by Breusch and Pagan (1980), and is distributed as  $\chi_2^2$  under the null. Honda (1985) does not derive a uniformly most powerful one-sided test for  $H_0^b$ , but he suggests a 'handy' one-sided test which is distributed as  $N(0,1)$  under  $H_0^b$ . Later Honda (1991) derives the SLM version of this 'handy' one-sided test. Baltagi, Chang and Li (1992) derive a locally mean most powerful (LMMP) one-sided test for  $H_0^b$  and its SLM version is given by Baltagi (1995b). Under  $H_0^b$ ;  $\sigma_\mu^2 = \sigma_\lambda^2 = 0$ , these Standardized Lagrange Multiplier statistics are asymptotically  $N(0,1)$  and their asymptotic critical values should be closer to the exact critical values than those of the corresponding unstandardized tests. Alternatively, one can perform a likelihood ratio test or an ANOVA-type F-test. Both tests have the same asymptotic distribution as their LM counterparts. Moulton and Randolph (1989) find that although the F-test is not locally most powerful, its power function is close to the power function of the exact LM test and is therefore recommended. A comparison of these various testing procedures using Monte Carlo experiments is given by Baltagi, Chang and Li (1992). Recent developments include a generalization by Li and Stengos (1994) of the Breusch-Pagan test to the case where the remainder error is heteroskedastic of unknown form. Also, Baltagi and Chang (1995) who propose a simple ANOVA F-statistic based on recursive residuals to test for random individual effects.

For incomplete (or unbalanced) panels, the Breusch-Pagan test can be easily extended, see Moulton and Randolph (1989) for the one-way error components model and Baltagi and Li (1990) for the two-way error components model. For non-linear models, Baltagi (1996) suggests a simple method for testing for zero random individual and time effects using a Gauss-Newton regression. In case the regression model is linear, this test amounts to a variable addition test, i.e., running the original regression with two additional regressors. The first is the average of the least squares

residuals over time, while the second is the average of the least squares residuals over individuals. The test statistic becomes the F-statistic for the significance of the two additional regressors.

Baltagi and Li (1995) derive three LM test statistics that jointly test for serial correlation and individual effects. The first LM statistic jointly tests for zero first-order serial correlation *and* random individual effects, the second LM statistic tests for zero first-order serial correlation assuming fixed individual effects, and the third LM statistic tests for zero first-order serial correlation assuming random individual effects. In *all* three cases, Baltagi and Li (1995) showed that the corresponding LM statistic is the *same* whether the alternative is AR(1) or MA(1). In addition, Baltagi and Li (1995) derive two simple tests for distinguishing between AR(1) and MA(1) remainder disturbances in error components regressions and perform Monte Carlo experiments to study the performance of these tests. For the fixed effects model, Bhargava, Franzini and Narendranathan (1982) derived a modified Durbin-Watson test statistic based on FE residuals to test for first-order serial correlation and a test for random walk based on differenced OLS residuals. Chesher (1984) derived a score test for neglected heterogeneity, which is viewed as causing parameter variation. Also, Hamerle (1990) and Orme (1993) suggest a score test for neglected heterogeneity for qualitative limited dependent variable panel data models.

Holtz-Eakin (1988) derives a simple test for the presence of individual effects in dynamic (auto-regressive) panel data models, while Holtz-Eakin, Newey and Rosen (1988) formulate a coherent set of procedures for estimating and testing VAR (vector autoregression) with panel data. Arellano and Bond (1991) consider tests for serial correlation and over-identification restrictions in a dynamic random effects model, while Arellano (1990) considers testing covariance restrictions for error components or first-difference structures with White noise, MA or AR schemes.

Chamberlain (1982, 1984) finds that the fixed effects specification imposes testable restrictions on coefficients from regressions of all leads and lags of the dependent variable on all leads and lags of independent variables. These over-identification restrictions are testable using minimum chi-squared statistics. Angrist and Newey (1991) show that, in the standard fixed effects model, this over-identification test statistic is simply the degrees of freedom times the  $R^2$  from a regression of *within residuals* on all leads and lags of the independent variables. They apply this test to models of the union-wage effect using five years of data from the National Longitudinal Survey of Youth and to a conventional human capital earnings function estimating the return to schooling. They do not reject a fixed effect specification in the union-wage example, but they do reject it in the return to schooling example.

Testing for unit roots using panel data has been recently reconsidered by Quah (1994), Levin and Lin (1996) and Im, Pesaran and Shin (1996). This has been applied by MacDonald (1996) to real exchange rates for 17 OECD countries based on a wholesale price index, and 23 OECD countries based on a consumer price index, all over the period 1973-1992. The null hypothesis that real exchange rates contain a unit root is rejected. Earlier applications include: Boumahdi and Thomas (1991) who apply a likelihood ratio unit root panel data test to assess efficiency of the French capital market. Using 140 French stock prices observed weekly from January 1973 to February 1986 ( $T=671$ ) on the Paris Stock Exchange, Boumahdi and Thomas (1991) do not reject the null hypothesis of a unit root. Also, Breitung and Meyer (1994) who apply panel data unit roots test to contract wages negotiated on firm and industry level in Western Germany over the period 1972-1987. They find that both firm and industry wages possess a unit root in the autoregressive representation. However, there is weak evidence for a cointegration relationship.

#### **IV. Generalizations of the Error Components Model**

The error components disturbances are homoskedastic across individuals. This may be an unrealistic assumption and has been relaxed by Mazodier and Trognon (1978) and Baltagi and Griffin (1988). A more general heteroskedastic model is given by Randolph (1988) in the context of unbalanced panels. Also, Li and Stengos (1994) who proposed estimating a one-way error component model with heteroskedasticity of unknown form using adaptive estimation techniques.

The error components regression model has been also generalized to allow for serial correlation in the remainder disturbances by Lillard and Willis (1978), Revankar (1979), MaCurdy (1982) and more recently Baltagi and Li (1991, 1995) and Galbraith and Zinde-Walsh (1995). Chamberlain (1982, 1984) allows for arbitrary serial correlation and heteroskedastic patterns by viewing each time period as an equation and treating the panel as a multivariate set up. Also, Kiefer (1980), Schmidt (1983), Arellano (1987) and Chowdhury (1994) extend the fixed effects model to cover cases with an arbitrary intertemporal covariance matrix.

The normality assumption on the error components disturbances may be untenable. Horowitz and Markatou (1996) show how to carry out nonparametric estimation of the densities of the error components. Using data from the Current Population Survey, they estimate an earnings model and show that the probability that individuals with low earnings will become high earners in the future are much lower than that obtained under the assumption of normality. One drawback of this nonparametric estimator is its slow convergence at a rate of  $1/(\log N)$  where  $N$  is the number of individuals. Monte Carlo results suggest that this estimator should be used for  $N$  larger than 1000.

Micro panel data on households, individuals and firms are highly likely to exhibit measurement error, see Duncan and Hill (1985) who found serious measurement error in average



hourly earnings in the Panel Study of Income Dynamics. Using panel data, Griliches and Hausman (1986) showed that one can identify and estimate a variety of errors in variables models *without* the use of external instruments. Griliches and Hausman suggest differencing the data  $j$  periods apart,  $(y_{it}-y_{i,t-j})$ , thus generating 'different lengths' difference estimators. These transformations wipe out the individual effect, but they may aggravate the measurement error bias. One can calculate the bias of the 'different lengths' differenced estimators and use this information to obtain consistent estimators of the regression coefficients. Extensions of this model include Kao and Schnell (1987 a,b), Wansbeek and Koning (1989), Hsiao (1991), Wansbeek and Kapteyn (1992) and Biorn (1992). See also Baltagi and Pinnoi (1995) for an application to the productivity of the public capital stock.

The error components model has been extended to the seemingly unrelated regressions case by Avery (1977), Baltagi (1980), Magnus (1982), Prucha (1984) and more recently Kinal and Lahiri (1990). Some applications include Howrey and Varian (1984) on the estimation of a system of demand equations for electricity by time of day, and Sickles (1985) on the analysis of productivity growth in the U.S. Airlines industry.

For the simultaneous equation with error components, Baltagi (1981b) derives the error component two-stage (EC2SLS) and three-stage (EC3SLS) least squares estimators, while Prucha (1985) derives the full information MLE under the normality assumption. These estimators are surveyed in Krishnakumar (1988). Monte Carlo experiments are given by Baltagi (1984) and Mátyás and Lovrics (1990). Recent applications of EC2SLS and EC3SLS include: (i) an econometric rational-expectations macroeconomic model for developing countries with capital controls, see Haque, Lahiri and Montiel (1993), and (ii) an econometric model measuring income and price elasticities of foreign trade for developing countries, see Kinal and Lahiri (1993).

Mundlak (1978) considered the case where the endogeneity is solely attributed to the individual effects. In this case, Mundlak showed that if these individual effects are a linear function of the averages of *all* the explanatory variables across time, then the GLS estimator of this model coincides with the FE estimator. Mundlak's (1978) formulation assumes that *all* the explanatory variables are related to the individual effects. The random effects model on the other hand assumes no correlation between the explanatory variables and the individual effects. Instead of this 'all or nothing' correlation among the  $x_{it}$ 's and the  $\mu_i$ 's, Hausman and Taylor (1981) consider a model where *some* of the explanatory variables are related to the  $\mu_i$ 's. In particular, they consider the following model:

$$y_{it} = x_{it}'\beta + z_i'\gamma + \mu_i + v_{it} \quad (4)$$

where the  $z_i$ 's are cross-sectional time-invariant variables. Hausman and Taylor (1981), hereafter HT, split the matrices  $X$  and  $Z$  into two sets of variables:  $X = [X_1; X_2]$  and  $Z = [Z_1; Z_2]$  where  $X_1$  is  $n \times k_1$ ,  $X_2$  is  $n \times k_2$ ,  $Z_1$  is  $n \times g_1$ ,  $Z_2$  is  $n \times g_2$  and  $n = NT$ .  $X_1$  and  $Z_1$  are assumed exogenous in that they are not correlated with  $\mu_i$  and  $v_{it}$  while  $X_2$  and  $Z_2$  are endogenous because they are correlated with the  $\mu_i$ 's, but not the  $v_{it}$ 's. The within transformation would sweep the  $\mu_i$ 's and remove the bias, but in the process it would also remove the  $Z_i$ 's and hence the within estimator will not give an estimate of the  $\gamma$ 's. To get around that, Hausman and Taylor (1981) suggest an instrumental variable estimator that uses  $\bar{X}_1, \bar{X}_2, \bar{X}_1$  and  $Z_1$  as instruments. Therefore, the matrix of regressors  $X_1$  is used twice, once as averages and another time as deviations from averages. This is an advantage of panel data allowing instruments from *within* the model. The order condition for identification gives the result that the number of  $X_1$ 's ( $k_1$ ) must be at least as large as the number of  $Z_2$ 's ( $g_2$ ). With stronger exogeneity assumptions between  $X$  and the  $\mu_i$ 's, Amemiya and MaCurdy (1986) and Breusch, Mizon and Schmidt (1989) suggest more efficient

instrumental variable estimators. Cornwell and Rupert (1988) apply these IV methods to a returns to schooling example based on a panel of 595 individuals drawn from the PSID over the period 1976-1982. Recently, Metcalf (1996) shows that for the Hausman-Taylor model given in (4), using less instruments may lead to a more powerful Hausman specification test. Asymptotically, more instruments lead to more efficient estimators. However, the asymptotic bias of the inefficient estimator will also be greater as the null hypothesis of no correlation is violated. The increase in bias more than offsets the increase in variance. Since, the test statistic is linear in variance but quadratic in bias, its power will increase.

Cornwell, Schmidt and Wyhowski (1992) consider a simultaneous equation model with error components that distinguishes between two types of exogenous variables, namely *singly exogenous* and *doubly exogenous* variables. A singly exogenous variable is correlated with the individual effects but not with the remainder noise, while a doubly exogenous variable is uncorrelated with both the effects and the remainder disturbance term. For this encompassing model with two types of exogeneity, Cornwell, Schmidt and Wyhowski (1992) extend the three instrumental variable estimators considered above and give them a GMM interpretation. Wyhowski (1994) extend these results to the two-way error components model, while Revankar (1992) establishes conditions for exact equivalence of instrumental variables in a simultaneous equation two-way error components model.

## V. Dynamic Panel Data Models

The dynamic error components regression is characterized by the presence of a lagged dependent variable among the regressors, i.e.,

$$y_{it} = \delta y_{i,t-1} + x'_{it} \beta + \mu_i + v_{it} \quad i=1, \dots, N; t=1, \dots, T \quad (5)$$

where  $\delta$  is a scalar,  $x_{it}'$  is  $1 \times k$  and  $\beta$  is  $k \times 1$ . This model has been extensively studied by Anderson and Hsiao (1982) and Sevestre and Trognon (1985).<sup>2</sup> Since  $y_{it}$  is a function of  $\mu_i$ ,  $y_{i,t-1}$  is also a function of  $\mu_i$ . Therefore,  $y_{i,t-1}$ , a right hand regressor in (5), is correlated with the error term. This renders the OLS estimator biased and inconsistent even if the  $v_{it}$ 's are not serially correlated. For the fixed effects (FE) estimator, the within transformation wipes out the  $\mu_i$ 's, but  $\tilde{y}_{i,t-1}$  will still be correlated with  $\tilde{v}_{it}$  even if the  $v_{it}$ 's are not serially correlated. In fact, the within estimator will be biased of  $O(1/T)$  and its consistency will depend upon  $T$  being large, see Nickell (1981) and more recently Kiviet (1995) who shows that the bias of the FE estimator in a dynamic panel data model has an  $O(N^{-1}T^{-3/2})$  approximation error. The same problem occurs with the random effects GLS estimator. In order to apply GLS, quasi-demeaning is performed, and  $y_{i,t-1}^*$  will be correlated with  $u_{it}^*$ . An alternative transformation that wipes out the individual effects, yet does not create the above problem is the first difference (FD) transformation. In fact, Anderson and Hsiao (1982) suggested first differencing the model to get rid of the  $\mu_i$ 's and then using  $\Delta y_{i,t-2} = (y_{i,t-2} - y_{i,t-3})$  or simply  $y_{i,t-2}$  as an instrument for  $\Delta y_{i,t-1} = (y_{i,t-1} - y_{i,t-2})$ . These instruments will not be correlated with  $\Delta v_{it} = v_{i,t} - v_{i,t-1}$ , as long as the  $v_{it}$ 's themselves are not serially correlated. This instrumental variable (IV) estimation method leads to consistent but not necessarily efficient estimates of the parameters in the model because it does not make use of all the available moment conditions, see Ahn and Schmidt (1995) and it does not take into account the differenced structure on the residual disturbances ( $\Delta v_{it}$ ). Arellano (1989) finds that for simple dynamic error components models the estimator that uses differences  $\Delta y_{i,t-2}$  rather than levels  $y_{i,t-2}$  for instruments

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<sup>2</sup>In particular, the assumptions made on the initial values are of utmost importance, see Anderson and Hsiao (1982), Bhargava and Sargan (1983) and Hsiao (1986). Hsiao (1986) summarizes the consistency properties of the MLE and GLS under a RE dynamic model depending on the initial values assumption and the way in which  $N$  and  $T$  tend to infinity.

has a singularity point and very large variances over a significant range of parameter values. In contrast, the estimator that uses instruments in levels, i.e.,  $y_{i,t-2}$ , has no singularities and much smaller variances and is therefore recommended. Additional instruments can be obtained in a dynamic panel data model if one utilizes the orthogonality conditions that exist between lagged values of  $y_{it}$  and the disturbances  $v_{it}$ , see Holtz-Eakin (1988), Holtz-Eakin, Newey and Rosen (1988) and Arellano and Bond (1991). Based on these additional moments, Arellano and Bond (1991) suggest a GMM estimator and propose a Sargan-type test for over-identifying restrictions.<sup>3</sup> Arellano and Bover (1995) develop a unifying GMM framework for looking at efficient IV estimators for dynamic panel data models. They do that in the context of the Hausman and Taylor (1981) model given in (4). Ahn and Schmidt (1995) show that under the standard assumptions used in a dynamic panel data model, there are additional moment conditions that are ignored by the IV estimators suggested by Arellano and Bond (1991). They show how these additional restrictions can be utilized in a GMM framework. Ahn and Schmidt (1995) also consider the dynamic version of the Hausman and Taylor (1981) model and show how one can make efficient use of exogenous variables as instruments. In particular, they show that the strong exogeneity assumption implies more orthogonality conditions which lie in the deviations from mean space. These are irrelevant in the static Hausman-Taylor model but are relevant for the dynamic version of that model.

An alternative approach to estimating dynamic panel data models have been suggested by Keane and Runkle (1992). Drawing upon the forward filtering idea from the time-series literature, this method of estimation first transforms the model to eliminate the general and arbitrary serial

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<sup>3</sup>Bhargava (1991) gives sufficient conditions for the identification of both static and dynamic panel data models with endogenous regressors.

correlation pattern in the data. By doing so, one can use the set of original predetermined instruments to obtain consistent parameter estimates of the model. First differencing is also used in dynamic panel data models to get rid of individual specific effects and the resulting first differenced errors are serially correlated of an MA(1) type with unit root if the original  $v_{it}$ 's are classical errors. In this case, there will be gain in efficiency in performing the Keane and Runkle filtering procedure on the first-differenced (FD) model. Underlying this estimation procedure are two important hypotheses that are testable. The first is  $H_A$ ; the set of instruments are *strictly exogenous*. In order to test  $H_A$ , Keane and Runkle propose a test based on the difference between fixed effects 2SLS (FE-2SLS) and first difference 2SLS (FD-2SLS). FE-2SLS is consistent only if  $H_A$  is true. In fact if the matrix of instruments contain predetermined variables then FE-2SLS would not be consistent. In contrast, FD-2SLS is consistent whether  $H_A$  is true or not. If  $H_A$  is not rejected, one should check whether the individual effects are correlated with the set of instruments. In this case, the usual Hausman and Taylor (1981) test applies. However, if  $H_A$  is rejected, the instruments are predetermined and the Hausman-Taylor test is inappropriate. In this case, the test will be based upon the difference between FD-2SLS and 2SLS. Under the null, both estimators are consistent, but if the null is not true, FD-2SLS remains consistent while 2SLS does not. These two tests are Hausman (1978) type tests except that the variances are complicated because Keane and Runkle do not use the efficient estimator under the null, see Schmidt, Ahn and Wyhowski (1992). Keane and Runkle (1992) apply their testing and estimation procedures to a simple version of the rational expectations life-cycle consumption model. See also Baltagi and Griffin (1995) for another application to liquor demand.

Alternative estimation methods of a static and dynamic panel data model with arbitrary error structure are considered by Chamberlain (1982, 1984). Chamberlain (1984) considers the

panel data model as a multivariate regression of  $T$  equations subject to restrictions and derives an efficient minimum distance estimator that is robust to residual autocorrelation of arbitrary form. Chamberlain (1984) also first-differences these equations to get rid of the individual effects and derives an asymptotically equivalent estimator to his efficient minimum distance estimator based on 3SLS of the  $(T-2)$  differenced equations. Building on Chamberlain's work, Arellano (1990) develops minimum chi-square tests for various covariance restrictions. These tests are based on 3SLS residuals of the dynamic error component model and can be calculated from a generalized linear regression involving the sample autocovariance and dummy variables. The asymptotic distribution of the unrestricted autocovariance estimates is derived without imposing the Normality assumption. In particular, Arellano (1990) considers testing covariance restrictions for error components or first-difference structures with white noise, moving average or autoregressive schemes. If these covariance restrictions are true, 3SLS is inefficient and Arellano (1990) proposes a GLS estimator which achieves asymptotic efficiency in the sense that it has the same limiting distribution as the optimal minimum distance estimator. More recently, Li and Stengos (1995a) derived a  $\sqrt{N}$ -consistent estimator for a semi-parametric dynamic panel data model, while Li and Stengos (1995b) proposed a non-nested test for parametric versus semi-parametric dynamic panel data models.

## **VI. Incomplete Panel Data Models**

Incomplete panels are more likely to be the norm in typical economic empirical settings. For example, if one is collecting data on a set of countries over time, a researcher may find some countries can be traced back longer than others. Similarly, in collecting data on firms over time, a researcher may find that some firms have dropped out of the market while new entrants emerged over the sample period observed. For randomly missing observations, unbalanced panels have

been dealt with in Fuller and Battese (1974), Baltagi (1985), Wansbeek and Kapteyn (1989) and Baltagi and Chang (1994).<sup>4</sup> For the unbalanced one-way error component model, GLS can still be performed as a least squares regression. However, BQU estimators of the variance components are a function of the variance components themselves. Still, unbalanced ANOVA methods are available, see Searle (1987). Baltagi and Chang (1994) performed extensive Monte Carlo experiments varying the degree of unbalancedness in the panel as well as the variance components. Some of the main results include the following: (i) As far as the estimation of regression coefficients are concerned, the simple ANOVA type feasible GLS estimators compare well with the more complicated estimators such as MLE and MINQUE and are never more than 4% above the MSE of true GLS. (ii) For the estimation of the remainder variance component  $\sigma_v^2$ , these methods show little difference in relative MSE performance. However, for the individual specific variance component estimation,  $\sigma_u^2$ , the ANOVA type estimators perform poorly relative to MLE and MINQUE methods when the variance component  $\sigma_u^2$  is large and the pattern is severely unbalanced. (iii) Better estimates of the variance components, in the MSE sense, do not necessarily imply better estimates of the regression coefficients. This echoes similar findings in the balanced panel data case. (iv) Extracting a balanced panel out of an unbalanced panel by either maximizing the number of households observed or the total number of observations lead in both cases to an enormous loss in efficiency and is not recommended.<sup>5</sup> For an empirical application,

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<sup>4</sup>Other methods of dealing with missing data include: (i) imputing the missing values and analyzing the filled-in data by complete panel data methods, (ii) discarding the nonrespondents and weighting the respondents to compensate for the loss of cases, see Little (1988) and the section on non-response adjustments in Kasprzyk, et. al. (1989).

<sup>5</sup>Chowdhury (1991) showed that for the fixed effects error component model, the within estimator based on the entire unbalanced panel is efficient relative to any within estimator based on a sub-balanced pattern. Also, Mátyás and Lovrics (1991) performed some Monte Carlo



see Mendelsohn, et. al. (1992) who use panel data on repeated single family home sales in the harbor area surrounding New Bedford, Massachusetts over the period 1969 to 1988 to study the damage associated with proximity to a hazardous waste site. Mendelsohn, et. al. (1992) find a significant reduction in housing values, between seven and ten thousand (1989 dollars), as a result of these houses proximity to hazardous waste sites. The extension of the unbalanced error components model to the two-way model including time effects is more involved. Wansbeek and Kapteyn (1989) derive the FE, MLE and a feasible GLS estimator based on quadratic unbiased estimators of the variance components and compare their performance using Monte Carlo experiments.

Rotating panels attempt to keep the same number of households in the survey by replacing the fraction of households that drop from the sample in each period by an equal number of freshly surveyed households. This is a necessity in surveys where a high rate of attrition is expected from one period to the next. For the estimation of general rotation schemes as well as maximum likelihood estimation under normality, see Biorn (1981). Estimation of the consumer price index in the U.S. is based on a complex rotating panel survey, with 20% of the sample being replaced by rotation each year, see Valliant (1991). With rotating panels, the fresh group of individuals that are added to the panel with each wave provide a means of testing for time-in-sample bias effects. This has been done for various labor force characteristics in the Current Population Survey. For example, several studies have found that the first rotation reported an unemployment rate that is 10% higher than that of the full sample, see Bailer (1975). While the findings indicate a pervasive

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experiments to compare the loss in efficiency of FE and GLS based on the entire incomplete panel data and complete sub-panel. They find the loss in efficiency is negligible if  $NT > 250$ , but serious for  $NT < 150$ .

effect of rotation group bias in panel surveys, the survey conditions do not remain the same in practice and hence it is hard to disentangle the effects of time-in-sample bias from other effects.

For some countries, panel data may not exist. Instead the researcher may find annual household surveys based on a large random sample of the population. Examples of some of these cross-sectional consumer expenditure surveys include: the British Family Expenditure Survey which surveys about 7000 households annually. Examples of repeated surveys in the U.S. include the Current Population Survey and the National Crime Survey. For these repeated cross-section surveys, it may be impossible to track the same household over time as required in a genuine panel. Instead, Deaton (1985) suggests tracking cohorts and estimating economic relationships based on cohort means rather than individual observations. One cohort could be the set of all males born between 1945 and 1950. This age cohort is well defined, and can be easily identified from the data. Deaton (1985) argued that these pseudo panels do not suffer the attrition problem that plagues genuine panels, and may be available over longer time periods compared to genuine panels.<sup>6</sup> For this pseudo-panel with  $T$  observations on  $C$  cohorts, the fixed effects estimator  $\tilde{\beta}_{FE}$ , based on the within 'cohort' transformation, is a natural candidate for estimating  $\beta$ . However, Deaton (1985) argued that these sample-based averages of the cohort means can only estimate the unobserved population cohort means with measurement error. Therefore, one has to correct the within estimator for measurement error using estimates of the errors in measurement variance-

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<sup>6</sup>Blundell and Meghir (1990) also argue that pseudo panels allow the estimation of life-cycle models which are free from aggregation bias. In addition, Moffitt (1993) explains that a lot of researchers in the U.S. prefer to use pseudo panels like the Current Population Survey because it has larger more representative samples and the questions asked are more consistently defined over time than the available U.S. panels.

covariance matrix obtained from the individual data. Details are given in Deaton (1985). There is an obvious trade-off in the construction of a pseudo panel. The larger the number of cohorts, the smaller is the number of individuals per cohort. In this case,  $C$  is large and the pseudo panel is based on a large number of observations. However, the fact that the average cohort size  $n_c = N/C$  is not large implies that the sample cohort averages are not precise estimates of the population cohort means. In this case, we have a large number  $C$  of imprecise observations. In contrast, a pseudo panel constructed with a smaller number of cohorts and therefore more individuals per cohort is trading a large pseudo panel with imprecise observations for a smaller pseudo panel with more precise observations. Verbeek and Nijman (1992b) find that  $n_c \rightarrow \infty$  is a crucial condition for the consistency of the within estimator and that the bias of the within estimator may be substantial even for large  $n_c$ . On the other hand, Deaton's estimator is consistent for  $\beta$ , for finite  $n_c$ , when either  $C$  or  $T$  tend to infinity.

Moffitt (1993) extends Deaton's (1985) analysis to the estimation of dynamic models with repeated cross-sections. Moffitt illustrates his estimation method for the linear fixed effects life-cycle model of labor supply using repeated cross-sections from the U.S. Current Population Survey (CPS). The sample included white males, ages 20-59, drawn from 21 waves over the period 1968 to 1988. In order to keep the estimation problem manageable, the data was randomly subsampled to include a total of 15,500 observations. Moffitt concludes that there is a considerable amount of parsimony achieved in the specification of age and cohort effects. Also, individual characteristics are considerably more important than either age, cohort or year effects. Blundell, Meghir and Neves (1993) use the annual U.K. Family Expenditure Survey covering the period 1970-1984 to study the intertemporal labor supply and consumption of married women. The total number of households considered was 43,671. These were allocated to ten different

cohorts depending on the year of birth. The average number of observations per cohort was 364. Their findings indicate reasonably-sized intertemporal labor supply elasticities. More recently, Collado (1995) propose a GMM estimator corrected for measurement error to deal with a dynamic pseudo-panel data model. This estimator is consistent as  $C$  tends to infinity, for a fixed  $T$  and  $n_C$ .

### VII. Limited Dependent Variables and Panel Data

In many economic studies, the dependent variable is discrete, indicating for example that a household purchased a car or that an individual is unemployed or that he or she joined the union. For example, let  $y_{it} = 1$  if the  $i$ -th individual participates in the labor force at time  $t$ . This occurs if  $y_{it}^*$ , the difference between the  $i$ -th individual's offered wage and his unobserved reservation wage is positive. This can be described more formally as follows:

$$\begin{aligned} y_{it} &= 1 && \text{if } y_{it}^* > 0 \\ &= 0 && \text{if } y_{it}^* \leq 0 \end{aligned} \quad (6)$$

where

$$y_{it}^* = \mathbf{x}_{it}'\boldsymbol{\beta} + \mu_i + v_{it} \quad (7)$$

i.e.,  $y_{it}^*$  can be explained by a set of regressors  $\mathbf{x}_{it}$  and error components disturbances. In this case:

$$\Pr[y_{it} = 1] = \Pr[y_{it}^* > 0] = \Pr[v_{it} > -\mathbf{x}_{it}'\boldsymbol{\beta} - \mu_i] = F(\mathbf{x}_{it}'\boldsymbol{\beta} + \mu_i)$$

The last equality holds as long as the density function describing the cumulative distribution function  $F$  is symmetric around zero. For panel data, the presence of individual effects complicates matters significantly. For the one-way error component model with random individual effects  $E(u_{it}u_{is}) = \sigma_u^2$  for any  $t, s = 1, 2, \dots, T$ , and the joint likelihood of  $(y_{1t}, \dots, y_{Nt})$  can no longer be written as the product of the marginal likelihoods of the  $y_{it}$ 's. This complicates the derivation of maximum likelihood and will now involve bivariate numerical integration. On the

other hand, if there are no random individual effects, the joint likelihood will be the product of the marginals and one can proceed as in the usual cross-sectional limited dependent variable case. For the fixed effects model, with limited dependent variable, the model is non-linear and it is not possible to get rid of the  $\mu_i$ 's by taking differences or performing the FE transformation, as a result  $\beta$  and  $\sigma_v^2$  cannot be estimated consistently for  $T$  fixed, since the inconsistency in the  $\mu_i$ 's is transmitted to  $\beta$  and  $\sigma_v^2$ , see Hsiao (1986). The usual solution around this incidental parameters ( $\mu_i$ 's) problem is to find a minimal sufficient statistic for the  $\mu_i$ 's which does not depend on the  $\beta$ 's. Since the maximum likelihood estimates are in general functions of these minimum sufficient statistics, one can obtain the latter by differentiating the log likelihood function with respect to  $\mu_i$ . For the logit model, this yields the result that  $\sum_{t=1}^T y_{it}$  is a minimum sufficient statistic for  $\mu_i$ . Chamberlain (1980) suggests maximizing the *conditional* likelihood function

$$L_c = \prod_{i=1}^N \Pr(y_{i1}, \dots, y_{iT} / \sum_{t=1}^T y_{it}) \quad (8)$$

rather than the unconditional likelihood function. For the fixed effects logit model, this approach results in a computationally convenient estimator. However, the computations rise geometrically with  $T$  and are excessive for  $T > 10$ .

In order to test for fixed individual effects one can perform a Hausman-type test based on the difference between Chamberlain's conditional maximum likelihood estimator and the usual logit maximum likelihood estimator ignoring the individual effects. The latter estimator is consistent and efficient only under the null of no individual effects and inconsistent under the alternative. Chamberlain's estimator is consistent whether  $H_0$  is true or not, but it is inefficient under  $H_0$  because it may not use all the data. Both estimators can be easily obtained from the usual logit maximum likelihood routines. The constant is dropped and estimates of the asymptotic variances are used to form Hausman's  $\chi^2$  statistic. This will be distributed as  $\chi_k^2$  under  $H_0$ . For

an application studying the linkage between unemployment and mental health problems in Sweden using the Swedish Level of Living Surveys, see Björklund (1985).

In contrast to the fixed effects logit model, the conditional likelihood approach does not yield computational simplifications for the fixed effects *probit* model. In particular, the fixed effects cannot be swept away and maximizing the likelihood over all the parameters including the fixed effects will in general lead to inconsistent estimates for large  $N$  and fixed  $T$ .<sup>7</sup> Heckman (1981b) performed some limited Monte Carlo experiments on a probit model with a single regressor. For  $N = 100$ ,  $T = 8$ ,  $\sigma_v^2 = 1$  and  $\sigma_\mu^2 = 0.5, 1$  and  $3$ , Heckman computed the bias of the fixed effects MLE of  $\beta$  using 25 replications. He found at most 10% bias for  $\beta = 1$  which was always towards zero.

Although the probit model does not lend itself to a fixed effects treatment, it has been common to use it for the random effects specification. For the random effects probit model, maximum likelihood estimation yields a consistent and efficient estimator of  $\beta$ . However, MLE is computationally more involved. Essentially, one has to compute the joint probabilities of a  $T$  variate normal distribution which involves  $T$ -dimensional integrals, see Hsiao (1986). This gets to be infeasible if  $T$  is big. However, by conditioning on the individual effects, this  $T$  dimensional integral problem reduces to a single integral involving the product of a standard normal density and the difference of two normal cumulative density functions. This can be evaluated using the Gaussian quadrature procedure suggested by Butler and Moffitt (1982). This approach has the advantage of being computationally feasible even for fairly large  $T$ . For an application, see

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<sup>7</sup>In cases where the conditional likelihood function is not feasible as in the fixed effects probit case, Manski (1987) suggests a conditional version of his maximum score estimator which under fairly general conditions provides a strongly consistent estimator of  $\beta$ .

Sickles and Taubman (1986) who estimate a two equation structural model of the health and retirement decisions of the elderly using five biennial panels of males drawn from the Retirement History Survey. For a recent Monte Carlo study on the random effects probit model, see Guilkey and Murphy (1993). Underlying the random effects probit model is the equicorrelation assumption between successive disturbances belonging to the same individual. In a study of labor force participation of married women, Avery, Hansen and Hotz (1983) reject the hypothesis of equicorrelation across the disturbances, and suggest a method of moments estimator that allows for a general type of serial correlation among the disturbances. Chamberlain (1984) applies both a fixed effects logit estimator and a minimum distance random effects probit estimator to a study of the labor force participation of 924 married women drawn from the Panel Study of Income Dynamics. Lechner (1995) suggests several specification tests for the panel data probit model. These are generalized score and Wald tests employed to detect omitted variables, neglected dynamics, heteroskedasticity, non-normality and random-coefficient variations. The performance of these tests in small samples is investigated using Monte Carlo experiments. Also, an empirical example on the probability of self-employment in West Germany is given using a random sample of 1926 working men selected from the German Socio-Economic Panel and observed over the period 1984-1989.

Heckman and MaCurdy (1980) consider a fixed effects tobit model to estimate a life cycle model of female labor supply. They argue that the individual effects have a specific meaning in a life cycle model and therefore cannot be assumed independent of the  $x_{it}$ 's. Hence, a fixed effects rather than a random effects specification is appropriate. For this fixed effects tobit model, the model is given by (7), with  $v_{it} \sim \text{IIN}(0, \sigma_v^2)$  and

$$y_{it} = y_{it}^* \quad \text{if } y_{it}^* > 0 \quad (9)$$

$$= 0 \quad \text{otherwise.}$$

where  $y_{it}$  could be the expenditures on a car. This will be zero at time  $t$ , if the  $i$ -th individual does not buy a car. In the latter case all we know is that  $y_{it}^* \leq 0$ .<sup>8</sup> As in the fixed effects probit model, the  $\mu_i$ 's cannot be swept away and as a result  $\beta$  and  $\sigma_v^2$  cannot be estimated consistently for  $T$  fixed, since the inconsistency in the  $\mu_i$ 's is transmitted to  $\beta$  and  $\sigma_v^2$ . Heckman and MaCurdy (1980) suggest estimating the log-likelihood using iterative methods. Recently, Honoré (1992) suggested trimmed least absolute deviations and trimmed least squares estimators for truncated and censored regression models with fixed effects. These are semiparametric estimators with no distributional assumptions necessary on the error term. The main assumption is that the remainder error  $v_{it}$  is independent and identically distributed conditional on the  $x_{it}$ 's and the  $\mu_i$ 's, for  $t=1, \dots, T$ . Honoré (1992) exploits the symmetry in the distribution of the latent variables and finds that when the true values of the parameters are known, trimming can transmit the same symmetry in distribution to the observed variables. This generates orthogonality conditions which must hold at the true value of the parameters. Therefore, the resulting GMM estimator is consistent provided the orthogonality conditions are satisfied at a unique point in the parameter space. Honoré (1992) shows that these estimators are consistent and asymptotically normal. Monte Carlo results show that as long as  $N \geq 200$ , the asymptotic distribution is a good approximation of the small sample distribution. However, if  $N$  is small, the small sample distribution of these estimators is skewed. Honoré (1993) extends his analysis to the *dynamic* Tobit model with fixed effects, i.e.,

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<sup>8</sup>Researchers may also be interested in panel data economic relationships where the dependent variable is a count of some individual actions or events. For example, the number of patents filed, the number of drugs introduced or the number of jobs held. These models can be estimated using Poisson panel data regressions, see Hausman, Hall and Griliches (1984).



$$y_{it}^* = \delta y_{i,t-1} + x_{it}'\beta + \mu_i + v_{it} \quad (10)$$

with  $y_{it} = \max\{0, y_{it}^*\}$  for  $i = 1, \dots, N$ ;  $t = 1, \dots, T$ . The basic assumption is that  $v_{it}$  is IID(0,  $\sigma_v^2$ ) for  $t = 1, \dots, T$ , conditional on  $y_{i0}$ ,  $x_{it}$  and  $\mu_i$ . Honoré (1993) shows how to trim the observations from a dynamic Tobit model so that the symmetry conditions are preserved for the observed variables at the true values of the parameters. These symmetry restrictions are free of the individual effects and no assumption is needed on the distribution of the  $\mu_i$ 's or their relationship with the explanatory variables. These restrictions generate orthogonality conditions which are satisfied at the true value of the parameters. The orthogonality conditions can be used in turn to construct method of moments estimators. Using Monte Carlo experiments, Honoré (1993) shows that MLE for a dynamic Tobit model fixed effects performs poorly, whereas the GMM estimator performs quite well, when  $\delta$  is the only parameter of interest.

Recently, Keane (1994) derived a computationally practical simulation estimator for the panel data probit model. Simulation estimation methods replace intractable integrals by unbiased Monte-Carlo probability simulators. This is ideal for limited dependent variable models where for a multinomial probit model, the choice probabilities involve multivariate integrals.<sup>9</sup> In fact, for cross-section data, McFadden's method of simulated moments (MSM) involves an M-1 integration problem, where M is the number of possible choices facing the individual. For panel data, things get more complicated, because there are M choices facing any individual at each period. This means that there are  $M^T$  possible choice sequences facing each individual over the panel. Hence the MSM estimator becomes infeasible as T gets large. Keane (1994) side-steps this

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<sup>9</sup> For good surveys of simulation methods, see Hajivassiliou and Ruud (1994) for limited dependent variable models and Gourieroux and Monfort (1993) with special reference to panel data. The methods surveyed include simulation of the likelihood, simulation of the moment functions and simulation of the score.

problem of having to simulate  $M^T$  possible choice sequences by factorizing the method of simulated moments first order conditions into transition probabilities. The latter are simulated using highly accurate importance sampling techniques. This method of simulating probabilities is referred to as the Geweke, Hajivassiliou and Keane (GHK) simulator because it was independently developed by these authors. Keane (1994) performs Monte Carlo experiments and finds that even for large  $T$  and small simulation sizes, the bias in the MSM estimator is negligible. When maximum likelihood methods are feasible, Keane (1994) finds that the MSM estimator performs well relative to quadrature-based maximum likelihood methods even where the latter are based on a large number of quadrature points. When maximum likelihood methods are not feasible, the MSM estimator outperforms the simulated maximum likelihood estimator even when the highly accurate GHK probability simulator is used. Keane (1993) applies the MSM estimator to the same data set used by Keane, Moffitt and Runkle (1988) to study the cyclical behavior of real wages. He finds that the Keane, Moffitt and Runkle conclusion of a weakly procyclical movement in the real wage appears to be robust to relaxation of the equicorrelation assumption.

Heckman (1981a,b,c) emphasizes the importance of distinguishing between 'true state dependence' and 'spurious state dependence' in dynamic models of individual behavior. In the 'true' case, once an individual experiences an event, his preferences change and he or she will behave differently in the future as compared with an identical individual that has not experienced this event in the past. In the 'spurious' case, past experience has no effect on the probability of experiencing the event in the future. However, one cannot properly control for all the variables that distinguish one individual's decision from another. In this case, past experience which is a good proxy for these omitted variables shows up as a significant determinant of the future probability of occurrence of this event. Testing for true versus spurious state dependence is

therefore important in these studies, but it is complicated by the presence of the individual effects or heterogeneity. In fact, even if there is no state dependence,  $\Pr[y_{it}/x_{it}, y_{i,t-1}] \neq \Pr[y_{it}/x_{it}]$  as long as there are random individual effects present in the model. If in addition to the absence of the state dependence, there is also no heterogeneity, then  $\Pr[y_{it}/x_{it}, y_{i,t-1}] = \Pr[y_{it}/x_{it}]$ . A test for this equality can be based on a test for  $\gamma = 0$  in the model

$$\Pr[y_{it} = 1/x_{it}, y_{i,t-1}] = F(x_{it}'\beta + \gamma y_{i,t-1}) \quad (11)$$

using standard maximum likelihood techniques. If  $\gamma = 0$  is not rejected, we ignore the heterogeneity issue and proceed as in conventional limited dependent variable models not worrying about the panel data nature of the data. However, rejecting the null does not necessarily imply that there is heterogeneity since  $\gamma$  can be different from zero due to serial correlation in the remainder error or due to state dependence. In order to test for time dependence one has to condition on the individual effects, i.e., test  $\Pr[y_{it}/y_{i,t-1}, x_{it}, \mu_i] = \Pr[y_{it}/x_{it}, \mu_i]$ . This can be implemented following the work of Lee (1987) and Maddala (1987). In fact, if  $\gamma = 0$  is rejected, Hsiao (1996) suggests testing for time dependence against heterogeneity. If heterogeneity is rejected, the model is misspecified. If heterogeneity is not rejected then one estimates the model correcting for heterogeneity. See Heckman (1981c) for an application to married women's employment decisions based on a three-year sample from the Panel Study of Income Dynamics. One of the main findings of this study is that neglecting heterogeneity in dynamic models overstate the effect of past experience on labor market participation.

In many surveys, non-randomly missing data may occur due to a variety of self-selection rules. One such self-selection rule is the problem of non-response of the economic agent. Non-response occurs, for example, when the individual refuses to participate in the survey, or refuses to answer particular questions. This problem occurs in cross-section studies, but it gets

aggravated in panel surveys. After all, panel surveys are repeated cross-sectional interviews. So, in addition to the above kinds of non-response, one may encounter individuals that refuse to participate in subsequent interviews or simply move or die. Individuals leaving the survey cause attrition in the panel. This distorts the random design of the survey and questions the representativeness of the observed sample in drawing inference about the population we are studying. Inference based on the balanced sub-panel is inefficient even in randomly missing data since it is throwing away data. In non-randomly missing data, this inference is misleading because it is no longer representative of the population. Verbeek and Nijman (1996) survey the reasons for non-response and distinguish between 'ignorable' and 'non-ignorable' selection rules. This is important because, if the selection rule is ignorable for the parameters of interest, one can use the standard panel data methods for consistent estimation. If the selection rule is non-ignorable, then one has to take into account the mechanism that causes the missing observations in order to obtain consistent estimates of the parameters of interest.

We now consider a simple model of non-response in panel data. Following the work of Hausman and Wise (1979), Ridder (1990) and Verbeek and Nijman (1996), we assume that  $y_{it}$  given by equation (1) is observed if a latent variable  $r_{it}^* \geq 0$ . This latent variable is given by

$$r_{it}^* = z_{it}'\gamma + \epsilon_i + \eta_{it} \quad (12)$$

where  $z_{it}$  is a set of explanatory variables possibly including some of the  $x_{it}$ 's. The one-way error components structure allows for heterogeneity in the selection process. The errors are assumed to be normally distributed  $\epsilon_i \sim \text{IIN}(0, \sigma_\epsilon^2)$  and  $\eta_{it} \sim \text{IIN}(0, \sigma_\eta^2)$  with the only non-zero covariances being  $\text{cov}(\epsilon_i, \mu_i) = \sigma_{\mu\epsilon}$  and  $\text{cov}(\eta_{it}, v_{it}) = \sigma_{\eta v}$ . In order to get a consistent estimator for  $\beta$ , a generalization of Heckman's selectivity bias correction procedure from the cross-sectional to the panel data case can be employed. The conditional expectation of  $u_{it}$  given selection now involves

two terms. Therefore, instead of one selectivity bias correction term, there are now two terms corresponding to the two covariances  $\sigma_{\mu\epsilon}$  and  $\sigma_{\eta\nu}$ . However, unlike the cross-sectional case, these correction terms cannot be computed from simple probit regressions and require numerical integration. Fortunately, this is only a one-dimensional integration problem because of the error component structure. Once the correction terms are estimated, they are included in the regression equation as in the cross-sectional case and OLS or GLS can be run on the resulting augmented model. For details, see Verbeek and Nijman (1996) who also warn about heteroskedasticity and serial correlation in the second step regression if the selection rule is non-ignorable. Verbeek and Nijman (1996) also discuss MLE for this random effect probit model with selection bias.

Before one embarks on these complicated estimation procedures one should first test whether the selection rule is ignorable. Verbeek and Nijman (1992a) consider a Lagrange-Multiplier test for  $H_0: \sigma_{\nu\eta} = \sigma_{\mu\epsilon} = 0$ . The null hypothesis is a sufficient condition for the selection rule to be ignorable for the random effects model. Unfortunately, this also requires numerical integration over a maximum of two dimensions and is cumbersome to use in applied work. In addition, the LM test is highly dependent on the specification of the selectivity equation and the distributional assumptions. Alternatively, Verbeek and Nijman (1992a) suggest some simple Hausman-type tests based on GLS and within estimators for the unbalanced panel and the balanced sub-panel. All four estimators are consistent under the null hypothesis that the selection rule is ignorable and all four estimators are inconsistent under the alternative. In practice, Verbeek and Nijman (1992a) suggest including three simple variables in the regression to check for the presence of selection bias. These are (i) the number of waves the  $i$ -th individual participates in the panel, (ii) a binary variable taking the value 1 if and only if the  $i$ -th individual is observed over the entire sample, and finally, (iii) a binary variable indicating whether the

individual was present in the last period. Testing the significance of these variables is recommended as a check for selection bias. Intuitively, one is testing whether the pattern of missing observations affects the underlying regression. Wooldridge (1995) derives some simple variable addition tests of selection bias as well as easy-to-apply estimation techniques that correct for selection bias in linear fixed effects panel data models. The auxiliary regressors are either Tobit residuals or inverse Mill's ratios and the disturbances are allowed to be arbitrarily serially correlated and unconditionally heteroskedastic.

There are many empirical applications illustrating the effects of attrition bias, see Hausman and Wise (1979) for a study of the Gary Income Maintenance experiment. For this experimental panel study of labor supply response, the treatment effect is an income guarantee/tax rate combination. People who benefit from this experiment are more likely to remain in the sample. Therefore, the selection rule is non-ignorable, and attrition can overestimate the treatment effect on labor supply. For the Gary Income Maintenance Experiment, Hausman and Wise (1979) found little effect of attrition bias on the experimental labor supply response. Similar results were obtained by Robins and West (1986) for the Seattle and Denver Income Maintenance Experiments. For the latter sample, attrition was modest (11% for married men and 7% for married women and single heads during the period studied) and its effect was not serious enough to warrant extensive correction procedures. More recently, Keane, Moffitt and Runkle (1988) studied the movement of real wages over the business cycle for a panel data drawn from the National Longitudinal Survey of Young Men (NLS) over the period 1966 to 1981. They showed that self selection biased the behavior of real wage in a procyclical direction.

### **VIII. Further Readings**

Supplementary readings on panel data include Hsiao's (1986) Econometric Society monograph. This is the standard reference on the subject. Maddala's (1993) two volumes collecting some of the classic papers in the field. A special issue of *Empirical Economics* edited by Raj and Baltagi (1992) and a special issue of the *Journal of Econometrics* edited by Baltagi (1995a). Two recent books on panel data are Baltagi (1995b) and Mátyás and Sevestre (1996).

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