Dynamic Competition in Telecom Networks under the Receiver Pays Principle

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This paper analyzes dynamic network competition under the Receiver Pays Principle. We investigate how the implementation of the Receiver Pays Principle affects the networks operators' pricing strategies in a model of dynamic competition. We characterize the equilibrium and provide sufficient conditions under which it exists and is unique. In the region where the equilibrium exists we show that networks price calls at their off-net cost. We further show that, even in this dynamic setting, the off-net cost pricing equilibrium neutralizes the potential role that future reciprocal access charges could play as an instrument to soften retail competition. Last, we argue that while the level of the access price does not affect networks' profits, it clearly distorts consumer welfare.

Keywords: dynamic competition, interconnection, receiver pays principle, access pricing.

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1. INTRODUCTION

Since the introduction of competition in telecommunications, network interconnection has been one of the most controversial issues of telecoms regulation. The need for interconnection stems from the fact that networks need to connect their subscribers with those on other networks; indeed it is one of the keys, but not the only, to achieving effective competition in the market. This involves "two-way" access agreements whereby networks provide call origination, transit and termination services to each other. It then raises the non-trivial question of how accepting traffic from or delivering traffic to other networks should be priced.

"One-way" access refers to the case where an incumbent monopolizes the local network and must provide a bottleneck input to new entrants that compete with it in a downstream market. Since the incumbent could use the bottleneck to expel competitors from the market, there is a wide consensus in the literature that regulation is socially desirable. In the case of two-way access, however, networks operate at the same level of network hierarchy, that is, they do not only compete for subscribers in the retail market but do depend on each other to supply the retail service. Thus one may at first sight be induced to think that regulation is unnecessary. In practice, access charges are frequently set cooperatively, while cooperation over retail prices is in general considered to be illegal. Some might wonder whether networks could not agree on setting a specific access charge that softens competition in the retail market.

In order to develop an optimal policy it is key to determine whether unconstrained interconnection negotiations over access charges can undermine retail competition or on the contrary are socially optimal, in which case no regulation is needed. Indeed, this question has been studied by the seminal papers of Armstrong (1998) and Laffont, Rey and Tirole (1998a,b). Assuming symmetric networks, reciprocal access charges and linear retail pricing these papers show that competition in the retail market can be undermined by collusion over the access charge. This result stems from the fact that if a network lowers its retail price, then its subscribers will make more calls, which provokes an access deficit whenever the access charge is above the cost. Then, by agreeing to high access charges, networks reduce the incentive to undercut each other. More surprisingly, Laffont, Rey and Tirole (1998a) show that in the same setting but under two-part pricing, the collusive power of the access charge vanishes, that is, networks' equilibrium profits do not depend on the level of the access charge. This result comes about because of an intense waterbed effect. Intuitively, an increase in the access charge leads to an increase in the usage price, which makes it more desirable for networks to build market share. In the linear pricing case, networks cannot build market share without incurring an access deficit, but under two-part tariffs they can by lowering their fixed fees while keeping usage prices constant.

This waterbed effect occurs in the limit where networks find it worthwhile to spend the full revenue from access fees in order to attract subscribers. That is, higher usage prices are offset by lower fixed fees such that networks' profits remain independent of the reciprocal access charge.

This striking result has become the focus of much research,[2] and also has been proved to hold when customers are heterogeneous.[3] Indeed, this neutrality result depends crucially on three assumptions: full-participation, no termination-based price discrimination and symmetry.[4] Carter and Wright (2003) allow asymmetric networks by providing for brand loyalty and show that the incumbent strictly prefers the access charge to be set at marginal cost, and that both networks prefer cost-based access charges when there is a sufficient degree of asymmetry. Intuitively, the larger network or incumbent faces a higher proportion of intra-network calls, whereas the smaller network faces a higher proportion of inter-network calls. Then, since networks price calls at the *perceived* marginal cost, a reciprocal access charge above cost increases the *perceived* marginal cost of the smaller network (because of most of its calls are inter-network) and hence its call price also increases. This, consequently, implies that the larger network will face a net outflow of calls with an above-cost access charge and hence a deficit in the wholesale market. We show below how this last result partially explains our non-neutrality result in a dynamic model of competition even though networks are symmetric. To sum up, established telecoms networks under nonlinear pricing and no termination-based price discrimination cannot use reciprocal access charges as an instrument of collusion as long as there is full participation or an exogenous participation rate, and thereby unconstrained interconnection negotiations over reciprocal access charges might be a socially optimal policy.

So far we have only considered the literature that studies competition in a static model. What about dynamic competition? Does it alter our conclusion? De Bijl and Peitz (2000, 2002) study dynamic network competition but focusing only on myopic behaviour or, in other words, on the per-period profit maximizing equilibria. They study the asymmetric case and find in the short term a similar result to that of Carter and Wright (2003), and in the long term a result that is very close to neutrality.[5] Our previous work (López, 2005) however depicts

[2] See Armstrong (2002) and Vogelsang (2003) for a survey of this literature.

[3] Dessein (2003) and Hahn (2004) introduce heterogeneity in volume demand and shows that equilibrium profits are still independent of the reciprocal access charge under second-degree price discrimination. De Bijl and Peitz (2000, chpt. 7) allow for third-degree price discrimination and still find the same result.

[4] Firstly, Poletti and Wright (2004) by allowing customers' participation constraint to be binding in equilibrium show that access charges above cost can play a collusive role. In addition, Schiff (2002) show that under partial consumer participation and some other assumptions, as for instance an exogenous participation rate, networks prefer the access charge equal to the marginal cost, but when these assumptions are relaxed, networks instead prefer either cost-based or below-cost access prices depending on the case that is under consideration. Secondly, Gans and King (2001) show that networks prefer access charges below cost when they can price discriminate.

[5] It is worth to remark, however, that they make numerical analyses of a wide range of

the competition between two differentiated networks in a two-period model and under the subgame-perfect equilibrium concept. We show that even symmetric networks with full participation can use reciprocal access charges to soften competition when they compete in a dynamic setting. In particular, the networks' overall profits are neutral with respect to the first-period access charge but increase when the second-period access charge departs away from the marginal cost. A robust economic argument supports this result: in the second period the profits of the larger firm decrease when the access charge departs away from the marginal cost, which in turn decreases the incentives to fight for market share in the first period. This result holds both when consumer expectations are naive and when they are rational. Thus regulation might be needed in order to prevent anticompetitive behaviour since cost-based access charges maximize the full-period welfare surplus. Price controls of course is a draconian policy that regulators normally avoid if others alternatives are available, in particular because it is not clear whether regulation costs are lower than the potential benefits derived from price controls. A possible solution that avoids direct intervention in the market is moving towards a Receiver Party Pays system.

The Receiver Pays Principle (RPP) is already applied to mobile call pricing in U.S and some Asian countries, and it has also been widely adopted in international roaming, although in both cases for non-economic reasons.[6] An important economic argument that may support its implementation is the existence of call externalities, which occur because both callers and receivers may benefit from a phone call. In practice, RPP has been recently invoked as an instrument to reduce mobile termination charges.[7] Despite the spectacular growth of mobile telephony in recent years, mobile termination charges have remained high in Europe, where the Caller Pays Principle (CPP) applies. These high termination rates are from fixed to mobile calls, and have become a serious concern in most European countries; they do not only affect negatively the consumer welfare but are also perceived to be damaging the fixed telecoms sector's ability to innovate and invest in new technologies.[8] In this respect, some observers see RPP as a good alternative to price controls and predict that its implementation in the telecoms industry would exert downward pressure on mobile termination charges.[9]

interesting scenarios that are not considered here, as for instance the non-reciprocal access price case and the process of entry (De Bijl and Peitz, 2004.)

[6] In the former case it has been so mainly because of technological reasons: the access codes of the mobile service providers are not distinct to those of the fixed network.

[7] Those charges that mobile operators levy on each other and on fixed network operators for terminating calls on their networks.

[8] In France, Germany and the UK, the total transfer of funds for fixed to mobile calls (computing the excess of termination charges paid over costs and including a normal return on capital employed) from fixed networks to the mobile sector is estimated to be €19 billion between 1998 and 2002 (see Cave et al. 2003.)

[9] Intuitively, under the receiver pays regime, if a mobile network sets high termination charges it will decrease the utility of its own subscribers and so its attraction. Consequently, competition in reception charges should result in lower termination charges.

We are primarily interested in determining whether future reciprocal access charges can still soften first-period competition when networks compete in fixed fees, call prices and reception charges. Obviously, adopting RPP will significantly change the networks' incentives. This in turn makes important to develop a conceptual framework in which the resulting industry can be analyzed. We build on previous literature to propose such a framework, and aim to investigate how networks' pricing strategies react to the adoption of the receiver pays regime when they compete in a multi-period setting. Our starting point is that callers and call receivers derive utility from making and receiving calls. Moreover, networks are allowed to charge customers for receiving calls. The analysis faces the problem of *sovereignty:* who decides to end the call? It will be argued that in a deterministic framework allowing receivers to hang up makes the model discontinuous. We thus generalize this setup by assuming that both the caller's and receiver's utilities are subject to a random noise, the purpose of this is to smooth the demand, in fact this makes the model even more realistic.

The receiver pays principle has been studied under different settings (all of them focusing on static competition) by Berger (2001), Fabrizi (2005), Hermalin and Katz (2001, 2005), Kim and Lim (2001), Jeon, Laffont and Tirole (2004) and Laffont et al. (2003).[10] Nevertheless, the most related papers to the problem we study are the last two papers. Laffont et al. analyze Internet backbone competition. In their framework there are two types of customers: senders or websites and receivers or consumers. In our model however every consumers both send and receive traffic, and get surplus from and are charged for making and receiving calls. On the other hand, Jeon, Laffont and Tirole (2004) and our paper analyze three-part tariff competition in a telecommunications environment where the volume of traffic between each caller and receiver is endogenously determined by one of them though subject to a random noise. More specifically, Jeon, Laffont and Tirole assume that only the receiver's utility is subject to a noise and a certain proportionality between the receiver's and the caller's utilities. Our setup however generalizes their work by allowing a random noise in both the callers and receivers' utilities, and by removing the assumption of a given proportionality between the utility functions. Yet, the main contributions of our paper are that in this general setup we easily show that the off-net-cost pricing principle is a candidate equilibrium, and more importantly we prove that under general conditions the off-net-cost pricing equilibrium exists and is the unique possible equilibrium. Instead, Jeon, Laffont and Tirole (2004) establish only the existence (and not uniqueness) of the off-net-cost pricing equilibrium in a very specific case that will be commented later. Finally, we extend our results to a multi-period setting. In concrete, our main insights are as follows:

[10] For an overview of this literature see Jeon, Laffont and Tirole (2004).

Existence. Under linear demands, low enough substitutability between networks and a random noise with a wide enough support, there exists a unique equilibrium, which is interior and where networks choose the same call and reception prices over the time.

Pricing. In equilibrium, networks price calls at their off-net cost, whatever the sizes of the installed bases. Fixed fees and full-period profits are neutral with respect to the level of the *per-period* access markup.

Role of access charges. Should one ban unconstrained interconnection negotiations over reciprocal access charges? The off-net-cost pricing equilibrium neutralizes the potential anticompetitive role that future reciprocal access charges could play. In other words, under RPP networks cannot use access charges as an instrument to soften retail competition, whereas under CPP they can increase profits and decrease consumers surplus by setting future reciprocal access charges different from marginal costs.

Welfare. Should one set cost-bases access charges? As already noted, full-period profits do not depend on the level of the access markups, nevertheless an increase in the access markup raises the call price and decreases the reception charge. These two effects introduce a clear distortion in the consumer welfare. Given we have assumed a random noise in the utility functions, we look for the level of the access markup that maximizes the *expected* social welfare. We conclude that the optimal value of the access markup depends on the characteristics of each market in particular. Although, we can demonstrate that if the caller's and receiver's utility functions are identical, then starting from zero access markup, a decrease in the access charge raises the expected social welfare. Indeed, we show that 'bill and keep' might be optimal in this situation.

The article is organized as follows. Section 2 presents the main insights of our previous work. Section 3 describes the model and makes the main assumptions. Section 4 analyzes the two-period game, characterizes the equilibrium, studies the role of the access charge and extends the basic model to a multi-period setting. Section 5 investigates how the access charge affects the social welfare and studies its optimal level. Section 6 summarizes the main insights.

2. HOW TO SOFTEN NETWORK COMPETITION UNDER THE CPP

To provide a motivation for our analysis, it is convenient to introduce briefly the main insights of our previous work, where CPP is assumed throughout. In particular, we show below that under CPP networks can soften retail competition by setting access charges different from marginal costs. To that end, let $\widehat{\pi}_2^i$ denote the equilibrium second-period profits, which depends on the second-period access markup m_2, and the network $i's$ first-period market share α_1^i provided that switching costs exist. In the second period the model is similar to the traditional static model in which the symmetric equilibrium profits are neutral with respect to the access markup[11]. Moreover, the equilibrium second-period call price p_2^i is equal to the cost of an average call originating on network i, that is,

$$p_2^i = c + \alpha_2^j m_2,$$

where c is the industry's marginal cost of a call. Recall that in the first period, each network sets prices taking into account its first-period profitability, but also the effect that its first-period market share will have on its second-period profits. In particular, network i chooses the first-period call price p_1^i and the first-period fixed fee F_1^i so as to maximize its total discounted profits, taking network $j's$ first-period call price and fixed fee as given. As already pointed out in Laffont et al. (1998a), it is analytically convenient to view network competition as one in which the networks pick usage fees and net surpluses rather than usage fees and fixed fees, so that market shares are determined directly by net surpluses. The net surplus that a network $i's$ subscriber derives in the first period is: $w_1^i \equiv v(p_1^i) - F_1^i$, where $v(p_1^i)$ is the subscriber's indirect utility function who faces a call price of p_1^i. Thus network i solves:

$$\max_{p_1^i, w_1^i} \Pi \equiv \pi_1^i(p_1^i, p_1^j, w_1^i, w_1^j) + \delta \widehat{\pi}_2^i(m_2, \alpha_1^i(w_1^i, w_1^j)),$$

where π_1^i denotes the network $i's$ first-period profits and δ the discount factor. The first-order condition with respect to p_1^i yields $p_1^i = c + \alpha_1^j m_1$, that is, networks choose their call prices in the same way as they do in the second period. Further, the first-order condition with respect to w_1^i is

$$0 = \frac{\partial \pi_1^i}{\partial w_1^i} + \delta \frac{\partial \widehat{\pi}_2^i}{\partial \alpha_1^i}(m_2, \alpha_1^i) \frac{\partial \alpha_1^i}{\partial w_1^i}. \tag{1}$$

That is, the equilibrium first-period fixed fees are given as a function of m_2 through the term $\partial \widehat{\pi}_2^i / \partial \alpha_1^i$. In addition, we can show that in a symmetric equilibrium the full-period profits are equal to

$$\Pi(m_2) = \frac{1+\delta}{4\sigma} - \frac{\delta}{2} \frac{\partial \widehat{\pi}_2^i}{\partial \alpha_1^i}(m_2, 1/2),$$

where σ is the degree of substitutability between the two networks. Moreover, $\partial \widehat{\pi}_2^i / \partial \alpha_1^i(0, 1/2) > 0$, thus so as to satisfy (1) it must hold that $\partial \pi_1^i / \partial w_1^i < 0$, that is, in the neighborhood of $m_2 = 0$, networks compete more aggressively in the first period than they would do in a market without switching costs. More

[11] Indeed, the symmetric equilibrium profits are equal to the profits that networks would obtain under unit demands.

importantly,

$$\frac{\partial \widehat{\pi}_2^i}{\partial m_2 \partial \alpha_1^i}(0, 1/2) = 0, \qquad \frac{\partial}{\partial m_2}\left(\frac{\partial \widehat{\pi}_2^i}{\partial m_2 \partial \alpha_1^i}(0, 1/2)\right) < 0.$$

That is, slightly moving m_2 away from zero reduces the value of having a higher market share in the second period: $\partial \widehat{\pi}_2^i / \partial m_2 \partial \alpha_1^i$; this in turn softens competition for market share in the first period and increases full-period profits. An explanation for this result can be found in the Proposition 1 of Carter and Wright (2003), which proves that the profits of the larger network decrease when the access charge departs away from the marginal cost. Intuitively, as higher or lower is the second-period access markup with respect to the marginal costs, lower the second-period profits for the larger firm will be, and consequently the competition for first-period market share is disincentived. Notice that equilibrium first-period profits are independent of the reciprocal access charges, thus m_1 does not have to be different from m_2 to undermine network competition, that is, $m_1 = m_2 = m \neq c_0$ will also increase networks profits and decrease consumer welfare. The analysis below shows that under RPP networks can no longer increase full-period profits by colluding over the level of the future reciprocal access charges. Indeed, under RPP: $\partial \widehat{\pi}_2^i / \partial m_2 \partial \alpha_1^i = 0 \ \forall m_2, \alpha_1^i$; in effect, competition in call prices, fixed fees and reception prices neutralizes the effects that access charges have on equilibrium full-period profits.

3. THE MODEL

There are two networks indexed by i and j. Each has its own full coverage network and competes for a consumer set of measure 1. It is assumed that every consumer joins one of the networks, that is, there is full participation. In addition, networks are assumed to be interconnected, therefore a consumer who subscribes to one network can call any other consumer on either network. The usual balanced-traffic assumption is maintained throughout the analysis, which implies that the percentage of calls originating on a network and completed on the same network is equal to the market share of this network. Networks compete in nonlinear prices, and offer a three-part tariff: $\{F^i, p^i, r^i\}$, where F^i denotes network $i's$ fixed fee, and p^i and r^i represent respectively the per-unit call and reception charge. For off-net calls, the originating network must pay a reciprocal access charge a per unit of termination to the terminating network.[12] Moreover, networks are not allowed to price discriminate between calls that terminate on- and off-net.

Cost structure. Symmetric costs are assumed for simplicity. The cost of

[12] Reciprocity means that a network pays as much for termination of a call on the rival network as it receives for completing a call originated on the rival network.

serving a customer is $f \geq 0$, which reflects the cost of connecting the customer's home to the network and of billing and servicing the customer. The marginal cost of terminating or originating a call is denoted by c_0, and the marginal trunk cost of a call by c_1. The total cost is c. The access mark-up is thus

$$m \equiv a - c_0.$$

Demand structure. Networks sell a differentiated but substitutable product, they are differentiated à la Hotelling. Consumers are uniformly located on the segment $[0, 1]$ and the two networks are located at the two ends of the interval. Thus, consumers' tastes for networks are represented by their position on the segment and taken into account through the transportation costs τ. Given income y a consumer located at x and joining network i has utility

$$y + v_0 - \tau |x - x_i| + w^i,$$

where v_0 represents a fixed surplus from being connected to either network,[13] $\tau |x - x_i|$ is the cost of subscribing to a network located at x_i, and w^i is the net surplus of a network i subscriber from making and receiving calls on that network.

Timing. We consider three stages. In the first stage or period zero, reciprocal access charges are set by a regulator or negotiated between carriers; a flexible regulation is allowed, so that access charges may differ over time. In the first and second periods, which are indexed by $t \in \{1, 2\}$, networks compete in retail prices, taking as given the access charges.

Dynamics. Every customer incurs a cost $s > 0$ when switching networks.[14] Note that if $s > \tau$ every consumer remains with the same network in a symmetric equilibrium. We assume instead that $s < \tau$, so that at least some consumers switch. In addition, we shall make the following two assumptions:

A.1. *Preferences are independent across periods.*

A.2. *Consumers have naive expectations.*

The first assumption only says that preferences may change over time.[15] On the other hand, A.2. imposes a strong condition on the consumers' behavior.

[13] y and v_0 are assumend to be large enough such that the full participation assumption is satisfied.

[14] Quite obviously, in the absence of switching costs, networks are per-period profit maximizing. There is however much evidence suggesting that switching costs are significant (see for instance De Bijl and Peitz, 2000)

[15] This case might also arise when the customers are different in different periods and second-period customers are exposed to the choice of first-period customers. Actually, asumming constant preferences over time introduces technical problems when the Hotelling model is used: for some variations in prices, market shares remain constant.

It will however be argued that rational consumer expectations would not affect the main insights of the paper. From now on and without any loss of generality assume network i is located at the beginning of the segment $[0,1]$ and network j at the end. Then, a consumer located at $x=\alpha_1$ is indifferent between the two networks in the first period if and only if

$$w_1^i - \tau\alpha_1 = w_1^j - \tau(1-\alpha_1).$$

Therefore, the network $i's$ market share is

$$\alpha_1^i = \frac{1}{2} + \sigma\left(w_1^i - w_1^j\right),$$

where $\sigma = 1/2\tau$ is the index of substitutability between the two networks. At the beginning of the second period there is a fraction α_1^i of consumers initially attached to network i. For these and given A.1 and A.2, a consumer located at $x \in [0,1]$ will remain associated with network i if $w_2^i - \tau x \geq w_2^j - \tau(1-x) - s$. A consumer initially attached to network j, say x, will instead switch to network i if $w_2^i - \tau x - s \geq w_2^j - \tau(1-x)$. Therefore, the network $i's$ second-period market share is

$$\begin{aligned} \alpha_2^i &= \alpha_1^i \left[\frac{1}{2} + \sigma\left(w_2^i - w_2^j + s\right)\right] + \alpha_1^j \left[\frac{1}{2} + \sigma\left(w_2^i - w_2^j - s\right)\right] \\ &= \frac{1}{2} + (2\alpha_1^i - 1)\sigma s + \sigma\left(w_2^i - w_2^j\right). \end{aligned} \tag{2}$$

Finally, networks have rational expectations and discount second-period revenues and costs by a factor δ.

Demand for traffic. Subscribers derive a surplus from making and receiving calls. The utility from placing q calls is denoted by $\mu(q)$, whereas the utility from receiving $\widetilde{q}$ calls is denoted by $\widetilde{\mu}(\widetilde{q})$; we assume that these utility functions are twice continuously differentiable, with $\mu' > 0$, $\mu'' < 0$, $\widetilde{\mu}' > 0$, and $\widetilde{\mu}'' < 0$.[16] The analysis faces the problem of *sovereignty:* who decides to end the call? The receiver's demand function $\widetilde{q}(r)$ is given by $\widetilde{\mu}'(\widetilde{q}) = r$, whereas the caller's demand function $q(p)$ is given by $\mu'(q) = p$; When receivers are allowed to hang up the volume of calls from network i to network j is thus given by $Q(p^i, r^j) = \min\{q(p^i), \widetilde{q}(r^j)\}$; In a deterministic framework, this makes the model discontinuous and complicates its analysis.[17] In order to get around this

[16] Throughout this paper the apostrophe symbol means the first derivative of the considered function with respect to its argument. In this case for instance $\mu' = d\mu/dq$ and $\mu'' = d^2\mu/(dq)^2$.

[17] When reception charges are regulated or contractually determined before networks compete in retail tariffs, an assumption that simplifies much the analysis is that the caller determines the volume of calls. However, when reception charges are set by the networks at the same time they chose call prices and fixed fees, this assumption introduces a potential problem in the analysis due to the *multiplicity of equilibria:* from the viewpoint of networks and subscribers, only the sum $\{F^i + r^i\widetilde{q}\}$ matters, not its composition; hence different combinations of F^i and

problem we assume that both the caller's and receiver's utilities are subject to a random noise, which smooths the demand. To that end, let ε and $\widetilde{\varepsilon}$ denote, respectively, the random term of the caller's and receiver's utilities, and assume that: i) they follow respectively the distribution functions $F(\cdot)$ and $\widetilde{F}(\cdot)$ with supports $[\underline{\varepsilon}, \overline{\varepsilon}]$ and $[\underline{\widetilde{\varepsilon}}, \overline{\widetilde{\varepsilon}}]$ where $\overline{\varepsilon} - \underline{\varepsilon} > 0$ and $\overline{\widetilde{\varepsilon}} - \underline{\widetilde{\varepsilon}} > 0$, and strictly positive density functions $f(\cdot)$ and $\widetilde{f}(\cdot)$; ii) they are identically and independently distributed for each caller-receiver pair. We then make the following assumption:

A.3. *The caller's utility is given by:* $u = \mu(q) + \varepsilon q$, *whereas the receiver's utility is given by:* $\widetilde{u} = \widetilde{\mu}(\widetilde{q}) + \widetilde{\varepsilon}\widetilde{q}$.

Assumption A.3. allows the willingness to stay on the phone to be state-contingent for both callers and receivers. In addition, demands q and $\widetilde{q}$ are assumed to be bounded, hence for a given $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ there exist price levels $\underline{p}$ and $\overline{p}$ such that if $p \leq \underline{p}$ then $q = \overline{q}$, where $0 < \overline{q} < \infty$, and if $p \geq \overline{p}$ then $q = 0$. Similarly, for a given $\widetilde{\varepsilon} \in [\underline{\widetilde{\varepsilon}}, \overline{\widetilde{\varepsilon}}]$ there exist price levels $\underline{r}$ and $\overline{r}$ such that if $r \leq \underline{r}$ then $\widetilde{q} = \overline{\widetilde{q}}$, where $0 < \overline{\widetilde{q}} < \infty$, and if $r \geq \overline{r}$ then $\widetilde{q} = 0$. Therefore, $p^i \in [\underline{p}, \overline{p}]$, $r^i \in [\underline{r}, \overline{r}]$, and since $\alpha^i \in [0,1]$ the networks' profit functions are also bounded.

4. ANALYSIS

Under A.3., and for a given pair of prices (p_t^i, r_t^j), the *expected* volume of calls from a network i subscriber to a network j subscriber at period t is given by:[18]

$$\begin{aligned} Q(p_t^i, r_t^j) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\underline{\widetilde{\varepsilon}}}^{\overline{\widetilde{\varepsilon}}} [q(p_t^i, \varepsilon)\Im_{\text{if } q(p_t^i,\varepsilon) \leq \widetilde{q}(r_t^j,\widetilde{\varepsilon})} \\ &\quad + \widetilde{q}(r_t^j, \widetilde{\varepsilon})\Im_{\text{if } q(p_t^i,\varepsilon) > \widetilde{q}(r_t^j,\widetilde{\varepsilon})}] f(\varepsilon)\widetilde{f}(\widetilde{\varepsilon}) d\varepsilon d\widetilde{\varepsilon}. \end{aligned}$$

Further, the *expected* utility that a network i subscriber derives from calling a network j subscriber at period t is

$$\begin{aligned} U(p_t^i, r_t^j) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\underline{\widetilde{\varepsilon}}}^{\overline{\widetilde{\varepsilon}}} [u(q(p_t^i, \varepsilon))\Im_{\text{if } q(p_t^i,\varepsilon) \leq \widetilde{q}(r_t^j,\widetilde{\varepsilon})} \\ &\quad + u(\widetilde{q}(r_t^j, \widetilde{\varepsilon}))\Im_{\text{if } q(p_t^i,\varepsilon) > \widetilde{q}(r_t^j,\widetilde{\varepsilon})}] f(\varepsilon)\widetilde{f}(\widetilde{\varepsilon}) d\varepsilon d\widetilde{\varepsilon}, \end{aligned}$$

r^i are feasible equilibria but nonequivalent since each combination may affect differently the rival network.

[18] Throughout the analysis the symbol $\Im_{if}$ means that the double integral of the term that is located at its left side is defined if and only if the condition that is located at its right side is satisfied.

while the *expected* utility that a subscriber from network j derives from receiving calls from a network i subscriber at period t is given by:

$$\begin{aligned}\widetilde{U}(p_t^i, r_t^j) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\underline{\widetilde{\varepsilon}}_r}^{\overline{\widetilde{\varepsilon}}} [\widetilde{u}(q(p_t^i, \varepsilon))\Im_{\text{if } q(p_t^i,\varepsilon)\leq \widetilde{q}(r_t^j,\widetilde{\varepsilon})} \\ &\quad + \widetilde{u}(\widetilde{q}(r_t^j, \widetilde{\varepsilon}))\Im_{\text{if } q(p_t^i,\varepsilon) > \widetilde{q}(r_t^j,\widetilde{\varepsilon})}] f(\varepsilon)\widetilde{f}(\widetilde{\varepsilon})d\varepsilon d\widetilde{\varepsilon}.\end{aligned}$$

Therefore, the volume of traffic from network i to network j depends on two usage prices and is sometimes determined by the caller and at other times by the receiver. In this framework we still find the following standard results:

$$\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} = \int_{\underline{\widetilde{\varepsilon}}}^{\overline{\widetilde{\varepsilon}}} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{\partial q(p_t^i - \varepsilon)}{\partial p_t^i} \Im_{\text{if } q(p_t^i,\varepsilon)\leq \widetilde{q}(r_t^j,\widetilde{\varepsilon})} f(\varepsilon)\widetilde{f}(\widetilde{\varepsilon})d\varepsilon d\widetilde{\varepsilon},$$

$$\frac{\partial U(p_t^i, r_t^j)}{\partial p_t^i} = p_t^i \frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i}, \tag{3}$$

where we have used $\partial u(q(p_t^i - \varepsilon))/\partial q = \mu'(\mu'^{-1}(p_t^i - \varepsilon)) + \varepsilon = p_t^i$. And,

$$\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} = \int_{\underline{\widetilde{\varepsilon}}}^{\overline{\widetilde{\varepsilon}}} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{\partial \widetilde{q}(r_t^i - \widetilde{\varepsilon})}{\partial r_t^i} \Im_{\text{if } q(p_t^j,\varepsilon)\leq \widetilde{q}(r_t^i,\widetilde{\varepsilon})} f(\varepsilon)\widetilde{f}(\widetilde{\varepsilon})d\widetilde{\varepsilon} d\varepsilon,$$

$$\frac{\partial \widetilde{U}(p_t^j, r_t^i)}{\partial r_t^i} = r_t^i \frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i}, \tag{4}$$

where we have used $\partial \widetilde{u}(\widetilde{q}(r_t^i - \widetilde{\varepsilon}))/\partial \widetilde{q} = \widetilde{\mu}'(\widetilde{\mu}'^{-1}(r_t^i - \widetilde{\varepsilon})) + \widetilde{\varepsilon} = r_t^i$. For the sake of the presentation, we will write $Q_t^{ij} = Q(p_t^j, r_t^j)$, $U_t^{ij} = U(p_t^i, r_t^j)$ and $\widetilde{U}_t^{ij} = \widetilde{U}(p_t^i, r_t^j)$ $\forall i, j$. Recall that network $i's$ second-period market share is

$$\alpha_2^i = \frac{1}{2} + (2\alpha_1^i - 1)\sigma s + \sigma(w_2^i - w_2^j), \tag{5}$$

where the *expected* net surplus of a network i consumer at period t is defined as

$$w_t^i = \phi_t^i - F_t^i, \tag{6}$$

with

$$\begin{aligned}\phi_t^i(\alpha_t^i, p_t^i, p_t^j, r_t^i, r_t^j) &= \alpha_t^i U_t^{ii} + \alpha_t^j U_t^{ij} + \alpha_t^i \widetilde{U}_t^{ii} + \alpha_t^j \widetilde{U}_t^{ji} \\ &\quad - p_t^i \left(\alpha_t^i Q_t^{ii} + \alpha_t^j Q_t^{ij}\right) - r_t^i \left(\alpha_t^i Q_t^{ii} + \alpha_t^j Q_t^{ji}\right).\end{aligned} \tag{7}$$

4.1. THE SECOND-PERIOD CASE

In the second period networks maximize profits with respect to call prices, reception charges and fixed fees; thus, any network i solves:

$$\begin{aligned}\max_{p_2^i, r_2^i, F_2^i} \pi_2^i \quad \equiv \quad & \alpha_2^i\{\alpha_2^i(p_2^i - c)Q_2^{ii} + \alpha_2^j(p_2^i - c - m_2)Q_2^{ij} + \alpha_2^j m_2 Q_2^{ji} \\ & + r_2^i(\alpha_2^i Q_2^{ii} + \alpha_2^j Q_2^{ji}) + F_2^i - f\}.\end{aligned} \tag{8}$$

We can solve (8) by maximizing it with respect to p_2^i and r_2^i for a given α_2^i, adapting fixed fees so that market shares remain constant. For this to hold, net surpluses must satisfy $w_2^i - w_2^j = (1/\sigma)(\alpha_2^i - 1/2) - (2\alpha_1^i - 1)s$; using (6) it follows that the fixed fee must be equal to

$$F_2^i = \phi_2^i - \phi_2^j + F_2^j - \frac{1}{\sigma}\left(\alpha_2^i - \frac{1}{2}\right) + (2\alpha_1^i - 1)s.$$

By substituting this last expression into the profit function we have:

$$\begin{aligned}\overline{\pi}_2^i(p_2^i, r_2^i) \quad = \quad & \alpha_2^i\{\alpha_2^i(p_2^i - c)Q_2^{ii} + \alpha_2^j(p_2^i - c - m_2)Q_2^{ij} + \alpha_2^j m_2 Q_2^{ji} \\ & + r_2^i(\alpha_2^i Q_2^{ii} + \alpha_2^j Q_2^{ji}) + \phi_2^i - \phi_2^j + F_2^j - \frac{1}{\sigma}\left(\alpha_2^i - \frac{1}{2}\right) \\ & + (2\alpha_1^i - 1)s - f\}.\end{aligned} \tag{9}$$

For given $r_2^i = r_2^j = r_2$ and p_2^j, the call price p_2^i determines the volume of calls generated by network i when callers are sovereign on average, network i incurs a unit cost $c + \alpha_2^j m_2$ from delivering these calls to network i and network j. However, since the call price affects subscribers' net surplus as well, fixed fees must be adapted in order to maintain markets shares constant; more precisely, a decrease in the call price p_2^i :

- affects network $i's$ revenue, but at the expense of consumers; hence to keep market shares constant fixed fees must be adapted so as to neutralize this transfer.
- allows network i to increase its fixed fee by $U(p_2^i, r_2)$, which is the utility that network $i's$ subscribers obtain from making calls (call prices also affect the utility from receiving calls but they affect both networks' consumers in the same way, so that fixed fees do not need to be adapted to maintain market shares.)
- affects the quantity of money that network $j's$ subscribers pay for calls received from network i : this effect is called *pecuniary externality* in Jeon et al. (2004), and allows network i to increase its fixed fee by $r_2\alpha_2^i$ and keep market shares constant.

We can summarize in the following expression the terms that are affected by the level of the network $i's$ call price, when adjusting the fixed fee so as to maintain market shares constant:

$$\alpha_2^i\{[-(c+\alpha_2^j m_2)+r_2\alpha_2^i]Q(p_2^i,r_2)+U(p_2^i,r_2)\}. \tag{10}$$

For given $p_2^i = p_2^j = p_2$ and r_2^j, setting the reception charge r_2^i similarly determines the volume of calls generated by network i when receivers are sovereign. For this volume of calls, network i incurs a cost $\alpha_2^i c$, but earns $\alpha_2^j m_2$ again from off-net calls. The reception charge r_2^i also affects subscribers' net surpluses, which requires fixed fees to be adapted so as to maintain market shares constant:

- First, network i gains revenue from reception charges, but its fixed fee must be altered by the same amount to keep market shares constant.
- Keeping market shares constant, network i can increase its fixed fee to reflect the utility obtained from receiving calls: $\widetilde{U}(p_2, r_2^i)$ (reception charges affect similarly both networks' subscribers for the calls they place to network $i's$ subscribers: $\alpha_2^i U(p_2, r_2^i)$; this therefore does not require fixed fees to be adapted).
- Finally, we find a new sort of *pecuniary externality:* the reception charge r_2^i determines how much network $j's$ consumers must pay for the calls they make to network $i's$ consumers. This externality allows network i to increase its fixed fee by $p_2\alpha_2^i$ while keeping market shares constant.

The following expression summarizes the terms that are affected by the level of network $i's$ reception charge, when adjusting the fixed fee so as to maintain market shares constant:

$$\alpha_2^i\{[-\alpha_2^i c+\alpha_2^j m_2+p_2\alpha_2^i]Q(p_2,r_2^i)+\widetilde{U}(p_2,r_2^i)\}. \tag{11}$$

By differentiating (10) with respect to p_2^i and (11) with respect to r_2^i, and using (3) and (4) we obtain the first-order conditions:

$$p_2^i = c+\alpha_2^j m_2-\alpha_2^i r_2, \tag{12}$$

$$r_2^i = \alpha_2^i c-\alpha_2^j m_2-\alpha_2^i p_2. \tag{13}$$

Essentially, we see that networks price calls and call receptions at their *strategic marginal cost:*[19] network $i's$ equilibrium call prices are equal to the average unit cost of a call originating on network i, minus the pecuniary externality imposed on network $j's$ subscribers; likewise, network $i's$ equilibrium reception charges are equal to the average cost of receiving calls on network i, minus the pecuniary externality imposed on network $j's$ consumers. Using $p_2^i = p_2$ and $r_2^i = r_2$, we

[19] Using the terminology of Jeon et al. (2004).

obtain the equilibrium call and reception prices:

$$p_2 = c + m_2, \tag{14}$$

$$r_2 = -m_2. \tag{15}$$

We shall emphasize that this symmetric solution is valid for any given level of market shares: hence (14) and (15) characterize the equilibrium second-period usage prices, which are symmetric whatever the sizes of customer installed bases. Now, setting call and reception prices at the equilibrium level, we can rewrite network $i's$ second-period profits as follows:

$$\pi_2^i = \left(\frac{1}{2} + (2\alpha_1^i - 1)\sigma s - \sigma(F_2^i - F_2^j) \right) (F_2^i - f). \tag{16}$$

By differentiating this last expression with respect to F_2^i we obtain the following first-order condition:

$$F_2^i = \frac{1}{2} \left[f + \frac{1}{2\sigma} + (2\alpha_1^i - 1)s + F_2^j \right]. \tag{17}$$

Similarly, we can obtain network $j's$ first-order condition with respect to its fixed fee, and by solving that system of two equations we obtain the equilibrium second-period fixed fees as a function of the first-period market shares:

$$\widehat{F}_2^i(\alpha_1^i) = f + \frac{1}{2\sigma} + \frac{(2\alpha_1^i - 1)s}{3}. \tag{18}$$

By substituting $\widehat{F}_2^i$ and $\widehat{F}_2^j$ into (16), we then obtain the equilibrium second-period profits as a function of first-period market shares:

$$\widehat{\pi}_2^i(\alpha_1^i) = \frac{1}{4\sigma} + (2\alpha_1^i - 1)\frac{s}{3} + (2\alpha_1^i - 1)^2 \frac{\sigma s^2}{9}. \tag{19}$$

Notice that equilibrium second-period profits do not depend on m_2. Moreover, note from (19) that if $\alpha_1^i = 1/2$ the equilibrium second-period profits are equal to the profits that networks would obtain under unit demands, that is, $\pi_2^i(1/2) = 1/4\sigma$. In order to prove the existence and uniqueness of this equilibrium we will have to be more specific about the noise and the caller's and receiver's demand. We then make the following assumption:

A.4. $\mu(q) = aq - (b/2)q^2$ and $\widetilde{\mu}(\widetilde{q}) = d\widetilde{q} - (e/2)\widetilde{q}^2$, where $a, b, d, e > 0$. Moreover, $\varepsilon, \widetilde{\varepsilon} \in [\underline{\varepsilon}, \overline{\varepsilon}]$, where $\underline{\varepsilon} < 0 < \overline{\varepsilon}$, $E(\varepsilon) = E(\widetilde{\varepsilon}) = 0$, and both random terms follow a uniform distribution with density function: $f(\varepsilon) = \widetilde{f}(\widetilde{\varepsilon}) = 1/\Delta$, where $\Delta = \overline{\varepsilon} - \underline{\varepsilon}$.

Notice that A.3. and A.4. implies linear demand functions: $q = (a - p + \varepsilon)/b$

and $\widetilde{q} = (d - r + \widetilde{\varepsilon})/e$. Then we have the following proposition:

PROPOSITION 1. *(Existence and Uniqueness) Under A.1, A.2, A.3 and A.4, for a small enough σ and a large enough Δ there exists a unique second-period equilibrium, which is interior and where networks choose:*

$$p_2^i = c + m_2,$$

$$r_2^i = -m_2,$$

$$F_2^i = f + \frac{1}{2\sigma} + \frac{(2\alpha_1^i - 1)s}{3}.$$

Proof. See Appendix. ∎

In summary, networks price calls at their off-net cost, that is, each network sets prices for making and receiving calls equal to the marginal cost that it could incur if all other subscribers belonged to the rival network. The off-net-cost pricing principle dates back to Laffont et al. (2003), which found this pricing rule in a framework for Internet backbone competition. In contrast, Jeon, Laffont and Tirole (2004) and our paper analyze three-part tariff competition in a telecommunications environment. At the expense of assuming linear demands, our setup however generalizes their work by allowing a random noise in both the callers and receivers' utilities, and by removing the assumption of a given proportionality between the utility functions. Moreover, Jeon, Laffont and Tirole only establish the existence of the off-net-cost pricing equilibrium when the noise on the receiver side converges to zero, so that the volume is determined by callers with probability converging to one. Instead, we have showed that the off-net-cost pricing equilibrium exists and is unique for a small enough σ and a large enough Δ. Indeed, a small (enough) σ (i.e., networks are relatively poor substitutes) is a standard assumption in the "two-way" access literature; and a large (enough) Δ is not a too restrictive assumption since extreme situations might happen in reality. For example, there exist many situations in which a person may not want to receive or make a call even though it is free. Finally, it is worth to remark that in the second period networks do not have incentives to corner the market by choosing a strategy different to that of the off-net-cost pricing one.

4.2. THE FIRST PERIOD

Recall that networks are assumed to be initially symmetric; thus, first-period market shares are given by

$$\alpha_1^i = \frac{1}{2} + \sigma(w_1^i - w_1^j),$$

where $w_1^i = \phi_1^i - F_1^i$. In the first period, network i chooses first-period usage price, reception charge and net surplus in order to maximize its total discounted profits:

$$\Pi^i(p_1^i, r_1^i, F_1^i) = \pi_1^i(p_1^i, r_1^i, F_1^i) + \delta\widehat{\pi}_2^i(\alpha_1^i(p_1^i, r_1^i, F_1^i)), \tag{20}$$

$$\begin{aligned}\text{with } \pi_1^i \quad = \quad & \alpha_1^i\{\alpha_1^i(p_1^i - c)Q_1^{ii} + \alpha_1^j\left[p_1^i - c - m_1\right]Q_1^{ij} + \alpha_1^j m_1 Q_1^{ji} \\ & + r_1^i\left[\alpha_1^i Q_1^{ii} + \alpha_1^j Q_1^{ji}\right] + F_1^i - f\},\end{aligned}$$

and where $\widehat{\pi}_2^i$ is given by (19). As above, we can maximize first Π^i with respect to p_1^i and r_1^i for a given α_1^i, adjusting fixed fees so as to keep α_1^i constant. Then, $\partial\Pi^i/\partial p_1^i = \partial\pi_1^i/\partial p_1^i$ and $\partial\Pi^i/\partial r_1^i = \partial\pi_1^i/\partial r_1^i$; therefore, networks choose their retail prices and reception charges in the same way as they do in the second period, that is, $p_1^i = c + m_1$ and $r_1^i = -m_1$. Now, we may proceed similarly to the analysis of the previous section, assume that first-period call and reception prices are at the equilibrium level, and rewrite the full-period profits as follows:

$$\Pi^i = \left(\frac{1}{2} - \sigma(F_1^i - F_1^j)\right)\{F_1^i - f\} + \delta\widehat{\pi}_2^i(\alpha_1^i(F_1^i, F_1^j)). \tag{21}$$

By differentiating this last expression with respect to F_1^i we obtain the following first-order condition:

$$0 = -\sigma\{F_1^i - f\} + \left(\frac{1}{2} - \sigma(F_1^i - F_1^j)\right) - \delta\sigma\frac{d\widehat{\pi}_2^i(\alpha_1^i)}{d\alpha_1^i}. \tag{22}$$

From (19) we have that $d\widehat{\pi}_2^i(\alpha_1^i)/d\alpha_1^i = (2s/3) + (4/9)(\alpha_1^i - \alpha_1^j)\sigma s^2$, therefore:

$$F_1^i = \frac{f}{2} + \frac{1}{4\sigma} + \frac{F_1^j}{2} - \frac{\delta s}{3} - \frac{4\sigma^2 s^2\delta}{9}(F_1^j - F_1^i). \tag{23}$$

Given the symmetry of the game in the first period, we may look for a symmetric solution where $F_1^i = F_1^j$, then it is easy to see from (23) that in equilibrium network i chooses:

$$F_1^i = f + \frac{1}{2\sigma} - \frac{2\delta s}{3}. \tag{24}$$

The following proposition gives the conditions for the existence and uniqueness of the first-period equilibrium:

Proposition 2. *Under A.1, A.2, A.3, and A.4, for a small enough σ and a large enough Δ : i) there exists a unique interior equilibrium where networks choose their first-period call and reception prices in the same way as they do in the second period:*

$$p_1 = c + m_1, \quad r_1 = -m_1,$$

ii) the equilibrium first-period fixed fees and full-period profits do not depend on *the level of the first or second-period access markup:*

$$F_1 = f + \frac{1}{2\sigma} - \frac{2s\delta}{3}, \qquad \Pi = \frac{1+\delta}{4\sigma} - \frac{s\delta}{3},$$

iii) there exists no "cornered-market" equilibrium if switching costs are small enough.

Proof. See Appendix. ∎

We may then conclude that networks can no longer use future reciprocal access charges as an instrument to soften first-period competition. Notice that as long as $\partial\widehat{\pi}_2^i/\partial\alpha_1^i > 0$ networks compete more aggressively in the first period, so as to build market share that is profitable in the second period. From López (2005) we know that when networks only compete in call prices and fixed fees, $\partial\widehat{\pi}_2^i/\partial\alpha_1^i$ depends on both α_1^i and m_2, and in a symmetric equilibrium slightly moving m_2 away from zero can reduce the value of having a higher market share in the second-period, $\partial\widehat{\pi}_2^i/\partial\alpha_1^i$ is strictly concave in m_2 at $m_2 = 0$, and therefore increase their full-period profits by softening first-period competition for market share. In contrast, when networks compete also in reception charges, $\widehat{\pi}_2^i$ depends only on first-period market shares, implying that $\partial^2\widehat{\pi}_2^i/\partial m_2\partial\alpha_1^i = 0 \ \forall m_2, \alpha_1^i$; hence, first-period competition does not depend on m_2, and neither do the full-period profits. In the rational consumer expectations case the expressions for the second-period equilibrium are the same as with naive expectations: (14), (15) and (18). In the first period, however, consumers recognize that a network with higher market share will charge higher prices in the second-period whenever switching costs are positive. Nevertheless, since the value of having a higher second-period market share is neutral with respect to the level of m_2, first-period prices are also neutral, and hence m_2 does not affect the subscribers' first-period net surpluses. In summary, with both naive and rational consumers expectations, networks cannot increase their full-period profits by departing m_2 away from zero when competition is in call prices, fixed fees and reception charges.

4.3. THE MULTI-PERIOD CASE

Assume networks compete in (finite) T discrete periods of time. Our setup is as follows: in each period $t = 1...T$, networks can condition their play at time t on the history of play until that date h_{t-1} (closed-loop or feedback strategies). Let $V_t^i(\cdot)$ denote the value function for network i at time t, with $V_{T+1}^i = 0$. We will provide sufficient conditions under which there exists a unique subgame-perfect equilibrium.

CLAIM 1: *Suppose networks compete in finite $T > 1$ discrete periods of time, and assume A.1, A.2, A.3 and A.4 holds, then for a small enough σ and a large enough Δ there exists an interior subgame-perfect equilibrium such that in any* *continuation equilibria (even off the equilibrium path): (i) networks price calls at their off-net cost, (ii) the fixed fees and per-period profits depend on h_t only through α^i_{t-1}, and moreover (iii) do not depend on the access markup levels.*

The proof of Claim 1 will proceed in several steps. First of all, note that the analysis of the game in period T is the same as in the two-period case, thus from proposition 1 we know that in period T, under A.1, A.2, A.3 and A.4, for a small enough σ and a large enough Δ there exists a unique equilibrium, which is interior and where networks price calls at the off-net cost. Moreover, we know that V^i_T exists, depends on h_{T-1} only through α^i_{T-1}, and is quadratic. Consider now period $T-1$ where α^i_{T-2} is given, network i knows h_{T-2}, and solves:

$$\begin{aligned}\max_{p^i_{T-1},r^i_{T-1},F^i_{T-1}} \Pi^i_{T-1} \quad &\equiv \quad \pi^i_{T-1}(p^i_{T-1},r^i_{T-1},F^i_{T-1},p^j_{T-1},r^j_{T-1},F^j_{T-1},\alpha^i_{T-2}) \\ &\quad +\delta V^i_T(\alpha^i_{T-1}),\end{aligned}$$

where π^i_{T-1} is given by (8) and

$$\begin{aligned}\alpha^i_{T-1} \quad &= \quad 1/2+(2\alpha^i_{T-2}-1)\sigma s+\sigma(\phi^i_{T-1}(p^i_{T-1},r^i_{T-1},p^j_{T-1},r^j_{T-1},\alpha^i_{T-1}) \\ &\quad -\phi^j_{T-1}(p^j_{T-1},r^j_{T-1},p^i_{T-1},r^i_{T-1},\alpha^i_{T-1})+F^j_{T-1}-F^i_{T-1}).\end{aligned}$$

The analysis can again be simplified by invoking the one-to-one relationship between F^i_t and α^i_t : network i choosing a tariff (p^i,r^i,F^i) given network $j's$ tariff (p^j,r^j,F^j), is equivalent to choosing (p^i,r^i,α^i). We can thus rewrite network $i's$ problem as follows:

$$\begin{aligned}\max_{p^i_{T-1},r^i_{T-1},\alpha^i_{T-1}} \overline{\Pi}^i_{T-1} \quad &\equiv \quad \overline{\pi}^i_{T-1}(p^i_{T-1},r^i_{T-1},\alpha^i_{T-1},p^j_{T-1},r^j_{T-1},F^j_{T-1},\alpha^i_{T-2}) \\ &\quad +\delta V^i_T(\alpha^i_{T-1}),\end{aligned}$$

where $\overline{\pi}^i_{T-1}$ is given by (9). It then follows that $\partial\overline{\Pi}^i_{T-1}/\partial p^i_{T-1}=\partial\overline{\pi}^i_{T-1}/\partial p^i_{T-1}$ and $\partial\overline{\Pi}^i_{T-1}/\partial r^i_{T-1}=\partial\overline{\pi}^i_{T-1}/\partial r^i_{T-1}$; thus a candidate solution for the four first-order equilibrium conditions with respect to usage prices in period $T-1$ is $p^{i\,*}_{T-1}=p^{j\,*}_{T-1}=c+m_{T-1}$ and $r^{i\,*}_{T-1}=r^{j\,*}_{T-1}=-m_{T-1}$. Replacing these expressions into $\overline{\Pi}^i_{T-1}$, we can derive the corresponding candidate equilibrium fixed fees $F^{i\,*}_{T-1}$, which solve

$$\max_{F^i_{T-1}} \alpha^i_{T-1}\left\{F^i_{T-1}-f\right\}+\delta V^i_T(\alpha^i_{T-1}), \tag{25}$$

subject to

$$\alpha^i_{T-1}=(1/2)+(2\alpha^i_{T-2}-1)\sigma s-\sigma(F^i_{T-1}-F^j_{T-1}).$$

Note that (25) is a quadratic optimization problem, which implies that $F^{i\,*}_{T-1}$

is a linear function of α^i_{T-2}, and hence

$$\widetilde{V}^i_{T-1} \equiv \overline{\Pi}^i_{T-1}(p^{i\ *}_{T-1}, p^{i\ *}_{T-1}, F^{i\ *}_{T-1}, p^{j\ *}_{T-1}, r^{j\ *}_{T-1}, F^{j\ *}_{T-1}, \alpha^i_{T-2})$$

is a quadratic function of α^i_{T-2}. Consequently, if V^i_T exists, depends on h_{T-1} only through α^i_{T-1}, and is quadratic, there exists in period $T-1$ a candidate equilibrium where networks price calls at the off-net cost and where fixed fees depend on h_{T-2} only through α^i_{T-2}, and do so linearly, so that for this candidate equilibrium the valuation function $\widetilde{V}^i_{T-1}$ is also quadratic and depends on h_{T-2} only through α^i_{T-2}. Therefore, by mathematical induction we can derive a sequence of candidate equilibria for all t where networks price calls at the off-net cost and $F^{i\ *}_t$ depends on h_{t-1} only through α^i_{t-1}, and so a sequence of candidate valuation functions $\widetilde{V}^i_t$ that are quadratic and depends on h_{t-1} only through α^i_{t-1}.

Since we exhibit a candidate equilibrium by solving the first-order conditions, this candidate equilibrium is indeed an equilibrium if the Hessian of $\overline{\Pi}^i_{T-1}$ is definite negative since in that case second-order conditions are satisfied. That is, if the Hessian of $\overline{\Pi}^i_{T-1} \equiv \overline{\pi}^i_{T-1} + \delta V^i_T(\alpha^i_{T-1})$ is definite negative, then our candidate equilibrium is an equilibrium from $t = T-1$ onwards and $\widetilde{V}^i_{T-1}$ is well defined and the valuation function. It then follows that if the Hessian of $\overline{\Pi}^i_{T-2} \equiv \overline{\pi}^i_{T-2} + \delta\widetilde{V}^i_{T-1}$ is also definite negative, our candidate equilibrium is an equilibrium from $t = T-2$ onwards and $\widetilde{V}^i_{T-2}$ is also well defined and the valuation function, and so on. Before providing conditions under which this is so we first derive the sequence of candidate equilibrium fixed fees and valuation functions. In each period, the two networks each solve a linear-quadratic dynamic programming problem, and thus the candidate value functions $\widetilde{V}^i_t(\alpha^i_{t-1})$ are quadratic and characterized by coupled Ricatti equations that can be solved recursively. Let us define $y_t = (F^1_t, F^2_t, \alpha^1_t, 1)'$ and $x_t = (F^1_t, F^2_t)'$; the optimization problem for networks 1 and 2 in period t can be formulated, respectively, as follows:

$$\max_{F^1_t} \frac{1}{2} y'_t I y_t + \delta\widetilde{V}^1_{t+1}(y_t),$$

$$\max_{F^2_t} \frac{1}{2} y'_t J y_t + \delta\widetilde{V}^2_{t+1}(y_t),$$

subject to $y_t = Ay_{t-1} + Bx_t$, where

$$I = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -f \\ 0 & 0 & -f & 0 \end{bmatrix}, \ J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & f \\ 0 & 1 & f & -2f \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\sigma s & 1/2 - \sigma s \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\sigma & \sigma \\ 0 & 0 \end{bmatrix}.$$

Moreover, since $\widetilde{V}_t^i(y_{t-1})$ is quadratic for any $t < T$ we can write

$$\widetilde{V}_t^i(y_{t-1}) = \frac{1}{2} y'_{t-1} S_t^i y_{t-1}. \tag{26}$$

The matrix S_t^i can be obtained as follows[20]: define $B = [b_1, b_2]$, let

$$\Phi_t = \begin{bmatrix} b'_1 \Sigma_t^1 \\ b'_2 \Sigma_t^2 \end{bmatrix},$$

where

$$\Sigma_t^1 = I + \delta S_{t+1}^1,$$

$$\Sigma_t^2 = J + \delta S_{t+1}^2.$$

Note that network 1 and 2 solve, respectively,

$$\max_{F_t^1} \left\{ \frac{1}{2} (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2)' \Sigma_t^1 (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2) \right\}$$

$$\max_{F_t^2} \left\{ \frac{1}{2} (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2)' \Sigma_t^2 (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2) \right\}$$

Consequently, the couple of first-order conditions are

$$b'_1 \Sigma_t^1 (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2) = 0,$$

$$b'_2 \Sigma_t^2 (Ay_{t-1} + b_1 F_t^1 + b_2 F_t^2) = 0.$$

Finally, by solving this system of two linear equations we might find the rule for the *candidate* equilibrium fixed fees, given by

$$x_t = E_t y_{t-1}, \tag{27}$$

where $E_t = -(\Phi_t B)^{-1} \Phi_t A y_{t-1}$. In addition, S_t^1 and S_t^2 are determined by

$$S_t^1 = (A + BE_t)'(I + \delta S_{t+1}^1)(A + BE_t),$$

$$S_t^2 = (A + BE_t)'(J + \delta S_{t+1}^2)(A + BE_t).$$

[20] We follow here the same procedure as in Kydland (1975), although we use the trick of including the constant 1 in the list of state variables so as to express networks' profits in a simple quadratic form.

By iterating the above process one can compute the candidate value function for both networks at any $t \in [1, T-1]$. In fact, we can compute recursively the unique solution in fixed fees for any period t and hence guarantee the existence of a well-defined $\widetilde{V}_t^1(y_{t-1})$ only if $|\Phi_t B| \neq 0$ for $t = 1, ...T$, which is satisfied here since networks compete for market share. That is, we would have $|\Phi_t B| = 0$ only if reaction functions had the same slope, in which case there would be infinitely many solutions, or no solution (see Kydland, 1975.) Note that both $\widetilde{V}_t^1$ and $F_t^{1\ *} \equiv x(1)$ do not depend on m_t. Recall that if the Hessians of $\overline{\Pi}_t^1$ and $\overline{\Pi}_t^2$ are definite negative in own strategies for all t, our candidate equilibrium will be a subgame-perfect equilibrium. Let H_t^i denote the Hessian matrix of $\overline{\Pi}_t^i$ under the candidate equilibrium, and let $(H_t^i)_k$ denote the $k-th$ principal minor of the Hessian matrix H_t^i. To prove Claim 1, it suffices to apply the following proposition,

Proposition 3. *Under A.1, A.2, A.3 and A.4, for a small enough σ and a large enough Δ, $\left|\left(H_t^i\right)_1\right| < 0$, $\left|\left(H_t^i\right)_2\right| > 0$, and $\left|\left(H_t^i\right)_3\right| < 0$ $\forall t$.*

Proof. See Appendix. ∎

A couple of remarks are in order:

Remark 1. (Uniqueness) To prove uniqueness we can follow a similar reasoning to that of proof of proposition 2: by assuming a large enough Δ we can reduce the set of candidate equilibria in usage prices to a singleton where usage prices are set at their off-net cost: $p_t^{i\ *} = p_t^{j\ *} = c + m_t$ and $r_t^{i\ *} = r_t^{j\ *} = -m_t$. Moreover, at this level we have that $[F_t^{i\ *}(\alpha_{t-1}^i), F_t^{j\ *}(\alpha_{t-1}^i)]$ are uniquely determined and given by (27). Finally we know from above that this unique candidate equilibrium is indeed an equilibrium for a (positive) small enough σ.

Remark 2. (Corner Equilibrium) We now show that no cornered-market equilibrium exists when switching costs are not too high. Suppose there exists an equilibrium where network i corners the market in any period t by setting $(p_t^{i\ *}, r_t^{i\ *}, F_t^{i\ *})$ given that network j sets $(p_t^{j\ *}, r_t^{j\ *}, F_t^{j\ *})$. Then, $\pi_t^{j\ *} = 0$ and $\Pi_t^{j\ *} = \delta V_{t+1}^j(0)$. And $\Pi_t^{i\ *} = \pi_t^{i\ *} + \delta V_{t+1}^i(1)$, where $\pi_t^{i\ *} = (p_t^{i\ *} - c + r_t^{i\ *})Q(p_t^{i\ *}, r_t^{i\ *})$. But in order to corner the market network i must sacrifice present profits so as to attract consumers. It means that $\pi_t^{i\ *}$ is lower than the static equilibrium profits, which is always interior. Moreover, as switching costs decrease, the link between the present and the future vanishes, that is, $\lim_{s\to 0} V_{t+1}^i(1) = V_{t+1}^i(1/2)$. Therefore, $\lim_{s\to 0} \Pi_t^{i\ *} < \lim_{s\to 0} \widehat{\Pi}_t^i = \widehat{\pi}_t^i(\alpha_{t-1}^i) + \delta V_{t+1}^i(1/2)$, where as before $\widehat{\pi}_t^i(\alpha_{t-1}^i)$ denote the equilibrium profits of network i in period t as a function of α_{t-1}^i. Thus, a (positive) small enough s is a sufficient condition under which no "cornered-market" equilibrium exists.

5. SOCIAL OPTIMUM

Jeon et al. (2004) already pointed out that efficiency cannot be achieved in the presence of noise since marginal utilities have a random term, which in turn requires price instruments to be contingent on the realization of this term. We address this problem in a different way, and we look for the level of the access markup that maximizes the *expected* social welfare.

We begin by considering symmetric networks, that is, $\alpha_1^i = \alpha_1^j = 1/2$. It follows from proposition 1 that the symmetric equilibrium is the unique possible one so the market is again equally divided in the second period. This symmetric solution minimizes the average consumer's disutility from not being able to join to his preferred network, and hence allow us to rule out this social cost from the analysis for the moment. Since payments are only transfers from one agent to another, from a social-welfare viewpoint what matters is the utility that consumers derive from making and receiving calls, and the costs of these calls. Consider a call from a network i consumer to a network j consumer, its length is given by Q^{ij}, and the total utility derived by both the caller and the receiver from this call is: $U^{ij} + \widetilde{U}^{ij}$. Let $W(m_t)$ denote the expected welfare arising from this call, in equilibrium

$$W(m_t) = U(c + m_t, -m_t) + \widetilde{U}(c + m_t, -m_t) - cQ(c + m_t, -m_t). \tag{28}$$

The first-order condition is

$$\begin{aligned} \frac{dW}{dm_t}(m_t) &= \left(\frac{\partial U}{\partial p_t}(m_t) - \frac{\partial U}{\partial r_t}(m_t)\right) + \left(\frac{\partial \widetilde{U}}{\partial p_t}(m_t) - \frac{\partial \widetilde{U}}{\partial r_t}(m_t)\right) \\ &\quad -c\left(\frac{\partial Q}{\partial p_t}(m_t) - \frac{\partial Q}{\partial r_t}(m_t)\right) \\ &= 0. \end{aligned} \tag{29}$$

A small increase in m_t implies two opposite effects: it increases p_t^i but also decreases r_t^i. Moreover, a small increase in p_t^i reduces the callers' willingness to stay on the phone, and consequently it decreases both the callers' utility $\partial U/\partial p_t^i$ and the receivers' utility $\partial \widetilde{U}/\partial r_t^i$. On the other hand, a small decrease in r_t^i increases the receivers' willingness to stay on the phone, which in turn increases the utility of both callers and receivers: $-(\partial U/\partial r_t + \partial \widetilde{U}/\partial r_t)$. Thus, on one hand, it decreases the volume of traffic in which callers are sovereign and hence the costs incurred in these calls; this social gain is given by: $-c(\partial Q/\partial p_t)$. At the same time, however, it increases the volume of traffic in which receivers are sovereign, which implies a social cost equal to $c(\partial Q/\partial r)$. Using (3) and (4) yields, in equilibrium, $\partial U/\partial p_t = (c + m_t)(\partial Q/\partial p_t)$ and $\partial \widetilde{U}/\partial r_t = -m_t(\partial Q/\partial r_t)$; so equation (29) boils down to:

$$\begin{aligned}\frac{dW}{dm_t}(m_t) &= m_t\left(\frac{\partial Q}{\partial p_t}(m_t)+\frac{\partial Q}{\partial r_t}(m_t)\right)+c\frac{\partial Q}{\partial r_t}(m_t) \\ &\quad+\left(\frac{\partial \widetilde{U}}{\partial p_t}(m_t)-\frac{\partial U}{\partial r_t}(m_t)\right) \\ &= 0.\end{aligned} \tag{30}$$

Moreover, under the existence and uniqueness conditions of proposition 1:

$$d^2W/(dm_t)^2 \simeq -(1/2)(1/b+1/e+e/b^2+b/e^2)<0.$$

Letting m_t^* denote the optimal access markup we thus have that any m_t^* such that $(dW/dm_t)(m_t^*)=0$ is socially optimal. In order to be more precise let us state the following proposition,

PROPOSITION 4. *Under A.1, A.2, A.3, A.4, for a small enough σ and a large enough Δ, in equilibrium:*

$$\frac{\partial \widetilde{U}}{\partial p}(m_t) \simeq -\left(\frac{1}{b}\right)\left[\frac{e}{2b}(c+m_t-a)+\frac{d}{2}+\frac{\Delta}{8}\left(\frac{e}{b}+\frac{b}{3e}\right)\right],$$

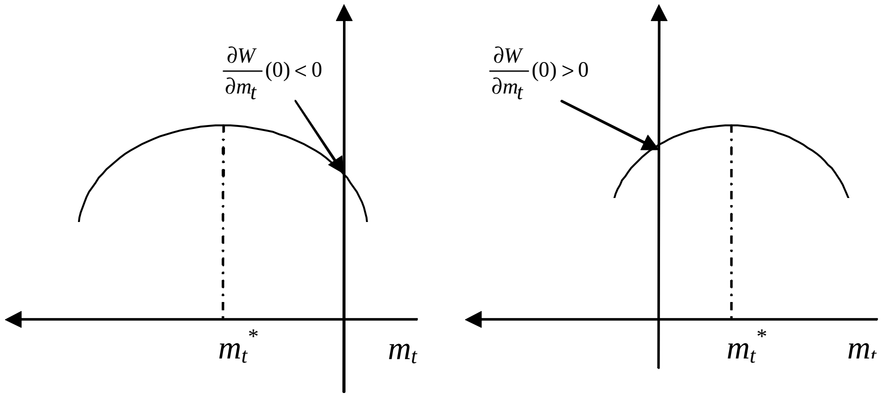
$$\frac{\partial U}{\partial r}(m_t) \simeq -\left(\frac{1}{e}\right)\left[-\frac{b}{2e}(m_t+d)+\frac{a}{2}+\frac{\Delta}{8}\left(\frac{b}{e}+\frac{e}{3b}\right)\right].$$

Proof. See Appendix. ∎

Clearly, the above proposition points out that the optimal value of the access markup depends on the characteristics of each market in particular. Consider now a small increase in the access markup starting from $m_t=0$, it slightly increases call prices and slightly decreases the reception price. More precisely,

$$\frac{dW}{dm_t}(0)=\left(c\frac{\partial Q}{\partial r_t}(0)+\frac{\partial \widetilde{U}}{\partial p_t}(0)\right)-\frac{\partial U}{\partial r_t}(0). \tag{31}$$

This expression is in general different from zero. Roughly speaking, $m_t=0$ is (generically) never optimal. Indeed, if $(dW/dm_t)(0)<0$ it follows that $m_t^*<0$, and conversely $m_t^*>0$ if $(dW/dm_t)(0)>0$ (see figure below.) Assume that $u(x)=\widetilde{u}(x)$, that is, $a=d$ and $b=e$. Now, making use of Proposition 4 yields: $(\partial\widetilde{U}/\partial p)(0)-(\partial U/\partial r)(0)\simeq -c/2b<0$. Consequently, the consumers' surplus decreases: the small decrease in r_t increases the callers' utility less than the decrease in the receivers' utility that is driven by the small increase in p_t. This social cost together with the cost incurred by the increase in the average length of calls yield $(dW/dm_t)(0)\simeq -c/b<0$. Conversely, a small decrease in m_t will decrease p_t and increase r_t such that the receivers' utility increases more than the loss in the callers' utility, moreover the average length of calls decreases since



r_t increases, which indeed decreases costs in $|c\partial Q/\partial r_t|$. Then, it is optimal to decrease m_t, that is, $m_t^* < 0$. Given this, we have proved the following proposition:

PROPOSITION 5. *If* $u(x) = \widetilde{u}(x)$, *then* $m_t^* < 0$ *and is given by (30) if* $a_t^* = m_t^* + c_0 > 0$. *Otherwise, 'bill and keep' is socially optimal and* $m_t^* = -c_0$.

So far we have assumed symmetric networks; let us now turn to the asymmetric case. The utility that any network $i's$ subscriber derives from calls is $\alpha_t^i U^{ii} + \alpha_t^j U^{ij} + \alpha_t^i \widetilde{U}^{ii} + \alpha_t^j \widetilde{U}^{ji}$, and the costs incurred by his calls are $(\alpha_t^i Q^{ii} + \alpha_t^j Q^{ij})c$. Since there are α_t^i consumers attached to network i and α_t^j consumers attached to network j, the total utility that consumers derive is:

$$\begin{aligned} &\alpha_t^i(\alpha_t^i U^{ii} + \alpha_t^j U^{ij} + \alpha_t^i \widetilde{U}^{ii} + \alpha_t^j \widetilde{U}^{ji}) + \alpha_t^j(\alpha_t^j U^{jj} + \alpha_t^i U^{ji} \\ &+\alpha_t^j \widetilde{U}^{jj} + \alpha_t^i \widetilde{U}^{ij}) - \alpha_t^i(\alpha_t^i Q^{ii} + \alpha_t^j Q^{ij})c - \alpha_t^j(\alpha_t^j Q^{jj} + \alpha_t^i Q^{ji})c. \end{aligned} \tag{32}$$

But in equilibrium expression (32) boils down to (28). Therefore, the above analysis remain valid in the asymmetric case. The intuition is very simple: since usage prices are identical in both networks whatever the market shares are, we have that in equilibrium $U^{ii} = U^{ij}$, $\widetilde{U}^{ij} = \widetilde{U}^{ji}$ and $Q^{ij} = Q^{ji}$; it then follows that consumers derive from calls the same utility in both networks. Let us now turn back to the consumer's disutility from not being able to join to his preferred network and the switching costs issue. Given first-period market shares α_1^i and α_1^j, the socially optimal configuration of market shares $(\alpha_2^{i\,*}, \alpha_2^{j\,*})$ minimizes both social costs. Suppose that $s = 0$ and the market is initially unequal divided between the two competitors (i.e., $\alpha_1^i \neq \alpha_1^j$), then $\alpha_2^{i\,*} = \alpha_2^{j\,*} = 1/2$ will still minimize the average consumer's disutility since preferences are assumed to be independent across periods. Nevertheless, if every subscriber incurs a cost when switching networks, then $\alpha_2^i = \alpha_2^j = 1/2$ is not necessarily optimal if $\alpha_1^i \neq \alpha_1^j$.

Note however that in equilibrium $\phi_t^i = \phi_t^j$, which amounts to

$$w_t^i - w_t^j = \widehat{F}_t^j(\alpha_{t-1}^j) - \widehat{F}_t^i(\alpha_{t-1}^i).$$

That is, net surpluses in the equilibrium do not depend on the access markup, so neither do the market shares. We thus need one more instrument or a direct regulation of fixed fees so as to achieve $(\alpha_2^{i\,*}, \alpha_2^{j\,*})$.

6. CONCLUSION

This article has studied the implications of adopting the receiver pays regime when networks compete in a dynamic framework. We allowed callers and call receivers to derive utility from making and receiving calls, and networks to price calls and charge customers for receiving calls. Assuming the existence of a random noise in the caller's and receiver's utility, we first showed that the off-net-cost pricing principle is a candidate equilibrium.

Second, we showed that under linear demands q and $\widetilde{q}$, this candidate equilibrium is indeed the unique equilibrium provided that the degree of substitutability between networks is low enough and the random noise has a wide enough support. Other insights were derived. In the region where the equilibrium exists, an increase in the access charge raises the call price and decreases the reception charge, but does not affect the networks' full-period profits. Instead, the access charge level clearly affects the consumer welfare; indeed its optimal level from the social welfare viewpoint depends on the characteristics of each market. In the particular case where the linear demand functions q and $\widetilde{q}$ are the same, starting from zero access markup, a small decrease in the access charge decreases the call price and raises the reception charge. As a result, the receivers' utility increases more than the loss in the callers' utility, and the average length of calls decreases, which in turn decreases costs. Consequently, we find optimal to decrease access charges so that either a interior solution is reached, or 'bill and keep' might be socially optimal.

Third and finally, in our previous work (López, 2005) we showed that networks are able to soften present competition by departing away future reciprocal access charges from marginal costs. Under the receiver pays regime we showed however that in a multi-period setting the off-net-cost pricing equilibrium neutralizes the potential anticompetitive role that reciprocal access charges could play.

Our article is a further step in the research agenda; it has characterized the equilibrium that arises in dynamic network competition under the receiver pays regime, and has studied how networks operators' pricing strategies might react to the adoption of such regime. We expect further research extending our analysis. Three key directions are noteworthy. Firstly, as already pointed out by Jeon et al. (2004), the "noncooperative volume setting" assumption should be extended to allow more cooperative behaviors, as for instance the maximization of joint surplus over the call length. Secondly, asymmetric calling patterns should be analyzed. It is not difficult to find cases in which the calling pattern is unbalanced, which might affect the incentives of the networks in the industry. Thirdly, it would be interesting to check whether the off-net-cost pricing principle still applies to the case of multiple networks competing for market share.

7. APPENDIX

LEMMA 1. *Under A.1, A.2, A.3 and A.4:*

$$\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} = -\frac{1}{2b} - \frac{1}{\Delta}\left[\frac{d - r_t^j}{e} + \frac{p_t^i - a}{b}\right]$$

$$\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} = -\frac{1}{2e} - \frac{1}{\Delta}\left[\frac{a - p_t^j}{b} + \frac{r_t^i - d}{e}\right]$$

$$\frac{\partial^2 U(p_t^j, r_t^i)}{(\partial r_t^i)^2} = -\left(\frac{b}{2e^2}\right) - \left(\frac{1}{e\Delta}\right)\left[a - \frac{b}{e}(d - r_t^i)\right]$$

$$\frac{\partial^2 \widetilde{U}(p_t^i, r_t^j)}{(\partial p_t^i)^2} = -\left(\frac{e}{2b^2}\right) - \left(\frac{1}{b\Delta}\right)\left[d - \frac{e}{b}(a - p_t^i)\right]$$

$$\frac{\partial^2 U(p_t^j, r_t^i)}{\partial p_t^j \partial r_t^i} = \frac{p_t^j}{b\Delta}$$

$$\frac{\partial^2 \widetilde{U}(p_t^i, r_t^i)}{\partial r_t^i \partial p_t^i} = \frac{r_t^i}{\Delta e}$$

Proof. Let us construct $Q(p_t^i, r_t^j)$ by means of several illustrative steps. First of all, notice that for a given pair of prices (p_t^i, r_t^j) and a given pair of realized values $(\varepsilon, \widetilde{\varepsilon})$, the length of a call from a network i consumer to a network j consumer is given by $Q(p_t^i, r_t^j, \varepsilon, \widetilde{\varepsilon}) = \min[q(p_t^i - \varepsilon), \widetilde{q}(r_t^j - \widetilde{\varepsilon})]$, where $q = \mu'^{-1}(p_t^i - \varepsilon)$ and $\widetilde{q} = \widetilde{\mu}'^{-1}(r_t^j - \widetilde{\varepsilon})$, that is,

$$q = \frac{a - (p_t^i - \varepsilon)}{b}, \qquad \widetilde{q} = \frac{d - (r_t^j - \widetilde{\varepsilon})}{e}.$$

Step 1. Assume for the moment $\widetilde{\varepsilon}$ is exogenous and takes value $\widetilde{\varepsilon}'$.

Step 2. Note that $q(p^i - \varepsilon)$ is strictly increasing in ε, which means that it will exist an ε^* such that $q(p_t^i - \varepsilon^*) = \widetilde{q}(r_t^j - \widetilde{\varepsilon}')$, namely

$$\varepsilon^* = p_t^i - q^{-1}(\widetilde{q}(r_t^j - \widetilde{\varepsilon}')).$$

Moreover, if $\varepsilon^* \notin [\underline{\varepsilon}, \overline{\varepsilon}]$ then $f(\varepsilon^*) = 0$.

Step 3. For any $\varepsilon \leq \varepsilon^*$, the caller will be sovereign, whereas the receiver will be sovereign provided that $\varepsilon > \varepsilon^*$. Therefore, we can write the demand as follows:

$$\begin{aligned} d(p_t^i, r_t^j, \widetilde{\varepsilon}') &= \int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} q(p_t^i - \varepsilon) f(\varepsilon) d\varepsilon + \int_{\varepsilon^*(\cdot)}^{\overline{\varepsilon}} \widetilde{q}(r_t^j - \widetilde{\varepsilon}') f(\varepsilon) d\varepsilon \\ &= \int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} q(p_t^i - \varepsilon) f(\varepsilon) d\varepsilon + \widetilde{q}(r_t^j - \widetilde{\varepsilon}')[F(\overline{\varepsilon}) - F(\varepsilon^*(\cdot))], \end{aligned}$$

which can be rewritten for any value of $\widetilde{\varepsilon} : d = d(p_t^i, r_t^j, \widetilde{\varepsilon})$.

Step 4. Therefore, under A.3., for a given pair of prices (p_t^i, r_t^j), the volume of calls from a network i consumer to a network j consumer at period t is given by:

$$\begin{aligned} Q(p_t^i, r_t^j) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} d(p_t^i, r_t^j, \widetilde{\varepsilon}) \widetilde{f}(\widetilde{\varepsilon}) d\widetilde{\varepsilon} \\ &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} q(p_t^i - \varepsilon) f(\varepsilon) d\varepsilon \\ &\quad + \widetilde{q}(r_t^j - \widetilde{\varepsilon})[F(\overline{\varepsilon}) - F(\varepsilon^*(\cdot))]) \widetilde{f}(\widetilde{\varepsilon}) d\widetilde{\varepsilon} \end{aligned}$$

Now, for a given r_t^j we can differentiate $Q(p_t^i, r_t^j)$ with respect to p_t^i :

$$\begin{aligned} \frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} \frac{\partial q(p_t^i - \varepsilon)}{\partial p_t^i} f(\varepsilon) d\varepsilon \right) \widetilde{f}(\widetilde{\varepsilon}) d\widetilde{\varepsilon} \\ &= -\left(\frac{1}{b\Delta^2} \right) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\varepsilon^*(\cdot) - \underline{\varepsilon}) d\widetilde{\varepsilon}, \end{aligned}$$

where $\varepsilon^* = (b/e)(d - r_t^j + \widetilde{\varepsilon}) + p_t^i - a$. Then,

$$\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} = -\frac{1}{2b} - \frac{1}{\Delta} \left[\frac{d - r_t^j}{e} + \frac{p_t^i - a}{b} \right]$$

In a similar way, we can assume p_t^i as given and differentiate $Q(p_t^i, r_t^i)$ with respect to r_t^i. To that end, we can rewrite the demand as follows:

$$\begin{aligned} Q(p_t^j, r_t^i) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}^*(\cdot)} \widetilde{q}(r_t^i - \widetilde{\varepsilon}) \widetilde{f}(\widetilde{\varepsilon}) d\widetilde{\varepsilon} \\ &\quad + q(p_t^i - \varepsilon)[\widetilde{F}(\overline{\varepsilon}) - \widetilde{F}(\widetilde{\varepsilon}^*(\cdot))]) f(\varepsilon) d\varepsilon, \end{aligned}$$

, where $\widetilde{\varepsilon}^* = (e/b)(a - p_t^j + \varepsilon) + r_t^i - d$. Then,

$$\begin{aligned} \frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}^*(\cdot)} \frac{\partial \widetilde{q}(r_t^i - \widetilde{\varepsilon})}{\partial r_t^i} \widetilde{f}(\widetilde{\varepsilon}) d\widetilde{\varepsilon} \right) f(\varepsilon) d\varepsilon \\ &= -\left(\frac{1}{e\Delta^2} \right) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\widetilde{\varepsilon}^*(\cdot) - \underline{\varepsilon}) d\varepsilon \end{aligned}$$

Thus,

$$\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} = -\frac{1}{2e} - \frac{1}{\Delta}\left[\frac{a-p_t^j}{b} + \frac{r_t^i - d}{e}\right]$$

Assume a given p_t^i and rewrite $U(p_t^i, r_t^i)$ as follows:

$$\begin{aligned} U(p_t^i, r_t^i) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}^*(\cdot)} u(\widetilde{q}(r_t^i - \widetilde{\varepsilon}))\widetilde{f}(\widetilde{\varepsilon})d\widetilde{\varepsilon} \\ &\quad + u(q(p_t^i - \varepsilon))[\widetilde{F}(\overline{\varepsilon}) - \widetilde{F}(\widetilde{\varepsilon}^*(\cdot))])f(\varepsilon)d\varepsilon, \end{aligned}$$

Then,

$$\frac{\partial U(p_t^i, r_t^i)}{\partial r_t^i} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}^*(\cdot)} u'(\widetilde{q}(r_t^i - \widetilde{\varepsilon}))\frac{\partial \widetilde{q}(r_t^i - \widetilde{\varepsilon})}{\partial r_t^i}\widetilde{f}(\widetilde{\varepsilon})d\widetilde{\varepsilon})f(\varepsilon)d\varepsilon$$

Note that $u'(\widetilde{q}(r_t^i - \widetilde{\varepsilon})) = a - b\widetilde{q}(r_t^i - \widetilde{\varepsilon}) + \varepsilon$. Thus,

$$\begin{aligned} \frac{\partial U(p_t^i, r_t^i)}{\partial r_t^i} &= -\left(\frac{1}{e\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}^*(\cdot)} a - b\left[\frac{d - (r_t^i - \widetilde{\varepsilon})}{e}\right] + \varepsilon d\widetilde{\varepsilon})d\varepsilon \\ &= -\left(\frac{1}{e\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[a - \frac{b}{e}(d - r_t^i) + \varepsilon - \frac{b}{2e}(\widetilde{\varepsilon}^*(\cdot) + \underline{\varepsilon})\right](\widetilde{\varepsilon}^*(\cdot) - \underline{\varepsilon})d\varepsilon \end{aligned}$$

It follows that:

$$\begin{aligned} \frac{\partial^2 U(p_t^i, r_t^i)}{(\partial r_t^i)^2} &= -\left(\frac{1}{e\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} a - \frac{b}{e}(d - r_t^i + \underline{\varepsilon}) + \varepsilon d\varepsilon \\ &= -\left(\frac{b}{2e^2}\right) - \left(\frac{1}{e\Delta}\right)\left[a - \frac{b}{e}(d - r_t^i)\right] \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 U(p_t^i, r_t^i)}{\partial p_t^i \partial r_t^i} &= -\left(\frac{1}{e\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \widetilde{\varepsilon}^*(\cdot) - \frac{e}{b}(a + \varepsilon) + (d - r_t^i)d\varepsilon \\ &= \frac{p_t^i}{b\Delta} \end{aligned}$$

For a given p_t^i we can write $\widetilde{U}(p_t^i, r_t^i)$ as follows:

$$\begin{aligned} \widetilde{U}(p_t^i, r_t^i) &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} \widetilde{u}(q(p_t^i - \varepsilon))f(\varepsilon)d\varepsilon\right) \\ &\quad + \left(\widetilde{u}(\widetilde{q}(r_t^i - \widetilde{\varepsilon}))\left[F(\overline{\varepsilon}) - F(\varepsilon^*(\cdot))\right]\right)\widetilde{f}(\widetilde{\varepsilon})d\widetilde{\varepsilon} \end{aligned}$$

Then,

$$
\begin{aligned}
\frac{\partial \widetilde{U}(p_t^i, r_t^i)}{\partial p_t^i} &= -\left(\frac{1}{b\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} (\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} d - e\left[\frac{a-(p_t^i-\varepsilon)}{b}\right] + \widetilde{\varepsilon} d\varepsilon) d\widetilde{\varepsilon} \\
&= -\left(\frac{1}{b\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[d - \frac{e}{b}(a-p_t^i) + \widetilde{\varepsilon} - \frac{e}{2b}(\varepsilon^*(\cdot)+\underline{\varepsilon})\right](\varepsilon^*(\cdot)-\underline{\varepsilon}) d\widetilde{\varepsilon}
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
\frac{\partial^2 \widetilde{U}(p_t^i, r_t^i)}{(\partial p_t^i)^2} &= -\left(\frac{1}{b\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} d - \frac{e}{b}(a-p_t^i+\underline{\varepsilon}) + \widetilde{\varepsilon} d\widetilde{\varepsilon} \\
&= -\left(\frac{e}{2b^2}\right) - \left(\frac{1}{b\Delta}\right)\left[d - \frac{e}{b}(a-p_t^i)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \widetilde{U}(p_t^i, r_t^i)}{\partial r_t^i \partial p_t^i} &= -\left(\frac{1}{b\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \varepsilon^*(\cdot) - \frac{b}{e}(d+\widetilde{\varepsilon}) + (a-p_t^i) d\widetilde{\varepsilon} \\
&= \frac{r_t^i}{e\Delta}
\end{aligned}
$$

∎

Lemma 2. *Under A.1, A.2, A.3, A.4., and for a large enough* Δ :

$$\partial^2 \overline{\pi}_t^i/(\partial p_t^i)^2 \simeq -\alpha_t^i/2b$$

$$\partial^2 \overline{\pi}_t^i/(\partial r_t^i)^2 \simeq -\alpha_t^i/2e$$

$$\frac{\partial^2 \overline{\pi}_t^i}{(\partial \alpha_t^i)^2} = 2\lambda_{\alpha,t}(p_t^i, r_t^i, p_t^j, r_t^j) - 2/\sigma,$$

$$\partial^2 \overline{\pi}_t^i/\partial r_t^i \partial p_t^i \simeq 0$$

$$\partial^2 \pi_t^i/\partial p_t^i \partial r_t^i \simeq 0$$

$$\frac{\partial \overline{\pi}_t^i}{\partial p_t^i \partial \alpha_t^i} \simeq [2(p_t^i - c)\alpha_t^i + ((p_t^i - c - m_t)(\alpha_t^j - \alpha_t^i) + 2\alpha_t^i r_t^j)]\left(-\frac{1}{2b}\right) \equiv \lambda_{p^i,t}(p_t^i, r_t^j, \alpha_t^i)$$

$$\frac{\partial \overline{\pi}_t^i}{\partial r_t^i \partial \alpha_t^i} \simeq [2\alpha_t^i(r_t^i - c) + ((\alpha_t^j - \alpha_t^i)(r_t^i + m_t) + 2p_t^j \alpha_t^i)]\left(-\frac{1}{2e}\right) \equiv \lambda_{r^i,t}(p_t^j, r_t^i, \alpha_t^i),$$

where $\lambda_{\alpha,t} = -cQ_t^{ii} + (c+m_t)Q_t^{ij} - m_t Q_t^{ji} + p_t^j(-Q_t^{jj} + Q_t^{ji}) + r_t^j(-Q_t^{jj} + Q_t^{ij}) + (U_t^{ii} + \widetilde{U}_t^{ii} - U_t^{ji} - \widetilde{U}_t^{ij}) - (U_t^{ij} + \widetilde{U}_t^{ji} - U_t^{jj} - \widetilde{U}_t^{jj})$ *is a bounded function.*

Proof. Using the market share definition, we can rewrite the second-period profits in terms of p_2^i, r_2^i and α_2^i :

$$\begin{aligned}\overline{\pi}_t^i(p_t^i, r_t^i, \alpha_t^i) &= \alpha_t^i\{-c\alpha_t^i Q_t^{ii} - (c+m_2)\alpha_t^j Q_t^{ij} + \alpha_t^j m_t Q_t^{ji} + p_t^j(\alpha_t^j Q_t^{jj} \\ &\quad + \alpha_t^i Q_t^{ji}) + r_t^j(\alpha_t^j Q_t^{jj} + \alpha_t^i Q_t^{ij}) + \alpha_t^i(U_t^{ii} + \widetilde{U}_t^{ii} - U_t^{ji} \\ &\quad - \widetilde{U}_t^{ij}) + \alpha_t^j(U_t^{ij} + \widetilde{U}_t^{ji} - U_t^{jj} - \widetilde{U}_t^{jj}) + F_t^j - \frac{1}{\sigma}\left(\alpha_t^i - \frac{1}{2}\right) \\ &\quad + (2\alpha_{t-1}^i - 1)s - f\}\end{aligned} \tag{33}$$

From expression (9) we have that:

$$\begin{aligned}\frac{\partial \overline{\pi}_t^i}{\partial p_t^i} &= \alpha_t^i\{(-\alpha_t^j(c+m_t) + \alpha_t^j p_t^i + \alpha_t^i r_t^j)\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} \\ &\quad + \alpha_t^i\left(-c + p_t^i\right)\frac{\partial Q(p_t^i, r_t^i)}{\partial p_t^i} + \alpha_t^i\left(\frac{\partial \widetilde{U}(p_t^i, r_t^i)}{\partial p_t^i} - \frac{\partial \widetilde{U}(p_t^i, r_t^j)}{\partial p_t^i}\right)\}\end{aligned} \tag{34}$$

$$\begin{aligned}\frac{\partial^2 \overline{\pi}_t^i}{(\partial p_t^i)^2} &= \alpha_t^i\{(-\alpha_t^j(c+m_2) + \alpha_t^j p_t^i + \alpha_t^i r_t^j)\frac{\partial Q^2(p_t^i, r_t^j)}{\left(\partial p_t^i\right)^2} + \alpha_t^j\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} \\ &\quad + \alpha_t^i\frac{\partial Q(p_t^i, r_t^i)}{\partial p_t^i} + \alpha_t^i(-c + p_t^i)\frac{\partial Q^2(p_t^i, r_t^i)}{(\partial p_t^i)^2}\}\end{aligned}$$

$$\begin{aligned}\frac{\partial \overline{\pi}_t^i}{\partial r_t^i} &= \alpha_t^i\{\alpha_t^i\left(-c + r_t^i\right)\frac{\partial Q(p_t^i, r_t^i)}{\partial r_t^i} + (\alpha_t^j(m_t + r_t^i) + \alpha_t^i p_t^j)\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} \\ &\quad + \alpha_t^i\left(\frac{\partial U(p_t^i, r_t^i)}{\partial r_t^i} - \frac{\partial U(p_t^j, r_t^i)}{\partial r_t^i}\right)\}\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{\partial^2 \overline{\pi}_t^i}{(\partial r_t^i)^2} &= \alpha_t^i\{\alpha_t^i\frac{\partial Q(p_t^i, r_t^i)}{\partial r_t^i} + \alpha_t^i\left(-c + r_t^i\right)\frac{\partial Q^2(p_t^i, r_t^i)}{(\partial r_t^i)^2} + \alpha_t^j\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} \\ &\quad + (\alpha_t^j(m_t + r_t^i) + \alpha_t^i p_t^j)\frac{\partial^2 Q(p_t^j, r_t^i)}{(\partial r_t^i)^2}\}\end{aligned}$$

Then, using Lemma 1:

$$\frac{\partial^2 \overline{\pi}_t^i}{(\partial p_t^i)^2} = \alpha_t^i\{\alpha_t^j\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} + \alpha_t^i\frac{\partial Q(p_t^i, r_t^i)}{\partial p_t^i} - \left(\frac{1}{\Delta b}\right)(p_t^i - c - \alpha_t^j m_t + \alpha_t^i r_t^j)\}$$

$$\frac{\partial^2 \overline{\pi}_t^i}{(\partial r_t^i)^2} = \alpha_t^i\{\alpha_t^i\frac{\partial Q(p_t^i, r_t^i)}{\partial r_t^i} + \alpha_t^j\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} - \left(\frac{1}{\Delta e}\right)(r_t^i - \alpha_t^i c + \alpha_t^j m_t + \alpha_t^i p_t^j)\}$$

Moreover,

$$\begin{aligned}
\frac{\partial^2 \overline{\pi}_t^i}{\partial r_t^i \partial p_t^i} &= \alpha_t^i \{ \alpha_t^i \left(-c + p_t^i\right) \frac{\partial Q^2(p_t^i, r_t^i)}{\partial r_t^i \partial p_t^i} + \alpha_t^i \frac{\partial^2 \widetilde{U}(p_t^i, r_t^i)}{\partial r_t^i \partial p_t^i} \} \\
&= \frac{(\alpha_t^i)^2}{\Delta e}(p_t^i + r_t^i - c)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \overline{\pi}_t^i}{\partial r_t^j \partial p_t^i} &= \alpha_t^i \{ \alpha_t^i \frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} + (-\alpha_t^j (c + m_t) + \alpha_t^j p_t^i + \alpha_t^i r_t^j) \frac{\partial Q(p_t^i, r_t^j)}{\partial r_t^j \partial p_t^i} \\
&\quad -\alpha_t^i \frac{\partial \widetilde{U}(p_t^i, r_t^j)}{\partial r_t^j \partial p_t^i} \} \\
&= \alpha_t^i \{ \alpha_t^i \frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} + \frac{(-\alpha_t^j (c + m_t) + \alpha_t^j p_t^i)}{\Delta e} \}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \overline{\pi}_t^i}{\partial p_t^i \partial r_t^i} &= \alpha_t^i \{ \alpha_t^i \left(-c + r_t^i\right) \frac{\partial^2 Q(p_t^i, r_t^i)}{\partial p_t^i \partial r_t^i} + \alpha_t^i \frac{\partial^2 U(p_t^i, r_t^i)}{\partial p_t^i \partial r_t^i} \} \\
&= \frac{(\alpha_t^i)^2}{\Delta b}(p_t^i + r_t^i - c)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \overline{\pi}_t^i}{\partial p_t^j \partial r_t^i} &= \alpha_t^i \{ \alpha_t^i \frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} + (\alpha_t^j (m_t + r_t^i) + \alpha_t^i p_t^j) \frac{\partial Q(p_t^j, r_t^i)}{\partial p_t^j \partial r_t^i} \\
&\quad -\alpha_t^i \frac{\partial U(p_t^j, r_t^i)}{\partial p_t^j \partial r_t^i} \} \\
&= \alpha_t^i \{ \alpha_t^i \frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} + \frac{\alpha_t^j (m_t + r_t^i)}{\Delta b} \}
\end{aligned}$$

On the other hand,

$$\frac{\partial \overline{\pi}_t^i}{\partial \alpha_t^i} = \frac{\overline{\pi}_t^i}{\alpha_t^i} + \alpha_t^i \{ \lambda_{\alpha,t}(p_t^i, r_t^i, p_t^j, r_t^j) - \frac{1}{\sigma} \},$$

where

$$\begin{aligned}
\lambda_{\alpha,t} &= -cQ_t^{ii} + (c + m_t) Q_t^{ij} - m_t Q_t^{ji} + p_t^j(-Q_t^{jj} + Q_t^{ji}) + r_t^j(-Q_t^{jj} + Q_t^{ij}) \\
&\quad +(U_t^{ii} + \widetilde{U}_t^{ii} - U_t^{ji} - \widetilde{U}_t^{ij}) - (U_t^{ij} + \widetilde{U}_t^{ji} - U_t^{jj} - \widetilde{U}_t^{jj})
\end{aligned}$$

And,

$$\frac{\partial^2 \overline{\pi}_t^i}{(\partial \alpha_t^i)^2} = 2\lambda_{\alpha,t}(p_t^i, r_t^i, p_t^j, r_t^j, \alpha_t^i) - 2/\sigma,$$

Moreover,

$$\begin{aligned}
\frac{\partial \pi_t^i}{\partial p_t^i \partial \alpha_t^i} &= 2(p_t^i - c)\alpha_t^i \frac{\partial Q(p_t^i, r_t^i)}{\partial p_t^i} + ((p_t^i - c - m_t)(\alpha_t^j - \alpha_t^i) + 2\alpha_t^i r_t^j)\frac{\partial Q(p_t^i, r_t^j)}{\partial p_t^i} \\
&\quad +2\alpha_t^i \left(\frac{\partial \widetilde{U}(p_t^i, r_t^i)}{\partial p_t^i} - \frac{\partial \widetilde{U}(p_t^i, r_t^j)}{\partial p_t^i} \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_t^i}{\partial r_t^i \partial \alpha_t^i} &= 2\alpha_t^i (r_t^i - c) \frac{\partial Q(p_t^i, r_t^i)}{\partial r_t^i} + ((\alpha_t^j - \alpha_t^i)(r_t^i + m_t) + 2p_t^j \alpha_t^i)\frac{\partial Q(p_t^j, r_t^i)}{\partial r_t^i} \\
&\quad +2\alpha_t^i \left(\frac{\partial U(p_t^i, r_t^i)}{\partial r_t^i} - \frac{\partial U(p_t^j, r_t^i)}{\partial r_t^i} \right),
\end{aligned}$$

where:

$$\left(\frac{\partial \widetilde{U}(p_t^i, r_t^j)}{\partial p_t^i} - \frac{\partial \widetilde{U}(p_t^i, r_t^i)}{\partial p_t^i} \right) = \frac{1}{2e\Delta}[(r_t^j)^2 - (r_t^i)^2] \tag{36}$$

$$\left(\frac{\partial U(p_t^i, r_t^j)}{\partial r_t^i} - \frac{\partial U(p_t^i, r_t^i)}{\partial r_t^i} \right) = \frac{1}{2b\Delta}[(p_t^j)^2 - (p_t^i)^2] \tag{37}$$

Thus, for a large enough Δ and using Lemma 1 it follows the stated results. ∎

Proof. Proposition 1.

We first focus on network $i's$ best response to given prices of the rival: p_t^j, r_t^j and F_t^j. Note first that, for given p_t^i and r_t^i, $\Delta\phi_t \equiv \phi_t^i - \phi_t^j : [0,1] \to R$ is an affine function of the market share at period $t : \Delta\phi_t(\alpha_t^i) = \kappa\alpha_t^i + y$, where κ and y are real numbers. Note further that relevant fixed fees are bounded: given the pair (p_2^i, r_2^i) there exists an upper bound $\overline{F}$ such that $\overline{\alpha}_t^i(\overline{F}) = 0$ and thus $F_t^i > \overline{F}$ cannot be a best response; similarly there exists a lower bound $\underline{F}$ such that $\overline{\alpha}_t^i(\underline{F}) = 1$ and hence for any $F_t^i < \underline{F}$ we still have that $\overline{\alpha}_t^i = 1$ but lower network $i's$ profits, thus $F_t^i < \underline{F}$ cannot be a best response. Therefore, the F_t^i that can be a best response to the triple (p_t^j, r_t^j, F_t^j), for given (p_t^i, r_t^i), belongs to the interval $[\underline{F}, \overline{F}]$ (see figure below.) Accordingly, for given (p_t^i, r_t^i), $\overline{\alpha}_t^i : [\underline{F}, \overline{F}] \longrightarrow [0,1]$ is one-to-one or injective in F_t^i iff $\kappa \neq 1/\sigma$:

$$\overline{\alpha}_t^i = \frac{1}{1-\sigma\kappa}\left(\frac{1}{2} + (2\alpha_1^i - 1)\sigma s + \sigma(F_t^j - F_t^i + y)\right),$$

that is, $\overline{\alpha}_t^i$ is well-defined and monotonically increasing or monotonically decreasing in F_t^i iff $\kappa \neq 1/\sigma$. The *degenerate case* where $\kappa = 1/\sigma$ could exist for given usage prices and σ, however as long as q and $\widetilde{q}$ are bounded, which is assumed, there will always be a small enough σ such that this *degenerate case* cannot occur. Consequently, for a small enough σ, $\overline{\alpha}_t^i$ is well-defined and injective, and thus invertible on its domain; its inverse $\overline{\alpha}_t^{i-1} = \overline{F}_t^i$ is then uniquely defined.

Each network i maximizes π_2^i with respect to p_2^i, r_2^i and F_2^i, for given p_2^j, r_2^j

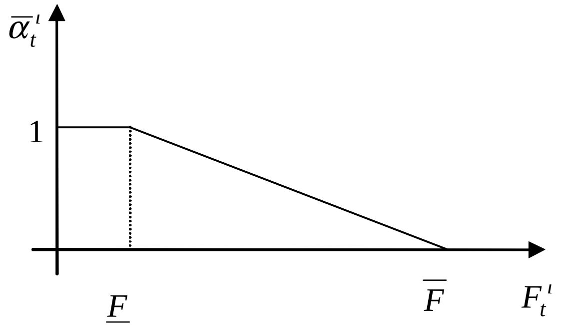


FIG. 1 .

and F_2^j, subject to $\alpha_2^i = (1/2) + (2\alpha_1^i - 1)\sigma s + \sigma(\phi_2^i - F_2^i - \phi_2^j + F_2^j)$, where π_2^i is given by (8) and ϕ_2^i is given by (7). Using the market share definition, we can rewrite the second-period profits in terms of p_2^i, r_2^i and α_2^i : $\overline{\pi}_2^i(p_2^i, r_2^i, \alpha_2^i)$, which is given in (33). Moreover, since for any (p_t^i, r_t^i) $\overline{\alpha}_2^i$ is one-to-one, for given p_2^j, r_2^j and F_2^j, maximizing π_2^i with respect to p_2^i, r_2^i and F_2^i is equivalent to maximizing $\overline{\pi}_2^i$ with respect to p_2^i, r_2^i and α_2^i; that is, there exists a one-to-one correspondence between both best response correspondences (p_2^i, r_2^i, F_2^i) and $(p_2^i, r_2^i, \alpha_2^i)$, to a given triple (p_2^j, r_2^j, F_2^j). Now, we check whether such a best response correspondence $(p_2^i, r_2^i, \alpha_2^i)$ is well-defined, in other words whether the Hessian of the network $i's$ profit function $\overline{\pi}_2^i$ is negative definite:

$$H^i = \begin{bmatrix} \frac{\partial^2 \overline{\pi}_2^i}{(\partial p_2^i)^2} & \frac{\partial^2 \overline{\pi}_2^i}{\partial r_2^i \partial p_2^i} & \frac{\partial^2 \overline{\pi}_2^i}{\partial \alpha_2^i \partial p_2^i} \\ \frac{\partial^2 \overline{\pi}_2^i}{\partial p_2^i \partial r_2^i} & \frac{\partial^2 \overline{\pi}_2^i}{(\partial r_2^i)^2} & \frac{\partial^2 \overline{\pi}_2^i}{\partial \alpha_2^i \partial r_2^i} \\ \frac{\partial^2 \overline{\pi}_2^i}{\partial p_2^i \partial \alpha_2^i} & \frac{\partial^2 \overline{\pi}_2^i}{\partial r_2^i \partial \alpha_2^i} & \frac{\partial^2 \overline{\pi}_2^i}{(\partial \alpha_2^i)^2} \end{bmatrix}$$

Let H_k^i denote the $k-th$ principal minor of the Hessian matrix H^i. Using Lemma 2, for a large enough Δ, we have that $\left|H_1^i\right| \simeq -\alpha_2^i/2b$ and $\left|H_2^i\right| \simeq (\alpha_2^i)^2/4be$, moreover

$$\left|H_3^i\right| \simeq \lambda_{\alpha,2}\frac{(\alpha_2^i)^2}{2be} - \frac{1}{\sigma}\frac{(\alpha_2^i)^2}{2be} + \frac{\alpha_2^i}{2e}(\lambda_{p^i,2})^2 + \frac{\alpha_2^i}{2b}\left(\lambda_{r^i,2}\right)^2$$

Then, for any $\alpha_2^i \in (0,1]$ and a large enough Δ : $\left|H_1^i\right| < 0$ and $\left|H_2^i\right| > 0$, moreover since demands are bounded by assumption, λ_α, λ_{p^i} and λ_{r^i} are also bounded functions, and hence there exists a small enough σ such that $\left|H_3^i\right| < 0$. Let us now show that no cornered-market equilibrium exists. Suppose that network i corners the market by setting $(p_2^{i\,*}, r_2^{i\,*}, F_2^{i\,*})$. Then, $\pi_2^j = 0$ and using (8): $\pi_2^{i\,*} = [(p_2^{i\,*} - c + r_2^{i\,*})Q(p_2^{i\,*}, r_2^{i\,*}) + F_2^{i\,*} - f]$, with $\pi_2^{i\,*} \geq 0$, otherwise cornering the market would not be an optimal strategy. But network j could charge $p_2^{j\,*} = p_2^{i\,*}$, $r_2^{j\,*} = r_2^{i\,*}$

and $F_2^{j\,*} = F_2^{i\,*} + \epsilon$, where $\epsilon > 0$. It follows that $\alpha_2^{j\,*} = (1/2) + (2\alpha_1^j - 1)\sigma s - \sigma\epsilon$, and if $\alpha_1^j = 0$ we have that $\alpha_2^{j\,*} = (1/2)(1 - s/\tau) - \sigma\epsilon$, then since $s < \tau$ it exists a small enough and positive ϵ such that $\alpha_2^{j\,*} > 0$ for any $\alpha_1^j \in [0,1]$. It follows that for such a small enough ϵ and using (8), the network $j's$ profits would then be

$$\begin{aligned}
\pi_2^j &= \alpha_2^{j\,*}[(p_2^{j\,*} - c + r_2^{j\,*})Q(p_2^{j\,*}, r_2^{j\,*}) + F_2^{j\,*} - f] \\
&= \alpha_2^{j\,*}(\pi_2^{i\,*} + \epsilon) \geq \alpha_2^{j\,*}\epsilon > 0,
\end{aligned}$$

a contradiction. In summary, for a large enough Δ there exists a small enough σ such that profit functions are strictly concave whatever the rival prices are, which means that the network $i's$ best response is a continuos function. Therefore, any candidate equilibrium must satisfy the first-order conditions, and any solution that satisfy the first-order conditions is an equilibrium. The set of first-order conditions can be written as follows:

$$\begin{array}{ll}
\frac{\partial \pi_2^i}{\partial p_2^i}(p_2^i, r_2^i, \alpha_2^i, r_2^j) = 0 \quad (C.1), & \frac{\partial \pi_2^j}{\partial p_2^j}(p_2^j, r_2^j, \alpha_2^j, r_2^i) = 0 \quad (C.3), \\
\frac{\partial \pi_2^i}{\partial r_2^i}(p_2^i, r_2^i, \alpha_2^i, p_2^j) = 0 \quad (C.2), & \frac{\partial \pi_2^j}{\partial r_2^j}(p_2^j, r_2^j, \alpha_2^j, p_2^i) = 0 \quad (C.4),
\end{array}$$

$$\frac{\partial \pi_2^i}{\partial \alpha_2^i}(p_2^i, r_2^i, \alpha_2^i, p_2^j, r_2^j, F_2^j) = 0, \quad \frac{\partial \pi_2^j}{\partial \alpha_2^j}(p_2^j, r_2^j, \alpha_2^j, p_2^i, r_2^i, F_2^i) = 0.$$

Together with the market share definitions, we have 8 equations and 8 unknown variables. Consider the first four first-order conditions derived from maximizing profits with respect to usage prices *(C.1-C.4),* notice that fixed fees do not enter these conditions (as can been seen from (34) and (35).) Using lemma 1 and expressions (36) and (37) we can write:

$$\begin{aligned}
\frac{\partial \pi_2^i}{\partial p_2^i} &= -\alpha_2^i\left(\frac{1}{2b}\xi_p^i(p_2^i, r_2^j) + \frac{1}{\Delta}\omega_p^i(p_2^i, r_2^i, r_2^j)\right), \\
\frac{\partial \pi_2^i}{\partial r_2^i} &= -\alpha_2^i\left(\frac{1}{2e}\xi_r^i(r_2^i, p_2^j) + \frac{1}{\Delta}\omega_r^i(r_2^i, p_2^i, p_2^j)\right), \\
\frac{\partial \pi_2^j}{\partial p_2^j} &= -\alpha_2^j\left(\frac{1}{2b}\xi_p^j(p_2^j, r_2^i) + \frac{1}{\Delta}\omega_p^j(p_2^j, r_2^j, r_2^i)\right), \\
\frac{\partial \pi_2^j}{\partial r_2^j} &= -\alpha_2^j\left(\frac{1}{2e}\xi_r^j(r_2^j, p_2^i) + \frac{1}{\Delta}\omega_r^j(r_2^j, p_2^j, p_2^i)\right),
\end{aligned}$$

where $\xi_p^i(p_2^i, r_2^j) = -c - \alpha_2^j m_2 + p_2^i + \alpha_2^j r_2^j$, $\xi_r^i(r_2^i, p_2^j) = -\alpha_2^i c + \alpha_2^j m_2 + r_2^i + \alpha_2^i p_2^j$, and ω_p^i and ω_r^i are nonlinear functions that do not depend on Δ. That is, each one of these equations can be written as the sum of a linear function (ξ) and a nonlinear function (ω). Moreover, this system of equations have at least one solution, which is given by $p_2^i = p_2^j = c + m_2$, and $r_2^i = r_2^j = -m_2$, and where

$\xi_p^i = \xi_p^j = 0$, $\omega_p^i = \omega_p^j = 0$, $\xi_r^i = \xi_r^j = 0$, $\omega_r^i = \omega_r^j = 0$. Let Ξ denote the set of solutions to the system (C.1-C.4), which we already know is non-empty. Note that by increasing Δ the nonlinear components of this system tend to vanish, indeed the non-linear equations tend to be linear as Δ increases. Therefore, by assuming a large enough Δ we can make vanish all those solutions that might come from nonlinearities and thereby make Ξ tend to be finite and have *at most* one element or to have infinite elements, which is/are the solution/s that would come from the linear system: $\xi_p^i(p_2^i, r_2^j) = 0$, $\xi_r^i(r_2^i, p_2^j) = 0$, $\xi_p^j(p_2^j, r_2^i) = 0$, $\xi_r^j(r_2^j, p_2^i) = 0$. Indeed, we know that for a large enough Δ the set Ξ is non-empty nor infinite but tend to a singleton since $\xi_p^i(c+m_2, -m_2) = \xi_p^j(c+m_2, -m_2) = 0$ and $\xi_r^i(-m_2, c+m_2) = \xi_r^j(-m_2, c+m_2) = 0$. Therefore, for any $\alpha_2^i \in (0,1)$ and a large enough Δ there exists a unique equilibrium in usage prices, where networks price calls at their off-net cost. Let us now return to the original formulation of the profit function that is given in (8) and where the strategic variables are p_2^i, r_2^i and F_2^i. Substituting $p_2^i = p_2^j = c + m_2$ and $r_2^i = r_2^j = -m_2$ into (8) gives us the expression (16). By maximizing this expression with respect to the network $i's$ fixed fee we obtain linear reaction functions: $F_2^i(F_2^j)$, which are given in (17). Moreover, $dF_2^i/dF_2^j = 1/2$, therefore there exists a unique equilibrium in fixed fees that is given in (18). ∎

Proof. Proposition 2.

Following Proposition 1 and using the market share definition, we can rewrite the first-period profits in terms of p_1^i, r_1^i and α_1^i. Moreover, since for a small enough σ, $\overline{\alpha}_1^i$ is one-to-one for any (p_1^i, r_1^i) and given p_2^j, r_2^j and F_2^j, maximizing π_1^i with respect to p_1^i, r_1^i and F_1^i is equivalent to maximizing $\overline{\pi}_1^i$ with respect to p_1^i, r_1^i and α_1^i; that is, there exists a one-to-one correspondence between both best response correspondences (p_1^i, r_1^i, F_1^i) and $(p_1^i, r_1^i, \alpha_1^i)$, to a given triple (p_1^j, r_1^j, F_1^j). Hence, we only need to check whether the Hessian of the network $i's$ full-period profit function: $\overline{\Pi}^i(p_1^i, r_1^i, \alpha_1^i) = \overline{\pi}_1^i(p_1^i, r_1^i, \alpha_1^i) + \delta\widehat{\pi}_2^i(\alpha_1^i)$, with $\widehat{\pi}_2^i(\alpha_1^i)$ given by (19), is negative definite. Let H_k^i denote the $k-th$ principal minor of the Hessian matrix. Using Lemma 2, for a large enough Δ, we have that $\left|H_1^i\right| \simeq -\alpha_1^i/2b$ and $\left|H_2^i\right| \simeq (\alpha_1^i)^2/4be$, moreover

$$\left|H_3^i\right| \simeq \lambda_{\alpha,1}\frac{(\alpha_1^i)^2}{2be} - \frac{1}{\sigma}\frac{(\alpha_1^i)^2}{2be} + \delta\sigma\frac{2s^2\left(\alpha_1^i\right)^2}{9be} + \frac{\alpha_1^i}{2e}(\lambda_{p^i,1})^2 + \frac{\alpha_1^i}{2b}\left(\lambda_{r^i,1}\right)^2$$

Then, for any $\alpha_1^i \in (0,1]$ and a large enough Δ : $\left|H_1^i\right| < 0$ and $\left|H_2^i\right| > 0$, and since $\lambda_{\alpha,1}$, $\lambda_{p^i,1}$ and $\lambda_{r^i,1}$ are bounded functions, there exists a small enough σ such that $\left|H_3^i\right| < 0$. We now show that in the first period no cornered-market equilibrium exists if switching costs are small enough. Suppose that network i corners the market by setting $(p_1^{i\,*}, r_1^{i\,*}, F_1^{i\,*})$, then $\Pi^{i\,*} = \pi_1^{i\,*} + \delta\widehat{\pi}_2^i(1)$, where $\widehat{\pi}_2^i(1) = 1/4\sigma - s/3 - \sigma s^2/9$. Note that in order to corner the market network i

must sacrifice present profits so as to build market share. This implies that $\pi_1^{i\ *}$ is lower than the static equilibrium profits, which is always interior. As switching costs decrease, the link between the present and the future vanishes, that is, $\lim_{s\to 0}\widehat{\pi}_2^i(1) = \widehat{\pi}_2^i(1/2) = 1/4\sigma$. Thus, $\lim_{s\to 0}\Pi^{i\ *} = \pi_1^{i\ *} + \delta/4\sigma < \lim_{s\to 0}\Pi = 1/4\sigma + \delta/4\sigma$. Finally, notice that $\partial\overline{\Pi}^i/\partial p_1^i = \partial\overline{\pi}_1^i/\partial p_1^i$ and $\partial\overline{\Pi}^i/\partial r_1^i = \partial\overline{\pi}_1^i/\partial r_1^i$, therefore we can construct a system of equations similar to the system $C.1 - C.4$ given in the proof of proposition 1 with the unique difference that the time index subscript takes now value 1. Then, following a similar reasoning to that used in the proof of proposition 1, one can show that for a large enough Δ there exists a unique equilibrium in usage prices, which is given by $p_1^i = p_1^j = c + m_1$ and $r_1^i = r_1^j = -m_1$, and hence do not depend on the level of the market shares. Given this, we can return to the original formulation of the full-period profit function that is given in (20) and where the strategic variables are p_1^i, r_1^i and F_1^i. By substituting the equilibrium usage prices into (20) we obtain the expression (21). Finally, maximizing this expression with respect to the network $i's$ fixed fee yields linear reaction functions $F_1^i(F_1^j)$ that are given in (23) and have got a unique intersection point that is given in (24). ∎

Proof. Proposition 3.

Let $s_{t+1}^i(m,n)$ denote the $(m,n).th$ entry of the matrix S_{t+1}^i. The following lemma will be needed: ∎

LEMMA 3. $\lim_{\sigma\to 0} s_t^i(3,3) = 0$ *and* $\lim_{\sigma\to 0} s_t^j(3,3) = 0$ $\forall t$.

Proof. By matrix computation we can show that

$$\text{if } S_{t+1}^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & s_{t+1}^i(3,3) & s_{t+1}^i(3,4) \\ 0 & 0 & s_{t+1}^i(4,3) & s_{t+1}^i(4,4) \end{bmatrix} \text{ then } S_t^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & s_t^i(3,3) & s_t^i(3,4) \\ 0 & 0 & s_t^i(4,3) & s_t^i(4,4) \end{bmatrix},$$

where

$$s_t^i(3,3) = \frac{-4\sigma s^2\left[-2 + \sigma\delta s_{t+1}^i(3,3)\right]}{\left[-3 + \sigma\delta(s_{t+1}^j(3,3) + s_{t+1}^i(3,3))\right]^2}.$$

Thus if $\lim_{\sigma\to 0} s_{t+1}^i(3,3) = 0$ then $\lim_{\sigma\to 0} s_t^i(3,3) = 0$. Now, noting that $s_T^i(3,3) = (8/9)\sigma s^2$ the lemma is proved by mathematical induction. ∎

Our candidate equilibrium is $p_t^{i\ *} = p_t^{j\ *} = c + m_t$, $r_t^{i\ *} = r_t^{j\ *} = -m_t$, and $(F_t^{i\ *}, F_t^{j\ *})$, which are given by (27) if $|\Phi_t B| \neq 0$ for $t = 1, ...T$, which is satisfied since reaction functions have different slopes. Thus there exists a unique closed-loop sequence of candidate equilibria, and hence $F_t^{i\ *}(\alpha_{t-1}^i) \equiv x(1)$, which is given by (27), is uniquely determined and define $\widetilde{V}_t^i(\alpha_{t-1}^i)$, which is given by (26). Therefore, $d\widetilde{V}_{t+1}^i/d\alpha_t^i = s_{t+1}^i(3,3)\alpha_t^i + s_{t+1}^i(3,4)$. The proposition will be proved by mathematical induction. First, assume there exists an equilibrium in any period $t+1$, so that $\widetilde{V}_{t+1}$ is a true valuation function, using lemma 2 and for a large enough Δ, we have that $\left|\left(H_t^i\right)_1\right| \simeq -\alpha_t^i/2b$, $\left|\left(H_t^i\right)_2\right| \simeq (\alpha_t^i)^2/4be$, and

$$\begin{aligned}\left|\left(H_t^i\right)_3\right| &\simeq \lambda_{\alpha,t}\frac{(\alpha_t^i)^2}{2be} - \frac{1}{\sigma}\frac{(\alpha_t^i)^2}{2be} + \delta\frac{(\alpha_t^i)^2}{4be}\frac{d^2\widetilde{V}_{t+1}^i}{(d\alpha_t^i)^2} \\ &\quad + \frac{\alpha_t^i}{2e}(\lambda_{p^i,t})^2 + \frac{\alpha_{T-1}^i}{2b}\left(\lambda_{r^i,t}\right)^2,\end{aligned}$$

where $d^2\widetilde{V}_{t+1}^i/(d\alpha_t^i)^2 = s_{t+1}^i(3,3)$. Thus for any $\alpha_t^i \in (0,1]$ and a large enough Δ : $\left|\left(H_t^i\right)_1\right| < 0$ and $\left|\left(H_t^i\right)_2\right| > 0$, and since $\lambda_{\alpha,t}, \lambda_{p^i,t}$ and $\lambda_{r^i,t}$ are bounded functions, and by lemma 3 $\lim_{\sigma\to 0} s_{t+1}^i(3,3) = 0$, there exists a (positive) small enough σ such that $\left|\left(H_t^i\right)_3\right| < 0$. In short, given that there exists an interior equilibrium in period $t+1$, we can construct the Hessian matrix of the network $i's$ profit function in period t, and by assuming *i)* a large enough Δ obtain that for any $\alpha_t^i \in (0,1]$: $\left|(H_t^i)_1\right| < 0$, $\left|(H_t^i)_2\right| > 0$, and *ii)* a (positive) small enough σ obtain that $\left|\left(H_t^i\right)_3\right| < 0$ since $\lambda_{\alpha,t}$, $\lambda_{p^i,t}$ and $\lambda_{r^i,t}$ are bounded functions, and $\lim_{\sigma\to 0} s_{t+1}^i(3,3) = 0$. Therefore, the existence of this candidate equilibrium can be proved by mathematical induction as long as we prove its existence in the last period of the game. In this respect, using proposition 1 we have that for a (positive) small enough σ and a large enough Δ there exists a unique equilibrium in period T, which is interior.

Proof. Proposition 4.

Making use of the proof of Lemma 2 we can write:

$$\frac{\partial\widetilde{U}^{ij}}{\partial p_t^i} = -\left(\frac{1}{b\Delta^2}\right)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}[\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} d - e\left(\frac{a - p_t^i + \varepsilon}{b}\right) + \widetilde{\varepsilon}d\varepsilon]d\widetilde{\varepsilon},$$

where $\varepsilon^* = (b/e)(d - r_t^j + \widetilde{\varepsilon}) + p_t^i - a$. Note that,

$$\begin{aligned}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)}\widetilde{\varepsilon}d\varepsilon d\widetilde{\varepsilon} &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\widetilde{\varepsilon}(\varepsilon^* - \underline{\varepsilon})d\widetilde{\varepsilon} \\ &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\widetilde{\varepsilon}\left[(\frac{b}{e})(d - r_t^j + \widetilde{\varepsilon}) + p_t^i - a - \underline{\varepsilon}\right]d\widetilde{\varepsilon} \\ &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\frac{b}{e}\widetilde{\varepsilon}^2 + \widetilde{\varepsilon}\left[(\frac{b}{e})(d - r_t^j + \widetilde{\varepsilon}) + p_t^i - a - \underline{\varepsilon}\right]d\widetilde{\varepsilon} \\ &= \frac{b}{e}\frac{\overline{\varepsilon}^3 - \underline{\varepsilon}^3}{3} \\ &= \frac{2b}{24e}\Delta^3\end{aligned}$$

On the other hand,

$$
\begin{aligned}
\xi &\equiv \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} [\int_{\underline{\varepsilon}}^{\varepsilon^*(\cdot)} d - e\left(\frac{a-p_t^i+\varepsilon}{b}\right) d\varepsilon] d\widetilde{\varepsilon} \\
&= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(d - \frac{e}{b}(a-p_t^i)\right)(\varepsilon^* - \underline{\varepsilon}) - \frac{e}{b}\frac{(\varepsilon^*)^2 - \underline{\varepsilon}^2}{2} d\widetilde{\varepsilon} \\
&= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left[d - \frac{e}{b}(a-p_t^i) - \frac{e}{2b}(\varepsilon^* + \underline{\varepsilon})\right](\varepsilon^* - \underline{\varepsilon}) d\widetilde{\varepsilon}
\end{aligned}
$$

Replacing the definition of ε^* into last expression yields

$$
\xi = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} [y + \frac{e}{2b}\frac{\Delta}{2} - \frac{\widetilde{\varepsilon}}{2}][v + \frac{\Delta}{2} + \frac{b}{e}\widetilde{\varepsilon}] d\widetilde{\varepsilon},
$$

where $y = (e/2b)(p_t^i - a) + (d + r_t^j)/2$ and $v = (b/e)(d - r_t^j) + p_t^i - a)$. Then,

$$
\xi = yv\Delta + \frac{y}{2}\Delta^2 + \frac{ev}{4b}\Delta^2 + \frac{e}{8b}\Delta^3 - \frac{b}{24e}\Delta^3
$$

And,

$$
\begin{aligned}
\frac{\partial \widetilde{U}^{ij}}{\partial p_t^i} &= -\left(\frac{1}{b\Delta^2}\right)\left[\xi + \frac{2b}{24e}\Delta^3\right] \\
&= -\left(\frac{1}{b\Delta^2}\right)\left[yv\Delta + \frac{y}{2}\Delta^2 + \frac{ev}{4b}\Delta^2 + \frac{\Delta^3}{8}\left(\frac{e}{b} + \frac{b}{3e}\right)\right] \\
&= -\left(\frac{1}{b}\right)\left[\frac{yv}{\Delta} + \frac{y}{2} + \frac{ev}{4b} + \frac{\Delta}{8}\left(\frac{e}{b} + \frac{b}{3e}\right)\right]
\end{aligned}
$$

Thus, for a large enough Δ we can write

$$
\frac{\partial \widetilde{U}^{ij}}{\partial p_t^i} \simeq -\left(\frac{1}{b}\right)\left[\frac{e}{2b}(p_t^i - a) + \frac{d}{2} + \frac{\Delta}{8}\left(\frac{e}{b} + \frac{b}{3e}\right)\right]
$$

In equilibrium: $p_t^i = c + m_t$, which proves the first part of the proposition. Last, using the same steps as before one can show that for a large enough Δ :

$$
\frac{\partial U^{ij}}{\partial r_t^j} \simeq -\left(\frac{1}{e}\right)\left[\frac{b}{2e}(r_t^j - d) + \frac{a}{2} + \frac{\Delta}{8}\left(\frac{b}{e} + \frac{e}{3b}\right)\right],
$$

and using that in equilibrium $r_t^j = -m_t$ it is proved the second part of the proposition. ∎

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