Quasi–Monte Carlo Methods in Stochastic Simulations*

An Application to Fiscal Policy Simulations using an Aggregate Disequilibrium Model of the West German Economy 1960–1994

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Abstract

Different stochastic simulation methods are used in order to check the robustness of the outcome of policy simulations with a macroeconometric model.

A macroeconometric disequilibrium model of the West German economy is used to analyze a reform proposal for the tax system. The model was estimated with quarterly data for the period 1960 to 1994, the presently possible margin.

Because of nonlinearities confidence intervals for the simulation results have to be obtained by means of stochastic simulations.

The main contribution of this paper consists in presenting the simulation results. The robustness of these results is analyzed using different approaches to stochastic simulation. In particular, different methods for the generation of uniform error terms and their conversion to normal variates are applied. These methods include standard approaches as well as quasi–Monte Carlo methods.

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1 Introduction

The present study extends earlier work of the authors in two directions. As in Schellhorn and Winker (1994) a stochastic simulation framework is used to assess the significance of the outcomes of some policy simulations within a macroeconometric model. However, the simulation methodologies are analyzed in much more detail. In particular, the role of the error generation mechanism is emphasized. On the other side, the most recent version of a macroeconometric disequilibrium model for the West German economy is used for the application. A full documentation of this model version can be found in Franz, Göggelmann and Winker (1996).

In this version, the model covers the period 1960 to 1994 with quarterly data. Hence, it includes the first years after German reunification, but keeps being restricted to the West German economy due to data limitations. As the German federal statistical office stops producing regionally disaggregated data for the post 1994 period, the next update of the model has to consider Germany as a whole after 1989.

Besides the extension of the sample period the model has been extended by the inclusion of important aspects of fiscal and monetary policy. However, as in Schellhorn and Winker (1994), the main goal of this paper is not a detailed description of the model and potential future developments, but a careful analysis of its simulation properties and of the instrument of stochastic simulation itself. The macroeconometric disequilibrium model and the simulation of different policies with regard to the taxation of labour will be used as a real life application we are interested in.

Nevertheless, first, section 2 is devoted to the description of the main features of the model, or – in other words – presents the aggregated disequilibrium model in a nutshell. Then, a discussion of different simulation methods is presented in section 3. A central aspect of stochastic simulation methods, the generation of the error terms, is analyzed in section 4. In particular, we are interested in the question whether deficiencies in the error generation methods may result in misleading simulation outcomes. Consequently, section 5 turns back to the application and compares the results of stochastic simulation for a tax policy experiment using different methods.

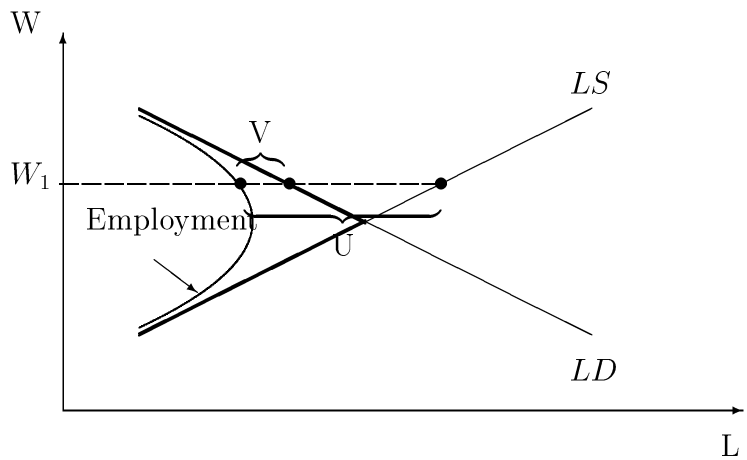
2 A short glance at the model

The macroeconometric disequilibrium model for West Germany used in this paper can be described as an econometric implementation of a temporary equilibria in a rationing context model. It was first developed under the framework of the European Employment Programme.[1] As it is not the aim of this paper to present a detailed description of the model and its functioning,[2] only a sketch of its conception, construction and some chosen results will be given.

2.1 Basic conceptions of the model

Figure 1 provides a comparatively easy first look at the "philosophy" of the model. The strict "minimum condition" on labour and goods market serves as the model's starting point. This means, e.g., that observed employment is given as the minimum of labour supply LS and labour demand LD, and corresponds to the lines in bold. However, this strict minimum condition did not take the coexistence of unemployment and vacancies ("mismatch")into consideration, and resulted therefore in abrupt switches between different disequilibrium situations for the aggregate economy practically from one day to the next.

Figure 1: Minimum condition



Therefore, the generation of disequilibrium models under discussion here restricts the minimum conditions to the "micro–markets", and thus avoids the two above mentioned points of criticism through the use of a suitable aggregation process. One instance of such a micro–market would be a single firm which produces a homogeneous product. Many such micro–markets exist in an economy, whereby each one on its own is characterised either by excess demand or by excess supply. Realistically, the disequilibrium situation ("regime") in a single micro–market could change abruptly. In view of the many micro–markets therefore, regime change takes place continuously and gradually at the aggregate level.

[1] The results are published in Drèze (1990).

[2] The interested reader will find it in Franz, Göggelmann and Winker (1996) and the exhaustive list of references therein.

A mismatch occurs because the coordination between these micro–markets is neither timeless nor perfect. Therefore, in figure 1, actual employment lies under the labour demand curve and above the labour supply curve, illustrating the coexistence of vacancies (V) and unemployed (U) - hence the mismatch.

The derivation and usage of an aggregate method to portray this employment curve, and with it the gradual change of regime, is one of the central building blocks of the model. By the "smoothing by aggregation" method conceived by Lambert (1988) the following aggregate relationship results:[3]

$$LT = [LD^{-\rho} + LS^{-\rho}]^{-1/\rho} \tag{1}$$

where LT denotes aggregate employment, LD labour demand and LS labour supply, and ρ is a measure of the disturbances on the micro–markets and is in inverse proportion to the variance of these disturbances. The advantage of equation (1) to the econometrician is obvious. The excess supply or demand situation on the different micro–markets does not need to be known – the equation can be estimated using the aggregate variables LT, LD and LS exclusively, and ρ is therefore the estimated regression parameter. However, while actual employment LT and – with some reservations – labour supply LS are both observable, there are no reliable aggregate data available on labour demand LD. Thus, LD must be estimated. Using the estimates of (1) it becomes possible to calculate the share of firms for which the employment is limited by labour supply, expected goods demand and capacities, respectively. Then, according to these shares the regime on the labour market can be described as a more demand or supply determined one. A similar model is estimated for the goods market taking into account possible spill–overs between the markets.

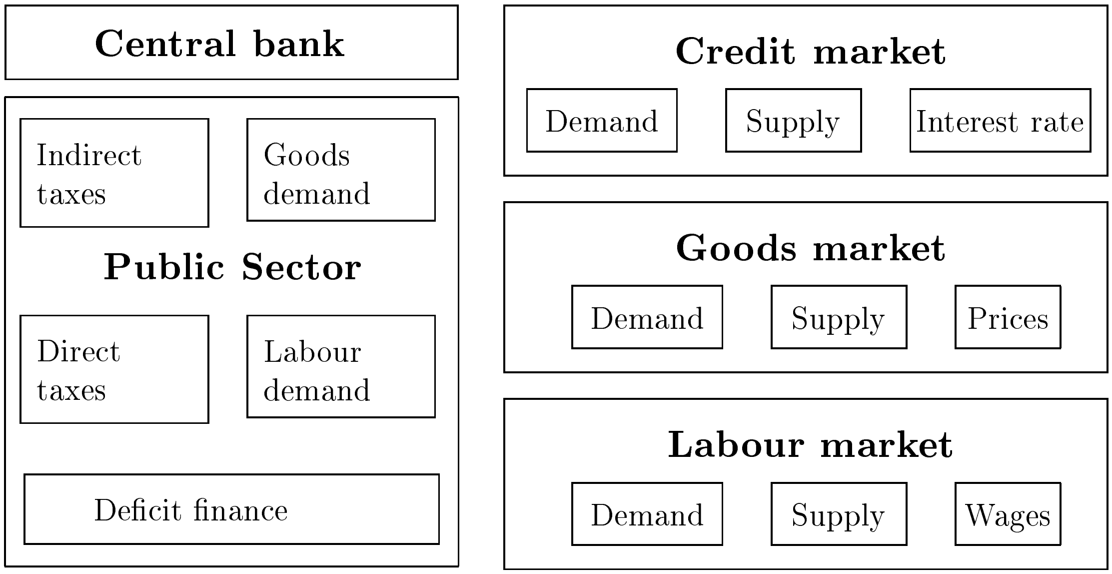
2.2 The structure of the model

Figure 2 gives a schematic illustration of the central building blocks of the model. Alongside the already completed integrated components, the diagram shows extensions of the model, in particular with regard to the behavioural equations of the public sector and central bank, some of which are still the object of on–going research.

Three markets, modeled respectively on demand, supply and price, can be distinguished, namely the goods market including foreign trade, the labour market and the credit market. The state can be said to be already portrayed in that it affects demand in one of the three markets. As the budget constraints and tax revenues of the state bring about a linking of activity in all three markets, total state activity is surmised into a separate sector. The central bank also makes up another major component of the model because of its influence, on the one side,

[3] For the details of the derivation the reader is referred to Smolny (1993).

Figure 2: Central building blocks of the model



over the money market through movements in interest rates, and most likely on the other over the credit market through the availability of credit.

To mirror the explicit dynamic nature of the behavioural equations of the model and to take care of the time series characteristics of the data for most of the equations an error correction framework is used for estimation.

Real aggregate demand Y^d is made up of the demand components consumption, investment, effective exports demand minus effective imports demand, and government expenditures. While it is taken for granted that no rationing of consumption or investment demand will appear, it is assumed that existing disequilibria will probably manifest themselves in the goods market, diverging actual trade flows from their notional goals. Therefore, effective import demand, M^d, minus effective exports, X^d, is seen as relevant for demand. Actual imports, M, are disaggregated into two components – the first is the effective import demand, M^d, and the other is the amount of spill–over imports M^u, induced by excess demand for imports on the home goods market. Because M is the only variable that can be observed, the other components, M^d and M^u, have to be estimated. This is done using the assumption that $M^d = M$ for that time period when the capacity utilization rate in the home country is at its historical minimum.

The optimal factor inputs were derived with the help of a profit–maximising approach, under the assumption of a linear homogeneous CES technology. Due to the medium to long–term nature of the decision process, expectations about the development of relative factor costs had to be formed. This was done using all relevant information available up to the moment of the decision $t - \tau$. As the time of realisation t is known, but not the moment of decision $t - \tau$, adaptive expectations with different values of τ were formed. Finally, the productivity equations were estimated using these expected values and τ was chosen so as to minimize the standard errors of these estimates.

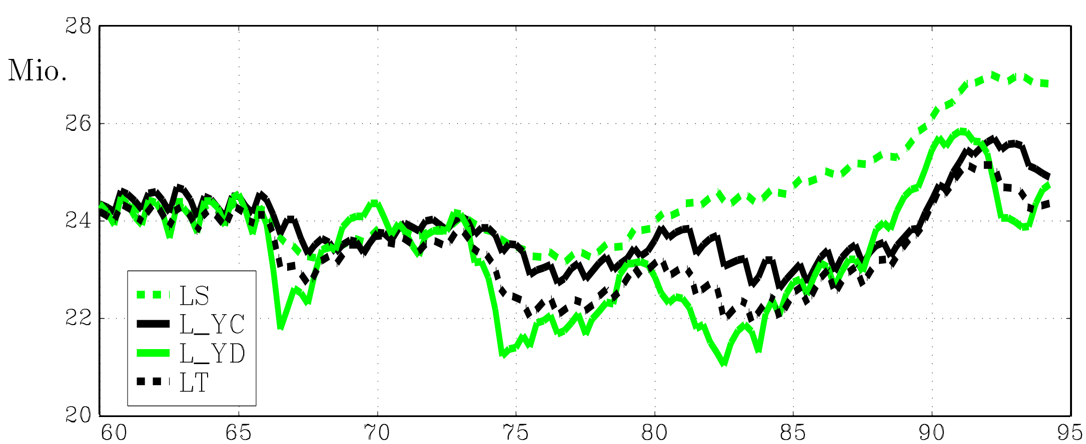
Because of fluctuations in the utilization rate, the technical productivities of labour and capital do not correspond to the observed productivities. They are determined by an adjustment of the observed productivities to the utilization rate.

Many variables important for the model can be determined with the help of productivities. Thus the product of technical labour productivity and employment corresponds to the maximum amount of goods that can be produced given the number of employees. In the short run it represents the aggregate supply of goods. Looked the other way around, the ratio of goods demand and technical labour productivity gives the number of employees L_{Y^d} required for the production of aggregate demand.

2.3 Some estimation results

Based on the sketched conception, the estimation results for the model allow to calculate regime shares for the labour market. The starting point is the single firm at the micro–market level, restricted either by labour supply, expected demand for goods, or by available capacities. The regime share, defined as the firms' share of a regime in proportion to the total number of firms, is calculated by aggregation from the respective employment series L_{Y^c}, L_{Y^d}, LS and LT depicted in figure 3.

Figure 3: Employment series



It is easy to see that, with the exception of the recession in 1967, the employment series ran very close to one another until the beginning of the seventies. Only after the two oil price shocks, the series began to drift apart.

It is also noteworthy that LT always lies below the L_{Y^c} series, which corresponds to optimal employment based on available capacities. This confirms the assumption that labour adjusts quicker in comparison to capital as input factor. In contrast to L_{Y^c}, optimal employment due to available demand L_{Y^d} often falls below LT, indicating a delayed adjustment of labour, as well as labour hoarding.

3 Policy Simulation

3.1 The simulation framework

Any backward looking model such as the macroeconometric disequilibrium model introduced in the previous section can be formally given by a system of, in general non–linear, equations

$$\begin{aligned} y_{i,t} &= F_{i,t}(y_{1,t}, \dots, y_{i-1,t}, y_{i+1,t}, \dots, y_{n,t}, x_t, \beta_t, \varepsilon_{i,t}) \,, \\ & \quad \text{for } i = 1, \dots n, \ \ t = 1, \dots T \,, \end{aligned} \tag{2}$$

where $y_t \equiv (y_{1,t}, \dots, y_{n,t})$ denotes the vector of dimension n of endogenous variables, x_t the vector of predetermined variables including lagged endogenous variables as they appear in the model equations, β_t the coefficient vector, and $\varepsilon_t \equiv (\varepsilon_{1,t}, \dots, \varepsilon_{m,t}, 0, \dots, 0)$ the vector of errors at time t. Without loss of generality, it has been assumed that the first m equations of the system are stochastic,[4] whereas the remaining equations are deterministic, i.e. with $\varepsilon_{i,t}$ identically zero for $i > m$ and all t. These deterministic equations are definitory equalities or equalities derived from national accounts identities. Finally, $F_{i,t}$ describes the functional form of the equations.

In the model used for the analysis in this paper, neither the functional form nor the parameters β_t change over time, hence the subscript t for F_i and β will be skipped for ease of notation. Although being fixed over time, the functional form is not simply linear due to the non–linear specification of the transacted quantities on goods and labour markets and the disequilibrium modelling of the credit market.

Given the functional relationships F_i and values for the variables x_t and y_t the estimation of the model provides a solution to the system of equations (2) in terms of β and ε_t. Sometimes, our interest in the empirical model ends here, if we are interested mainly in a description or diagnosis of the past behaviour of the economy or want to draw inference on some hypotheses based on the estimated sign or value of the coefficients in β.

However, once established that the empirical representation of the economy by the model is adequate after a careful econometric testing procedure and not in too

[4]The version of the disequilibrium model used in this paper contains 29 stochastic equations.

sharp contrast to the theoretical underpinnings, the applied macroeconometrician is interested in further conclusions.

In particular, the effects of changes in some exogenous variables or parameter estimates on the endogenous variables are of central interest. While changes in the exogenous variables might be related to policy measures, a change in some parameter β_i could reflect shifts in behavioural relationships. In both cases, the system (2) has to be solved again for the new set of parameters and values for the exogeneous variables. Due to the non–linearity of the model the calculation of these new solutions, in general, has to rely on numerical methods. As the model under study does not exhibit any particular problems of solvability for reasonable parameter values, we will not go into details of the numerical procedure which essentially is a Gauss–Seidel algorithm.[5]

Our main interest in this paper is in the estimation of the employment effects of some fiscal policy measures related to the taxation of the factor labour. The obvious approach to estimate these effects is to compare a solution to the original system of equations (2) with the solution to a modified system of equations where changes in some exogeneous parameters or variables reflect the different stance of fiscal policy. The differences in the endogenous variables are attributed to the policy effects. However, in order to perform such a simulation on a dynamic non–linear macroeconometric model some additional decisions have to be made. The reason for this necessity lies in the presence of nuisance parameters in the system of equations (2), namely the lagged endogenous variables in x_t, the estimated coefficients β and the stochastic error terms ε_t which influence the outcome of the just described difference simulation.

3.2 Simulation methods

In order to scope with the different nuisance parameters the standard simulation approach has to be adapted. As the model is highly dynamic in its specification, it is certainly not admissible to perform *static simulations* by fixing the lagged values of the endogenous variables to their actual historical values. Instead the simulation has to be performed period by period using the simulated values of the endogenous variables in past periods for the current simulation. In such a *dynamic simulation* any influence on the system can evolve according to the estimated dynamics of the model.

While it seems reasonable to use the simulated values of the endogenous variables of past periods as inputs to the simulation of the current period, such a straightforward solution does not exist for the exogenous variables x_t. Two cases might be distinguished. First, the simulation period can be restricted to a subsample of the estimation period. Then, of course the historically observed values

[5] For larger scale models a reordering of the equation might help to improve performance and achieve convergence. Cf. Gilli and Pauletto (1997).

for x_t can be used in an *ex post simulation*. Second, simulations could be used to predict future developments. For such *ex ante simulations*, forecasts of the x_t have to be produced first. Both kind of simulations rely on the assumption that the system behaves similar in other circumstances caused by changes in exogenous variables or policy measures. As this assumption is even more challengable in ex ante simulations, when the values of the x_t can become quite different from those during the estimation period, we restrict ourselves to ex post simulations for this paper.

A final source of nuisance is given by the error terms ε_t. As by definition we do not have an economic rational for the sign or value of these error terms it is not clear which values to use for simulation purposes. As only differences of simulations are of interest for the study of policy effects one is well out of it if the model is linear as the error terms cancel by taking differences. Consequently, in many if not most simulation studies *deterministic* simulations are performed setting all ε_t to their expected value of zero. Despite the ease of implementation and its merits in the linear case several aspects challenge its usefulness in the general case.

For non–linear and/or dynamic models the solution values of the endogenous variables, in general, are not equal to the mathematical expectation of these variables. Using a deterministic simulation may result in biased outcomes. Furthermore, it is not possible to present statistics on the distribution of the simulated endogenous variables such as standard deviations or confidence intervals which might be of central interest. Hence, in a highly non–linear model as the one used in this study, the use of *stochastic* simulation seems to be adequate. This stochastic simulation with regard to the ε_t is performed by iterating the difference simulation described above for many drawings of the ε_t out of their assumed distribution. Then, an empirical approximation to the conditional distribution of the policy effects can be obtained. The next section is devoted to the approaches and problems of generating the drawings for this kind of stochastic simulation.

Of course, besides a stochastic simulation with regard to the distribution of the error terms ε_t similiar considerations apply to the estimated parameters β, the exogenous variables x_t and misspecifications of the F_i. However, the inclusion of stochastic components in these factors is beyond the scope of this paper.[6]

4 Generation of error terms

The attention of Schellhorn and Winker (1994) was concentrated on the correlation structure of the error terms between stochastic equations. They performed simulations both with error terms assumed to be independent between equations and using the estimated sample covariance structure. The differences in the estimated

[6] For some arguments see Schellhorn and Winker (1994).

means and for some measures of forecast errors were rather small. Therefore, only the version based on the estimated covariance structure is used in this paper.

While neglecting the influence of the covariance structure, more emphasis is given to the generation of the error terms themselves. Ripley (1987) and Winker and Fang (1998) give several examples that the pseudo–random number generators in standard statistical software packges – such as MicroTSP, Eviews or GAUSS used in this paper – may generate artifical correlation structures in higher dimension. As the dimension of our model is 29 stochastic equations times the number of simulation periods, it appears natural to allow for the possibility of such effects. Hence, different methods for the generation of error terms were used.

We start with a standard normal random number generator as is used, e.g. in GAUSS 3.2, which is based on a linear congruential uniform random number generator

$$seed_{n+1} = (a \cdot seed_n + c) \text{mod} M \quad \text{and} \quad x_n = seed_n / M \, ,$$

where M is a positive (large) integer, $1 \leq a < M$ without common divisor with M and $0 \leq c, seed_0 \leq M - 1$. The standard values in GAUSS 3.2 are $M = 2^{31} - 1$, $a = 397204094$ and $c = 0$. $seed_0$ is initialised by the system clock. We use the value $a = 16807$ and $seed_0 = 12345$ as proposed by Bratley, Fox and Schrage (1983), p. 201f, and included in Algorithm 659 of the ACM.[7]

As an alternative set of error terms we have recourse to quasi–Monte Carlo approaches. A description of these approaches being beyond the scope of this paper, the reader is referred to Niederreiter (1992) and Winker and Fang (1998) for an overview and further references. We will content ourselves with a short argument. While in standard Monte Carlo approaches it is aimed at reproducing "randomness" without necessarily giving an exact definition of what is meant by this feature for a pseudo–random number generator, quasi–Monte Carlo point sets are generated by explicit deterministic algorithms. The goal is to generate an as close as possible approximation to the uniform distribution. In contrast to the term "randomness" the quality of the approximation can be measured by some discrepancy function.[8]

For our application we use three different quasi–Monte Carlo point sets generated by the algorithms in Bratley and Fox (1988), namely a Fauré sequence, a Sobol sequence and a Halton sequence, all in dimension 29. Furthermore, we used a generalized Fauré sequence in dimension 101 available through the world wide web.[9] Unfortunately, even the dimension of this sequence is not large enough to cover the dimensionality of our simulation problem. Hence, we have to find some adequate reduction of the dimension of our problem. We will come back to this problem later. The same problem arises for other quasi–Monte Carlo sequences,

[7] Cf. Bratley and Fox (1988).
[8] Cf. Niederreiter (1992).
[9] `http://www.sequences.com/download_csv.html`.

since the dimension of the sequences generated by public domain software,[10] in general, is still rather small.

Both the standard random number generators and the quasi–Monte Carlo sequences produce more or less uniformly distributed numbers in the unit interval or multidimensional unit cube, respectively. As the simulation of our macromodel requires normal deviates, they, therefore, have to be transformed. The built–in generator of GAUSS 3.2 uses a fast acceptance–rejection algorithm for this purpose,[11] for which the most frequently sampled distribution is the sum of two uniforms.[12] Other transformation algorithms include the sum of twelve uniforms and the Box and Muller (1958) method which generates two independent normals from two independent uniforms. The drawback of all transformation methods using more than one uniform for the generation of one normal is that some "good" multivariate behaviour of the uniform pseudo–random numbers does not necessarily carry over to the normals.[13]

The only transformation which preserves the multivariate structure of the uniform error terms is the inversion method.[14] This method makes use of the fact that the c.d.f. of any distribution only takes on values on the interval [0,1]. If a variable y is distributed according to the strictly increasing c.d.f. $F(y)$, then the variable $x = F(y)$ is distributed $U \sim (0,1)$. For every realisation of x the c.d.f. can be inverted to get $y = F^{-1}(x)$. The transformation method is applicable whenever $F^{-1}(x)$ is easy to calculate. It is rarely used to generate standard normals because $\Phi^{-1}(x)$ does not have a closed form and therefore has to be approximated numerically, which sometimes is regarded as being too costly in terms of computing time. Nevertheless, for the reason mentioned above it seems to be the only adequate method for the comparison of different sets of uniforms in our simulation framework. We use the approximation of Beasley and Springer (1977) for the calculation of F^{-1}.

Given vectors of m i.i.d. standard normal random numbers, the approximation of $\mathcal{N}(0, \Sigma)$, where Σ denotes the estimated covariance matrix, can be achieved by different methods. First, a Cholesky factorization ψ of Σ, i.e. $\psi'\psi = \Sigma$, can be calculated. Then, premultiplying the i.i.d. random numbers by ψ' generates error vectors with the required distribution.[15] This is the method used for our simulation experiments. An alternative has been proposed by McCarthy (1972). He uses the estimated residuals of a model. Suppose that for the m structural equations of the model residuals have been estimated for T sample periods. If the $T \times m$ Matrix E of these estimated residuals is pre–multplied by a T–dimensional row

[10] Cf. `http://www.math.hkbu.edu.hk/qmc/software.html` for links to generators for Fauré–, Sobol–, and Halton–sequences.

[11] Cf. Kinderman and Ramage (1976).

[12] Cf. Ripley (1987), p. 87.

[13] Cf. Ripley (1987), pp. 56ff, for some ill behaved examples.

[14] Cf. Ripley (1987), p. 59f., and Davidson and MacKinnon (1993), p. 736f.

[15] Cf. Ripley (1987), p. 98f, and Davidson and MacKinnon (1993), p. 737.

vector of uncorrelated standard normals and rescaled by $1/\sqrt{T}$, then the resulting m–dimensional row vector has the same covariance matrix as the estimated residuals.

We will not use methods to approximate higher moments of the estimated residuals such as proposed by Sterbenz and Calzolari (1990) or Brown and Mariano (1994). A study of such higher order effects is left for future research. Finally, we do not apply any variance reduction methods as we are mainly interested in the variance of the simulated policy effects. Variance reduction techniques often improve the efficiency of estimates of mean or bias, but are not adequate for this purpose.[16]

5 Results for a policy simulation

The methodology of stochastic simulation using different error generation mechanisms was applied to a fiscal policy simulation of present interest. The current debate about policy measures to improve the employment situation in Germany concentrates a lot on the tax burden on the factor labour including contributions to the social security system. It is argued that a shift of this direct tax elements towards indirect taxation might contribute to a rise in employment. The model was used to analyse the effectiveness of such fiscal policy reforms in the different regimes reflected by the model.[17] It should be noted, however, that due to the restriction to West Germany and the period before 1994, the simulations might give rather qualitative statements on than quantitative dimensions of such effects.

Compensated by a budget neutral increase of indirect taxes a 10 percent cut in direct taxes and a 10 percent cut in social security contributions were analysed, respectively. Each analysis was conducted using two different incidence assumptions implicitly defined by the behaviour of unions in wage negotiations. As an example, we only present the results for the cut in direct taxes and "normal" union reactions, i.e. reactions according to the estimated wage functions which do not take into account that part of the price increases are induced by the budget neutral increase of indirect taxes. It should be noted, however, that the employment results obtained by deterministic simulations were more favourable for a cut in social security contributions and when the nature of the policy measure is explicitly taken into account in the wage negotiations.

The simulation of this policy measure was repeated for two periods characterised by different regimes on the labour market. During the first chosen period, 1981 to 1985, the labour market was first dominated by the goods demand regime which lost importance only at the end when capacity constraints became more

[16]Cf. Ericsson and Marquez (1997). Ripley (1987), p. 119, states that "variance reduction techniques [...] are often known as swindles", but at least "variance reduction obscures the essential simplicity of simulation."

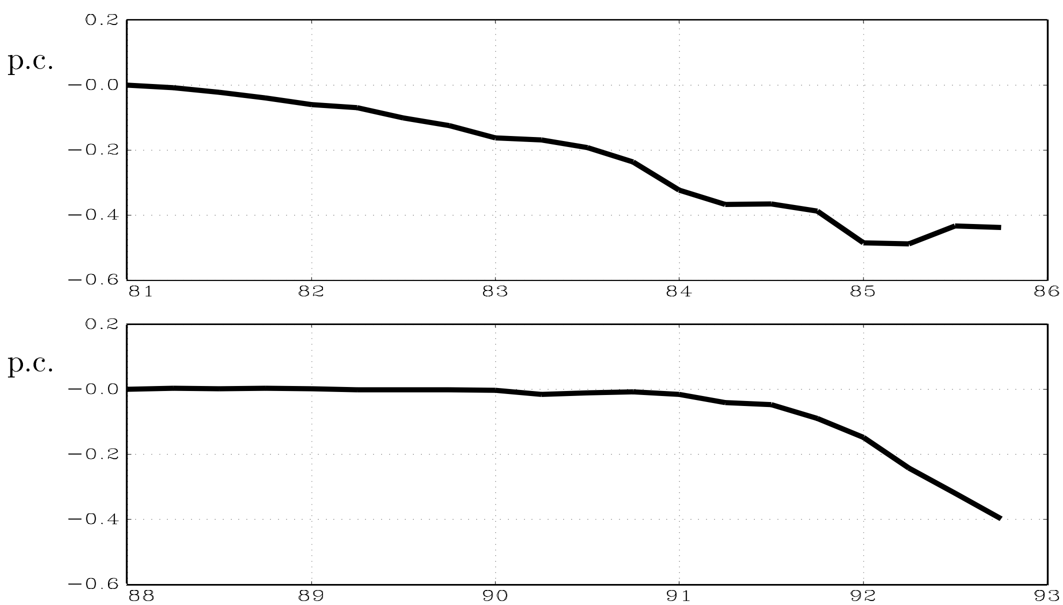
[17]Cf. Franz, Göggelmann and Winker (1997).

relevant.[18] Nevertheless, we will describe this period as mainly demand determined. The second period, 1988 to 1992, in contrast, is first clearly dominated by the capacity regime. Only from 1992 on, the goods demand regime gained importance.

5.1 Deterministic simulations

As the policy experiment was conducted with a strong emphasis on the employment outcome we will only present simulation results for total employment and wage rates in this paper. Figures 4 and 5 show first results for the two variables. The solid lines depict the outcome of a deterministic dynamic simulation with all error terms set to their expected value of zero.

Figure 4: Employment effects (deterministic simulation)



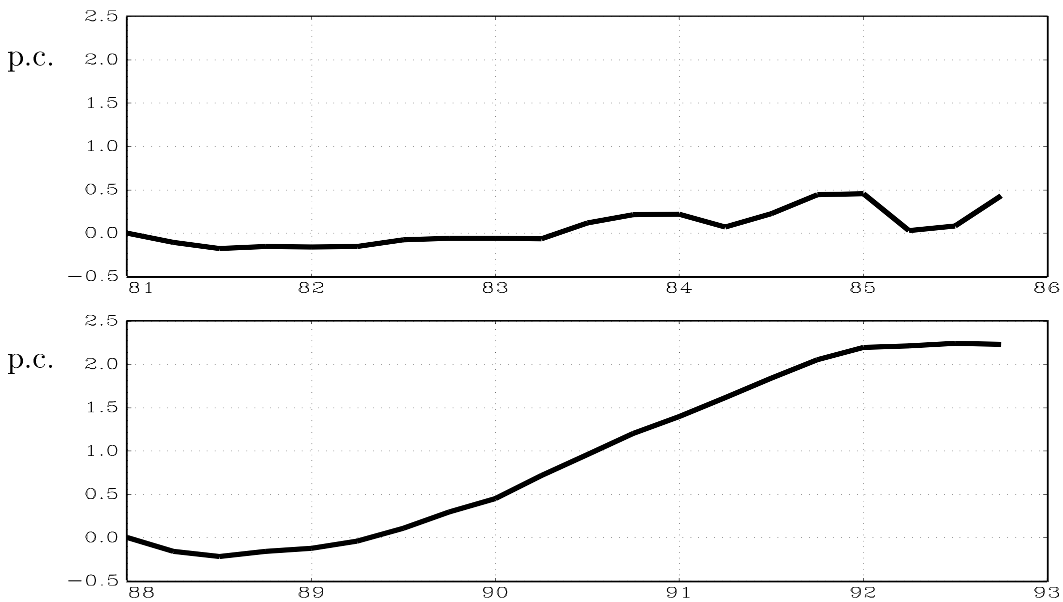
On the whole, the employment effects of a tax reduction turn out to be negative in the first subperiod shown in the upper panel of figure 4. In the case of a tax reduction, firms only directly profit from the lowering of those direct taxes relevant to them, and not from the increase in net wages caused by the tax cut. On the other hand, the firms will try to bypass the increased indirect taxes, which finance the income relief. The ensuing higher prices result in sinking real wages and higher wage claims, which is only slightly moderated by cooperative trade union behaviour. Sinking real wages and the ensuing fall in demand combine to increase the strength of the demand for goods regime. Consequently, firms reduce

[18] See figure 3 on page 6.

their labour demand and employment cut–backs result in the demand determined subperiod 1981–1985.

In contrast to the goods demand regime, almost no employment reactions can be detected in the first three years of the second subperiod as represented in the lower panel of figure 4. Only from 1991 on, when the goods demand regime gained strength, small deviations can be detected. The effect of the fiscal policy reform, though smaller than in the goods demand regime, is still negative for the deterministic simulation. The reason for the different outcome in comparison to the first simulation subperiod is the fact that the demand for goods is not the rationing factor in the capacity regime. Therefore, a decrease in real demand has little or no effect on the firms' labour demand.

Figure 5: Wage effects (deterministic simulation)



The different effects on wages[19] as depicted in figure 5 can be explained in the model context by the following reasoning. While contributions to the social insurance system are splitted between employees and employers, wage taxes are fully paid by the employees. Therefore, the simulated reduction in taxes will lead to lower wage costs only when employees reduce their wage claims. But in our estimations, reductions in taxes or contribitutions paid by employees are found to have only short run effects on wage bargaining. On the other hand, increasing prices caused by higher indirect taxes lead to higher wage claims. Because in the long run the price effect is much stronger than the tax effect, this leads to significantly higher wages after a period of three years compared with the control solution. The effect of indirect taxes differs within the two regimes under consid-

[19]Gross monthly wages per employee in the private sector.

eration. So lacking goods demand in the first period leaves less space for higher prices compared to the capital constrained regime in the second. Therefore wages increase stronger in the latter period.

5.2 Stochastic simulations

Before turning to a discussion of the results of different methods of stochastic simulation, we have to come back to the problem of dimensionality. For both simulations the number of stochastic equations times the number of simulation periods amounts to $29 \times 20 = 580$. The first obvious conclusion is that any simulation with a few hundred or thousand iterations will cover only a negligible part of this space. That is exactly the situation when the advantage of quasi–Monte Carlo methods becomes most pronounced in other applications. Nevertheless, a dimension of more than 500 is beyond the scope of any quasi–Monte Carlo method currently available. Hence, we have to find some restrictions to the error space. The following approaches seem to be possible:

1. Reduce the number of simulation periods to two or one. Unfortunately, this would remove the dynamic aspect of the model from the simulation.

2. Reduce the number of stochastic equations. In fact, we could fix some of the error terms either to zero or to their estimated values and concentrate on the uncertainty evolving from parts of the model. This approach might be helpful to detect weak points in the predictive quality of the model. Hence, we will follow such an approach in future research.

3. Restrict the simulation to the set of error terms obtained by estimation and use bootstraping.[20]

4. Neglect the intertemporal correlation structure, i.e. assume that all the drawings of 29 error terms belong to only one period. Thus, if we perform 1000 iterations for the 20 simulation periods, we generate 20 000 error vectors of dimension 29, which should approximate the 29–dimensional normal distribution with the estimated covariance structure as close as possible. This is the approach followed for the stochastic simulations in this paper.

For all five methods for the generation of error terms discussed in the previous section about 20 000 uniform error vectors of dimension 29 were generated.[21] Next, we calculated a measure of uniformity for these point sets. The L_2–discrepancies range between $0.24267 \cdot 10^{-5}$ and $0.24507 \cdot 10^{-5}$ with smaller values for the Fauré and Sobol sequences and the pseudorandom number generator. The differences

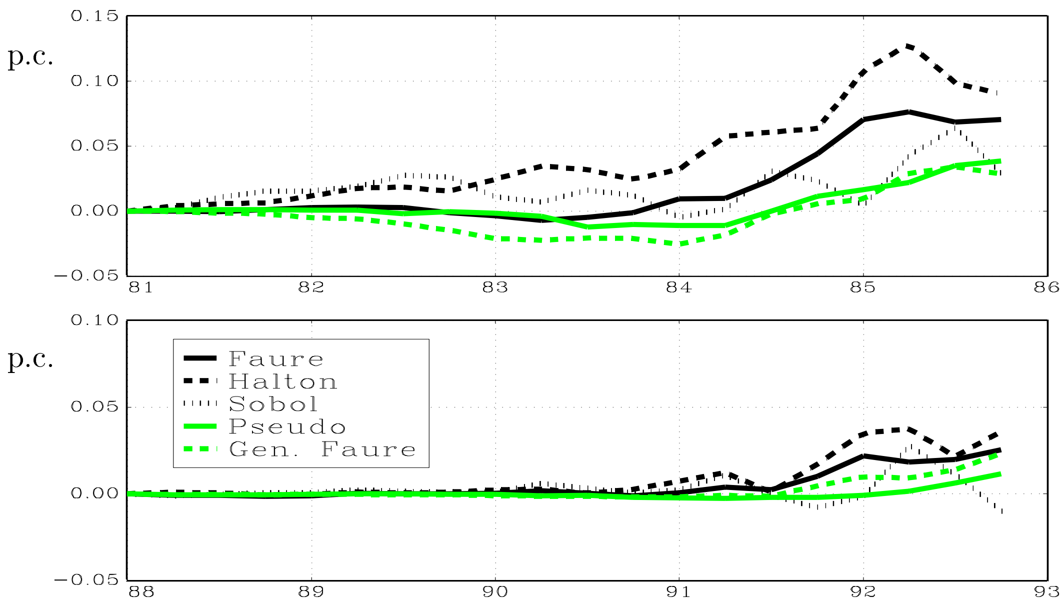
[20] Cf. Ripley (1987), p. 174ff.

[21] The actual numbers range between 20200 and 20300 as for the quasi–random sequences exact multiples of the dimension of the process are required.

using this measure of uniformity are small. It might be useful to study further measures as proposed in Hickernell (1995). Finally, the inverse method was applied to transform the uniform to normal variates.

Figures 6 and 7 give first insights to the behaviour of the model under stochastic simulation. They show the differences between the median of the stochastic simulations for the different error sets and the result of the deterministic simulation. These differences can be interpreted as an indicator of the bias of using deterministic simulation.

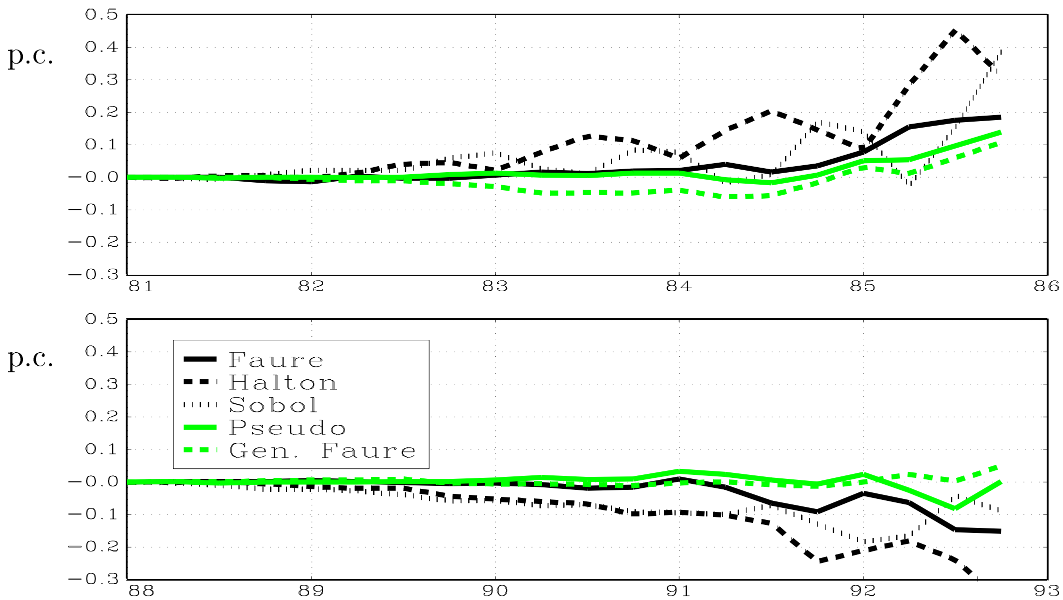
Figure 6: Bias of employment simulation (median of stochastic simulation vs. deterministic simulation)



In principle, the same figures could be generated based on the means of the stochastic simulations. Then, differences in the bias for the means and the medians could be attributed to non symmetric reactions of the model. However, these differences were much smaller than the ones of the deterministic simulations. Hence, only results for the latter are plotted.

The overall impression for both variables is that the bias measured by the difference between median and solution of the deterministic simulation is much larger for the first simulation period. In fact, for the employment series it reaches about 25% of the total effect for the Fauré and Halton sequences and nearly 100% for the Fauré and Sobol sequences for the wage series. Furthermore, the sign of the bias changes in the second period for the wage series. Hence, the results of Schellhorn and Winker (1994) are strongly supported as neglecting the nonlinear characteristics of a model might lead to heavily biased estimates of the effects of some policy measures.

Figure 7: Bias of wage simulation (median of stochastic simulation vs. deterministic simulation)

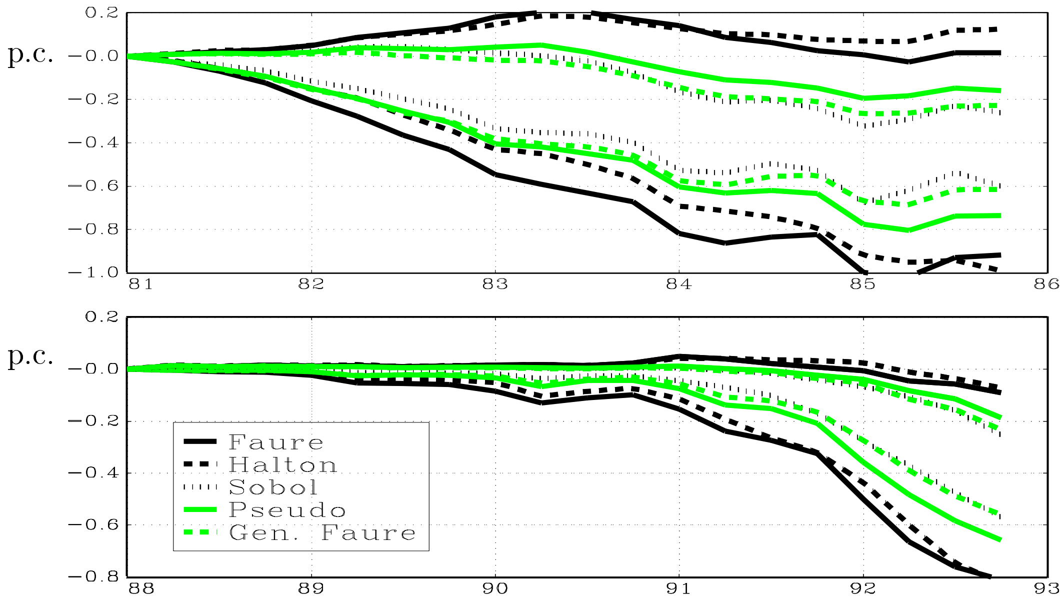


However, the main purpose of this paper consists in an analysis of the impact of different methods for generating the error terms in stochastic simulations. The figures on the estimated bias also contribute to an answer to this question. The estimates of the bias depend on the method for generating the error terms. While the use of standard pseudo random number generators such as the linear congruential uniform random number generator (Pseudo) described in the previous section gives rather small estimates of the bias, the use of Fauré or Halton sequences tend to give sensibly larger ones. Unfortunately, it is not possible to give the exact bias for our model and to decide on this basis which of the error sets is most suitable. Only for a very large number of stochastic simulations the estimated bias approximates the true bias closely for all error sets used in this study. Due to the complexity of our model it was not possible given constraints imposed by available computing resources to obtain a “reasonable” estimate of the true bias. Consequently, the only conclusion we can draw so far is that the choice of the error sets matters in stochastic simulations.

Even if deterministic simulations are not biased, stochastic simulations are necessary to obtain information on the significance of estimated responses. Figures 8 and 9 contain the 5– and 95–percentiles from the stochastic simulations for different error sets. If the null is not contained in the range spanned by the two percentiles, a significant effect is present.

Again, the results differ markedly between the two periods and for the different methods to construct the error sets used for stochastic simulation. However, the overall impression can be described as follows. First, the confidence intervals

Figure 8: Confidence intervals for employment simulation (5% and 95%)



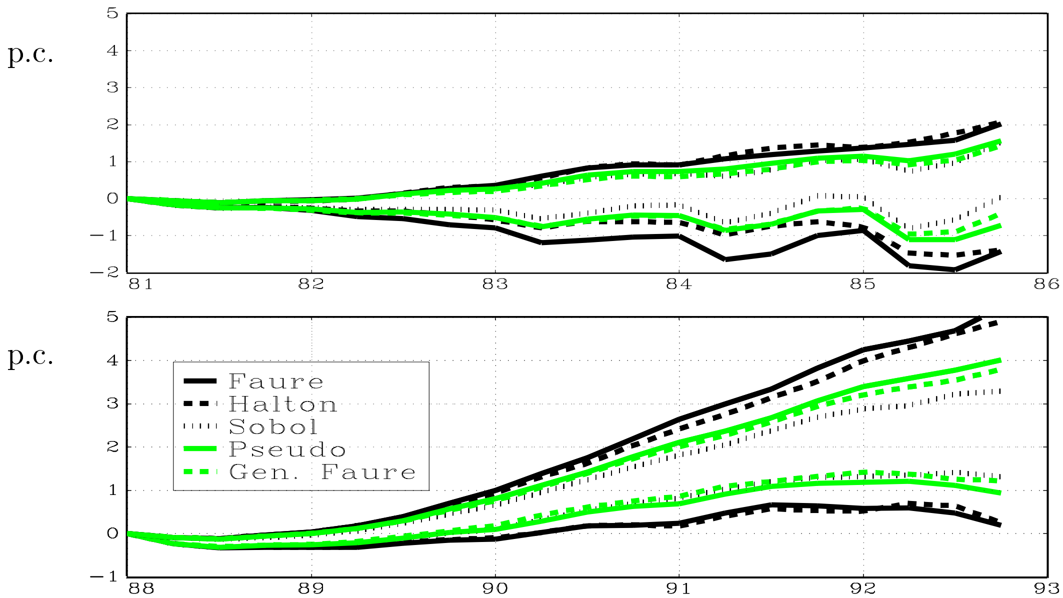
and the difference between the distinct methods widen as the simulated effect becomes larger as is the case in the first period for employment and in the second period for wages. Second, the error terms generated from the Fauré and Halton sequences result in the largest error bounds. The confidence intervals generated from the pseudorandom number generator and the generalized Fauré sequence are substantially smaller. The Sobol sequence gives the smallest confidence bands.

As in the case of the bias, the actual size of the confidence region is not known. Hence, unless we perform a much larger simulation study, it is not possible to judge which of the methodologies used in this study provides the best fit. Nevertheless, a general conclusion seems possible. The choice of error sets for stochastic simulations does not only influence the quantitative outcomes, but may change the qualitative interpretation. While the negative employment effect in the first simulation period is found to be significant based on the standard pseudorandom number generators, it is not necessarily the case for Fauré or Halton sequences. Consequently, the use of different methodologies for error term generation can give if not a better than at least a more conservative estimate of confidence regions.

6 Conclusions

In this paper we studied the impact of some fiscal policy reforms on the West German economy using stochastic simulations. In particular, we were interested in potential employment effects.

Figure 9: Confidence intervals for wage simulation (5% and 95%)



The first conclusion is that – as we had expected from prior simulation experiments – the employment effects depend on the current rationing situation on goods and labour markets. While for a period such as in the early 80s negative employment effects would result, no marked impact could be found for the first few years of a period starting in 1988.

The second conclusion is, that not only quantitative information but even the qualitative content of simulation studies using non–linear models may depend on the employed methodology. While it is common knowledge that the use of deterministic simulations in this context may result in biased estimates of the effects, a result which is confirmed in our study, a comparison of different Monte Carlo and quasi–Monte Carlo approaches to stochastic simulation indicates that the influence on the outcome might reach the same order of magnitude as the switch from deterministic to stochastic simulation.

Unfortunately, for our complex model it was not possible either to calculate or at least to estimate the "true" response. Hence, we cannot give a recommendation on which methodology to use. Nevertheless, using different methodologies can give some information about the uncertainty introduced in the result by the methodology and a more conservative estimate of the confidence region. Hence, the use of quasi–Monte Carlo methods is strongly recommended not only because of their asymptotic superiority. Further research will apply the methods to simpler settings in order to make it possible to compare their relative performance.

Nevertheless, we stick to the point that any policy simulation of the kind performed in our paper should be accompanied by a careful analysis in a stochastic setting both to improve the modelling and to check the robustness of the outcomes.

Stochastic simulation matters, maybe even more than we thought before starting this exercise.

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Quasi–Monte Carlo Methods in Stochastic Simulations

Non technical summary

Simulation is a standard tool for the analysis of policy effects in various modelling frameworks. It is used either for forecasting future developments or for assessing the potential benefits of different economic policies in the past using so–called contrafactual scenarios. Independent of the class of models employed for the analysis – including computable general equilibrium models, pure time series approaches and structural macroeconometric models – different methodologies can be used for such simulations.

First, specific values have to be assigned to some variables treated as exogenous and representing the specific policy measure and – if necessary – the state of the world. Second, some assumptions have to be made with regard to the stochastic components (error terms) of the model. Most frequently, the error terms are set to their expected values. However, it is well known, that in a non linear model the resulting deterministic simulation results are biased. Furthermore, such a deterministic framework generates only a point estimate of the model response without any information about its robustness or confidence regions. One way to overcome these problems is to use stochastic simulations. Therefore, the simulation itself is repeated for many different drawings from the error distribution. In this case, unbiased estimates of the mean response are possible, and some estimates of confidence intervals are also provided.

In this paper, a fiscal policy simulation using a macroeconometric disequilibrium model of the West Germany economy is used to analyze a reform proposal for the tax system. A budget neutral cut in direct taxes compensated by an equivalent raise in indirect taxes is simulated as contrafactual for the time periods 1981–1985 and 1990-1994.

The special emphasis of the anylsis is the assessment of the robustness of point estimates, i.e. measuring the bias of deterministic simulations, and of estimates for confidence intervals using different methods for the generation of error terms.

We find that due to nonlinearities in the model deterministic simulation produces biased estimates as expected. Furthermore, the extent of the estimated bias depends heavily on the error terms employed for the stochastic simulation. A similar impact is found for the estimated confidence intervals. Hence, we conclude that it is not only necessary to employ stochastic simulations in order to reduce bias and get some information on the robustness of the results, but that one has to take care about the generation of error terms. The use of different approaches can help to obtain more reliable results.