Robust GMM Estimation of an Euler Equation Investment Model with German Firm Level Panel Data

by

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Abstract: In this paper the outlier robust GMM panel data estimator recently proposed by LUCAS, VAN DIJK, AND KLOEK (1994) is applied to an Euler equation model of firm investment behaviour with imperfectly competitive product markets for a small panel of German nonfinancial stock companies. Plots for checking distributional implications and the selection of tuning constants are provided. Whereas the estimation results from the usual GMM estimator would contradict the theory, the empirical results using the robust GMM estimator largely support it.

Keywords: Business Fixed Investment, Euler Equation Models, Panel Data Analysis, Robust Estimation, Generalized Method of Moments

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1 Introduction

The analysis of the determinants of business fixed investment has long been one of the most challenging and controversial topics in applied econometrics. Due to the microeconomic foundation of macroeconomic theory, models based on the dynamic optimization behaviour of a representative firm had been developed. Different specifications of the Euler equation model and the neoclassical version of Tobin's q–model were applied to macroeconomic and sectoral data, although with little success. See Chirinko (1993) for a comprehensive survey on investment theory and empirical evidence.

The rising number of suitable micro data sets and strong improvements in computing facilities supported the application of the theories to the level for which they were originally constructed, the individual firm. Panel data studies emerged for a variety of countries, leading to some encouraging, but not fully convincing results. For Germany, firm level studies of q–type investment models were carried out by Elston (1993,1996) and Elston and Albach (1995). Euler equation models were analyzed by Harhoff (1996).

One of the most crucial problems in panel data investment analysis is the assumption of a unique empirical model for all firms, with the exception of a firm specific effect. In this paper we loosen this assumption, allowing for an unknown fraction of outliers. We apply the outlier robust Generalized Method of Moments (Robust GMM) estimator recently proposed by Lucas, van Dijk, and Kloek (1994) to a Bond and Meghir (1994) type Euler equation model of firm investment behaviour for a small panel of German nonfinancial stock companies. We find that the investment equation, based on dynamic optimization with imperfectly competitive product markets is able to explain the investment behaviour of the bulk of the firms.

The outline of the paper is as follows. In Section 2 the empirical Euler equation model is developed. The Robust GMM estimator is presented in Section 3. In Section 4 we apply plotting techniques common in robust statistics for checking distributional implications and for the selection of tuning constants. Empirical results are stated and Section 5 concludes.

2 The Euler Equation Model

We consider a model of the firm which is usually applied in the literature (see Blundell, Bond, and Meghir, 1996 and especially Bond and Meghir, 1994). A firm i is maximizing the expected present value of dividend flows. Regarding the identity of sources and use of liquid funds and neglecting debt, financial assets, and taxation, this leads to the maximization of

$$V_{i0} = \mathrm{E}_{i0}\left\{\sum_{t=0}^{T}(1+r_i)^{-t}\left[p_{it}(Q_{it})Q_{it} - w_{it}L_{it} - c_{it}I_{it}\right]\right\}, \quad (1)$$

where Q_{it} indicates firms output, L_{it} the amount of hired labour, I_{it} gross investment in fixed capital, and p_{it}, c_{it}, w_{it} the prices of output goods, investment goods and labour, respectively. The markets for investment goods and labour are assumed to be perfect. The output price is allowed to depend on the firm's output due to imperfectly competitive product markets. The expectations operator $\mathrm{E}_{i0}\{\cdot\}$ points out that decisions are made conditional on information available to firm i in period 0. $(1+r_i)^{-1}$ is the firm's time invariant discount factor.

The firm's capital stock K_{it} develops according to the transition equation

$$K_{it} = (1-\delta_i)K_{i,t-1} + I_{it} \qquad (t = 0, \ldots, T) \tag{2}$$
$$K_{i,-1} = \bar{K}_{i,-1} \quad \text{given} \tag{3}$$

with δ_i as the time invariant rate of physical depreciation.

The output Q_{it} depends on the firm's capital stock, the amount of hired labour, and the current gross investment according to a linear homogeneous neoclassical production function F and a linear homogeneous convex adjustment cost function G:

$$Q_{it} = F(K_{it}, L_{it}) - G(K_{it}, I_{it}) \qquad (t = 0, \ldots, T). \tag{4}$$

The adjustment cost function G is usually assumed to be quadratic:

$$G(K_{it}, I_{it}) = \frac{b}{2}\left(\frac{I_{it}}{K_{it}} - a\right)^2 K_{it} \qquad (t = 0, \ldots, T), \tag{5}$$

where a, b are finite constants with $b > 0$.

The firm's control variables are the amount of labour L_{it} and gross investment I_{it}. The structure of the optimization problem implies that we can confine ourselves to closed loop problems. The optimal values of the control variables do only depend on the capital stock of the previous period $K_{i,t-1}$ which acts as the state variable covering the complete history of past decisions. Invoking the discrete Maximum Principle (see Arkin and Evstigneev, 1987) we can use equations (1)–(4) to form a discrete current–value Hamiltonian:

$$\begin{aligned}
&\mathcal{H}_{i,t+1}\left(L_{it}, I_{it}, K_{i,t-1}, \lambda_{i,t+1}\right) \\
&= \mathrm{E}_{it}\left\{p_{it}(Q_{it})Q_{it} - w_{it}L_{it} - c_{it}I_{it} + \lambda_{i,t+1}\left[(1-\delta_i)K_{i,t-1} + I_{it}\right]\right\} \qquad (t = 0, \ldots, T)
\end{aligned}$$

with the costate variable $\lambda_{i,t+1}$ as the shadow price of the installed capital stock at the beginning of period $t+1$.

The necessary conditions characterizing a maximum are given by

$$\mathrm{E}_{it}\left\{(p_{it} + p_{it,Q}Q_{it})F_L(K_{it}, L_{it}) - w_{it}\right\} = 0 \qquad (t = 0, \ldots, T) \tag{6}$$
$$\mathrm{E}_{it}\left\{(p_{it} + p_{it,Q}Q_{it})[F_K(K_{it}, L_{it}) - G_K(K_{it}, I_{it}) - G_I(K_{it}, I_{it})] - c_{it} + \lambda_{i,t+1}\right\} = 0 \qquad (t = 0, \ldots, T) \tag{7}$$
$$\mathrm{E}_{it}\left\{(1-\delta_i)(p_{it} + p_{it,Q}Q_{it})[F_K(K_{it}, L_{it}) - G_K(K_{it}, I_{it})] + (1-\delta_i)\lambda_{i,t+1}\right\} = \mathrm{E}_{it}\left\{(1+r_i)\lambda_{it}\right\} \qquad (t = 1, \ldots, T) \tag{8}$$
$$\mathrm{E}_{it}\left\{\lambda_{i,t+1}\right\} = 0 \qquad (t = T) \tag{9}$$
$$\mathrm{E}_{it}\left\{(1-\delta_i)K_{i,t-1} + I_{it}\right\} = \mathrm{E}_{it}\left\{K_{it}\right\} \qquad (t = 1, \ldots, T-1) \tag{10}$$

where e.g. F_L indicates the partial derivation of F with respect to L. Equations (6) and (7) follow from setting to zero the partial derivatives of the Hamiltonian with respect to the control variables. The equations of motion for the costate variable (8) and the state variable (10) follow from the partial derivatives with respect to the state and costate variable, respectively. The transversality condition for the costate variable is given by equation (9).

Substituting equation (7) in equation (8), substituting the expectation in $t-1$ of the resulting equation back in equation (7), and shifting the time index one period foreward, we obtain the Euler equation of firm investment behaviour:

$$
\begin{aligned}
\mathrm{E}_{it} \Bigg\{ \left(1+\frac{1}{\eta_{it}}\right) p_{it} \left[F_K(K_{it},L_{it}) - G_K(K_{it},L_{it}) - G_I(K_{it},I_{it})\right] - c_{it} \\
+ \frac{1-\delta_i}{1+r_i}\left(1+\frac{1}{\eta_{i,t+1}}\right) p_{i,t+1}\left[G_I(K_{i,t+1},I_{i,t+1}) + c_{i,t+1}\right]\Bigg\} \\
= \quad 0 \qquad (t=0,\ldots,T-1),
\end{aligned}
$$

where η_{it} denotes the price elasticity of demand. Recognizing the homogeneity properties of F and G, equations (5) and (6), and additionally assuming that the expectations are formed rationally, the Euler equation can be transformed to an expression in observable variables (Bond and Meghir, 1994):

$$
\begin{aligned}
\frac{I_{i,t+1}}{K_{i,t+1}} \quad = & \quad -a\rho + ((1+a)(1+\rho))\frac{I_{it}}{K_{it}} - (1+\rho)\left(\frac{I_{it}}{K_{it}}\right)^2 \\
& -\left(\frac{1+\rho}{b}\frac{\eta}{1+\eta}\right)\left[\frac{Q_{it}}{K_{it}} - \frac{w_{it}}{p_{it}}\frac{L_{it}}{K_{it}} - \left(1-\frac{1-\delta_i}{1+r_i}\frac{c_{i,t+1}}{c_{it}}\right)\frac{c_{it}}{p_{it}}\right] - \left(\frac{1+\rho}{b}\frac{1}{1+\eta}\right)\frac{Q_{it}}{K_{it}} \\
& -\left(\frac{1}{b}\frac{\eta}{1+\eta}\frac{1+r}{1-\delta_i}\right)\frac{u_{i,t+1}}{p_{i,t+1}} \qquad (t=0,\ldots,T-1),
\end{aligned}
$$

where both $1+\rho = (1+r_i)/[(1+\pi_{it})(1-\delta_i)]$ and the demand elasticity η are assumed to be constant over individual firms and time for simplicity. π_{it} indicates the rate of inflation of output goods. $u_{i,t+1}$ stands for the expectations error, which has zero mean and is uncorrelated with information available to firm i in period t.

The optimization procedure results in an equation where the investment rate is seen as a function of the lagged investment rate, the lagged investment rate squared, the lagged rate of real profit to capital, and the lagged ratio of output to capital:

$$
\frac{I_{i,t+1}}{K_{i,t+1}} \quad = \quad \mu + \beta_1\frac{I_{it}}{K_{it}} + \beta_2\left(\frac{I_{it}}{K_{it}}\right)^2 + \beta_3\frac{P_{it}}{K_{it}} + \beta_4\frac{Q_{it}}{K_{it}} + \epsilon_{i,t+1} \qquad (t=0,\ldots,T-1), \quad (11)
$$

with

$$
P_{it} = Q_{it} - \frac{w_{it}}{p_{it}}L_{it} - \left(1-\frac{1-\delta_i}{1+r_i}\frac{c_{i,t+1}}{c_{it}}\right)\frac{c_{it}}{p_{it}}K_{it}
$$

as real profit adjusted for the user costs of capital. The explanatory variables are predetermined. The coefficient β_1 is strictly positive and should be near one, depending on the value assumed for a. The coefficient β_2 is less than -1 for reasonable values of ρ. If the demand for output is elastic, the coefficient β_3 is expected to be strictly negative and the coefficient β_4 strictly positive and less than β_3 in absolute values. The parameters of the economic model can be calculated as

$$
a = -\left(1+\frac{\beta_1}{\beta_2}\right), \quad b = \frac{\beta_2}{\beta_3+\beta_4}, \quad \rho = -\beta_2 - 1, \quad \eta = \frac{\beta_3}{\beta_4}. \qquad (12)
$$

3 The Robust GMM Estimator

For estimation purposes we stack the observations for the dependent variable into a T–dimensional column vector y_i, the observations for the $K = 4$ explanatory variables into a $T \times K$–matrix X_i, the coefficients into a K–dimensional column vector β, and the error terms into a T–dimensional column vector of errors ϵ_i. As usual in panel data econometrics, we add an individual random effect to equation (11) reflecting unobserved individual heterogeneity and obtain

$$y_i \quad = \quad X_i\beta + e_T(\alpha_i + \mu) + \epsilon_i \qquad (i = 1, \ldots, N),$$

where e_T indicates a T–dimensional column vector of ones. The elements ϵ_{it} of the vector of errors and the individual effect α_i are assumed to be iid$(0, \sigma^2_\epsilon)$ and iid$(0, \sigma^2_\alpha)$, respectively, and statistically independent. The explanatory variables are predetermined by assumption and possibly correlated with the individual effects, i.e. $\mathrm{E}\{x'_{is}\epsilon_{it}\} = 0$ for $s \leq t$, and $\mathrm{E}\{x'_{it}\alpha_i\} \neq 0$ in general, defining x_{it} as the t–th row of X_i.

Since the individual effects cannot be estimated consistently for finite T they have to be filtered by a suitable $(T-1) \times T$ filter matrix with rank $T-1$, such as the first difference filter matrix F^D (Anderson and Hsiao, 1982) with $F^D e_T = 0$:

$$F^D\,(y_i - X_i\beta) \quad = \quad F^D\epsilon_i \qquad (i = 1, \ldots, N).$$

The usual procedure in GMM panel data estimation is to construct a $(T-1) \times M$ $(M \geq K)$ matrix of instruments $W_i = \mathrm{diag}(w_{i1}, \ldots, w_{iT})$ to form the theoretical moment conditions

$$\mathrm{E}\left\{W'_i F^D \epsilon_i\right\} \quad = \quad 0 \qquad (i = 1, \ldots, N), \tag{13}$$

where w_{it} denotes the m_t–dimensional row vector of explanatory variables orthogonal to the filtered error terms in period t with $\sum_{t=1}^{T} m_t = M$. Replacing the theoretical moments by their empirical counterparts defined on the cross section $N^{-1} \sum_{i=1}^{N} W'_i F^D(y_i - X_i\beta)$, the GMM estimator (Hansen, 1982) is defined to be the minimizing argument of the quadratic criterion function

$$N^{-2}\,(y - X\beta)'\,F'WA_NW'F\,(y - X\beta)$$

with $y = (y'_1, \ldots, y'_N)'$, $X = (X'_1, \ldots, X'_N)'$, $W = (W'_1, \ldots, W'_N)'$ and $F = I_N \otimes F^D$, I_N representing an N–dimensional unity matrix. A_N is an asymptotically nonstochastic, positive definite weighting matrix of dimension M. The resulting estimator is consistent and asymptotically normal for any matrix A_N, and asymptotically efficient for given moment conditions, if a matrix converging to the inverse of the covariance matrix of the theoretical moments is used as the weighting matrix (see Arellano and Bond, 1991).

However, Lucas, van Dijk, and Kloek (1994) proved that one single outlier in the space of instruments or error terms can make the usual GMM panel data estimator grow above all bounds, i.e. that the GMM estimator has an unbounded influence function and an infinite gross error sensitivity (see Hampel, et al., 1986). They construct a Robust GMM estimator with a bounded influence function, replacing (13) by a robustly weighted theoretical moment condition:

$$\mathrm{E}\left\{W'_i \Phi_i F^D \epsilon_i\right\} \quad = \quad 0 \qquad (i = 1, \ldots, N).$$

Φ_i is a $(T-1)$ dimensional diagonal matrix of robust weights depending on W_i and $F^D\epsilon_i$, sufficiently downweighting aberrant values of the instruments or the filtered error terms to ensure a bounded influence function of the estimator. Proceeding as in the usual GMM case, they construct a quadratic criterion function

$$N^{-2}(y-X\beta)'F'\Phi W A_N W'\Phi F(y-X\beta)$$

with $\Phi = \mathrm{diag}(\Phi_1, \dots, \Phi_N)$, which is to be minimized with respect to β. The resulting estimator has been shown to be consistent and asymtotically normal under suitable regularity conditions, including the assumption that the instruments for a given period t are sampled from a multivariate normal population. The estimator is asymptotically efficient for given robustly weighted moment conditions, if a matrix converging to the inverse of the covariance matrix of the robustly weighted theoretical moments is used as the weighting matrix A_N.

In selecting an appropriate weighting matrix Φ_i LUCAS, VAN DIJK, AND KLOEK (1994) make use of the decomposition proposed by MALLOWS (see LI (1985)) for the general M–estimation of a linear regression model:

$$\Phi_i = \Phi_i^W \Phi_i^\epsilon$$

where Φ_i^W denotes a $(T-1)$ dimensional diagonal weighting matrix for the instruments with typical element ϕ_{it}^W and Φ_i^ϵ a $(T-1)$ dimensional diagonal weighting matrix for the error terms with typical element ϕ_{it}^ϵ.

The weights ϕ_{it}^ϵ of the error terms are functions of the scale adjusted errors

$$\phi_{it}^\epsilon = \phi^\epsilon\left[\frac{\epsilon_{it}^f}{s(\epsilon^f)}\right],$$

where ϵ_{it}^f is the t–th element of the vector of filtered error terms and $s(\epsilon^f)$ a measure of the scale of the filtered errors. The weights ϕ_{it}^W of the instruments are defined as functions of a measure of the distance of the instruments to their own mean:

$$\phi_{it}^W = \phi^W\left[d_t(w_{it})\right],$$

where $d_t(w_{it})$ is the distance of the i–th firm's instruments within the t–th period.

Since the weights depend on the parameter vector β through the weights of the error terms, the Robust GMM estimator is nonlinear and has to be computed iteratively. We follow LUCAS, VAN DIJK, AND KLOEK (1994) and use

$$\hat\beta^0 = \left(X'F'\Phi^W W A_N^0 W'\Phi^W F X\right)^{-1} X'F'\Phi^W W A_N^0 W'\Phi^W F y$$

with $A_N^0 = \left(N^{-1}\sum_i^N W_i'\Phi_i^W H \Phi_i^W W_i\right)^{-1}$ as the starting estimator. H is a Toeplitz matrix built by the $(T-1)$–dimensional vector $(2,-1,0,\dots,0)'$, representing a matrix proportional to the covariance matrix of the filtered error terms $F^D\epsilon_i$ in the iid case (ARELLANO AND BOND, 1991). We use one fully iterative procedure, updating the weighting matrix Φ^ϵ in every iteration step for a given initial estimate of the scale $\hat{s}(\epsilon^f)$. Since the instruments' weights ϕ_{it}^W

are independent of β, they can be estimated in advance. We calculate the weighting matrix A_N^j of the j-th step of iteration by

$$A_N^j = \left(N^{-1} \sum_i^N W_i' \Phi_i^W \Phi_i^\epsilon(\hat{\beta}^{j-1}) \hat{\epsilon}_i^{f,j-1} \hat{\epsilon}_i^{f,j-1\prime} \Phi_i^\epsilon(\hat{\beta}^{j-1}) \Phi_i^W W_i \right)^{-1}.$$

where $\hat{\beta}^{j-1}$ and $\hat{\epsilon}^{f,j-1}$ are the vectors of coefficients and residuals of the estimated filtered model of the previous step.

In estimating the scale of the residuals of the starting estimate, a convenient robust equivariant estimator of scale is needed. We regard the median of the absolute deviation from the median (MAD estimator) as suitable,

$$\hat{s}[\epsilon^f] = \text{med} \left[\mid \epsilon_{it}^f - \text{med}(\epsilon_{it}^f) \mid \right],$$

because it has a high breakdown point, e.g. it can cope with a relatively large number of outliers (see Donoho and Huber, 1983). As usual, the MAD estimator is divided by 0.6745 to give a consistent estimator of the normal scale in the case of no outliers (Goodall, 1983).

As the weighting function of the residuals we use the function proposed by Huber (1964), because it allows a clear distinction between outliers and non–outliers:

$$\phi^\epsilon \left[\frac{\epsilon_{it}^f}{\hat{s}(\epsilon^f)} \right] = \mathrm{I}_{\{|\epsilon_{it}^f| < k_1 \hat{s}(\epsilon^f)\}}(\mid \epsilon_{it}^f \mid) + \frac{k_1 \hat{s}(\epsilon^f)}{\epsilon_{it}^f} \mathrm{I}_{\{|\epsilon_{it}^f| \geq k_1 \hat{s}(\epsilon^f)\}}(\mid \epsilon_{it}^f \mid)$$

with I representing the well known indicator function. The tuning constant k_1 defines an interval where the residuals are given full weights. Scale adjusted residuals which exceed k_1 in absolute values are downweighted. The weighting scheme implies a left and right censoring of the residuals' empirical distribution at $-k_1\hat{s}(\epsilon^f), k_1\hat{s}(\epsilon^f)$. Plotting techniques for selecting the tuning constant are provided in section 4.

As a measure of the instruments' distances the square root of the Mahalanobis distance is used, which is known to be the Rao distance in the multivariate normal case (Jensen, 1995):

$$d_t(w_{it}) = \sqrt{(w_{it}' - \bar{w}_t)(\Sigma_t^W)^{-1}(w_{it}' - \bar{w}_t)'}$$

In calculating the distances robust estimates of the location vector $\bar{w}_t$ and the scatter matrix Σ_t^W are needed. Lucas, van Dijk, and Kloek (1994) apply the iterative S–estimator developed by Lopuhaä (1989). This procedure provides a high breakdown point, but is computationally extremely expensive. To avoid this, we simplify this procedure and use the iterative robust M–estimator (Maronna, 1976) in the specification of Campbell (1980) to estimate the location vector $\bar{w}_t$ and the scatter matrix Σ_t^W of the instruments within the t–th period:

$$\hat{\bar{w}}_t^j = \frac{1}{\sum_{i=1}^N \phi^W \left[\hat{d}_t^{j-1}(w_{it}) \right]} \sum_{i=1}^N \phi^W \left[\hat{d}_t^{j-1}(w_{it}) \right] w_{it}'$$

$$\hat{\Sigma}_t^{W,j} = \frac{1}{\sum_{i=1}^N \left(\phi^W \left[\hat{d}_t^{j-1}(w_{it}) \right] \right)^2 - 1} \sum_{i=1}^N \phi^W \left[\hat{d}_t^{j-1}(w_{it}) \right]^2 (w_{it}' - \hat{\bar{w}}_t^{j-1})(w_{it}' - \hat{\bar{w}}_t^{j-1})'.$$

The ordinary empirical moments are used as starting values. This procedure has a lower breakdown point, but the additional advantage that the resulting weighting scheme within the procedure can directly be used as the weights ϕ_{it}^{W}. CAMPBELL (1980) uses a redescending generalization of the HUBER (1964) function as the robust weighting function:

$$\phi^{W}[d_t(w_{it})] = \mathrm{I}_{\{d_t(w_{it})<k_2\}}[d_t(w_{it})] + \frac{k_2}{d_t(w_{it})} \exp\left[-\frac{1}{2}\left(\frac{d_t(w_{it})-k_2}{k_3}\right)^2\right]\mathrm{I}_{\{d_t(w_{it})\geq k_2\}}[d_t(w_{it})]$$

with k_2, k_3 as suitable tuning constants. To make the tuning constants depend on the row dimension m_t of w_{it}', we use Fisher's square root approximation of a χ^2 variable by a standard normal and calculate $k_2(t) = \sqrt{m_t} - n_2/\sqrt{2}$ where n_2 is a given κ–percentage point of the standard normal distribution. k_2 implies a right truncation of the asymptotic distribution of the squared distances d_{it}^2 which are given full weights at the κ–percentage point of the $\chi^2(m_t)$–distribution. Distances which exceed k_2 are decreasingly downweighted, where k_3 defines the rate of decrease. The Huber function is contained as the special case, when k_3 tends to infinity. Plotting techniques for checking the distributional implications are provided in the next section.

4 Diagnostic Plots and Empirical Results

The empirical analysis is based upon a small panel of German nonfinancial stock companies, whose shares were traded continually at the Frankfurt stock exchange during the period 1987 to 1992. We use the companies' balance sheet data to construct the relevant variables (see data appendix). After applying a number of exclusion restrictions, we are left with a set of $N=96$ firms with 6 observations which reduce to $T=3$ after calculation of the variables, accounting for the lag structure and first differencing.

Following SCHMIDT, AHN, AND WYHOWSKI (1992) we use all linear moment restrictions implied by the rational expectations hypothesis. Therefore, we exclude the squared lagged endogenous variable from the set of instruments. With $K=3$ remaining predetermined variables this means that we obtain $m_1 = 3$, $m_2 = 6$, and $m_3 = 9$, in sum $M = T(T+1)K/2 = 18$ moment restrictions, leaving $M-K = 14$ degrees of freedom for the test of overidentifying restrictions.

In estimating the instruments' distances, more precisely the location vector and the scatter matrix, by the procedure proposed by CAMPBELL (1980) we tried different combinations of the tuning constants n_2 and k_3. We checked the distributional properties of the instruments' distances which are not regarded to be outliers by the use of truncated quantile–quantile (QQ) probability plots and observed that the values of the 95–percentage point of the standard normal distribution for n_2 and 1.25 for k_3 performed best.

Insert figure 1 about here.

In the three left diagrams of figure 1 the ordered distances are plotted against the quantiles of a $\chi^2(m_t)$ distribution truncated at the 95–percentage point. In the three right diagrams they are plotted against the quantiles of a truncated Beta $[m_t/2, (N-m_t-1)/2]$ distribution which performs better in small samples (GNANADESIKAN AND KETTENRING, 1972). The

i–th quantile is evaluated at $\frac{i-0.5}{N}$ in the truncated χ^2 case (WILK, GNANADESIKAN, AND HUYETT, 1962) and at $(i-p_1)/(N-p_1-p_2-1)$ with $p_1=(m_t-2)/(2m_t)$, $p_2=(N-m_t-3)/[2(N-m_t-1)]$ in the truncated Beta case (SMALL, 1978). If the empirical and theoretical distribution coincide, the observations should lay on a straight line through the origin at 45°. We still find some curvature in the QQ–plots. This may be due to the relatively small number of observations in each period or the relatively large number of outliers. $O_1=20$, $O_2=26$, and $O_3=45$ from $N=96$ observations were declared outliers in the instruments' space by the CAMPBELL (1980) procedure. However, since the distributional properties of the instruments have to be assumed for estimation purposes but are not predicted by the economic model, outliers in the instruments' space do not contradict the underlying theory.

For the selection of the tuning constant k_1 of the HUBER (1964) function, we apply the two plotting techniques proposed by DENBY AND MALLOWS (1977). They suggest to systematically vary the tuning constant across reasonable values, and to plot the standardized robust residuals as well as the scale adjusted robustly estimated coefficients against the tuning constants.

Insert figure 2 about here.

In the first 3 diagrams of figure 2 the standardized robustly estimated residuals of the $N=96$ observations are plotted against the tuning constants for each period $t=1,2,3$. We have added two straight x–marked lines through the origin at ±45°. For a given tuning constant the residuals lying on and above the upper line and on and below the lower line are regarded as outliers and trimmed by the HUBER (1964) function. In the fourth diagram of figure 2 the robustly estimated coefficients standardized by the interquartile range of the corresponding explanatory variables to remove scale effects are plotted against the tuning constants.

The tuning constant is systematically varied from 1.00 to 7.00 at 0.25 steps. If we move downwards from 7.00 to 4.75 we observe only few outliers and moderate movements of the residuals in the first three diagrams and of the coefficients in the fourth diagram. From 4.50 to 3.25 the residuals and coefficients move stronger. This means that the trimmed residuals have great influence on the estimation results and therefore are possibly outliers. The values of $k_1=2.75$ in the first period and $k_1=3.00$ in the second and third period seem to be thresholds for characterizing extremely behaving residuals on the negative half plane as outliers. From $k_1=2.25$ downward residuals and coefficients again begin to move strongly, since more and more observations are declared outliers.

For that reasons we report the results of the Robust GMM estimation with tuning constants $k_1=2.50, 2.75, 3.00$. In table 1 the results of the Robust GMM estimation are presented and compared with the ordinary two–step GMM estimates (ARELLANO AND BOND, 1991). Robustly estimated standard errors and tests for overidentifying restrictions (see LUCAS, VAN DIJK, AND KLOEK, 1994) are added.

Bearing in mind that the estimated variances, especially of the coefficients representing the dynamics of the model, are relatively high and downward biased in two–step GMM techniques, we use the ordinary GMM results to recalculate the parameters of the underlying model. According to (12) we would obtain

$$a=-1.169, \quad b=7.502, \quad \rho=-1.476, \quad \eta=0.502,$$

Table 1: Estimation Results

method	GMM	Robust GMM		
k_1	—	2.50	2.75	3.00
$\left(\frac{I}{K}\right)_{-1}$	0.0804	0.9384	0.8974	0.8438
	(0.2347)	(0.9267)	(0.8475)	(0.8453)
$\left(\frac{I}{K}\right)^2_{-1}$	0.4756	−1.1988	−1.0896	−0.9590
	(0.3887)	(2.3074)	(2.1164)	(2.2236)
$\left(\frac{P}{K}\right)_{-1}$	0.0212	−0.3558	−0.3651	−0.3762
	(0.0341)	(0.1094)	(0.1111)	(0.1127)
$\left(\frac{Q}{K}\right)_{-1}$	0.0422	0.1528	0.1542	0.1573
	(0.0188)	(0.0388)	(0.0361)	(0.0365)
J–Test	12.024	13.683	13.773	13.844
p–value	0.6044	0.4736	0.4667	0.4614

():   estimated variances in brackets
k_1:   tuning constant for the Huber (1964) function
J–Test: test statistic for the Hansen (1982) test for overidentifying restrictions

implying a negative discount rate ρ and a positive price elasticity of demand η, which would contradict the theory. Recalculating the parameters of the underlying economic model from the RGMM($k_1 = 2.75$) results, we obtain

$$a = -0.176, \quad b = 5.166, \quad \rho = 0.090, \quad \eta = -2.367,$$

which is in line with the economic theory. We see that the Robust GMM results support the theory, especially when using the tuning constant $k_1 = 2.75$ recommended by the plotting techniques, whereas the GMM results would contradict it.

5 Concluding Remarks

In this paper we applied the outlier robust GMM panel data estimator of Lucas, van Dijk, and Kloek (1994) to an Euler equation investment model with imperfectly competitive product markets for a small panel of German nonfinancial stock companies. Plotting techniques common in robust statistics are used to check distributional implications and to select the relevant tuning constants. Whereas the estimation results of the ordinary GMM procedure would contradict the theory, the results of the Robust GMM largely support it.

The loosening of the assumption of a unique empirical model for all firms seems to be a step into the right direction. In a next step the applied plotting techniques or other methods should be used to identify the firms for which the Euler equation does not hold, e.g. which are declared outliers by the estimation procedure. Careful inspection of the situation of these firms, would indicate reasonable extensions of the theoretical model, which is left for future research.

A Data Appendix

The companies' balance sheet data are used to construct the relevant variables. The balance sheets come from the German "Jahresabschlußdatenbank Aachen" (Möller et al. 1992), which is part of the "Deutsche Finanzdatenbank" (Bühler, Göppl, and Möller, 1993). We concentrate on nonfinancial stock companies, whose shares were traded continually at the Frankfurt stock exchange during the period 1987 and 1992. We exclude pure or predominant holding companies and firms which changed their balance sheet date. After removing obviously faulty records, we are left with a set of $N = 96$ firms.

Since the estimated model is formulated in real terms and balance sheet data are nominal, we transform equation (11) in nominal terms:

$$\frac{(cI)_{i,t+1}}{(cK)_{i,t+1}} = \beta_0 + \beta_1 \frac{(cI)_{it}}{(cK)_{it}} + \beta_2 \left(\frac{(cI)_{it}}{(cK)_{it}}\right)^2 + \beta_3 \frac{(pP)_{it}}{(pK)_{it}} + \beta_4 \frac{(pQ)_{it}}{(pK)_{it}} + \epsilon_{i,t+1}$$

with $(pP)_{it} = (pQ)_{it} - (wL)_{it} - (c^*K)_{it}$. $(c^*K)_{it}$ indicates the user cost of capital. The variables were constructed as follows:

Gross investment in fixed capital $(cI)_{it}$ was calculated by correcting the differences in book values of subsequent periods by the nominal depreciation.

Replacement costs of the fixed capital stock $(cK)_{it}$ were calculated separately for structures and equipment by perpetual inventory methods: $(cK)_{it} = (1-\delta)\frac{c_t}{c_{t-1}}(cK)_{it-1} + (cI)_{it}$. As prices c_t we use the aggregate price deflators for gross investment in structures and equipment of the German "Statisches Bundesamt". As depreciation rates we use the values 0.0456 for structures and 0.1566 for equipment as proposed by King and Fullerton (1984). Following Schaller (1990) we use book values multiplied by the aggregate ratio of net capital stock at current cost to net capital stock at historical cost for the economy as the whole as starting values.

Fixed capital evaluated at output prices $(pK)_{it}$ was calculated by multiplying the replacement costs of capital by the ratio of the aggregate price deflator for output prices in the manufacturing sector of the "Statistisches Bundesamt" to the above mentioned aggregate price deflators for gross investment for structures and equipment.

Profits corrected for the user costs of capital $(pP)_{it}$ were calculated by adding the nominal deprecation to the firms net worth ("Betriebsergebnis") and subtracting the user costs of capital. The user costs of capital were calculated separately for structures and equipment using the above mentioned price deflators and depreciation rates in $c_{it} - \frac{1-\delta_i}{1+r} c_{i,t+1}$. As the discount rate r we use the yield of fixed income securities published by the "Deutsche Bundesbank".

Nominal Ouput $(pQ)_{it}$ was calculated indirectly by adding nominal deprecation and gross wages to the the firms net worth.

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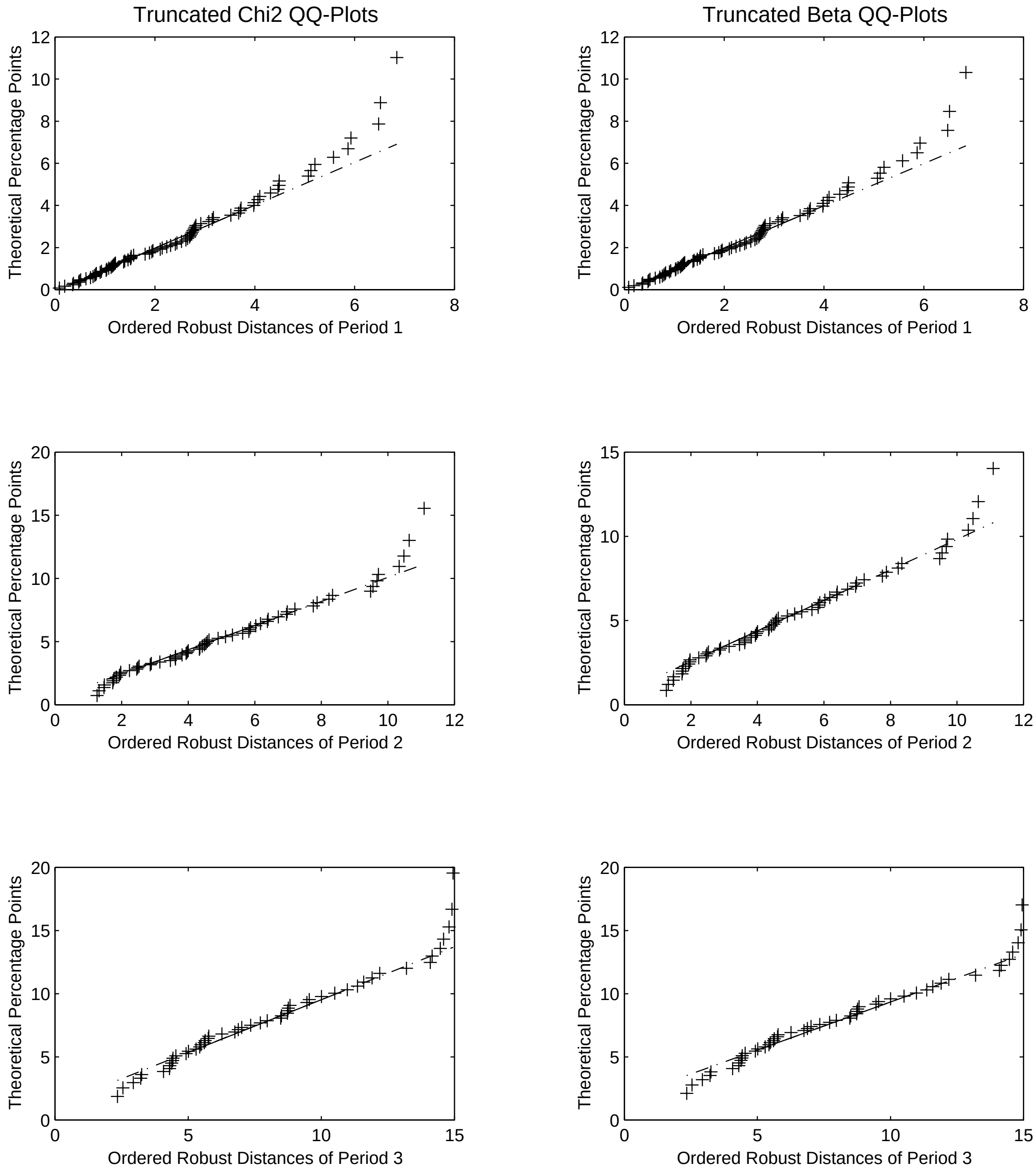

Figure 1: QQ–plots of Robust Distances

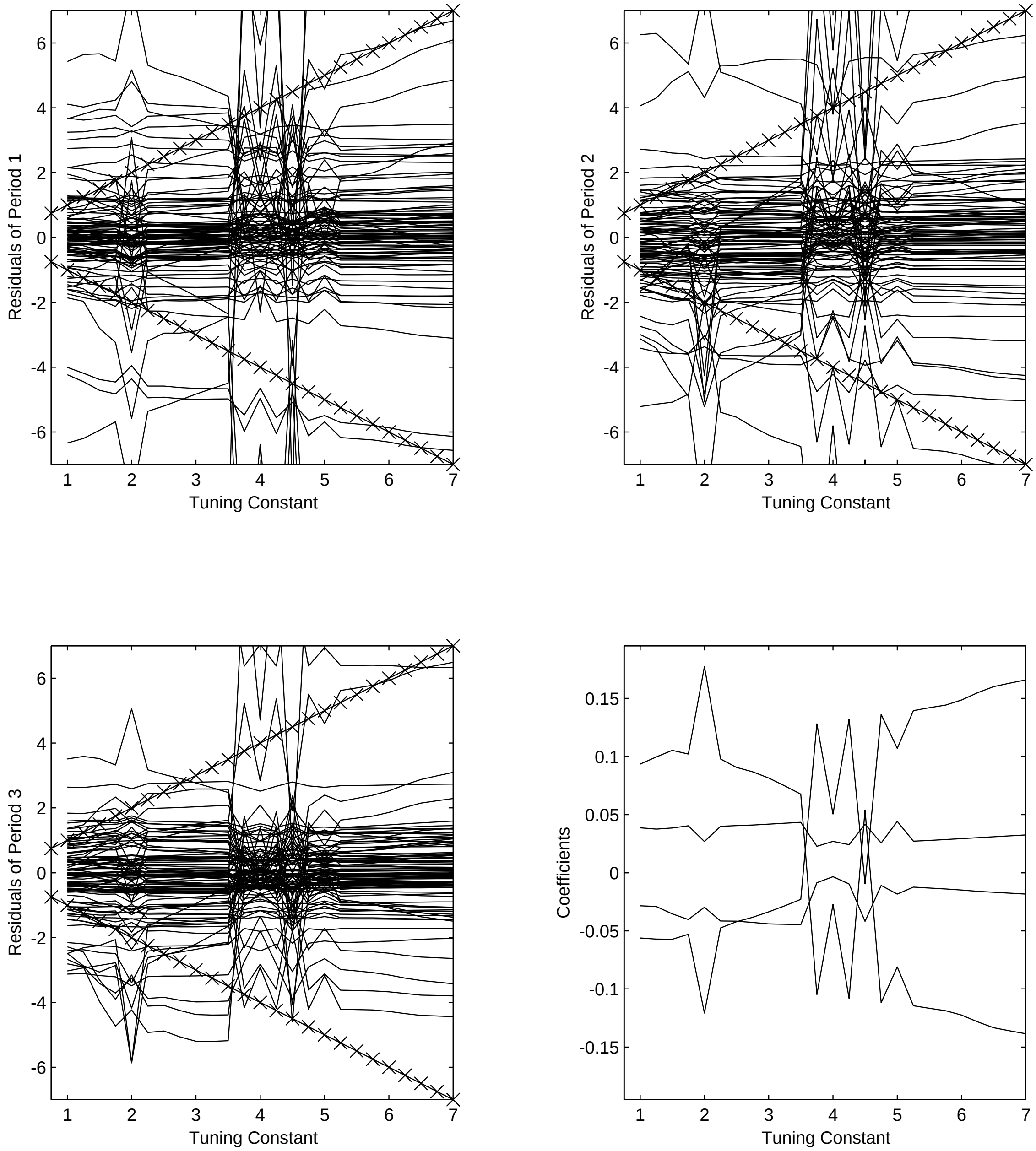

Figure 2: Denby–Mallows Plots of Robust Residuals and Coefficients