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Tests of Representative Firm Models: Results for German Manufacturing Industries

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Abstract

Many studies of producer behavior consider cost and input demand functions derived from microeconomic theory and estimate them on the basis of aggregate data. If firms' characteristics differs, the neglect heterogeneity can lead to estimation bias. An alternative is to restrict individual behavioral functions to be linear in the firm specific parameters. The aim of this paper is to describe "aggregate producer" behavior without placing too strong restrictions on the functional form and to explicitly take account of firm heterogeneity. Estimation for German manufacturing sectors confirms that neglected heterogeneity is an important source of bias in representative agent models.

Keywords: exact aggregation, representative firm, heterogeneity, demand system.

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1. Introduction

Many studies involved with the modeling of producer behavior consider cost and input demand functions derived from *microeconomic* theory and estimate them on the basis of *aggregate data*. Following the work of Diewert (1973), the selected functional forms are often required to provide a second order local approximation to an arbitrary function. However, when the aggregation of individual cost functions is not made explicitly and aggregate data appear instead in microeconomic relations in a purely ad hoc fashion, heterogeneity bias can emerge as soon as firms differ in any explicatory variable.

In order to alleviate heterogeneity bias, some authors suggest to restrict individual behavior to be linear in the variable subject to heterogeneity. In this case the aggregate form will only depend on aggregate variables. Doing so, Appelbaum (1982) or Borooah and Van der Ploeg (1986) model producer behavior on the basis of cost functions linear in the production level. The resulting independence of marginal costs from the production level seems however restrictive, and could now lead to an approximation bias.

An alternative solution is to adopt the flexible functional forms from the first approach and also to proceed to linear aggregation as done by the second approach. This route is followed by Lewbel (1988) or by Heineke (1992) among others. In production economics, Dickson (1994) offers one of the few contributions along those lines. In particular, he considers both aggregation and approximation issues simultaneously, and shows that when firms differ in the production level, a Herfindahl index emerges in the aggregated form. Another originality of this approach is to give economic foundations to the relationship between concentration, costs and input demands. In his study however, Dickson only considers differences in the level of production. Heterogeneity also characterizes firms' production process. Moreover on economic grounds, it seems plausible that both kind of heterogeneity are interrelated.

The original framework developed by Fortin (1988, 1991) allow heterogeneity in both production process and explanatory variables. We choose to adapt this last approach to optimized relations (firms are minimizing costs) and to flexible functional forms for each firm. After exact aggregation across all firms belonging to the same industry, some distributional statistics appear in the aggregate relations. When not available, these statistics are assumed constant over the period and estimated along with technology parameters. Then, aggregation bias can be identified and the representative firm model, nested within ours, can be confronted to rejection tests. This is done for 27 German manufacturing industries.

The first section is devoted to the discussion of some aggregation results, the second describes the model. Presentation of the database and of available concentration statistics is the focus of a third section. Then the estimation procedure is presented and results are discussed.

2. Exact aggregation of cost functions

We assume that every firm has a cost function belonging to the same functional family and parameterized by a vector α_{it}

$$c\left(p_t, y_{it}; \alpha_{it}\right) = \min_{x_{it}} \left\{p_t' x_{it} : f_{it}\left(x_{it}\right) \geq y_{it}\right\} \tag{1}$$

where $p_t \in \mathbb{R}_{++}^{\ell}$ and $x_{it} \in \mathbb{R}_{+}^{\ell}$, f is a production function, y_{it} the output level and t a time index also reflecting the effect of technological change on production technology since it indexes the production function and the cost parameters α_{it}. The subscript i characterizes variables specific to firms: input prices are assumed identical to all n_t firms belonging to an industry.

The exact aggregated cost function (across the n_t firms) can then be defined as:

$$C_t = \sum_{i=1}^{n_t} c\left(p_t, y_{it}; \alpha_{it}\right) = C\left(p_t, y_{1t}, \ldots, y_{nt}; \alpha_{1t}, \ldots, \alpha_{nt}\right). \tag{2}$$

In addition, it is assumed that the joint distribution of firm characteristics (y_{it}, α_{it}) can be parameterized by a finite number of vectors

$$\Phi_1\left(y_{1t}, \ldots, y_{nt}\right), \Phi_2\left(\alpha_{1t}, \ldots, \alpha_{nt}\right), \text{ and } \Phi_3\left(y_{1t}, \ldots, y_{nt}; \alpha_{1t}, \ldots, \alpha_{nt}\right).$$

In the present case, like Fortin (1988, 1991) or Heineke and Shefrin (1988), we parametrize the distribution of (y_{it}, α_{it}) by a finite vector of moments $M_t = (n_t, y_t, \alpha_t, h_t)$, where n_t is the moment of order zero, $y_t = \sum_{i=1}^{n_t} y_{it}$ and $\alpha_t = \sum_{i=1}^{n_t} \alpha_{it}$ are the first order moments and h_t a vector of higher order moments.[1] Then we can rewrite

$$C_t = \overline{C}\left(p_t, M_t\right). \tag{3}$$

In the practice, the variables entering M_t will depend on the form of the microeconomic functions c and on the form of the distribution of heterogeneity characteristics (y_{it}, α_{it}).

2.1 The representative firm and aggregation bias

The assumption of a representative firm is often taken for granted in macroeconomic work starting with a function $C^m\left(p_t, y_t, \alpha_t^m\right)$ satisfying microeconomic properties. Two drawbacks appears in these approaches: first the relation between the macroeconomic parameters α_t^m and their corresponding microeconomic ones does not appears explicitly, nothing says that generally $\alpha_t^m = \sum_{i=1}^{n_t} \alpha_{it}$. Secondly, nothing says that the function C^m still satisfies microeconomic properties. Before to come to this last issue, several manner to define a representative firm are presented and discussed.

Definition: The representative agent framework holds when the aggregate cost function C_t verifies either the conditions a), b) or c):

a) $C_t = \sum c = \widetilde{C}\left(p_t, y_t, \alpha_t\right)$, where $\widetilde{C}\left(p_t, y_t, \alpha_t\right) = c\left(p_t, \sum_{i=1}^{n_t} y_{it}; \sum_{i=1}^{n_t} \alpha_{it}\right)$

b) $B_t = \overline{C}\left(p_t, M_t\right) - \widetilde{C}\left(p_t, y_t, \alpha_t\right) = 0$ for a set of p_t, y_t, α_t.

c) Condition b) holds and the aggregate cost function $\overline{C}$ satisfies all microeconomic properties.

The definition a), requires that only first order moments are arguments of the aggregate cost func-

[1] Heineke et Shefrin (1988) show that the only microeconomic functional forms satisfying such a reparametrization (from (2) to (3)) belong to the following class:

$$c\left(p_t, y_{it}; \alpha_{it}\right) = \sum_{j=1}^{J} g_j\left(p_t\right) \Phi_j\left(y_{it}, \alpha_{it}\right).$$

Further the authors show that under firm's rationality, the summation index J is finite (see also Heineke, 1992 for a discussion). Here, the validity of such a reparametrization is assumed.

tion and is perhaps thereby very common. When in formula (3) $M_t = (y_t, \alpha_t)$, we obtain $\overline{C} = \widetilde{C}$.

Several kind of restrictions can support definition a). A first possibility to avoid the emergence of distributional statistics in the aggregate form is to restrict the microeconomic cost functions to take the Gorman polar form (see e.g. Lau 1982):

$$c(p_t, y_{it}; \alpha_{it}) = g_1(p_t)\alpha_{it} + g_2(p_t)y_{it} \tag{4}$$

where the vectors g_j for $j = 1, 2$ are identical for all firms. In this case $M_t = (y_t, \alpha_t)$, the (linear) aggregate parameters and the production level are the only statistics emerging in the aggregate form $\widetilde{C}$. Varieties of this form are for example adopted by Appelbaum (1982) or by Borooah and van der Ploeg (1986). However, the linearity in the output level appears particularly restrictive; first a redistribution of production from one firm to another has no effect on industry costs, second (4) implies identical marginal costs for all firms within the industry.

As yet, all restrictions refer to individual behavior. Lewbel (1992) among other, shows that another possibility to avoid aggregation bias is to restrict only the *distribution* of individual characteristics, allowing more general functional forms.

Definition b) is presented and discussed by Fortin (1988, 1991). It is implied by the definition a) but less demanding. The term B_t, measuring the gap between the exact aggregated cost function and the one based only on y_t, is called an aggregation bias. Of course, if one requires b) to hold for every p_t, y_t, α_t then b) would coincide with a). For some points, however, B_t may vanish.

Lewbel (1992) requires in addition to b) that the aggregate cost function have the same economic properties as the microeconomic ones (definition c).[2] From (2), it is immediately seen that the aggregate cost functions conserves some of the properties of their microeconomic counterparts (continuity, positivity, linear homogeneity in prices). Chavas (1993) show that the assumption of identical input prices is crucial to find the homogeneity in prices for aggregate costs. But for C to satisfy all properties of c, additional assumption are necessary.

Lewbel (1987, 1992) underlines that an independence assumption between M_t and input prices is additionally required for Shephard's lemma to hold at the aggregate level.[3] For instance, by derivation of C with respect to prices:

$$\frac{dC}{dp_t} = \frac{\partial \overline{C}}{\partial p_t} + \frac{\partial \overline{C}}{\partial M_t}\frac{dM_t}{dp_t}, \tag{5}$$

then $dM_t/dp_t = 0$ is sufficient to imply $dC/dp_t = \sum_{i=1}^{n_t} \partial c/\partial p_t$. In the short term, n_t and y_t being given, it amounts to assume the independence of α_t and h_t from p_t.

Assuming both price identity and independence between heterogeneity distribution and input prices, definitions b) and c) become equivalent. Thus only the definition a) and b) will be considered in the tests of the representative firm.

2.2 The representativity of marginal effects

In what follows, an aggregate statistic, if it is equal to the sum of their microeconomic corresponding

[2] Lewbel present the definition in the consummer context, here we adapt it to the producer.

[3] It is also asumed that Shephard's lemma holds at the microeconomic level; that is demand for input i is given by $x_i = \partial c(p_t, y_{it}; \alpha_{it}) / \partial p_{it}$, which requires that all input prices can be changed independently from other input prices ($\partial p_t / \partial p_t$ is the identity matrix).

statistics will be called representative. By definition (2) ensures the representativity of the cost function, whereas from the price independence assumption the representativity of input demand functions follows. For other statistics, representativity is not preserved in general. For instance by taking the derivative of (3) with respect to time:

$$\begin{aligned}\frac{dC}{dt} &= \frac{\partial \overline{C}}{\partial p_t}\frac{dp_t}{dt}+\frac{\partial \overline{C}}{\partial M_t^l}\frac{dM_t}{dt} \\ &= \frac{\partial \overline{C}}{\partial p_t}\frac{dp_t}{dt}+\frac{\partial \overline{C}}{\partial \alpha_t}\frac{d\alpha_t}{dt}+\frac{\partial \overline{C}}{\partial n_t}\frac{dn_t}{dt}+\frac{\partial \overline{C}}{\partial y_t}\frac{dy_t}{dt}+\frac{\partial \overline{C}}{\partial h_t}\frac{dh_t}{dt},\end{aligned} \tag{6}$$

The marginal effect of time can be decomposed into four distinct effects: an influence of technological change and three trend effects, respectively in prices, in market entry–exit process, in total product demand and in distributional statistics. Thus it seems necessary to unravel each influences in order to identify the actual effect of technological change as also underlined by Gouriéroux (1990) in a similar context. As for the marginal effect of prices, restrictions on $\partial\overline{C}/\partial M_t^l$ (that is on the microeconomic functions c) or on dM_t/dt and dp_t/dt can guarantee that $dC/dt = \sum_{i=1}^{n_t} \partial c/\partial \alpha_{it}$.

Similarly, the effect of a marginal variation in total production on aggregate cost is given by

$$\begin{aligned}\frac{dC}{dy_t} &= \frac{\partial \overline{C}}{\partial p_t}\frac{dp_t}{dy_t}+\frac{\partial \overline{C}}{\partial y_t}+\frac{\partial \overline{C}}{\partial \alpha_t}\frac{d\alpha_t}{dy_t}+\frac{\partial \overline{C}}{\partial n_t}\frac{dn_t}{dy_t}+\frac{\partial \overline{C}}{\partial h_t}\frac{dh_t}{dy_t} \\ &= \sum_{i=1}^{n_t}\frac{\partial c}{\partial y_{it}}\frac{\partial y_{it}}{\partial y_t}+\left(\sum_{i=1}^{n_t}\frac{\partial c}{\partial \alpha_{it}}\frac{d\alpha_t}{dy_{it}}+\frac{\partial \overline{C}}{\partial p_t}\frac{dp_t}{dy_{it}}+\frac{\partial \overline{C}}{\partial n_t}\frac{dn_t}{dy_{it}}+\frac{\partial \overline{C}}{\partial h_t}\frac{dh_t}{dy_{it}}\right)\frac{dy_{it}}{dy_t}.\end{aligned} \tag{7}$$

Aggregate marginal costs appear as a sum of a geometric mean of individual marginal costs and of some indirect effects of aggregate production variation. Even in case of independence between α_t, n_t, h_t and y_t the aggregate marginal cost does not correspond to the sum of microeconomic marginal costs but to a weighted average. The terms $\partial y_{it}/\partial y_t$ and dy_{it}/dy_t reflect how an additional increase in total output is distributed between firms. For example, even if cost functions take the Gorman polar form (4), only $\partial\widetilde{C}/\partial h_t = 0$ and $\sum_{i=1}^{n_t} \partial c/\partial y_t = g_2(p_t)$ are ensured.

3. From microeconomic to aggregate functions

The objective of this section is to derive from a microeconomic system of cost and demand functions the corresponding aggregate system. In order to avoid too many a priori restrictions, we consider flexible functional forms providing a local approximation to an arbitrary cost function (Diewert 1973). Then we establish the relations between microeconomic parameters and their aggregate counterparts, and study the potential bias emerging in costs when heterogeneity is neglected.

We assume that microeconomic cost functions belong to the class of normalized quadratic forms described by Diewert and Wales (1987, 1992):

$$\begin{aligned}c(p_t,t,y_{it};\alpha_i) &= p_t'A_{ip}+\frac{1}{2}(\eta' p_t)^{-1}p_t'A_{ipp}p_t+p_t'A_{ipt}t \\ &\quad +p_t'A_{ipy}y_{it}+\theta' p_t\left(\alpha_{itt}t^2+\alpha_{ity}ty_{it}+\alpha_{iyy}y_{it}^2\right).\end{aligned} \tag{8}$$

$A_{ip} = [\alpha_{ip}]$, $A_{ipp} = A_{ipp}' = [\alpha_{ipp}]$, $A_{ipy} = [\alpha_{ipy}]$, $A_{ipt} = [\alpha_{ipt}]$, are respectively $\ell \times 1$, $\ell \times \ell$, $\ell \times 1$ and $\ell \times 1$ matrices containing some subset of the parameters α_i to be estimated. The vectors η

and θ are introduced for normalization and can be estimated or arbitrarily fixed without destroying flexibility as discussed by Diewert and Wales. Among the usual properties of cost functions, the linear homogeneity and the price symmetry are directly imposed to (8). The linear price homogeneity implies in addition that A_{ipp} contains only $(\ell-1)\,\ell/2$ independent parameters instead of $(\ell+1)\,\ell/2$. This is directly imposed in the form of the following ℓ equality constraints:

$$(1,\ldots,1)\,A_{ipp}=0. \tag{9}$$

Diewert and Wales (1987) also show that the price concavity of c is equivalent to the negative semi–definiteness of the matrix A_{ipp}. Moreover they present a method to impose directly this last condition and show that the normalized quadratic form still remains flexible with concavity imposed, contrary to other usual specifications. All these adaptations are adopted in the present study.

The parameters α_i are constant over the time period, their time dependence is conveyed by the presence of the variable t (a time trend increasing by one each year). This amounts to specifying a linear time dependence for the parameters in A_{ip} and A_{ipy}. The additional α_{itt} component enables the cost function (8) to be fully flexible with respect to time as shown by Diewert and Wales (1992).

When the functional form (8) is aggregated linearly across the n_t firms forming an industry, the resulting aggregate form is

$$\begin{aligned}\overline{C} &= p_t'A_p+\frac{1}{2}\left(\eta'p_t\right)^{-1}p_t'A_{pp}p_t+p_t'A_{pt}t \\ &\quad +p_t'\sum_{i=1}^{n_t}A_{ipy}y_{it}+\theta'p_t\left(\alpha_{tt}t^2+\sum_{i=1}^{n_t}\alpha_{ity}ty_{it}+\sum_{i=1}^{n_t}\alpha_{iyy}y_{it}^2\right),\end{aligned} \tag{10}$$

where $A_p=\left[\sum\limits_{i=1}^{n_t}\alpha_{ip}\right]$, $A_{pj}=\left[\sum\limits_{i=1}^{n_t}\alpha_{ipj}\right]$, $j\in\{p,t\}$, $\alpha_{tt}=\sum\limits_{i=1}^{n_t}\alpha_{itt}$. The aggregate demand functions $x_t=\sum\limits_{i=1}^{n_t}x_{it}$ are given by:

$$\begin{aligned}x_t &= A_p+\left(\eta'p_t\right)^{-1}A_{pp}p_t-\frac{1}{2}\left(\eta'p_t\right)^{-2}p_t'A_{pp}p_t\eta+A_{pt}t \\ &\quad +\sum A_{ipy}y_{it}+\theta\left(\alpha_{tt}t^2+\sum\alpha_{ity}ty_{it}+\sum\alpha_{iyy}y_{it}^2\right).\end{aligned} \tag{11}$$

As discussed in the first section $x_t=\partial C/\partial p_t=\sum_{n_t}\partial c/\partial p_t$ is verified. These aggregate relations (10) and (11) depends on non-observable variables $\sum A_{ipy}y_{it}$, $\sum\alpha_{iyy}y_{it}^2$ and $\sum\alpha_{iy}ty_{it}$. However, following Fortin (1991) and using the definition of second order non-centered moments $E[ab]=cov(a,b)+E[a]\,E[b]$, we can rewrite

$$\sum A_{ipy}y_i = n\Omega_{py}+E\left[A_{ipy}\right]\sum y_i, \tag{12a}$$

$$\sum \alpha_{ity}y_i = n\omega_{yt}+E\left[\alpha_{ity}\right]\sum y_i, \tag{12b}$$

$$\sum \alpha_{iyy}y_i^2 = n\omega_{yy}+E\left[\alpha_{iyy}\right]\sum y_i^2, \tag{12c}$$

where Ω_{py} is a vector with ℓ components ω_{p_ky}, $k=1,\ldots,\ell$. All ω_{jy} for $j=p_h,\ t,\ y$, symbolize the covariance between two microeconomic characteristics: the parameter α_{ijy} and the level of production (or its square in the last case).

Using these relations, the aggregated costs can be expressed as:

$$\overline{C}\left(p_t, t, y_t; \alpha, n_t, \sum y_{it}^2, \Omega_{py}, \omega_{ty}, \omega_{yy}\right), \tag{13}$$

which depends only on a limited number of distributional statistics. The value of $\sum y_{it}^2$ is deduced from the Herfindahl index H_t as $\sum y_{it}^2 = H_t \left(\sum y_{it}\right)^2$, the other distribution statistics are however not simply observable, thus they are considered as parameter of our model and estimated along with α.

4. Data description

Most of the data was provided by the Statistisches Bundesamt, the German federal statistical office. They are available for 27 West-German industrial sectors at the two-digit level of classification and cover the years 1960 to 1992. Three inputs are considered, material, labor and capital, so that in our case ℓ=3, $x_{it} = (m_{it}, l_{it}, k_{it})'$, $x_t = (m_t, l_t, k_t)'$, and $p_t' = (p_{mt}, p_{lt}, p_{kt})$. Conrad and Unger (1987) split the material data in energy and other materials. Then they can estimate one additional demand function. However, the energy data come from their own computation and official energy data are available from the Statistisches Bundesamt only from 1981 onward (at this level of aggregation).

Further computation appears necessary to define some variables. The price of materials is not published. Since production prices are published, we calculate the material input in constant prices as the difference between production and value added (in constant prices). The labor input is evaluated in men–hour. The yearly average hours of work are collected by the Institut für Arbeitsmarkt- und Berufsforschung for the same aggregation level. All prices are normalized to one in 1991.

The capital input has the particularity of not disappearing instantaneously in the production process, but progressively. Thus, to take account of capital consumption, the real net stock of capital is retained. Since these real net values are only available since 1970 for every two-digit industrial sector, we choose to approximate the missing data by using the investment definition based on the permanent inventory rule:

$$i_{kt} = k_{t+1} - (1 - \delta_t)\, k_t \tag{14}$$

where δ_t is the depreciation rates and $k_t = \sum_{i=1}^{n_t} k_{it}$. Since the aggregate gross real investment values i_{kt} are known for the whole period, the net capital stock can now be computed for 1969, given its 1970 value (using 14) if the depreciation rates for this year is known. These depreciation rates are available over the whole period only in the aggregate, but by supposing that in each branch they vary at the same rate as in the total manufacturing, we recover the missing depreciation rates (years 1960 to 1969), and then the data on the net real capital stock. The user costs for capital are derived according to $p_{k_t} = (1 + r_t)\, p_{ik_t} - (1 - \delta_t)\, p_{ik_{t+1}}$, where p_{ik_t} is the acquisition price of the capital, r_t is the nominal long run rate of interest. The data described so far correspond to an updated dataset also used by Flaig and Steiner (1993a, 1993b).

In addition, the framework developed above requires data on the number of firms n_t, and for $\sum y_{it}^2$. The first emerging question concerns the decision unit considered. Since all published data are collected on the basis of firms, this last unit is retained rather than establishments. From 1977 onwards, data for firms with more than 20 employees are available for every year; for 1960 to 1976, data pertain relative to firms with 10 employees or more. For the year 1960–1976, data relative to firms with 20 employees and more are calculated on the basis of the data for 1977 and variation rates available for the years before for firms with 10 employees and more.

Data on the number of small firms (with less than 20 employees) are only to be found for the year 1961, 1970 and 1987, when the German firms census took place.[4] By linear interpolation, data on the number of small firms are recovered for the remaining years. For the years 1987 to 1992, the trend before 1987 is used.

The needed indicator $\sum_i y_i^2$ reflecting the variance of the production level across firms can be computed through an Herfindahl index H_t. This index unfortunately present three drawbacks: it refers to nominal production, is available only since 1977 and is computed only for establishments with more than 20 employees. The first drawback is neglected here; in fact it is negligible only if production prices are identical among firms belonging to the same industry. How we recover Herfindahl indexes over the whole period and for all the firms is described in the appendix.

5. Empirical results for German manufacturing industries

The model consists of the aggregate input demand functions (11) where the unobservable sums are replaced using (12). The covariances ω are assumed to be constants over the estimation period and estimated along with other parameters. Since the cost and input demand functions are linearly dependent, only the three demand functions are needed. Further, the demand are divided by the production level to make the homoscedasticity of the added disturbance vector ε_t more plausible. Thus the system becomes

$$\begin{aligned}\frac{x_t}{y_t} &= \left(A_p + (\eta' p_t)^{-1} A_{pp} p_t - \frac{1}{2} (\eta' p_t)^{-2} p_t' A_{pp} p_t \eta + A_{pt} t\right) / y_t \\ &\quad + n_t \Omega_{py} / y_t + A_{py} \\ &\quad + \theta \left(\alpha_{tt} t^2 + (n_t \omega_{yt} + \alpha_{yt} y_t) t + n_t \omega_{yy} + \alpha_{yy} y_t^2 H_t\right) / y_t + \varepsilon_t .\end{aligned} \tag{15}$$

The variable t is defined as a time trend equal to one in 1960 and increasing yearly by one. The vectors θ and η are defined as x_{1991}/C_{1991} so that $\theta' p_t = \eta' p_t$ can be interpreted as Laspeyres price indices for total costs. With this specification however, the identification of ω_{yy} is no longer possible since it is perfectly proportional to the coefficients in Ω_{py}. Thus, this parameter is deleted, and then the model contains 19 parameters which will be estimated on the basis of $3 \times 33 = 99$ observations. If the number of firms would be constant over the period or distributed around a constant mean, no heterogeneity parameters at all would be identifiable. The same conclusion would be reached if instead of estimating parameters α corresponding to sums of micro–parameters one would estimate averages of the corresponding micro–parameters (noted $\overline{\alpha}$).[5] In this last case, we would simply have to reparametrize (15) using $A_j = n_t \overline{A}_j$, $j = p, pp, pt$, and $\alpha_{tt} = n_t \overline{\alpha}_{tt}$, and then to estimate the $\overline{A}_j$ and $\overline{\alpha}_{tt}$ instead of the A_j and α_{tt}. However, the macroeconomic models presented in the literature usually

4 For these years, the numbers of small firms are listed for several classes of employment (with 1, 2 to 4, 5 to 9, 10 to 19 employees).

5 Such a situation appears in Fortin (1991a, b), who could not identify the aggregation bias or the involved covariances for each industry, but only across industries.

do not use any data on the number of firms n_t, so that to measure the aggregation bias emerging in these model, the specification (15) is the appropriate to be considered.

The parameters are estimated in two stages. First, the unrestricted model (A_{pp} is not restricted to be negative semi definite) is estimated according to the *SUR* procedure, iterating on the residual covariance matrix. The estimated variance-covariance matrix $V(\hat{\alpha})$ of the unrestricted parameter vector $\hat{\alpha}$ is consistent. In the second stage, parameters $\hat{\alpha}^0$ satisfying the restrictions are calculated using minimum distance:

$$\hat{\alpha}^0 = \arg\min \left(\hat{\alpha} - \hat{\alpha}^0\right)' \left[n_{obs} V(\hat{\alpha})\right]^{-1} \left(\hat{\alpha} - \hat{\alpha}^0\right),$$

where n_{obs} is the number of observations in the model (33 in our case). Gouriéroux and Monfort (1989) show that the resulting estimator is asymptotically equivalent with the constrained *SUR* estimator.[6]

We test the representative firm assumption relying on definitions a) and b). For instance, the aggregate form (15) nests both models described above. The definition a), requires that the Gorman polar form holds for each individuals. In this case the parameters A_{ipy}, α_{ity} are identical for each firm, that is, all covariances vanish in (15), and $\alpha_{iyy} = 0$.

The second tests refers to the weaker definition b) (or equivalently c) of the representative firm. Between the two kind of restrictions discussed above, we consider the one on individual firm behavior. The aggregation bias B_t corresponds to the difference between the aggregate cost function (13) and the corresponding aggregate Gorman polar form:[7]

$$\begin{aligned} \widetilde{C}(p_t, y_t, \alpha_t) &= p_t' A_p + \frac{1}{2}(\eta' p_t)^{-1} p_t' A_{pp} p_t + p_t' A_{pt} t \\ &\quad + p_t' A_{py} y_t + \theta' p_t \left(\alpha_{tt} t^2 + \alpha_{ty} t y_t\right). \end{aligned} \tag{16}$$

Thus

$$B_t = p_t' n_t \Omega_{py} + p_t' \theta \left(n_t \omega_{ty} + \alpha_{yy} \sum_i y_{it}^2 \right). \tag{17}$$

Clearly, rejection of the definition a) does not imply the rejection of b) since even if some parameters Ω_{py}, ω_{ty}, or α_{yy} are significantly different from zero, they may be compensated in (17) by opposite effects and B_t may vanish. The results of the tests of the two assumptions are summarized in table 1 Both tests are relative to the year 1976, the middle year of our sample.

The definition a), corresponding to wide accepted definition of the representative agent is always rejected at the exception of two industries.

Tests of definition b) could appear more optimistic, although the aggregation bias is shown to be important in absolute value (B represents frequently more than a third of total costs) and is still significative in several industries (in 12 out of 27 industries).

[6] Convergence turns out to be much more difficult to obtain when iteration on both the restricted parameters and the residual covariance matrix (the *SUR* model) are done simultaneously.

[7] This is the aggregate of (4) with $g_1(p) = \left(p_t', \frac{1}{2}(\eta' p_t)^{-1} \operatorname{vec}(p_t p_t'), p_t' t, \theta' p_t t^2\right)$, $\alpha_t = (A_p, \operatorname{vec}(A_{pp}), A_{pt}, \alpha_{tt})$, and $g_2(p) = \left(p_t' A_{py} + \theta' p_t \alpha_{ty} t\right)$.

Table I: Tests of representative agent models[a]

Nr.	Industry[b]	definition a) χ^2 test	definition b) B in % of C	t-test
14	Chemical products	649.59	-55.1	-2.48
15	Mineral oil refining	5.44	-17.9	-0.34
16	Plastic products	50.54	53.2	2.20
17	Rubber products	48.41	-3.7	-0.31
18	Stones and clay	193.16	97.0	8.92
19	Ceramics	406.54	-27.4	-2.81
20	Glass	97.01	-12.6	-0.69
21	Iron and steel	7.90	2.2	0.27
22	Non-ferrous metal	61.37	4.31	0.23
23	Founderies	165.81	120.6	6.32
24	Drawning plants	37.71	-22.3	-1.94
25	Structural metal products	61.59	3.6	0.23
26	Mechanical engineering	45.60	11.8	1.30
27	Office machinery[c]	109.49	-63.6	-1.36
28	Road vehicles	150.11	-33.6	-5.44
29	Shipbuilding	28.27	46.6	2.15
31	Electical engineering	45.25	-25.9	-1.28
32	Precision, optical instruments	232.82	7.3	0.94
33	Finished metal goods	63.57	59.2	2.15
34	Musical instruments,toys, etc.	101.88	-30.4	-2.00
35	Wood working	144.43	287.1	7.44
36	Wood products	137.65	18.7	1.46
37	Paper manufacturing	95.34	32.4	2.62
38	Paper processing	79.18	73.9	1.75
39	Printing and duplicating	50.75	-11.5	-0.31
40	Leather	424.05	33.1	6.50
41	Textile	476.40	29.6	2.28
42	Clothing	555.26	6.4	2.01
43	Food and beverages	160.56	-11.5	-1.28

[a] χ^2 values for the hypothesis that the five coefficients reflecting aggregation bias are simultaneously zero, the critical value at the 5% level is 11.07 and at the 1% level 15.09. For the Student tests for the hypothesis that the aggregation bias is zero, the critical value at the 5% level is 2.04.

[b] For the aicraft industry and the tobacco industry (number 30 and 45), no producer price are available, thus we leave these industries out. For the food and beverages, product price are not available separately, thus we aggregate these industries together (number 43 and 44).

[c] For the office machinery industry, the data are only available from 1970 ownwards.

6. Conclusion

Investigating exact aggregation of cost and demand functions, we show that studies estimating micro-economic behavioral functions on the basis of aggregate data could lead to aggregation bias affecting not only the cost function but also its derivatives and the interpretation of several estimations relying on representative firm models.

Restrictions on microeconomic functional forms appear often an issue to avoid the emergence of distributional statistics (often not available by statistical offices) in the aggregate. Thus, Gorman polar forms satisfying such restrictions are often assumed for individuals. In this paper, on the basis of a flexible quadratic cost function at the firm level, we show that the required conditions to find the Gorman form are rejected for 25 out of 27 German industries considered.

Some less restrictive assumptions could also avoid the apparition of distributional statistics in the aggregate. Several distributional effects may in fact be compensated, and the aggregation bias may then vanish without necessary postulating the Gorman polar form for individuals. After identifying, we estimate the importance of this aggregation bias; the resulting test appears a little more optimistic, but the hypothesis of vanishing bias is still rejected for several industries.

Briefly, the best way to avoid aggregation bias and implied bias in parameter estimations, is to undertake exact aggregation. The heterogeneity of firms (here in their production process and in the level produced) appears a good reason to extend standard producer behavior models. Apart from avoiding biases, this allow also to study the effect of heterogeneity and concentration on costs and input demand.

Appendix

Conditionally on the 6-firm concentration ratios CR_6 available over the whole period, the Herfindahl index is within the interval $[H_{\geq 20}^{\min}, H_{\geq 20}^{\max}]$, where $H_{\geq 20}^{\min}$ is the conditional minimal Herfindahl index equal to:[8]

$$H_{\geq 20}^{\min} = (CR_6)^2/6 + (1 - CR_6)^2/(n_{\geq 20} - 6).$$

The upper limit results from the application of the transfer principle, and a direct adaptation from the results of Sleuwaegen and Dehandschutter (1986) leads to:

$$H_{\geq 20}^{\max} = \begin{cases} \left(CR_6 - 5\frac{1-CR_6}{n_{\geq 20}-6}\right)^2 + (n_{\geq 20} - 1)\left(\frac{1-CR_6}{n_{\geq 20}-6}\right)^2 & \text{if } CR_6 \geq s \\ CR_6/6 & \text{if } CR_6 < s \end{cases}.$$

where the step s, is given by:[9]

$$s = \frac{1/6 + 5/(n_{\geq 20} - 6)}{1 + 5/(n_{\geq 20} - 6)}.$$

Herfindahl index are only available for the period 1977 to 1992 for firms with 20 employees or more. To recover the missing values for the period 1960 to 1976, we regress the Herfindahl index on variables CR_6, $y_{\geq 20}$ and $n_{\geq 20}$ available for the whole period.[10] Then we use this estimated relation to "forecast" their values for 1960 to 1976. The estimated relation for $H_{\geq 20}$ is chosen as a convex combination of its extreme values as:

$$H_{\geq 20} = \frac{1}{1 + \exp(-m)} H_{\geq 20}^{\min} + \left(1 - \frac{1}{1 + \exp(-m)}\right) H_{\geq 20}^{\max},$$

with $m = a_0 + a_1 CR_6 + a_2 CR_6^2 + a_3 y_{\geq 20} + a_4 n_{\geq 20}$. For only five industries, the coefficient of correlation between actual and predicted value was below 0.9. Now Herfindahl indexes for firms with more than twenty employees are recovered for the whole period and $\sum_i y_{\geq 20i}^2$ is deduced by multiplying $H_{\geq 20}$ by $y_{\geq 20}^2$.

For firms with less than twenty employees, data on their number are available for several employment classes (see the data description). We compute $\sum_i y_{<20i}^2$ after distribution of $y_{<20}$ ($y_{<20} = y - y_{\geq 20}$) in the four employment classes assuming that firms with less than twenty employees produce proportionally to their number of employees. Further we suppose that inside each employment class the corresponding number of firms is uniformly distributed. Such an assumption is often made in industrial economics to compute surrogates of concentration indices, even for the distribution of the whole production among firms (see Schmalensee 1977). Here, the way this correction is made appears not important; in fact $\sum_i y_{<20i}^2$ represents always less than 1% of $\sum y_i^2$, which results from the very small production per firm ratio for small firms.

8 The subscript ≥ 20 refers to variables relative to firms with more than twenty employees.

9 When $n_{\geq 20}$ tends to infinity, the results of Sleuwaegen and Dehandschutter are retrieved.

10 In fact these values are only partially available from 1960 to 1976, for some years CR_6 or $n_{\geq 20}$ have been linearly interpolated and $y_{\geq 20}$ deduced from available information on y.

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