Discussion Paper

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Value at Risk: – Proposals on a Generalization –

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von

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Abstract

The Value at Risk approach (VaR) is more and more used as a tool for risk measurement. The approach however has shortcomings both from a theoretical and a practical point of view. VaR can be classified within existing concepts of risk measurement: it is particularly interpretable as a special measure of shortfall risk. From that point of view VaR will be extended and improved. Eventually return distributions and shortfall measures are calculated for portfolios including option strategies. Though VaR is held constant across the resulting return distributions quite different valuations of risk arise depending on the shortfall measure used for the comparison.

Keywords: Value at Risk, Shortfall Risk, Risk Management, Options

1 Introduction

The control of the risks of banking operations is more and more understood as an important public task. International organisations and professional associations are busy to develop new concepts to measure and control risks. The aim is to achieve a global standard in risk controlling. The numerous publicly known problems of banks and industrial companies to handle financial derivatives have further stressed the importance of efficient control mechanisms.

The Basle Committee on Banking Supervision is responsible for the development and improvement of internationally standardized concepts and measures to regulate banking operations. The Basle Committee is dealing with the problems of risk regulation and capital requirements for many years.

In its publication „Amendment to the Capital Accord to incorporate Market Risks" from January 1996 the Basle Committee emphasized the importance of internal risk models. The internal risk models are methods to measure the risk of the trading book of banks. These models are developed by banks themselves and adjusted for specific applications.

The so-called Value at Risk (VaR) approach plays an important role in measuring the risk potential of the trading book. The Basle Committee itself recommends VaR as a suitable method for risk measurement. In the above mentioned publication the Basle Committee gives detailed instructions about the conditions necessary for the application of Value at Risk, instead of using the standard methods developed by the Basle Committee.[1]

A shortcoming of Value at Risk is the lack of a sound theoretical foundation. In the following chapters it is shown that the VaR approach is interpretable as a special

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[1] Basle Committee on Banking Supervision (1996), Part B.

measure of shortfall risk. The shortfall approach gives the theoretical foundation and shows that VaR does not differentiate between different attidutes to risk.

Using the shortfall approach it is straightforward to develop a generalized Value at Risk approach that eliminates major shortcomings of the existing Value at Risk. The generalized VaR approach is suitable to all kinds of risk averse investors. It also widens the applications in the fields of performance measurement and capital allocation.

2 Value at Risk: Description and Problems with its Use

Value at Risk is a general method to measure risks. The usual application is measuring the risks of the trading book of banks. VaR is the value of the potential loss of a capital investment that is exceeded by only a given very small probability (1% or 5%). VaR therefore indicates an exceptionally high loss.

The mathematical formula for Value at Risk is as follows:

$$\boxed{\begin{aligned} &\Pr(R \le \mathrm{VaR}) &&= \alpha \\ \Leftrightarrow\ &F(\mathrm{VaR}) &&= \alpha \\ \Rightarrow\ &\mathrm{VaR} &&= F^{-1}(\alpha) \end{aligned}}$$

The cumulative distribution function F(R) indicates the probability (Pr) that the portfolio return (R) is less than a given value (α). Calculating VaR the value of α is given exogenously. Usual values for the confidence level α are 1% or 5%. VaR then gives the return that is exceeded on average in (1-α)% of all time periods.

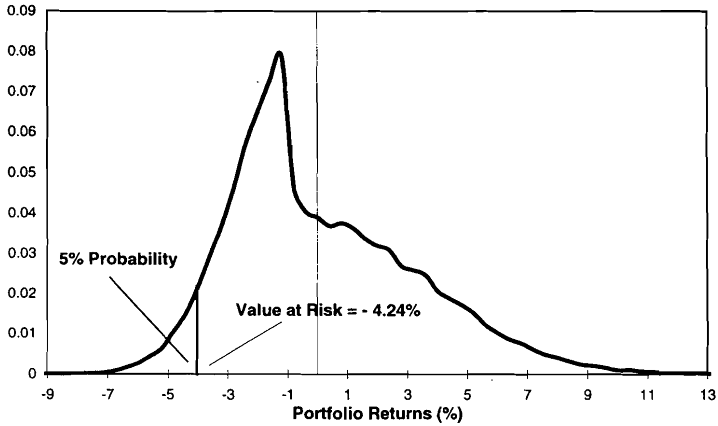
The Basle Committe uses Value at Risk to calculate the capital requirements for the trading operations of banks. For that purpose VaR is multiplied by the investment volume. The resulting amount gives the loss potential in local currency. In some articles this amount is called „Value at Risk". In our article portfolios are represented only by their return distribution. Therefore in our article Value at Risk means a specific return value. The required capital for trading operations proposed by the Basle Committee is the amount of potential loss multiplied by three. The multiplication is chosen to account for the inherent „model risk" due to insufficient experience using the VaR approach.

To calculate VaR it is necessary to have an estimation of the return distribution. Figure 1 illustrates graphically the calculation of VaR. In this example VaR is -4.24%. Left of VaR the area below the curve is equal to a probability of 5%. This means that portfolio returns less than -4.24 will on average occur with a probability of 5%.

Fig. 1: The Value at Risk Approach

Results of a Portfolio Simulation: a normally distributed Asset together with a Put Option (purchased on 50% of the underlying Asset, at the money)

Distribution of the Returns (Period: 10 Days)



Usually it is assumed that stock- and bond-indices are approximately normally distributed.[2] Then portfolios combining different stock- and bond-indices are also approximately normally distributed. In this case VaR is easily calculated as the 1%-

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[2] It is assumed that the continously compounded return has a normal distribution. Then the simple discrete return follows a lognormal distribution.

or 5%-quantile of the normal distribution with the properly chosen mean and standard deviation.

Controlling risk is a very important task if the portfolio contains a considerable amount of financial derivatives. Assuming that the underlyings of the options are normally distributed the price of the options included in the portfolio can be calculated using the well known Black-Scholes formula.

In the following chapters simulation results are often used to give examples for the Value at Risk approach. It is therefore necessary to describe in detail how the simulations have been constructed.[3] Figure 1 shows the return distribution of a portfolio that contains normally distributed asset and put options purchased on 50% of the underlying asset. The strike price of the put option is adjusted to the price of the underlying asset after each period (= rolling hedge using at the money put options).

Table 1: The Simulations

1. Normal distribution: (= Distribution of the underlying asset)	
(a) Number of periods: (b) Mean return (annualised): (c) Standard deviation (annualised)	50000 10% from 3 to 25
2. Strategy with put options:	
(a) Strike price of the put options:	at the money (depending on the price of the underlying asset)
(b) Holding period: (c) Rolling hedge:	10 days (= time to maturity of the option) a new put option is purchased after 10 days
(d) Part of the underlying asset on which put options are purchased: (e) Proportion (asset : put option): (f) Interest Rate: (g) Volatility (= annualised standard devitation):	50% 1 : 1 5% from 3 to 25 (depending on the volatility of the underlying asset)

[3] Table 1 summarizes the assumptions underlying the simulations.

The holding period of the options is 10 days, according to the recommendations of the Basle Committee for the calculation of VaR. The return distributions are calculated from the portfolio values of 50000 periods. Every distribution or calculated parameter refers to a period length of 10 days.

The use of only Value at Risk to compare the risk of two or more portfolios can be highly misleading. The two distributions in figure 2 have only one characteristic number in common: the VaR value calculated with a confidence level of 5%. In both cases VaR is equal to -4.87%. That is both portfolios have the characteristic that the return falls short of -4.87% with a probability of 5%.

Fig. 2: Two Distributions with the same Value at Risk (=-4.87%)
1. Normal Distribution, 2. Portfolio with Put-Options
(purchased on 50% of the underlying Asset, at the money)

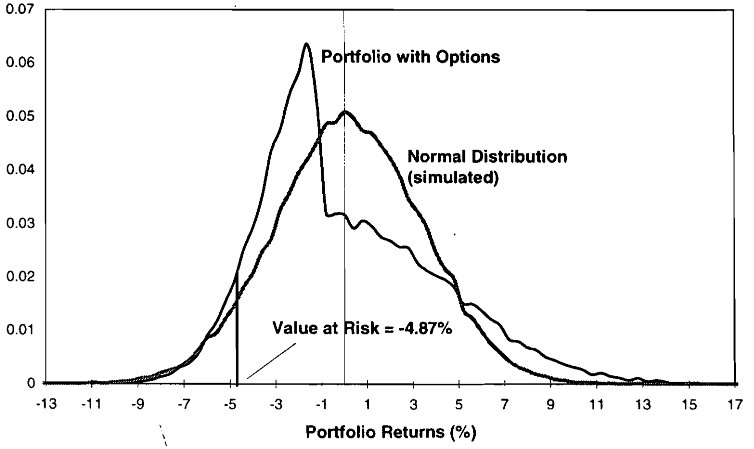


Table 2 compares important characteristic numbers of the two return distributions. The first distribution described in table 2 is a simulated normal distribution. The mean return for a period of 10 days is 0.396% (= approx. 10% annualised) and the standard deviation is 3.4. The second distribution contains a normally distributed asset (mean: 0.396%, standard deviation: 5) and put options purchased on 50% of the underlying asset[4]. Due to the costs of the put option the mean return (= 0.359%) is now slightly below the mean return of the normal distribution.

Table 2: Statistical Characteristics of the two Distributions from Fig. 2

(Returns calculated for periods of 10 days)

All numbers in %	**Simulated Normal Distribution**	**Simulated Portfolio with Put Options**
Value at Risk	-4.87	-4.87
Mean Return	0.396	0.359
Standard Deviation	3.2	3.9
Minimum Return	-13	-11.2
Maximum Return	13.2	19.4
1% Quantil	-7.1	-6.6
5% Quantil	-4.87	-5.87
10% Quantil	-3.7	-4
Median	0.4	-0.6
75%Quantil	2.6	2.8
90% Quantil	4.5	5.9
95% Quantil	5.6	7.7
99% Quantil	7.8	11

One important difference between the two distributions is the skewness. Table 2 compares the quantiles of the distributions. The portfolio containing put options is skewed to the right. Below the VaR the probability of a loss is smaller than compared to the normal distribution. Choosing a VaR-confidence level of 1% instead of 5%, the portfolio containing the put options has a smaller VaR than the normal distribution, choosing a VaR-confidence level of 10% the normal distribution exhibits a smaller VaR, however. Below the VaR the portfolio with put options has not only a smaller loss probability but also a smaller average loss. The

4 See table 1 for a detailed description of the hedging strategy.

loss below the VaR is on average 5.8%. The normal distribution however has an average loss of 6.2% left to the VaR value. It is a major shortcoming of the Value at Risk approach that it considers only loss probabilities but not the possible amount of the losses. Therefore VaR can have undesirable incentives on the construction of portfolios, because a more risk avoiding portfolio does not necessarily have a smaller capital requirement than a more aggressive portfolio.

3 The Concept of Shortfall Risk and Value at Risk

The concept of shortfall risk refers to the possibility to fall short of a desired minimum return. Other expressions for the same concept are downside risk and lower partial moments. The term lower partial moments (lpm) means that in the concept of shortfall risk statistical moments are calculated below the desired minimum return.

Fig. 3: The Shortfall Risk - Approach
Same Portfolio as Fig. 1

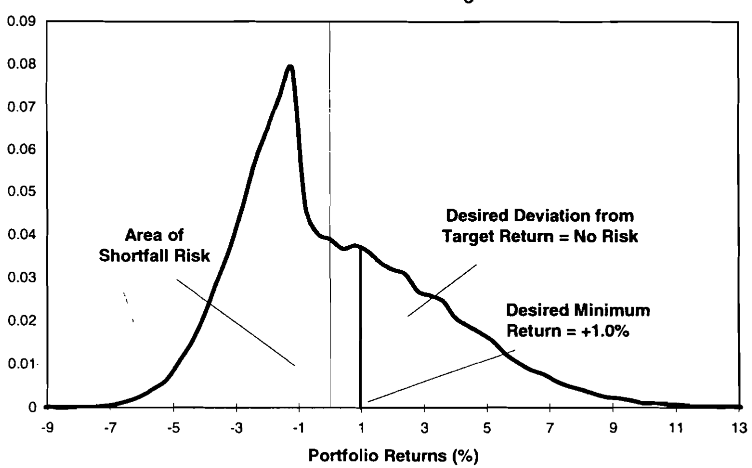


Figure 3 gives an illustration of the shortfall risk concept. Before calculating the lower partial moments it is necessary to fix the minimum return. The minimum return will often have a value of 0% or be equal to a current money market interest rate. Another possible choice is the required return for liabilities. Insurance companies may have a preference to choose the return of competitors as the minimum return to calculate shortfall risk. In the example of figure 3 the desired minimum return is fixed arbitrarily to +1%.

The lower partial moments are calculated for that part of the return distribution below the minimum return. The return above the minimum return are desired by the investor and are therefore not used to calculate the risk measures. The use of variance or standard deviation as measures of risk is often critisized by investors because negative and positive returns are equally used in the calculation. In the concept of shortfall risk only undesired returns are used to calculate risk. Shortfall risk is therefore close to most investors feeling about the meaning of risk.

Using options the return distribution of a portfolio will become skewed to the right or left. In such cases it is necessary to use lower partial moments because the standard deviation will indicate a misleading amount of risk, either too small (using call options) or too high (using put options).

The general formula for the calculation of lower partial moments is as follows:

$$\boxed{\mathrm{lpm}_n(z) = \int_{-\infty}^{z} (z - R)^n \, dF(R)}$$

Lower partial moments are calculated as the integral of the weighted return distribution (=F(R)) from minus infinity to the minimum return z. The weights (z-R) are always positive and are set to the n-th power. In case of n = 0 the lower partial moment is the integral of the unweighted return distribution. lpm_0 is therefore equal to the cumulative distribution function F at the return R = z.

Using VaR as minimum return to calculate lpm_0 it is straightforward to express the similarity between the two concepts:

$$
\boxed{\begin{aligned} lpm_0 &= F(z) \\ &= \Pr(R \le z) \\ &= \Pr(R \le VaR) \\ &= F(VaR) \\ \Rightarrow VaR &= F^{-1}(lpm_0) \end{aligned}}
$$

The formulas showing lpm_0 and VaR are directly related via the cumulative destribution function. Fixing VaR lpm_0 gives the probability to fall short of the VaR value. Fixing the probability lpm_0 the corresponding VaR can be calculated.

The other lower partial moments are calculated choosing n = 1, 2, 3, etc. In case of n = 1 the lower partial moment is called target shortfall and it gives the expected loss below the minimum return. If e.g. lpm_1 is equal to 1% than the deviation from the minimum return to the left is 1% on average. In case of n = 2 the differential between the portfolio return and the minimum return is squared. The resulting target semivariance (= lpm_2) is therefore calculated similar to the variance.

Figures 4 and 5 illustrate the differences between lpm_n-measures. Figure 4 shows values of lpm_1 and the square root of lpm_2 calculated for different normal distributions. The normal distributions all have the same mean return (10% annualized), but different standard deviations. The graphs start with a standard deviation of 3 and ends with a standard deviation of 25. The minimum return necessary for the calculation of the lower partial moments is fixed to the VaR values of each normal distribution given a confidence level of 5%. As can be seen the lpm_2 value is always above the corresponding lpm_1 value. Normal distributions with higher standard deviation also exhibit higher values for the lower partial moments.

In figure 5 the graphs show the ratios of the lower partial moments between a portfolio with put options and the corresponding normal distribution. The normal distribution chosen is therefore the same as the distribution of the underlying asset of the portfolio with put options.

Fig. 4: Lower Partial Moments calculated for Normal Distributions
Annualised Standard Deviations from 3 to 25
LPMn -Target Return = VaR (Confidence Level = 5%)

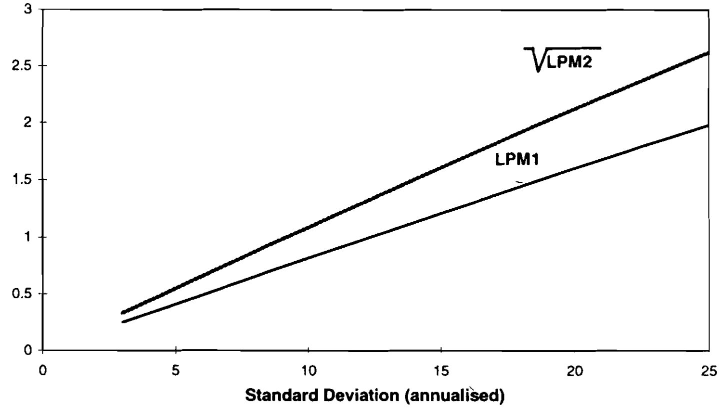
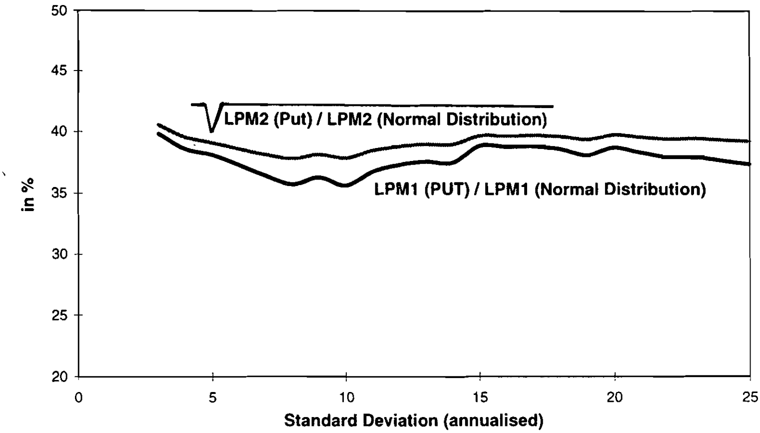


Fig. 5: Normal Distribution compared to Portfolio with Put-Options Proportion of LPMn-Values
LPMn -Target Return = VaR of the Normal Distribution (Confidence Level = 5%)



The minimum return for the calculation of the lpm values is equal to the VaR value of the normal distribution (confidence level = 5%). Figure 5 shows that the lpm values of the two portfolios are of quite different magnitude. The lpm_1 values of the portfolio with put options are only 38% of the lpm_1 values of the normal distribution and the analogous ratios to the square root of lpm_2 values are approximately 39%. These results demonstrate the big amount of possible risk reduction using the implemented hedging strategy with put options.

Lower partial moments can and should be used as a measure of risk instead of the standard deviation. Using lower partial moments in practice the suitable degree of n has to be chosen. Fortunately the concept of shortfall risk has a sound foundation in economic decision theory. Instead of the well known μ/σ criterion from Markowitz an μ/lpm_n criterion is used.[5] The decision problem is the same: find all combinations of assets that minimize portfolio risk given a specific portfolio return. The result is an efficient frontier in the μ/lpm_n -space.[6]

The shape of the utility function of the investor, especially the kind of risk aversion, determines which shortfall measure should be chosen. If the utility function can be characterised by only a positive first derivative ($U' > 0$) then lpm_0 is the suitable risk measure. If the investor is risk averse so that the first derivative of the utility function is positive and the second derivative is negative ($U' > 0$, $U'' < 0$) then the investor should choose lpm_1 as the measure of risk and if the investor is still more risk averse ($U' > 0$, $U'' < 0$, $U''' > 0$) he should choose lpm_2.[7]

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5 See Bawa/Lindenberg (1977) and Harlow/Rao (1989).

6 Under the condition: $n>0$. See Bawa/Lindenberg (1977).

7 See Bawa (1978) and Fishburn (1977). LPM_0 is the general risk measure. It is applicable to all utility functions ($u'>0$). The analogue in the theory of decisions under uncertainty is the 1. order Stochastic Dominance rule. LPM_1 and LPM_2 are more special than LPM_0. They are applicable if the investor is risk averse. LPM_1 and LPM_2 are analogous to the 2. and 3. order Stochastic Dominance rule, respectively.

The concept of shortfall risk makes no assumptions about the return distribution.[8] Therefore the shortfall risk approach is a generalization of the Markowitz approach. As the requirement of normally distributed returns is a considerable restriction using the Markowitz approach in practice, lower partial moments are not only an improvement in theory but also in practice.[9]

The assumption of normally distributed returns is fully inappropriate if the portfolio contains a considerable amount of options. The shortfall approach however is applicable to any kind of return distribution. Additionally the specific risk aversion of the investor is taken into account choosing the minimum return and the suitable lower partial moment.

4 A Generalized Value at Risk Approach

The concept of shortfall risk gives a decision-theoretic foundation of the Value at Risk approach. It is straightforward to develop a generalized VaR approach using lower partial moments. Value at Risk is the same kind of measure as is lpm0. Therefore VaR is suitable as a measure of risk only for risk neutral investors ($U' > 0$). The use of lpm_n-measures ($n > 0$) is the basis for the development of generalized VaR measures that take into account risk aversion.

Instead of choosing a probability (or confidence level) to calculate VaR, the value of a general integral (S_n) has to be given:

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8 Lower partial moments are applicable as generalized Safety-First rules to any return distribution. In this application lower partial moments are only an aproximation to the exact solution. If the distribution belongs to the two-parameter location-scale family (e.g. the t-distribution, stable distribution) then lower partial moments give the exact solution to Safety-First problems. See Bawa (1978) and Harlow/Rao (1989).

9 Alternatively it could be assumed that the utility function of the investor is quadratic. Much less restrictive and therefore much more common is the assumption of normally distributed asset returns. In this case the utility function can have any shape.

$$\boxed{S_n = \int_{-\infty}^{VaR_n} (VaR_n - R)^n \, dF(R)}$$

First, the value of S_n has to be given. Then the integral can be solved to the generalized VaR_n value. In case of $n = 0$ S_0 is equal to the confidence level of the usual VaR. S_1 can be interpreted as the expected deviation to the left of the VaR_n value. In case of $n > 1$ the meaning of S_n is more difficult to interpret. To make the choice of S_n easier it may be useful to look at the S_n values of a suitable normal distribution. In the following it is described how this could work. Some examples are added for further illustration.

S_n can take any real value. In the following example a normal distribution is used which has the same mean return and the same VaR_0 value as the distribution of a portfolio with put options. Now the integrals S_1 and S_2 are calculated using this so constructed normal distribution. The upper boundary of the integral is the value of VaR_0 calculated with given confidence levels of 1%, 5% and 10%, respectively. Given these values of S_1 and S_2 the values of VaR_1 and VaR_2 can be calculated for the distribution of the portfolio containing put options. The figures 6 and 7 show some examples. In figure 6 the graphs show theVaR_0 values (= the usual VaR) and the VaR_2 values of the portfolio with put options. The values of S_2 are calculated as described above using a corresponding normal distribution. The VaR_2 is significantly smaller than VaR_0. This means that now the relative defensive portfolio with the put options also has a smaller VaR_2 value than the corresponding normal distribution.[10]

[10] Using the method to calculated VaR_n as described in the text the VaR_n values of the normal distribution are the same for all n. That means the procedure described takes into account only deviations relative to the normal distribution.

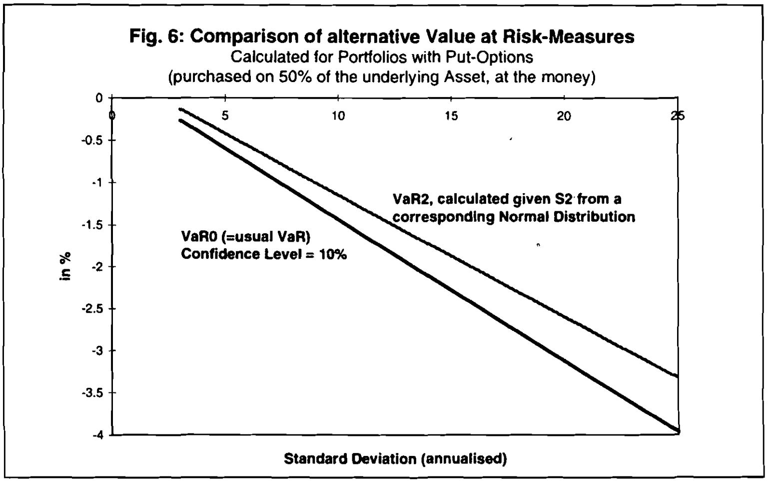
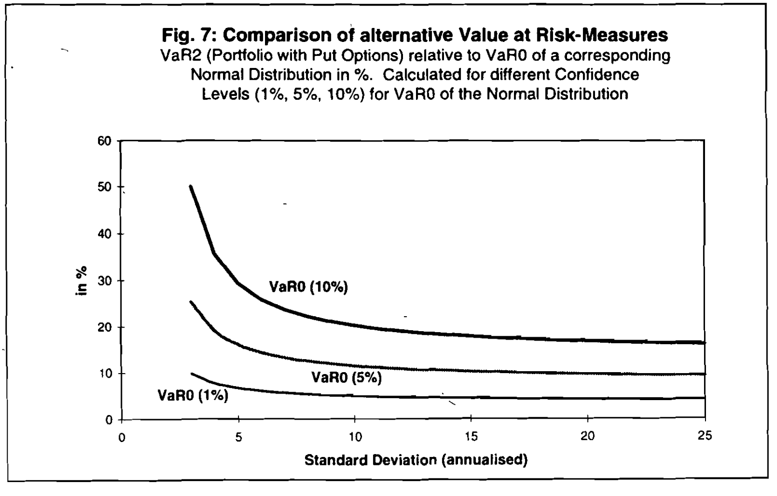


Figure 7 shows the reduction using S_2 instead of S_0. The graphs are based on the ratio $(1 - VaR_2/VaR_0)*100\%$. For each chosen confidence level (S_0 = 1%, 5% or 10%) the results are very similar: The largest reduction of the Value at Risk reveals portfolios with low standard deviation. But for all relatively defensive portfolios the use of VaR_1 or VaR_2 instead of VaR_0 could reduce the capital requirements significantly.



5 The Use of Lower Partial Moments as a Risk Management Tool

The usual Value at Risk measure (= VaR_0) will probably be the most important tool for public risk control in the future. Not only does the Basle Committee recommend the use of VaR in internal risk models but there are also commercial software products that facilitate the use of VaR in the bank's day-to-day business.[11]

There are also sound theoretical arguments why VaR is a suitable risk measure used in public risk control: VaR or equivalently lpm_0 are the least restrictive shortfall risk measures possible. They are a suitable risk measure for all investors having a utility with a positive first derivative. Risk averting behaviour is not a necessary condition.

But the use of VaR as the only measure of risk in the internal risk management of banks may be inadequate. One of the major tasks of risk management is the optimal allocation of capital. The Sharpe ratio is a well known measure to evaluate the performance of portfolios. The Sharpe ratio is directly derived from the Capital Asset Pricing Model (CAPM) and is equal to the slope of the efficient frontier. In mathematical terms the Sharpe ratio divides the mean return less the riskless interest rate by the standard deviation of the portfolio returns. The result is therefore equal to the risk adjusted mean return of the portfolio and can be used to rank the performance of different portfolios.

But there is an important requirement for the successful use of the Sharpe ratio: the returns of the portfolio have to be normally distributed, at least approximately. The Sharpe ratio is therefore not an appropriate performance measure if the portfolios contain options. The reason why portfolio return have to be normally distributed is the use of the standard deviation in the denominator.

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[11] See e.g. the software tool RiskMetrics from JPMorgan (JPMorgan (1995)).

Using the lower partial moments as measures of risk as described in chapter 3 modified Sharpe ratios can be constructed that are suitable to any return distribution.[12] The modified Sharpe ratios SR_1 and SR_2 defined below use lpm_1 and lpm_2 as measures of risk[13]. Therefore SR_1 and SR_2 take into account correctly deviations from the normal distribution. The formulas for SR_1 and SR_2 are as follows:

$$SR_1 = \frac{\mu - r}{lpm_1}$$

$$SR_2 = \frac{\mu - r}{\sqrt{lpm_2}}$$

μ = average portfolio return
r = riskless interest rate

Figures 8 and 9 illustrate the importance of the choice of the correct risk measure. Figure 8 shows Sharpe ratios of a normal distribution and of a portfolio containing put options. The graphs start with a standard deviation of the normal distribution (= distribution of the underlying asset in the second portfolio) of 3 and ends with a value of 25. Both distributions have the same mean return and the same VaR_0 at a confidence level of 1%. The portfolio without options seems to outperform the other one. But using the standard deviation overestimates the risk of the partly insured portfolio: the standard deviation does not take into account that the

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12 See Zimmermann (1994) and Albrecht/Maurer/Stephan (1995). Bawa/Lindenberg (1977) derive a generalised CAPM using lower partial moments instead of the standard deviation as risk measure. They assume that the target return is equal to a riskless interest rate r. The generalised CAPM under the assumption that the target rate can take any value is derived in Harlow/Rao (1989).

13 The investor can fix any target rate to calculate the lower partial moments. The risk adjusted performance of the portfolio and the ranking of different portfolios therefore depend on the LPM_n measure chosen and on the target return.

portfolio is skewed to the right. As a consequence the risk adjusted return of this portfolio seems to be relatively small.

Fig. 8: Comparison of Sharpe-Ratios
1. Normal Distribution, 2. Portfolio with Put-Options
(purchased on 50% of the underlying Asset, at the money)
Corresponding Distributions have equal Mean and equal VaR0

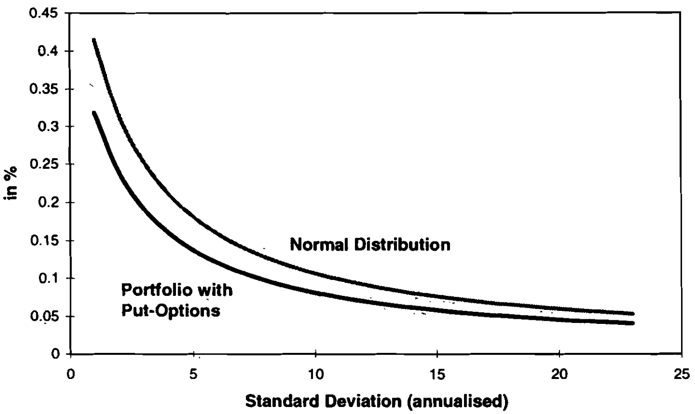

Figure 9 shows the correct values for the risk adjusted performance of the two portfolios using SR_2 as performance measure. In this example the minimum return to calculate lpm_2 is the VaR_0 value using a confidence level of 1%. Now the portfolio containing put options has the higher risk adjusted performance because it is taken into account that the high losses are less probable than compared to the normal distribution.

A ratio often used in risk management is the so-called RORAC (= Return On Risk Adjusted Capital). RORAC is defined as the mean return of a portfolio divided by the usual Value at Risk.[14] RORAC is therefore similar to a Sharpe ratio using VaR_0 as risk measure in the denominator. Due to the use of VaR_0 (= lpm_0) to calculate RORAC this performance measure is inadequate if the risk manager is risk averse.

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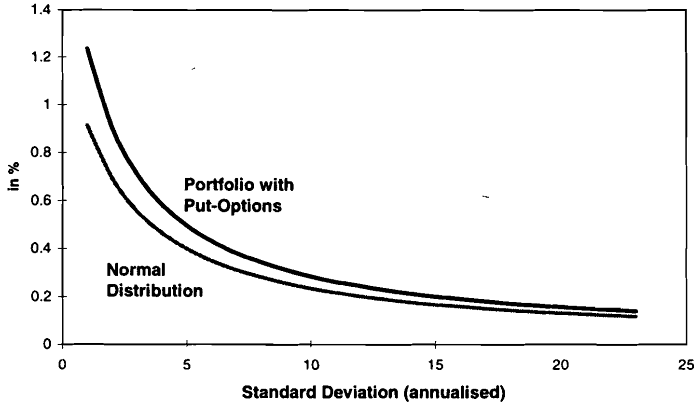
[14] See Bürger (1995), p. 250.

A risk averse risk manager should better use lpm_1 or lpm_2 to calculate the risk adjusted performance.

Fig. 9: Comparison of modified Sharpe Ratios (SR2)
1. Normal Distribution, 2. Portfolio with Put-Options
(purchased on 50% of the underlying Asset)
Corresponding Distributions have equal Mean and equal VaR0



The importance of the modified Sharpe ratios in risk management are the induced incentives to construct defensive portfolios. Portfolios insured using e.g. put options will have a higher risk adjusted return compared to more aggressive portfolios with the same unadjusted return (= mean return - riskless interest rate). Using lpm_2 this effect is more pronounced than using lpm_1. Using only VaR_0 as risk measure as in case of RORAC the skewness of the return distribution is not taken into account. Then there is no incentive to construct portfolios that are relatively defensive (= skewed to the right). If the bank´s management is risk averse and has a preference for relatively defensive portfolios then lpm_1 or lpm_2 have to be used as measure of risk in calculating the risk adjusted performance of portfolios.

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