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## DAX Index Futures: Mispricing and Arbitrage in German Markets

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by

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## Abstract

The paper reports the results of an empirical study of the price relation between the German Performance Stock Index, DAX, and DAX futures. An ex-ante arbitrage strategy based on arbitrage signals is analyzed. The data set contains intraday bid- and ask futures quotes and index values on a minute by minute basis. It is found that the number and persistence of arbitrage opportunities differs considerably for futures nearest to deliver as compared to futures which are not nearest to deliver. The findings suggest that arbitrageurs trade mainly in futures nearest to deliver. The risk associated with arbitrage trading is found to be very small so that arbitrage profits are nearly risk free.

# 1. Introduction

Since the introduction of stock index futures in US-markets in 1992, researchers and practitioners have been interested in the relationship between index futures prices and the underlying stock indices. In particular, two questions have been raised. First, can the price relation be described by the cost-of-carry model? Second, do prices in one market lead those of the other market? This paper contributes to the first line of research using a new data set for the German stock index, DAX, and related DAX futures. The second question is not pursued in this paper, but for an analysis see, for example, Stoll/Whaley (1990) and Chan (1992) for US-markets and Grünbichler/Longstaff/Schwartz (1992) and Kempf/Kaehler (1993) for German markets.

The existence of mispriced futures contracts has been documented at length for American, English, and Japanese markets. Many previous studies report significant differences between observed stock index futures prices and theoretical futures prices derived from the cost-of-carry model.[1] Most of these studies also determine ex post arbitrage opportunities by computing the difference between the absolute value of the mispricing and the round trip transaction costs. Thus, these studies test whether arbitrageurs trading instantaneously can earn arbitrage profits. In addition, ex ante arbitrage studies typically take into account the fact that arbitrageurs undergo an execution lag. This leads to orders which are executed at prices differing, in general, from the prices which originally indicated arbitrage opportunities. From this viewpoint arbitrage trading is risky and the arbitrage profits are uncertain.

In previous studies of ex ante arbitrage strategies, such as Yadav/Pope (1990), Chung (1991), and Klemkosky/Lee (1991), it is assumed that arbitrageurs place their orders as soon as an ex post arbitrage opportunity is observed, i.e. as soon as the mispricing is larger than the round trip transaction costs. This implies that the risk associated with the order execution lag is not considered in the trading strategy of an arbitrageur. However, these studies find that there is considerable risk resulting from

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[1]   See for example Cornell/French (1983a, 1983b), Modest/Sundaresan (1983), Figlewski (1984a, 1984b), MacKinley/Ramaswamy (1988), Brenner/Subrahmanyam/Uno (1989, 1990), Bhatt/Cakici (1990), Stulz/Wasserfallen/Stucki (1990), Yadav/Pope (1990, 1992), Chung (1991), Klemkosky/Lee (1991), Puttonen/Martikainen (1991), Lim (1992), and Puttonen (1993).

this execution lag. Consequently, ex ante arbitrage profits are substantially smaller than those indicated by ex post arbitrage opportunities.

In this study, ex ante arbitrage opportunities are reexamined. The paper differs from the related papers of Yadav/Pope (1990), Chung (1991), and Klemkosky/Lee (1991) in three main respects. First, a different arbitrage strategy is investigated. Based on interviews with arbitrage traders that were carried out for this study, it is not assumed that arbitrageurs open arbitrage positions whenever an ex post arbitrage opportunity is observed. Instead, it is assumed that arbitrageurs require a risk premium to cover the execution risk they bear. Second, arbitrage profits are not measured relative to different, somewhat arbitrary, levels of transaction costs. Here, the arbitrage profits of a low cost arbitrageur are determined. Finally and most importantly, the German stock index, DAX, and its associated futures are studied.

Although the German stock and futures markets are among the largest in the world and the Dax index differs in several important respects from other well-known stock indices, the existence of arbitrage profits in these markets has received relativeley little attention. Hohmann (1991), Loistl/Kobinger (1992), and Prigge/Schlag (1992) analyze the mispricing of Dax futures and the related ex post arbitrage opportunities, whereas Bamberg/Röder (1994) focus on the impact of taxes on arbitrage opportunities. Grunbichler/Callahan (1993) analyze ex ante arbitrage profits, restricting themselves to the rare case of overvalued futures.

There are three specific features which lead to a relatively low level of risk associated with ex ante arbitrage strategies in German markets as compared with other markets. First, unlike other well-known stock indices, e.g. S&P 500, FTSE-100, and Nikkei 225, the DAX index measures the total performance of the underlying stock portfolio. As a consequence of this performance feature, there is no dividend risk for arbitrageurs. Second, the DAX index is narrow, consisting of only 30 blue chips of the German stock market which represent about 60% of the market capitalization and 85% of the trade volume. Therefore, arbitrageurs are able to trade a perfect matching basket at reasonable costs and in a reasonable span of time. Consequently, the tracking error risk can be avoided and the execution risk is relatively low. Third, there is no execution risk in the futures market as the German Futures and Options Exchange (DTB) is an electronic screen-trading market. Based on these specific features of the German markets, one would expect that arbitrage

opportunities will be exploited very quickly, and that ex ante arbitrage strategies are nearly risk free. As a result, the price relation between stock and futures markets should not allow for large and long lasting arbitrage opportunities.

This hypothesis is studied using a new data set which includes all time stamped bid and ask quotes in the futures market and the value of the DAX index on a minute by minute basis. As with nearly all studies cited above, a "conservative" cash and carry arbitrage strategy is modelled. All arbitrage positions are held until the maturity of the futures contracts. Neither rolling over nor early unwinding of arbitrage positions is considered.[2] An arbitrageur using these more sophisticated strategies may open arbitrage positions even when a cost of carry arbitrageur would not. This behavior may be rational, because he receives the potentially valuable options to unwind or to rollover his positions at lower costs in the future.[3]

The remainder of this paper is organized as follows. Section 2 describes the trading mechanism for DAX futures and stocks. Section 3 reviews the valuation theory of stock index futures and transforms it to DAX futures. The data set used in the study is described in Section 4. Section 5 tests whether the price relation between DAX and DAX futures can be described by the cost-of-carry model. The results of the ex ante arbitrage study are detailed in Section 6. Briefly, the main results,summarized in Section 7, are as follows: Futures contracts at the German Futures and Options Exchange are significantly undervalued. The absolute value of the udervaluation increases in all contracts with time to maturity. There is a large number of arbitrage signals - most indicating short-arbitrage opportunities. The arbitrage strategies used by arbitrageurs are profitable in more then 95% of all cases, even if execution lags up to 5 minutes are assumed. Arbitrage profits are lowest when trading in futures nearest to delivery. If arbitrageurs are assumed to determine the mispricing, this result suggests that they concentrate their trading on futures nearest to delivery.

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2    Empirical studies in which these strategies are considered include Merrik (1989), Yadav/Pope (1990), and Sofianos (1993). They provide evidence that unwinding is a strategy heavily used and that it accounts for an important part of the total arbitrage profit.

3    See Brennan/Schwartz (1988, 1990), Cooper/Mello (1990), and Bühler/Kempf (1994) for pricing models of the early unwind option included in arbitrage positions.

## 2. Institutional Settings

In terms of market capitalization, the German stock market is the fifth largest in the world. At the end of 1992, its market capitalization was 562 billion Deutsch Marks (DM), i.e. about 351 billion US Dollars.[4] In terms of trading volume relative to the market capitalization, the German stock market is the most active market in Europe and one of the most active markets in the world. In 1992, the turnover ratio was 119 % in the German stock market compared to 42 % at the New York Stock Exchange and 34 % in the UK market. Within the German stock market, about 85 % of trading consists of stocks included in the DAX index.

The DAX index comprises 30 German stocks selected according to market capitalization, turnover, and availability of early opening prices. The DAX index is a capital-weighted performance index which is adjusted for price changes caused by subscription rights, stock splits, and dividends. Most of the well-known stock indices are corrected for subscription rights and stock splits etc. while the adjustment procedure for dividend payments is a specific feature of the DAX. The adjustment for dividends is obtained by reinvesting the total amount of dividend payments into the dividend paying stock. This means that with a dividend payment, the total shares of stock in the portfolio underlying the DAX index increases, while its total value remains unchanged.

The DAX index is calculated at an accuracy of 0.01 index points based on transaction prices of the underlying stocks. The Frankfurt Stock Exchange from which the index data are obtained is the largest of the 8 regional German stock exchanges, accounting for about two thirds of total trading volume. The DAX is calculated as soon as at least 16 of the underlying stocks have been traded. As a result, at the beginning of a trading day, the index may include up to 14 prices resulting from transactions from the previous day. However, Grunbichler/Longstaff/Schwartz (1992) report that, on average, trading in stocks included in the DAX starts within four minutes. This suggests that the problem of including overnight prices occurs only at the very beginning of a trading day. During trading hours the index is calculated every minute based on the latest transaction

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[4]      For the calculation of the US Dollar amounts, an exchange rate of 1.6 Deutsch Marks per US Dollar is assumed throughout.

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price of each stock reported by official brokers via a real time price information system. Official brokers are obliged to transmit transaction prices immediately, so that there is essentially no time difference between a new price and its inclusion in the index.

There are three groups of traders at the Frankfurt Stock Exchange. First, there are employees of banks trading on the bank's or on a customer's account. Second, there are floor brokers who trade for their own account and are permitted to trade whatever they want, but generally specialize in certain market segments. For example, there are floor brokers who provide quotes for trading stock baskets reflecting the DAX index. Third, there are official brokers who provide a continuous auction in the most liquid stocks as well as a noon auction in all stocks. Official brokers also run limit order books. Unlike floor traders, official brokers are legally restricted in trading on their own account. They are appointed by the government and paid by a fixed brokerage fee of 0.06 % of the trade volume. Official brokers do not set up bid and ask quotes, but they match the orders submitted by banks and floor brokers.

Index arbitrage trading in the spot market is usually carried out by banks in the following way. As soon as an arbitrage opportunity is perceived, several employees of the bank located at the exchange rush to official brokers assigned to the stocks to be traded, or to a single floor broker specialized in trading baskets. In general, the stock market transactions are carried out within about one or two minutes. Arbitrageurs are able to sell stocks short by borrowing them from an in-house securities lending desk or from an institution called "Deutsche Kassenverein." They are charged a time dependent lending fee, but the proceeds of short sales can be invested. Securities transactions of banks are delivered by crediting or debiting their accounts kept at the Deutsche Kassenverein. A transaction fee of 0.7 DM per trade is charged by this institution, independent of the size of the trade.

The second part of the index arbitrage trading consists of buying or selling DAX futures. DAX futures are screen traded at the German Futures and Options Exchange (DTB). The DTB runs a fully automated trading system combining quotation, trading, clearing, and settlement. The system is open from 7:30 AM to 5:00 PM where each trading day consists of a pre-trading period, an opening period, a continuous trading period, and a post-trading period. During the pre- and post-

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trading period orders are placed into the market, but no trading takes place. The opening period consists of a pre-opening period during which indicative prices are provided and then a single opening auction where the price is set to maximize turnover. The trading period ranges from 9:30 AM to 4:00 PM, so that there is futures trading before the floor opens and after the floor closes. In terms of trading volume, the DTB was the fifth largest index futures exchange in the world in 1992, with about 13,000 index futures contracts traded per day. More than 80 percent of these trades concentrate on the futures contract with the shortest time to maturity. Each DAX futures contract has a volume of 100 DM per index point. Given an index value of 1600, this leads to a contract size of $100,000. The minimal price change in DAX futures is 0.5 index points - a tick size of 50 DM.

The index futures market is a dealer market with voluntary market makers providing liquidity for the market. Alongside market makers, regular market participants trade on their own or customers' accounts. Each trader submits orders via his trading terminal to the exchange where the orders are matched automatically based on price and time priority. On the trading screen of a trader, prices and quantities of bid and ask quotes, the last transaction price, and the value of the DAX index are displayed. Thus, an arbitrageur can immediately calculate the mispricing of a futures contract and build up the desired futures position at known bid or ask prices. As an arbitrageur places his order by pressing a button on his computer, there is essentially no order execution lag in the futures market. One way trading costs in the index futures markets are 4 DM per contract which equals about 0.0025 % of the contract volume.

## 3.    Valuation of DAX Futures

The standard approach of pricing stock index futures, the cost-of-carry model, rests on the following assumptions: First, all markets where arbitrageurs trade are free of restrictions, i.e. there are no transaction costs, assets can be perfectly divided, interest rates for lending and borrowing are equal, orders are executed instantaneously, and there are no tax effects. Second, arbitrageurs prefer more to less and are not restricted in the size of their arbitrage positions. Third, interest rates are

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non-stochastic so that futures can be treated as forward contracts. Fourth, all dividend payments from the underlying stock portfolio occurring until maturity of the futures contract are known. Given all of these assumptions, it is well-known that arbitrageurs will insure that the following no arbitrage relation holds.

$$(1) \qquad F(t,T) = I(t)e^{r(t,T)[T-t]} - \sum_{j=1}^{J} D(t_j)e^{r(t_j,T)[T-t_j]},$$

where F(t,T) is the price of the index future at time t with maturity at time T. I(t) denotes the value of the stock index at time t, r(t,T) the interest rate per year for borrowing or lending money for the period of time $[t,T]$, and T-t is the time to maturity of the futures contract. $D(t_j)e^{r(t_j,T)[T-t_j]}$ denotes the dividend payment at time $t_j$ compounded to the maturity date T of the future.

Among other things, it is essential in deriving equation (1) that arbitrageurs know at time t all future dividend payments resulting from their stock positions until time T when the futures contract matures. This assumption of known dividend payments can be relaxed when valuing DAX futures. As pointed out above, the DAX is a performance index which measures the total performance of the underlying stock portfolio. It is adjusted for stock price changes caused by subscription rights, stock splits, and dividend payments.[5] This adjustment is achieved, for example, by reinvesting dividend payments into the dividend paying stock and rebalancing the portfolio once a year.[6] So, the increasing number of total shares of stocks in the portfolio couterbalances exactly the decrease in the stock price, so that the total value of the index portfolio and the index itself are unchanged by dividends.

As a consequence of this index correction, DAX futures relate to an augmented stock portfolio and an arbitrageur has to follow the index reinvestment strategy to avoid unbalanced arbitrage positions. If the arbitrageur reinvests the dividend

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[5]     In the following, the index adjustment for dividend payments is discussed as dividend payments occur much more often than subscription rights and stock splits. The adjustments in the latter cases follow in a straightforward manner the procedure in the case of dividends.

[6]     In order to avoid an impact of dividends onto the weightening of the index, there is a second index correction once a year in September. This correction assumes implicitely that all stocks purchased during the last year are sold and the proceeds are reinvested into the stock portfolio according to the original weightening scheme.

payment into the dividend paying stocks and rebalances his portfolio once a year, his stock portfolio increases in the same way as the index does and it can be shown that it exactly cancels out the obligation caused by the futures contract at maturity. Therefore, the no arbitrage relation between DAX index and DAX futures does not depend on dividend payments and can be written as:

(2)     $F(t,T) = DAX(t)e^{r(t,T)[T-t]}$.

Note that the arbitrage profits only depend on variables known at time t. In particular, they are independent of the level of dividend payments and the point of time when dividends are paid. Therefore, the arbitrageur reinvesting in the way described above, runs no dividend risk.


It is essential for this result that the reinvestment of dividends increases the arbitrageur's portfolio by the same number of shares of stocks as with the DAX portfolio. This condition will always be met when there are no tax effects yielding to different dividend payments and when the investor can reinvest the dividend payment instantaneously. Both requirements are covered by the standard assumptions of the cash and carry model, as described above.

However, in real markets, dividends may have an impact on the fair DAX futures price if arbitrageurs have to pay taxes, and if they take tax payments into considerationin their trading decisions. In this paper tax effects are neglected for two reasons. First, it can be safely assumed that arbitrage positions are mainly taken by institutional investors which are taxed according to the German tax laws. Depending on their payout policy, profits of these investors are taxed by a tax rate between 36% and 56%. If the profits and losses from arbitrage operations have a full impact on the dividends then the relevant tax rate is 36%. As a result, in this case there will be no tax effects, since this tax rate is implicitly applied on dividends when the DAX is computed.[7] Second, if there exist several groups of arbitrageurs differing in their tax rates, this will result in a   tax clientele effect, as the cost of carry value of a futures

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[7]     The DAX index is adjusted for 64% of the gross dividend. Thus, it is assumed that tax burden of a gross dividend equals 36 % which is exactly the tax rate for distributed profits of German companies.

contract differs for each group.[8] Thus, the relation between the spot and futures price has to be determined in an equilibrium setup. This is beyond the scope of this paper.

In section 5 price differences between the observed futures prices, $\overline{F}(t,T)$, and the arbitrage free futures price $F(t,T)$ are analyzed. The present value of that difference in relation to the index value is denoted as relative mispricing:

$$(3) \quad RMIS(t,T) = \frac{e^{-r(t,T)[T-t]}\left[\overline{F}(t,T) - F(t,T)\right]}{DAX(t)} * 100 = \frac{e^{-r(t,T)[T-t]}\overline{F}(t,T) - DAX(t)}{DAX(t)} * 100$$

A positive mispricing is called an overvaluation and a negative mispricing an undervaluation of the futures contract. Without transaction costs, an overvaluation leads to long arbitrage positions (stocks long, futures short), and an undervaluation results in short arbitrage trading (stocks short, futures long). Under real conditions, arbitrageurs have to take transaction costs into consideration so that not every mispricing will yield arbitrage profits. In this paper, the term mispricing always refers to differences between the futures price observed and the fair price developed under the assumptions stated above, whereas the term arbitrage is used only when market imperfections are considered as well.

## 4. Data

The interest rates and DAX futures data are supplied by the Deutsche Finanzdatenbank, Karlsruhe and Mannheim, and the DAX index data by the Frankfurt Stock Exchange.

The study covers the time period between the first trading day in stock index futures (Nov 23, 1990) and the expiration day of the DEC92 stock index future (Dec 17, 1992). At the DTB, four DAX futures with maturities at the third Friday of March, June, September and December are established, while only the three contracts with

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[8]     The impact of dividend taxation on the fair futures value is discussed in detail in Kempf/Spengel (1993) and Bamberg/Röder(1993).

the shortest time to maturity are traded. Thus, eleven contracts are traded during the research period, although three of them are not studied. The futures contract with maturity in December 1990 (DEC90) is not included in the sample because of its very short time to maturity when trading started. The contracts with maturities in March 1993 (MAR93) and June 1993 (JUN93) are omitted, because the data set does not cover their expiration dates.

The futures data set consists of about 6 million data records including transaction prices, quotes, trading volume, and time recorded in seconds. As the data records are real time stamped, the time differences between two subsequent records vary.

The DAX index is calculated every minute. The data set contains 93,336 values of the DAX index, but unlike Chung (1991), it does not include the prices of the underlying stocks. The DAX index is calculated from about 10:30 AM to 1:30 PM. In order to exclude index values based partly on stock prices from the previous trading day, the period of time from 10:30 AM to 10:45 AM is excluded from the data set.

The data set of interest rates consists of daily bid and ask interbank interest rates for overnight, one month, three month, six month and twelve month money. For the pricing of a futures contract, the two interest rates which match the maturity of the futures contract best are interpolated linearly. In calculating the mispricing series, the median bid and ask quotation is used, while the relevant bid or ask quote is considered when analyzing arbitrage opportunities.

For calculating a mispricing series, it is necessary to use simultaneous prices from the spot and futures market. In this study, almost time simultaneous pairs of prices are obtained as follows. To every value of the DAX index, the immediately preceding bid and ask quotes of the futures contract considered are assigned. An investor can trade on this quotes as long as they are not changed. From these time series of price pairs, those are deleted for which the time difference between the index value and the futures quotes is longer than one minute. Without reducing the number of observations in the way described, there would be pairs of prices in the sample where the same futures quotes are assigned to several index values. In that case, one mispriced future would possibly indicate several mispricings and arbitrage opportunities. As a result of the matching procedure for a given futures contract,

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both the values of the index and the futures prices in two pairs of prices will be different. The average time difference between an index value and a futures price matched into one pair ranges from 17 seconds to 20 seconds for different futures contracts. The number of price pairs obtained for each futures contract in this way is shown in Table I.

## 5. Mispricing Series

The relative mispricing of futures contracts is calculated according to equation (3) using the mean of the current bid-ask-quotes for futures contracts and interest rates of matching maturities. Futures prices and DAX values are paired as described in section 4. If the cost-of-carry model describes the data well, the average mispricing should not significantly differ from zero.

Table I provides summary statistics for each contract separately. The results are shown for total time to maturity and for the period of time during which each futures contract is nearest to delivery. On average, 38 percent of all pairs of prices occur during the later subperiod

The average size of mispricing is significantly[9] different from zero for each contract and each period, thus providing strong evidence against the hypothesis that the price relation between DAX index and DAX futures can be described by the cost-of-carry model. For total time to maturity, all futures show a significant average undervaluation between -0.46 and -1.18 percent. During the subperiod when the futures are nearest to deliver the mispricing is closer to zero, but still negative and significant. In this period the average mispricing ranges from -0.04 to -0.49 percent. In all contracts, the absolute size of mispricing is smaller in the latter subperiod than for total time to maturity.

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[9]     As there is considerable autocorrelation in the mispricing series, all significance tests are carried out using the standard errors obtained by the method of Newey/West (1987) which allows for heteroskedasticity and autocorrelation in the data. The size of autocorrelation is shown for lag 1 and lag 10 in table I.

Table I:    Descriptive Statistics on Relative Mispricing Series: Total Time to Maturity / Futures Nearest to Delivery

| RMIS(t.T) | T = 03/91 | T = 06/91 | T = 09/91 | T = 12/91 | T = 03/92 | T = 06/92 | T = 09/92 | T = 12/92 |
|---|---|---|---|---|---|---|---|---|
| n | 10231 / 7625 | 19493 / 9373 | 24991 / 8386 | 24627 / 7513 | 22113 / 6933 | 20425 / 6476 | 21934 / 8945 | 24477 / 9415 |
| Mean ( % ) | -0.63*/-0.47* | -1.13*/-0.49* | -1.18*/-0.29* | -0.95*/-0.26* | -0.86*/-0.29* | -0.91*/-0.24* | -0.68*/-0.04* | -0.46*/-0.07* |
| SDV ( % ) | 0.54 / 0.52 | 0.80 / 0.30 | 0.93 / 0.24 | 0.63 / 0.16 | 0.51 / 0.24 | 0.56 / 0.26 | 0.66 / 0.13 | 0.53 / 0.17 |
| < 0 | 8896 / 6290 | 18701 / 8582 | 24129 / 7525 | 24200 / 7089 | 20833 / 5654 | 18988 / 5039 | 18100 / 5111 | 20635 / 5597 |
| > 0 | 1330 / 1330 | 789 / 788 | 857 / 856 | 423 / 420 | 1274 / 1273 | 1435 / 1435 | 3815 / 3815 | 3837 / 3813 |
| = 0 | 5 / 5 | 3 / 3 | 5 / 5 | 4 / 4 | 6 / 6 | 2 / 2 | 19 / 19 | 5 / 5 |
| UV ratio | 6.7 / 4.7 | 23.7 / 10.9 | 28.2 / 8.8 | 57.2 / 16.9 | 16.4 / 4.4 | 13.2 / 3.5 | 4.7 / 1.3 | 5.4 / 1.5 |
| AC (1) | 0.99 / 0.99 | 1.00 / 1.00 | 1.00 / 1.00 | 1.00 / 1.00 | 1.00 / 1.00 | 1.00 / 1.00 | 1.00 / 1.00 | 1.00 / 1.00 |
| AC (10) | 0.96 / 0.96 | 0.99 / 0.99 | 0.99 / 0.99 | 0.99 / 0.99 | 0.98 / 0.98 | 0.99 / 0.99 | 0.99 / 0.99 | 0.98 / 0.98 |
| $\Delta$ Mean*$10^3$ | 0.07 / 0.21 | 0.06 / 0.07 | 0.14 / 0.07 | 0.07 / 0.06 | 0.08 / 0.10 | 0.08 / 0.04 | 0.07 / 0.04 | 0.07 / 0.01 |
| $\Delta$ AC (1) | -0.26 / -0.26 | -0.23 / -0.23 | -0.22 / -0.22 | -0.17 / -0.17 | -0.17 / -0.17 | -0.18 / -0.18 | -0.13 / -0.13 | -0.08 / -0.08 |
| $\Delta$ AC (10) | -0.03 / -0.03 | -0.02 / -0.02 | -0.02 / -0.02 | -0.01 / -0.01 | -0.02 / -0.02 | -0.02 / -0.02 | -0.02 / -0.02 | -0.02 / -0.02 |

RMIS:           Relative mispricing as defined in equation (3);
T:              Maturity of the futures contract;
n:              Number of observations;
SDV:            Standard deviation of mispricings;
< 0/> 0/= 0:    Number of undervalued/overvalued/fair valued futures;
UV ratio:       Number of undervalued futures relative to the number of overvalued futures;
AC(i):          Autocorrelation of order i;
$\Delta$ Mean:  Average of first differences in mispricing (%);
$\Delta$ AC (i) Autocorrelation of order i in first differences;
*               Significant at the 0.1 percent level;

Comparing the average mispricing of different futures contracts shows that the size of mispricing is in general closer to zero in later contracts than in earlier contracts. This is true for total time of maturity as well as for futures nearest to delivery. Over time, the observed price relation approaches the fair price relation determined by the cost of carry model.

In terms of numbers, Table I shows that futures are mostly undervalued. At maximum, in the contract DEC91 only about one out of 58 mispriced futures is overvalued. That relation decreases to about one out of 18 when the future is the contract nearest to delivery. By the end of 1992, the undervaluation ratio approaches but stays consistently above one. Furthermore, overvaluation occurs in almost all cases only in futures nearest to delivery.

Remarkable is the high degree of autocorrelation in the time series of mispricing. The last three lines of Table I show results for first differences in mispricing. On average, they are not significantly different from zero. The autocorrelation in these differences is slightly negative at lag 1 and is close to zero at lag 10. This indicates, in combination with persistent high autocorrelations in levels, that mispricing is a non-stationary variable.

This non-stationarity may result from a time dependent trend in mispricing. One possible explanation may be time dependent holding costs which are not considered in formula (3). One might think about the costs of lending stocks which have to be considered in Germany, when a short arbitrage position is opened. This leads to an asymmetric, no-arbitrage window, so that the undervaluation may increase with time to maturity without allowing for arbitrage profits.

Another explanation for a time dependent trend in mispricing is given by Cornell/French (1983a, 1983b). They suggest that there is a tax timing option in spot markets which is not available in futures markets. They conclude that futures should be undervalued compared to the cost-of-carry model, i.e. there should be a negative mispricing, and that the absolute size of undervaluation should increase with time to maturity. To test this hypothesis a simple linear regressions of the following type are run for each futures contract separately:

(4)    $RMIS^A(t,T) = a + b*(T-t) + \tilde{\varepsilon}$

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## Table II: Summary Statistics of Linear Regressions on Time to Maturity

| Contract | T=03/91 | T=06/91 | T=09/91 | T=12/91 | T=03/92 | T=06/92 | T=09/92 | T=12/92 |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|
| n | 72 | 136 | 180 | 186 | 173 | 168 | 171 | 180 |
| $a*100$ | 10.0 | 11.3 | 34.3*** | 11.8 | -1.1 | 4.8 | 30.6*** | 32.8** |
| $b*100$ | -1.3* | -1.2* | - 1.1* | - 0.8* | -0.6* | -0.7* | -0.8* | -0.6* |
| $R^2$ | 0.69 | 0.85 | 0.89 | 0.93 | 0.90 | 0.92 | 0.90 | 0.80 |

a:          Constant;
b:          Coefficient of time to maturity;
T:          Maturity of the future contract;
$R^2$:      Goodness-of-fit statistic;
*/**/***:   Significant at the 0.1/1/5 percent level;

where $RMIS^A(t,T)$ equals the average relative mispricing of one day and (T-t) equals time to maturity in days. The existence of a tax timing option suggests that the parameter $a$ should be zero and the parameter $b$ should be negative.

The results of these regressions, reported in Table II, strongly reject the hypothesis that the mispricing does not depend on time to maturity. It can be concluded from Table II that futures are the more undervalued, the longer their time to maturity is. This result holds for all futures under investigation. The results concerning the parameter b support the existence of a valuable tax timing options in the spot instrument which is not available in futures contracts. 3 of the 8 regressions lead to a significantly positive constant a which cannot be explained by a tax timing option. This suggests that there may be other unidentified variables influencing the mispricing. The explanatory power of the regression is fairly high in all contracts. In Figure 1, a typical form of this relation is drawn using data of the futures contract maturing in June 1991.

Figure 1:    Relative Mispricing of the Futures Contract with Maturity June 1991.
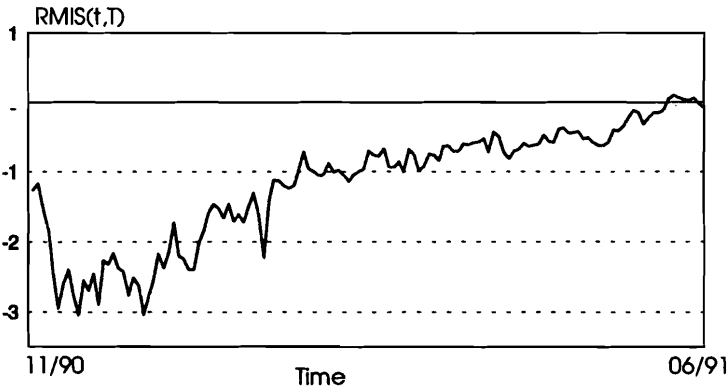


The results presented here are similar to the findings in other markets. An undervaluation of stock index futures is, for example, reported by Figlewski (1984a, 1984b) and MacKinlay/Ramaswamy (1988) for US markets, by Bren-

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ner/Subrahmanyam/Uno (1989, 1990) and Lim (1992) for Japanese markets, by Yadav/Pope (1990) for UK markets, by Stulz/Wasserfallen/Stucki (1990) for Swiss markets, and by Puttonen (1993) for Finish markets. Several of these papers report that the absolute value of the undervaluation of futures decreases when markets mature and even turn into an overvaluation. For German markets, it is found that the mispricing is closer to zero in later contracts than in earlier contracts, but that the average mispricing stays below zero. In contrast to the studies cited above and the results reported above, Cornell (1985) reports for US markets and Bailey (1989) for the Japanese markets that there are on average no significant deviations of futures prices from their fair values. Even more contrary to the results presented here, Bhatt/Cakici (1990) find on average a small premium in futures prices for US markets, which occurs more often in futures with longer time to maturity.

Mixed results have been found concerning the relation between time to maturity and mispricing of futures. On the one hand, Yadav/Pope (1990) report that the absolute size of undervaluation of futures increases significantly with time to maturity for most of the contracts at the beginning of the research period while later the effect is insignificant or shows the opposite sign. Similar to Yadav/Pope (1990), MacKinlay/Ramaswamy (1988) recognize a positive effect of time to maturity on the absolute value of the mispricing. On the other hand, neither Lim (1992) nor Brenner/Subrahmanyam/Uno (1989) find a significant relation between mispricing and time to maturity. Contrary to these studies, in German markets a persistent trend of an increasing size of undervaluation with time to maturity is found. Neither the size of the regression parameter $b$, nor the explanatory power of the regressions change consistently over time.

## 6. Arbitrage

In Section 5, it is shown that the cost-of-carry model is unable to describe the observed relationship between futures and spot market prices. However, this still leaves open the question of whether the mispricing is large enough for arbitrageurs

to earn profits when they are obliged to pay transaction costs. This question is addressed in the remainder of this paper.

A typical two step approach is used in order to simulate an arbitrage strategy. First, specific arbitrage signals are determined. Second, taking into account the execution lag, the profits from an ex ante arbitrage strategy based on these arbitrage signals are studied.


## 6.1. Arbitrage Signals

Transaction costs are an important factor determining the no arbitrage windows within which arbitrageurs are not willing to trade. In general transaction costs, and consequently, the no arbitrage windows, are not of the same size for different investors. In this paper, arbitrage opportunities of the investor with the smallest no arbitrage window, subsequently called the low cost arbitrageur, are studied. In Germany institutional investors, like commercial and investment banks, can be identified as low costs arbitrageurs. Therefore, the transaction costs of this group of investors will be subsequently determined. It is essential for arbitrage studies that the level of these costs are accurately determined as the results from arbitrage studies are very sensitive to this specification.

There are in general five elements of transaction costs, namely brokerage fees, settlement fees, stock lending fees, the bid ask spread, and market impact costs. The costs associated with the former two are specified in Section 2. One way transaction costs are 0.06 % of the trade size in the stock market and 4 DM per futures contract. Settlement fees to be paid to the Deutsche Kassenverein are 0.7 DM per trade in the spot market - i.e. 21 DM when trading the index portfolio.

There are costs of borrowing stocks when the arbitrage strategy requires short selling of stocks. As a low cost arbitrageur is assumed to belong to a bank, he may borrow stocks from an in-house securities lending desk or from the Deutsche Kassenverein lending system. In general, the borrowing rates are significantly smaller when the in-house system is used. Therefore, it is assumed that an arbitrageur borrows the stock portfolio needed from an in-house securities lending desk. Unfortunately, data of in-house borrowing rates are not available on a frequent

basis. They are supplied to the authors for six points of time in 1991 and 1992 by a large German commercial bank. The levels of borrowing rates are decreasing over the period of time under investigation, ranging from 2.5 percent per year at the beginning of 1991 to 0.75 percent per year at the end of 1992. Borrowing rates at points of time where no data are available are computed by linear interpolation of the two closest borrowing rates.

An explicit spread is quoted in the futures market but not in the stock market. When opening a futures position in a long or short arbitrage strategy the bid or ask price of the futures is considered. As the futures is settled at the spot price at maturity day T no spread in the futures market has to be taken into account when closing an arbitrage position. No bid ask spreads are quoted in the stock market by official brokers. However, there may be an effective bid ask spread in that market as suggested by Roll (1984). Haller/Stoll (1989) estimate this effective spread in German stock prices using the model of Roll (1984). They report that the spread is not significantly different from zero when trading large stocks. Therefore no spread in trading stocks is considered in this study.

An arbitrageur is able to trade a known number of futures at the bid or ask quote, in general five or ten contracts. In the trading strategy analyzed in the remainder of this paper it is assumed that the arbitrageur is trading one futures contract when observing an arbitrage signal. Therefore, no market impact costs in the futures market have to be considered.As the stocks included in the DAX are very liquid it is assumed in addition that the opening and closing of an arbitrage position has no price impact on the individual stocks in the DAX.

Using the data described above, round trip transaction costs for trading one arbitrage position are approximated in the following way. It is assumed that round trip transaction fees are 0.12 percent of the current trading volume in the spot market (in DM). They have to be paid when the arbitrage position is build up. Trading costs resulting from the portfolio adjustment due to dividend payments are neglected. Furthermore, it is assumed that arbitrageurs have to borrow stocks when they want to sell them. This assumption insures that the no arbitrage window does not depend on the number of stocks bought by the arbitrageur in the past. Therefore, the no arbitrage window is independent of the former arbitrage behaviour. Putting

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everything together, one unit short arbitrage (sell stocks, buy one future) results in transaction costs

(5)    $C(t,T)^s = 0.12 * DAX\ (t) + (21+4)*(1+e^{-r(T-t)}) + B_s[T-t]*DAX\ (t)*100$

and one unit long arbitrage yields transaction costs $C(t,T)^L$ of the following size:

(6)    $C(t,T)^L = 0.12*DAX(t) + (21+4)*(1+e^{-r(T-t)}),$

where $DAX(t)$ is the index value. One futures contract represents a trading volume of 100 DM - i.e. $ 62.5 per index point. $B_s*[T-t]*DAX(t)*100$ are the borrowing costs when selling a comparable stock portfolio short for the period of time T-t. $0.12*DAX(t)$ are the round trip transaction fees for trading this stock portfolio, and $(21+4)(1+e^{-r(T-t)})$ represents the present value of the round trip stock settlement fees and transaction fees of trading one futures contract. A rough calculation yields that the total transaction costs of a long arbitrage position are about 0.125 percent of the value of the stock portfolio. The transaction costs of a short arbitrage position are larger and depend heavily, due to the stock lending fee, on the time to maturity of the futures contract. For a futures contract with 90 days until maturity, this lending fee is between 0.1875 percent of the value of the stock portfolio at the end of 1992 and 0.625 percent at the beginning of 1991. The bid ask spread is considered by using the bid or the ask price in the arbitrage signals (7) and (8).

Transaction costs define the minimum size of the no arbitrage window. However, based on interviews with market participants, it is assumed in this study that arbitrageurs are willing to trade only if, in addition to transaction costs, a required risk premium, RP, is covered by the mispricing. This risk premium is taken as four index points, which is the lowest level specified by an arbitrageur. Given an index value of 1600 index points, which was the average level of the DAX index in 1991 and 1992, four index points equal 0.25 percent of the index value. Thus, the risk premium is even larger than the transaction costs associated with a long arbitrage strategy.

When analyzing the profitability of arbitrage strategies, the relevant bid or ask quote for the futures price and interest rates are used to calculate the mispricing. The relative mispricing in determining a long (short) arbitrage signal, $RMIS^L(RMIS^s)$, is

computed using the ask (bid) interest rate and the bid (ask) futures quote. A long arbitrage signal is observed if

(7)  $$RMIS(t,T)^L > \frac{C(t,T)^L + RP*100}{DAX(t)*100},$$

and a short arbitrage signal if:

(8)  $$-RMIS(t,T)^S > \frac{C(t,T)^S + RP*100}{DAX(t)*100}.$$

Note that the no arbitrage window is not symmetric around zero, but larger at negative mispricing due to stock borrowing costs. The size of a long arbitrage signal is defined as

(9)  $$RMIS(t,T)^L - \frac{C(t,T)^L + RP*100}{DAX(t)*100},$$

and the size of a short arbitrage signal as

(10)  $$-RMIS(t,T)^S - \frac{C(t,T)^S + RP*100}{DAX(t)*100}.$$

Table III shows the average number and size of arbitrage signals for total time to maturity and for the period of time when the contract is nearest to delivery. There are a large number of arbitrage signals, most of them indicating a short arbitrage opportunity (SAS). Long arbitrage signals (LAS) are observed very rarely and occur in nearly all cases during the subperiod when the future is the contract nearest to delivery. In terms of absolute numbers, the predominance of short arbitrage signals is not surprising since most of the futures are undervalued (see Table I). Even after controlling for the different frequencies of positive and negative mispricings, Table III indicates still relatively more short arbitrage signals than long arbitrage signals. The ratio of the number of short (long) arbitrage signals and undervalued futures is denoted by SAS (LAS) ratio. For example, it can be seen that 27.6 percent of all undervalued futures with maturity in September 1991 provide a short arbitrage signal, while only 0.2 percent of the overvalued futures provide a long arbitrage signal.

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Table III:   Signals Indicating Long Arbitrage or Short Arbitrage Opportunities:
             Total Time to Maturity / Futures to Delivery

| Contract | T=03/91 | T=06/91 | T=09/91 | T=12/91 | T=03/92 | T=06/92 | T=09/92 | T=12/92 |
|---|---|---|---|---|---|---|---|---|
| SAS No | 2799 / 1578 | 7235 / 886 | 6654 / 150 | 1540 / 0 | 1033 / 318 | 7729 / 133 | 7329 / 3 | 4705 / 18 |
| SAS Size (%) | 0.29 / 0.36 | 0.45 / 0.06 | 0.48 / 0.08 | 0.07 / 0 | 0.04 / 0.04 | 0.15 / 0.03 | 0.23/ 0.14 | 0.25 / 0.05 |
| LAS No | 10 / 10 | 0 / 0 | 2 / 2 | 1 / 0 | 0 / 0 | 0 / 0 | 5 / 5 | 0 / 0 |
| LAS Size (%) | 0.10 / 0.10 | 0 / 0 | 0.29 / 0.29 | 0.02 / 0 | 0 / 0 | 0 / 0 | 0.06 / 0.06 | 0 / 0 |
| Signal Ratio (%) | 27.5 / 20.8 | 37.1 / 10.6 | 26.6 / 1.8 | 6.3 / 0 | 4.7 / 4.6 | 37.8 / 2.1 | 33.4 / 0.0 | 19.2 / 0.2 |
| SAS Ratio (%) | 31.5 / 25.1 | 38.7 / 10.3 | 27.6 / 2.0 | 6.4 / 0 | 5.0 / 5.6 | 40.7 / 2.6 | 40.5 / 0.1 | 22.8 / 0.3 |
| LAS Ratio (%) | 0.8 / 0.8 | 0 / 0 | 0.2 / 0.2 | 0.2 / 0 | 0 / 0 | 0 / 0 | 0.1 / 0.1 | 0 / 0 |
| Subsequent SAS | 16.1 / 18.3 | 18.0 / 4.4 | 14.7 / 3.1 | 5.1 / 0 | 3.8 / 4.2 | 24.4 / 3.0 | 31.3 / 3 | 92.3 / 1.8 |
| Subsequent LAS | 2.5 / 2.5 | 0 / 0 | 2.0 / 2.0 | 1.0 / 0 | 0 / 0 | 0 / 0 | 2.5 / 2.5 | 0 / 0 |

T:              Maturity of the futures contract;
SAS No:         Number of signals indicating short arbitrage opportunities;
SAS Size (%)    Average size of short arbitrage signals;
LAS No:         Number of signals indicating long arbitrage opportunities;
LAS Size (%):   Average size of long arbitrage signals;
Signal Ratio:   Total number of arbitrage signals relative to the total number of mispricings;
SAS Ratio:      Number of signals that indicate short arbitrage opportunities relative to the number of observed undervalued futures;
LAS Ratio:      Number of signals that indicate long arbitrage opportunities relative to the number of observed overvalued futures;

There are more arbitrage signals when futures are not nearest to delivery than during the period of time when they are nearest to delivery. The results for futures not nearest to delivery are not explicitly shown in the table, but they can easily be calculated from the numbers provided. For example, consider the futures contract maturing in September 1991. There are 152 arbitrage signals (150 SAS + 2 LAS) when it is the nearest contract, and 6656-152 = 6504 arbitrage signals when it is not the contract nearest to delivery. The arbitrage signals in the nearest to delivery period relate to 8381 mispriced futures in that period (see Table I) yielding a signal ratio of 1.8 percent. In this subperiod, only 1.8 percent of all mispriced futures with maturity September 1991 provide an arbitrage signal. Over total time to maturity this ratio is 26.6 percent. From these numbers it follows that the signal ratio in that contract is 39.2 percent for the period of time when it is not the contract nearest to delivery. In this subperiod, more than one out of three futures quotes provide an arbitrage signal, although the no arbitrage window widens with time to maturity.

The average size of short arbitrage signals is in most contracts much smaller for the nearest to delivery subperiod than for total time to maturity. In the SEP91 futures contract, the average short arbitrage signal is 0.48 percent for total time to maturity and only 0.08 percent for the period of time when it is nearest to delivery. From that result, one might suspect that arbitrageurs concentrate in trading futures nearest to delivery and prevent prices from deviating far from the no arbitrage window as defined by equations (7) and (8).

If arbitrageurs trade actively in the way described above, arbitrage signals should be exploited quickly forcing the mispricing back into the no arbitrage window. Therefore, the number of subsequent arbitrage signals is an indicator for the unobservable trading activity of arbitrageurs. Table III shows that on average more subsequent short arbitrage signals occur in most contracts during the period of time when they are not nearest to delivery.[10] This result suggests that arbitrageurs respond less to arbitrage signals in futures not nearest to delivery than in futures nearest to delivery. This finding is consistent with theoretical results of Holden (1990). He shows that the optimal trading strategy of a non-competitive arbitrageur depends on time to maturity since the arbitrageur considers the impact of current arbitrage trading on future arbitrage opportunities.

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[10]   As there are only a few long arbitrage signals, they are not discussed in detail. The numbers are shown in table III.

When comparing different futures contracts, it is remarkable that the numbers of subsequent arbitrage signals after March 1992 are high in futures not nearest to delivery and low in futures nearest to delivery. This suggests a stronger concentration effect - compared to earlier contracts -on futures nearest to delivery.

## 6.2. Ex Ante Arbitrage Profitability

In this section, the profitability of an ex ante arbitrage strategy conditional on arbitrage signals is studied. It is assumed that an arbitrageur behaves according to the following strategy. The arbitrageur places orders at the spot and futures markets whenever an arbitrage signal is observed. The futures market order is executed without delay, and the spot market transaction is executed with an order execution lag. The length of the order execution lag at the spot market is specified by most arbitrageurs at one to two minutes. However, execution lags ranging from one to 15 minutes are considered. Order execution lags of this length are incorporated in the study in order to test the sensitivity of the results to the execution lag.

Table IV presents the results of an arbitrage strategy conditional on short arbitrage signals. The case of long arbitrage signals is not pursued further since only a few long arbitrage signals are observed. The result of a short arbitrage strategy, conditional on the existence of a short arbitrage signal, is given as

$$(11) \quad \left[-e^{-r_{bid}(T-\hat{t})}\overline{F}_{ask}(t,T) + DAX(\hat{t})\right]*100 - C(\hat{t},T)^S$$

where $t$ is the point of time at which a short arbitrage signal is observed and the futures market trade is executed. $\hat{t}$ is the point of time at which the spot market order is executed. $\hat{t}$ and $t$ differ by the order execution lag. Note that the risk premium has an impact only on the arbitrage signal, but not on the arbitrage profit.

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Table IV: Ex Ante Short Arbitrage Profits: Total Time to Maturity / Futures Nearest to Delivery

| Contract | T=03/91 | T=06/91 | T=09/91 | T=12/91 | T=03/92 | T=06/92 | T=09/92 | T=12/92 |
|---|---|---|---|---|---|---|---|---|
| SAS | 2799 / 1578 | 7235 / 886 | 6654 / 150 | 1540 / 0 | 1033 / 318 | 7729 / 133 | 7329 / 3 | 4705 / 18 |
| SA (1) (%) | 100 / 100 | 100 / 100 | 100 / 100 | 100 / - | 100 / 100 | 100 / 100 | 100 / 100 | 100 / 100 |
| SA (2) (%) | 100 / 100 | 100 / 100 | 100 / 98.7 | 99.9 / - | 100 / 100 | 100 / 100 | 100 / 100 | 100 / 100 |
| SA (5) (%) | 99.6 / 99.5 | 99.6 / 99.2 | 99.5 / 94.7 | 98.9 / - | 98.6 / 99.7 | 99.6 / 100 | 99.5 / 100 | 99.3 / 94.4 |
| SA (10) (%) | 96.3 / 96.4 | 96.5 / 93.8 | 96.6 / 86.7 | 95.2 / - | 94.1 / 95.3 | 96.6 / 96.2 | 96.8 / 100 | 96.9 / 100 |
| SA (15) (%) | 93.3 / 94.1 | 93.0 / 88.6 | 92.9 / 82.7 | 91.5 / - | 90.8 / 90.1 | 93.6 / 90.2 | 94.1 / 100 | 94.4 / 94.4 |
| PSA (1) (%) | 0.69 / 0.76 | 0.87 / 0.36 | 0.88 / 0.37 | 0.41 / - | 0.35 / 0.31 | 0.47 / 0.29 | 0.53 / 0.48 | 0.55 / 0.36 |
| PSA (2) (%) | 0.69 / 0.76 | 0.87 / 0.35 | 0.88 / 0.33 | 0.40 / - | 0.34 / 0.30 | 0.46 / 0.28 | 0.53 / 0.44 | 0.55 / 0.31 |
| PSA (5) (%) | 0.68 / 0.76 | 0.86 / 0.34 | 0.88 / 0.26 | 0.38 / - | 0.32 / 0.29 | 0.46 / 0.27 | 0.53 / 0.63 | 0.55 / 0.27 |
| PSA (10) (%) | 0.66 / 0.74 | 0.83 / 0.31 | 0.85 / 0.20 | 0.36 / - | 0.29 / 0.26 | 0.45 / 0.24 | 0.51 / 0.96 | 0.54 / 0.34 |
| PSA (15) (%) | 0.65 / 0.73 | 0.81 / 0.30 | 0.83 / 0.18 | 0.34 / - | 0.27 / 0.26 | 0.43 / 0.22 | 0.50 / 0.92 | 0.52 / 0.31 |
| TPSA (1) (1000 $) | 1688 / 1031 | 5611 / 316 | 5215 / 53 | 631 / 0 | 367 / 104 | 3672 / 41 | 4150 / 1 | 2814 / 6 |
| TPSA (2) (1000 $) | 1684 / 1029 | 5605 / 311 | 5206 / 49 | 621 / 0 | 357 / 102 | 3668 / 41 | 4146 / 1 | 2814 / 5 |
| TPSA (5) (1000 $) | 1670 / 1019 | 5567 / 298 | 5165 / 38 | 592 / 0 | 332 / 96 | 3640 / 39 | 4117 / 2 | 2798 / 4 |
| TPSA (10) (1000 $) | 1618 / 992 | 5393 / 277 | 5019 / 30 | 554 / 0 | 304 / 88 | 3523 / 34 | 3991 / 3 | 2736 / 5 |
| TPSA (15) (1000 $) | 1585 / 979 | 5231 / 265 | 4887 / 26 | 532 / 0 | 290 / 86 | 3413 / 31 | 3876 / 3 | 2667 / 5 |

T:        Maturity of the futures contract;
SAS:      Number of short arbitrage signals;
SA(i):    Number of successful short arbitrage trades with a lag of i minutes relative to the number of short arbitrage signals;
PSA(i):   Average size of arbitrage profit with a lag of i minutes relative to the index value;
TPSA(i):  Total arbitrage profit with a lag of i minutes;

It is shown in Table IV that all short arbitrage signals result in an arbitrage profit when the spot order is executed after a delay of one minute. Arbitrage trading is risk free when the spot market order is executed within such a short period of time. Increasing the length of the execution lag in the spot market leads, as expected, to a reduction of the success ratio. This result is found for total time to maturity and during the subperiod when the futures are the contracts nearest to delivery.[11] Arbitrage trading is more risky the longer it takes to build up the spot position. Since the futures position is built up without delay, this finding suggests that the spot market moves on average in such a manner that arbitrage profits decrease. The level of mispricing seems to have an impact on the future spot price movement.

The success ratios found in this study are much larger than those reported by Chung (1991) for the US-market. In his study, even with an execution lag of only 20 seconds, less than 92 % percent of all arbitrage trades are profitable. The differences in results can mainly be attributed to the different trading strategies studied. While in this study arbitrage trading starts when the mispricing exceeds the transaction costs by more than a risk premium required, Chung (1991) assumes that arbitrageurs do always trade whenever the mispricing exceeds transaction costs.

The average size of arbitrage profits relative to the value of the stock portfolio, $DAX(\hat{t})*100$, is denoted by PSA in Table IV. It is positive in all cases, which shows that the losses suffered by the arbitrage strategy are smaller than the arbitrage profits. Comparing the values of PSA with the size of the short arbitrage signals shown in Table III, it is found that the average arbitrage profit is larger than the arbitrage signal, even at an execution lag of 15 minutes. This suggests that the market participants overstate the execution risk and demand a risk premium larger than necessary to cover that risk.

Finally, Table IV shows the total arbitrage profit of an arbitrageur who opens one arbitrage position whenever an arbitrage signal is observed. The ex ante arbitrage profit is determined by the number of arbitrage signals as well as by the arbitrage

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[11] Due to the small numbers of short arbitrage signals, the futures contracts nearest to delivery with maturity in December 1991, September 1992, and December 1992 are not discussed when analyzing the success ratio, SA, and the average arbitrage profit, PSA.

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profit resulting from each transaction. These results exhibit two characteristics First, arbitrageurs trading according to the strategy specified above earn high arbitrage profits in most contracts. Second, large arbitrage profits can mostly be realized in futures contracts during the period of time when they are not the contracts nearest to delivery. However, this finding also suggests that most arbitrageurs concentrate on futures nearest to delivery. In futures nearest to delivery large arbitrage profits occur only in the first two contracts suggesting that the futures market exhibits a maturation effect. But, this maturation effect is only found in futures nearest to delivery.

## 7. Summary

The paper contains the results of a study of the price relation between the German stock performance index, DAX, and DAX futures. It is shown that for these markets there is no dividend risk for an arbitrageur. The execution risk in the spot market is considered in the arbitrage strategy by taking a risk premium into account. The relative mispricing has to exceed round trip transaction costs plus the risk premium before arbitrageurs are willing to trade.

The main results of this paper are the following. First, it is found that the relation between index and futures prices cannot be described by the model of cash and carry arbitrage. Futures contracts are significantly undervalued. This finding is similar to results reported from other markets. The absolute value of the undervaluation increases in all contracts with time to maturity. Second, there are a large number of arbitrage signals - most indicating short arbitrage opportunities. This is true particularly for futures contracts which are not nearest to delivery. Third, arbitrage signals disappear quickly in futures when they are the contracts with shortest time to maturity. This suggests that arbitrageurs exploit arbitrage signals rapidly, but only in the nearest contracts. Fourth, there is very limited risk associated with the simulated arbitrage strategy. With a reasonable order execution lag, more than 95 % of all arbitrage trades are profitable. Finally, with the exception of the first half of 1991, the total arbitrage profit of an arbitrageur is low when trading in futures nearest to delivery. It is much larger when trading in futures which are not nearest to delivery.

As one might suspect that arbitrageurs determine the mispricing by their trading strategies, this result suggests that arbitrageurs concentrate their trading on futures nearest to delivery.

# Bibliography

Bailey, Warren (1989): "The Market for Japanese Stock Index Futures: Some Preliminary Evidence," *The Journal of Futures Markets*, 9:283-295.

Bamberg, Günter; and Röder, Klaus (1994): "Arbitrage am DAX-Futures Markt unter Berücksichtigung von Steuern", *Kredit und Kapital*, 26:575-607.

Bhatt, Swati; and Cakici, Nusret (1990): "Premiums on Stock Index Futures - Some Evidence", *The Journal of Futures Markets*, 10:367-375.

Brennan, Michael J.; and Schwartz, Eduardo S. (1988): "Optimal Arbitrage Strategies under Basis Variability", *Studies in Banking and Finance*, 5:167-180.

Brennan, Michael J.; and Schwartz, Eduardo S. (1990): "Arbitrage in Stock Index Futures", *Journal of Business*, 63: S7-S31.

Brenner, Menachem; Subrahmanyam, Marti G.; and Uno, Jun (1989): "The Behavior of Prices in the Nikkei Spot and Futures Market", *Journal of Financial Economics*, 23:363-383.

Brenner, Menachem; Subrahmanyam, Marti G.; and Uno, Jun (1990): "Arbitrage Opportunities in the Japanese Stock and Futures Markets", *Financial Analysts Journal*, March-April 1990:14-24.

Buhler, Wolfgang; and Kempf, Alexander (1994): "The Value of the Early Unwind Option in Futures Contracts with an Endogenous Basis", ZEW Discussion Paper # 94-06, Mannheim.

Chan, Kalok (1992): "A Further Analysis of the Lead-Lag Relationship between Cash Markets and Stock Index Futures Markets", *Review of Financial Studies*, 5:123-152.

Chung, Peter Y. (1991): "A Transactions Data Test of Stock Index Futures Market Efficiency and Index Arbitrage Profitability", *The Journal of Finance*, 46:1791-1809.

Cooper, Ian; and Mello, Antonio (1990): "Futures/Cash Arbitrage with Early Unwinding Opportunities and Inelastic Liquidity Demand", Paper presented at ESF Workshop on Options and Futures.

Cornell, Bradford (1985): "Taxes and the Pricing of Stock Index Futures: Empirical Results," *The Journal of Futures Markets*, 5:89-101.

Cornell, Bradford; and French, Kenneth R. (1983a): "The Pricing of Stock Index Futures", *The Journal of Futures Markets*, 3:1-14.

Cornell, Bradford; and French, Kenneth R. (1983b): "Taxes and the Pricing of Stock Index Futures", *The Journal of Finance*, 38:675-694.

Figlewski, Stephen (1984a): "Hedging Performance and Basis Risk in Stock Index Futures: Empirical Results", *The Journal of Finance*, 39:657-669.

Figlewski, Stephen (1984b): "Explaining the Early Dicounts on Stock Index Futures: The Case of Disequilibrium", *Financial Analysts Journal*, July-August 1984:43-47, 67.

Grunbichler, Andreas; and Callahan, Tyrone W. (1993): "Stock Index Futures Arbitrage in Germany: The Behavior of DAX Index Futures Prices", Working Paper # 20-93, University of California, Los Angeles.

Grunbichler, Andreas; Longstaff, Francis; and Schwartz, Eduardo S. (1992): "Electronic Screen Trading and the Transmission of Information: An Empirical Examination", Working Paper # 27-92, University of California, Los Angeles.

Haller, Andreas; and Stoll, Hans R. (1989): "Market Structure and Transaction Costs: Implied Spreads in the German Stock Market", *Journal of Banking and Finance*, 13:697-708.

Hohmann, Ralf (1991): "Der Einfluß der Wertpapierleihe auf die Bewertung des DAX-Future", *Kapitalanlagen, Recht und Steuern*, Heft 8/9:574-582.

Holden, Craig W. (1990): "Intertemporal Arbitrage Trading: Theory and Empirical Evidence", Discussion Paper # 474, Indiana University, Bloomington.

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Kempf, Alexander; and Kaehler, Jürgen (1993): "Informationsverarbeitung auf Kassa- und Terminmärkten: DAX versus DAX-Futures", *ZEW-Wirtschaftsanalysen*, 1:359-380.

Kempf, Alexander; and Spengel, Christoph (1993): "Die Bewertung des DAX-Futures: Der Einfluß von Dividenden", ZEW Discussion Paper # 93-12, Mannheim.

Klemkosky, Robert C.; and Lee, Jae Ha (1991): "The Intraday Ex Post and Ex Ante Profitability of Index Arbitrage", *The Journal of Futures Markets*, 11:291-311.

Lim, Kian-Guan (1992): "Arbitrage and Price Behavior of the Nikkei Stock Index Futures", *The Journal of Futures Markets*, 12:151-161.

Loistl, Otto; and Kobinger, Martin (1992): "Index-Arbitrage insbesondere mit DAX-Futures", *DFVA Beiträge zur Wertpapieranalyse*, Dreieich.

MacKinlay, A. Craig; and Ramaswamy, Krishna (1988): "Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices", *Review of Financial Studies*, 1:137-158.

Merrick, John J. Jr. (1989): Early Unwindings and Rollovers of Stock Index Futures Arbitrage Programs: Analysis and Implications for Predicting Expiration Day Effects, *The Journal of Futures Markets*, 9:101-111.

Modest, David M.; and Sundaresan, Mahadevan (1983): "The Relationship Between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence", *The Journal of Futures Markets*, 3:15-41.

Newey, Whitney K.; and West, Kenneth D. (1987): "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, 55:703-708.

Prigge, Jutta; and Schlag, Christian (1992): "Die Bewertung des DAX-Future-Kontraktes an der Deutschen Terminbörse, *Zeitschrift für Bankwirtschaft und Bankrecht*, 4:299-307.

Puttonen, Vesa (1993): "Stock Index Futures Arbitrage in Finland: Theory and Evidence in a New Market", *European Journal of Operational Research*, 68:304-317.

Puttonen, Vesa; and Martikainen, Teppo (1991): "Short Sale Restrictions: Implications for Stock Index Arbitrage", *Economics Letters*, 37:159-163.

Roll, Richard (1984): "A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market", *The Journal of Finance*, 39:1127-1139.

Sofianos, George (1993): "Index Arbitrage Profitability", *The Journal of Derivatives*, 1:6-20.

Stoll, Hans R.; and Whaley, Robert E. (1990): "The Dynamics of Stock Index and Stock Index Futures Returns", *Journal of Financial and Quantitative Analysis*, 25:441-468.

Stulz, René M.; Wasserfallen, Walter; and Stucki, Thomas (1990): "Stock Index Futures in Switzerland: Pricing and Hedging Performance", *Review of Futures Markets*, 9:577-592.

Yadav, Pradeep K.; and Pope, Peter F. (1990): "Stock Index Futures Arbitrage: International Evidence", *The Journal of Futures Markets*, 6:573-603.

Yadav, Pradeep K.; and Pope, Peter F. (1992): "Intraweek and Intraday Seasonalities in Stock Market Risk Premia: Cash and Futures", *Journal of Banking and Finance*, 16:233-270.