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Forward Trading and Collusion in Supply Functions

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Abstract

This paper studies the effect of forward contracts on the stability of collusion among firms, competing in supply functions on the spot market. A forward market can increase the range of discount factors which allow to sustain collusion. On the contrary, collusion is destabilised when a potential deviator sells a significant amount forward. Results do not depend on the type (financial or physical) of contract fulfilment and are robust to different levels of demand uncertainty. As a policy implication, the study finds that liquid and anonymous forward markets are incompatible with collusion.

1 Introduction

Many commodities are forward traded such that the time of contract closure significantly pre-dates the time of delivery. Forward trading allows producers to hedge price risk by locking in future revenues well ahead of actual production. However, by selling forward, firms not only hedge risk, but also change their strategic position in the subsequent spot market as pointed out by Allaz and Vila (1993) in the case of Cournot oligopoly. The following analysis provides insights into the effect of forward contracts on the stability of collusion when the spot market clears in supply functions, a setting that is especially close, but not restricted, to electricity wholesale markets.

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I find that forward trading is either limited or ceases completely when collusion has to be sustained, depending on the observability of firms' forward positions. This is due to the possibility of a deviating firm to boost its profits from deviation through forward sales. The finding has important implications for competition policy and transparency regulation: when the producers' forward positions are not observable, the existence of liquid and anonymous forward markets is incompatible with the prevalence of collusion. However, the possibility of selling forward can help sustain collusion if firms fear the pro-competitive effect of such forward sales after the collapse of a collusive agreement.

The empirical relevance of my setting is demonstrated by Mercadal (2018), who shows that the mark-ups in the US Midwest electricity market were too high to be consistent with one-shot Nash-equilibrium behaviour. The author interprets this finding as an evidence of tacit collusion. The excessive mark-ups disappear, however, in response to a policy change that substantially increases the liquidity in the financial forward market while leaving the physical market conditions unchanged – a finding which is in line with the results of my paper. Further evidence is provided by Sweeting (2007) who shows that two dominating producers in the UK electricity market exerted substantial market power, leaving behind the possibility of increasing short run profits. The same UK power sector has served as a major example for a market where firms compete in supply functions on a short term market and can close forward contracts several months ahead (see e.g. Green, 1999). In fact, most electric power exchanges worldwide impose some kind of supply function bidding for spot sales, and allow for hedging of the price risk through forward contracts. (See e.g. for the UK Green and Newbery (1992), Wolak (2003) for California, Wolak (2007) for Australia, Hortaçsu and Puller (2008) for Texas, or Fabra and Reguant (2014) for Spain.) Another empirical example where competition on the spot market is in price-quantity curves and with wide-ranging forward hedging possibilities are various financial markets, such as the security auctions of the Treasury or liquidity auctions held by central banks (see Vives, 2011, for a discussion of these cases). A rigorous analytical discussion of the potential for collusion in this setting is – however – missing.

The pro-competitive effect of forwards in Allaz and Vila's model is due to repeated interactions of rival suppliers before the final clearing of the market. By selling forward, a firm can lock in a fixed quantity for the upcoming spot market, thereby achieving a sort of Stackelberg advantage over its competitor. The total supply in the spot market is therefore larger with forward trading and the equilibrium price is lower. Given that both the firms are allowed to trade forward, they both face the same incentive to expand their output by selling forward, although they would be better off without. Mahenc and Salanie (2004) find an opposite result in their study of price competition in the spot market. By buying forward, the producer commits to raise prices in the spot market. A similar result is obtained by Ferreira (2003) for Cournot competition with infinitely many openings of the forward market. Green (1999) and Newbery (1998) show that the pro-competitive effect of forward sales found by Allaz and Vila (1993) carry over to the case of supply function bidding in the spot market. Brandts et al. (2008) show supporting evidence from laboratory experiments. Holmberg and Willems (2015) consider supply functions and the strategic use of option contracts. None of these contributions however addresses the possibility of collusion. Ciarreta and Gutiérrez-Hita (2006) model collusion in supply functions, but do not consider forward contracts.

Liski and Montero (2006) in contrast study a repeated oligopoly game with several openings of a forward and a spot market. They show that the possibility of forward trading facilitates collusion in both, price and quantity competition in the spot market. But, somewhat parallel to the dichotomy of Allaz and Vila (1993), and Mahenc and Salanie (2004), the strategic effect of forward positions differs fundamentally between the two cases. For a price-setting oligopoly, selling forward locks in the corresponding quantities and thus reduces the size of the market that could be captured by a deviating firm. This contrasts with competition in quantities where a deviating firm can never capture the complete spot market because other firms set their quantities in advance. So Liski and Montero (2006) conclude that when spot market competition is in prices, the critical discount factor for collusion decreases in the forward sales of the firms. However, when firms compete in quantities, collusion is harder to sustain when firms have sold forward.

From the fundamental work of Klemperer and Meyer (1989), it is well known that a Supply Function Equilibrium (SFE) lies between the theoretical Cournot and Bertrand prices and quantities. One might therefore conjecture that the effect of forward trading on collusion in supply functions will be somehow contained between the results for price and quantity competition. But in light of the contrary results for both the cases in the literature, there is no evident intuition about the effect the forward positions ought to have.

The linear supply function equilibrium model allows to represent competition in supply functions in a practical manner (Vives, 2011). The unique and analytically solvable equilibria led to its widespread adoption in the literature. With respect to forward trading, however, the linear model is known to exert a somewhat artificial strategic neutrality of forward contracts. Newbery (1998) and Green (1999) therefore calibrate firms' forward positions with conjectural variations. Holmberg (2011) shows how endogenous forward positions emerge if the assumption of linear supply functions is relaxed. This paper first follows Newbery (1998) and Green (1999) in studying the effect of forward sales on collusion for the linear model. It then discusses a generalisation in the line of Holmberg (2011) for non-linear supply functions with endogenous forward contracting.

The outline for the rest of the paper is as follows: Section 2 introduces the model and derives spot market strategies for the one-shot game and for joint profit maximisation. The repeated game with several forward and spot market openings is studied in Section 3. Demand uncertainty and its effect on the sustainability of collusion is addressed in Section 4. Section 5 shows the equivalence of financial and physical settlement of forward contracts, discusses the impact of different informational regimes, and the generalisation to non-linear supply functions in the spot market. Section 6 is the conclusion.

2 The Static Game

2.1 The Model

Two firms i, j produce a homogeneous good at symmetric costs $C(q) = c_1 q_k + \frac{c_2}{2} q_k^2$, $c_1, c_2 > 0$, with $q_k \in \mathbb{R}^+$ denoting the quantities of firms $k = i, j$. They compete in a spot market with supply functions $q_k(p) = \alpha_k + \beta_k p$, with $\alpha_k, \beta_k \in \mathbb{R}$, $\beta_k > 0$, $k = i, j$, where $p \in \mathbb{R}$ denotes the spot market price.[1] Demand is given by $D(p) = A - bp + \varepsilon$ with $A, b > 0$. $\varepsilon : \Omega \to \mathbb{R}$ is a random variable defined on a probability space Ω with a continuous distribution function that is common knowledge to all market participants. ε has a zero mean, a finite second moment σ^2, and a lower bound $\varepsilon > bc_1 - A$ that ensures positive demand at a price equal to the intercept of marginal costs.

Before bidding in the spot market, the firms can sell 'contracts-for-difference' (financial forwards), specifying that once the spot market clears, the seller of the contract receives from the buyer the contracted quantity times the forward price f, and pays to the buyer the contracted quantity times the realised spot price p. The forward market follows the model of Allaz and Vila (1993): Both firms simultaneously disclose the quantities x_i, $x_j \geq 0$ they intend to sell forward.[2] There are $S \geq 2$ risk neutral, not liquidity constraint speculators who observe the total amount of open interest $x_i + x_j$. Speculators simultaneously choose the per-unit price $f_s \in \mathbb{R}$, $s = 1, ..., S$ they are willing to pay for the offered contracts. The complete game structure is as follows:

1. Firms i and j simultaneously choose the quantities x_i, x_j they intend to sell in forward contracts. $x_i + x_j$ becomes common knowledge.

2. Each speculator $s = 1, 2, ..., S$ chooses the per-unit price $f_s \in \mathbb{R}$ she intends to pay for the offered forward quantities. The highest price bid $f = \max_{s=1,...,S} \{f_s\}$ wins all offered contracts. If there are $N \leq S$ identical highest price bids, each of these bids wins a quantity $N^{-1}(x_i + x_j)$ in forward contracts.

3. Firms simultaneously choose their spot market supply functions $q_i(p)$, $q_j(p)$ of the linear form $q_k(p) = \alpha_k + \beta_k p$, $k = i, j$.

4. The realisation of the demand shock ε is observed. The spot price p is determined by solving the market clearing condition $D(p) = q_i(p) + q_j(p)$. Forward contracts are settled. Firms produce and sell quantities according to their supply function at the market clearing price.

[1] Negative prices are allowed and are actually observed in a number of electricity spot markets around the world.

[2] In line with Green (1999), I do not consider producers going long in the forward market, e.g. buying forward their own output.

The market clearing spot price in terms of demand and supply parameters is

$$p = \frac{A + \varepsilon - \alpha_i - \alpha_j}{b + \beta_i + \beta_j} \tag{1}$$

The ex-post profit of every speculator with a winning price bid is $N^{-1}(x_i + x_j)(f - p)$. The profit of all other speculators is zero. The ex-post realised profit of firm i is

$$\pi_i = (f - p)x_i + pq_i(p) - C\left(q_i(p)\right), \tag{2}$$

The solution concept applied in the following is subgame-perfect Nash equilibrium.

2.2 Nash Strategies

The Spot Market

Following Klemperer and Meyer (1989), and more specifically Newbery (1998) and Green (1999), we determine the price firm i would choose optimally for an arbitrary realisation of the demand shock ε, and then derive the corresponding supply function.

Express firm i's profit in terms of its residual demand, using $q_i = D(p) - q_j(p)$.

$$\pi_i = (f - p)x_i + p\,\left(D(p) - q_j(p)\right) - C\left(D(p) - q_j(p)\right)$$

The first order condition for an optimal price is

$$\frac{d\pi_i}{dp} = -x_i + D(p) - q_j(p) + \left(D' - q_j'\right)\left(p - C'\left(D(p) - q_j(p)\right)\right) = 0. \tag{3}$$

Using $D(p) - q_j(p) = q_i(p) = \alpha_i + \beta_i p$ we obtain

$$\frac{d\pi_i}{dp} = -x_i + \alpha_i + \beta_i p - (b + \beta_j)\left(p - c_1 - c_2(\alpha_i + \beta_i p)\right) = 0. \tag{4}$$

To be true for any possible price, the first order condition must hold as an identity, implying that all factors multiplying p at the same power are equal. this yields the following optimal supply function parameters.

$$\beta_i^* = \frac{b + \beta_j}{1 + c_2(b + \beta_j)} \tag{5}$$

$$\alpha_i^*(x_i, \beta_i, \beta_j) = \frac{x_i - c_1(b + \beta_j)}{1 + c_2(b + \beta_j)} = \beta_i^*\left(\frac{x_i}{b + \beta_j} - c_1\right) \tag{6}$$

Note that α_i^* depends on the slope parameters β_i^*, β_j, and on the firm's own forward position x_i. The slope parameter β_i^* depends on the slope β_j of the rival's supply function, but does not depend on either firms' supply function intercept α_i, α_j, or the forward positions x_i, x_j. So all strategic interactions between the rival firms occur through the slope parameters β_i and β_j. Use the expression in (5) to define the best response function of firm i to the slope parameter of firm j as $\beta_i^* = BR(\beta_j)$. Due to symmetry of both firms, the best response function for j is equivalent, thus $BR(.)$ is not indexed by i or j. It can be solved for the unique positive Nash equilibrium slope parameter which will be denoted by β_n in the remainder of the paper:

$$\beta_n = BR(\beta_n) > 0 \tag{7}$$

The inverse of the supply function given by (5) and (6) is such that the price equals marginal costs at the level of forward contracts. Larger quantities are sold above marginal costs and smaller quantities below it. Figure 1 illustrates the Nash spot market strategies with and without forward sales together with the joint profit maximising supply function which will be discussed in Section 2.3.

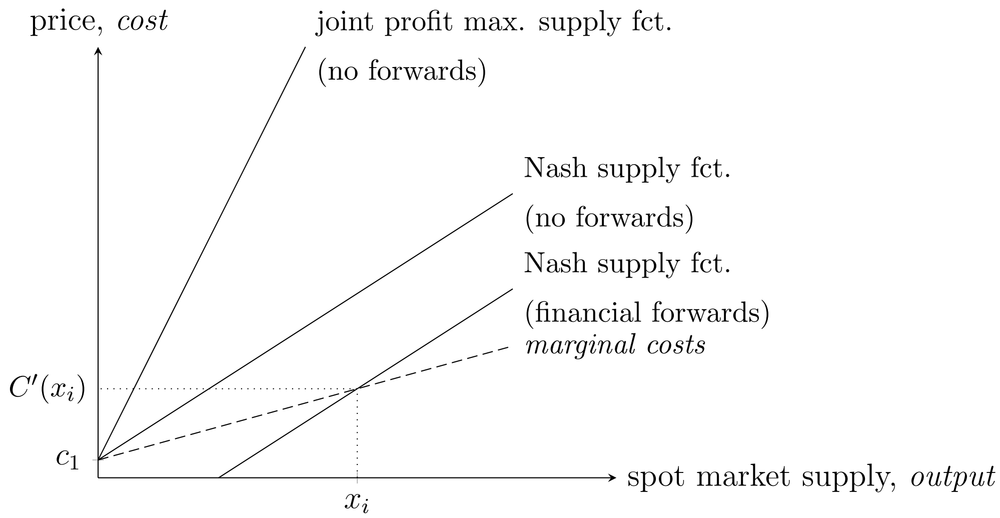

Figure 1: Joint profit maximisation and Nash spot market supply functions with and without forward contracts.

The Forward Market

Speculators observe the total open interest $(x_i + x_j)$, anticipate the optimal supply functions of firms in the spot market, and submit their forward price bid f_s based on this expectation.[3] The expected spot price $\bar{p}_n$ obtains from Equation (1) with $\varepsilon = 0$, where the subscript n indicates that one-shot Nash strategies are expected in the spot market.

$$\bar{p}_n = \mathrm{E}(p_n) = \frac{A - \alpha_i^* - \alpha_j^*}{b + \beta_i^* + \beta_j^*} \tag{8}$$

[3] With $\beta_i = \beta_j = \beta_n$, it is evident from (6) and (1) that $x_i + x_j$ is a sufficient statistic to predict the expected spot price.

Any winning forward price bid $f_s < \bar{p}_n$ would imply negative expected profits for the winning speculator, whereas any winning price bid $f_s > \bar{p}_n$ could be profitably undercut by a rival speculator. The only pure strategy equilibrium outcome is that the winning forward price bid equals the expected spot price, $f = \bar{p}_n$, given x_i and x_j.

In the first stage, firms i and j choose their forward positions x_i and x_j, anticipating the effect on the spot market outcome and the forward price f. The expected profits of firms depend on the range of possible demand realisations. Consider the profit of firm i for a specific realisation of ε as given in (2) and use $q_i = \alpha_i + \beta_i p$ to rewrite π_i in terms of powers of p.

$$\pi_i = (fx - c_1\alpha_i) + p\left(-x + \alpha_i - c_1\alpha_i - \frac{c_2}{2}\alpha_i^2\right) + p^2\left(\beta_i - \frac{c_2}{2}\beta_i^2\right) \tag{9}$$

The ex-ante expected profit $\hat{\pi}_i$ of firm i is

$$\begin{aligned}\hat{\pi}_i &= (fx - c_1\alpha_i) + \mathrm{E}(p)\left(-x + \alpha_i - c_1\alpha_i - \frac{c_2}{2}\alpha_i^2\right) + \mathrm{E}(p^2)\left(\beta_i - \frac{c_2}{2}\beta_i^2\right)\\ &= (fx - c_1\alpha_i) + \mathrm{E}(p)\left(-x + \alpha_i - c_1\alpha_i - \frac{c_2}{2}\alpha_i^2\right) + \left(\mathrm{E}(p)^2 + \mathrm{Var}(p)\right)\left(\beta_i - \frac{c_2}{2}\beta_i^2\right)\\ &= \pi_{i,\varepsilon=0} + \mathrm{Var}(p)\left(\beta_i - \frac{c_2}{2}\beta_i^2\right)\end{aligned}$$

where the last step used the expression (9) for ex-post realised profits and $\pi_{i,\varepsilon=0}$ denotes realised profits when the demand shock is exactly at its expected value. Equation (1) allows to derive $\mathrm{Var}(p) = \sigma^2(b + \beta_i + \beta_j)^{-2}$. Thus,

$$\hat{\pi}_i = \pi_{i,\varepsilon=0} + \sigma^2\left(\frac{\beta_i - \frac{c_2}{2}\beta_i^2}{(b + \beta_i + \beta_j)^2}\right). \tag{10}$$

Lemma 1 summarises the effect of selling forward on expected profits when both firms play Nash in the spot market.

Lemma 1.

(a) The expected one-shot Nash equilibrium profit $\hat{\pi}_i^n$ of firm i is monotonically decreasing in the number of forward contracts it sells. Precisely,

$$\frac{d\hat{\pi}_i^n}{dx_i} < 0 \ \text{ for } \ x_i > 0 \qquad \text{and} \qquad \frac{d\hat{\pi}_i^n}{dx_i} = 0 \ \text{ for } \ x_i = 0$$

(b) The expected one-shot Nash equilibrium profit $\hat{\pi}_i^n$ of firm i is monotonically decreasing and concave in a joint increase of the forward contracts $x = x_i = x_j$ sold by both firms.

A proof is given in Appendix A.1. Lemma 1.b is needed for the discussion in Section 3. Lemma 1.a implies that firms don't sell forward in equilibrium. The reason for this is twofold: on the one hand, selling forward means committing to expand the output in the spot market, thus making the market more competitive. On the other hand, selling forward does not provide a strategic leverage to the behaviour of the competitor. All strategic interactions between the two firms work through the supply functions' slope, but forward sales only affect the intercept. Selling forward therefore does not allow to strategically alter the rivals supply function.

2.3 Joint Profit Maximisation

Assume for the moment, that producers can effectively collude to maximise joint profits. Such a cartel would act as a monopolist that controls all the supply in the spot market, but still faces competitive speculators in the forward market. Knowing that $f = \mathrm{E}(p)$, one could naïvely presume that forward positions cancel out from the profit function (2). Thus, the cartel would be indifferent to the number of forward contracts it sells; optimal spot market strategies would be equivalent to Ciarreta and Gutiérrez-Hita (2006).[4]

$$\beta_c = \frac{b}{2 + c_2 b} \tag{11}$$

$$\alpha_c = \frac{-bc_1}{2 + c_2 b} \tag{12}$$

However, backward induction shows that there is a commitment issue for such a virtual monopolist. Let π^c denote the ex-post one period profit of a firm given that both firms collude and $\hat{\pi}^c$ its expected value. Lemma 2 states that these are unaffected by the possibility of trading forward for two reasons.

LEMMA 2. *Expected collusive profits $\hat{\pi}^c$ are unaffected by the possibility of trading forward because*

(a) absent any additional commitment mechanism, a cartel will not sell forward.

(b) when firms credibly commit to play supply functions $q_i = q_j = \alpha_c + \beta_c p$ defined in (11) and (12) in the subsequent spot market, selling forward has no effect on expected profits and the cartel is indifferent to the number of forward contracts it sells.

A proof for Lemma 2 is given in Appendix A.2. The reason for Lemma 2.a is that selling forward is equivalent to committing to a higher output in the spot market which negatively affects the cartels profit. Lemma 2.b just summarises what happens if this commitment effect is ruled out. Because firms are risk neutral and expected profits remain unchanged, the cartel is then indifferent to its forward positions. Section 3 discusses whether a collusive agreement in a repeated game provides a commitment as described in Lemma 2.b. Either way, joint profit

[4]Ciarreta and Gutiérrez-Hita (2006) assume c_1 to be zero. (12) is adjusted to account for a positive intercept of marginal costs.

maximising firms will stick to the supply functions given by (11) and (12). The resulting expected spot price is denoted as $\bar{p}_c$ in the following.

$$\bar{p}_c = \mathrm{E}(p_c) = \frac{A - 2\alpha_c}{b + 2\beta_c}, \tag{13}$$

3 Collusion

3.1 Collusion in the repeated game

Consider now the case of infinitely many interactions of the same two firms on, first, the forward market, then the corresponding spot market, then the forward market for the next period, and so on. For simplicity, forward sales are restricted to the upcoming spot market.[5] Firms discount profits from one spot market to the next by a constant factor $\delta \in (0,1)$. The one-shot Nash-equilibrium is also an equilibrium for the repeated game. But the joint profits would be higher (maximal) if firms could implement the cartel solution in every period. Sustainability of such collusion is studied with standard grim trigger punishment strategies where firms revert permanently to the one-shot Nash equilibrium whenever a firm deviates from the collusive supply function. A deviating firm can thus at best earn deviation profits once. Let $\hat{\pi}^d$ denote the expected one-period profit of a deviating firm and recall that $\hat{\pi}^c$ and $\hat{\pi}^n$ are the corresponding profits of collusion and the one-shot Nash equilibrium respectively. The well-known incentive constraint for sustained collusion is

$$\frac{\hat{\pi}^c}{1-\delta} \geq \hat{\pi}^d + \hat{\pi}^n \frac{\delta}{1-\delta}$$

With $\hat{\pi}^d > \hat{\pi}^c > \hat{\pi}^n$, the critical discount factor $\overline{\delta}$ for which the incentive constraint is binding is

$$\overline{\delta} = \frac{\hat{\pi}^d - \hat{\pi}^c}{\hat{\pi}^d - \hat{\pi}^n} \tag{14}$$

Whenever $\delta \geq \overline{\delta}$, producers maintain collusion. Under this condition, speculators can rationally expect the spot market price to be at the collusion level and buy any quantity forward at a per unit price $f = \bar{p}_c$. From Lemma 2.b we know that colluding firms are then indifferent to their forward sales with respect to profits. Thus they are free to choose forward positions so as to minimise the incentive for deviation.

3.2 Deviation

We first consider deviation on the spot market, then study the effect of forward sales on profits from such deviation, and finally discuss the resulting choices of

[5] A generalisation to several openings of the forward market for the same spot market or to forward contracts spanning several spot market openings as in Green and Le Coq (2010) does not provide fundamentally different insights.

firms and speculators in the forward market.

Suppose a collusive agreement is in place and firm i sells forward some quantity x_i at a price $f = \bar{p}_c$. We consider a situation wherein the deviation in the spot market comes as a surprise. (Regardless of this outcome not being a rational expectations equilibrium, the threat is relevant for the colluding firms and speculators to take into account.) A deviator in the spot market plays the best response given by (5) and (6) against the collusive supply function of its rival.

$$\beta_d = BR(\beta_c) \quad \text{and} \quad \alpha_d = \alpha_i^*(x_i, \beta_d, \beta_c). \tag{15}$$

Lemma 3 summarises the effect of x_i on the profits from deviation.

LEMMA 3. *Expected profits $\hat{\pi}^d$ of a firm, deviating in the spot market, are convex and increasing with the number of its forward sales, irrespective of the forward position of its rival.*

A proof of Lemma 3 is in Appendix A.3. In brief, it states that a deviator in the spot market would like to have sold forward as much as possible to earn a rent from the mistaken beliefs of the speculators. The speculators need to consider this threat before buying forward when a collusive agreement is in place. Section 5.1 discusses this aspect in further detail.

3.3 Sustainability of collusion with forward contracts

Consider the definition of $\overline{\delta}$ in (14) and an increase in the number of forward contracts x_i sold during collusion by a potential deviator. According to Lemma 2, collusive profits $\hat{\pi}^c$ remain unchanged. According to Lemma 3, profits of deviation $\hat{\pi}^d$ increase. Profits during the punishment phase $\hat{\pi}^n$ are unaffected. Evidently, the more that has been sold forward during collusion, the higher the incentive to deviate and the higher the critical discount factor. Now consider an increase in the number of forwards that firms sell during the punishment phase. Such a change will solely affect the expected profits of punishment $\hat{\pi}^n$, and it will do so negatively (see Lemma 1). Thus, the critical discount factor $\overline{\delta}$ will decrease with increasing forward sales in the punishment phase.

Let $(x_{i,c}, x_{j,c})$ denote the forward positions of firms during collusion and $(x_{i,n}, x_{j,n})$ the forward position of firms during Nash reversion. The above given argument is summarised in Proposition 1.

PROPOSITION 1.

(a) Collusion is harder to sustain when firms sell forward during collusion. More precisely, the critical discount factor $\overline{\delta}$ increases in $\max(x_{i,c}, x_{j,c})$.

(b) Collusion is easier to sustain when firms expect significant forward sales during the punishment phase, meaning, the critical discount factor decreases with a joint increase of $x_{i,n}$ and $x_{j,n}$.

Proposition 1 implies that, in order to minimize the incentive to deviate, the colluding firms should not sell forward at all. Nevertheless, collusive equilibria with positive forward sales exist. The complete characterisation of the collusive equilibrium is as follows: For every forward market opening, firms i and j submit offers $x_{i,c}, x_{j,c} \geq 0$ which are bounded from above by the condition that the incentive constraint $\delta \geq \overline{\delta}$ holds even for $x_i = 2x_{i,c}$ or $x_j = 2x_{j,c}$. Speculators $s = 1, ..., S$ observe $(x_i + x_j)$, deduce that collusion will hold in the spot market, and bid $f_s = \bar{p}_c$. In every spot market, firms bid the joint profit maximising supply function given by (11) and (12).

The off-equilibrium beliefs and strategies are as follows: whenever the total amount of offered contracts exceeds $x_{i,c} + x_{j,c}$, speculators infer deviation and bid f_s equal to the expected spot price that obtains if both firms play their one-shot Nash strategies (6) and (7) for some x_i, x_j, matching the observed amount of offered contracts.[6] Whenever the forward market clears at a price other than $\bar{p}_c$, it is common knowledge that collusion broke. The firms i and j then bid their one-shot Nash strategies in the spot market. Section 5.1 discusses the range of possible forward sales during collusion against the backdrop of different degrees of transparency.

Proposition 1 also allows to sort the case of competition in supply functions among those cases that have been studied in the literature. Liski and Montero (2006) have found diametrical effects of forward sales on collusion in price versus quantity competition. The result for competition in supply functions summarised in Proposition 1 resembles that for quantity competition found by Liski and Montero. It clearly contrasts with their result for price competition in the spot market.

Figure 2 illustrates the mechanics behind Proposition 1, depicting the expected profits in three regimes over varying levels of contracted quantities. The critical discount factor $\overline{\delta}$ has a graphical interpretation: for a given level of contracting during collusion, $\overline{\delta}$ is equivalent to the distance between deviation profits and collusive profits, divided by the vertical distance between deviation profits and one-shot Nash profits at the expected level of contracting during Nash reversion.

The argument which leads to Proposition 1 also implies Corollary 1 which is needed for the discussion in Section 4. Let $\overline{\delta}_0$ denote the critical discount factor with zero forward sales of the potential deviator and zero forward sales $x_i = x_j = x_n$ during the punishment phase.

$$\overline{\delta}_0 = \overline{\delta} \quad \text{such that} \quad x_c = x_n = 0 \text{ and } \overline{\delta} \text{ as in (14)} \tag{16}$$

In short, $\overline{\delta}_0$ describes the critical discount factor which would prevail when there are no forward markets.

Corollary 1. *There are infinitely many combinations (x_c, x_n) of forward positions x_c held by a deviating firm, and forward positions $x_i = x_j = x_n$ held by both firms during the punishment phase, such that $\overline{\delta} = \overline{\delta}_0$.*

[6] Note that with symmetric slopes of supply functions in the spot market, $x_i + x_j$ is a sufficient statistic to predict the expected spot price.

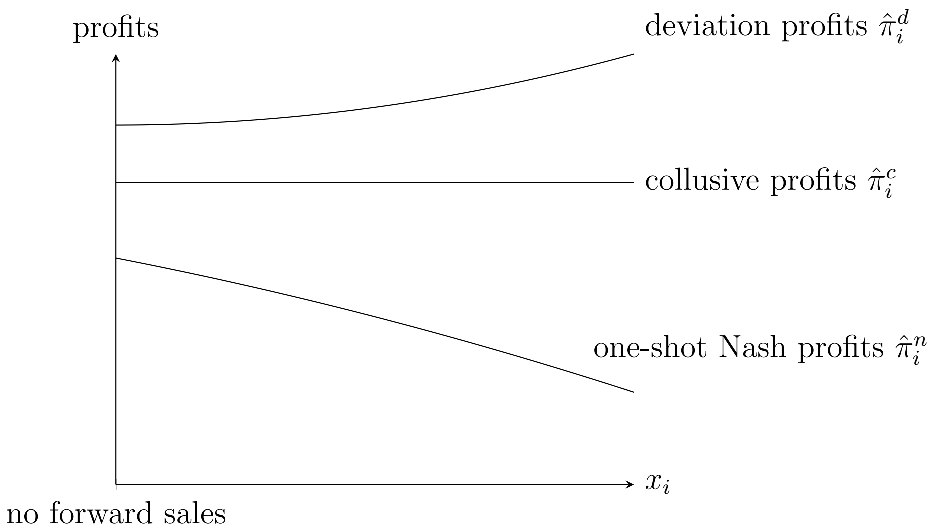

Figure 2: Profits across varying levels of forward contracting (symmetric forward positions in the case of Nash equilibrium profits).

Proof. Immediate from definition (14), Lemma 1 to 3 (especially the monotonicity plus concavity/convexity results in Lemma 1.b and Lemma 3), and the fact that profits are continuous in the level of forward sales. □

4 The impact of demand uncertainty on collusion

The literature on supply function equilibrium often assumes some variation of demand to identify optimal price-quantity schedules, but the profit effect of that variation is ignored. Ciarreta and Gutiérrez-Hita (2006), for example, do not discuss the effect of demand shocks on collusion, although demand uncertainty is present in their model. At first glance this is surprising because the range of potential profit margins increases with higher variability of demand. The critical discount factor $\overline{\delta}$ is a ratio of profits in different regimes, and it is not obvious that this ratio needs to be constant across varying levels of demand uncertainty. The following Proposition therefore complements the results from the preceding section and those of Ciarreta and Gutiérrez-Hita (2006).

PROPOSITION 2.

a. *When the critical level of the discount factor is equivalent to that, which would prevail without any forward sales,* $\overline{\delta} = \overline{\delta}_0$*, then it is unaffected by a change in the variance* σ^2 *of the demand shock* ε*.*

b. *When forward sales of firms during collusion or during the punishment phase are such that* $\overline{\delta} \neq \overline{\delta}_0$*, then the critical factor decreases with* σ^2 *for any* $\overline{\delta} > \overline{\delta}_0$*. It increases with* σ^2 *for any* $\overline{\delta} < \overline{\delta}_0$

Proposition 2.a implies that in the study of Ciarreta and Gutiérrez-Hita (2006) it is safe to ignore the effect of demand uncertainty. It simply cancels from the incentive constraint when there are no forward markets. Proposition 2.b can be resumed a bit more intuitively as: the larger the variability of demand, the smaller the effect of forward positions on the critical discount factor. A proof of Proposition 2 is given in Appendix A.5. The general reasoning for the proof is explained below. It requires the following Lemma.

LEMMA 4. *The derivatives of expected profits of deviation, collusion, and Nash-reversion with respect to* σ^2 *can be ordered as follows:*

$$\frac{d\hat{\pi}^d}{d\sigma^2} > \frac{d\hat{\pi}^c}{d\sigma^2} > \frac{d\hat{\pi}^n}{d\sigma^2}$$

The proof for Lemma 4 is in Appendix A.4. It is illustrated in Figure 3. Expected profits of deviation are larger than collusive profits, which are larger than one-shot Nash equilibrium profits. Lemma 4 states that the same ordering applies to the derivatives of these profits with respect to demand variability: profits of deviation rise stronger with σ^2 than those from collusion, which again rise stronger with σ^2 than those from the one-shot Nash equilibrium. Due to the overall linearity of expected profits (see Eq. 10), we can use the Basic Proportionality Theorem to evaluate the change of ratios of profit differences over σ^2. Consider the three lines tracing through the points B, C, and D in Figure 3. They represent the expected one-period profits in the three regimes without any forward sales. There is a common point of intersection at $\sigma^2 = -(A - bc_1)^2$. Obviously, a negative value for the variance has no practical interpretation. But the common point of intersection implies the following: Any parallel line to the one that intersects expected profits in B, C, and D, will also hit the lines representing expected profits, and it will be segmented in equivalent proportion to the segments spanning between B, C, and D, respectively.

As an example, take two vertical lines, one at σ^2, one at $\sigma^{2'}$. The basic proportionality theorem implies that the ratio of the distances $\overline{\text{BC}}/\overline{\text{BD}}$ is the same as the ratio of $\overline{\text{B'C'}}/\overline{\text{B'D'}}$. These ratios are equivalent to the fraction $(\hat{\pi}^d - \hat{\pi}^c)/(\hat{\pi}^d - \hat{\pi}^n)$, evaluated at two different level of σ^2. Because only the case without any forward sales is considered here, this ratio is equivalent to $\overline{\delta}_0$ defined in (16). Therefore, without forward sales during collusion, deviation or Nash reversion, the discount factor $\overline{\delta}_0$ is the same for any level of σ^2.

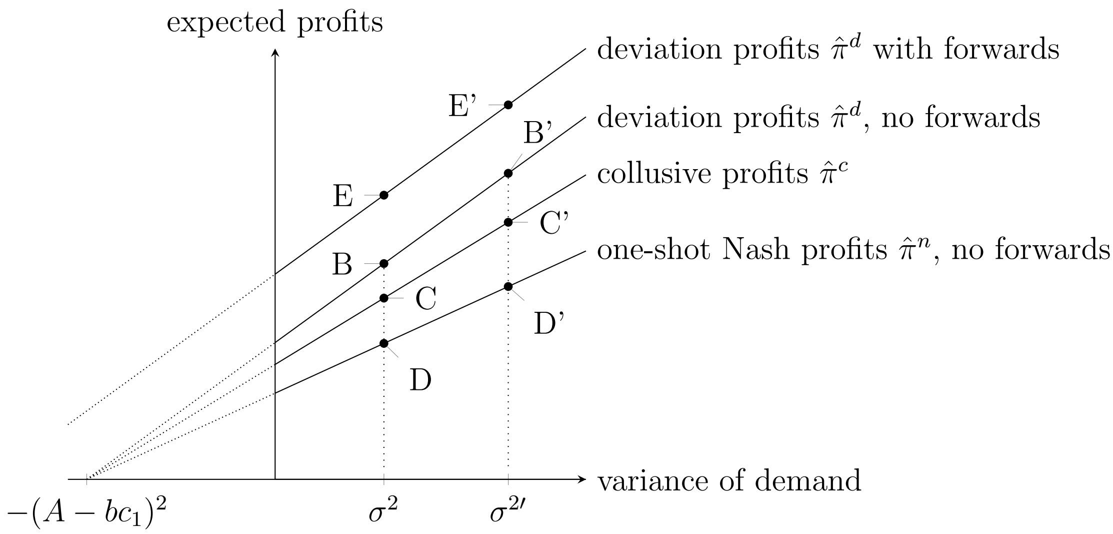

Figure 3: Expected profits over variance of demand shocks σ^2

The proof in the Appendix A.5 shows that there is always a common point of intersection when there are no forward sales during collusion or during the punishment phase, leading to Proposition 2.a. It also extends the argument to cases with positive forward sales, leading to Proposition 2.b.

5 Generalisations

5.1 Implications of different informational environments

The following paragraphs provide a short, non-formal discussion about the observability of forward positions and the implications for regulation.

We have seen that colluding firms should have no interest in selling forward, but some forward sales (e.g. imposed by regulatory obligations) might still be compatible with sustained collusion. The same condition – given its full observability – implies that speculators can rationally expect collusion to hold in the subsequent spot market. When speculators observe the individual quantities of forward contracts offered by each firm (x_i, x_j), they can deduce the incentive constraint for the firm that is most prone to deviation, evaluate whether the incentive constraint holds for this firm, and thus determine if they can rationally buy forward contracts at the expected collusive price.

If speculators observe only the aggregated forward volume $(x_i + x_j)$ as assumed before, it could be that all these contracts are from just one firm, and that this firm is the potential deviator. The upper bound on $x_{i,c}, x_{j,c}$ is strictly lower compared to the situation with fully observable forward offers. Less transparency therefore reduces the amount of contracts that speculators are willing to buy at the expected collusive price.

In the extreme case, consider the situation where each speculator receives an individual offer to buy forward contracts. No speculator knows how much is offered by a particular firm to the other speculators. Considering that the offer might come from a potential deviator who sells as an insider, no speculator can plausibly predict an average spot price that won't be prone to undercutting by a deviator. Thus no speculator will buy forward whatever the price (e.g. $f_s \to -\infty$ for all $s = 1, ..., N$) to ensure profits of at least zero.[7] Anticipating this outcome of the forward market, the producing firms will not offer any forward contracts in the first place.

This has important implications for anti-trust and financial transparency regulations: When commodity producers cannot credibly signal a sufficiently tight upper bound of their forward positions to speculators and rivals, the mere existence of liquid and anonymous forward markets rules out the possibility of tacit collusion. Indeed, Mercadal (2018) provides evidence for collusive conduct in the US Midwest electricity market which came to an end when new regulation substantially increased the liquidity of forward markets, which is exactly in line with the argument made in this paper. On the contrary, collusion might be eased by financial regulation that obliges firms to disclose timely and truthfully all the forward positions they take, or if it puts specific limits on these positions, such as e.g. the European Union Markets in Financial Instruments Directive / Regulation (MIFID / MIFIR).

5.2 Physical vs. Financial Forwards

Nowadays, most commodities are traded forward for both physical and financial delivery. Liski and Montero (2006) point out that in homogeneous goods and with price competition, financial contracts might have a substantially different effect on collusion as compared to physical contracts. The following paragraphs show that in my setting, considering physical forward contracts is equivalent in terms of equilibrium price and revenues to pure financial forward contracts which have been studied before.

Let $x_{i,f}$, $x_{j,f}$ denote financially contracted quantities, cleared by a balancing payment equal to the (positive or negative) difference $f - p$ from the buyer to the seller when the spot market clears. Physical forward sales $x_{i,\phi}$, $x_{j,\phi}$ oblige the seller to produce the corresponding output in addition to its spot market sales and allow the buyer to consume this output without buying it on the corresponding spot market. The buyer pays the contracted price to the seller upon delivery. Thus, buying physically forward is equivalent to buying on the spot market plus a financial forward contract as a hedge. We assume the buyers of physical forward contracts

[7] Admittedly, this argument relies on the questionable assumption that firms have unlimited capacity and therefore unlimited capability to manipulate the spot price. Here, its purpose is to illustrate the relevance of information for a functioning forward market when a collusive agreement is in place.

to act rationally and competitively, imposing a basic no-arbitrage condition that guarantees identical forward prices for the physically and the financially settled contracts, jointly denoted by f. Spot market demand is now $D(p) - x_{i,\phi} - x_{j,\phi}$ because some demand has already been contracted in advance. q_i, q_j still denote total output of firms, but this is not necessarily equal to spot market supply any more. Let $s_i(p)$, $s_j(p)$ denote the spot market supply functions of the linear form, $s_i = \alpha_i + \beta_i p$, and note that $q_i = x_{i,\phi} + s_i(p)$. Although the firms' output must be non-negative[8], supply in the spot market can be either positive or negative; in other words, firms might buy back on the spot market what they have sold forward before. The profit function then becomes

$$\pi_i = (f - p)x_{i,f} + f x_{i,\phi} + p s_i(p) - c_1(s_i(p) + x_{i,\phi}) - \frac{c_2}{2}(s_i(p) + x_{i,\phi})^2$$

The derivation of optimal strategies follows the same procedure as in Section 2. The first order condition, equivalent to Equ. (4), is now

$$\frac{d\pi_i}{dp} = s_i - x_{i,f} - (b + \beta_j)(p - c_1 - c_2(x_{i,\phi} + s_i)) = 0,$$

Using $s_i = s_i(p) = \alpha_i + \beta_i p$ and setting all factors multiplying p at the same power to be equal yields the same optimal slope β_i as in (5) which is independent of the firms' forward positions. The intercept, however, now accounts for the physically forward contracted quantities. Its definition (6) is generalised as follows:

$$\alpha_i^* = \beta_i \left(\frac{x_{i,f}}{b + \beta_j} - c_1 - c_2 x_{i,\phi} \right) \tag{6'}$$

The spot price is determined by the market clearing condition.

$$p = \frac{A - \alpha_i - \alpha_j - x_{i,\phi} - x_{i,\phi} + \varepsilon}{b - \beta_i - \beta_j},$$

Take the definition in (6′) and note that with Equ. (5), $\beta_i/(b+\beta_j)$ can be expressed as $1 - \beta_i c_2$. The market clearing price becomes:

$$p = \frac{A + \varepsilon - (x_{i,\phi} + x_{j,\phi} + x_{i,f} + x_{j,f})(1 - \beta_n) + 2\beta_n c_2}{b - 2\beta_n}.$$

This shows that physical ($x_{i,\phi}$, $x_{j,\phi}$) and financial forward positions ($x_{i,f}$, $x_{j,f}$) have an identical effect on the one-shot Nash equilibrium price in the spot market, and consequently on total output. It is also easy to verify that the produced quantities q_i and q_j are the same for a given level of forward sales, independent if these are financial or physical forward contracts. Moreover, the optimal collusive strategy obviously does not change due to the mere possibility of selling physically forward,

[8]Non-negativity of firms' output is still secured by the assumption, $(A + \varepsilon)/b > c_1) \forall \varepsilon$, but this is less evident from the notation here.

and the same applies for the incentives to deviate. The strategic effect of either form of contracting is equivalent.

The main difference between physical and financial forwards is that the former substitute trade in the spot market, thus reducing the observed volumes. The spot market 'supply function' now extends to negative quantities as illustrated in Figure 4. This might be of relevance when studying real world markets where a-priori firms forward contracts might specify a financial or physical settlement.

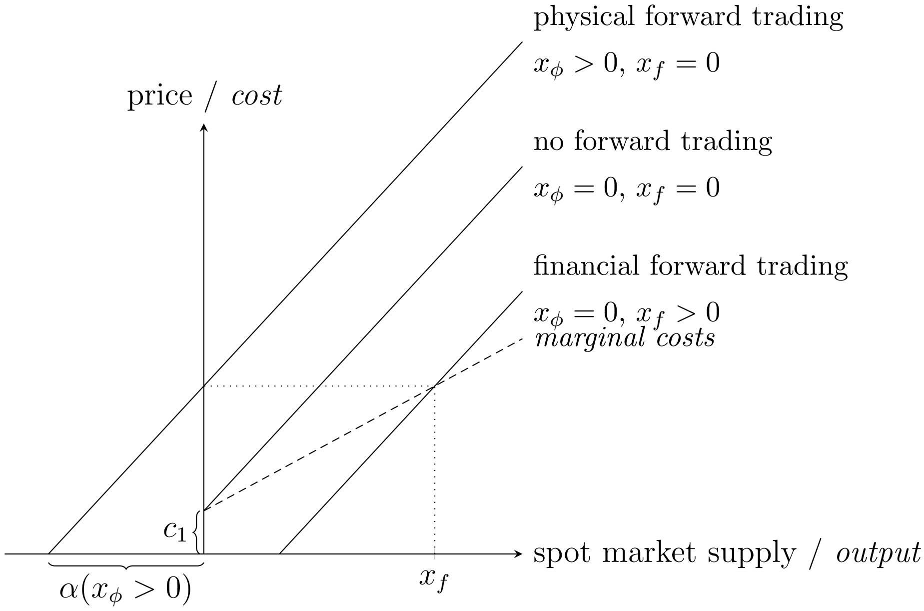

Figure 4: Spot supply of firms with physical vs. financial forward positions

5.3 Non-linear supply functions

The linear SFE model – although widely used – represents a special case where forward sales are strategically neutral and firms have no strategic incentive to engage in the forward market. Empirically, however, it is observed that most producers sell large proportions of their output forward, e.g. in electricity markets. Furthermore, the supply functions in these markets are typically non-linear e.g. due to non-linear marginal costs or limited production capacities. Klemperer and Meyer (1989) show that, if supply functions are non-linear, a multitude of Nash equilibria exists, where supply functions are bound from above by the curve that finally becomes inelastic and hits the Cournot price and quantity for the maximum realisation of demand. A lower bound is given by the supply function that intersects marginal costs at maximum demand. The following paragraphs will argue that the main insight from the previous analysis, summarised in Proposition 1, remains true even for a more general setting with non-linear supply functions. The discussion, however, will remain fairly general without determining firms' spot market strategies in very detail.

Consider a setting equivalent to the one described in Section 2in which the cost can take the form of any continuous, monotonically increasing, convex, and twice differentiable function $C(q)$, and firms i and j can bid any continuous, twice differentiable, monotonically increasing supply function $q_i(p) =, q_j(p) > 0$, respectively. Assume that the demand shock ε has sufficiently wide support such that the firms will optimise their supply function for all possible prices studied in the following. The first order condition for profit maximisation is given in (3). We can rewrite it as

$$q_i - x_i = \left(q_j' - D'\right)\left(p - C'\left(q_i\right)\right).$$

Because $(q_j' - D')$ is strictly positive, we know that in equilibrium i will bid a price equal to marginal costs for $q_i = x_i$ and no other quantity. Take a point (p_0, q_0) in the price-quantity plane such that $p_0 > C(q_0)$. This point will be on the optimal supply curve of i if and only if $q_i > x_i$. Conversely, $q_i < x_i$ implies a price bid of i such that $p < C'(q_i)$. Figure 1 can still serve as an illustration: positive forward sales shift the supply function outwards to hit marginal costs at the contracted quantity. But due to the potential non-linearity, the overall shape of the curve is not specified.

An increase of x_i for a given quantity q_i will reduce the left hand side of the equation given above. For the equation to hold, the right hand side needs to decrease by the same amount, e.g. by a lower price p or by a less elastic residual demand (a change in $(q_j' - D')$. In summary, an increase of forward sales x_i will make firm i bid more aggressively.

So how would this affect the possibility of collusion? The results for the linear case are summarised in Proposition 1. Necessary and sufficient conditions for Proposition 1 are given in Lemmas 1 to 3. The following paragraphs discuss how these Lemmas apply to the non-linear case.

Consider Lemma 1: The profits of firms in the one-shot Nash equilibrium decrease with forward sales because contracting increases competition in the spot market. As argued above, and as already discussed by e.g. Green (1999, pp. 115-116) and Holmberg (2011), the same is true when supply functions are non-linear. Selling forward will make firms bid more aggressively in the spot market, which increases competition and decreases profits. Moreover, Holmberg (2011) shows under fairly general conditions that with non-linear supply functions, forward sales are not strategically neutral anymore. By selling forward, firms can commit to a spot market strategy that alters the spot market strategy of the rival. As a result, both firms will sell forward and both firms will be worse off than they would be without a forward market.

Consider Lemma 2: Collusive profits are unaffected by the possibility of selling forward. This is also true when supply functions are non-linear. The monopoly solution maximises joint profits in the spot market, and the forward market does not bring additional demand. Additional profits from selling forward could only be obtained from a difference between spot and forward prices, which contradicts the assumption of a competitive forward market. Thus, colluding firms optimally implement the joint profit maximising supply function (which can be linear or otherwise) independent of their forward positions.

Consider Lemma 3: A deviating firm earns both at the expense of its rivals and that of the speculators. After the forward price is locked in at the collusive level, the deviator plays its spot market best response $q_i^*(p)$ to the collusive supply function $q_j^c(p)$ of its rival, where $q_i^*(p)$ fulfils the first order condition (3). Implicitly, the best response of i therefore solves the equivalent first order condition

$$\frac{d\pi_i}{dq_i^*(p)} = 0 \quad \text{for all } p.$$

The derivative of profits of deviation with respect to the number of forwards sold by the deviating firm can be decomposed as follows (with f fixed in advance):

$$\frac{d\pi_i^d}{dx_i} = \frac{\partial \pi_i^d}{\partial x_i} + \left(\frac{\partial \pi_i^d}{\partial p_i} \frac{dp}{dq_i(p)} + \frac{\partial \pi_i^d}{\partial q_i(p)} \right) \frac{dq_i^*(p)}{dx_i}.$$

In the decomposition above, the whole term in parentheses multiplyed by $q_i^*(p)/x_i$ equals zero by definition of $q_i^d(p)$ as firm i's best response. This allows us to simplify:

$$\frac{d\pi_i^d}{dx_i} = \frac{\partial \pi_i^d}{\partial x_i} = f - p$$

which is equivalent to the finding in Lemma 3 for the linear case (see Appendix A.3).

Taken together, we find that Lemmas 1 to 3 also hold for the case of non-linear supply functions. These were the necessary and sufficient conditions for Proposition 1. Thus, Proposition 1 holds even for non-linear supply functions in the spot market.

But Holmberg (2011) shows that non-linear supply function equilibria provide endogenous incentives to sell forward. Thus, in contrast to the model in Section 3, the non-linear case gives rise to forward positions that emerge from the firms strategic considerations: Colluding firms will not sell forward as they have nothing to gain. A deviating firm could raise additional profits by selling forward at the collusive price and then deviating in the spot market. But speculators with that suspicion will not buy forward. And the one-shot Nash equilibrium is characterised by positive strategic forward sales of the firms, trying to tilt the supply function of their rival. With this endogenous incentive to sell forward during the punishment phase, the punishment is credibly harsher than it would be without a forward market. Thus, forward markets increase the range of discount factors for which collusion can be sustained.

Proposition 2 is less straight-forward to generalise. In a non-linear SFE, the price-cost margin evolves non-linearly along the supply function and therefore profits are non-linear in the demand shock. Depending on the functional form of collusive, deviation, and Nash profits, increasing uncertainty might potentially stabilise or destabilise a collusive agreement.

6 Conclusion

This paper has shown how forward sales of firms affect the stability of a collusive agreement when firms compete in supply functions in the spot market. Forward markets can facilitate collusion by increasing competition during the punishment phase. On the contrary, a forward market can destabilise collusion when firms find the possibility of selling forward as an insider during collusion. A deviating firm would seek to sell forward at an expected collusive price, and profit at the expense of rivals and speculators by depressing the spot price. For to maintain collusion, firms will not sell forward at all, or carefully control the volumes of forward sales in relation to the overall output of the firms. The other market side is governed by a similar concern: Speculators will refrain from buying forward contracts when they suspect the collusive agreement to be infringed in the subsequent spot market. Either way, a collusive agreement is incompatible with the existence of liquid and anonymous forward markets where a deviating firm could sell large amounts forward undetectedly. In my view, this is an important insight for anti-trust and transparency regulation of oligopolistic commodity markets.

The findings are robust against a number of generalisations. Uncertainty of demand does not necessarily work against collusion. Instead, it has an ambiguous effect such that increasing demand uncertainty reduces the impact of forward positions on the critical discount factor. Moreover, the physical versus financial forward contracts distinction made in the literature (Allaz and Vila, 1993; Ferreira, 2003; Liski and Montero, 2006; Green and Le Coq, 2010) is irrelevant in a setting in which firms bid supply functions in the spot market. Researchers conducting empirical studies, however, should bear in mind that financial forwards can incentivise firms to bid below their marginal costs, while physical forwards can put producers in the buyer-side on the spot market. Both these effects together can yield a large variety of possible bidding functions in empirical examples. Most importantly, the core result of the effect of forward contracts on collusion also holds for a more general case of non-linear supply functions in the spot market.

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A Appendix

A.1 Proof of Lemma 1

Lemma 1.a: We know from the firm' s best response (6) and (5) that the forward position x_i affects the spot market strategy of firm i solely through α_i, and that there is no strategic reaction from the rival firm j to a change that solely affects i's intercept. Moreover, the effect of demand uncertainty on expected profits is additively separable and does not depend on α_i or α_j (see (10) in the paper). Based on a simple envelope argument, we can decompose the derivative of firm i's expected profit with respect to x_i as follows:

$$\frac{d\hat{\pi}_i}{dx_i} = \frac{\partial \pi_{i,\varepsilon=0}}{\partial x_i} + \frac{d\pi_{i,\varepsilon=0}}{d\alpha_i}\frac{d\alpha_i}{dx_i}.$$

In the one-shot Nash equilibrium, firm i chooses its supply function in the spot market optimally, thus $\alpha_i = \alpha_i^*$ and $d\pi_i^n/d\alpha_i^* = 0$, conditional on a predetermined forward price f. Again using a simple envelope argument, the decomposition above becomes

$$\begin{aligned} \frac{d\hat{\pi}_i^n}{dx_i} &= \frac{\partial \pi_{i,\varepsilon=0}^n}{\partial x_i} + \frac{\partial \pi_{i,\varepsilon=0}^n}{\partial f} \cdot \frac{df}{d\alpha_i} \cdot \frac{d\alpha_i^*}{dx_i} \\ &= (f - \bar{p}_n) + x_i \frac{df}{d\alpha_i}\frac{d\alpha_i^*}{dx_i}. \end{aligned} \tag{A.1}$$

The competitive forward market equilibrium implies $f = \mathrm{E}(p) = \bar{p}_n$, thus,

$$\frac{d\hat{\pi}_i^n}{dx_i} = x_i \frac{d\bar{p}_n}{d\alpha_i^*}\frac{d\alpha_i^*}{dx_i}.$$

From (8) we know $d\bar{p}_n/d\alpha_i < 0$ and from (6) $d\alpha_i^*/dx_i > 0$. So $d\hat{\pi}_i^n/dx_i$ is zero for $x_i = 0$ and negative for any $x_i > 0$.

Lemma 1.b: A change in forward sales x_j of i's rival translates into an isolated change of firm j's supply function intercept α_j, which affects the spot market clearing price p. Speculators observe x_j and update their expectations accordingly, which affects the forward price f.

$$\frac{d\hat{\pi}_i^n}{dx_j} = \left(\frac{d\pi_{i,\varepsilon=0}}{df}\frac{df}{d\alpha_j^*}\frac{d\alpha_j^*}{dx_j} + \frac{d\pi_{i,\varepsilon=0}}{d\bar{p}_n}\frac{d\bar{p}_n}{d\alpha_j^*} \right) \frac{d\alpha_j^*}{dx_j}$$

Because $df d\alpha_j^* = d\bar{p}_n d\alpha_j^*$, the expected revenues from i's forward sales $x_i(f - \bar{p}_n)$

are unaffected. The above derivative then becomes

$$\begin{aligned}\frac{d\hat{\pi}_i^n}{dx_j} &= \frac{d}{d\bar{p}_n}\left(\bar{p}_n q_i - C(q_i)\right)\\ &= \left(q_i + q_i'\left(\bar{p}_n - c_1 - c_2 q_i\right)\right)\frac{d\bar{p}_n}{d\alpha_j^*}\frac{d\alpha_j^*}{dx_j}\end{aligned}$$

Consider the first order condition (4) which has to hold for any spot price p and use $q_i = \alpha_i + \beta_i p$ to express the profit margin $(p - c_1 - c_2 q_i)$ as $(q_i - x_i)/(b + \beta_j)$.

$$\frac{d\hat{\pi}_i^n}{dx_j} = \left(q_i + \beta_i \frac{q_i - x_i}{b + \beta_j}\right)\frac{d\bar{p}_n}{d\alpha_j^*}\frac{d\alpha_j^*}{dx_j}$$

For a joint variation $dx_i = dx_j = dx$ and with symmetric spot market strategies β_n and $\alpha_i^* = \alpha_j^*$,

$$\begin{aligned}\frac{d\hat{\pi}_i^n}{dx} &= \left(x + q_i + \beta_n \frac{q_i - x}{b + \beta_n}\right)\frac{d\bar{p}_n}{d\alpha_i}\frac{d\alpha^*}{dx}\\ &= \left(\frac{bx + (b + 2\beta_n)q_i}{b + \beta_n}\right)\frac{-1}{b + 2\beta_n}\frac{\beta_n}{b + \beta_n}\\ &= \frac{-\beta_n}{(b + \beta_n)^2}\left(\frac{b}{b + 2\beta_n}x + q_i\right),\end{aligned}$$

which is negative because the production q_i is always strictly positive and $x \geq 0$, therefore, the profit of i decreases with a joint increase of forwards sales x.

For the second derivative,

$$\frac{d^2\hat{\pi}_i^n}{dx^2} = \frac{-\beta_n}{(b + \beta_n)^2}\left(\frac{dq_i}{dx} + \frac{b}{b + 2\beta_n}\right), \tag{A.2}$$

use the definition of the optimal strategies in (5) and (6) to substite α_i^* and express the best response supply function of i given β_j as:

$$q_i^* = \beta_i^*\left(p - c_1 + \frac{x_i}{b + \beta_j}\right).$$

Then obtain

$$\frac{dq_i^*}{dx} = \beta_i^*\left(\frac{1}{b + \beta_j} + \frac{dp}{d\alpha_i}\frac{d\alpha_i}{dx_i} + \frac{dp}{d\alpha_j}\frac{d\alpha_j}{dx_j}\right).$$

In the symmetric case, substitute $\beta_i = \beta_j = \beta_n$ and $\alpha_j = \alpha_i^*$ into (1) to derive

$$\begin{aligned}\frac{dq_i^*}{dx} &= \beta_n\left(\frac{1}{b + \beta_n} + \frac{-2\beta_n}{(b + \beta_n)(b + 2\beta_n)}\right)\\ &= \frac{b\beta_n}{(b + \beta_n)(b + 2\beta_n)}.\end{aligned}$$

Substituting into (A.2) yields

$$\frac{d^2\hat{\pi}_i^n}{dx^2} = \frac{-\beta_n}{(b+\beta_n)^2}\left(\frac{b\beta_n}{(b+\beta_n)(b+2\beta_n)} + \frac{b}{b+2\beta_n}\right)$$

which is negative due to the negative sign of the first factor whereas $\beta_n, b > 0$.

$\square$

A.2 Proof of Lemma 2

The joint profit maximising supply function is set to implement optimal quantities for any possible spot price, given forward positions x_i, x_j and the forward price f. Following the procedure as for the one-shot Nash case, we find the optimal slope parameter to be the one in (11), whereas the optimal intercept is

$$\alpha_{ij} = \frac{x_i + x_j - bc_1}{2 + c_2 b}, \tag{A.3}$$

which increases with larger forward sales x_i, x_j. So the cartel will find it profitable to expand output after having sold forward. Assume, for notational convenience, that the colluding firms have symmetric forward sales $x_i = x_j = x_c$. Note that from Equation (10), the effect of demand uncertainty σ^2 on expected profits is additively separable and does not depend on x_i, x_j, α_i, or α_j, thus

$$\frac{d\hat{\pi}^c}{dx_c} = \frac{d\pi^c_{\varepsilon=0}}{dx_c}.$$

For a decomposition, note that forward sales can affect profits directly, and through their effect on spot market strategies, which in turn determine prices. Using a simple envelope argument, the derivative of the expected joint profits with respect to x_c can be decomposed as follows

$$\frac{d2\hat{\pi}^c}{dx_c} = \frac{\partial 2\pi^c_{\varepsilon=0}}{\partial x_c} + \left(\frac{\partial 2\pi^c_{\varepsilon=0}}{\partial f}\frac{df}{d\alpha_{ij}} + \frac{d2\pi^c_{\varepsilon=0}}{dp_{\varepsilon=0}}\frac{dp_{\varepsilon=0}}{d\alpha_{ij}}\right)\frac{d\alpha_{ij}}{dx_c},$$

We know that the derivative $\frac{d2\pi^c_{\varepsilon=0}}{dp_{\varepsilon=0}}$ is zero because the cartel chooses its supply functions optimally for any price p given f and x_c. Moreover, the forward market is in equilibrium if and only if $f = \mathrm{E}(p)$, thus $\frac{\partial \pi^c_{\varepsilon=0}}{\partial x_c} = f - p_{\varepsilon=0} = 0$ and $\frac{df}{d\alpha_{ij}} = \frac{dp_{\varepsilon=0}}{d\alpha_{ij}}$. With $\frac{\partial 2\pi^c_{\varepsilon=0}}{\partial f} = 2x_c$, the decomposition above reduces to

$$\frac{d2\hat{\pi}^c}{dx_c} = 2x_c\frac{dp_{\varepsilon=0}}{d\alpha_{ij}}\frac{d\alpha_{ij}}{dx_c}.$$

$\frac{dp_{\varepsilon=0}}{d\alpha_{ij}}$ describes the adjustment of the expected spot price to a change in the symmetric supply functions intercept of the colluding firms. Consider the following two cases which correspond to Lemma 2.a and 2.b.

(a) When firms adjust their supply functions in the spot market optimally to their forward positions by chosing α_{ij} as given in (A.3), we have $\frac{dp_{\varepsilon=0}}{d\alpha_{ij}}\frac{d\alpha_{ij}}{dx_c} \neq 0$. Thus, profit maximisation requires $x_c = 0$.

(b) However, when firms do have a credible commitment mechanism to stick to the supply function defined by β_c and α_c in (11) and (12), this implies $\frac{d2\hat{\pi}^c}{dx_c} = 0$ for any x_c. Thus the cartel is indifferent to selling forward.

□

A.3 Proof of Lemma 3

A firm deviating in the spot market is playing its best response. Following the same argument as before, we can decompose the derivative of firm i's expected profit with respect to its own forward sales as in Equation (A.1):

$$\frac{d\hat{\pi}_i^d}{dx_i} = \frac{\partial \pi_{i,\varepsilon=0}^d}{\partial x_i} + \frac{\partial \pi_{i,\varepsilon=0}^d}{\partial f} \cdot \frac{df}{d\alpha_i} \cdot \frac{d\alpha_i^*}{dx_i}$$

Before deviating in the spot market, firm i has sold forward at the collusive price $f = \bar{p}_c$. Because we assume that deviation is a surprise, the forward price does not reflect the spot market strategy of the deviator, thus $df/d\alpha_i = 0$, and the above decomposition reduces to

$$\frac{d\hat{\pi}_i^d}{dx_i} = \frac{\partial \pi_{i,\varepsilon=0}^d}{\partial x_i} = f - p_{\varepsilon=0} = \bar{p}_c - \bar{p}_d$$

where $\bar{p}_d$ describes the spot price (1) at the expected level of demand ($\varepsilon = 0$) when i deviates as in (15) and j plays the collusive supply function as in (11) and (12). And because the forward price during collusion $f = \bar{p}_c$ is larger than the expected realised spot price when i deviates, the difference above is positive.

Knowing that f is constant in x_i, the second derivative is

$$\frac{d^2\hat{\pi}^d}{dx_i^2} = -\frac{d\bar{p}_d}{d\alpha_i}\frac{d\alpha_i^*}{dx_i}.$$

As $dp/d\alpha_i < 1$ and $d\alpha_i^*/dx_i > 0$, we can conclude that the expected profits of a deviating firm i are convex and increasing with its own forward sales x_i.

With the rival firm j sticking to the collusive supply function, x_j has no effect on j's spot market strategy and consequently no effect on the profit of firm i. □

A.4 Proof of Lemma 4

Consider the definition of expected profits in (10) and its derivative with respect to σ^2

$$\frac{d\hat{\pi}_i}{d\sigma^2} = \frac{\beta_i - \frac{c_2}{2}\beta_i^2}{(b + \beta_i + \beta_j)^2}.$$

The numerator is positive for $0 < \beta_i < 2/c_2$, which is true for the collusive bid as well as for any best response bid (straight-forward to check from definitions (5) and (11)). Thus, we know that the derivative of $\hat{\pi}_i$ with respect to σ^2 is positive in all the relevant cases.

The order of derivatives of collusive, deviation, and one-shot Nash profits can be studied conveniently in terms of the strategy parameters β_i and β_j. Consider the following cross derivative:

$$\begin{aligned}\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta_i} &= \frac{(1-c_2\beta_i)(b+\beta_i+\beta_j)^2 - 2(b+\beta_i+\beta_j)(\beta_i - \frac{c_2}{2}\beta_i^2)}{(b+\beta_i+\beta_j)^4} \\ &= \frac{(b+\beta_j) - \beta_i(1+c_2(b+\beta_j))}{(b+\beta_i+\beta_j)^3}\end{aligned}$$

Note that the latter expression is exactly zero whenever β_i is the best response β_i^*, as given in Equation (5). It is strictly positive when β_i is less than the best response. We know that the deviator plays its best response to the collusive supply function, and that this best response is a more elastic supply curve than the collusive one, $\beta_d = BR(\beta_c) > \beta_c$, therefore

$$\frac{d\hat{\pi}^d}{d\sigma^2} > \frac{d\hat{\pi}^c}{d\sigma^2}.$$

Now, consider another cross derivative, this time for the effect of a joint variation of slopes $\beta_i = \beta_j = \beta$.

$$\begin{aligned}\frac{d^2\hat{\pi}_i}{d\sigma^2 d\beta} &= \frac{(1-c_2\beta)(b+2\beta)^2 - 4(b+2\beta)(\beta - \frac{c_2}{2}\beta^2)}{(b+2\beta)^4} \\ &= \frac{b - \beta(2+c_2 b)}{(b+2\beta)^3}\end{aligned}$$

This derivative is zero for $\beta = \beta_c$ given by Equation (11) and negative for all $\beta > \beta_c$. Firms play symmetric strategies both in collusion and in one-shot Nash equilibrium (the punishment phase), but supply increases less with higher spot prices during collusion than it does in one-shot Nash equilibrium, $\beta_n > \beta_c$. Therefore

$$\frac{d\hat{\pi}^c}{d\sigma^2} > \frac{d\hat{\pi}^n}{d\sigma^2}.$$

□

A.5 Proof of Proposition 2

Proposition 2.a: Section 4 explains how the Basic Proportionality Theorem can be applied, given that expected one period profits of deviation, collusion and one-shot Nash equilibrium as functions of σ^2 have a common point of intersection. The following paragraphs show that this is always the case when there are no forward sales. Consider Equation (2) for the ex-post realised profits of firm i. Without forward sales ($x_i = x_j = x = 0$), this can be rewritten as

$$\pi_{i,x=0} = q_i(p)\left(p - c_1 - \frac{c_2}{2}q_i(p)\right). \tag{A.4}$$

Note that with $x_i = 0$, the supply function intercept α_i reduces to $\alpha_i = -\beta_i c_1$ (equivalently $\alpha_j = \beta_j c_1$), irrespective of firm i being in collusion (12) or playing its best response (6). Thus, i's quantity can be expressed as $q_i = \alpha_i + \beta_i p = \beta_i(p - c_1)$. Substituting into (A.4) gives

$$\pi_{i,x=0} = (\beta_i - \frac{c_2}{2}\beta_i^2)(p - c_1)^2$$

Now consider the spot market clearing price p from Equation (1) and set $\varepsilon = 0$:

$$\begin{aligned} p &= \frac{A - \alpha_i - \alpha_j}{b + \beta_i + \beta_j} \\ &= \frac{A + \varepsilon - (\beta_i + \beta_j)c_1}{b + \beta_i + \beta_j}, \end{aligned}$$

thus,

$$\begin{aligned} p - c_1 &= \frac{A - (\beta_i + \beta_j)c_1}{b + \beta_i + \beta_j} - c_1 \\ &= \frac{A - bc_1}{b + \beta_i + \beta_j}. \end{aligned}$$

Substituting $p - c_1$ into the the expression for $\pi_{i,x=0}$ above yields

$$\pi_{i,x=0} = \left(\frac{A - bc_1}{b + \beta_i + \beta_j}\right)^2 (\beta_i - \frac{c_2}{2}\beta_i^2).$$

Reconsider the expression for expected profits $\hat{\pi}_i$ from Equation (10), and replace $\pi_{i,\varepsilon=0}$ with $\pi_{i,x=0}$ as defined above.

$$\begin{aligned} \hat{\pi}_{i,x_i=0} &= \pi_{i,x=0} + \sigma^2\left(\frac{\beta_i - \frac{c_2}{2}\beta_i^2}{(b + \beta_i + \beta_j)^2}\right) \\ &= \frac{\beta_i - \frac{c_2}{2}\beta_i^2}{(b + \beta_i + \beta_j)^2}\left((A - bc_1)^2 + \sigma^2\right) \end{aligned}$$

It is obvious that $\hat{\pi}_{i,x=0}$ has a null for $\sigma^2 = -(A - bc_1)^2$, no matter what strategies β_i, β_j the firms play. Therefore, without forward trading, $\hat{\pi}^d$, $\hat{\pi}^c$ and $\hat{\pi}^n$ have a common point of intersection in the plane spanning across $\hat{\pi}_i$ and σ^2. So the intercept theorem applies and the fraction $\left(\hat{\pi}^d - \hat{\pi}^c\right) / \left(\hat{\pi}^d - \hat{\pi}^n\right)$ is constant with respect to σ^2 which is the statement in Proposition 2.a.

Proposition 2.b: Now, suppose the deviating firm has sold forward a quantity $x_d > 0$. By Lemma 3, its ex-post profits will be higher than those without forward contracting. The critical discount factor will also be higher. But the derivative of profits with respect to σ^2 is unchanged (see Eq. 10, the multiplyer of σ^2 does not depend on x_i or α_i). Therefore, the line depicting the expected profits of deviation $\hat{\pi}^d$ over σ^2 is shifted upwards as shown in Figure 3 by a line through point E. The lines for collusive and one-shot Nash profits keep their intersection at $\sigma^2 = -(A - bc_1)^2$, but – due to the ordering of derivatives – the expected profits of deviation $\hat{\pi}^d$ intersect the punishment profits $\hat{\pi}^n$ at some level of σ^2 which is larger (less negative) compared to the intersection of $\hat{\pi}^d$ with collusive profits $\hat{\pi}^c$. Now, with distinct points of intersections, the one for deviation and collusive profits being more negative than the one for deviation and punishment profits, we know that $\hat{\pi}^d - \hat{\pi}^n$ increases in greater proportion with σ^2 than does $\hat{\pi}^d - \hat{\pi}^c$ for any $\sigma^2 \geq 0$. This implies that the critical discount factor $\overline{\delta}$ now decreases with σ^2.

$$(x_d, x_n) \text{ such that } \overline{\delta} > \overline{\delta}_0 \quad \Leftrightarrow \quad \frac{d\overline{\delta}}{d\sigma^2} < 0$$

Conversely, consider the case when there are no forward sales during collusion, but the firms expect positive forward sales in the punishment phase: $x_d = 0$ and $x_n > 0$. By Lemma 1, the profits of Nash-reversion π^n are lower compared to the situation without forwards. The corresponding line in Figure 3 shifts downwards. A reverse argument to the one before applies. Now, $\hat{\pi}^d - \hat{\pi}^c$ will increase in greater proportion with σ^2 compared to $\hat{\pi}^d - \hat{\pi}^n$.

$$(x_d, x_n) \text{ such that } \overline{\delta} < \overline{\delta}_0 \quad \Leftrightarrow \quad \frac{d\overline{\delta}}{d\sigma^2} > 0$$

So it is evident, that there are infinitely many combinations of x_d and x_n which shift deviation profits upwards and punishment profits downwards such that the three lines still intersect at a common point (see Corollary 1). This common point of intersection moves along the unaffected line of collusive profits towards more negative values of σ^2. Again from the basic proportionality theorem, we know that for a given positive level of σ^2, a shift of $\hat{\pi}^d$ upwards and $\hat{\pi}^n$ downwards such that all three lines still intersect in a common point will keep the ratio $(\hat{\pi}^d - \hat{\pi}^c)/(\hat{\pi}^d - \hat{\pi}^n)$ constant. By simply taking $x_i = x_j = 0$ we are back to the case without forward contracts as in Proposition 2.a. Therefore,

$$(x_d, x_n) \text{ such that } \overline{\delta} = \overline{\delta}_0 \quad \Leftrightarrow \quad \frac{d\overline{\delta}}{d\sigma^2} = 0.$$

□

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