

Discussion Paper No. 14-092

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Computable General Equilibrium Models
Using Optimal Fingerprint
Detection Methods**

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Specifying Parameters in Computable General Equilibrium Models using Optimal Fingerprint Detection Methods

Simon Koesler*

October 23, 2014

Abstract

The specification of parameters is a crucial task in the development of economic models. The objective of this paper is to improve the standard parameter specification of computable general equilibrium (CGE) models. On that account, we illustrate how Optimal Fingerprint Detection Methods (OFDM) can be used to identify appropriate values for various parameters. This method originates from climate science and combines a simple model validation exercise with a structured sensitivity analysis. The new approach has three main benefits: 1) It uses a structured optimisation procedure and does not revert to ad-hoc model improvements. 2) It allows to account for uncertainty in parameter estimates by using information on the distribution of parameter estimates from the literature. 3) It can be applied for the specification of a range of parameters required in CGE models, for example for the definition of elasticities or productivity growth rates.

Keywords: CGE modelling, sensitivity analysis, model validation
parameter specification, substitution elasticities

JEL-Classification: C52, C68, D58

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1 Introduction

The development of computable general equilibrium (CGE) models requires many assumptions regarding their theoretical setup (e.g. the underlying factor market specification) as well as the definition of required parameters (e.g. the specification of substitution elasticities). While, without doubt, both elements of model design are important and require utmost accuracy to avoid false model results, in this paper we discuss the amelioration of the process of parameter specification and present an alternative approach that can be used to parameterise CGE models.

For the specification of parameters, modelers normally make use of calibration techniques (c.f. Dawkins et al., 2001) or build on estimates from the literature. These approaches entail some important limitations. Standard benchmark calibration for instance does not account for fluctuations over time and is thus prone to errors when special events or situations in the benchmark year are not specifically taken into account. Picture for example building on a biased economic structure because of an inflated tourism and construction sector in a year where the Olympic Games take place. Applying estimates from the literature is also not straightforward. If for instance parameters are not specifically estimated for the implementation in models or at least studied on the basis of the same underlying theoretical structure, conceptual and definitional mismatches may lead to the misspecification of parameter values (Browning et al., 1999). McKittrick (1998) illustrates the issue for the case of substitution elasticities. What is more, simply using values from the literature neglects that in most cases the information originates from estimation procedures and must thus be associated with some degree of uncertainty. While for instance most of the substitution elasticities estimated by Koesler and Schymura (forthcoming) feature small standard deviations, some estimates imply an important amount of variability of which modelers should be aware of. All too often, modelers are also confronted with a situation in which no estimates or data is available for the definition of required parameters. In this case, they have to build on their experience and intuition and have only few options to truly evaluate their model specification. This leads directly to the critique of McKittrick (1998) that CGE models lack empirical foundations.

These difficulties motivate the main objective of this paper which is to improve the parameter specification of CGE models. In the following we illustrate how Optimal Fingerprint Detection Methods - an approach originally used in climate science (e.g. IPCC, 2007; Forest et al., 2000, 2001) - can inspire the identification of appropriate parameter values for CGE models. This method builds on a generalised multivariate regression analysis and combines a simple model validation exercise with a structured sensitivity analysis. Compared to other procedures, the new approach has three main benefits: 1) It uses a structured optimisation procedure and does not revert to ad-hoc model improvements. 2) It allows to account for uncertainty in parameter estimates by using information on the distribution of parameter estimates from the literature. 3) It can be applied for the specification of various parameters required in CGE models, for example for the definition of elasticities or productivity growth rates.

The paper is structured as follows. First, we briefly provide a background on Optimal Fingerprint Detection Methods, henceforth referred to as OFDM. Second, we demonstrate how OFDM can be applied in the context of CGE modelling and explore its capabilities in a CGE framework using a stylised small scale CGE model. Subsequently we apply the new approach to the still simple but full-fledged Basic WIOD CGE Model developed by Koesler and Pothen (2013) and derive a set of substitution elasticities for the specification of production in the model. Finally, we summarise and conclude in the last section.

2 Optimal Fingerprint Detection Methods (OFDM)

OFDM originate from climate science (e.g. Hasselmann, 1997; Allen and Tett, 1999). Above all, they are used to detect climate change and to identify climate change drivers (c.f. IPCC, 2007). On that account, a multivariate regression analysis is set up which generally has the following form:

$$\mathbf{Y} = \mathbf{X}\mathbf{a} + \mathbf{e}, \quad (1)$$

where the vector \mathbf{Y} includes data from observations (i.e. the climate record), \mathbf{X} is a vector of (expected) response patterns which determine the climate system in the model, \mathbf{a} is a vector of scaling factors which are used to adjust the response patterns so that the simulation outcomes correspond the observational data and \mathbf{e} is a vector with error terms that is to be minimised. The underlying logic is thereby that if the estimated response patterns in vector \mathbf{X} is capable of replicating real world observations under normal circumstances, that is in a situation with no climate change, then if the elements in the vector \mathbf{a} do not equal one when trying to replicate the current climate, there is some disturbance of the climate system. Deviations can then potentially be attributed to climate change.

OFDM have however also been applied to specify parameters in models simulating the climate. To that end, Forest et al. (2000, 2001) use a multivariate regression analysis as described in Equation 1, but with the difference that in their work the vector \mathbf{X} includes simulation results instead of expected response patterns. This regression setup relates the climate record one to one to the climate model output (e.g. observed temperature to simulated temperature) and allows for a structured validation of model results with observed data. Also in this context, the underlying logic of the OFDM approach is straightforward. As long as not all elements in the scaling vector \mathbf{a} are equal to one, the model is not perfectly capable of replicating the observed data and thus needs to be refined.

To judge the overall performance of the model when contrasting its output to observed data, Forest et al. (2000, 2001) use a goodness-of-fit criterion r^2 which builds on the difference between actual observations and model results without scaling. That is $\tilde{\mathbf{u}} = \mathbf{Y} - \mathbf{X}\mathbf{a}$ with \mathbf{a} being a unity vector ($a_i = 1\forall i$). The error $\tilde{\mathbf{u}}$ captures element

that are not taken into account by the model (e.g. internal climate variability) as well as deviations that occur because of a non-perfect model specification. While the first type of variability is intrinsic in any modeling approach - after all models are always a simplification of the real world - it is the latter that the method eventually seeks to minimise. The criterion r^2 itself is defined as:

$$r^2 = \tilde{\mathbf{u}}^T \mathbf{COV}^{-1} \tilde{\mathbf{u}}, \quad (2)$$

where \mathbf{COV} is the error covariance matrix, which - as we illustrate in the next section - can be estimated using model control runs. The aim of the modeler must then be to minimise r^2 , respectively the deviations resulting from any model misspecification. This can be done by means of a sensitivity analysis implementing different parameter specifications and reevaluating each model setup using the goodness-of-fit criteria. The most apt parameter specification will then be the one which provides the lowest r^2 .

3 Illustrative application of OFDM to CGE framework

3.1 Process

As indicated in the previous section, OFDM consists basically of a validation exercise combined with a sensitivity analysis. The process of using OFDM to find adequate parameter specifications for a CGE model involves three main steps.

To begin with, modelers have to choose a set of parameters for which they require guidance regarding their specification and must create a portfolio of different specifications that should be tested. While in principle any parameter value can be evaluated using OFDM, the choice of possible values can be guided in particular by available estimates from the literature. In this case, it is recommendable to build the portfolio of different parameter values not just using the actual estimates, but in addition any available information of the distribution of the parameter value (i.e. information on standard deviations and other higher moments). This allows to implement a more informed sensitivity analysis similar to the structured sensitivity analyses described by Harrison and Vinod (1992) or Hermeling et al. (2013) later in the process. This brings the additional benefit of being able to account for the uncertainty attached to parameter estimates when setting up the model.

The next step implements a validation exercise and investigates whether the CGE model with a specific parameter setup from the portfolio developed in the previous step is capable of replicating an observed record. Thereby a validation procedure as described by Kehoe et al. (1995) and Kehoe (2005) is applied which contrasts historical developments to model predictions. Although instead of using correlation and deviation coefficients to judge the fit of the model output, here we use the goodness-of-fit criteria presented in Equation 2. The procedure requires information on key economic indicators at two points in time and knowledge of changes in variables exogenous to the model that

have taken place in the meantime.¹ The model is then calibrated to the earlier point in time and equipped with the parameter setup that is to be tested. Subsequently, to generate a set of predicted changes, the model is shocked with all observed changes in exogenous variables. Finally the simulation result is compared to the observations from the second point in time on the basis of the OFDM criteria from Equation 2. The resulting value of r^2 provides a first indication of the quality of the parameter setup under investigation.

The third and final step can be referred to as the sensitivity analysis part of the OFDM. Basically, it involves repeating the previous step for all parameter specifications that are to be tested and to compare the respective r^2 values. The parameters combination featuring the lowest r^2 and thus providing the best fit to the observed data without having to scale the model output, can then be considered the most adequate parameter specification.

3.2 Stylised CGE model

Before applying the OFDM method to a full scale CGE model, we demonstrate the process and capabilities of OFDM in a CGE setting by making use of a small stylised CGE model. The model is deliberately simple and features only one region, one final demand agent and two production sectors. The model covers the time period t to $t + n$ with $n \geq 1$. Agents are assumed behave myopic and do not link different periods through saving or investment. In accordance to this, in the following, we drop the time subscript when describing agents behaviour.

The final demand agent supplies capital K and labour L and consumes two different commodities A and B . Its consumption function is characterised by a constant elasticity of substitution (CES) function of the form:

$$C = (\alpha_C A^{\rho_C} + (1 - \alpha_C) B^{\rho_C})^{\frac{1}{\rho_C}} \quad (3)$$

where ρ_C is the substitution parameter of final consumption which relates to the elasticity of substitution for final consumption through $\rho_C = \frac{\sigma_C - 1}{\sigma_C}$ and α_C the input share of final consumption goods. The factor endowments grow at the constant rate γ every period.

In addition, there are two sectors A and B that produce commodities A and B on the basis of two CES production functions:

$$A = \left(\alpha_{KLM}^A (\alpha_{KL}^A K^{\rho_{KL}} + (1 - \alpha_{KL}^A) L^{\rho_{KL}})^{\frac{\rho_{KLM}}{\rho_{KL}}} + (1 - \alpha_{KLM}^A) (\alpha_M^A A^{\rho_M} + (1 - \alpha_M^A) B^{\rho_M})^{\frac{\rho_{KLM}}{\rho_M}} \right)^{\frac{1}{\rho_{KLM}}} \quad (4)$$

¹While in general it is fairly easy to have access to data describing two points in time, it is difficult to account for all changes that have taken place in between. We are aware of this problem, which we believe is intrinsic in any validation exercises, and discuss this issue in more detail in Section 3.5.

Table 1: Structure of generic model economy in period t and $t+1$

Period t	Sector A	Sector B	Period $t+1$	Sector A	Sector B
Input A	30.00	10.00	Input A	30.48	10.16
Input B	10.00	30.00	Input B	10.37	31.11
Capital	25.00	75.00	Capital	26.25	78.75
Labor	75.00	25.00	Labor	75.00	25.00
Output	140.00	140.00	Output	142.10	145.02
Final Demand	100.00	100.00	Final Demand	101.46	103.54

and

$$\begin{aligned}
 B = & \left(\alpha_{KLM}^B (\alpha_{KL}^B K^{\rho_{KL}} + (1 - \alpha_{KL}^B) L^{\rho_{KL}})^{\frac{\rho_{KLM}}{\rho_{KL}}} \right. \\
 & \left. + (1 - \alpha_{KLM}^B) (\alpha_M^B A^{\rho_M} + (1 - \alpha_M^B) B^{\rho_M})^{\frac{\rho_{KLM}}{\rho_M}} \right)^{\frac{1}{\rho_{KLM}}}, \tag{5}
 \end{aligned}$$

where again ρ are the substitution parameters and α the input share parameters for the different production nests.

For the sake of being able to assess the potential of OFDM, we assume that the 'true' setup of the model involves all substitution elasticities being equal to one ($\sigma_C = \sigma_{KL} = \sigma_M = \sigma_{KLM} = 1$), the capital growth rate γ_K is 5%, and there is no change in the endowments of labour ($\gamma_L = 0$). However, to make the case for the need of an approach to find an adequate parameter specification, we also assume that the true values of the elasticities and the endowment growth rates are initially unknown. The objective of the OFDM is then to identify the 'true' parameter values. The input share parameters α are calibrated to the overall structure of the economy which is given in Table 1 for the periods t and $t+1$. The data for $t+1$ has been generated by running the model featuring the aforementioned 'true' parameter specification for one period.

Besides illustrating how OFDM can be applied to the CGE framework, the stylised CGE model allows to explore its potential in a general equilibrium setting and how it is best applied in this context. On that note, we seek to answer three main questions: 1) Is OFDM successful in identifying an apt parameter specification and for what parameters can it be applied? 2) What output variables should be included in the computation if the goodness-of-fit criterion? 3) What type of shocks can be used in the validation process necessary for OFDM?

3.3 Computation of covariance matrix

As becomes clear from Equation 2, OFDM requires knowledge of the interrelationship between model output variables or more formally the covariance matrix **COV**. Ideally **COV** would emerge from actual observations, but given the artificial nature of the stylised model used in this section this is obviously no option. As a matter of fact,

deriving \mathbf{COV} is also a problem that climate scientist face when applying OFDM. The size of their models does not allow inferring \mathbf{COV} from the relatively short available climate record as there are not enough degrees of freedom available and the record might be affected by external forcings which would lead to a bias (IPCC, 2007). The first issue is also a problem when applying OFDM in the context of CGE modelling, as also here there rarely exist appropriate time series data that could be used. The problem can however be overcome by using 'pseudo-observations' generated by control runs of the model (Allen and Tett, 1999). The underlying idea is thereby to use the model itself and a series of simulations to generate a data set that mimics the missing actual observations.

We apply this approach and generate a series of pseudo observations in the form of an artificial time series by solving the stylised CGE model described above with the 'true' parameter setting for the period $t = 0$ to $t = 150$.² Note, although we make use of a change in factor endowments by applying the growth rate γ for different points in time, the pseudo observations could in principle also be generated by using a change in any other exogenous variable as a shock. Subsequently, to break the direct relationship between the reported variables and to overcome the deterministic nature of the data generated by the model, we multiply all reported variables with a parameter which follows a normal distribution of the form $\mathcal{N}(1, 0.01)$. Finally, \mathbf{COV} can be estimated by:

$$\widehat{\mathbf{COV}} = \frac{1}{n} \mathbf{Y}_{PO} \mathbf{Y}_{PO}^T, \quad (6)$$

where n is the number of observation vectors (here 151) and \mathbf{Y}_{PO} a matrix including all observation vectors derived from the generated pseudo observations. The choice of variables that is included in the observation vectors depends on the variable that will be used in the actual OFDM process. Exploring which these should be in order to have optimal results of the OFDM is one of the objectives of the next section. For the illustrative example of this section, we eventually use all input variables, sectoral output and total final demand.

3.4 Potential and best practice of OFDM in CGE context

3.4.1 Type of parameters that can be specified using OFDM

In CGE models there exists multitude of different parameters that need to be specified. This includes in particular elasticities, input shares and growth rates. For the objective of this paper we focus on substitution elasticities and the growth rates of productivity or endowments. However, in principle, the approach could also be applied to other required parameters.

To explore if applying the OFDM to the stylised model reveals the 'true' underlying substitution elasticities and growth rates we first generate a portfolio of different param-

²We demonstrate later in the paper that the pseudo observations could also be generated using a model with other parameter settings.

eter setups. For this we choose for each elasticity of substitution $(\sigma_C, \sigma_{KL}, \sigma_M, \sigma_{KLM})$ 250 different values on the basis of a normal distribution of the form $\mathcal{N}(1, 0.5)$. If this process provides negative values for any of the elasticities, we repeat the draw and eventually implement a truncated normal distribution. Analogous to this but assuming a distribution of the form $\mathcal{N}(0.05, 0.05)$, we also determine 250 different values for the growth rates of capital γ_K and labour γ_L . Here negative values are not discarded. Besides using these stochastic process to generate parameter values, we also include the 'true' parameter values presented above in the portfolio. Subsequently the model is run several times and for each simulation one or two different parameter setting from the parameter portfolio is applied. Parameters values that are not iterated remain at their 'true' values. For each run the model output is then contrasted to the 'observations' in time $t + 1$ presented in Table 1 and we compute the goodness-of-fit criterion r^2 of the OFDM. Thereby and for the time being, we focus on the model predictions for factor input, intermediate input, sectoral output and overall final demand. If the OFDM approach works, then r^2 should be minimal - or even zero - for all model runs that apply a parameter setup close to the 'true' parameter values of the stylised model.

Figure 1 presents the r^2 values for different model runs. Each dot represents a different parameter specification. The axes depict the parameter values and the color of the dots indicate the value of r^2 . Green dots translate into low levels of r^2 and red dots to high levels of r^2 . As becomes clear from all graphs and also when exploring the underlying numerical values, there is only one parameter specification with a minimal r^2 . Moreover, the parameter specification with the minimal r^2 - in this illustrative the situation where $r^2 = 0$ - corresponds to the 'true' parameter values of the stylised model. Therefore we can conclude that the OFDM is capable of identifying the (here by definition) most apt parameter setup for all substitution elasticities and growth rates. Another important insight from all graphs in Figure 1 is that as the tested parameter values approach their 'true' value, deviations of model results and observed data become smaller and r^2 decreases.³ This suggests that even in a situation where the number of parameter setups that can be tested is limited (e.g. because of long solving times) OFDM is useful, because even then it can give guidance in what direction parameters should be adjusted.

The combination of parameters that are iterated in the model runs is of no importance for the accuracy of the OFDM procedure. As becomes clear from Graphs 1d and 1e, OFDM is in both cases capable of identifying the 'true' parameter value of the growth rate of capital. This holds regardless of whether the capital growth rate is tested jointly with a substitution elasticity or the labour growth rate.

While all other graphs in Figure 1 have been generated using the 'true' parameter specification described in the previous section, Graph 1f emerges from model setup

³At first sight, this may not be the case in Graph 1d. Note however that this is due to the fact that as closer the growth rate of capital is to zero, the less important is the level of the substitution elasticity σ_C and thus potentially any value of σ_C provides the same result.

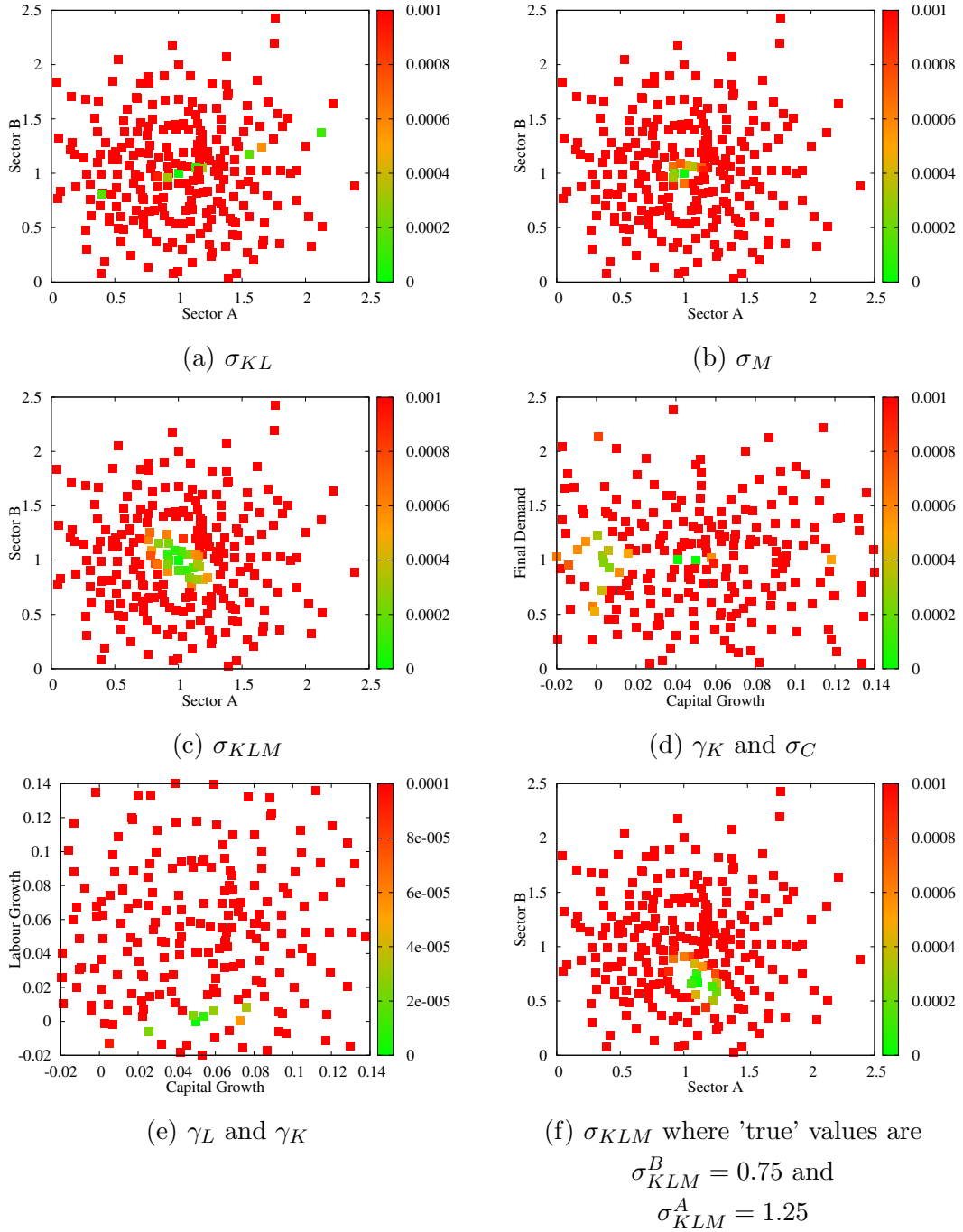


Figure 1: Goodness-of-fit criteria of OFDM for different parameters [axes give parameter values, color gives value of r^2]

where the 'true' parameter values for the substitutability between intermediates and value added (the capital-labour-composite) is no longer $\sigma_{KLM}^A = \sigma_{KLM}^B = 1$ but is set to be $\sigma_{KLM}^A = 1.25$ and $\sigma_{KLM}^B = 0.75$. Since we use the same covariance matrix as before, this also implies that here **COV** has been derived from pseudo-observations which have been generated using an 'incorrectly' specified model. What is more, in this particular OFDM process, all substitution elasticities that are not tested have been set so that they deliberately not match their 'true' value, that is for this analysis we specify the model such that ($\sigma_C = \sigma_M = \sigma_{KLM} = 0.9$). As OFDM is also in this case capable of giving an identifying of the true parameter values, this allows to reveal three important capabilities of OFDM. First, the approach is not limited to a situation where where all elasticities are equal to one. Second, OFDM also works in a setting where not just the tested parameters are unknown and potentially not correctly specified. Third, the method is not affected by the (mis)specification of parameter values in the model that is used to generate the pseudo-observations required for the estimation of the **COV**. However, it must be noted that in this case the precision of the process is reduced. The lowest r^2 is achieved for $\sigma_{KLM}^A = 1.11$ and $\sigma_{KLM}^B = 0.67$, so the 'true' values are slightly missed.

3.4.2 Choice of output variables included in goodness-of-fit criteria

The computation of the goodness-of-fit measure r^2 and therefore also the covariance matrix $\widehat{\mathbf{COV}}$ requires choosing a set of relevant output variables. CGE models generally provide a wide range of simulation results, including information on prices, output levels, trade activities, factor use, employment, environmental indicators. In addition, potentially all data is available on a sectoral and/or regional level therewith increasing the number of output variables. This raises the question, which of the output variables are crucial and should be used in the OFDM process. While at first sight it seems tempting to include all available variables, it soon becomes clear that even for small models this involves processing a large amount of data. Especially for the computation of the covariance matrix including a large number of output variables is problematic as it requires to increase the amount of observations accordingly in order to ensure that enough degrees of freedom are available. The issue is aggravated by the use of pseudo-observations when deriving the covariance matrix. In a general equilibrium context, many of the output variables feature linear relationships, thus feature a high correlation, and therefore make it impossible to compute the inverse of the covariance matrix required by Equation 2. For this reason for example total factor input and total final demand cannot be used simultaneously in the computation of r^2 .

But if more is not better, what is the least amount of variables that should be considered? Figure 2 provides the results of an analysis of σ_M , whereas once more the OFDM procedure has been applied to the stylised model with the original 'true' values of $\sigma_C = \sigma_{KL} = \sigma_M = \sigma_{KLM} = 1$, $\gamma_K = 0.05$ and $\gamma_L = 0$, but with the difference that here various output variables are used to compute $\widehat{\mathbf{COV}}$ and r^2 . For Graph 2a

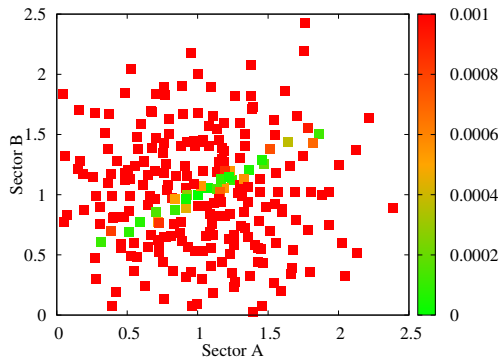
only intermediate inputs into the production of good A and B are considered. Although $\sigma_M = 1$ is part of the parameter sets that can be deemed to provide a good fit, the 'true' value can not be identified as the only apt specification. But if in addition to the intermediate inputs also factor inputs to the two sectors are considered, as it is in Graph 2b, the 'true' value of σ_M is revealed unambiguously. Then again, using only output variables in the OFDM process that are not directly related to the tested parameter, such as for example sectoral output and total final demand in Graph 2c, makes it impossible to find an adequate parameter specification. However, as becomes clear from Graph 2d, adding these variables to the analysis using all input variables does not affect the good result of the OFDM process. This allows us to reason that in order to ensure that OFDM works well, at least the directly affected variables should be included in the process and more variables do not harm the process - as long as the number of variables is still tractable and variables are not a linear combination of each other. In accordance to this and if not stated otherwise, we use for the OFDM applications in this paper all input variables, sectoral output and total final demand to compute r^2 and $\widehat{\text{COV}}$.

3.4.3 Type of shocks that can be used for OFDM

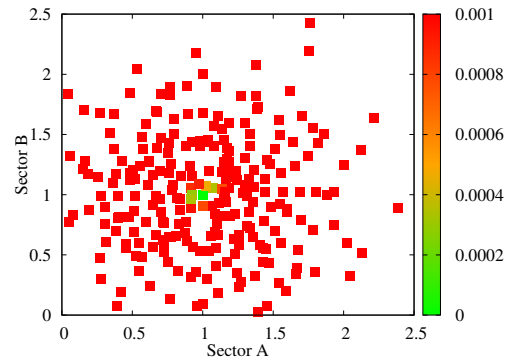
For a real world application, the validation step in the OFDM procedure will eventually require keeping track of various types of changes and using these in the replication attempt. This implies that OFDM must be able to identify the 'true' underlying parameter values independent of the type of shock that is applied. To explore this issue, we run yet another series of OFDM procedures and seek to identify the 'true' value of σ_{KLM} for sector A and B, but this time use three different types of shocks. Figure 3 presents the corresponding results. The first shock used for Graph 3a is an increase in the available endowments, which is the type of shock that we have used so far in our deliberations. Note that this type of shock corresponds to a change in factor productivity. For the second Graph 3b we apply a tax on output of sector A and for the third Graph 3c we consider a tax on capital inputs in sector A. Thereby it can be expected that the effect of taxes will be similar to that of tariffs, although due to the limited scope of our single region model we cannot undertake a true analysis of this here. As can be seen from all graphs, OFDM always succeeds in identifying the 'true' value of σ_{KLM} . Thus we can conclude that OFDM appears to work with a variety of different shocks. It must be noted however, that the shock that is applied must have a certain magnitude to allow OFDM to work reliably. In our stylised example for instance, the results become blurry if a tax of 5% or less is applied.

3.5 Discussion

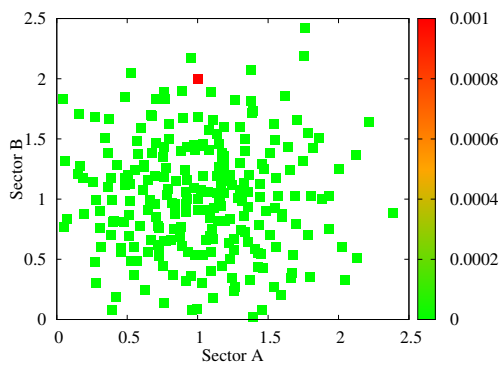
Limits of validation OFDM builds on a series of validation exercises. Therefore its results strongly depend on the availability of data for two different points in time and information on the exogenous shocks that moved the economy from one state to the other. In particular the latter is generally hard to come by, because at any



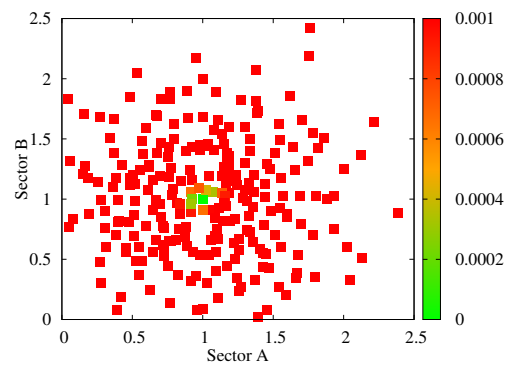
(a) only intermediate input variables



(b) factor and intermediate input variables

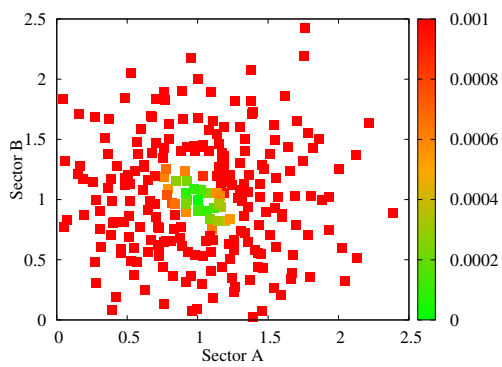


(c) sectoral output and final demand variables

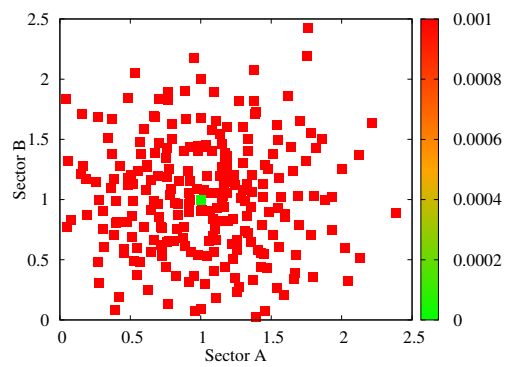


(d) factor and intermediate input, sectoral output and final demand variables

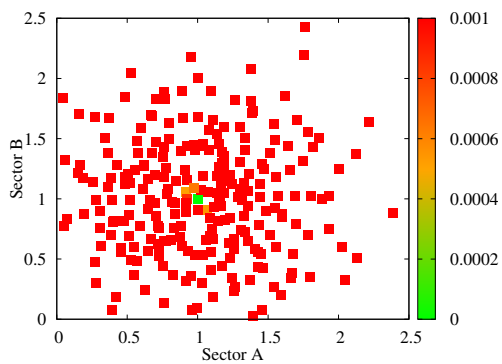
Figure 2: Goodness-of-fit criteria of OFDM for σ_M using different output variables for the computation of r^2 and $\widehat{\mathbf{COV}}$ [axes give parameter values, color gives value of r^2]



(a) increase of capital stock



(b) output tax in sector A



(c) tax on capital input in sector A

Figure 3: Goodness-of-fit criteria of OFDM for σ_{KLM} using different shocks in the process [axes give parameter values, color gives value of r^2]

moment in time there exists a multitude of different shocks that influence the economy and it is clearly impossible to account for all of them. This however implies that any validation exercise will always miss a potentially important element of change and will as a consequence attribute the adjustment of the system to a different (but accounted) channel. Ultimately this will also affect the capabilities of the OFDM method. Research can however confine the problem by limiting the number of relevant changes that are not accounted for. For this purpose using comprehensive datasets such as the World Input-Output Database (WIOD, Timmer et al. (2012); Dietzenbacher et al. (2013)) offer an opportunity to modelers. WIOD offers a rich and consistent representation of most important economies and their trade linkages in the form a time series. This allows to infer many changes that have taken place over time, for example with respect to changes in endowments, taxes and tariffs, trade structure, interregional and intertemporal saving and borrowing, and to take them into account when validating models.

Multiple adequate parameter setups In a general equilibrium model, the value of some parameters may influence the level or importance of other parameters. To be able to judge the flexibility of production for example, it is necessary to consider all substitution elasticities related to the process. If the substitutability between intermediate and value added is very low, then the elasticity of substitution between capital and labour may become less important when assessing a shock in factor supply and could in an extreme case take any value without affecting the model outcome. Potentially this may lead to a situation where there exists more than one 'true' parameter setup. For the OFDM process, this would imply that there is not just one minimal r^2 , but many locally minimal r^2 . Although such a situation has never occurred when using OFDM in our small stylised model framework, it is in principle a possible outcome of OFDM. In this case either parameter setup seems equally valid, as from a modelling perspective they all minimise the deviation of model output to observations. To be able to judge whether there are multiple adequate parameter setups, modellers should always consider in their parameter portfolio a sufficiently large range of values and ideally include all possible parameter values in the OFDM process. Taking a broader perspective with regard to possible parameter values also helps to prevent identifying a locally minimal r^2 as the optimal value by mistake.

Inaccuracy of OFDM if parameters are misspecified As illustrated when discussing Graph 1f, OFDM appears become imprecise if some of the model parameters are misspecified when seeking an adequate specification for another set of parameters. Unfortunately, due to the lack of information on adequate parameter values criticised in the outset of this paper, in any real world application this will most likely be an issue for most applications of OFDM. The problem can however be overcome by including all parameters of which modelers are unsure of in the OFDM procedure. While this may require to enlarge the portfolio of parameter setups that is to be tested and thus will be more demanding from a computational perspective, it increases the degree of freedom

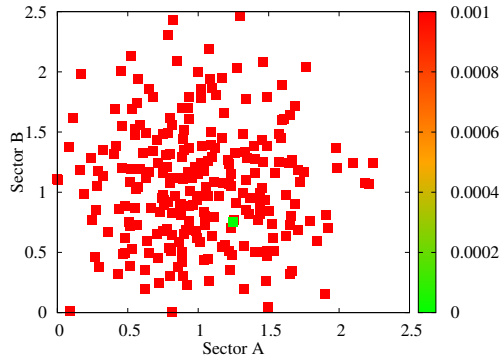


Figure 4: Goodness-of-fit criteria of OFDM for σ_{KLM} which includes all other elasticities in the analysis [axes give parameter values, color gives value of r^2]

and therefore the likelihood of applying the 'true' parameter setup in one of the model runs. This in turn will allow to find a model setup with truly minimal deviations and thus most adequate parameter values. Figure 4 demonstrates the functioning of this comprehensive approach. Here, the setup is similar to the OFDM process used to generate Graph 1f, but instead of applying the false parameter specifications, we include all elasticities in the process. Eventually and in contrast to the earlier attempted, the 'true' values of $\sigma_{KLM}^A = 1.25$, $\sigma_{KLM}^B = 0.75$ and although not pictured $\sigma_C = \sigma_{KL} = \sigma_M = 1$ are identified without any inaccuracy.

Optimisation vs. sensitivity analysis Instead of using a structured sensitivity analysis with different parameter setups to identify the most apt parameter setup, researchers could also apply an optimisation process which minimises the goodness-of-fit criterion r^2 to derive a suitable parameter specification. Such an idea would follow an approach presented by Liu et al. (2004) in a paper seeking to find a set of optimum Armington elasticities. Compared to a self-contained optimisation, a sensitivity analysis has two main advantages. First, it does not require to solve a complex system of equations and can be expected to be much less computationally demanding. Second, it allows for an easy and straightforward implementation of additional information on potentially good parameter values that has been supplied for example by estimates from the literature. However, using the goodness-of-fit criteria from OFDM in an optimisation approach and contrasting the results to a standard OFDM procedure is an interesting question for future research.

4 Application of OFDM to Basic WIOD CGE model

After having presented and illustrated OFDM on the basis of a small stylised CGE model, in this section we apply the method to a full scale CGE model. On that account, we seek to identify adequate substitution elasticities for the specification of production (σ_{KL} , σ_{KLE} and σ_{KLEM}) for the Basic WIOD CGE Model. This model is a static,

multi-region, multi-sector CGE model. With regard to the basic economic structure, it builds on the comprehensive World Input-Output Database (WIOD, Timmer et al. (2012); Dietzenbacher et al. (2013) which will be an advantage for the validation part of OFDM.⁴ Details on the Basic WIOD CGE Model are provided in the Appendix and in Koesler and Pothen (2013).

Most importantly for our analysis, the model distinguishes between three groups of commodities: energy commodities $Y_{(eg,r)}$, industry commodities $Y_{(ind,r)}$ and services $Y_{(ser,r)}$. The production of these goods is characterised by production functions with constant elasticities of substitution (CES) and constant returns to scale. Nested functions with three levels are employed to specify the substitution possibilities between capital K , labour L , energy inputs $A_{(eg,r)}$ and non-energy $A_{(neg,r)}$ inputs (including intermediates from industry and services). We apply a KLEM production structure, thus capital and labour enter the production function on the lowest level, on the second level value added is combined with energy and finally on the top level of the CES function the energy-value-added composite is combined with a non-energy material aggregate. An overview of the production structure is given in Figure 5.

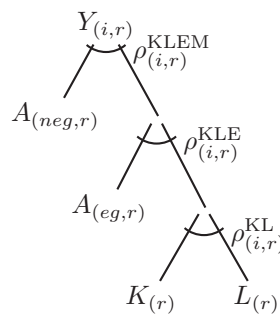


Figure 5: Structure of KLEM production function in Basic WIOD CGE Model

For our purpose, the WIOD data is aggregated into two regions (Europe (EUR) and 'Rest of the World' (ROW)), three sectors (energy goods (EG), industry (IND) and services (SER)) and two final demand agents (households (FC_HH) and government (GOV)). Additional information on the aggregation is given in the Appendix of this paper. With regard to the specification of parameters, the model is calibrated to the year 2003. We choose 2003 to avoid possible distortions from the economic crisis in later years. The required Armington elasticities are taken from GTAP7 (Badri and Walmsley, 2008; Hertel et al., 2007, 2008) and are mapped to the sectors we consider prior to the implementation into the model. Consumption and the intermediate mix in production are characterised by a Leontief function. In its initial setup and if not stated otherwise, we use estimates from Koesler and Schymura (forthcoming), henceforth abbreviated as KS, to specify the flexibility of production with regard to different inputs. The respective substitution elasticities are given in Table 3. But eventually OFDM is applied

⁴The WIOD database is available at <http://www.wiod.org>. We use data downloaded on the 17th of April 2013.

to determine an adequate specification of the substitution elasticities σ_{KL} , σ_{KLE} and σ_{KLEM} .

For the descriptive purpose of this paper, we undertake three different OFDM processes. The first is limited to an investigation of the elasticity of substitution between capital and labour in the energy sector (σ_{KL}^{EG}). The second explores substitutability on a more general basis and considers different values for σ_{KL} , σ_{KLE} and σ_{KLEM} for all sectors on the basis of a OFDM process without starting values. The third repeats the second process but this time takes estimates from KS as starting values.

For the reasons presented before, it is also not possible to use the time series data provided in WIOD to derive the required covariance matrix. Therefore, we generate a set of 250 pseudo observations by shocking the model with a series of different changes in total factor productivity, respectively a uniform increase in the endowment of labour and capital of households to be able to estimate the covariance matrix. Furthermore, following the insights from the previous section, we use output variables for total final demand, sectoral output as well as total factor and intermediate input in production to compute $\widehat{\text{COV}}$.

For the validation step in all three OFDM processes, we seek to replicate with the model the economy of the year 2004. An that account, we first compute all changes from 2003 to 2004 that we can observe in the WIOD dataset and subsequently apply the changes to the model in the form of a series of simultaneous shocks. This involves changes in household labour and capital endowments, intertemporal and interregional saving or borrowing, the prevailing tax structure, and international transport margins. As discussed before, our approach will clearly miss some changes that have occurred during this period. But given the comprehensive coverage of WIOD we hope to limit the number of omitted variables to a minimum.

4.1 Factor substitutability in the energy sector

In the first of three applications of OFDM, we seek to determine σ_{KL} for the sector producing the energy good (EG) in both regions EUR and ROW. On that account, we generate a portfolio of 250 different specifications of $\sigma_{KL}^{EG, EUR}$ and $\sigma_{KL}^{EG, ROW}$ that are to be tested. Thereby we arbitrary draw parameter values from a distribution of the form $\mathcal{N}(1, 1)$, whereas we repeat the draw if values smaller than zero or bigger than ten occur.⁵

The results of applying the parameter setups in the validation of is presented in Figure 6. To ease the presentation, we standardised the goodness-of-fit measure using:

$$r_{Standard}^2 = \frac{|r^2|}{|r_{MAX}^2|}, \quad (7)$$

such that $0 \leq r_{Standard}^2 \leq 1$. Thus parameter specifications featuring a $r_{Standard}^2$ of zero

⁵The CES functional form used in the model requires all elasticities to be weakly positive and as in the context of CGE models a substitution elasticity of ten already implies a very high substitutability, we do not consider values bigger than ten.

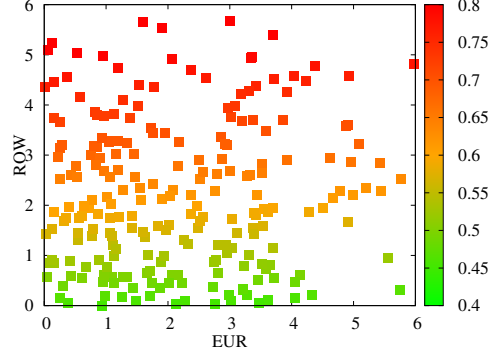


Figure 6: Standardised goodness-of-fit criteria of OFDM for σ_{KL} of the energy sector (EG) in Europe (EUR) and 'Rest of the world' (ROW) [axes give parameter values, color gives value of $r_{Standard}^2$]

achieve a perfect fit and a $r_{Standard}^2$ value of one indicates that the parameter setup in question is the worst of all tested specifications. Although no clear locus with adequate parameter values can be identified in Figure 6, the OFDM clearly suggest that low values for $\sigma_{KL}^{EG,ROW}$ are better than high values. The parameter values featuring the smallest $r_{Standard}^2$ are $\sigma_{KL}^{EG,EUR} = 2.76$ and $\sigma_{KL}^{EG,ROW} = 0.03$. But given the big range of $\sigma_{KL}^{EG,EUR}$ with relatively similar low $r_{Standard}^2$ values, the factual best result for $\sigma_{KL}^{EG,EUR}$ should not be overrated.

4.2 General input substitutability in production

For the next application we broaden the scope of the OFDM process and consider different values for σ_{KL} as well as σ_{KLE} and σ_{KLEM} for all sectors. We once more generate a portfolio of 250 parameter setup using the aforementioned distribution and constraints. Thereby all 18 parameters are iterated simultaneously.

Figure 7 presents the result of this OFDM application for the Basic WIOD CGE Model. From the different graphs it becomes clear that for all sectors and elasticities, some parameter specification are better suited than others to replicate the 2004 situation. But in this bigger application, the graphs are not as informative in our previous applications and in most cases we cannot identify a parameter area around which the fit is better than elsewhere. Only for σ_{KL} in Graphs 7a-c we can identify patterns. It suggest that for in EG and SER lower values seem to fit better for σ_{KL} , while in IND higher values seem more appropriate. The reason for the graphical ambiguity is that because all 18 values for σ_{KL} , σ_{KLE} and σ_{KLEM} are iterated simultaneously, even parameter settings that seem similar in one of the graphs potentially feature very different values for the other 16 parameter values. The graphical interpretation of the results is therefore limited. Still, from looking at the numerical values of $r_{Standard}^2$ we can derive the parameter setup which provides the best model fit. The respective values for σ_{KL} , σ_{KLE} and σ_{KLEM} are given in Table 2. Note also that compared to the previous

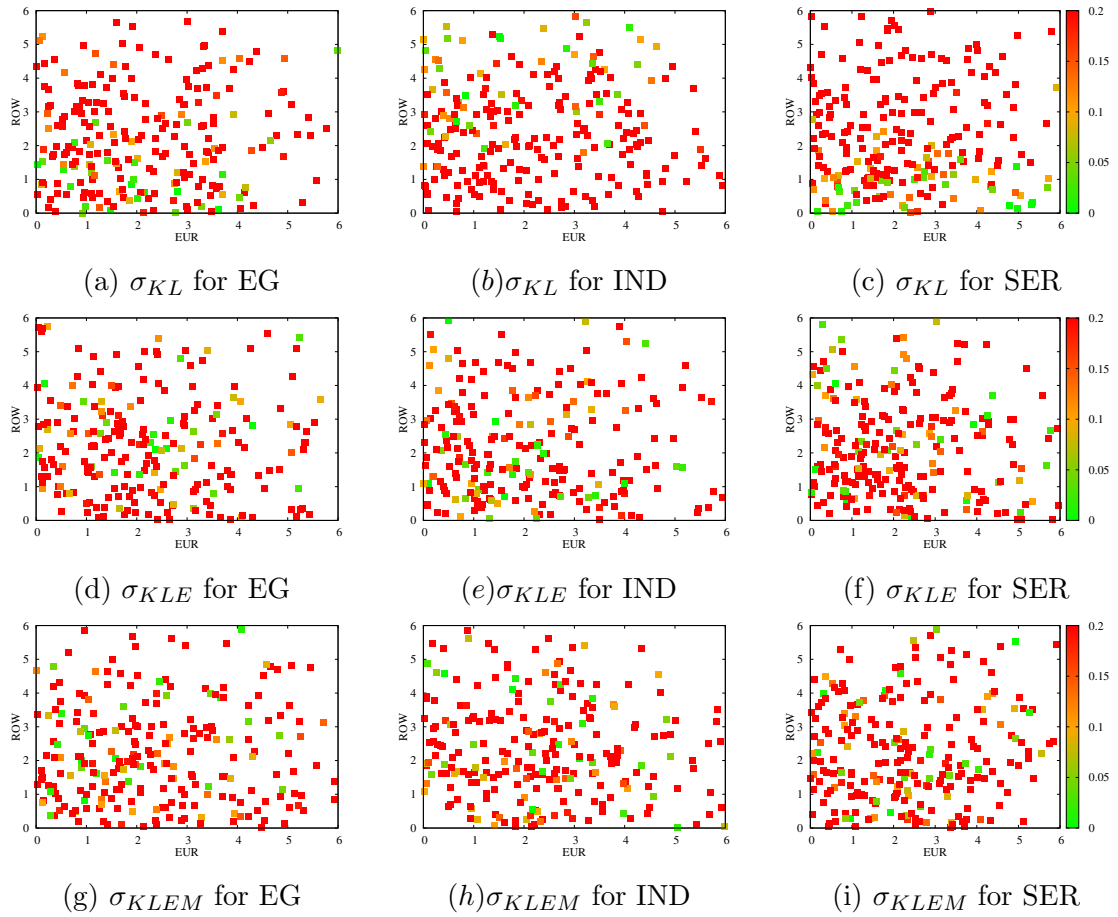


Figure 7: Standardised goodness-of-fit criteria of OFDM for σ_{KL} , σ_{KLE} and σ_{KLEM} when applied to Basic WIOD CGE Model [axes give parameter values, color gives value of r^2]

Table 2: Results of OFDM for σ_{KL} , σ_{KLE} and σ_{KLEM} when applied to Basic WIOD CGE Model

	ROW			EUR		
	EG	IND	SER	EG	IND	SER
σ_{KL}	0.12	6.82	0.31	3.62	4.54	5.33
σ_{KLE}	4.25	5.92	4.06	8.37	0.49	0.60
σ_{KLEM}	0.83	4.10	4.07	0.97	1.77	1.79

analysis the overall goodness-of-fit tends to be better which results in lower $r_{Standard}^2$ values. The reason for this is straightforward, including more parameters in the OFDM process increases the degrees of freedom and therewith the possibilities to adjust the model so that it can eventually generate a good fit.

4.3 General input substitutability in production with starting values

For the third application of OFDM, we build a portfolio of 250 different parameter specifications for σ_{KL} , σ_{KLE} and σ_{KLEM} on the basis of the estimates and standard deviations provided by KS. That is we use their estimates as initial values and iterate the parameters around these starting points assuming a normal distribution with the standard deviation presented also in their study.⁶ Again we apply the constraints for parameter values smaller than zero and higher than ten and, as before, repeat the draw in such a case. As this paper uses of a different aggregation than KS, we aggregate their estimates and standard deviations on the basis of the following equations:

$$\sigma_{Aggregate} = \sum_i (\alpha_i \sigma_i) \quad (8)$$

and

$$VAR(\sigma_{Aggregate}) = \sum_i (\alpha_i^2 VAR(\sigma_i)), \quad (9)$$

where α_i is the relative sector size in the aggregate and the later assumes that the elasticities between sectors are not correlated. Note that although KS reject variations across regions and over time for the substitution elasticities they estimate, here changes in the sector share may lead to elasticities that vary across regions and over time. The estimates we eventually use as starting values and the related standard deviations are given in Table 3 and correspond to the aggregated 2004 values for Europe. Note also that here we iterate σ_{KL} , σ_{KLE} and σ_{KLEM} again simultaneously for the generation of the different parameter setups.

⁶KS do not provide a substitution elasticity between capital and labour for the Coke Refined Petroleum and Nuclear Fuel (CPN) sector, here we assume that this elasticity is equal to the corresponding elasticity for the chemical and chemical products sector (0.24). For estimates that equal *Inf* we take an elasticity value of 10. Furthermore, for elasticities were no standard deviation is provided or were it is bigger than 10 we assume that it is equal to 2.5.

Table 3: Results of OFDM for σ_{KL} , σ_{KLE} and σ_{KLEM} when applied to Basic WIOD CGE Model using estimated starting values from KS, standard deviations are given in parentheses

	ROW			EUR		
	EG	IND	SER	EG	IND	SER
σ_{KL}	3.59	0.38	1.24	5.13	0.35	1.21
σ_{KLE}	3.83	0.44	0.25	3.37	0.39	0.26
σ_{KLEM}	0.52	0.61	1.60	0.52	0.71	1.37
σ_{KL}^{KS}	3.44 (0.61)	0.35 (0.02)	0.82 (0.09)	5.75 (0.93)	0.34 (0.02)	1.17 (0.12)
σ_{KLE}^{KS}	2.85 (1.34)	0.43 (0.03)	0.29 (0.04)	3.30 (0.59)	0.40 (0.02)	0.27 (0.03)
σ_{KLEM}^{KS}	0.41 (0.01)	0.59 (0.19)	1.95 (0.55)	0.53 (0.01)	0.58 (0.14)	1.47 (0.27)

As described before, the graphical interpretation of the results of the OFDM process applied here is only of limited value. Therefore we move directly to the presentation of the parameter setup featuring the best model fit. The corresponding values are given in Table 3 together with the starting values and standard deviations from KS. Compared to KS in particular the values for EG and SER in ROW seem to be higher. The other parameters are rather stable with only few minor adjustments. But it must be noted that of course the standard deviation attached to the original estimate critically influence the potential for updating the parameter values. This is also the reason why the OFDM process with starting values results in an overall less good fit relative to a OFDM process without starting values. Again this is due to the fact that if the tested parameter values are not restrained because of low standard deviations, the likelihood that a fitting parameter setup is included in the investigated portfolio is higher and thus the overall model fit is potentially better.

Ultimately, the availability of a set of suitable elasticity values from the literature raises the question why a OFDM process should be applied in the first place. There are two reasons for this. For one thing, to be able to use parameter values from the literature these should ideally have been estimated specifically for the use in the underlying model or at least build on the same theoretical structure. Otherwise this can result in a misspecification of the model (Browning et al., 1999). Unfortunately, although this is the case for the estimates from KS which have been estimate using the same dataset and functional form as the Basic WIOD CGE Model, this favourable situation is unlikely to apply for most models and parameters. What is more, estimates must always be associated with some degree of uncertainty. Directly applying estimates in a model neglects that they are basically also a - admittedly well informed and elaborated - 'best guess'. Modelers should be aware of these issues and if possible take measures that account for the limitations of estimates form the literature. Applying OFDM allows this.

5 Summary and conclusion

This paper is devoted to the enhancement of CGE modelling and presents OFDM as an alternative method to the specification of parameter values in CGE models. We first provide some background information on OFDM and outline how it has been used in climate science to detect distortions in the climate system and to specify climate models. Next we illustrate how the process of OFDM can be applied within a CGE framework and apply it to a stylised CGE model with the aim of demonstrating OFDM and exploring its potential in a CGE context. We show that OFDM is capable of identifying the 'true' parameter values for substitution elasticities as well as growth rates of endowments respectively factor productivity. Furthermore, our results suggest that the process can be applied using a range of different types of shocks such as changes in endowments or taxes. Finally we apply the OFDM approach to a full scale CGE model and derive a set of substitution elasticities for the Basic WIOD CGE Model.

Overall, using OFDM to specify parameters in CGE models allows to secure three main benefits: 1) OFDM employs a structured optimisation procedure and does not require modelers to update the model specification on the basis of their intuition as is the case for most other validation exercises or sensitivity analyses. 2) It enables modelers to account the uncertainty that is associated with parameter estimates from the literature. 3) OFDM is versatile and can be used to identify adequate parameter specifications for a range of different parameters such as elasticities or growth rates.

There remain some limitations however. In its process, OFDM involves model validation and because of the difficulty to account for all changes that take place over a certain period of time, the results might be somewhat distorted. The issue might however be alleviated by using datasets such as WIOD which provide comprehensive and consistent information on changes throughout economies. In addition, OFDM requires information on the relationship between model output in the form of a covariance matrix, this information might also prove hard to provide, in particular when many of the model output variables are to be used in the OFDM process. Furthermore, the choice of which parameter values are to be included in OFDM process and the question within which range these should be tested confronts modelers with a tradeoff. On the one hand, exploring a wide range of parameters and values increases the likelihood of achieving better results in the validation exercise and potentially provides values which are highly suitable according to the goodness-of-fit criteria of OFDM. Using additional information on parameter values from estimates found in the literature on the other hand, decreases the parameter space and may result in a less good model model fit. This implies that the process provides parameter values that are less adequate according to the OFDM criteria, but allows to include information from previous studies in the analysis.

With regard to future research, one obvious next step would be to apply the OFDM approach to a full scale CGE model and to use the resulting parameter specification in a CGE analysis. This would help overcome some of the critique CGE models are frequently confronted with and eventually will make CGE simulations more reliable.

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A Appendix

A.1 The Basic WIOD CGE model - Short description

The Basic WIOD CGE model is a static, multi-region, multi-sector computable general equilibrium (CGE) model. It has been developed within the project 'WIOD World Input-Output Database: Construction and Applications' funded by the European Commission, Research Directorate General as part of the 7th Framework Programme and has been designed deliberately as flexible as possible in order to allow researchers to use the World Input-Output Database (WIOD) in the framework of a CGE model in various applications. While a comprehensive description of the Basic WIOD CGE Model and the data it uses is provided by Koesler and Pothen (2013), the following provides a concise description of the model.

The model distinguishes between two groups of commodities in region r : energy commodities $Y_{(eg,r)}$ and non-energy commodities $Y_{(neg,r)}$. The production of these goods is characterised by production functions with constant elasticities of substitution (CES) and constant returns to scale. Nested CES functions with three levels are employed to specify the substitution possibilities between capital $K_{(r)}$, labour $L_{(r)}$, energy inputs $A_{(eg,r)}$ and non-energy intermediate inputs $A_{(neg,r)}$ of sectoral production. A KLEM production structure is applied for all sectors i , thus capital and labour enter the production function on the lowest level, on the second level value added is combined with energy and finally on the top level of the CES function the energy-value-added composite is combined with a non-energy material aggregate. An overview of the production structure is given in Figure 5 and the corresponding zero-profit condition is given in Equation A.1. Thereby and for all following CES functions, π denotes profits and CES stands for a constant elasticity of substitution function. The arguments of the CES function is given in parentheses and the corresponding elasticity of substitution in the upper index. Small p 's are prices of commodities and factors.

$$\begin{aligned} \pi_{(r,i)}^Y \leq & CES_{(r,i)}^0 \left[CES_{(r,i)}^0 (pem_{(em,ETSGroup)}), CES_{(r,i)}^{\sigma^{klem}} \left[CES_{(r,i)}^{\sigma^{ms}} (pa_{(neg,r,i)}), \right. \right. \\ & CES_{(r,i)}^{\sigma^{kle}} \left[CES_{(r,i)}^{\sigma^e} (pe_em_{(eg,r,i)}), \right. \\ & \left. \left. \left. CES_{(r,i)}^{\sigma^{kl}} (pl_{(r)}, pk_{(r)}) \right] \right] \right] \end{aligned} \quad (A.1)$$

Sectoral output can be used for intermediate use and/or final consumption domestically and/or exported to other regions. Perfect competition is assumed in all markets. Interregional trade is fully flexible and need not be balanced as long as the agent's overall budget is balanced.

As is the case for many other models, the choice among imports and domestically produced commodities is based on Armington's idea of regional product differentiation (Armington, 1969), i.e. domestic and foreign goods are not necessarily perfect substitutes and in combination form an Armington aggregate. However, in the Basic WIOD CGE

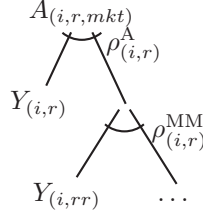


Figure A.1: Structure of Armington aggregate

$$\begin{array}{c}
 U_{(r,fd)} \\
 \hline
 \left[\rho_{(r,fd)}^U = 0 \right] \\
 \begin{array}{cc}
 A_{(eg,r)} & A_{(neg,r)}
 \end{array}
 \end{array}$$

Figure A.2: Structure of utility of representative agents

Model, Armington goods are not only region specific to account for regional differences in preference for domestic and foreign goods, but also sector specific in order to allow intermediates to be traced from their origin to their destination. Figure A.1 gives an overview of the underlying Armington structure and Equations A.2 and A.3 present the zero-profit and market clearance conditions for international commodity markets. $Y_{(r,i)}$ is domestic production, $Y_{(rr,i)}$ is production by foreign regions, small p 's are prices and $M_{(i,rr, mkt)}$ are imports of commodity i of market mkt (final demand and sectors) in region rr . While the Armington elasticity $\sigma_{(r,i)}^{es-a}$ governs the substitutability between domestic and foreign goods, $\sigma_{(r,i)}^{es-mm}$ controls the substitution between the same good from different regions. Apart from this, the basic model abstracts from other potential trade distortions.

$$\pi_{(i,r, mkt)}^A \leq CES_{(i,r, mkt)}^{\sigma_{(r,i)}^{es-a}} \left[py_{(r,i)}, CES_{(i,r, mkt)}^{\sigma_{(r,i)}^{es-mm}} (py_{(rr,i)}) \right] \text{ with } rr \neq r \quad (\text{A.2})$$

$$M_{(i,r, mkt)} \geq \sum_{rr; rr \neq r} \left(\frac{\partial \pi_{(i,r, mkt)}^A}{\partial py_{(rr,i)}} A_{(i,r, mkt)} \right) \quad (\text{A.3})$$

Each region may be represented by up to five aggregated representative agent who embraces all final demand types available in WIOD. The representative agents maximise their utility by purchasing bundles of consumption goods subject to their budget constraint. Utility of representative agents $U(fd, r)$ is given as a Leontief composite of energy $A_{(eg,r)}$ and a non-energy commodities $A_{(neg,r)}$. The structure of the utility functions is given in Figure A.2 and the related zero-profit condition is given in Equation A.4.

$$\pi_{(r,fd)}^U \leq CES^0 \left[CES^0(pa_{(neg,r)}), CES^0(pa_{(eg,r)}) \right] \quad (\text{A.4})$$

As described exemplarily for households and a government agent in Equation A.5 and A.6, the budget is determined by factor and tax income along with (intertemporal and interregional) borrowing or saving. In the basic version, agents supply a fix amount of capital and labour. Factors are mobile across sectors within regions but not across regions and therefore the model in its basic version abstracts from interregional factor mobility and investment.

$$B_{(r,FC_HH)} = pk_{(r)} \sum_i (K_{(r,i)}) + pl_{(r)} \sum_i (L_{(r,i)}) - Saving_{(r,FC_HH)} + Borrowing_{(r,FC_HH)} \quad (\text{A.5})$$

$$B_{(r,GOV)} = Tax(r) - Saving_{(r,GOV)} + Borrowing_{(r,GOV)} \quad (\text{A.6})$$

Besides standard economic activity, the model makes provisions for the accounting of CO₂ and other air emissions (N₂O, CH₄, NO_x, SO_x, NH₃, NMVOC, CO) caused by economic activity. For CO₂, the model distinguishes between energy related emissions and process emissions from sectoral production as well as consumption. Because the WIOD dataset does currently not allow to tie any of the other air emissions to particular inputs, these emissions are considered only as process emissions from production and consumption. From a modelling perspective, when emissions are related to energy, they occur during the production process parallel to the use of energy. That is they are associated with the second nest of the production structure outlined in Figure 5 and the first branch of Figure A.2 depicting the structure of utility. Process emission in turn are understood as a byproduct of production and consumption and are thus tied to sectoral output and final demand. If required, an emission trading system or a taxing scheme can be applied to all types of emissions.

Following Rutherford (2005) and Böhringer et al. (2003), the equilibrium in our model is characterised through three types of equilibrium conditions, namely market clearance conditions for all commodities and factors (supply = demand), income balances (net income = net expenditure) and zero profit conditions (cost of inputs = value of output). The variables defining the equilibrium are activity levels for the constant-returns-to-scale production, commodity and factor prices, and the price of final consumption. The market clearance condition related to the production of commodities is illustrated in Equation A.7.

$$Y_{(r,i)} \geq \sum_{ii} \left(\frac{\partial \pi_{(r,ii)}^Y}{\partial py_{(r,i)}} Y_{(r,ii)} \right) + \sum_{fd} \left(\frac{\partial \pi_{(r,fd)}^U}{\partial py_{(r,i)}} U_{(r,fd)} \right) + \sum_{rr; r \neq rr} \sum_{mkt} \left(\frac{\partial \pi_{(i,rr,mkt)}^A}{\partial py_{(r,i)}} A_{(i,rr,mkt)} \right) \quad (\text{A.7})$$

The market clearance condition for final demand is given in Equation A.8.

$$B_{(r,fd)} \geq U_{(r,fd)} \quad (\text{A.8})$$

For factor markets the following market clearance conditions must hold.

$$K_{(r,i)} \geq \sum_{ii} \left(\frac{\partial \pi_{(r,ii)}^Y}{\partial p_{k(r)}} Y_{(r,ii)} \right) \quad (\text{A.9})$$

and

$$L_{(r,i)} \geq \sum_{ii} \left(\frac{\partial \pi_{(r,ii)}^Y}{\partial p_{l(r)}} Y_{(r,ii)} \right) \quad (\text{A.10})$$

Numerically, the model is formulated as a mixed complementarity problem (MCP) in the mathematical optimization program GAMS, a program that is frequently used to develop and run CGE models. It is written in GAMS using the MPSGE syntax (c.f. Rosenthal, 2010; Rutherford, 1999). The model is solved using the PATH algorithm (c.f. Dirkse and Ferris, 1993).

Regarding the basic economic structure and information on emissions, the model builds on data from the World Input-Output Database (WIOD) (Timmer et al., 2012; Dietzenbacher et al., 2013) and can be calibrated to any year WIOD covers. The required Armington elasticities are taken from GTAP7 (Badri and Walmsley, 2008; Hertel et al., 2007, 2008) and are mapped to WIOD sectors prior to the implementation into the model. For substitution elasticities determining the flexibility of production with regard to inputs, estimates from Koesler and Schymura (forthcoming) are applied.

A.2 Additional tables

Table A.1: List of regions

Short	Regions	Associated WIOD Regions
EUR	Europe	AUT, BEL, BGR, CYP, CZE, DNK, ESP, EST, FIN, FRA, GBR, GER, GRC, HUN, IRL, ITA, LTU, LUX, LVA, MLT, NLD, POL, PRT, ROM, SVK, SVN, SWE
ROW	Rest of the World	AUS, BRA, CAN, CHN, IDN, IND, JPN, KOR, MEX, ROW, RUS, TUR, TWN, USA

Table A.2: List of sectors

Short	Sector	Associated WIOD Sector
EG	energy goods	C, 23, E
IND	industry	AtB, 15t16, 17t18, 19, 20, 21t22, 24, 25, 26, 27t28, 29, 30t33, 34t35, 36t37, F, 60, 61, 62
SER	services	50, 51, 52, H, 63, 64, J, 70, 71t74, L, M, N, O, P