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International Trade and the Adaptation to Climate Change and Variability

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Non-technical summary

There are different reasons, why countries trade. The most important one, economists typically refer to, is that trade is a source of wealth. There is, however, a further reason, which usually is not mentioned in the economic literature, but is in the center of this analysis. Since the impact of global warming considerably varies across countries, trade is a kind of insurance against the risks of climate change. For example, if in some region because of weather extremes food production is reduced, over the short-run the resulting losses might be substituted by imports. Over the long-run production could even shift to regions, which have the comparative advantage of being less vulnerable to climate change and variability. Seen in this way, trade is a means to adapt to the increasing risks of global warming.

Moderating climate impacts is one side of the coin. The second one is that trade can spread the cost of climate change across regions. After six years of drought Australia's rice production almost collapsed in 2008. This was one of several factors contributing to a doubling of the world market price of rice, which led to panicked hoarding and violent protests in low income countries. In other words, output losses in a single region might cause higher world market prices, and the resulting terms-of-trade effects could pertain to real income losses in almost any country. In countries, where direct impacts of climate change are relatively moderate and where sufficient financial and technical resources for adaptation exist, terms-oftrade effects can be responsible for a significant fraction of a country's total costs from climate change. This provides an argument to policy makers, why funding adaptation in the poor developing countries can be in the self-interest of the rich, industrialized ones.

This is to our knowledge the first paper which analyses in a systematic manner the interaction between adaptation to climate change, international trade and terms-of-trade effects in a simple model. It has three messages mainly: First, trade can be viewed as a kind of adaptation to climate change and variability, as trade can help to reduce direct impacts of global climate change on a region's welfare. In particular, the less affected and the richer nations are, the more they can profit from moderating the impacts of global climate change through trade. Second, if regions are rich enough to adapt optimally to climate change, the resulting allocation of adaptation measures is Pareto-efficient. In this case funding of adaptation does not make sense from an economic perspective. Finally, since the regions of the South typically lack the resources for adapting optimally to climate change, because of terms of trade effects, it might be in the self-interest of the industrialized nations to fund adaptation can be Pareto-improving only, if the benefits of funding, i.e., damages, which are moderated through adaptation, are big enough, and hence, if the recipient's own expenditure for adaptation is low. If not, the paradoxical effect of recipient immiserization through tied transfers can occur.

Das Wichtigste in Kürze

Die Gründe, weshalb Staaten miteinander handeln, sind vielfältig. Als wichtigsten Grund berufen sich Ökonomen seit Ricardo meistens auf die wohlfahrtsfördernde Wirkung von internationalem Handel für die involvierten Staaten. Daneben gibt es aber einen weiteren Grund, der in der ökonomischen Literatur meistens ignoriert wird, hier aber im Zentrum der Analyse stehen soll. Da sich die Klimaschäden zwischen unterschiedlichen Ländern stark unterscheiden, kann Handel als eine Art von Versicherung gegen die Risiken des Klimawandels fungieren. Führen beispielsweise häufigere Wetterextreme in einem Land zu einem Rückgang der Lebensmittelproduktion, kann das in der kurzen Frist durch eine Importausweitung ausgeglichen werden. In der langen Frist kann die Lebensmittelproduktion sogar in Länder, die durch geringere Klimavulnerabilität einen komparativen Vorteil haben, verlagert werden. In diesem Sinne kann vermehrter internationaler Handel als Anpassungsmaßnahme an den Klimawandel interpretiert werden.

Die Reduktion von Klimaschäden ist die eine Seite der Medaille. Auf der anderen Seite kann internationaler Handel die Auswirkungen des Klimawandels auf andere Regionen übertragen. Nach sechs Jahren der Trockenheit ist Australiens Reisproduktion 2008 fast vollständig zusammengebrochen. Das gilt als einer von verschiedenen Faktoren, die zu einer Verdoppelung des Weltmarktpreises und schließlich zu gewaltsamen Ausschreitungen in mehreren Entwicklungsländern führten. Das zeigt, dass Produktionsausfälle in einem Land zu höheren Weltmarktpreisen führen und die resultierenden Terms-of-Trade Effekte reale Einkommensverluste in anderen Staaten verursachen können. In Ländern, die nur geringe direkte Klimaschäden zu erwarten haben und die über genügend Mittel zur Anpassung verfügen, können solche Terms-of-Trade Effekte für einen signifikanten Teil der totalen Klimakosten verantwortlich sein. Die Finanzierung von Anpassungsmaßnahmen in Entwicklungsländern kann aus diesem Grund im Eigeninteresse der Industriestaaten liegen.

Diese Studie ist nach unserem Wissen die erste, die systematisch die Interaktion zwischen Anpassung, internationalem Handel und Terms-of-Trade Effekten untersucht. Drei Punkte sind von besonderer Relevanz: Erstens, internationaler Handel hilft den stark betroffenen Regionen die Klimakosten zu reduzieren, Zweitens, sind Regionen wohlhabend genug sich an den Klimawandel anzupassen ist die Finanzierung von Anpassungsmaßnahmen durch andere Staaten wirkungslos. Drittens, da die Entwicklungsländer oft nicht genügend Mittel haben, sich genügend anzupassen, kann es durch die erwähnten Terms-of-Trade Effekte für die Industrieländer Sinn machen, solche Anpassungsanstrengungen finanziell zu unterstützen. Solche Unterstützung ist aber nur Pareto-verbessernd wenn die Klimaschäden hoch und die Ressourcen des Empfängerlandes gering sind. Andernfalls kann es zum paradoxen Ergebnis kommen, dass die Finanzierung von Anpassungsmaßnahmen für das Empfängerland zu Wohlfahrtsverlusten führt.

International Trade and the Adaptation to Climate Change and Variability

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Abstract

This paper has three messages mainly, which are observed in a simple model of climate change, international trade and regional adaptation. First, trade can be viewed as a kind of adaptation to climate change and variability, as trade can help to reduce direct impacts of global climate change on a region's welfare. In particular, the less affected and the richer nations are, the more they can profit from moderating the impacts of global climate change, the resulting allocation of adaptation measures is Pareto-efficient. In this case funding of adaptation, which is an element of international climate policy, does not make sense from an economic perspective. Third, since the regions of the South typically lack the resources for adapting optimally to climate change, because of terms of trade effects, it might be in the self-interest of the industrialized nations to fund adaptation can be Pareto-improving only, if the benefits of funding, i.e., damages, which are moderated through adaptation, are big enough, and hence, if the recipient's own expenditure for adaptation is low. If not, the paradoxical effect of recipient immiserization through tied transfers can occur.

Keywords: funding of adaptation, climate change, international trade

JEL-Classification: F18, Q56, Q54

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1 Introduction

There are different reasons, why countries trade. The most important one, economists typically refer to, is that trade is a source of wealth. There is, however, a further reason, which usually is not mentioned in the economic literature, but is in the center of this analysis. Since the impact, global warming can have on regional societies, considerably varies across countries, trade is a kind of insurance against the risks of climate change and variability. For example, if in some region because of weather extremes food production is reduced, over the short-run the resulting losses might be substituted by imports. Over the long-run production could even shift to regions, which have the comparative advantage of being less vulnerable to climate change and variability. Seen in this way, trade is a means to adapt to the increasing risks of global warming (see Julia and Duchin, 2007).

Moderating climate impacts is one side of the coin. The second one is that trade can spread the cost of climate change across regions. Here is a recent example. After six years of drought Australia's rice production almost collapsed in 2008. This was one of several factors contributing to a doubling of the world market price of rice, which led to panicked hoarding and violent protests in low income countries (see Bradsher, 2008). In other words, output losses in a single region might cause higher world market prices, and the resulting terms-of-trade effects could pertain to real income losses in almost any country. This is a problem for the poorest in particular. Typically, these countries are heavily exposed to climate change, but they neither own the resources for coping with the associated risks, nor are they able to cover increasing expenses for imports through increasing their exports (see Cline, 2007).

Both the direct impacts and the terms-of-trade-effects depend on the societies' vulnerability to climate change. In particular, the less sensitive production in export oriented sectors is to climate change, and the less dependent a country is on imports of vulnerable goods, the lower will be the term-of-trade effects. Consequently, investing into measures for reducing vulnerability seems in the interest of countries. On the one hand this could be mitigation, which refers to policy interventions such as the reduction of anthropogenic greenhouse gases emissions. On the other hand it could be adaptation, which refers to investment into processes, practices, or structures to moderate or offset potential damages of global climate change, as well as to reduce the climate vulnerability of communities, regions, or countries (see Parry et al., 2007).

Some adaptation is done autonomously by the market; some has the property of a regional public good and requires strategic investment. Many poor countries, however, lack the capability to adapt. This is not only because they are short of financial resources. Poor countries typically have weaker market institutions, and their governments routinely undersupply local public goods. Therefore, it must be expected that without financial and technical assistance climate change will pronounce the existing inequalities between the industrialized and the developing world (see Barrett, 2008). For that reason three adaptation funds have been established at the 2001 COP6 meeting: (1) the Least Developed Countries Fund (LDCF), which aims to support the 49 least developed countries, (2) the Special Climate Change Fund (SCCF), which provides financial support to all developing countries, and (3) the Adaptation Fund, which is based on Article 12 of the Kyoto Protocol (for details, see Dellink et al., 2009). And the Green Climate Fund (GCF), which has the goal to provide 100 billion US Dollar annually for mitigation and adaptation activities in developing countries, came closer to start operating at the recent COP17 meeting in Durban, South Africa.

Despite of that till today the contributions of industrialized countries are small compared to the financial resources, the developing countries need for adapting efficiently to global climate change. As Buob and Stephan (2011) discuss, one reason might be that as long as there is no international cooperation in the solution of the global climate problem, and as long as mitigation is voluntary, both the industrialized and the developing countries have low interest in funding adaptation. More precisely Buob and Stephan observe a paradox in the following sense: For economic reasons industrialized countries will financially assist adaptation in developing ones only, if the burden of mitigation is shifted from the developed to the developing countries, which, however, will harm the South's welfare.

As is often the case in game theory settings, Buob and Stephan (2011) neglect international trade. But terms-of-trade effects can be important for at least two reasons. First, as Schenker (2010) shows, in countries, where direct impacts of climate change are relatively moderate and where sufficient financial and technical resources for adaptation exist, terms-of-trade effects are responsible for a significant fraction of a country's total costs from climate change. This not only reinforces what we already stated above, namely that investing into measures for reducing own vulnerability seems in the interest of countries. It also provides an argument to policy makers, why funding adaptation in the poor developing countries can be in the self-interest of the rich, industrialized ones.

Second, providing technical and financial assistance for adapting to the risk of climate change can be viewed as a kind of tied transfer from industrialized to developing countries. Since the pioneering work of Bhagwati et al. (1983) it is recognized that terms-of-trade deterioration could lead to what literature calls a transfer paradox. That means, (1) a donor country might gain by giving aid to a recipient country, which then loses welfare through this transfer, and (2), changes in the international terms-of-trade caused by a transfer is the principal reason for such perverse results. This paper contributes to this literature, but is to our knowledge the first one, which in a systematic manner analyses the interaction between adaptation to climate change, international trade and terms-of-trade effects.

Generally it is argued that climate policy requires a long-term perspective. Undeniable this applies, if mitigation is the policy option under consideration, where, because of the inertia of the climate system, costs are borne early, but benefits accrue in the distant future. For the same reason it should be clear, however, that the climate will change, even if greenhouse gas emissions are drastically reduced immediately. In what follows, we take global climate change as given. One argument is that we adopt a mid-term perspective. A second one could be that an internationally coordinated mitigation policy exists, by which the development of the atmospheric greenhouse gas concentration is exogenously determined. Based on this assumption Section 2 presents a simple analytical framework, which is designed to analyze the interaction between global climate change, adaptation and international trade. Section 3 discusses three different cases: (1) the pure trade effect of climate change, (2) optimal autonomous adaptation, and (3) funding adaptation in the presence of international trade. Section 4 concludes.

2 A North-South model of climate change, trade and adaptation

In the following we develop a static model of climate change, adaptation expenditures and international trade. Economic activities differ with respect to their sensitivity to climate change. For example, agricultural production is more responsive to climate change and weather extremes than manufacturing cars or personal computers. Furthermore, the impact of climate change will be larger and more harmful in the developing countries of the South than in the industrialized ones in the North. This follows mainly from limited adaptive capacities, and since the fraction of economic activities, which are at risk of climate change disruption, is larger in poor than in rich countries (see Adger et al., 2003). Taken together this motivates (1)

to discriminate between sectors and commodities, which are vulnerable to climate change, and those which are not, and (2), to divide the world economy into at least two regions: North (N) and South (S). North corresponds to the OECD countries and represents a region of relatively high wealth, but relatively low exposure to climate change. South is an acronym for the developing part of the world, which is poor, but highly vulnerable to climate change.

Let vulnerable be the nickname for the aggregate of goods and services, which are produced in climate sensitive sectors such as agriculture, animal husbandry, water supply or forestry. Vulnerable and capital are traded on open international markets, and both are inputs into the regions' gross production. Thereby, gross production of region *n* is characterized by a linear homogenous function $f_n(x_n, k_n)$, where x_n and k_n are the inputs of vulnerable and capital, respectively.

The production of vulnerable is described by cost functions. Costs are expressed in units of gross output and are a strictly increasing function $g_n(Q, a_n, z_n)$ of the output z_n and impacts of climate change, which in turn depend on two parameters: (1) the global climate, which is represented by atmospheric carbon concentration Q, and (2), the region's expenditure a_n for regional adaptation. That means, the higher is the atmospheric carbon concentration Q and the lower is the region's investment a_n into adaptation measures, the higher will the regional climate impact and hence the costs for producing a certain output of vulnerable goods and services. More precisely, we assume¹

Assumption 1:

$$\begin{aligned} \frac{\partial f_n}{\partial x_n} &> 0, \frac{\partial^2 f_n}{\partial x_n^2} < 0, \frac{\partial f_n}{\partial k_n} > 0, \frac{\partial^2 f_n}{\partial k_n^2} < 0, \\ \frac{\partial g_n}{\partial Q} &> 0, \frac{\partial^2 g_n}{\partial Q^2} > 0, \frac{\partial g_n}{\partial a_n} < 0, \frac{\partial^2 g}{\partial a_n^2} > 0, \frac{\partial g_n}{\partial z_n} > 0, \frac{\partial^2 g}{\partial z_n^2} > 0, \\ \frac{\partial^2 g_n}{\partial Q \partial a_n} &= \frac{\partial^2 g_n}{\partial a_n \partial Q} < 0, \frac{\partial^2 g_n}{\partial Q \partial z_n} = \frac{\partial^2 g_n}{\partial z_n \partial Q} > 0, \frac{\partial^2 g_n}{\partial a_n \partial z_n} = \frac{\partial^2 g_n}{\partial z_n \partial a_n} < 0 \end{aligned}$$

As such costs of producing vulnerable are strictly increasing with atmospheric carbon concentration and decreasing, but at declining rate with adaptation expenditure. Moreover, the more

For illustrative purposes let $g_n(Q, a_n, z_n) = z_n^2 [1 + \theta_n(Q - a_n)]$, where $\theta'_n > 0, \theta''_n > 0$.

is invested into adaptation the lower is the marginal impact of climate change on costs of exporting vulnerable, while climate change intensifies the cost reduction effect of adaptation.

At this point, three remarks on our modeling of climate impacts and adaptation seem justified. First, our analysis focuses on market impacts only, i.e., impacts of climate change, which can be expressed in terms of losses (or gains) in the regions' ability to produce private goods in vulnerable sectors. This implies that in contrast to the majority of the literature on interaction between international trade and the environment (for example, see Copeland and Taylor, 2005) we are neglecting non-market damages, i.e., impacts of climate change such as species losses, for which no market value exists.² Second, as already mentioned above, adaptation covers a wide range variety of measures, such as building of dykes, installing of early warning systems or the shifting to more heat and drought resistant crops. There exists no common metric for these diverse measures. Therefore, similar to Ebert and Welsch (2011), expenditure for investing into adaptation measures is used as argument in the cost function of vulnerable. Finally, climate change results from the accumulation of carbon dioxide and other greenhouses gases in the atmosphere and primarily materializes in three forces, which affect economic activities: mean or seasonal temperature change, changes in mean or seasonal precipitation and sea level rise. As was shown by Mendelssohn et al. (2006) in contrast to atmospheric carbon concentration precipitation as well as temperature change very much depends on the geographical location. Given the high level of abstraction, it seems reasonable therefore to represent global climate change through changes in the atmospheric stock of carbon dioxide and capture differences in regional patterns through region specified costs functions.

Each region can invest in its own as well as the other region's adaptation. I.e., $a_n = a_n^n + a_n^s$, where a_n^n denotes own adaptation expenditure and a_n^s is the investment of region *s* into adaptation measures of region *n*. Suppose further that the world markets both for vulnerable and capital are in equilibrium

- $(2.1) z_N + z_S x_N x_S = 0,$
- $(2.2) K_N + K_S k_N k_S = 0,$

² Non-market damages are by definition damages which are not directly expressed in units of a national accounting system. As such non-market damages are not transmitted by changes in economic variables as prices, but use rather different channels as the media, etc.

where K_n denotes the exogenously given capital stock of region n = N, S.

Let p and r denote the world market price of vulnerable and capital, respectively. Gross production can be consumed domestically and might be used to cover the costs of investing into adaptation as well as of producing vulnerable. Therefore, for any region n the budget constraint is

(2.3)
$$c_n = f_n(x_n, k_n) - g_n(Q, a_n, z_n) - a_n^n - a_s^n + p(z_n - x_n) + r(\overline{K}_n - k_n)$$

where c_n is domestic consumption. $p(z_n - x_n)$ and $r(\overline{K}_n - k_n)$ are the net-deficits from trading vulnerable and capital, respectively.

3 Analysis

Let sectors behave as price takers. Then profit maximization leads to the following optimality conditions for an interior solution, which are both sufficient and necessary

(3.1)
$$\frac{\partial f_n}{\partial x_n}(x_n,k_n) - p = 0, n = N, S,$$

(3.2)
$$p - \frac{\partial g_n}{\partial z_n}(Q, a_n, z_n) = 0, n = N, S,$$

(3.3)
$$\frac{\partial f_n}{\partial k_n}(x_n,k_n) - r = 0, n = N, S,$$

Condition (3.1) and (3.2) together state that the marginal productivity of vulnerable equals the world market price, which in turn is equal to the marginal costs of producing vulnerable. Condition (3.3) implies that the marginal productivity of capital has to be identical across regions.

These conditions allow to define (1) exports $z_n = Z_n(Q, a_n, p)$ as function of the world market price p and the regional climate impact (Q, a_n) , (2) imports $x_n = X_n(p, k_n)$ as function of price p and capital inputs k_n , and (3), capital inputs $k_n = K_n(r, x_n)$ as function of the world capital market price r and vulnerable inputs x_n , respectively.

Based on this and by taking the total differential of condition (2.1) we get after some manipulations (see Appendix, Proposition 1)

(3.4)
$$Adp = \left(\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}\right) dQ + \frac{\partial z_N}{\partial a_N} da_N + \frac{\partial z_S}{\partial a_S} da_S,$$

where $A = \left(\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} - \frac{\partial z_S}{\partial p}\right)$ is the slope of the world excess demand function of vulnerable. Since the slope is negative, climate change as well as regional adaptation affects the terms-of-trade: the higher (lower) is the impact of climate change on the regions' production of vulnerable, the less (more) will be exported, and hence, the world market price will rise (fall).

As condition (3.4) suggests (for a proof, see Appendix, Corollary 1), there will be no reallocation of capital across regions, if atmospheric carbon concentration and/or adaptation expenditure change only marginally. Intuitively, this can be explained as follows. Suppose that the initial capital endowment K_n , n = N, S, corresponds to the equilibrium stocks, i.e., suppose that the marginal productivity of capital initially is identical across regions. Global climate change, which does not directly affect capital inputs, will cause a reallocation of capital between regions only, if it induces differences in the regions' rate of return on capital. Now, consider

$$d\frac{\partial f_n}{\partial k_n}(x_n,k_n) = \frac{\partial^2 f_n}{\partial k_n^2} dk_n + \frac{\partial^2 f_n}{\partial k_n \partial x_n} dx_n,$$

and recall that $k_n = K_n(r, x_n)$, hence $dk_n = \frac{\partial k_n}{\partial x_n} dx_n + \frac{\partial k_n}{\partial r} dr$. By using conditions (A1) and (A3) (see Appendix) we observe

$$d\frac{\partial f_n}{\partial k_n}(x_n,k_n)=dr, \ n=N,S.$$

Consequently, since all sectors and regions behave as price takers, climate change will not lead to a reallocation of capital across regions.

An immediate consequence is that inputs of vulnerable into regional production are not directly affected by climate change, i.e.,

$$\frac{dx_n}{dQ} = \frac{\partial x_n}{\partial p} \frac{dp}{dQ} < 0, n = N, S.$$

There are, however, indirect effects. For, if the world market price of vulnerable rises due to global climate change, less will be put into regional production.

In what follows, let us assume that there is no additional expenditure for adaptation, neither in the North nor in the South, i.e. $da_n = 0$, n = N,S. Then condition (3.4) turns into

(3.4a)
$$\frac{dp}{dQ} = \frac{\left(\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}\right)}{\frac{\partial z_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} - \frac{\partial z_S}{\partial p}} > 0,$$

whereas condition (3.2) implies (see Appendix, conditions (A5), (A6))

$$\frac{dz_n}{dQ} = \frac{\partial z_n}{\partial Q} + \frac{\partial z_n}{\partial p} \frac{dp}{dQ}.$$

The first expression on the right represents the direct impact of climate change on exports of vulnerable, which is negative. The second one reflects trade effects and is positive (see (3.4a)). As such, the overall effect depends upon which of the two dominates. Moreover we have

$$(3.5) \qquad \frac{d(z_n - x_n)}{dQ} = \frac{\partial z_n}{\partial Q} + \left(\frac{\partial z_n}{\partial p} - \frac{\partial x_n}{\partial p}\right) \frac{dp}{dQ} = \frac{\partial z_n}{\partial Q} \left[\frac{\frac{\partial x_s}{\partial p} - \frac{\partial z_s}{\partial p}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_s}{\partial p} - \frac{\partial z_N}{\partial p} - \frac{\partial z_s}{\partial p}}\right] + \frac{\partial z_s}{\partial Q} \left[\frac{\frac{\partial z_n}{\partial p} - \frac{\partial x_n}{\partial p}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_s}{\partial p} - \frac{\partial z_N}{\partial p}}\right].$$

Given conditions (A2), (A5) and (A6) (see Appendix) the first expression on the right hand is negative, while the second one is positive.

As mentioned above, countries in the North are less vulnerable to climate change that those in the South. Mendelsohn et al. (2006) predict that the poor countries of the South will suffer the largest part of the damages from climate change. Although adaptation, economic wealth, and technology may take influence on how market damages are distributed across countries, over the mid-term regional market impacts of global warming are essential zero in the US, Japan and Russia. India and many other low-income countries, however, might be confronted with significant losses. Therefore, it is reasonable to assume that over the mid-term the direct market impact of climate change on the production of vulnerable commodities is almost negligible in the North. I.e., let us assume for a moment

Assumption 2: If the global climate changes only slightly, impacts of climate change on costs of producing vulnerable outputs are zero in the North, but positive in the South.

Expressed in technical terms, Assumption 2 means that $\frac{\partial g_N}{\partial Q} = 0$, hence $\frac{\partial^2 g_N}{\partial Q \partial z_N} = \frac{\partial^2 g_N}{\partial z_N \partial Q} = 0$ and $\frac{\partial z_N}{\partial Q} = 0$ (see (A6)). This implies that in contrast to the North, the South, where direct effects matter, will now reduce its output of vulnerable. Or to phrase it differently, there is a change in the terms-of-trade in favor of the North. For if we apply Assumption 2, condition (3.5) gives

$$\frac{d(z_N-x_N)}{dQ} > 0 \text{ and } \frac{d(z_S-x_S)}{dQ} < 0.$$

Finally, let us consider how climate change affects regional welfare. Since welfare depends on consumption only, welfare can directly be measured in units of consumption. By differentiating totally, we get from condition (2.3)

$$dc_n = -\frac{\partial g_n}{\partial Q}dQ + dp(z_n - x_n), n = N, S.$$

The first term on the right hand side represents the direct effect, climate change has on regional welfare, and which typically is non-positive. The second term on the right hand side represents terms-of-trade effects. Now, since climate change implies rising world market prices, this effect is negative and accentuates the negative direct impacts, if region *n* is net importer of vulnerable, i.e., $(z_n - x_n) < 0$. However, if region *n* is net-exporter of vulnerable, then term-of-trade effects are positive and might offset the negative direct impact of climate change. In other words, if the South were net-exporter of vulnerable, and if the terms-of-trade effects are high enough, this could even overcompensate the direct impact of climate change.

3.2 Optimal autonomous adaptation

Next let us assume that both regions own the necessary resources to autonomously adapt to climate change. If adaptation is optimal in region n = N,S, then Kuhn-Tucker conditions imply

(3.6)
$$(\frac{\partial g_n}{\partial a_n} + 1)a_n = 0.$$

This indicates that in absence of budget constraints, in case of adaptation marginal adaptation expenditure equals marginal benefits, i.e., the marginal impact of adaptation on the costs of producing vulnerable. Since we explicitly discern between costs (expenditure) and benefits of

adaptation, i.e., effects of adaptation on costs of producing vulnerable, we are able to define a total cost function

$$h_n = g_n(Q, a_n, z_n) + a_n$$

Taking the total differential gives

$$\frac{dh_n}{dQ} = \frac{\partial g_n}{\partial Q} + \left(\frac{\partial g_n}{\partial a_n} + 1\right) \frac{da_n}{dQ} + \frac{\partial g_n}{\partial z_n} \frac{dz_n}{dQ}$$

Because of condition (3.6) this indicates that in case of optimal adaptation marginal production costs entailed by climate change do not include adaptation costs. They are determined by (1) residual damages of climate change, or as Tulkens and van Steenberghe (2009) phrase it, marginal suffering costs, and (2), by the marginal change in total costs of producing vulnerable because of a marginal change in exports of vulnerable.

Condition (3.6) furthermore indicates that if both regions are optimally adapted to climate change, then marginal benefits of adaptation are identical across regions. This is a sufficient and necessary condition for Pareto-efficiency in the allocation of adaption to global climate change as immediately follows from solving (see (2.3))

$$\max\{f_N(x_N, k_N) - g_N(Q, a_N, z_N) + f_S(x_S, k_S) - g_S(Q, a_S, z_S) - a_N - a_S\}$$

subject to conditions (2.1) and (2.2). The explanation is obvious. Both, benefits and costs of adaptation are regionally private. This means, seen from a global perspective, adaptation is private to the single region, and hence, if each region autonomously is optimally adapted to climate change, the resulting allocation must be optimal globally. Consequently, funding adaptation is not Pareto improving. In contrast, funding adaptation will crowd out autonomous adaptation and works like an income transfer only.

This becomes obvious, if we look at condition (3.6), which implicitly defines optimal autonomous adaptation $a_n^n = A_n(Q, z_n, a_n^s)$ as function of atmospheric carbon concentration Q, exports z_n and foreign expenditure a_n^s for adaptation in region n. Totally differentiating conditions (3.2) and (3.6) implies

$$dz_n = \frac{\partial z_n}{\partial a_n} (da_n^s + da_n^n) + \frac{\partial z_n}{\partial Q} dQ + \frac{\partial z_n}{\partial p} dp,$$

$$da_n^n + da_n^s = \frac{\partial a_n}{\partial z_n} dz_n + \frac{\partial a_n}{\partial Q} dQ,$$

which gives

(3.7)
$$\left(1 - \frac{\partial a_n}{\partial z_n} \frac{\partial z_n}{\partial a_n}\right) dz_n = \left(\frac{\partial z_n}{\partial Q} + \frac{\partial z_n}{\partial a_n} \frac{\partial a_n}{\partial Q}\right) dQ + \frac{\partial z_n}{\partial p} dp,$$

(3.8)
$$\left(1 - \frac{\partial a_n}{\partial z_n} \frac{\partial z_n}{\partial a_n}\right) \left(da_n^n + da_n^s\right) = \left(\frac{\partial a_n}{\partial Q} + \frac{\partial a_n}{\partial z_n} \frac{\partial z_n}{\partial Q}\right) dQ + \frac{\partial z_n}{\partial p} \frac{\partial a_n}{\partial z_n} dp.$$

Both conditions indicate that optimal autonomous adaptation as well as the exports of vulnerable is driven by climate change as well as changes in terms-of-trade. As the first bracket on the right hand side of condition (3.7) shows, climate change has two opposite effects on the exports of vulnerable: a direct and negative one, since climate change raises the costs of producing vulnerable (see (A.6)), and an indirect, but positive one, since climate change stipulates adaptation expenditure, which reduces the production costs (see (A.7), (A.9)). Furthermore, a marginal increase of the world market price of vulnerable positively affects the export of vulnerable (see (A5)).

As the first bracket on the right hand side of condition (3.8) indicates, climate change has a direct and positive effect on optimal adaptation expenditure (see (A.9)) as well as an indirect and negative one, since climate change reduces the exports (see (A.6), (A.8)). Furthermore increasing world market prices stipulates autonomous adaptation, provided $\frac{\partial a_n}{\partial z_n} \neq 0$ (see A.5). Now, since $\frac{\partial a_n}{\partial z_n} = -\frac{\partial^2 g_n}{\partial a_n \partial z_n} / \frac{\partial^2 g_n}{\partial a_n^2}$, this requires that, at optimum, marginal benefits of adaptation $\frac{\partial g_n}{\partial a_n}$ are not independent of exports.

Adaptation funding crowds out autonomous adaptation, as condition (3.8) shows. As such the North has no economic incentive for funding adaptation in the South. Form the North's perspective adaptation funding will pay only, if it affects the terms-of-trade. This is not the case, however, since optimal expenditure remains unchanged. Instead adaptation funding here turns out being some kind of income transfer from the North to the South. Through applying condition (3.6) we get from the budget constraint (2.3)

$$dc_n = -\frac{\partial g_n}{\partial Q}dQ + dp(z_n - x_n) - \frac{\partial g_n}{\partial a_n}da_n^s - da_n^s.$$

The right side of the last equation consists of four terms, which indicate that a region's wealth is directly affected by (1) climate change, (2) the terms-of-trade, and (3) the funding of adaptation, either, if the region is recipient (see the third term) or is donor (see the fourth term). More precisely, we have in case that North is the donor and South the recipient

$$dc_N = -\frac{\partial g_N}{\partial Q}dQ + dp(z_N - x_N) - da_S^N,$$

$$dc_{S} = -\frac{\partial g_{S}}{\partial Q}dQ + dp(z_{S} - x_{S}) - \frac{\partial g_{S}}{\partial a_{S}}da_{S}^{N}$$

Now, since changing transfers from North to the South do not change optimal adaptation, neither in the North nor in the South, funding adaptation has no effect, neither on terms-of trade nor regional production costs. Consequently, North would have cover a part of adaptation expenditure in South without gaining any benefits, whereas in the recipient's region the funding of adaptation works just like an untied income transfer.

3.3 Funding adaptation

The results from above suggest: If both regions are optimally adapted to climate change and variability, then it is not in the self-interest of the North to support strategically adaptation in the South. Generally it is argued, however, that although being more heavily exposed to the impacts of climate change, the developing countries do not own the necessary resources for coping with the associated risks (see Barrett, 2008). Or to phrase it differently: Countries in the South will not be able to optimally adapt to climate change on their own. Consequently it must be expected: (1) autonomous adaptation in the South remains sub-optimal. (2) Funding adaptation will not completely crowd out domestic adaptation, and hence, (3) funding adaptation might take influence on terms-of-trade, from which the North could profit. As such financing adaptation in the South would be a kind of facilitative adaptation.

For testing this hypothesis, let us assume:

Assumption 3: The South is not optimally adapted to climate change and does not extend it own adaptation expenditure. The North is optimally adapted and strategically investments into adaptation in the South. This implies in particular: $da_S^S = 0$, $da_S^N = da_S > 0$. And since we are interested in isolating the effects of funding adaptation in the South, for the sake of simplicity let us assume further: dQ = 0. Then, since $\frac{\partial z_S}{\partial a_S} > 0$ (see (A.7)), condition (3.4) implies

(3.4b)
$$\frac{dp}{da_S} = \frac{\frac{\partial z_S}{\partial a_S}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} - \frac{\partial z_S}{\partial p}} < 0.$$

In words: If there is additional adaptation expenditure, marginal costs of producing vulnerable in the South will be reduced, which ceteris paribus implies falling world market prices and rising exports from the South as will be shown in the following. Since there is no reallocation of capital across regions (see Appendix, Corollary 1) and since dQ = 0, we obtain from conditions (3.1) and (3.2)

$$\begin{aligned} \frac{dx_n}{da_S} &= \frac{\partial x_n}{\partial p} \frac{dp}{da_S} > 0, \\ \frac{dz_N}{da_S} &= \frac{\partial z_N}{\partial p} \frac{dp}{da_S} < 0, \\ \frac{dz_S}{da_S} &= \frac{\partial z_S}{\partial a_S} + \frac{\partial z_S}{\partial p} \frac{dp}{da_S} = \frac{\frac{\partial z_S}{\partial a_S} \left[\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} \right]}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} - \frac{\partial z_S}{\partial p}} > 0, \\ \frac{dz_S + dz_N}{da_S} &= \frac{\frac{\partial z_S}{\partial a_S} \left[\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p} \right]}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p}} + \frac{\frac{\partial z_S \partial z_N}{\partial a_S \partial p}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p}} > \end{aligned}$$

These results are intuitively clear. Because of falling costs of producing vulnerable in the South, world market prices will fall, and hence, demand for vulnerable will rise in any region. Since there is no change in production costs in the North, exports are now reduced in that region, while in the South, where production of vulnerable became more profitable, production is extended.

0.

Finally, let us consider how strategic adaptation funding regional consumption and hence welfare. In case of the North we obtain (see (2.3))

$$c_N = f_N(x_N, k_N) - g_N(Q, a_N, z_N) - a_N - a_S^N + p(z_N - x_N) + r(\overline{K}_N - k_N)$$

Since it is assumed that the North is optimally adapted and dQ = 0, profit maximization (see (3.1) - (3.3)) and condition (3.6) imply

$$(3.9) \qquad \frac{dc_N}{da_S} = \frac{dp}{da_S}(z_N - x_N) - 1.$$

Now, if the North is net-importer of vulnerable, i.e. $(z_N - x_N) < 0$, because of declining world market prices (see condition (3.4b)), the North might profit from investing into the South's adaptation infrastructure, if the resulting terms-of-trade effects are big enough. In contrast it is not clear a priory, how increasing adaptation expenditure will take influence on consumption in the South. Taking the total differential of the budget constraint now gives

(3.10)
$$\frac{dc_S}{da_S} = -\frac{\partial g_S}{\partial a_S} + \frac{dp}{da_S}(z_S - x_S),$$

which indicates that there are two countervailing effects. On the one hand, there is a positive effect on consumption and hence welfare, since the costs of producing vulnerable are reduced. This is reflected by the first term of the right side of condition (3.10). But on the other hand, if the South is net-exporter of vulnerable, declining world market prices reduce income in the South (see the second term of the right side of (3.10)).

Now, remember that $\frac{dp}{da_S}(z_N - x_N) + \frac{dp}{da_S}(z_S - x_S) = 0$. Consequently condition (3.10) turns into

$$\frac{dc_S}{da_S} = -\frac{\partial g_S}{\partial a_S} - \frac{dc_N}{da_S} - 1$$

Therefore, funding adaptation can be strictly Pareto-improving in the sense that marginal effects on consumption can be positive simultaneously in North and South only, if $-\frac{\partial g_S}{\partial a_S} > 1$. This means, in the South the foreign financed adaptation expenditure must be below optimal autonomous ones (see condition (3.6)). And since (see condition (3.10))

$$\frac{dc_S}{da_S} = -\frac{\partial g_S}{\partial a_S} - \frac{dp}{da_S}(z_N - x_N),$$

welfare can be positive in the South only, if the gains from a reduction of production costs outrange the donor's enrichment because of terms-of-trade effects. As such there are two messages: First, adaptation funding can be Pareto-improving only, if benefits of funding are

big enough. Given the usual properties of the cost function this implies that the level of adaptation is low in the recipient regions. Second, if not, the paradoxical effect of recipient immiserization through tied transfers can occur.

4 Conclusions

As our considerations indicate, economic losses, which climate change induces in vulnerable sectors, among which agriculture, forestry and animal husbandry are the most important ones, can be limited through shifting to imports rather than producing these goods at home. Or to phrase it differently, trade can reduce the market damages of global climate change. The problem of the poor countries is, however, that they may face difficulties to increase export earnings from other goods, which are necessary for paying for the additional imports. As Cline (2007) notes this gives the problem a "let them eat cake" flavor. And it implies in particular that if international trade is considered as mean of moderating the impacts of global warming, the corresponding terms-of-trade effects cannot be neglected.

One option of how to reduce the economic impact of climate change is to financially assist the developing countries in adapting to climate change. This is of particular importance, since many of the poor countries of the South lack the necessary resources for adapting optimally to the risk of climate change. As our analysis reveals providing financial assistance for adaptation can be Pareto-improving, if benefits of funding, i.e., damages, which are moderated through adaptation, are big enough, and if the recipient's expenditure for adaptation is low. If not, the paradoxical effect of recipient immiserization through tied transfers can occur.

Independent of that it must be mentioned that our analysis is based on the assumption that vulnerable products are traded on open and perfect world markets. Reality is far away from such a situation. In particular, this is not the case regarding international trade of agricultural products. For example, in 1973 the United States imposed an embargo on soybean exports in order to avoid inflationary effects of rising prices, and many nations are inclined to impose agricultural import barriers in the name of food self-sufficiency.

5 References

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Appendix 1

Remember: $k_n = K_n(r, x_n)$, $x_n = X_n(p, k_n)$, $z_n = Z_n(Q, a_n, p)$, $a_n = A_n(Q, z_n)$. If Assumption 1 is fulfilled, this implies for n = N, S

(A1)
$$\frac{\partial k_n}{\partial r} = \frac{1}{\frac{\partial^2 f_n}{\partial k_n^2}} < 0,$$

(A2)
$$\frac{\partial x_n}{\partial p} = \frac{1}{\frac{\partial^2 f_n}{\partial x_n^2}} < 0,$$

(A3)
$$\frac{\partial k_n}{\partial x_n} = -\frac{\frac{\partial^2 f_n}{\partial x_n \partial k_n}}{\frac{\partial^2 f_n}{\partial k_n^2}} > 0$$

(A4)
$$\frac{\partial x_n}{\partial k_n} = -\frac{\frac{\partial^2 f_n}{\partial x_n \partial k_n}}{\frac{\partial^2 f_n}{\partial x_n^2}} > 0$$

(A5)
$$\frac{\partial z_n}{\partial p} = \frac{1}{\frac{\partial^2 g_n}{\partial z_n^2}} > 0,$$

(A6)
$$\frac{\partial z_n}{\partial Q} = -\frac{\frac{\partial^2 g_n}{\partial z_n \partial Q}}{\frac{\partial^2 g_n}{\partial z_n^2}} < 0,$$

(A7)
$$\frac{\partial z_n}{\partial a_n} = -\frac{\frac{\partial^2 g_n}{\partial z_n \partial a_n}}{\frac{\partial^2 g_n}{\partial z_n^2}} > 0,$$

(A.8)
$$\frac{\partial a_n}{\partial z_n} = -\frac{\frac{\partial^2 g_n}{\partial a_n \partial z_n}}{\frac{\partial^2 g_n}{\partial a_n^2}} > 0,$$

(A.9)
$$\frac{\partial a_n}{\partial Q} = -\frac{\frac{\partial^2 g_n}{\partial Q \partial z_n}}{\frac{\partial^2 g_n}{\partial a_n^2}} > 0,$$

Finally note that

(A8)
$$\left(\frac{\partial^2 f_n}{\partial x_n k_n}\right)^2 = \frac{\partial^2 f_n}{\partial k_n^2} \frac{\partial^2 f_n}{\partial x_n^2},$$

which, because of the linear homogeneity of the production function f_n , n = N, S, directly follows from Euler's equation.

Proposition 1: Let sectors behave as price takers and let the world markets both for vulnerable and capital be in equilibrium. Then

(3.4)
$$\left(\frac{\partial z_N}{\partial p} + \frac{\partial z_S}{\partial p} - \frac{\partial x_N}{\partial p} - \frac{\partial x_S}{\partial p}\right) dp = \left(\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}\right) dQ + \frac{\partial z_N}{\partial a_N} da_N + \frac{\partial z_S}{\partial a_S} da_S.$$

Proof: By taking the total differential of condition (2.1) we get

$$\left(\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}\right) dQ + \frac{\partial z_N}{\partial a_N} da_N + \frac{\partial z_S}{\partial a_S} da_S - \frac{\partial x_N}{\partial k_N} dk_N - \frac{\partial x_S}{\partial k_S} dk_S = \left(\frac{\partial z_N}{\partial p} + \frac{\partial z_S}{\partial p} - \frac{\partial x_N}{\partial p} - \frac{\partial x_S}{\partial p}\right) dp$$

or, since $dk_N = -dk_S$ (see condition (2.2))

$$\left(\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}\right) dQ + \frac{\partial z_N}{\partial a_N} da_N + \frac{\partial z_S}{\partial a_S} da_S - \left(\frac{\partial x_N}{\partial k_N} - \frac{\partial x_S}{\partial k_S}\right) dk_N =$$
$$\left(\frac{\partial z_N}{\partial p} + \frac{\partial z_S}{\partial p} - \frac{\partial x_N}{\partial p} - \frac{\partial x_S}{\partial p}\right) dp$$

Next recall that condition (3.3) implies

$$\frac{\partial f_N}{\partial k_N}(x_N,k_N) = \frac{\partial f_S}{\partial k_S}(x_S,k_S),$$

hence by taking the total differential

$$\frac{\partial^2 f_N}{\partial k_N^2} dk_N + \frac{\partial^2 f_N}{\partial x_N \partial k_N} dx_N = \frac{\partial^2 f_S}{\partial k_S^2} dk_S + \frac{\partial^2 f_S}{\partial x_S \partial k_S} dx_S.$$

Now, since $dx_n = \frac{\partial x_n}{\partial k_n} dk_n + \frac{\partial x_n}{\partial p} dp$ and since conditions (A4) and (A8) together imply

$$\frac{\partial^2 f_n}{\partial x_n \partial k_n} \frac{\partial x_n}{\partial k_n} = -\frac{\partial^2 f_n}{\partial k_n^2},$$

we observe

(A9)
$$\frac{\partial^2 f_N}{\partial x_N \partial k_N} \frac{\partial x_N}{\partial p} dp = \frac{\partial^2 f_S}{\partial x_S \partial k_S} \frac{\partial x_S}{\partial p} dp,$$

which means (see (A2) and (A4))

(A10)
$$\frac{\partial x_N}{\partial k_N} = \frac{\partial x_S}{\partial k_S}$$

Consequently

$$dp = \frac{\frac{\partial z_N}{\partial Q} + \frac{\partial z_S}{\partial Q}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_S}{\partial p}} dQ + \frac{\frac{\partial z_N}{\partial a_N}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p}} da_N + \frac{\frac{\partial z_S}{\partial a_S}}{\frac{\partial x_N}{\partial p} + \frac{\partial x_S}{\partial p} - \frac{\partial z_N}{\partial p}} da_S$$

Corollary 1: Let sectors behave as price takers and let the world markets both for vulnerable and capital be in equilibrium. Then a marginal change in means of temperature and /or adaptation expenditure does not imply a reallocation of capital.

Proof: Consider the first order conditions

(3.1)
$$\frac{\partial f_n}{\partial x_n}(x_n,k_n) - p = 0, n = N, S,$$

(3.3)
$$\frac{\partial f_n}{\partial k_n}(x_n,k_n)-r=0, n=N,S.$$

which implies

$$\frac{\partial f_N}{\partial x_N} - \frac{\partial f_S}{\partial x_S} = 0$$
$$\frac{\partial f_N}{\partial k_N} - \frac{\partial f_S}{\partial k_S} = 0.$$

By taking the total differential and by taking into account that $dk_N = dk_S$, this gives the following system

$$\begin{pmatrix} \frac{\partial^2 f_N}{\partial x_N^2} & \frac{\partial^2 f_N}{\partial x_N \partial k_N} + \frac{\partial^2 f_S}{\partial x_S \partial k_S} \\ \frac{\partial^2 f_N}{\partial x_N \partial k_N} & \frac{\partial^2 f_N}{\partial k_N^2} + \frac{\partial^2 f_S}{\partial k_S^2} \end{pmatrix} \begin{pmatrix} dx_N \\ dk_N \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 f_S}{\partial x_S^2} \\ \frac{\partial^2 f_S}{\partial x_S \partial k_S} \end{pmatrix} dx_S.$$

Let *A* denote the determinate of the above matrix, i.e. $B = \frac{\partial^2 f_N}{\partial x_N^2} \frac{\partial^2 f_S}{\partial k_S^2} - \frac{\partial^2 f_N}{\partial x_N \partial k_N} \frac{\partial^2 f_S}{\partial x_S \partial k_S}$, which under the usual condition that cross derivatives do not dominate is positive. Then by using Cramer's rule, we obtain

$$dk_N = \frac{1}{B} \left[\frac{\partial^2 f_N}{\partial x_N^2} \frac{\partial^2 f_S}{\partial x_S \partial k_S} - \frac{\partial^2 f_N}{\partial x_N \partial k_N} \frac{\partial^2 f_S}{\partial x_S^2} \right] dx_S.$$

Condition (A9), however, implies $\frac{\partial^2 f_N}{\partial x_N^2} \frac{\partial^2 f_S}{\partial x_S \partial k_S} - \frac{\partial^2 f_N}{\partial x_N \partial k_N} \frac{\partial^2 f_S}{\partial x_S^2} = 0$, and hence $dk_N = 0$.