Discussion Paper No. 11-018

Fiscal Policy and Growth with Complementarities and Constraints on Government

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Non-Technical Summary

Governments are subject to a number of constraints that affect their ability to set fiscal policy optimally. This paper considers two. Firstly, governments are inevitably imperfectly informed about the production technology of firms and household preferences which both determine what level and what composition of public spending are optimal. Secondly, governments are constrained in their ability to change various elements of fiscal policy, due to, for example, quasi-fixed expenditure items such as social welfare benefits linked to entitlement conditions and interest payments that depend on the stock of public debt accumulated in the past. Other factors that constrain governments in this respect include limited administrative capacity (governments are only able to concentrate on a limited number of issues at a time) and limited political capital required for fiscal changes. We refer to this type of constraint as budget rigidities.

In this paper we derive the optimal level and the optimal composition of public spending in the situation in which governments are constrained in their ability to alter both and know little about the relative growth benefits of different public spending categories. As a theoretical framework, we use endogenous growth models with public finance in which fiscal policy affects the long-run growth rate of the economy. Previous papers that have used this class of models typically ignore these constraints on governments, and assume that private and public inputs are substitutes.

We generate a number of interesting results with respect to optimal fiscal policy in the presence of budget rigidities, informational limitations, or both in the realistic case when private and public inputs to private production that are provided by the government are complements. First, we show that the optimal level of productive public spending and the composition are interrelated: in particular, the optimal level of spending is higher when the composition is suboptimal, and the optimal share of public resources allocated to public investment may be very low when the level of spending is either too high or too low due to budget rigidities. This result contrasts with the common perception that public investment is the most important public spending category for long-run growth.

Secondly, we show that imperfect knowledge of the government is much more likely a constraining factor for fiscal policy with complementarity. The number of parameters that determine optimal fiscal policy increases compared to the case when public and private inputs are substitutes, and some model parameters which are commonly perceived not to impact on optimal fiscal policy in some cases are shown to play indeed a role. These results demonstrate that determining optimal fiscal policy even under growth maximization is highly complex in practice since for some of these parameters robust empirical estimates are not available and again contrast with the more simple but likely unrealistic case of substitutability.

The third contribution is to analyze fiscal reform when governments are imperfectly informed *and* when budget rigidities imply that the government is only able to implement piecemeal fiscal policy changes which take the existing fiscal policy as its starting point. Given that the optimal fiscal policy parameter values are unknown, the optimal size and direction of fiscal policy parameter changes are both unclear. We then show that by limiting the magnitude of policy parameter changes, budget rigidities in fact reduce informational requirement so that in most situations, the design of optimal fiscal reform only requires information about the direction of change but not about its magnitude. We further stress the need for better information by showing that with complementarity, fiscal reforms are more likely to reduce rather than to augment long-run growth.

Das Wichtigste in Kürze

Regierungen unterliegen Rahmenbedingungen, die ihre fiskalpolitischen Handlungsmöglichkeiten erheblich einschränken können. Diese Studie berücksichtigt solche Rahmenbedingungen, die auf verschiedenen Ebenen liegen. Regierungen verfügen nur über unvollständige Informationen. Das gilt gleichermaßen für die von den Unternehmen verwandte Produktionstechnologie wie für die Präferenzen der privaten Haushalte, beides wichtige Parameter für die optimale Höhe und Zusammensetzung von Staatsausgaben. Zudem ist der Spielraum für fiskalpolitische Reformen begrenzt. So sind große Ausgabenblöcke wie z.B. im Sozialbereich gesetzlich festgelegt und lassen sich nur bedingt ändern, und die Höhe von Zinszahlungen hängt von den in der Vergangenheit akkumulierten Schulden ab. Politische Restriktionen können sich zudem aus administrativen Kapazitätsengpässen der Regierung oder dem Mangel an politischem Kapital, das für Reformen benötigt wird, ergeben. Beides zusammen führt zu budgetärer Starrheit.

Die Studie bestimmt die optimale Höhe und die optimale Zusammensetzung von Staatsausgaben unter diesen Rahmenbedingungen. Unsere Untersuchung basiert auf einem endogenen Wachstumsmodell, wonach Fiskalpolitik die langfristige Wachstumsrate der Volkswirtschaft beeinflusst. Bisherige Arbeiten, die solche Modelle benutzen, ignorieren zumeist diese Restriktionen und unterstellen, dass private und öffentliche Inputs für private Produktion Substitute sind.

Die Resultate der Studie lassen sich wie folgt zusammenfassen. Erstens zeigen wir, dass die optimale Höhe und Zusammensetzung der Staatsausgaben interdependent sind. Die optimale Höhe staatlicher Ausgaben steigt, wenn die Zusammensetzung suboptimal ist, und der optimale Anteil von öffentlichen Investitionen im Gesamtbudget sinkt, wenn die Summe der Staatsausgaben aufgrund budgetärer Starrheit nicht dem optimalen Niveau entspricht. Das macht die weitverbreitete Annahme fraglich, wonach öffentliche Investitionen die wichtigste Ausgabenkategorie für langfristiges Wachstum sind.

Zweitens zeigen wir, dass unvollständige Information von der Regierung mit größerer Wahrscheinlichkeit eine Beschränkung bei Komplementarität darstellt. Die Anzahl der Parameter, die die optimale Fiskalpolitik bestimmen, ist signifikant höher als in dem Fall, wenn öffentliche und private Inputs Substitute sind. Unsere Analyse zeigt außerdem, dass einige Modellparameter, von denen bisher allgemein angenommen wird, dass sie in einigen Fällen keine Rolle für die optimale Fiskalpolitik spielen, doch von den Regierungen berücksichtigt werden müssen. Diese Ergebnisse demonstrieren, dass die Bestimmung optimaler Fiskalpolitik unter dem Ziel der Wachstumsmaximierung komplex ist, insbesondere, da oftmals keine robusten Schätzungen der Modellparameter vorliegen. Die Ergebnisse unterscheiden sich außerdem vom - unrealistischen - Fall, wenn Inputs Substitute sind.

Drittens analysieren wir fiskalpolitische Reformen bei unvollständiger Information der Regierung und bei budgetärer Starrheit, die impliziert, dass der Ausgangspunkt für Reformen die bestehende Fiskalpolitik ist, so dass Pfadabhängigkeit vorliegt. In dieser Situation ist sowohl die optimale Höhe als auch die optimale Richtung von Änderungen fiskalpolitischer Parameter unbekannt. Die Studie zeigt, dass in diesem Fall die Regierung vor allem Informationen zu der optimalen Richtung von Parameteränderungen benötigt. Wir heben zudem die Wichtigkeit von besseren Informationen der Regierung hervor indem wir zeigen, dass die Wahrscheinlichkeit, dass fiskalpolitische Reformen zu niedrigerem Wachstum führen, über 50 v.H. liegt.

Fiscal Policy and Growth with Complementarities and Constraints on Government

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Abstract

This paper considers the implications of complementarity in private production and constraints on government for optimal fiscal policy. Using an endogenous growth model with public finance, it derives three central results which modify findings in the literature under standard assumptions. First, it shows that optimal public spending composition and taxation are interrelated so that first- and second-best fiscal policies differ. Second, it shows that the growth-maximizing fiscal policy is affected by preference parameters. Third, it shows that with budget rigidities and informational limitations, knowledge about the optimal fiscal policy parameter values is not necessary for growth-enhancing fiscal policy adjustments.

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1 Introduction

Governments are subject to a number of constraints that affect their ability to set fiscal policy optimally. This paper considers two. Firstly, governments are inevitably imperfectly informed about the production technology of firms and household preferences. The importance of imperfect information for macroeconomic policy more generally is increasingly recognized. In the context of monetary policy, Greenspan (2004, p.39) notes that "policymakers often have to act, or choose not to act, even though we may not fully understand the full range of possible outcomes, let alone each possible outcome's likelihood". Phelps (2007, p.xix) proposes that "issues have to be rethought in a way that makes the ever-imperfect knowledge of [...] policymakers an integral part of the analysis".[1] In relation to fiscal policy, imperfect information is also often seen as an important source of second-best situations (Lipsey (2007)). Since Lipsey and Lancaster (1956), it is well known that second-best interactions may imply that first-best policies are not desirable, and that the rules for optimal policy change. There is a large literature on optimal taxation and other issues in public economics which addresses second-best problems. However, the existing literature on optimal taxation typically assumes that public spending requirements are exogenously given (Renström (1999)).

Secondly, governments are constrained in their ability to change various elements of fiscal policy, due to, for example, quasi-fixed expenditure items such as social welfare benefits linked to entitlement conditions, interest payments that depend on the previously accumulated stock of public debt, and the wages of public employees. Mattina and Gunnarsson (2007) estimate that the share of spending that is non-flexible due to legal obligations (which includes social benefits, interest payments, compensation of public employees, and subsidies) amounts to 72% in Slovenia. This figure may even be an underestimate of the true extent of non-flexible public spending: the ability of the government to change fiscal policy is also constrained by limited ad-

[1] Frydman and Goldberg (2007) provide an excellent survey of the evolution of the notion of imperfect knowledge in macroeconomics.

ministrative capacity (governments are only able to concentrate on a limited number of issues at a time) and by limited political capital required for fiscal changes (most public spending categories have beneficiaries and hence lobbies that may oppose change and therefore try to influence policy makers). We refer to this type of constraint as budget rigidities. Given the underlying causes of budget rigidities, it seems plausible that they persist in the long run.

In this paper we consider the impact of budget rigidities and imperfect knowledge on optimal fiscal policy, specifically the optimal *level* and the optimal *composition* of public spending, using an endogenous growth models with public finance *under growth maximization*.[2] Given that governments may be constrained in their ability to alter either total productive public spending or its mix and may know little about the relative growth benefits of different public spending categories, these models, because they allow for the inclusion of the productive effects of several public spending categories, offer a potentially interesting addition to the literature.[3] This paper therefore extends the endogenous growth-public finance literature by explicitly considering imperfect information and budget rigidities as constraints for the government. In some ways, it is similar to García Peñalosa and Turnovsky (2005) who consider enforcement problems as an alternative constraint on fiscal policy which makes capital income taxation desirable in contrast to a first-best situation where in the long run, it is optimal to completely shift the burden from factor income taxation from capital to labor.

The model we develop considers two distinct productive public spending categories, public services and public capital, so that the level of public

[2] As shown by Misch et al. (2008a), growth maximization is often a reasonable close proxy of welfare maximization and often easier to compute.

[3] Despite their prevalence in practice and despite the fact that they give rise to the possibility of second-best outcomes, informational limitations and budget rigidities have not previously been considered in models of this type. Existing papers instead derive the rules for the optimal volume and composition of public spending in the absence of such constraints on government. The only implicit constraint that these models impose on the government is the fact that lump sum taxation is not available and that economic agents take taxes and public spending as given so that we consider these rules for fiscal policy as first-best. We recognize however that frequently, the unavailability of lump-sum taxes within a market economy is considered as a source of second-best situations.

spending does not need to be exogenously fixed. The government uses a flat income tax to finance public spending; in this sense, we hold the structure of taxation constant. Our model differs from those found in the literature in the sense that it considers various model features introduced separately by the literature on endogenous growth models with public finance in a single framework: in the model we develop, the governments provides public services and accumulates public capital similarly to Tsoukis and Miller (2003) and by Ghosh and Roy (2004), private and public inputs are complements as in Devarajan et al. (1996), the efficiency of public spending is considered as Agénor (2010) for example, and we model the production of public services in greater detail similarly to Agénor (2008b) for example.

In this setup, it is realistic to assume that there are essentially information asymmetries: private agents are perfectly informed in the sense that they have full knowledge of their preference and technology parameters and that they obviously observe fiscal policy. In contrast, governments can reasonably be assumed to be imperfectly informed about the technology of production and household preferences in the sense that they do not know their exact values because exact empirical estimates are often difficult to find.

From these modifications to the standard setup, we generate a number of interesting results with respect to optimal fiscal policy in the presence of budget rigidities, informational limitations, or both. Firstly, in contrast to the case of CES technology, Cobb-Douglas technology assumed in most endogenous growth models with public finance has counterintuitive implications when either the level of public spending or the composition of public spending is fixed due to budget rigidities, or when public spending is not efficient under the objective of *growth maximization*. As a simple example, consider the case when there is one public expenditure category with productive effects and one that is not productive, and when the level of *productive* public spending is currently at its growth-maximizing level. Reallocating a greater share of public resources towards the unproductive public spending category implies that the level of *total* public spending is no longer optimal and must be increased to ensure that the level of *productive* public spending remains at its optimum. However, with two different productive public expenditure categories and Cobb-Douglas technology, the growth-maximizing level and the growth-maximizing composition are independent of each other contrary to what this simple example would suggest. Further, the technical efficiency of public spending does not have an impact either. In contrast, when CES technology is assumed, the optimal level of productive public spending depends on its composition and vice versa, and the technical efficiency of public spending matters. In particular, it is shown that the second-best level of taxation is higher when the composition is suboptimal, and that the second-best share of public resources allocated to public investment may be very low when the level of taxation (i.e. the level of public spending) is either below or above its first-best level (we assume that a first-best situation corresponds to the case where the government is fully informed about all technology and preference parameters and where all policy parameters are fully flexible). Similar results arise when public spending is not efficient. These are additional, but very simple and intuitive, cases of second-best interaction in public finance that have largely been ignored in the literature. These results are also consistent with Ghosh and Gregoriou (2008) who find that increasing the share of current spending at the expense of capital spending is growth-enhancing using data from developing countries.

Secondly, we show that imperfect knowledge of the government is much more likely a constraining factor for fiscal policy under CES technology when inputs to private production are complements. With Cobb-Douglas technology, the standard result is that only share parameters of the production function of final output determine the growth-maximizing tax rate (i.e. the volume of public spending) and the growth-maximizing expenditure composition. Moving away from the simple Cobb-Douglas case extends the number of parameters that determine optimal fiscal policy and thereby increases the informational requirements. We show that under CES technology and growth maximization, optimal policy is also determined by preference parameters, other technology parameters (in addition to the share parameters), and the stock-flow properties of public inputs (which can be interpreted as the rate of depreciation of public inputs to private production). These results demonstrate that determining optimal fiscal policy even under growth maximization is highly complex in practice, and again contrast with the more simple but likely unrealistic case of Cobb-Douglas technology, in particular since for some of these parameters robust empirical estimates are not available. This implies that governments face important informational limitations which in turn gives rise to a second-best situation where the government is unable to set fiscal policy parameters at their first-best values. The result is also an obvious analogy to the theory of taxation which demonstrates that even in simple static tax models, the optimal tax system depends on a wide range of factors for which it may be difficult to find empirical counterparts even when a range of simplifying assumptions is made (Creedy (2009)). These types of informational limitations are out of bounds for policy makers and cannot be removed directly so that the second-best situation persists.[4] The fact that the stock-flow properties of public inputs to private production do not affect growth-maximizing fiscal policy under Cobb-Douglas technology whereas they are important under CES technology is another example of why Cobb-Douglas technology is counterintuitive.

The third contribution is to analyze fiscal reform when budget rigidities and imperfect information simultaneously constrain fiscal policy. In contrast to above, we now assume that budget rigidities limit the magnitude of fiscal policy changes and do not imply that particular fiscal policy parameters are completely fixed. Therefore, the government can only implement piecemeal fiscal policy changes which take the existing fiscal policy as its starting point. It is shown that in line with standard second-best theory, designing growth-enhancing fiscal reforms is complex because fiscal policy parameters may have to be shifted away from their first-best to their second-best values. However, based on our previous arguments, the optimal fiscal policy parameter values are unknown in practice so that the optimal size and the optimal direction of fiscal policy parameter changes are both unclear. We then show that by limiting the magnitude of policy parameter changes, budget rigidities in fact reduce informational requirement so that in most situations, the design of

[4]In this sense, the commonly encountered argument which is that in a second-best situation the government should simply remove the distortion which gives rise to a second-best situation (Hoff (2001)) is not valid in this situation.

optimal fiscal reform only requires information about the *direction* of change but not about its *magnitude*. The reason is the concavity of the growth function which implies that with imperfect knowledge, the expected change of the growth rate of any fiscal reform is negative; it is only positive when there is greater certainty about the direction in which fiscal parameters must be changed so that they approach their optimal values. We further stress the need for better information by showing that with complementarity, even fiscal reforms which are relatively modest in size may result in sizeable and possibly negative changes of the growth rate.

These results have some strong policy implications. First, this new framework suggests that commonly held beliefs about what constitutes optimal fiscal policy are not valid. In particular, it is shown that household preferences not only affect the growth rate but also the growth-maximizing policy. This contrasts with standard models where the growth-maximizing fiscal policy is determined only by share parameters, and where governments can therefore ignore household preferences to set fiscal policy in a growth-maximizing way. More importantly, due to second-best interactions in the model, optimal public investment levels may be very low despite the fact that the output elasticity of public capital significantly differs from zero and even though public investment is often seen as the most important public spending category for long-term growth. While our model does not take into account the indirect effects of public capital as for instance in Agénor (2008b), our results still suggest that with low levels of revenue collection which is the case in many developing countries, optimal public investment is much lower than in a first-best situation. Second, our results show that the most important information for fiscal reform in practice is the direction of the policy parameter change which is likely easier to obtain than information about its magnitude.

The paper is organized as follows. Section 2 develops the model and derives the equilibrium of the market economy. Section 3 derives optimal fiscal policy under the assumption of Cobb-Douglas technology in the absence of constraints as a benchmark case. Section 4 demonstrates the impact of budget rigidities on optimal fiscal policy when some (but not all) fiscal policy parameters are exogenously set below or above their first-best levels. Section

5 shows that imperfect information is more likely a constraining factor under CES technology because the optimal fiscal policy responds to changes of a range of model parameters including the elasticity of substitution, preference parameters and the rate of depreciation of public inputs of which exact empirical estimates may not be available. Section 6 simultaneously considers budget rigidities and imperfect knowledge and analyzes the implications and the informational requirements of fiscal reform. Section 7 concludes.

2 The Model

The public finance growth framework we adopt in the paper is based on Devarajan et al. (1996). We extend their model by *simultaneously* considering public services and public capital as in Tsoukis and Miller (2003) and Ghosh and Roy (2004), CES technology as in the original model by Devarajan et al. (1996), the efficiency of public spending as in Agénor (2010) and a production function of public services in a similar way as Agénor (2008b) for example. We assume that there is a large number of infinitely lived households and firms that is normalized to one so that firm entry and exit cancel out or do not occur and population growth is zero.

The representative firm produces a single composite good using private capital (k) which is broadly defined to encompass physical and human capital, and two public inputs, G_1 and G_2, based on CES technology:

$$y = (\theta k^{\upsilon} + \alpha_1 G_1^{\upsilon} + \alpha_2 G_2^{\upsilon})^{\frac{1}{\upsilon}} \tag{1}$$

where θ, α_1 and α_2 are share parameters with $\theta = 1 - \alpha_1 - \alpha_2$. The productivity of private capital used by the individual firm therefore positively depends on G_1 and G_2 which can be conceived to be provided by different government sectors (e.g. education and transport infrastructure). For instance, private vehicles can be used more productively when the quality of the road network increases. G_1 and G_2 are non-rival and provided free of charge to the agents of the economy. υ determines the elasticity of substitution which corresponds to $\frac{1}{1-\upsilon}$. With $\upsilon = 0$, the production technology is Cobb-Douglas.[5]

[5] We recognize that a more general specification of (1) would be a nested CES function

G_1 denotes the amount of productive public services provided by the government (e.g. public law enforcement, public education services), whereas G_2 denotes the stock of public capital (e.g. public infrastructure) which the government accumulates through public investment, $\dot{G}_2$. In other words, G_1 can be interpreted as a public input to private production which fully depreciates over one period (i.e. the depreciation rate is 1), and G_2 can be seen as a public input with infinite lifetime that does not depreciate at all (i.e. the depreciation rate is 0). To capture the notion that factors of production are complements rather than substitutes, it is assumed that $\upsilon \leq 0$. This assumption seems justified when considering that public inputs provided by the government fundamentally differ from private inputs, such that it may be very costly for firms to substitute for them. For instance, privately generating electricity is typically much more expensive than using electricity from the public grid.

The government finances total public expenditure by levying a flat tax, τ, on income, and the government budget is assumed to be always balanced. We further assume that the technical efficiency of public spending may vary. For instance, inefficiencies arise if the government purchases the inputs for G_1 and G_2 at a high price, or if there is waste due to corrupt bureaucrats. While changing the level of technical efficiency may also involve a resource cost, we refrain from modelling this in greater detail for simplicity because this is not needed to derive our main results in later sections.

G_1 itself is produced using two different inputs, G_A and G_B, which can be interpreted as sub-sectoral public spending categories, based on CES technology:

$$G_1 = (\omega G_A^{\varepsilon} + \beta G_B^{\varepsilon})^{\frac{1}{\varepsilon}} \tag{2}$$

with $\omega = 1 - \beta$ and where ε determines the elasticity of substitution. This feature of the model allows for a richer specification of fiscal policy because the inter-sectoral allocation (i.e. the allocation of public resources between G_1 and G_2) and the sub-sectoral allocation of public resources (i.e. the

that allows for different elasticities of substitution between G_1 and G_2 on the one hand and between G_1 and G_2 taken together and private capital on the other. However, for the purpose of this paper, our specification of the production function is sufficient.

allocation of public resources between G_A and G_B) can be distinguished. It allows us to analyze the effects of misallocation at the sub-sectoral level on the growth-maximizing tax rate and the inter-sectoral composition below. In analogy to the production of final output, we assume that $\varepsilon \leq 0$ reflects the notion that G_A and G_B, are complements. For simplicity, we set $\varepsilon = \upsilon$ which facilitates the derivation of the results but does not change them qualitatively. G_A and G_B may represent the amounts of goods and services and spending on public administration used for the production of G_1.

Let ϕ_1 (ϕ_2) determine the inter-sectoral allocation of public resources and denote the share of total public expenditure that is allocated to G_1 ($\dot{G}_2$) with $\phi_1 + \phi_2 = 1$ (i.e. the share of resources allocated to public investment is $\phi_2 = 1 - \phi_1$) and let ϕ_A (ϕ_B) denote the share of public spending on G_1 that is allocated to G_A (G_B) with $\phi_A + \phi_B = 1$. Further, let κ_1 and κ_2 denote the technical efficiency of public spending on G_1 and G_2 which we assume to be different from the allocative efficiency. G_j (with $j = A, B$) can therefore be written as

$$G_j = \kappa_1 \phi_1 \phi_j \tau y \tag{3}$$

Using (2) and (3), the amount of G_1 can therefore be written as

$$G_1 = \kappa_1 \phi_1 (\omega \phi_A^{\varepsilon} + \beta \phi_B^{\varepsilon})^{\frac{1}{\varepsilon}} \tau y \tag{4}$$

The level of public investment, $\dot{G}_2$, can be written as

$$\dot{G}_2 = \kappa_2 \phi_2 \tau y \tag{5}$$

We are normalizing k_i so that at $k_i = 1$ (with $i = 1, 2$), public spending is assumed to be perfectly efficient in a technical sense. For simplicity, we assume that increasing the efficiency of public spending is possible at no cost (i.e. increasing k_i does not involve a trade-off). While in principle, this means that governments would never choose any value for k_i below one in the absence of budget rigidities, this assumption merely serves as a simplification and allows asking the hypothetical question about what *would* happen if public spending was *not* perfectly efficient. However, to capture the notion that efficiency gains are inevitably limited, we assume that $\kappa_1 \leq 1$ and that $\kappa_2 \leq 1$.

The households own the firms and therefore receive all their output net of taxation which they either reinvest in the firms to increase their capital stock or which they use for consumption depending on their preferences and the returns on private capital. Private investment by the representative household equals

$$\dot{k} = (1 - \tau)y - c \tag{6}$$

The representative household chooses the consumption path to maximize lifetime utility U given by

$$U = \int_0^\infty \left(\frac{c^{1-\sigma}}{1-\sigma} \right) e^{-\rho t} dt \tag{7}$$

subject to the household's resource constraint given by (6) taking τ, G_1, G_2 and $k_0 > 0$ as given.[6] From the first-order conditions, the growth rate of the household's consumption and of the economy can be written in familiar form as

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \left((1 - \tau) y_k - \rho \right) \tag{8}$$

In order to ensure that the transversality condition holds and does not constrain the choice of τ and $\phi_{1,2}$, it is assumed that $\sigma > 1$.[7]

Along the balanced growth path, output can be expressed as

$$y = \frac{\dot{y}}{\gamma} \tag{9}$$

Using (9) to substitute for y in (5), and integrating, yields

$$G_2 = \frac{\kappa_2 \phi_2 \tau}{\gamma} y \tag{10}$$

For the remainder of this section, we assume Cobb-Douglas technology as a means to simplify the analytical expressions. Hence $\upsilon = 0$ (and $\varepsilon = 0$), and the production function can then be written as

$$y = k^\theta G_1^{\alpha_1} G_2^{\alpha_2} \tag{11}$$

[6] The time subscript is omitted whenever possible. A dot over the variable denotes its derivative with respect to time. The initial stock of public capital must also be greater than zero.

[7] The transversality condition can be written as $\lim_{t \to \infty} [\lambda k] = 0$ where λ is the costate variable of the current-value Hamiltonian.

where $\theta = 1 - \alpha_1 - \alpha_2$. The marginal product of capital, y_k, can be written as

$$y_k = \theta \left(\frac{G_1}{y}\right)^{\alpha_1} \left(\frac{G_2}{y}\right)^{\alpha 2} \left(\frac{y}{k}\right)^{\alpha_1+\alpha_2} \tag{12}$$

Using (4), (10) and (11) to substitute for G_1/y, G_2/y and y/k in (12), and using (12) to substitute for y_k in (8) yields

$$\gamma = \frac{1}{\sigma}\left((1-\tau)\theta\left((\phi_{1A})^{\omega}(\phi_{1B})^{\beta}\kappa_1\phi_1\tau\right)^{\frac{\alpha_1}{\theta}}\left((\kappa_2\phi_2\tau)\frac{1}{\gamma}\right)^{\frac{\alpha_2}{\theta}} - \rho\right) \tag{13}$$

Note that (13) is not an expression of the growth rate but merely an equation that the growth rate satisfies because γ also appears on the RHS.

The Appendix shows that the equilibrium of the model is saddlepoint stable within relevant parameter ranges, and that the balanced growth path is unique. Along the balanced growth path, c, k, G_1, G_2 and y all grow at the same rate.

3 Benchmark Case: Optimal Fiscal Policy with Cobb-Douglas Technology

This section derives optimal fiscal policy when output (y) and public services (G_1) are produced using Cobb-Douglas technology and when the government does not face constraints under the objective of growth maximization. Apart from the budget constraint and the unavailability of lump-sum taxation, fiscal policy is hence not constrained by other factors so that this situation can be considered as first-best. This benchmark case will allow us to demonstrate the role of complementarities and government constraints in later sections.

For simplicity, we assume throughout the paper that the objective of the government is to maximize growth in contrast to papers that derive the welfare-maximizing fiscal policy in similar frameworks as Ghosh and Roy (2004) for example. While in these models, growth and welfare maximization are not identical, in practice, growth maximization is less complex and more common as changes in output are easier to observe than welfare. In addition, the differences in outcomes between growth and welfare maximization in similar models are often small (Misch et al. (2008a)). Optimal fiscal policy therefore refers to the growth-maximizing values of the tax rate and of the public spending shares of public services and public investment (denoted by τ^* and $\phi^*_{1,2}$, respectively).

Cobb-Douglas technology implies $\upsilon = \varepsilon = 0$. Since the model is based on the assumption that there is no cost to increase efficiency, the government sets $k_{1,2}$ at their maximum values $\kappa^*_{1,2}$ to ensure that public spending is fully efficient:

$$\kappa^*_{1,2} = 1 \tag{14}$$

(which obviously maximizes growth and welfare, and which does not depend on the underlying production technology). Implicitly differentiating (13) yields the growth-maximizing income tax rate, τ^*, which corresponds to

$$\tau^* = \alpha_1 + \alpha_2 \tag{15}$$

, the growth-maximizing inter-sectoral expenditure shares, $\phi^*_{1,2}$, which correspond to

$$\phi^*_{1,2} = \frac{\alpha_{1,2}}{\alpha_1 + \alpha_2} \tag{16}$$

where $\phi^*_1 + \phi^*_2 = 1$, and the growth-maximizing sub-sectoral expenditure shares within G_1, $\phi^*_{A,B}$, which correspond to

$$\phi^*_A = \omega \tag{17}$$

and

$$\phi^*_B = \beta \tag{18}$$

(15), (16), (17) and (18) suggest that with Cobb-Douglas technology, the growth-maximizing tax rate and expenditure shares only depend on share parameters of the production functions of final output and of public services. These results can be seen as representative of the existing literature: Similar results arise for instance in Agénor (2008a), Agénor (2008b) and Tsoukis and Miller (2003). They are also directly implied by Barro (1990) and Futagami et al. (1993) who first presented endogenous growth models with productive public service and public capital, respectively.

These results have important implications with respect to the effects of government constraints even though we did not consider them in this section. First, second-best interactions cannot arise in the sense that the optimal level of taxation, the optimal public spending composition and the efficiency of public spending are not interrelated. This means for instance that τ^* also represents the optimal level of taxation if $\phi_{1,2} \neq \phi^*_{1,2}$ and $\kappa^*_{1,2} < 1$. Therefore, the consideration of budget rigidities does not yield additional insights under Cobb-Douglas technology. In contrast, Section 4 demonstrates that with CES technology, budget rigidities have indeed important implications for optimal fiscal policy.

Second, with Cobb-Douglas technology, imperfect information may not play an important role for optimal fiscal policy either because the informational requirements to set fiscal policy optimally are limited: the calculation of growth-maximizing fiscal policy parameter values is straight forward, and they solely depend on few technology parameters. While their exact values may not be known, rough estimates of their magnitude are still likely available which then still enable to obtain reasonable estimates of τ^*, $\phi^*_{1,2}$ and $\phi^*_{A,B}$. With CES technology, it is much more likely that governments are affected by informational constraints as shown in Section 5 because additional parameters determine optimal fiscal policy.

4 Optimal Fiscal Policy and Budget Rigidities

This section analyzes optimal fiscal policy with CES technology and budget rigidities and therefore considers a more general setting than the previous section. We model budget rigidities by assuming that the government is unable to adjust one or more fiscal policy parameters which can then be considered as exogenously given. To some extent, budget rigidities persist in the long run: Major tax reforms and major reallocations of public resources are relatively rare events, even over longer time spans. In particular, we consider four distinct second-best situations. In each of them, one of the four types of fiscal policy parameters in the model (we distinguish the technical efficiency of public spending determined by κ_i, the rate of taxation τ, the inter-sectoral allocative efficiency determined by ϕ_i, and the sub-sectoral efficiency of public spending determined by ϕ_j) is exogenously given and cannot be adjusted by the government due to the presence of budget rigidities.

In the first three situations, we abstract from the policy problem related to sub-sectoral allocation within G_1 and set $\beta = 0$ and $\phi_A = 1$ for simplicity. In situation 1, the technical efficiency of public spending on G_1 is fixed at $\kappa_1 < 1$, whereas τ and $\phi_{1,2}$ are freely adjustable. In situation 2, the level of taxation is exogenously given and possibly suboptimal so that $\tau \neq \tau^*$ whereas public spending is fully efficient ($\kappa_{1,2} = 1$) in a technical sense, and the government sets the expenditure shares $\phi_{1,2}$ optimally. In situation 3, the expenditure shares of G_1 and G_2 in total public revenue are exogenously given and possibly suboptimal so that $\phi_{1,2} \neq \phi_{1,2}^*$ whereas public spending is fully efficient in a technical sense ($\kappa_{1,2} = 1$) and τ is freely adjustable. In situation 4, we set $0 < \beta < 1$ and assume that the sub-sectoral expenditure shares of G_A and G_B in spending on G_1 are exogenously given and possibly suboptimal so that $\phi_{A,B} \neq \phi_{A,B}^*$ whereas public spending is fully efficient ($\kappa_{1,2} = 1$) in a technical sense and τ as well as $\phi_{1,2}$ are freely adjustable. Although in Section 6, we model budget rigidities in greater detail, for the purpose of this section, it suffices to assume that the government is unable to address budget rigidities directly and that in each of these cases, they fully constrain government discretion with respect to the fiscal policy parameters in question.

Given that there is no cost to raise efficiency when the efficiency parameters $\kappa_{1,2}$ are adjustable (situations 2 and 3), their optimal values correspond to one so that

$$\kappa_{1,2}^* = 1 \tag{19}$$

As discussed above, (15) and (16) imply that under Cobb-Douglas technology, the growth-maximizing tax rate τ^* and the growth-maximizing expenditure shares $\phi_{1,2}^*$ and $\phi_{A,B}^*$ are independent of each other in the sense that deviations from the growth-maximizing tax rate have no impact on the growth-maximizing spending shares and vice versa. In addition, the sub-sectoral public resource allocations and the technical efficiency of public spending neither affect the optimal taxation nor the optimal inter-sectoral public spending composition.

With CES technology, these results fundamentally change in the sense that the first-best tax rate and the first-best expenditure shares, τ^* and $\phi^*_{1,2}$, are not necessarily identical to their second-best values denoted by τ^{**} and $\phi^{**}_{1,2}$, respectively. As closed-form solutions for the optimal policy parameters are not available with public capital, public services and CES production technology for the market economy, we resort to numerical examples to show that the value of τ^{**} (ϕ_1^{**}) is responsive to changes in $\kappa_{1,2}$, ϕ_1 and ϕ_A (to changes in $\kappa_{1,2}$, τ and ϕ_A). Figures 1, 2, 3 and 4 represent the four distinct second-best situations described in the beginning of this section.

Figure 1 is based on situation 1 and plots the second-best values of τ and $\phi_{1,2}$ as a function of the efficiency parameter κ_1 which is exogenously given and which varies between 0.5 and 1. It demonstrates that when $\kappa_1 < 1$, the second-best tax rate, τ^{**}, and the optimal share of resources allocated to G_1, ϕ_1^{**}, exceed the first-best tax rate, τ^*, and the first-best value of ϕ_1, ϕ_1^*, respectively. The intuition is that with complementarity of the inputs to private production, higher levels of taxation and increased resources allocated to G_1 serve to compensate for low public spending efficiency and thereby prevent the levels of G_1 from falling inefficiently low. This is a standard second-best result: Replicating first-best policies in a second-best situation may not be optimal. It can also be shown that the growth rate is still lower and does not attain its first-best value.

Figure 2 is based on situation 2 and plots the second-best value of ϕ_1, ϕ_1^{**}, as a function of τ which is exogenous and varies between 0 and 1 so that it may deviate from τ^*. It likewise demonstrates that when $\tau \neq \tau^*$, the optimal share of public resources allocated to G_1 (the optimal share of public resources allocated to public investment, $\dot{G}_2$) exceeds (falls short of) the one in first-best situations; hence $\phi_1^{**} > \phi_1^*$ ($\phi_2^{**} < \phi_2^*$). The intuition is as follows. G_2 represents the stock of public capital. Current public spending only affects the additions to the stock of capital and but not the existing stock of public capital. When $\tau < \tau^*$ and $\phi_1 = \phi_1^*$, G_1 drops relatively more than G_2. With complementarity, it is then efficient to allocate a larger share of public resources to G_1 to mitigate the decrease in overall public resources available. In the opposite case, when $\tau > \tau^*$ and $\phi_1 = \phi_1^*$, the intuition is less clear. Given the increase of public resources, the levels of G_1 and of G_2 are higher compared to the first-best situation. However, as G_2 is a stock variable, G_2 grows faster than G_1. With complementarity between G_1 and G_2, it is hence efficient to allocate a greater share of public resources to G_1 so that $\phi_1^{**} > \phi_1^*$.

Figure 3 is based on situation 3 and plots the second-best value of τ, τ^{**}, as a function of ϕ_1 which assumes values between 0 and 1 so that it may deviate from ϕ_1^*. It likewise demonstrates that when $\phi_1 \neq \phi_1^*$, the growth-maximizing level of taxation exceeds the one in a first-best situation; hence $\tau^{**} > \tau^*$. The intuition is similar to situation 1 when κ_1 is set below one. Under misallocation of public resources at the sectoral level, the overall effectiveness of public spending decreases. With complementarity between private and public inputs, it is efficient to compensate for this decrease by increasing the level of taxation (and thereby the level of total public spending).

Figure 4 is based on situation 4 and plots the second-best values of τ and ϕ_1, τ^{**} and ϕ_1^{**}, as a function of ϕ_A which is exogenously given. It demonstrates that under misallocation of resources at the sub-sectoral level ($\phi_A \neq \phi_A^*$), the growth-maximizing level of taxation and the growth-maximizing share of resources allocated to G_1 exceed the ones in a first-best situation (hence $\tau^{**} > \tau^*$ and $\phi_1^{**} > \phi_1^*$). The intuition is similar to situation 1: With sub-sectoral misallocation, the supply level of G_1 falls. With complementarity between private and public inputs, it is efficient to compensate for this decrease by increasing the resources available for spending on G_1 through higher taxation and through reallocation between G_1 and G_2.

These results demonstrate that under CES technology with complementary factor inputs, budget rigidities have important implications for optimal fiscal policy. With regard to optimal taxation and public spending composition, second-best fiscal policy parameters may significantly deviate from their first-best values. Assuming that in practice, it is unlikely that all fiscal policy parameters are set at their first-best values and that Cobb-Douglas technology is not common, first-best policies have little relevance. Determining second-best policy parameters is however more complex because they are not only affected by exogenous model parameters but also by the values of other policy parameters which hence become interrelated.

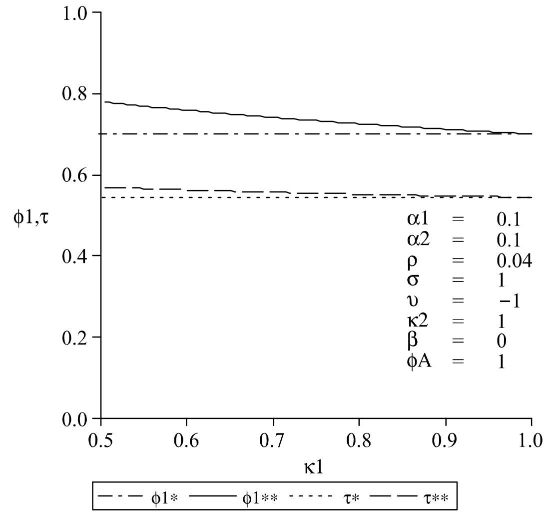
In our model, public capital may represent public infrastructure which is commonly assumed to play an important role in the process of economic development. However, even if the share parameter of public capital in private production significantly differs from zero (i.e. $\alpha_2 > 0$), the optimal share of public resources allocated to public capital ($\phi_2 = 1 - \phi_1$) in second-best situations may still be relatively small relative to α_2 or even close to zero as shown in Figure 2 depending on the rate of taxation which determines the level of public spending. In addition, the share of public resources allocated to public investment depends on the sub-sectoral allocation of public resources within the production of public services (G_1) as demonstrated in Figure 4.

Our results contrast with those of existing papers which do not examine the impact of budget rigidities of growth-maximizing fiscal policy. The results of Ghosh and Roy (2004) are closest to ours and imply that in a model with Cobb-Douglas technology, public capital and public services, the optimal tax rate depends on the composition of public spending and vice versa under welfare maximization. However, they do not discuss this result in detail, and they do not analyze optimal fiscal policy in the event when either the tax rate or the composition of public spending is not set at its first-best level which makes their results difficult to compare with ours.

5 Optimal Fiscal Policy and Imperfect Information

This section analyzes the determinants of optimal fiscal policy in the absence of budget rigidities with CES technology in greater detail and evaluates whether the assumption of imperfect information is more reasonable under CES technology compared to the benchmark case with Cobb-Douglas technology. The previous section has shown that with CES technology, optimal taxation and public spending composition are not only determined by tech-

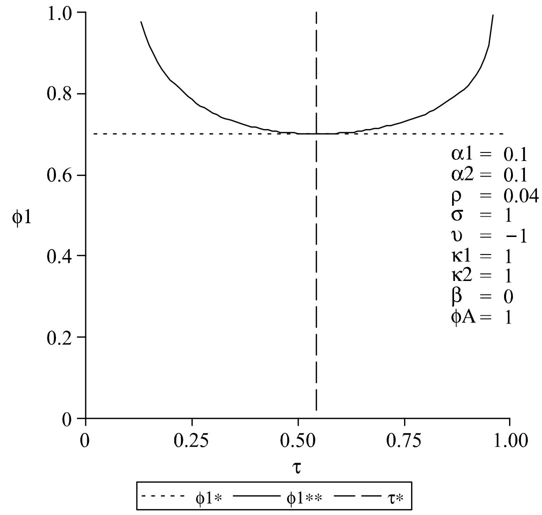
Figure 1: Optimal fiscal policy as a function of $\kappa 1$



nology parameters as in the case of Cobb-Douglas technology but also by the settings of the remaining fiscal policy parameters. In this sense, complexity increases and imperfect information more likely constrains the government.

We now use numerical examples to analyze whether the growth-maximizing fiscal policy parameters are responsive to changes in various *exogenous* model parameters which play no role under Cobb-Douglas technology (i.e. which do not enter (15), (16), (17) and (18)). Figure 5 plots the growth-maximizing tax rate, τ^*, the growth-maximizing expenditure share of total government revenue allocated to G_1, ϕ_1^*, and the growth-maximizing sub-sectoral share of resources allocated to G_A, ϕ_A, as a function of υ (which determines the elasticity of substitution) in a first-best situation. Given that the slopes deviate from zero, Figure 5 suggests that τ^* and $\phi_{1,2}^*$ are highly sensitive to the choice of the elasticity of substitution. In addition, with $\upsilon < 0$, the stock-flow properties of the public inputs also impact on the growth-maximizing fiscal policy. This can be seen by noting that even though α_2 (the share parameter associated with G_2) exceeds α_1, the optimal expenditure share ϕ_1^* may exceed 0.5 (and hence ϕ_2^*) when $\upsilon < 0$. In contrast, when Cobb-Douglas technology is assumed and when $\alpha_2 > \alpha_1$, (16) implies that $\phi_2^* > \phi_1^*$ always holds. This is another example of misleading implications of Cobb-Douglas

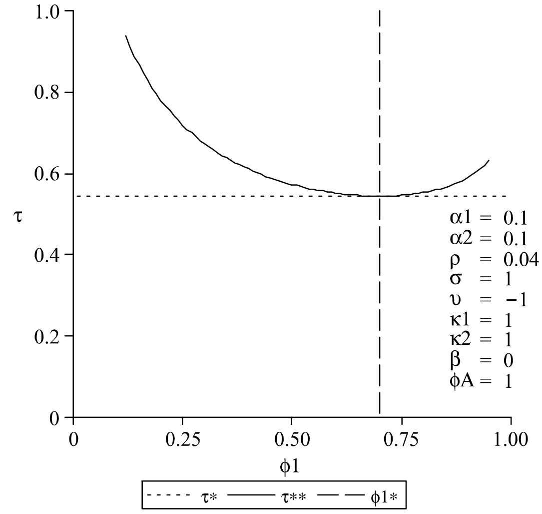
Figure 2: Optimal fiscal policy as a function of τ



technology. The intuition is that the level of G_2 (which is a stock variable) is typically higher than the level of G_1 (which is only derived from the flow of public spending). With complementarity, it is then optimal to increase the share of public resources allocated to G_1 and to increase overall public revenue through higher taxation. Both measures serve to increase the level of G_1. In contrast, the optimal sub-sectoral allocation represented by ϕ_A^* does not respond to exogenous changes in υ because G_A and G_B are both derived from the flow of public spending.

Figures 6 and 7 plot the growth-maximizing tax rate, τ^*, and the growth-maximizing expenditure shares, ϕ_1^* and ϕ_A^*, as a function of σ (which determines the households' intertemporal elasticity of substitution) and of the discount parameter ρ. Given that the slopes are not zero, it can be seen that with CES technology, both preference parameters also determine τ^* and ϕ_1^*. While Figures 6 and 7 suggest that the sensitivity of τ^* and ϕ_1^* to changes in σ and ρ is limited because the slope is not steep, this result is still novel. While under welfare maximization, it seems plausible that household preferences affect optimal fiscal policy, under growth maximization which is the case we consider it may be counterintuitive because in our model, fiscal policy only directly impacts on private production and income (and not on utility).

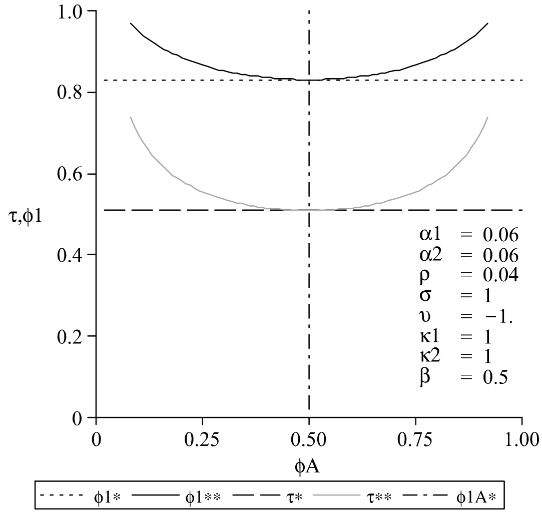
Figure 3: Optimal fiscal policy as a function of ϕ_1



Intuitively, this result directly follows from the model assumptions of complementarity and the fact that the government accumulates public capital. Complementarity essentially implies that in addition to the share parameters of the production function and the cost of generating public revenue, it is the level of private capital which determines the optimal level of the public inputs. However, the government is unable to manipulate the stock of public capital directly because contrary to public services, it is not derived from the flow of public spending but accumulated over time similarly to private capital. The growth-maximizing rate of public investment therefore depends on the rate of private investment which in turn can be shown to depend on preference parameters. This ensures that the level of public capital depends on the level of private capital as dictated by complementarity. In contrast and as above, the optimal sub-sectoral allocation represented by ϕ_A^* does not respond to exogenous changes in σ and ρ. This means that the allocation of public resources between two public services solely depends on share parameters in the production function even with CES technology.

These results further stress that even within simple models and under the simplifying assumption of growth maximization as the government objective and in the absence of budget rigidities, setting fiscal policy in an optimal

Figure 4: Optimal fiscal policy as a function of ϕ_A

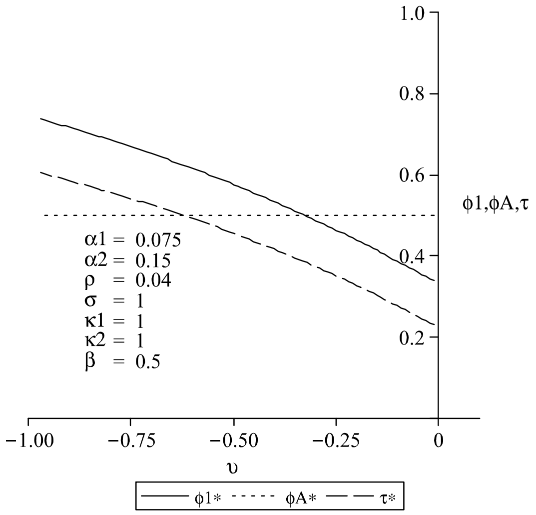


way is highly complex. The reason is that optimal fiscal policy depends on a range of exogenous model parameters of which robust empirical estimates may be hard to obtain in practice. While our model is highly abstract and excludes many features of fiscal policy, these results nevertheless suggest that the complexity of determining growth-maximizing fiscal policy is most likely to exceed government capacity in practice. In more realistic and hence more complex models, the range of parameters which determine optimal fiscal policy is likely to increase further. Thus, with CES technology, it is much more reasonable to assume that imperfect information constrains fiscal policy so that the government is unable to determine optimal taxation and expenditure composition.

6 Fiscal Reform with Budget Rigidities and Imperfect Knowledge

This section simultaneously considers budget rigidities and imperfect knowledge which are both modelled in greater detail and analyzes the implications of fiscal reform that takes current fiscal policy as its starting point using numerical examples. The previous sections have considered budget rigidities

Figure 5: Optimal fiscal policy as a function of υ

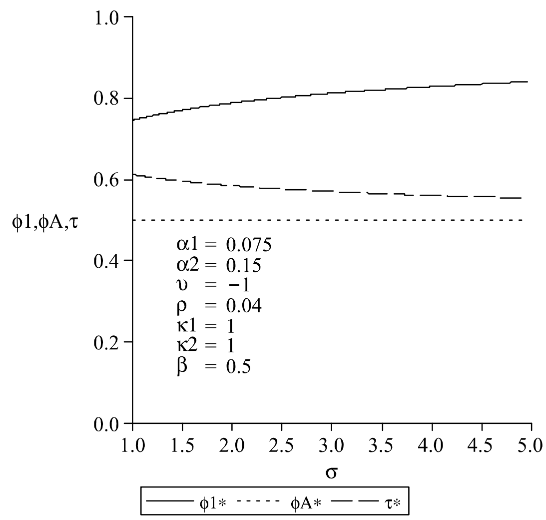


and imperfect information in isolation of each other. They have shown that with CES technology, budget rigidities may give rise to important second-best interactions and that imperfect information is more likely to represent a problem for governments than in the case of Cobb-Douglas technology.

We now extend the previous sections by assuming that both factors simultaneously constrain the government and prevent it from setting the fiscal policy parameters at their first-best values. For simplicity, we again abstract from sub-sectoral allocation by assuming that $\beta = 0$ and $\phi_A = 1$. First, the government faces informational limitations or imperfect knowledge: while we assume that the government knows that τ, G_1 and G_2 impact on the growth rate, that the growth rate is concave in τ and $\phi_{1,2}$ and that it is increasing in $\kappa_{1,2}$, it neither knows the first-best nor the second-best values of τ and $\phi_{1,2}$.[8] The previous sections showed that with CES technology, optimal fiscal pol-

[8] These assumptions mirror the fact that governments still have some knowledge about private agents even if they do not know the exact values of technology and preference parameters: economic theory and anecdotal evidence suggest that very low levels of public spending and very high levels of taxation are detrimental for private investment. It is this type of evidence that governments are aware of and which implies that optimal fiscal policy lies somewhere in between these extremes and that the growth rate is therefore a concave function. Our notion of imperfect information does therefore not imply that governments refrain from using fiscal policy to maximize the growth rate.

Figure 6: Optimal fiscal policy as a function of σ



icy depends on all model parameters which makes their determination highly complex even in simple models. At the same time, it seems natural to assume that government expertise is limited in practice and that exact empirical estimates of the model parameters are unknown which both make imperfect knowledge a reasonable assumption. With unknown τ^* and $\phi^*_{1,2}$, the government does not know whether τ and $\phi_{1,2}$ are currently set above or below their first-best values. Given that there may be second-best interactions as shown in the previous section, there is no guarantee that adjusting τ and $\phi_{1,2}$ in the direction of their first-best values, τ^* and $\phi^*_{1,2}$, enhances growth which increases complexity even further. In contrast, the government knows that increasing efficiency always raises the growth rate, and that $\kappa^*_{1,2} = 1$ (which is reasonable given that there are no cost involved in raising $\kappa_{1,2}$).

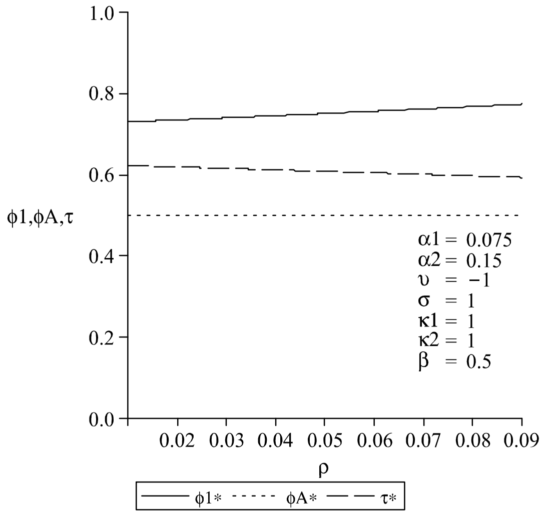
As a starting point, we assume that the economy is currently not at its growth optimum which implies that the fiscal policy parameters (τ, $\phi_{1,2}$ and $\kappa_{1,2}$) are not set at their first-best values:

$$\phi_{1,2} \neq \phi^*_{1,2} \tag{20}$$

$$\tau \neq \tau^* \tag{21}$$

$$\kappa_{1,2} < 1 \tag{22}$$

Figure 7: Optimal fiscal policy as a function of ρ



where $0 < \tau < 1$ and where $\phi_1 + \phi_2 = 1$. This implies that τ and $\phi_{1,2}$ are set either below or above their growth-maximizing levels, and that public spending is inefficient since $\kappa_{1,2} < 1$. In the case of τ and $\phi_{1,2}$, the reason may be imperfect information.

In addition, we assume that the government faces budget rigidities. In contrast to Section 4, we now assume that budget rigidities merely limit the extent to which fiscal policy parameters are adjustable and that they affect all policy parameters. On the one hand, they limit the *number of policy adjustments* that are feasible over an extended period of time.[9] This is realistic: government capacity and political capital are inevitably limited so that governments can only focus on few issues at a time. We count a change in the tax rate τ, an increase of one efficiency parameter ($\kappa_{1,2}$) and two offsetting changes in the composition of public setting (e.g. an increase in ϕ_1 and a decrease in ϕ_2 so that $\triangle\phi_1 = -\triangle\phi_2$) each as one policy change. For simplicity, we assume that governments are only able to make one policy adjustment. On the other hand, budget rigidities limit the *magnitude of*

[9]Obviously, there may be debate about what 'an extended period of time' means in an endogenous growth model with a continous time concept. We simply assume that the period is sufficiently long in the sense that wrong policy choices have significant adverse welfare which the government cannot undo.

each policy adjustment. While the degree of flexibility of each fiscal policy parameter differs, we assume that budget rigidities are meaningful in the sense that feasible policy adjustments do not allow a complete shift to the optimal parameter values (otherwise, budget rigidities would not constrain government policy). The largest feasible adjustments of $\phi_{1,2}$, τ and $\kappa_{1,2}$ in absolute terms (denoted by a_ϕ, a_τ and a_κ) are

$$a_\phi = r_\phi \left|\phi_{1,2}^{**} - \phi_{1,2}\right| \tag{23}$$

$$a_\tau = r_\tau \left|\tau^{**} - \tau\right| \tag{24}$$

$$a_\kappa = r_\kappa \left(1 - \kappa_{1,2}\right) \tag{25}$$

where r_ϕ, r_τ, r_κ define the flexibility of each parameter and where

$$0 < r_\phi, r_\tau, r_\kappa < 1 \tag{26}$$

(26) ensures that budget rigidities constrain fiscal policy parameters so that they cannot be set at their optimal values (otherwise budget rigidities would not be a constraint for fiscal policy).

In this type of situation, the policy problem is therefore to identify the fiscal reform (i.e. the fiscal policy adjustment) which enhances growth most and which is feasible under our specification of budget rigidities. Governments can either lower or raise ϕ_1 (offset by an adjustment of ϕ_2) by up to a_ϕ, they can raise or lower τ by up to a_τ, or they can raise κ_1 or κ_2 by up to a_κ, respectively. From (23), (24) and (25), the maximum feasible adjustment is always smaller than the adjustment required to reach the optimal parameter value which implies that it is optimal to adjust $\phi_{1,2}$, τ, or $\kappa_{1,2}$ by a_ϕ, a_τ, and a_κ, respectively (i.e. by the largest feasible amount and not by less in absolute terms). In other words, while there is an infinite number of distinct policy parameter adjustments, the policy problem is to choose one out of six different ones ($\phi_1 \pm a_\phi$ offset by corresponding changes in ϕ_2, $\tau \pm a_\tau$, $\kappa_1 + a_\kappa$ and $\kappa_2 + a_\kappa$).[10]

[10] For simplicity, we assume that the initial values of τ and $\phi_{1,2}$ are sufficiently far away from their minima (0) and their maxima (1) so that each of the policy adjustments could in principle be implemented.

Figure 8 shows the implications of different policy adjustments under several scenarios. The objective is first to illustrate the impact of complementarity on the change of the growth rate that results from alternative fiscal reforms and second to show that under imperfect knowledge, the expected change of the growth rate is negative as shown further below. The absolute magnitude of the policy adjustments considered depends on the assumptions about budget rigidities and is given by $|\triangle\tau| = a_\tau$, $\left|\triangle\phi_{1,2}\right| = a_\phi$ and $|\triangle\kappa_{1,2}| = a_\kappa$. The scenarios differ with respect to the flexibility of the fiscal policy parameters, the initial values of the fiscal policy parameters and the exogenous model parameters. Table 1 contains details about the scenarios. In order to ensure comparability between the scenarios, the change of the growth rate is expressed as a share of potential growth (i.e. the growth rate under growth-maximizing fiscal policy in a first-best situation). For instance, under scenario 1 which assumes Cobb-Douglas technology and which assumes $\tau < \tau^*$, $\phi_1 > \phi_1^*$ and $\kappa_{1,2} < 1$, raising the rate of taxation results in the greatest increase of the growth rate (the growth rate increases by about 5% in terms of potential growth), whereas lowering the tax rate results in a fall of the growth rate by about 14%. Comparing the changes of the growth rate under scenarios 1 and 2 which only differ with respect to the production technology (scenario 1 assumes Cobb-Douglas, whereas scenario 2 assumes CES technology) shows that complementarity between the inputs to private production implies that fiscal reform has much greater effects in relative terms under CES technology. Comparing scenarios 2 and 4 shows that obviously, the initial values of the fiscal policy parameters also impact on the optimal policy adjustment.

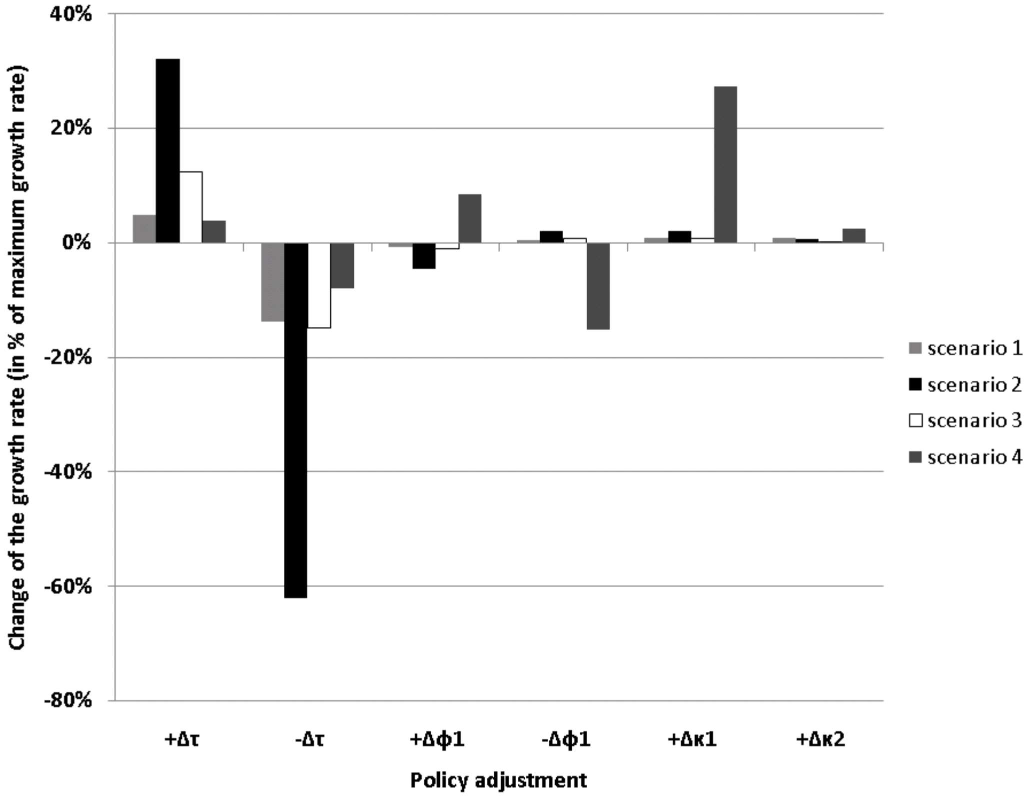
Under imperfect knowledge, the government does not know the response of the growth rate to policy parameter adjustments. Conceptually, it is useful to distinguish two types of mistakes the government can make. First, it may pick a policy adjustment which results in a reduction of the growth rate. Second, it may pick a policy adjustment which enhances growth but to a lesser extent compared to the case when the optimal policy adjustment is chosen. Obviously, the priority is to avoid the first mistake which can be seen as more costly than the second one.

Table 1: Scenarios

Scenario	1	2	3	4
α_1	0.15			
α_2	0.15			
υ	0	-1		
ρ	0.04			
σ	1			
β	0			
τ/τ^*	0.6	0.6	0.6	0.9
ϕ_1/ϕ_1^*	1.15	1.15	1.15	0.9
$\kappa_{1,2}$	0.9	0.9	0.9	0.5
r_τ	0.7	0.7	0.2	0.7
r_ϕ	0.7	0.7	0.2	0.7
r_κ	0.6	0.6	0.2	0.6
ϕ_A	1			

How likely is it that the growth rate will decrease as a result of policy parameter adjustments? While the government knows that increasing κ_1 and κ_2 unambiguously increases the growth rate, imperfect knowledge implies that it has no information about τ^{**} and $\phi_{1,2}^{**}$ which means that it does not know how changes of τ and $\phi_{1,2}$ affect the growth rate (i.e. the government does not know whether increasing or decreasing τ and $\phi_{1,2}$ raises the growth rate). A risk-neutral imperfectly informed government will therefore assign equal probability weights to either outcome (i.e. the government will assign a probability of 1/2 to both an increase and a decrease of the growth rate as a result of changing τ and $\phi_{1,2}$). These probabilities allow the calculation of the expected change of the growth rate that results from adjusting τ and $\phi_{1,2}$ which is simply the average of the absolute increase and the absolute decrease of the growth rate. Figure 8 implies that this average is unambiguously negative for τ and $\phi_{1,2}$ in all scenarios. In other words, while the government is unable to calculate the exact magnitude of the expected change of the growth rate due to imperfect knowledge, it *does* know that the expected change of the growth rate from adjusting τ and $\phi_{1,2}$ by a discrete amount is negative. For instance, under scenario 2, the *decrease of the growth rate* that results from decreasing τ by a_τ exceeds the *increase of the growth rate* that

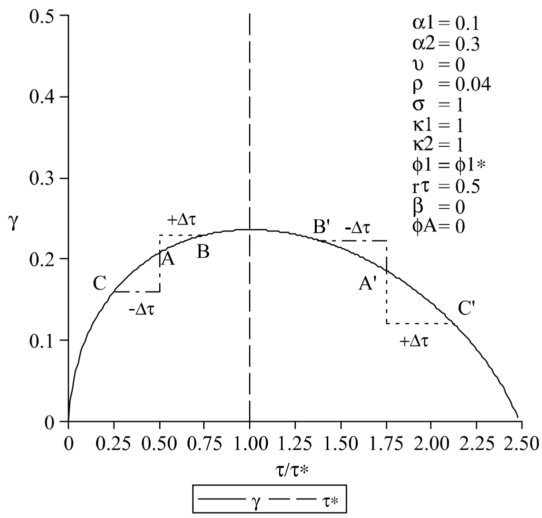
Figure 8: Changes of the growth rate as a result of fiscal policy adjustments



results from increasing τ by a_τ.

Figure 9 illustrates this point using a numerical example which assumes Cobb-Douglas technology. It plots the growth rate as a function of τ/τ^* (so that at $\tau/\tau^* = 1$, the tax rate is set at its first-best value). Suppose that the economy is initially at point A. *Increasing* the tax rate by a_τ raises the growth rate, and the economy attains point B, whereas in the opposite case with a *decrease* in the rate of taxation, the growth rate falls to point C. Imperfect knowledge implies that the government does not know whether $\tau/\tau^* < 1$ or $\tau/\tau^* > 1$ (i.e. whether the tax rate is below or above its optimal value). In other words, the government could equally assume that the economy is at point A' where *lowering* taxation results in a shift to point B' where the growth rate is higher. The concavity of the growth curve implies that shifting the rate of taxation away from its optimal value has always a greater absolute impact on the growth rate than shifting the rate of taxation towards its optimal value which explains why the expected change of the growth rate is negative. This result only holds when the adjustment of τ and $\phi_{1,2}$ are discrete (i.e. non-marginal).

Figure 9: Possible changes of the growth rate as a result of tax rate adjustments



The result of a negative expected change in the growth rate due to adjustments of τ and $\phi_{1,2}$ is that imperfectly informed governments refrain from adjusting them and therefore choose to increase $k_{1,2}$ only. The reason is that there is always full *certainty* that raising κ_1 and κ_2 will increase the growth rate. However, as Figure 8 demonstrates, governments then risk to commit the second mistake of not choosing the policy adjustment which increases the growth rate most.

The solution for this dilemma is to exploit the fact that budget rigidities constrain fiscal policy discretion and thereby reduce informational requirements to identify the optimal and feasible policy parameter adjustment. In order to avoid growth-reducing adjustments, information about the 'growth-enhancing' direction of the optimal adjustment of τ and $\phi_{1,2}$ is sufficient. This information appears to be more readily available than information about the exact values of the optimal policy parameters. Since budget rigidities imply that feasible policy adjustments are always smaller than policy adjustments required to attain the optimal values of τ and ϕ, governments do not require information about the optimal magnitude which would arguably be much harder to obtain. This allows governments to avoid the first mistake for all fiscal policy parameters.

Avoiding the second mistake then requires criteria to select the optimal fiscal reform among those which augment the growth rate. This does not require a comparison of the change in the growth rate of all policy adjustments in absolute terms. Rather, under budget rigidities, it may be sufficient to identify the policy adjustment which results in the largest increase of the growth rate which implies that only a comparison of the change of the growth rate in relative terms is required.

7 Conclusions

This paper has shown that Cobb-Douglas technology which is commonly assumed in most endogenous growth models with public finance has in some respect counterintuitive and misleading implications for growth-maximizing fiscal policy. This paper has also justified that second-best situations are likely to arise in the context of fiscal policy, and that with CES technology, second-best interactions have important implications for optimal fiscal policy under growth maximization. The main result here is that in second-best situations, the optimal level of taxation is likely higher, and the optimal share of investment is lower compared to a first-best situation. While in practice, public infrastructure is often seen as particularly important for economic development, this may only apply for a first-best situation. A natural extension would be to derive the welfare-maximizing fiscal policy within the same framework and compare the results to the growth-maximizing equivalent which we however leave for future research. In the same way, a useful extension of the model would be to examine whether second-best interactions also alter the optimal allocation between public expenditure categories other than those considered in this paper.

The paper has also considered the sources of divergence from a first-best situations that give rise to a second-best one. In addition to budget rigidities, complexity is likely to exceed government capacity and expertise which gives rise to imperfect knowledge. In turn, second-best interactions increase informational requirements to determine the optimal values of fiscal policy parameters further. However, our results also suggest that when fiscal policy discretion is limited due to budget rigidities, informational requirements are likely to decrease. In particular, in order to implement growth-enhancing fiscal policy reforms, only the fiscal policy parameter to be adjusted and the direction of the adjustment must be chosen. In contrast, the optimal magnitude of fiscal policy adjustments or the second-best values of fiscal policy parameters do typically not have to be known for the design of growth-enhancing fiscal reforms.

It is interesting to contrast our analysis of fiscal reform with the one by Ahmad and Stern (1984). Their objective is to identify welfare-improving tax changes which do not decrease tax revenue. They develop a simple static model with many goods which are all subject to specific taxes and which are consumed by the households who receive fixed and untaxed factor incomes. This framework enables to derive the marginal cost in terms of social welfare of raising an additional unit of government revenue from taxing a given good. If this marginal cost differs for two goods, welfare-improving tax reforms are feasible. Apart from the fact that they consider a welfare function which takes into account value judgements whereas we assume that aggregate growth is the objective function of the government, there are other important differences between their analysis and ours. First, our modelling framework is different. Whereas they consider different types of indirect taxes and implicitly take public spending requirements as exogenously given, we consider income taxation, different public expenditure categories and model the effects of public spending which implies that the optimal level of public spending is endogenously determined. Their condition for welfare-improving tax reforms is therefore not directly applicable in the context of this paper. Second, they consider the fact that their analysis is limited to the direction of fiscal adjustments (and excludes the size) as a disadvantage. In contrast, by explicitly modelling budget rigidities, we extend Ahmad and Stern (1984) in two important ways. On the one hand, we showed that the assumption that only small fiscal policy adjustments are feasible is realistic. On the other hand and more importantly, we presented compelling evidence that with concave objective functions, information about the direction of fiscal adjustment is most important.

A Appendix

A.1 Uniqueness and Stability of the Balanced Growth Path

Let $x = \frac{c}{k}$ and $z = \frac{G_2}{k}$. Together with the transversality condition, $\lim_{t \to \infty} [\lambda k] = 0$, and with the initial conditions, $x_0 > 0$ and $z_0 > 0$, the dynamics of the market economy can be expressed as a system of two differential equations (we assume that $\kappa_i = 1$):

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} \tag{A.1}$$

and

$$\frac{\dot{z}}{z} = \frac{\dot{G}_2}{G_2} - \frac{\dot{k}}{k} \tag{A.2}$$

From (8), (6) and (5), respectively,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left((1-\tau)y_k - \rho\right) \tag{A.3}$$

$$\frac{\dot{k}}{k} = (1-\tau)\frac{y}{k} - x \tag{A.4}$$

$$\frac{\dot{G}_2}{G_2} = \phi_2 \tau \frac{y}{G_2} \tag{A.5}$$

Setting $\frac{\dot{x}}{x} = 0$ in (A.1) and solving for x yields its steady state value, $\tilde{x}$:

$$\tilde{x} = (1-\tau)\frac{y}{k} - \frac{1}{\sigma}\left((1-\tau)y_k - \rho\right) \tag{A.6}$$

Using (A.6) to substitute for x in (A.4), and using (A.4) and (A.5) to substitute for $\frac{\dot{k}}{k}$ and $\frac{\dot{G}_2}{G_2}$ in (A.2) yields

$$F = \phi_2 \tau \frac{y}{G_2} - \frac{1}{\sigma}(1-\tau)y_k + \frac{\rho}{\sigma} \tag{A.7}$$

From (4) and (10),

$$\frac{G_1}{G_2} = \frac{\phi_1}{\phi_2}\gamma \tag{A.8}$$

From (1) and (A.8),

$$\frac{y}{G_2} = (\alpha z^{-v} + \alpha_1 \left(\frac{\phi_1}{\phi_2}\gamma\right)^v + \alpha_2)^{\frac{1}{v}} \tag{A.9}$$

Differentiating (1) for k, using (4) to substitute for G_1 and replacing $\frac{G_2}{k}$ by z yields

$$y_k = \left(\theta + \alpha_1 \left(\tau\phi_1 \frac{y}{k}\right)^v + \alpha_2 z^v\right)^{\frac{1}{v}-1} \theta \tag{A.10}$$

From (1) and (4),

$$\frac{y}{k} = \left(\frac{\theta + \alpha_2 z^v}{(1 - \alpha_1 \phi_1^v \tau^v)}\right)^{\frac{1}{v}} \tag{A.11}$$

After using (A.11) to substitute in (A.10) and (A.9) and (A.10) to substitute in (A.7), it can be seen that if $v \leq 0$, $\frac{dF}{dz} < 0$ implying that F is a monotonically decreasing function of z so that there is a unique positive value of $\tilde{z}$ that satisfies $F = 0$. From (A.6), there is a unique positive value of $\tilde{x}$ as well. Thus, the growth path is unique.

To investigate the dynamics in the vicinity of the unique steady state equilibrium, equations (A.1) and (A.2) can be linearized to yield

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x - \tilde{x} \\ z - \tilde{z} \end{bmatrix} \tag{A.12}$$

where $\tilde{x}$ and $\tilde{z}$ denote the steady state values of x and z. From (A.1) and (A.2), $\dot{x}$ and $\dot{z}$ can be rewritten as follows:

$$\dot{x} = \left(\frac{\dot{c}}{c} - \frac{\dot{k}}{k}\right)\tilde{x} \tag{A.13}$$

and

$$\dot{z} = \left(\frac{\dot{G}_2}{G_2} - \frac{\dot{k}}{k}\right)\tilde{z} \tag{A.14}$$

with $\frac{\dot{c}}{c}$, $\frac{\dot{k}}{k}$ and $\frac{\dot{G}_2}{G_2}$ defined according to (A.3), (A.4) and (A.5). Saddlepoint stability requires that the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A.12) must be negative:

$$\det J = a_{11}a_{22} - a_{12}a_{21} \tag{A.15}$$

Given the complexity of the matrix, it is easier to verify numerically that this condition holds. For most sensible examples with sensible parameter values that we used, this condition is satisfied.

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