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Non-Technical-Summary

Global warming, the pollution of the seas or a continuously increasing noise level are just a few examples of environmental issues that have alerted the global public in recent years. As a consequence, more and more scientific bodies and environmental groups are discussing whether the emission of pollution as a by-product of modern economic activity may endanger the exceptional economic growth path, the world has experienced during the last one and a half centuries.

To shed further light on this question, a standard Schumpeterian growth model is enlarged to include an environmental dimension. Thereby it explicitly links the pollution intensity of economic activity to the overall level of technological progress. More precisely, it is assumed that pollution arises as an externality of the intermediate good production process and the amount of pollution created in the production process crucially depends on the overall level of technological progress. Within the framework of the model, pollution has different effects on the agents present in the economy. In the basic model solely the households are directly affected by pollution. In a first extension, the model is enlarged to feature a pollution threshold above which no research is possible. A second extension enlarges the choice set of the households and allows for private pollution abatement.

In equilibrium, the economy follows a balanced growth path where output, research investment, consumption, as well as the level of technological progress grow at a constant and positive rate. The effect of pollution on the economic growth rate vitally depends on the households' degree of pollution aversion and even more on the link between pollution intensity and the technology level. All in all, pollution dampens the economic growth rate if a rise in the level of technological progress does not imply a big enough decrease of the pollution intensity. It fosters growth if the pollution intensity falls disproportionately fast when the economy advances. Pollution growth is proportional to the growth rate of the economy and also crucially depends on how the pollution intensity of the production process is linked to the level of technological progress. If the pollution intensity declines fast enough when the level of technological progress increases over time, pollution declines. Otherwise pollution grows continuously or is constant. Due to four types of deviations, namely monopolistic pricing, the appropriability effect, the business stealing effect and pollution externalities, the decentralized solution does not meet the social optimum. But, the social optimum can be implemented through the introduction of a subsidy to the final good sector, a subsidy towards the research sector and tradable pollution permits.

Given a pollution threshold above which no research is feasible, sustained economic growth is only possible if pollution does not increase over time. This implies that the pollution intensity must be decoupled from technological progress. Otherwise, at some point in time pollution will stall growth in the economy. The possibility of private pollution abatement enables the households to cope better with the pollution emitted during the production process and allows the economy to grow at a higher rate.

Nicht-Technische Zusammenfassung

Der Klimawandel, die Verschmutzung der Meere oder eine zunehmende Lärmbelastung sind nur einige wenige Beispiele für Umweltprobleme, die die Öffentlichkeit in jüngster Zeit alarmiert haben. Infolge dessen gewinnt die Frage, ob Umweltverschmutzung als eine Begleiterscheinung der modernen wirtschaftlichen Aktivität möglicherweise den außergewöhnlichen Wachstumspfad der letzten eineinhalb Jahrhunderte gefährdet, zunehmend an Bedeutung.

Um dieser Frage auf den Grund zu gehen wird ein schumpeterisches Wachstumsmodell erweitert und Umweltverschmutzung in das Kalkül der ökonomischen Agenten integriert. Dabei wird die Verschmutzungsintensität der wirtschaftlichen Aktivität explizit mit dem Niveau des technischen Fortschritts verknüpft. Im Rahmen des Modells hat Umweltverschmutzung verschiedene Auswirkungen auf die Agenten. Im Basismodell sind zunächst nur die privaten Haushalte von der Verschmutzung betroffen. In einer ersten Erweiterung wird das Modell um einen Verschmutzungsgrenzwert ergänzt, jenseits dessen die Innovationsfähigkeit des Forschungssektors abnimmt. Eine weitere Variante vergrößert die Handlungsmöglichkeiten der Haushalte und gibt ihnen die Möglichkeit zur aktiven Verschmutzungsminderung.

In dem beschriebenen Modellrahmen bewegt sich die Ökonomie im Gleichgewicht auf einem Balanced Growth Path. Dementsprechend steigen die Mengen an produzierten Gütern, die Investitionen in Forschung, der private Konsum und das Niveau des technischen Fortschritts im Gleichgewicht mit einer konstanten und positiven Rate. Der Effekt von Umweltverschmutzung auf das Wirtschaftswachstum ist nicht trivial. Er wird maßgeblich von dem Grad der Verschmutzungsaversion der Haushalte sowie der Art und Weise wie die Verschmutzungsintensität mit dem technischen Fortschritt verknüpft ist beeinflusst. Alles in allem wirkt Umweltverschmutzung dämpfend auf das Wirtschaftswachstum, sofern ein Anstieg des technologischen Fortschritts nicht mit einer ausreichend schnellen Absenkung der Verschmutzungsintensität einhergeht. Verschmutzung begünstigt Wachstum, wenn die Verschmutzungsintensität bei steigendem technischem Wissen überproportional schnell fällt. Das Ausmaß der Umweltverschmutzung wächst proportional zum Wirtschaftswachstum und wird auch maßgeblich davon beeinflusst, wie die Verschmutzungsintensität mit dem technischen Fortschritt verknüpft ist. Wenn diese schnell genug fällt, sinkt die Menge an Umweltverschmutzung. Andernfalls bleibt sie konstant oder steigt kontinuierlich an. Aufgrund von monopolistischer Preissetzung, dem Appropriations-Effekt, dem Business-Stealing-Effekt und der Verschmutzungsexternalität ist die dezentrale Lösung des Modells nicht Pareto-optimal. Das soziale Optimum kann jedoch durch die Einführung von Subventionen für den Forschungssektor und den Endproduktsektor sowie durch handelbare Emissionszertifikate erreicht werden.

Wird das Modell um einen Verschmutzungsgrenzwert erweitert, bei dessen erreichen die Innovationsfähigkeit des Forschungssektors nachlässt, ist anhaltendes Wirtschaftswachstum nur möglich, wenn die Menge der Umweltverschmutzung nicht im Zeitverlauf steigt. Dies setzt voraus, dass die Verschmutzungsintensität vom technischen Fortschritt entkoppelt ist. Ist dies nicht der Fall, so verhindert Umweltverschmutzung, ab dem Erreichen des Grenzwertes, weiteres Wirtschaftswachstum. Aktive Verschmutzungsbeseitigung wiederum erlaubt es den Haushalten besser mit der Umweltverschmutzung aus der Produktion umzugehen und ermöglicht allgemein größere Wachstumsraten.

Pollution Externalities in a Schumpeterian Growth Model

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Abstract

This paper extends a standard Schumpeterian growth model to include an environmental dimension. Thereby, it explicitly links the pollution intensity of economic activity to technological progress. In a second step, it investigates the effect of pollution on economic growth under the assumption that pollution intensities are related to technological progress. Several conclusions emerge from the model. In equilibrium, the economy follows a balanced growth path. The effect of pollution on the economic growth rate vitally depends on the households' degree of pollution aversion and on the link between pollution intensity and the technology level. The decentralized solution does not meet the social optimum, though the social optimum can be implemented through the introduction of subsidies and pollution permits. Expectedly, the introduction of a pollution threshold stalls growth if pollution is not decoupled from economic growth and the possibility of pollution abatement allows the economy to grow at a higher rate.

JEL Classification: O41, Q51, Q55, Q56
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1 Introduction

Global warming, the pollution of the seas or a continuously increasing noise level are just a few examples of environmental issues that have alerted the global public in recent years. As a consequence, more and more scientific bodies and environmental groups are discussing whether the emission of pollution as a by-product of modern economic activity may endanger the exceptional economic growth, the world has experienced during the last one and a half centuries.

To shed further light on this question, this paper extends a standard Schumpeterian growth model to include an environmental dimension. Thereby it explicitly links the pollution intensity of economic activity to the overall level of technological progress. More precisely, it is assumed that pollution arises as an externality of the intermediate good production process and the amount of pollution created in the production process crucially depends on the overall level of technological progress. The necessity of such a feature arises from the fact that today the pollution intensity is seen as a key factor in solving the problem of environmental degradation. Furthermore, pollution has an effect on the agents present in the economy. At first, solely the households are directly affected by pollution, though they take pollution as given. Later, the model is enlarged to feature a pollution threshold above which no research is possible and private pollution abatement is allowed.

In a second step, using the extended Schumpeterian growth model, this work investigates the effect of pollution on economic growth. In particular it seeks to resolve the following questions:

- How are the levels, the paths or the growth rates of crucial variables such as consumption, investment or pollution affected?
- What types of deviations can be observed when comparing the decentralized solution with the social optimum and what are the policy implications for these deviations?
- Is it possible to have sustained growth if there exists a pollution threshold?
- Is environmental protection in the form of pollution abatement compatible with economic growth?

This work is organized as follows. After the introduction, section 2 presents the basic model and briefly discusses the main aspects that have to be considered when incorporating pollution in modern economic growth models. Section 3 derives and characterizes the balanced growth path of the decentralized economy and explores the implications of pollution for the growth rate. Section 4 analyses the Pareto optimality of the decentralized economy and demonstrates how the optimal path can be implemented. Section 5 enlarges the basic model and discusses the implications of a pollution threshold as well as the effects of pollution abatement. This work ends with a summary of the results and concluding remarks in section 6. The appendix contains detailed derivations.

2 Description of the Model

2.1 Model Outline

The model is an advanced version of Aghion and Howitt's (1992) well-known Schumpetarian Model of Endogenous Growth and is set such that an equilibrium can be derived in a similar manner as Barro and Sala i Martin's (2004) interpretation of Aghion and Howitt's model.

As usual, there are two sectors, a final good sector and a combined R&D and intermediate production sector, as well as many identical households. Additionally, the standard model has been extended to include an environmental dimension. The model is set in infinite and continuous time and all agents are assumed to have perfect foresight.

2.2 Households

In the economy, there is a large but finite and constant number of L identical and infinitely living households. Every household consists of one adult who provides one unit of labor to the final good sector and holds assets in the form of a balanced portfolio of all firms in the economy. They earn wages, receive interest income on their assets and consume the final good. Furthermore, all households are affected by the public bad pollution, yet they take pollution as exogenously given. Their momentary utility is given by

$$u(t) = \begin{cases} \frac{1}{1-\theta}\left(\left(c(t)P(t)^{\eta}\right)^{(1-\theta)} - 1\right) & \text{for } 0 < \theta < \infty \text{ and } \theta \neq 1 \\ \ln c(t) + \eta \ln P(t) & \text{for } \theta = 1. \end{cases} \tag{1}$$

$c(t)$ is consumption per household of the final good at time t,[1] $P(t)$ the total amount of pollution present in the economy at time t, θ the multiplicative inverse of the elasticity of intertemporal substitution and $-\infty < \eta < 0$ a parameter determining the degree of pollution aversion of the households. The utility function ensures that the utility depends positively on the number of final goods consumed ($u_c > 0$) and negatively on the level of pollution they are exposed to ($u_P < 0$).[2]

[1] $C(t)$ is the total consumption at time t and since there are L households in the economy $c(t) = \frac{C(t)}{L}$ is the consumption per household.

[2] Here and throughout the text $u_c = \frac{\partial u(t)}{\partial c(t)}$, $u_P = \frac{\partial u(t)}{\partial P(t)}$, $u_{cc} = \frac{\partial\left(\frac{\partial u(t)}{\partial c(t)}\right)}{\partial c(t)}$ and $u_{cP} = \frac{\partial\left(\frac{\partial u(t)}{\partial c(t)}\right)}{\partial P(t)}$. Note also that whenever no ambiguity results, the time subscript of variables is omitted.

In this setting, it is assumed that every single household is affected by the total amount of pollution prevailing in the economy, yet it would also be possible to assume that the households are only affected by their particular pollution share, which is $\frac{P}{L}$. However, since L is constant over time, an alternative utility function using $\frac{P}{L}$ instead of P would not yield any additional insights.

2.3 Final Good Sector

The final good sector is competitive, and every firm i produces the final good Y according to the following Cobb-Douglas production function

$$Y_i = BL_i^{1-\alpha} \sum_{j=1}^{N} (q^{\kappa_j} X_{ij})^{\alpha} \,, \tag{2}$$

where $0 < \alpha < 1$, Y_i is the final output firm i produces, $B > 0$ is an overall measure of productivity in the final sector, L_i is the amount of labor hired by firm i and X_{ij} is the quantity of intermediate good j that firm i uses. N represents the number of varieties of intermediates available in the economy, whereas it is assumed that N is constant over time and very large. Finally, $1 \leq q < \infty$ represents a constant quality parameter and κ_j the quality rank of intermediate j. The final good production as a whole experiences diminishing returns, even tough the productivity of each intermediate good increases with its quality level.

In this model, the final good Y is a numeraire, so its price is equal to one, and it can be used for consumption, as an input in the intermediate sector or as investment in the R&D sector.

2.4 Technological Progress

Before describing the functioning of the combined R&D and intermediate sector in detail, it is important to explain what is meant by the term technological progress in this economy. In this context, technological progress consists of improvements regarding the quality of a certain type of intermediate good. Each innovation increases the quality rank κ_j of a intermediate good j by one and since innovations are irrevocable, as time passes, the intermediates climb a so-called quality ladder.

Quality improvements of a specific intermediate occur independently of the quality levels of other intermediate goods. While it is possible to have a big variety of quality ranks, it would also be thinkable to have a uniform quality level. In order to assess the overall level of technological progress in the economy,

$$Q(\kappa_j) = \sum_{j=1}^{N} q^{\frac{\kappa_j \alpha}{(1-\alpha)}} \tag{3}$$

is defined as the aggregated quality level prevailing in the economy.

2.5 Combined R&D and Intermediate Sector

The intermediate good sector and the research sector are unified and henceforth will be called the RDI sector.[3] As soon as a research firm is successful and innovates, it obtains a patent on its innovation and from then on is the sole producer of the most advanced intermediate good. Thus, after production, the intermediate good X_j is sold at a monopolistic price to the final good sector. What is more, innovations are always drastic. This means that the more advanced good replaces the older good completely until the next innovation for the same intermediate takes place and it is also replaced. This feature puts Schumpeter's idea of creative destruction into effect, as old vintages of a good are supplanted, or in the terms of Schumpeter, destroyed by goods of a higher quality (Schumpeter, 1912).

Intermediate goods are produced from the firms holding the corresponding patent for good j according to the simple production function

$$X_j = y_j, \tag{4}$$

where y_j is the number of final goods used to produce intermediate X_j. Accordingly, one final good Y is used to produce one intermediate good X_j. From this it follows that the total investment of final good Y towards the intermediate good production is

$$\sum_{j=1}^{N} y_j = \sum_{j=1}^{N} X_j = X. \tag{5}$$

Innovations occur with a probability of

$$l(\kappa_j) = Z(\kappa_j)\phi(\kappa_j), \tag{6}$$

where $Z(\kappa_j)$ is the amount of R&D investment of a firm for intermediate j subject to its quality rank κ_j and $\phi(\kappa_j)$ a term capturing the effect of the current quality rank on research. $\phi(\kappa_j)$ is assumed to be

$$\phi(\kappa_j) = Aq^{-(\kappa_j+1)\frac{\alpha}{(1-\alpha)}}, \tag{7}$$

[3]Note, the combined R&D and intermediate sector could also be modeled as two separate sectors without changing the results of the model. In this case, the firms in the intermediate sector would have to acquire a license for the blueprints of the highest quality intermediate from the R&D firms before they can produce the intermediate good at a quantity they desire and sell it to the final good sector (Romer, 1990).

where $A > 0$ represents an exogenous parameter measuring the productivity of the research sector. Hence, innovations become more difficult the more sophisticated the intermediate is; a feature that can be interpreted as some sort of "fishing-out".

There is free entry into the innovator business, and despite the patents, all inventors have access to the blueprints of the most advanced intermediate good. They are simply by law not allowed to produce it.

Although there are no assumptions regarding who innovates, Arrow's replacement effect, which states that an incumbent monopolist has always lower incentives to innovate than a potential entrant who can take over the monopolists rents once he successfully innovates, ensures that only outsiders or newcomers undertake research. The underlying logic of this is that, in contrast to an entrant, the incumbent monopolist already earns positive profits from the innovation he has generated earlier and by the means of another innovation he would only replace his own monopoly. Consequently, his benefit from another innovation is always strictly lower than the benefit of a newcomer. Moreover, the free entry condition requires all researching firms to be indifferent in undertaking research, no matter if they are incumbents or entrants. But if the outsiders with their higher research incentives fulfill the free entry condition, the incumbents always make a loss when engaging in the innovation business. Thus no research investment is the best choice for the incumbents and all innovations are generated by newcomers (Arrow, 1962).

2.6 Pollution

In general, the integration of pollution in an economic growth model requires the consideration of three aspects: the nature of pollution, the source of pollution and the effects of pollution on the agents in the economy.

The Nature of Pollution When specifying pollution in an economic growth model, there are two ways of interpreting pollution. Pollution can either be seen as a flow variable, which exclusively depends on the amount of pollution generated in the present period, or as a stock variable, so that the amount of pollution prevailing in an economy is determined by an evolution equation relating the present amount of pollution to past pollution (Stokey, 1998).

Commonly, the modeling of pollution in one or the other way is justified by the assumed composition of pollution, as done for instance by Stokey (1998). On the one hand, according to her, pollution may be associated with fugitive substances such as noise, smoke or electric smog. In this case pollution would be modeled as a flow variable. On the other hand, pollution can be seen as a stock of accumulated pollutants, as for example waste or greenhouse gases. If so, pollution is rather considered to be a stock variable.

Though, besides from the composition of pollutants, there are also other popular arguments for modeling pollution as a flow variable or as a stock variable. While the potential to generate transitional dynamics and the possibility to account for regenerative processes of the environment speak for pollution as a stock variable (Xepapadeas, 2005), the fact that avoiding another state variable mostly simplifies the computation of the equilibrium argues in favor of pollution as a flow variable (Elbasha and Roe, 1996).

Both approaches, pollution as a flow or as a stock variable, are frequently used throughout the literature on environmental growth models. For instance, Gradus and Smulders (1993) as well as Hart (2004) analyze the effect of pollution on growth using a flow variable, while Bovenberg and Smulders (1995) as well as Popp (2004) consider the consequences of a stock variable. Smulders and Gradus (1996) and Stokey (1998) have also worked on a comparison, contrasting the consequences of a flow variable with those of a stock variable. They conclude that in models with exogenous technological change and in the context of AK models, there is no substantial difference in the results when using a stock or a flow variable for pollution. However, a proof that this result does also apply in a Schumpeterian growth framework is still subject to current research.

The model presented in this work uses a flow variable for pollution in order avoid the intricacies of a new state variable. Hence, pollution is presumed to be a fugitive substance and no pollution is transfered from one period to the next.

The Source of Pollution According to Xepapadeas (2005), there are three main approaches of how pollution can be generated in an economic model.

In a first attempt pollution can be linked to the level of private consumption. In such a setting, pollution can be seen as a byproduct of consumption and the households are responsible for determining how much pollution is generated in the economy.

Alternatively, pollution may arise because it is needed as an input in the production process for the goods traded in the economy. Here, the firms decide directly how much pollution will be present in the economy when choosing which inputs they will use and what amount of output they will produce.

Yet typically, the flow of pollution is being related to an externality of the production process, so that pollution is a necessary byproduct of production. In this context, once more the firms decide how much pollution is created. Although, in such a setting, they do it rather indirectly by choosing an appropriate level of output.

One of the first to implement the externality approach was Forster (1973) and it gained widespread acceptance when it was used by Gradus and Smulders (1993) in their seminal contribution to the environmental growth theory. Stokey (1998) and Aghion and Howitt (1998) extended the traditional pollution output relation by explicitly introducing a parameter measuring the pollution intensity of output, even though this parameter remained exogenous, until for instance the work of Grimaud (1999) or Hart (2004) who finally endogenize the pollution intensity of the production process.

The model in this section is constructed in tradition to these models. It is assumed, that the pollution $P(t)$ arises as an externality of the production process of the intermediate goods $X(t)$. Furthermore, the pollution intensity of a good evolves in line with the general quality rank of a good. Correspondingly, the pollution $P(t)$ is assumed to be

$$P(t) = \left(\sum_{j=1}^{N} q^{\frac{\kappa_j(t)\alpha}{1-\alpha}} \right)^{-\zeta} \sum_{j=1}^{N} X_j(t) = Q^{-\zeta}(t) \sum_{j=1}^{N} X_j(t), \tag{8}$$

where $-\infty < \zeta < \infty$ is a finite and constant parameter determining the effect the level of technological progress $Q(t)$ has on pollution $P(t)$. Thus, the pollution intensity of the intermediate sector depends directly on the aggregated quality level and the parameter ζ is crucial regarding the magnitude of produced pollution. Consequently, if ζ is high (low), the production of intermediate goods entails a low (high) pollution generation. The fundamental idea behind this form of pollution function is that innovations are rarely bound to a single property, here output productivity or pollution intensity, and that progress in one specific area can have a broad effect on the economy as a whole.

Effects of Pollution The introduction of pollution into an economic model would be of no interest if pollution would not have an effect on the agents or markets in the economy. Generally speaking, one could think of a multitude of possibilities of how pollution might take effect in an economic growth model and consequently the following will have to focus on those approaches that have been discussed most in economic literature.

One possible way of introducing an effect of pollution, is to make the households dependent on the amount of pollution present in the economy. The simplest way of doing this is to incorporate pollution directly in the utility function of households, so that $u(c, P)$ with $u_P < 0$, as by definition pollution causes disutility. This approach is used in the majority of environmental growth models since it is highly intuitive and standard economic growth models can easily be adapted to feature such an extended utility function. A different method of rendering households dependent on pollution, is to postulate that the amount of labor they supply is vulnerable to pollution. The idea behind this is that an increasing amount of pollution harms the workers and therefore only a certain amount of effective labor can be employed in the production process. However, even though this approach has been elaborated empirically, so far little attention has been dedicated to this sort of pollution effect in the context of environmental growth models. For an exception see the work of Gradus and Smulders (1993).

An alternative approach of generating an effect of pollution consists in allowing pollution to have an influence on firms. One way of doing this is to specify the model that firms need pollution as an input for the production process. The consequence would be that firms may be forced to alter their production and innovation decisions if pollution is bound to stay below a certain threshold. Among scholars this approach has also proven to be very popular and it can be found in several seminal papers on environmental growth models ranging from Bovenberg and Smulders (1995) to Ricci (2007). Another method is to make the productivity of firms subject to the amount of pollution present in the economy. One can argue that pollution can affect the quality of the inputs used in the production of goods and therefore pollution will once more alter the production and innovation decisions of firms. On closer examination, this approach is similar to the one considering a decrease of the workforce due to pollution and in the end will have akin implications.

As mentioned above, there are numerous ways of introducing a pollution effect into growth models and this list could be continued with approaches allowing pollution to affect the reelection probability of policy makers or the market structure in the economy. Though this would go beyond the scope of this work.

The subsequent analysis assumes at first that pollution affects the economy through the utility of the households. In compliance with equation (1) households experience disutility from pollution. Later, the model is enlarged, and it is presumed that the RDI sector is vulnerable to pollution.

3 Equilibrium

After having presented the model in the previous section, the balanced growth path can now be derived in order to analyze the behavior of the model over time. In this context, a balanced growth path is defined as a state in which the variables of the model, namely $Y(t)$, $X(t)$, $C(t)$, $Z(t)$, $Q(t)$ and $P(t)$, grow at constant rates.

3.1 Decentralized Solution

Households' Problem Solving the households' problem and recalling that the population L is constant, leads to the extended Ramsey Rule[4]

$$r = \rho + \theta \frac{\dot{c}}{c} - \eta (1 - \theta) \frac{\dot{P}}{P} = \rho + \theta \frac{\dot{C}}{C} - \eta (1 - \theta) \frac{\dot{P}}{P}, \tag{9}$$

where $0 < \rho < \infty$ represents the rate of time preference and $r(t)$ the interest rate at time t. Its interpretation is almost standard. To forgo consumption in the current period the households require the marginal rate of return to equal the rate of time preference plus a premium since the faster consumption is expanded, the faster the marginal utility of consumption decreases and the less future returns on savings are appreciated. But here, the households additionally need to be compensated for the change in marginal utility that comes along with a change of pollution and interestingly, even though consumers take pollution as given, its evolution nevertheless affects their behavior. For the sake of completeness, the solution of the household problem determines a global optimum and is unique.

Output Decision of the Final Good Sector By maximizing the profits of the final good sector for all points in time and by acknowledging that there are no dynamic constraints,

$$X_j = L \left(\frac{\alpha B q^{\alpha \kappa_j}}{p_j} \right)^{\frac{1}{1-\alpha}} \tag{10}$$

can be derived as the aggregated demand function of the intermediate good X_j at any point in time, whereas p_j is the price of X_j.

Production and Innovation Decision of the RDI Sector There are two decisions the firms in the RDI have to cope with. First, they have to decide whether they should perform research and then, under the premise that they have been successfully innovating, they have to determine the optimal price at which they sell their patented good to the final sector. This sequential decision process can be solved through backward induction. Therefore, in a first step, the optimal price of the intermediate X_j is determined, and in a second step, the innovation decision is resolved.

Step 1: Pricing The optimization of the monopolistic profits a firm in the RDI sector can earn if it has been successfully innovating, leads to the price p_j that is charged for the

[4]The complete derivation of the solution of the household problem can be seen in section A.1 in the appendix.

intermediated good X_j and in combination with equation (10) to the corresponding demand for X_j

$$p_j = \frac{1}{\alpha} = p, \tag{11}$$

$$X_j = \alpha^{\frac{2}{1-\alpha}} L B^{\frac{1}{1-\alpha}} q^{\frac{\alpha\kappa_j}{1-\alpha}}. \tag{12}$$

So p_j is constant over time, intermediate good and quality level. Accordingly, the monopoly profit of a RDI firm π^{RDI} is

$$\pi^{\text{RDI}}(\kappa_j) = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} L B^{\frac{1}{1-\alpha}} q^{\frac{\alpha\kappa_j}{1-\alpha}} \tag{13}$$

$$= \bar{\pi} q^{\frac{\alpha\kappa_j}{1-\alpha}}, \tag{14}$$

where $\bar{\pi}$ is constant and can be seen as some sort of basic profit.

Step 2: Innovation Decision Firms in the RDI sector are only willing to undertake research if their expected return covers at least their research investment $Z(\kappa_j)$. Accordingly, given that there is free entry into the research business, the expected profit of every future innovation has to be zero, so

$$l(\kappa_j)\mathrm{E}\left[V(\kappa_j + 1)\right] - Z(\kappa_j) = 0. \tag{15}$$

Here $V(\kappa_j)$ is the present value of the κ_jth innovation and thus $\mathrm{E}\left[V(\kappa_j + 1)\right]$ is the expected value of the next technological breakthrough. Note, in equation (15) $Z(\kappa_j) > 0$ must be fulfilled, since firms who want to secure profits have to devote resources towards research.

When assuming that the interest rate is constant over time, as will be true in equilibrium, and by using equation (6) as well as (7), $l(\kappa_j)$ and $Z(\kappa_j)$ can be determined to be

$$l(\kappa_j + 1) = \bar{\pi} A - r = l \tag{16}$$

$$Z(\kappa_j) = q^{(\kappa_j+1)\frac{\alpha}{1-\alpha}} \left(\bar{\pi} - \frac{r}{A}\right). \tag{17}$$

Therefore, the innovation probability l is constant over time, intermediate good and quality rank as long as r is constant. Interestingly, due to the definition of $\phi(\kappa_j)$, the amount of research investment given by equation (17) varies across quality ranks, even though the innovation probability l remains constant regardless of κ_j.

The Balanced Growth Path From the analysis above, the values of the variables $Y(t)$, $X(t)$, $Z(t)$, $C(t)$ and $P(t)$ at the balanced growth path can be identified as variables depending on the aggregated quality level $Q(\kappa_j)$ prevailing in the economy:

$$X = L\alpha^{\frac{2}{1-\alpha}} B^{\frac{1}{1-\alpha}} Q \tag{18}$$

from aggregating equation (12) and using equation (3).

$$Y = L\alpha^{\frac{2\alpha}{1-\alpha}} B^{\frac{1}{1-\alpha}} Q \tag{19}$$

from using equation (12) and aggregating equation (2).

$$Z = q^{\frac{\alpha}{1-\alpha}} \left(\bar{\pi} - \frac{r}{A}\right) Q \tag{20}$$

from aggregating equation (17) and using equation (3).

$$C = \left(LB^{\frac{1}{1-\alpha}} \left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right) - q^{\frac{\alpha}{1-\alpha}} \left(\bar{\pi} - \frac{r}{A}\right)\right) Q \tag{21}$$

from using the resource constraint of the economy $Y = C + X + Z$.

$$P = L\alpha^{\frac{2}{1-\alpha}} B^{\frac{1}{1-\alpha}} Q^{1-\zeta} \tag{22}$$

from inserting equation (18) in (8). As a result, Y, X, C and Z all grow at the same rate as the aggregated quality level Q,

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\dot{Z}}{Z} = \frac{\dot{Q}}{Q}, \tag{23}$$

and the pollution P at the rate

$$\frac{\dot{P}}{P} = (1-\zeta)\frac{\dot{Q}}{Q}, \tag{24}$$

whereas a dot above a variable indicates the variables first derivative with respect to time. This leaves the growth rate of the aggregated quality level Q as the sole undetermined variable of the balanced growth path.

Recalling equation (3) and the fact that in equilibrium the innovation probability l is constant leads to the expected aggregated quality change per unit of time given by

$$\mathrm{E}\left[\Delta Q\right] = \sum_{j=1}^{N} l \left(q^{(\kappa_j+1)\frac{\alpha}{(1-\alpha)}} - q^{\kappa_j \frac{\alpha}{(1-\alpha)}}\right) \tag{25}$$

$$= l\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right) Q. \tag{26}$$

Dividing by Q and using equation (16) results in

$$\mathrm{E}\left[\frac{\Delta Q}{Q}\right] = l\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right) \tag{27}$$

$$= (\bar{\pi}A - r)\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right) \tag{28}$$

$$= \frac{\dot{Q}}{Q}, \tag{29}$$

whereas the last equation holds if the number of intermediate goods N is large enough to treat Q as differentiable.

In addition, when considering equation (24), the solution of the household problem (9) can be modified such that

$$r - \rho = (\theta - \eta(1-\theta)(1-\zeta))\,\frac{\dot{Q}}{Q}. \tag{30}$$

Now, combining equations (28) and (30) makes it possible to determine the interest rate

$$r = \frac{\rho + \bar{\pi}A\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta\,(1-\theta)\,(1-\zeta))}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta\,(1-\theta)\,(1-\zeta))}, \tag{31}$$

which is constant, so the simplification during the derivation of equation (17) has been appropriate. Finally, by inserting equation (31) into (28), the growth rate of the aggregate quality level $\frac{\dot{Q}}{Q}$ can be determined as

$$\frac{\dot{Q}}{Q} = \gamma = \frac{(\bar{\pi}A - \rho)\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta\,(1-\theta)\,(1-\zeta))}. \tag{32}$$

According to equation (23) this is also the growth rate of Y, X, Z as well as C, thus γ represents the prevailing growth rate of the economy and in compliance with equation (24), pollution P grows with the rate $(1-\zeta)\gamma$.

The balanced growth path is unique. This becomes obvious when normalizing the balanced growth path values of the endogenous variables given in equations (18), (19), (20), (21) and (22) with respect to Q and recalling equation (31).

3.2 Conditions on Parameters

In order to ensure the existence of the balanced growth path, the parameters determining the growth rate of aggregated quality, the transversality condition, the interest rate and the intertemporal substitution elasticity have to fulfill certain conditions.

In this model economy, once an innovation has taken place it can not be revoked or in other words, no technological knowledge can be forgotten. As a consequence, the growth rate of the aggregated quality level has to be positive and in equilibrium $\gamma = \frac{\dot{Q}}{Q} \geq 0$ has to be ensured. In accordance with equation (32) and the fact that $\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right) \geq 0$, the growth rate of aggregated quality is guaranteed to be positive, if either

$$(\bar{\pi}A - \rho) \geq 0 \quad \text{and} \quad 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) \geq 0 \tag{33}$$

or if

$$(\bar{\pi}A - \rho) \leq 0 \quad \text{and} \quad 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) \leq 0. \tag{34}$$

The transversality condition (A.5) in turn, is satisfied if and only if $(\mu(t)a(t))$ decreases over time. On the one hand, since the combined expected profits of all firms in the RDI sector grows with $\frac{\dot{Q}}{Q}$ and the fact that the final good sector is competitive, the value of the households assets $a(t)$, consisting of a balanced portfolio of all firms, grows at the same rate as the economy. On the other hand, under the terms of equation (A.4), $\mu(t)$ grows at the rate $(-r)$. Consequently, the transversality condition holds if

$$r > \gamma. \tag{35}$$

When introducing the balanced growth path values for r and γ from equations (31) and (32) in equation (35), this leads to two cases in which the transversality condition is fulfilled. Either

$$\begin{aligned} &\rho > \bar{\pi}A\left(1 - q^{\frac{-\alpha}{(1-\alpha)}}\right)(1 - \theta - \eta(1-\theta)(1-\zeta)) \\ &\text{if } 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) > 0 \end{aligned} \tag{36}$$

or

$$\begin{aligned} &\rho < \bar{\pi}A\left(1 - q^{\frac{-\alpha}{(1-\alpha)}}\right)(1 - \theta - \eta(1-\theta)(1-\zeta)) \\ &\text{if } 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) < 0. \end{aligned} \tag{37}$$

Last but not least, given equation (31) and the assumptions that all parameters in the equation are bound to be finite, the interest rate is finite if

$$1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) \neq 0. \tag{38}$$

In the end, combining the conditions from equations (33), (34), (36), (37) as well as (38) results in two cases in which the balanced growth path might exist.

$$\begin{aligned} \text{Case 1:} \quad & (\bar{\pi}A - \rho) > 0 \\ & 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) > 0 \\ & \rho > \bar{\pi}A\left(1 - q^{\frac{-\alpha}{(1-\alpha)}}\right)(1 - (\theta - \eta(1-\theta)(1-\zeta))) \\ \\ \text{Case 2:} \quad & (\bar{\pi}A - \rho) < 0 \\ & 1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta)) < 0 \\ & \rho < \bar{\pi}A\left(1 - q^{\frac{-\alpha}{(1-\alpha)}}\right)(1 - (\theta - \eta(1-\theta)(1-\zeta))) \end{aligned} \tag{39}$$

Though, on closer inspection, Case 2 appears to involve inconsistencies. This can best be seen, when assuming that $\theta = 1$.

$$\text{Case 2 with } \theta = 1: \quad \begin{aligned} &(\bar{\pi}A - \rho) < 0 \\ &q^{\frac{\alpha}{(1-\alpha)}} < 0 \\ &\rho < 0 \end{aligned} \tag{40}$$

Clearly, the second and third line are inconsistent with the assumptions in section 2.3 and 3.1 on the quality parameter q as well as on the rate of time preference ρ. Consequently, Case 1 must be satisfied in order to ensure the existence of the balanced growth path.

But Case 1 implies that $\gamma > 0$, which is only possible if the marginal utility of consumption is constant or declines over time. If this would not be the case, at some point in time u_c would be indefinitely large and households would consume the complete final good production, leaving nothing for research investment and therewith making growth impossible. Thus $u_{cc} = -\theta c^{-\theta-1} P^{\eta(1-\theta)} \leq 0$ and $u_{cP} = \eta\,(1-\theta)\,c^{-\theta} P^{\eta(1-\theta)-1} \leq 0$ have to be fulfilled on the balanced growth path. Since θ has already been defined to be positive, $u_{cc} \leq 0$ does not require any further assumptions. Then again, $u_{cP} \leq 0$ necessitates $\theta \leq 1$ and therewith demands that the intertemporal substitution elasticity is below unity.[5]

Thus, all in all, the existence of the balanced growth path requires:

$$\begin{aligned} &(\bar{\pi}A - \rho) > 0 \\ &1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta\,(1-\theta)\,(1-\zeta)) > 0 \\ &\rho > \bar{\pi}A\left(1 - q^{\frac{-\alpha}{(1-\alpha)}}\right)(1 - (\theta - \eta\,(1-\theta)\,(1-\zeta))) \\ &\theta \leq 1. \end{aligned} \tag{41}$$

3.3 Effects of Pollution on the Growth Rate

Before continuing the analysis of the equilibrium by investigating the transitional dynamics and the comparative statics of the model, it is convenient to study the effects of pollution in order to understand the implications of the pollution dimension of the model.

In this model, pollution exclusively affects the utility of households and hence the sole channel of transmission of pollution to economic growth must be the consumption decision of the households. The presence of pollution has two effects on the households. First, according to equation (1), an increase in pollution decreases the households' utility. But, as the households can not directly influence the amount of pollution they are exposed to, this direct effect of pollution is

[5] Stokey (1998) as well as Aghion and Howitt (1998) come to similar conclusions regarding the intertemporal substitution elasticity, albeit in a different class of environmental growth models.

limited to the households and does not provoke any consequences for the rest of the economy. What is more important, pollution also reduces the marginal utility of consumption u_c since $u_{cP} < 0$, hence the households consume less in the presence of pollution. Michel and Rotillon (1995) call this the "distaste" effect of pollution. The pollution-induced decrease of consumption triggers a reduction of demand for the final good, which then diminishes the need of intermediates and thus also the expected present value of firms undertaking research. This leads to a drop in research investment and under the terms of equation (6) to less innovations, the mainspring of economic growth in this setting. As a consequence, the aggregated quality level Q grows slower and in compliance with equation (23) the economic growth rate γ is reduced.

Whereas the propagation of the "distaste" effect throughout the economy is rather straightforward, the magnitude and the direction of this effect depends decisively on the parameter ζ and the term $\eta(1-\theta)$. On the one hand, the value of ζ is crucial in determining the amount of pollution present in the economy, metaphorically speaking, ζ determines the size of the transmission channel. On the other hand, $\eta(1-\theta)$ is responsible for the amount of consumption the households are willing to give up when faced with pollution. Hence $\eta(1-\theta)$ gives the power of pollution regarding its ability to change the households' consumption decision.

Therefore, in order to be able to assess the consequences of pollution on the economic growth rate, the effect of pollution must be analyzed using different constellations of ζ in relation to $\eta(1-\theta)$. In the following, five different scenarios will be explored, each investigating a different specification of ζ.[6] The different prevailing economic growth rates will be compared to the growth rate which would be present if there would exist no channel of pollution transmission.

In this simple setting, the absence of a channel of transmission of pollution to economic growth can be simulated by setting $\eta = 0$ or in words by rendering the households indifferent to pollution. In this case the economic growth rate would be

$$\gamma_{\text{no pollution}} = \frac{(\bar{\pi}A - \rho)\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)\theta}. \tag{42}$$

Scenario 1 ($\zeta < 0$)**:** In this constellation, due to the terms of equation (8), innovations lead to a high pollution intensity of the intermediate goods and according to equation (24) finally to a pollution growth $\frac{\dot{P}}{P}$ which is bigger than the overall growth rate of the economy γ. Consequently, an increase of the intermediate production due to technological progress generates an even bigger increase of pollution. Because of the "distaste" effect, this big increase in pollution causes a big reduction in household consumption and thereby a much lower growth rate compared to an

[6] Note, according to section 2.2 and 3.2, $\eta(1-\theta)$ has already been restricted to be negative.

economy without a pollution effect.[7]

$$\gamma_{(\zeta<0)} = \frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)}{1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta(1-\theta)(1-\zeta))} < \gamma_{\text{no pollution}} \tag{43}$$

Scenario 2 ($\zeta = 0$)**:** In this case, innovations do not influence the pollution intensity of the intermediate production and pollution grows at the same rate as the economy. Accordingly, more intermediates, as a result of ongoing innovations, cause a normal sized increase in pollution, which in turn induces a normal sized decrease in consumption. This leads to a normal sized decrease in research investment and thus to a lower growth rate compared to the growth rate without a pollution channel.

$$\gamma_{(\zeta=0)} = \frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)}{1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta(1-\theta))} < \gamma_{\text{no pollution}} \tag{44}$$

Scenario 3 ($0 < \zeta < 1$)**:** Now the aggregated quality level has a downsizing effect on the pollution intensity of the intermediate production, but the magnitude of this effect is rather small and pollution grows only slightly slower than the overall economy. Hence technological progress increases the amount of intermediates and also pollution, but the latter to a smaller extent. This triggers only a small decrease of consumption followed by a small fall in research investment and finally to a growth rate slightly smaller than the growth rate of an economy with no channel of pollution transmission.

$$\gamma_{(0<\zeta<1)} = \frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)}{1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta(1-\theta)(1-\zeta))} < \gamma_{\text{no pollution}} \tag{45}$$

Scenario 4 ($\zeta = 1$)**:** In this scenario, technological progress reduces the pollution intensity. In fact, it depletes the pollution intensity in such a way, that increases in the intermediate production due to innovations are completely compensated by a decrease in pollution intensity and do not lead to more pollution. Pollution is therefore constant over time and can not negatively affect the marginal utility of consumption. Hence, there is no "distaste" effect and there exists no transmission channel through which pollution can affect growth.

$$\gamma_{(\zeta=1)} = \frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)}{1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)\theta} = \gamma_{\text{no pollution}} \tag{46}$$

[7] In this section, the size of the pollution increase in the different cases are compared to the case where $\zeta = 0$, that is, a normal sized increase of pollution results directly and solely from the increase of the intermediate production.

Scenario 5 ($\zeta > 1$)**:** In this constellation, innovations also diminish the pollution intensity. But now, the fall in pollution intensity overcompensates the rise in intermediate production caused by progress and pollution recedes overtime. This triggers a reverse "distaste" effect, as the decrease in pollution causes the marginal utility of consumption to grow. Accordingly, households tend to consume more and therefore create more research incentives in form of higher monopolistic profits. In the end, this leads to more research and thus to bigger growth compared to the setting where households would not be affected from pollution.

$$\gamma_{(\zeta>1)} = \frac{(\bar{\pi}A - \rho)\left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1\right)(\theta - \eta(1-\theta)(1-\zeta))} > \gamma_{\text{no pollution}} \tag{47}$$

To summarize, the effect of pollution on the growth rate depends on the one hand on the households' degree of pollution aversion η and on the other hand, and even more importantly, on the link between pollution intensity and the overall level of technological progress, in this model mainly determined by ζ. Pollution reduces the overall economic growth rate if a rise in the level of technological progress does not imply a big enough decrease of the pollution intensity ($\zeta < 1$) and boosts growth if the pollution intensity falls disproportionately fast when the economy advances ($\zeta \geq 1$). In detail:

$$\gamma_{(\zeta<0)} < \gamma_{(\zeta=0)} < \gamma_{(0<\zeta<1)} < \gamma_{(\zeta=1)} = \gamma_{\text{no pollution}} < \gamma_{(\zeta>1)}. \tag{48}$$

3.4 Transitional Dynamics and Comparative Statics

3.4.1 Transitional Dynamics

From equation (23) it is straightforward to see that at any point in time, in equilibrium, Q, Y, X, Z and C all grow at the same rate γ. Consequently, the model has no transitional dynamics and the economy moves directly to the balanced growth path, whatever the initial conditions. This feature of the model is mainly due to the fact that there are no stock variables present in the economy. At any point in time, all the output is used for either consumption or investment towards the RDI sector and no stocks are build up. Furthermore, pollution is specified as a flow variable, thus no pollution is transfered from one period to the next and arises every period anew.

3.4.2 Comparative Statics

Scale Effects In accordance with equations (14) and (32), the prevailing growth rate γ of the economy increases with the size of the population L. This seems realistic in this context, since a bigger workforce makes it possible to produce more final good output, which in turn boosts innovations, the mainspring of economic growth in this setting, due to more disposable research investment. In reality though, this sort of scale effect can hardly be observed (Jones, 1999), and hence scale effects would have to be eliminated, for example by modifying the research technology or by allowing for horizontal and vertical innovations.[8] But, to simplify matters, in this model scale effects are accepted.

Changes in η The parameter η determines the degree of pollution aversion of the households and since

$$u_{cP} = \eta\,(1-\theta)\,C^{-\theta}P^{\eta(1-\theta)-1}, \tag{49}$$

also the magnitude of the "distaste" effect. While it is obvious from equation (49) that an increase (decrease) of η leads to an easing (strengthening) of the "distaste" effect,[9] the effect of a change in η on the growth rate is somewhat less straightforward. Deriving equation (32) with respect to η yields

$$\frac{\partial\gamma}{\partial\eta} = \frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2(1-\theta)\,(1-\zeta)}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta\,(1-\theta)\,(1-\zeta))\right)^2}. \tag{50}$$

Expectedly, equation (50) does not just depend on the η, but also on the specification of ζ. Consequently, the analysis of changes in η once more has to distinguish between five different specifications of ζ.

Scenario 1 ($\zeta < 0$):	An increase in η leads to more growth.
Scenario 2 ($\zeta = 0$):	An increase in η leads to more growth.
Scenario 3 ($0 < \zeta < 1$):	An increase in η leads to more growth.
Scenario 4 ($\zeta = 1$):	An increase in η has no effect on growth.
Scenario 5 ($\zeta > 1$):	An increase in η leads to less growth.

The reasoning behind these results is similar to the argumentation in section 3.3 regarding the effect of pollution on the growth rate and is therefore not repeated at this point. The sole difference here is that an increase (decrease) in η weakens (strengthens) the "distaste" effect and thus the pollution effect is less (more) severe.

[8] At this point, the interested reader is kindly referred to the work of Aghion and Howitt (1998) and (2009), Dinopoulos and Thompson (1998) or Young (1998).

[9] Keep in mind, that in section 2.2, η has been defined to be smaller than zero. Consequently, as η increases (η moves towards zero) the distaste of the households for pollution decreases and a reduction of η (η moves towards $-\infty$) induces a rise in the antipathy to pollution.

Changes in ζ The value ζ determines what effect the level of technological progress has on pollution and is crucial regarding the extent as well as the evolution of pollution in the economy. Under the terms of equations (8) and (24), an increase in ζ leads to a reduction of the amount of pollution present in the economy and over time to slower pollution growth or, if $\zeta > 1$, to a faster decrease of pollution. Following the argumentation in section 3.3, less pollution allows more consumption and, by the means of increased research incentives, in the end induces higher economic growth.

One arrives at the same conclusion when analyzing the derivative of the economic growth rate in equation (32) with respect to ζ,

$$\frac{\partial\gamma}{\partial\zeta} = -\frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2\eta\,(1-\theta)}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta\,(1-\theta)\,(1-\zeta))\right)^2} > 0. \tag{51}$$

Since it has been assumed that $(\bar{\pi}A-\rho) > 0$ and $\eta(1-\theta) < 0$ the term on the right hand side of equation (51) is always positive and therefore confirms the argument that a rise in ζ results in higher economic growth.

Changes in θ In this economy, θ represents the multiplicative inverse of the elasticity of intertemporal substitution. A change in θ has two implications, it changes the households' preferences towards consumption and in this setting additionally their attitude regarding pollution. Accordingly, the effect of a change in θ on the growth rate consists of two distinct effects.

$$\begin{aligned}\frac{\partial\gamma}{\partial\theta} =& -\underbrace{\frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta\,(1-\theta)\,(1-\zeta))\right)^2}}_{\text{consumption effect}} \\ & -\underbrace{\frac{\eta\,(1-\zeta)\,(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta\,(1-\theta)\,(1-\zeta))\right)^2}}_{\text{pollution effect}}\end{aligned} \tag{52}$$

The implications of the consumption effect (ce) on the growth rate is straightforward. If θ rises, households care more about consumption today and are less willing to delay consumption in favor of higher research investment. Therewith the households cripple the mainspring of economic growth and consequently growth falls. Correspondingly,

$$\text{ce} = -\frac{(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta\,(1-\theta)\,(1-\zeta))\right)^2} < 0. \tag{53}$$

The magnitude and the direction of the pollution effect (pe) is less obvious. Since $u_{cP} = \eta(1-\theta)\, c^{-\theta} P^{\eta(1-\theta)-1}$, an increase in θ weakens the "distaste effect" and as described in section 3.3 this can have positive or negative implications on the economic growth rate, depending on the assumptions regarding the effect of technological progress on pollution. Hereof it becomes clear why the pollution effect features the term $(1-\zeta)$. Whenever $\zeta > 1$ the easing of the "distaste effect", due to a raise of θ, entails a decrease of consumption and, in the long run, leads to less growth. The pollution effect is consequently negative. If $\zeta \leq 1$ the effect is reversed and the pollution effect is positive. To summarize

$$\text{pe} \;= -\frac{\eta(1-\zeta)(\bar{\pi}A-\rho)\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)^2}{\left(1+\left(q^{\frac{\alpha}{(1-\alpha)}}-1\right)(\theta-\eta(1-\theta)(1-\zeta))\right)^2}\begin{cases} <0 \text{ if } \zeta > 1 \\ \geq 0 \text{ if } \zeta \leq 1. \end{cases} \tag{54}$$

Therefore, while the consumption and output effect may go in the same direction, they may also oppose each other and the overall effect of a change in θ once more crucially depends on the parameter constellation of η and $(1-\zeta)$. From equation (52) it becomes clear that if $\eta(1-\zeta) \geq -1$, then the overall effect is positive and the growth rate increases when θ rises. On the other hand, if $\eta(1-\zeta) < -1$ the overall effect is negative.

Changes in A and B A and B both represent productivity parameters and, by definition, an increase in productivity always leads to an augmented output in the corresponding sector. Thus a rise in B increases the quantity of the final good in the economy, thereby allows more research investment and finally leads to higher growth. A rise in A on the other hand increases the probability of a successful innovation and therewith directly boosts the rate of technological progress, which in turn equals the economic growth rate according to equation (23).

4 Pareto Optimality

All in all, there are four possible sources of inefficiency throughout the model: monopolistic pricing in the intermediate good sector, the appropriability effect, the business stealing effect and finally pollution externalities.

4.1 Monopolistic Pricing

A first reason for suboptimality is monopolistic pricing. Owing to the fact that in this model innovations are always drastic and protected through patents, innovating firms in the intermediate good sector enjoy monopole power. This enables them to ask for higher prices compared to prices in a fully competitive environment. Therewith monopolistic pricing reduces the quantities of traded intermediate goods and induces a dead weight loss. What is more, a reduced quantity of intermediate goods implies less final output and in the end leads to inefficiently low economic growth as a result of less available goods for research investment.

4.2 Appropriability Effect

A second source of inefficiency results from insufficient property rights (Arrow, 1962) and is commonly referred to as the appropriability effect or the problem of appropriability (Acemoglu, 2009). The appropriability effect arises because innovators are not able to capture the entire social gain created by an innovation and as a consequence the private value of an innovation falls short of its social value. This problematic is based on the fact that an innovation does not only enable the innovator to capture temporary private profits but, as it is non-excludable, it also advances the public knowledge of a good so that future research can directly develop the next generation of the product. Consequently, research firms do not devote as much resources towards innovations as would be socially optimal and the appropriability effect abates growth in the decentralized economy.

4.3 Business Stealing Effect

A third cause of suboptimailty is the business stealing effect, which occurs as a result of different research incentives of incumbents and newcomers. In this economy innovations are assumed to be drastic and due to Arrow's replacement effect technological breakthroughs are always carried out by entrants (Arrow, 1962). The research incentives of a newcomer equal the profits he will make if he successfully innovates, since by means of a successful innovation an entrant replaces the incumbent as the sole producer of the enhanced good and can "steal" his monopolist rents. The social planner, on the other hand, values an innovation according to the profits of the incumbent monopolist as he can account for them as part of the producers' surplus. From applying once more Arrow's replacement effect it becomes clear that the research incentives of the social planner are lower than those of an entrant and hence lower than those in a laissez-faire setting. Accordingly, in the decentralized economy, the business stealing effect inflates research investment and in the end causes excessive growth.

4.4 Pollution Externalities

The fourth and last distortion in this economy is linked to the creation of pollution in the intermediate sector and it emerges because households and firms do not anticipate all the effects pollution has on the economy. As the computation of the social planner's choice with respect to pollution is rather complex, it is convenient to analyze the effect of an increase or a decrease of pollution on utility instead, in order to understand how the social planner would choose pollution.

In the equilibrium, pollution has two opposing effects on welfare. On the one hand, it is straightforward to see from u_P that pollution directly decreases the households' utility and in compliance with the pollution elasticity of utility, one unit less pollution increases the utility by

$$\left|\frac{\partial u}{\partial P}\frac{P}{u}\right| = \left|\eta\,(1-\theta)\,\frac{(cP^{\eta})^{(1-\theta)}}{(cP^{\eta})^{(1-\theta)}-1}\right|, \tag{55}$$

which can be simplified to

$$\left|\frac{\partial u}{\partial P}\frac{P}{u}\right| = |\eta\,(1-\theta)| \tag{56}$$

when assuming that the productivity of the final sector B is sufficiently large, that approximately $(cP^{\eta})^{(1-\theta)} \approx (cP^{\eta})^{(1-\theta)} - 1$.

On the other hand, as pollution is an imperative byproduct of the production of intermediates, a decrease in pollution is equivalent to a restriction of the intermediate production. A reduced intermediate production in turn entails a reduction in final output, due to the lack of inputs, and leads to a drop in research investment, intermediate investment and consumption. The decrease in consumption, as a result of less available final output and due to less economic growth because of the decrease in research investment, and the subsequent diminution of utility can then be seen as the indirect pollution effect on utility. To quantify the indirect pollution effect, it has to be determined by how much consumption decreases when pollution diminishes and additionally by how much utility changes when more or less consumption is available. The former is given by the pollution elasticity of consumption

$$\frac{\partial C}{\partial P}\frac{P}{C} = \frac{1}{1-\zeta}, \tag{57}$$

which becomes clear when recalling equation (21) and (22). The latter by the consumption elasticity of utility

$$\frac{\partial u}{\partial C}\frac{C}{u} = (1-\theta)\,\frac{(cP^{\eta})^{(1-\theta)}}{(cP^{\eta})^{(1-\theta)}-1}, \tag{58}$$

which can also be simplified to

$$\frac{\partial u}{\partial C}\frac{C}{u} = (1-\theta) \tag{59}$$

when using the same assumption on B as above.

Combined, the indirect pollution effect is consequently

$$\left(\frac{\partial C}{\partial P}\frac{P}{C}\right)\left(\frac{\partial u}{\partial C}\frac{C}{u}\right)=\frac{1}{1-\zeta}\left(1-\theta\right), \tag{60}$$

and a one unit decrease of pollution leads to a $\frac{1}{1-\zeta}(1-\theta)$ decrease of utility as a result of the indirect pollution effect.

The overall effect of a change in pollution on welfare is thus a combination of the direct and indirect pollution effect on welfare,

$$\text{overall effect } = (1-\theta)\left(\eta+\frac{1}{1-\zeta}\right). \tag{61}$$

Depending on the specifications of ζ, the two effects may oppose each other. The overall effect is positive if $\eta > -\frac{1}{1-\zeta}$ and negative if $\eta < -\frac{1}{1-\zeta}$. Therefore, if $\eta > -\frac{1}{1-\zeta}$, the social planner would choose a higher level of pollution than the level prevailing in the decentralized economy and if $\eta < -\frac{1}{1-\zeta}$, the planner would prefer a lower amount of pollution. This makes clear that apart from a setting in which $\eta = -\frac{1}{1-\zeta}$, the laissez-faire economy will exhibit a suboptimal amount of pollution and therewith an inefficient growth rate.

4.5 Overall Effect of the Distortions and Implementation of the Optimum

To summarize, seen on their own, monopolistic pricing and the appropriability effect imply a suboptimal low growth rate, the business stealing effect inflates the growth rate to an inefficient level and the pollution externalities expand or diminish the prevailing growth rate to a suboptimal amount, depending on the specification of ζ with respect to η. The overall implications of the four distortions remain opaque and, as in most Schumpeterian growth models, the prevailing growth rate in the laissez-faire equilibrium is Pareto suboptimal.

The socially optimal path can be implemented through the introduction of three policy tools: a subsidy to the final good sector, a subsidy towards the research sector and tradable pollution permits.

As in standard endogenous growth models, adequate subsidies to the firms in the final good sector as well as to research firms, financed by a lump sum tax on household income, allow to mitigate the distortions that are generated through the presence of monopolistic pricing and incorrect research incentives. Such subsidies take effect by appropriately adjusting the price of the intermediate goods and by installing adequate research incentives (i.a. Acemoglu, 2009).

The pollution permits in turn, which could also be replaced by a suitable pollution tax (Grimaud, 1998 and 1999),[10] enable the policy makers to control the amount of pollution present in the economy and can thereby counter the pollution externalities. The functioning of the pollution permits is straight forward. For every unit of pollution that firms in the intermediate sector emit, they need a pollution permit which they can either obtain to a certain amount for free from the government (grandfathering) or buy on a competitive market. This alters the production and innovation decision of the firms in question by introducing a new necessary input into the production function for the intermediate good. Their modified profit function is thus

$$\pi^{\text{RDI permit}}(\kappa_j) = p_j X_j - X_j - p_a \left(P_j - S_j\right), \tag{62}$$

where p_a is the price of a pollution permit and S_j the quantity of allowance a firm receives from the government for free. Clearly a tight (generous) regime regarding the distribution of pollution licenses increases (reduces) the costs for the intermediate firms and in the end reduces (increases) pollution. Accordingly, by choosing an appropriate distribution scheme policy makers can install the desired amount of pollution.

5 Discussion

5.1 Pollution Threshold

The next section seeks to resolve one of the main questions in the debate on environment and growth, namely whether economic growth is sustainable if the agents can only cope with a certain amount of pollution. Whereas in this context, sustainability is associated with the ability to support the growth rate prevailing in the equilibrium ad infinitum.

To resolve this question the underlying model is enlarged, so that not only the households are affected from pollution but also the research sector. It is assumed that if pollution exceeds a certain level, it interferes with the innovating process and in the end disrupts all research effort. Hence, there exists a pollution threshold above which the productivity parameter of the research sector A drops to zero. To justify this assumption, imagine for instance that the research sector uses highly sensitive instruments which become inaccurate in the presence of too much pollution in form of electric smog.

Additionally, the following analysis will also presume that the research firms do not anticipate the inefficiency of their undertaking when the pollution threshold is reached, so that their

[10]Note, Grimaud (1998, 1999) bases his analysis on models which allow for endogenous pollution intensities, but which do not link the pollution intensity to the level of overall technological progress in contrast to the model presented here. However, in this context this feature does not change Grimaud's reasoning.

innovation decision will not change and their demand for the intermediate good will not cease. However, note that this assumption is only for the sake of comprehensibility, the implications of a pollution threshold would remain the same if the research firms would be allowed to alter their innovation decision, solely the propagation mechanism would be less intuitive.

Before analyzing all the implications of a pollution threshold on the economy as a whole, it is reasonable to determine when, if at all, such a boundary is reached.

From equation (24) it is clear that pollution grows at the rate $(1-\zeta)\gamma$ in equilibrium. Obviously, if $\zeta \leq 1$, pollution will be constant or fall in equilibrium and the pollution threshold will be reached at no point in time as long as the pollution threshold is higher than the first pollution level. Consequently, in this case the level of pollution never interferes with the research effort and growth is truly sustainable.

If $\zeta < 1$, the amount of pollution emitted by the intermediate sector will inevitably reach its critical value at some point in time, whereas the exact moment depends on the specifications of the pollution threshold. Generally one could think of a multitude of different specification of the pollution threshold and while the pollution threshold could depend on the level of technological progress it might also be constant over time. Here, the moment where pollution attains its threshold will be computed exemplary for these two different specifications.

First, suppose that the pollution threshold is constant and equals $\bar{P}$. From the pollution production function (8) it is clear that $\bar{P}$ involves an intermediate production of

$$\bar{X} = \bar{P} Q^{\zeta}. \tag{63}$$

This can be combined with the total demand for intermediates from equation (10) and finally leads to the level of technological progress at which pollution has serious consequences for the research sector,

$$\bar{Q}_{\bar{P}} = \left(\frac{\bar{P}}{\alpha^{\frac{2}{1-\alpha}} L B^{\frac{1}{1-\alpha}}} \right)^{\frac{1}{1-\zeta}}. \tag{64}$$

On the other hand, assume that the pollution threshold is an increasing function of the level of technological progress. For example

$$\bar{P}(Q) = \bar{P}_0 m Q, \tag{65}$$

with m and $\bar{P}_0$ being both positive. Analog to the derivation above, this results in

$$\bar{Q}_{\bar{P}(Q)} = \left(\frac{\bar{P}_0 m}{\alpha^{\frac{2}{1-\alpha}} L B^{\frac{1}{1-\alpha}}} \right)^{\frac{1}{-\zeta}}, \tag{66}$$

as the level of technological progress at which the pollution threshold is reached.

Note that a part of determining the moment of a problematic pollution level, equation (64) and (66) once more stress the importance of the parameter ζ, as for higher (lower) ζ the economy can grow for a longer (shorter) period before the innovation process is affected from pollution. But, as long as $\zeta < 1$ at some point in time, which from now on is denoted as $t_{\bar{P}}$, pollution will attain its critical level and this will have severe implications for the economy.

Once the emissions from the intermediate good sector reach the pollution threshold at time $t_{\bar{P}}$, by assumption the productivity parameter of the research sector drops to zero. This implies that the firms in the RDI sector are no longer able to innovate and consequently the aggregated quality index is not advanced, so $Q(t_{\bar{P}}) = Q(t')$ for all $t' > t_{\bar{P}}$. Given that the level of technological progress Q has not changed from $t_{\bar{P}}$ to t', all the endogenous variables within the model, in particular the demand for the intermediates X, remain unchanged and the economy is frozen at its state in $t_{\bar{P}}$. This involves that the level of pollution P also remains unchanged compared to $t_{\bar{P}}$, and hence in t' the pollution threshold is reached once again. Therefore, from $t_{\bar{P}}$ on, no changes occur in the model and once the pollution threshold is reached in $t_{\bar{P}}$, the economy stalls at its current level. Thus in this setting growth is only sustainable if $\zeta \geq 1$, else the permanent pollution growth will eventually halt growth in the economy.

5.2 Pollution Abatement

Another important question in the discussion on environment and growth is whether mitigation efforts have an effect on economic growth rates. But, up to now, the underlying model has abstracted from the idea that pollution may be depleted. For this reason, the following section will overcome this simplification and will explore how waste disposal affects the economic growth rate in an environmental growth model.

Essentially, there are two approaches in the economic literature of how pollution abatement can be modeled in an environmental growth model. The first is to introduce a government into the model, which finances abatement spending. See Ligthart and van der Ploeg (1994) for an example. The other consists of allowing private firms to undertake abatement activities and has been elaborated for instance by Bovenberg and Smulders (1995). But given that the model presented in this work does not incorporate a government and the fact that the firms in the intermediate sector have hardly any incentive to spend resources on waste disposal, this model will use a new approach and will assume that the households can engage in mitigation activities.

Thus henceforth, the households can devote resources towards waste disposal and can thereby reduce the quantity of pollution they are effectively exposed to. Following the work of Greiner and Semmler (2008) effective pollution is

$$P_E(t) = \frac{P(t)}{D(t)^{\beta}}, \tag{67}$$

with $D(t)$ being the total amount of final good that is used for pollution abatement and $0 < \beta < 1$. In addition it is assumed that $D(t)^{\beta} > 1$ to ensure that effective pollution is always smaller than pollution without abatement. Correspondingly, the momentary utility of the households is now

$$u(t) = \begin{cases} \frac{1}{1-\theta}\left(\left(c(t)P_E(t)^{\eta}\right)^{(1-\theta)} - 1\right) & \text{for } 0 < \theta < \infty \text{ and } \theta \neq 1 \\ \ln c(t) + \eta \ln P_E(t) & \text{for } \theta = 1. \end{cases} \tag{68}$$

The possibility of waste disposal alters the household problem and in this context the households seek to maximize their overall utility by choosing an optimal amount of consumption and abatement in consideration of the usual constraints. Thus their optimization problem is given by

$$\begin{aligned} \max_{c(t),d(t)} \quad & U(0) = \textstyle\int_0^{\infty} e^{-\rho t} \frac{1}{1-\theta}\left(\left(c(t)P_E(t)^{\eta}\right)^{(1-\theta)} - 1\right) dt \\ \text{s.t.} \quad & \dot{a}(t) = r(t)a(t) + w(t) - c(t) - d(t) \\ & P_E(t) = \tfrac{P(t)}{D(t)^{\beta}} \\ & a(0) = a_0 \\ & \lim_{t\to\infty}\left(a(t)e^{-r(t)}\right) \geq 0, \end{aligned} \tag{69}$$

where $d(t)$ is the amount of abatement spending, each household contributes to $D(t)$. Note, although each household can only contribute to the economy-wide abatement effort with its own share $d(t)$, they nevertheless benefit from the amount of total waste disposal. This seems reasonable when thinking of a public park in which the people are urged to take care of their own litter in order to keep the park nice and clean for everybody.

Solving the optimization problem yields the optimal private and total abatement expenditures

$$d = -\eta\beta c \tag{70}$$

$$D = -\eta\beta C \tag{71}$$

as well as

$$r = \rho + \theta\frac{\dot{C}}{C} - \eta\left(1-\theta\right)\frac{\dot{P}}{P} + \left(\eta\beta\left(1-\theta\right)\right)\frac{\dot{D}}{D} \tag{72}$$

as a new extended Ramsey Rule.[11] Interestingly, equations (70) and (71) identify pollution abatement as some form of consumption and apparently when the households are given the possibility

[11]The complete derivation of the solution of the extended household problem can be seen in section A.2 in the appendix.

to invest in waste disposal there is a direct trade-off between consumption and abatement spending. This also implies that $\frac{\dot{d}}{d} = \frac{\dot{c}}{c} = \frac{\dot{D}}{D} = \frac{\dot{C}}{C}$, which will be useful when computing the growth rate of the economy.

Furthermore, since waste disposal can be done exclusively by the households and none of the other sectors is affected directly by pollution, the introduction of pollution abatement has no effect on the production and innovation decisions of the final good sector and the RDI sector. Correspondingly equations (23), (24) and (28) are also satisfied in this enlarged setting, whereas equation (23) can even be expanded by the term $\frac{\dot{D}}{D}$ because $\frac{\dot{D}}{D} = \frac{\dot{C}}{C}$. Hence, analog to section 3.1, the new extended Ramsey Rule (72) can be combined with the growth rate of the aggregated quality index (28) and the growth rate of pollution (24) in order to determine the prevailing growth rate of the economy. At first this leads to

$$r_{\text{abatement}} = \frac{\rho + \bar{\pi} A \left(q^{\frac{\alpha}{(1-\alpha)}} - 1 \right) (\theta - \eta (1-\theta)(1-\zeta-\beta))}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1 \right) (\theta - \eta (1-\theta)(1-\zeta-\beta))} \tag{73}$$

and then to

$$\gamma_{\text{abatement}} = \frac{(\bar{\pi} A - \rho) \left(q^{\frac{\alpha}{(1-\alpha)}} - 1 \right)}{1 + \left(q^{\frac{\alpha}{(1-\alpha)}} - 1 \right) (\theta - \eta (1-\theta)(1-\zeta-\beta))}. \tag{74}$$

According to this, the growth rate of the economy with pollution abatement is bigger than the growth rate in an economy that does not allow waste disposal and thus mitigation efforts have a positive effect on the economy. The reason for this is that when the households can deplete pollution, they can cope better with the pollution created in the RDI sector and are less negatively affected directly by pollution. Consequently, the households can bear a higher level of pollution which involves an increased demand of final goods and in the end enables the firms to undertake more research.

6 Conclusion

This work investigates the effect of pollution on economic growth using a Schumpeterian growth model which has been enlarged to include an environmental dimension. The model explicitly links the pollution intensity of economic activity to the overall level of technological progress in order to explore the role of pollution intensities in combination with technological progress on the growth environment-puzzle. Various questions regarding the balanced growth path of the economy, the social optimum, a pollution threshold and pollution abatement are studied and in the end several conclusions emerge from the analysis of the model.

In equilibrium, the economy follows a balanced growth path where output, research investment, consumption, as well as the level of technological progress grow at a constant, positive rate. Predictably the effect of pollution on the economic growth rate is not trivial. It vitally depends on the households' degree of pollution aversion and even more on the link between pollution intensity and the technology level. All in all, pollution dampens the economic growth rate if a rise in the level of technological progress does not imply a big enough decrease of the pollution intensity, more precisely for $\zeta < 1$, and fosters growth if the pollution intensity falls disproportionately fast when the economy advances ($\zeta \geq 1$). Pollution growth is proportional to the growth rate of the economy and also crucially depends on how the pollution intensity of the production process is linked to the level of technological progress. If the pollution intensity declines fast enough when the level of technological progress increases over time, this is the case for $\zeta > 1$, pollution declines over time, else pollution grows or is constant.

Due to four types of deviations, namely monopolistic pricing, the appropriability effect, the business stealing effect and pollution externalities, the decentralized solution does not meet the social optimum. But, the social optimum can be implemented through the introduction of a subsidy to the final good sector, a subsidy towards the research sector and tradable pollution permits.

Given a pollution threshold above which no research is possible, sustained economic growth is only feasible if pollution does not increase over time. This implies that ζ has to be above unity. Otherwise, at some point in time pollution will halt growth in the economy.

The possibility of pollution abatement enables the households to cope better with the pollution emitted during the production process and allows the economy to grow at a higher rate.

There remain various interesting extensions that could be incorporated into the model and many unresolved questions for further research. An interesting modification would be to introduce pollution as a stock variable instead of assuming that pollution can not be transfered from one period to the next. This would allow to study transitional dynamics and to account for regenerative processes. It would also be intriguing to extend the effect of pollution on the agents present in the economy. The labor supply could be vulnerable to pollution or environmental degradation might influence the production technology. Finally, up to now the analysis completely neglects uncertainty. However, the implications of uncertainty about future consequences of present actions are of vital interest, in particular in the context of environmental problems. Naturally, this list is far from exhaustive and the growth-environment puzzle certainly bears enough open questions to continue to be subject to further economic research.

A Appendix

A.1 Household Problem

The households strive to maximize their overall utility in consideration of their initial assets and their income, while taking the amount of pollution in the economy as exogenously given and obeying the No Ponzi Game condition. Thus the households' optimization problem is

$$\begin{aligned} \max_{c(t)} \quad & U(0) = \textstyle\int_0^\infty e^{-\rho t} \frac{1}{1-\theta} \left((c(t)P(t)^\eta)^{(1-\theta)} - 1 \right) dt \\ \text{s.t.} \quad & \dot{a}(t) = r(t)a(t) + w(t) - c(t) \\ & a(0) = a_0 \\ & \lim_{t \to \infty} \left(a(t) e^{-r(t)} \right) \geq 0, \end{aligned} \tag{A.1}$$

where $0 < \rho < \infty$ represents the rate of time preference, $a(t)$ the assets owned by a household, $r(t)$ the interest rate and $w(t)$ the wage, all three at time t. The present value Hamiltonian is accordingly

$$\begin{aligned} H(t, a(t), c(t), \mu(t)) = e^{-\rho t} \frac{1}{1-\theta} & \left((c(t)P(t)^\eta)^{(1-\theta)} - 1 \right) \\ & + \mu(t) \left(r(t)a(t) + w(t) - c(t) \right), \end{aligned} \tag{A.2}$$

with $\mu(t)$ being the Hamiltonian multiplier. The first order conditions are

$$\frac{\partial H}{\partial c} = e^{-\rho t} P^{\eta(1-\theta)} c^{-\theta} - \mu = 0 \tag{A.3}$$

$$\frac{\partial H}{\partial a} = \mu r = -\dot{\mu} \tag{A.4}$$

and the transversality condition is given by

$$\lim_{t \to \infty} \left(\mu(t) a(t) \right) = 0. \tag{A.5}$$

Finally, linerazing equation (A.3), deriving it with respect to time and combining the derivative with equation (A.4) yields equation (9).

A.2 Household Problem with Pollution Abatement

When allowing for pollution abatement, the households strive to maximize their overall utility by choosing an optimal amount of consumption and abatement spending, in consideration of their initial assets and their income, while taking the amount of pollution in the economy as exogenously given and obeying the No Ponzi Game condition. Thus the households' optimization problem is

$$\begin{aligned}
\max_{c(t),d(t)} \quad & U(0) = \int_0^\infty e^{-\rho t} \tfrac{1}{1-\theta} \left((c(t) P_E(t)^\eta)^{(1-\theta)} - 1 \right) dt \\
\text{s.t.} \quad & \dot{a}(t) = r(t)a(t) + w(t) - c(t) - d(t) \\
& P_E(t) = \tfrac{P(t)}{D(t)^\beta} \\
& a(0) = a_0 \\
& \lim_{t\to\infty} \left(a(t) e^{-r(t)} \right) \geq 0.
\end{aligned} \tag{A.6}$$

When exploiting the fact that all households are identical and will therefore choose the same quantity of abatement, so $d(t) = \frac{D(t)}{L}$, the present value Hamiltonian is given by

$$\begin{aligned}
& H\left(t, a(t), c(t), d(t), \mu(t)\right) \\
& = e^{-\rho t} \frac{1}{1-\theta} \left(c(t)^{1-\theta} \left(\frac{P(t)}{(d(t)L)^\beta} \right)^{\eta(1-\theta)} - 1 \right) \\
& \quad + \mu(t) \left(r(t)a(t) + w(t) - c(t) - d(t) \right),
\end{aligned} \tag{A.7}$$

with the Hamiltonian multiplier $\mu(t)$. The first order conditions are

$$\frac{\partial H}{\partial c} = e^{-\rho t} P^{\eta(1-\theta)} \left(dL \right)^{-\eta\beta(1-\theta)} c^{-\theta} - \mu = 0 \tag{A.8}$$

$$\frac{\partial H}{\partial d} = (-\eta\beta)\, e^{-\rho t} c^{1-\theta} P^{\eta(1-\theta)} d^{-\eta\beta(1-\theta)-1} L^{-\eta\beta(1-\theta)} - \mu = 0 \tag{A.9}$$

$$\frac{\partial H}{\partial a} = \mu r = -\dot{\mu} \tag{A.10}$$

and the transversality condition is given by

$$\lim_{t\to\infty} \left(\mu(t) a(t) \right) = 0. \tag{A.11}$$

Combining the first order conditions (A.8) and (A.9) enables to solve the optimal private and total abatement spending:

$$d = -\eta\beta c, \tag{A.12}$$

$$D = -\eta\beta C. \tag{A.13}$$

Finally, linerazing equation (A.8), deriving it with respect to time and combining the derivative with equation (A.10) yields equation (72).

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