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Consumption-Based Asset Pricing with a Reference Level: New Evidence from the Cross-Section of Stock Returns

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Non-technical Summary

According to classical asset pricing theory, differences in expected returns across assets must be ultimately accounted for by differences in the covariation of the asset's return with consumption growth. Despite its theoretical purity, however, the canonical consumption-based asset pricing model (CCAPM) has had a disappointing performance in past empirical tests.

In this paper, we formally estimate several extended versions of the model on common U.S. stock market data. In particular, we focus on the extension of the standard preferences by a reference level of consumption and investigate the empirical performance of this new approach in explaining the cross-sectional variation of average stock returns compared to well-established benchmark models. Several versions of a consumption-based model with a benchmark level of consumption have recently been proposed and estimated by Garcia, Renault and Semenov (2003). Their focus, however, was to test the conditional moment restrictions using the market portfolio and the Treasury-Bill as test assets. We extend their analysis by testing the unconditional moment restrictions of several of their proposed models on a broad cross-section of test assets, namely Fama and French's 25 portfolios sorted according to size and book-to-market.

Apart from employing this challenging set of test assets, this paper also motivates a specification of the reference level model which takes the return on human capital into account. Garcia et al. (2003) consider a specification, where the reference level is modelled as a function of the contemporaneous return of a market portfolio proxy. As emphasized by Roll (1977), a value-weighted stock market portfolio may not be an adequate proxy for the portfolio of total wealth since the human capital component of aggregate wealth is neglected. In this paper, we therefore consider an extended model in which the reference level does not only depend on the return of asset income, but also on the return of human capital. Following Jagannathan and Wang (1996), Lettau and Ludvigson (2001b) and Dittmar (2002) we use labor income growth as a proxy for the return on human capital.

We present empirical evidence that the model extensions by a reference level have the potential to improve the empirical performance of the consumption-based asset

pricing framework. The pricing errors of several of the new reference level models are considerably smaller than those of the original specification of the CCAPM. Furthermore, we find that the model augmented by human capital does a good job in explaining the cross-sectional variation in average returns across the 25 Fama-French portfolios with pricing errors close to those of the scaled factor model by Lettau and Ludvigson (2001b).

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Abstract

This paper presents an empirical evaluation of recently proposed asset pricing models which extend the standard preference specification by a reference level of consumption. The novelty is that we use a broad cross-section of test assets, which provides a level playing field for a comparison to well-established benchmark models. We also motivate a specification that accounts for the return on human capital as a determinant of the reference level. We find that this extension does a good job in explaining the cross-sectional variation in average returns across the 25 Fama-French portfolios with pricing errors close to those of Lettau/Ludvigson's celebrated scaled factor models.

JEL Classification: G12
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1 Introduction

Despite its theoretical appeal, the consumption-based asset pricing model (CCAPM) has as yet achieved little empirical success in calibration exercises or formal econometric testing [See e.g. Mehra and Prescott (1985), Hansen and Singleton (1982), Cochrane (1996) or Lettau and Ludvigson (2001b) etc.]. The empirical failure of the model has sparked a wave of research over the past 20 years aimed at improving the canonical CCAPM and making the model consistent with the empirical facts.[1]

This paper presents an empirical evaluation of recently proposed asset pricing models which extend the standard preferences by a reference level of consumption. The novelty of our paper is that we use a broad cross-section of test assets in order to evaluate the new models. So far, the conditional implications of asset pricing models with a reference level have been tested using a market portfolio proxy and the Treasury-Bill as basic test assets. In our empirical investigation we use Fama and French's 25 portfolios sorted by size and book-to-market, which provides a level playing field for a comparison of the new models to well-established benchmark models. We also motivate a specification that accounts for the return on human capital as a determinant of the reference level. This augmented model delivers a quite encouraging empirical performance with pricing errors which are close to those of the celebrated scaled CCAPM by Lettau and Ludvigson (2001b).

According to Cochrane (1997), the recently proposed theoretical modifications to the standard consumption-based framework can be primarily classified into two lines of research. One class of models tackles the empirical shortcomings of the standard CCAPM by abandoning the assumption of perfect capital markets. Models of this class focus on incomplete markets, survivorship bias, market imperfections, limited stock market participation on behalf of the population or are based on behavioral explanations. The second line of research maintains the framework of a representative investor and perfect capital markets but concentrates on mod-

[1] An overview and a critical reflection of recent approaches to solve the so-called "equity premium puzzle" is provided e.g. in Mehra and Prescott (2001) or Mehra (2003). An excellent recent survey on the topic is provided by Cochrane (2006).

ifications of investor preferences. Examples for modified preferences include for instance the model by Epstein and Zin (1991) which disentangles risk aversion and intertemporal substitution and the literature on habit formation [e.g. Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Campbell and Cochrane (1999), Campbell and Cochrane (2000)]. As pointed out by Chen and Ludvigson (2003), "habit-formation" constitutes a leading approach within this class of models. The central idea is that individuals get accustomed to a certain standard of living. They do not derive utility from consumption taken by itself as in the traditional economic models. Instead, the well-being of the individual depends on how much she consumes relative to a certain benchmark level.

Within the class of habit models, one can further distinguish between internal and external habit formation. Internal habit formation implies that the benchmark level is influenced by the investor's own consumption and saving decisions. By contrast, in models with external habit formation habit is not affected by the investor's decisions but depends on past aggregate consumption and can thus be interpreted as the benchmark level for the society as a whole. External habit formation expresses the idea that people want to maintain their relative standing in society often referred to as "Catching up with the Joneses" behavior, as noted in Abel (1990). The models considered in this paper are based on the concept of external habit formation. When habit is a function not only of past aggregate consumption but also current consumption, this leads to the more general "Keeping up with the Joneses" specification. This is true for instance in the model by Campbell and Cochrane (1999), where also contemporaneous consumption enters the habit function along with past consumption levels. An important feature of this model is counter-cyclical variation of risk-aversion that depends on the state of the economy. In a recession – a situation in which consumption has fallen close to the benchmark level – investors' level of risk aversion rises. In this way, the model offers interesting insights regarding the interplay of financial markets and the state of the economy.

The class of consumption-based asset pricing models with a reference level which we investigate in this paper has been proposed by Garcia et al. (2003). The main focus of their empirical investigation with U.S. monthly data was to test the conditional moment restrictions using a market portfolio proxy and the Treasury-Bill as test assets. We extend their analysis by estimating and testing several of their proposed models on a broad cross-section of test assets. We use Fama and French's 25 portfolios sorted according to size and book-to-market as our test assets.[2] Apart from employing this challenging set of test assets, this paper also proposes an extended version of the model originally proposed by Garcia et al. (2003). When specifying their reference level as a function of both past and current variables, the authors model their reference level also as a function of the contemporaneous return of a market portfolio proxy. As pointed out by Roll (1977), a value-weighted stock market portfolio may not be an adequate proxy for the total wealth portfolio since the human capital component of aggregate wealth is neglected. Dittmar (2002) has argued that integrating a measure of human capital into a non-linear stochastic discount factor is essential for pricing the cross-section of stock returns. Dittmar remains agnostic about the specific form of the utility function and approximates the SDF using a Taylor expansion. The SDF proxy he obtains is a polynomial in the return of aggregate wealth. An important factor of the empirical success of his model is to consider the return on human capital in the specification of the return on aggregate wealth. Drawing on that work, we therefore consider an extended model in which the reference level does not only depend on the returns of asset income, but also on the returns of human capital. As in Jagannathan and Wang (1996), Lettau and Ludvigson (2001b) and Dittmar (2002), we use labor income growth as a proxy for the return on human capital.

Another novelty of this paper is a performance comparison of the new models to well-established empirical asset pricing models. One of the most successful models in explaining the cross-section of stock returns is arguably the three-factor model by Fama and French (1993). Owing to its empirical success in explaining the cross-section of returns and its widespread use, it serves as a natural benchmark model.

[2]In order to check the robustness of our results we also estimated the models using ten size portfolios, which were employed in Cochrane (1996). The results are qualitatively similar and lead to the same conclusions.

An important contribution has been the work by Lettau and Ludvigson (2001a, 2001b) who investigate a conditional linearized version of the CCAPM. Specifically, Lettau and Ludvigson use the log consumption-wealth ratio (*cay*) as a conditioning variable, an instrument which has good forecasting properties for stock returns. Since Lettau Ludvigson's scaled CCAPM does a particularly good job in pricing the 25 Fama-French portfolios and is also solely based on macroeconomic factors it serves as an important benchmark for asset pricing models with a reference level.

Our paper is rooted in the empirical literature on representative agent models, which are estimated using aggregate consumption data, as pioneered by Hansen and Singleton (1982). Whereas earlier papers looked mainly at statistical rejections and only considered a few test assets, the recent focus has shifted to looking at economic pricing errors directly (RMSE, pricing error plots) and testing the models on the cross-section of size and book-to-market portfolios (Cochrane 2006). Most recently, there has been a renewed interest in empirical investigations of the consumption-based asset pricing approach in this contemporaneous empirical setting. Related work includes for instance Yogo (2006), who focuses on the consumption of durables, or Piazzesi, Schneider and Tuzel (2006), who analyze the influence of the share of housing consumption in total consumption. Ait-Sahalia, Parker and Yogo (2004) motivate a utility specification where households consume both basic and luxury goods. They are able to show that a linearized CCAPM based on luxury consumption data performs considerably better in explaining the cross-section of returns than the conventional specification based on aggregate consumption data. Parker and Julliard (2005) examine the long-run properties of the consumption-based model and also test their model using the 25 Fama-French portfolios.

The main results of this paper can be summarized as follows. Asset pricing models which account for a reference level of consumption can considerably improve the empirical performance of the standard CCAPM. However, our results show that it is important to account for the growth of human capital in the specification for the reference level. The performance of models with alternative specifications for the reference level is not that satisfactory. However, the human capital extended model delivers quite encouraging results, with pricing errors of the model close to those of Lettau-Ludvigson's scaled CCAPM and more sensible parameter estimates from an economic point of view. Enlarging the sample period, and using more recent data comprising the internet boom, the human capital extended model even outperforms this celebrated benchmark model and delivers pricing errors close to the theoretically much less appealing Fama-French three-factor model.

The remainder of the paper is organized as follows. In section 2 we present the data. We then lay out the theoretical framework with a brief discussion of the newly proposed consumption-based asset pricing models with a reference level. Section 4 presents estimation and test results for the new models and compares their empirical performance to benchmark asset pricing models. Section 5 concludes.

2 A Level Playground

This section describes the data used in our empirical analysis. By merging data sets which have already been used in a number of empirical studies [e.g. Fama and French (1993), Lettau and Ludvigson (2001b) etc.] we want to establish a level playing field on which the different models can show their relative merits and model performances can be compared. The base sample period is 1963:Q1-1998:Q3, the same as in Lettau and Ludvigson (2001b). As outlined above, Lettau and Ludvigson's scaled CCAPM provides a natural benchmark, so we chose the same sample period for the sake of comparability. We also report estimation results for the whole sample on which we have data available and which overlaps the "internet boom", 1952:Q2-2002:Q1.

Our test assets are Fama and French's 25 portfolios sorted by size and book-to-market characteristics.[3] Nominal data are converted into ex-post real returns by

[3]These data are regularly updated and provided by Kenneth French on his webpage (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`).

dividing nominal gross returns by the corresponding gross inflation rate. The inflation rate was calculated from the seasonally adjusted Consumer Price Index (CPI) for all urban consumers.[4] Since the data from Kenneth French's webpage are only available at a monthly frequency, the data were converted into quarterly data in order to match the frequency of the consumption data.

Table 1: Summary Statistics for the Test Assets (in %)

	Book-to-Market Equity Quintiles									
	$B1$	$B2$	$B3$	$B4$	$B5$	$B1$	$B2$	$B3$	$B4$	$B5$
Size Quintiles	Means					Standard Deviations				
$S1$	1.36	2.70	2.82	3.45	4.01	15.6	13.71	12.59	11.99	12.99
$S2$	1.72	2.42	3.14	3.43	3.69	14.09	12.20	11.05	10.49	11.25
$S3$	1.81	2.54	2.62	3.13	3.43	12.70	10.86	9.82	9.57	10.37
$S4$	1.86	1.79	2.52	2.98	3.25	10.95	10.01	9.09	9.05	10.08
$S5$	1.95	1.83	1.89	2.33	2.61	9.04	8.18	7.20	7.40	8.32

Note: The table reports means and corresponding standard deviations of the real quarterly returns of the 25 portfolios by Fama and French (1993). The table is organized as follows: for instance S1/B1 contains the average return of the portfolio that includes the smallest stocks in terms of market capitalization and at the same time with the lowest book-to-market ratio; S5/B5 contains the average return of the portfolio that includes the biggest stocks with the highest book-to-market ratios. Sample: 1963:Q1-1998:Q3.

The lower part of Table 1 reports average portfolio returns of the 25 Fama-French portfolios as well as the corresponding standard deviations. Table 1 illustrates the well known stylized facts, that average portfolio returns tend to decrease from small-stocks portfolios to big-stocks portfolios and the positive relationship between book-to-market and average returns.

Kenneth French's homepage also serves as our source for the benchmark Fama-French factors. Our return on the market portfolio taken from these data is the value-weighted return on all stocks traded on NYSE, AMEX and NASDAQ. The short term interest rate is the one-month Treasury-Bill from Ibbotson Associates. The factors SMB and HML are intended to mimic risk factors associated with size and book-to-market. SMB ("Small Minus Big") is the average return of a long position in portfolios with small stocks and a short position in portfolios with big stocks. HML ("High Minus Low") is the average return of a long position in portfolios with value stocks (high book-to-market ratio) and a short position in

[4]Obtained from the FRED database (`http://research.stlouisfed.org/fred2/`).

portfolios with growth stocks (low book-to-market ratio). For further details on the construction of SMB and HML see Fama and French (1993). All nominal data were converted into real returns as described above. In order to match the frequency of the consumption data, monthly returns had to be transformed into quarterly returns.

The consumption data (real and per capita) used in this empirical study were obtained from Sydney Ludvigson's website. These data are defined as consumption of nondurables and services excluding shoes and clothing. Time series of the conditioning variable *cay* and labor income (real, per capita) were also obtained from Sydney Ludvigson's website.[5] The time series of labor income was used for calculating the growth rate of labor income needed for the estimation of our human capital augmented reference level model.

3 Consumption-Based Asset Pricing with a Reference Level

In this section we review the theoretical framework of the consumption-based asset pricing models with a reference level introduced by Garcia et al. (2003). First, a few fundamental concepts are discussed. Then we turn to the modeling strategy for the reference level and discuss how the specification for the reference level can be augmented by human capital.

3.1 Basic concepts

Consumption-based asset pricing models with a reference level are best written in their stochastic discount factor representation. When the law of one price holds, there exists a stochastic discount factor (SDF) M_{t+1} that prices returns:

[5] `http://www.econ.nyu.edu/user/ludvigsons/`. The *cay* variable is obtained as the residual from the cointegrating relationship between consumption, asset income and labor income. See Lettau and Ludvigson (2001b, 2001a) for more information on data construction and on the theoretical motivation of the *cay* variable.

$$E[M_{t+1}R^i_{t+1}|\Phi_t] = 1. \tag{1}$$

R^i_{t+1} denotes the gross-return of asset i ($i = 1, \ldots, N$) and Φ_t represents the information set of the investor available as of t. The basic setting for asset pricing models with a reference level builds on classic consumption-based asset pricing where Equation (1) results from the first order conditions of an intertemporal consumption allocation problem with time-separable utility. The stochastic discount factor can then be interpreted as the marginal rate of substitution, $M_{t+1} = \delta\frac{U'(C_{t+1})}{U'(C_t)}$, where δ denotes the subjective discount factor and $U(\cdot)$ is the period utility function. Assuming a power utility specification $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$ with γ as the relative risk aversion (RRA) parameter the SDF is then given by

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{2}$$

Asset pricing models with a reference level retain this basic framework, but with a differently motivated period utility function $U(\cdot)$. Specifically, Garcia et al. (2003) advocate an approach in which utility does not only depend on consumption C_t, but also on consumption relative to a reference level X_t. Furthermore, the reference level X_t also enters the utility function in its absolute level,[6]

$$U(C_t/X_t, X_t) = \frac{\left(\frac{C_t}{X_t}\right)^{1-\gamma} X_t^{1-\psi}}{\text{sign}(1-\gamma)\text{sign}(1-\psi)}, \tag{3}$$

where $\text{sign}(z) = 1$ if $z \geq 0$ and $\text{sign}(z) = -1$ if $z < 0$, which ensures that utility is defined for all parameter values of interest. The parameter ψ controls the curvature of utility over the benchmark level. Several alternative specifications are nested as special cases. For instance with $\psi = \gamma$, Equation (3) reduces to the power-utility CCAPM. With $\psi = 1$, the reference level itself does not enter the utility function directly, and investor utility depends solely on consumption relative to her

[6]Campbell and Cochrane (1999) have pursued a different approach by assuming that the difference between consumption and reference level enters the utility function.

benchmark. The reference level is assumed to be related to aggregate consumption by identity in conditional expectations, i.e.

$$E_t(X_{t+\tau}) = E_t(C_{t+\tau}) \quad \forall\, \tau \geq 0. \tag{4}$$

Throughout this paper the reference level is considered as external by the investor. This implies that the reference level is a societal standard which the investor conceives as the benchmark for her consumption decision. Therefore, the stochastic discount factor which can be derived from Equation (3) takes the following form

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{X_{t+1}}{X_t}\right)^{\gamma-\psi}. \tag{5}$$

3.2 Modeling the Reference Level

To provide an empirically testable model, further assumptions regarding the evolution of the reference level X_t are necessary. Depending on the information set available to the investor, Garcia et al. (2003) distinguish between two possible modeling strategies. A first modeling strategy could assume that the investor only has information up to period t when forming her benchmark level $t+1$. Specifically, it is assumed that the reference level in $t+1$ equals the conditional expectation of the future consumption level, where the investor's information set in t only includes past realizations of consumption levels, i.e. $X_{t+1} = E(C_{t+1}|C_t, C_{t-1}, \ldots)$. This is consistent with (4) for horizon $\tau = 1$. In contrast to earlier papers which assumed that habit only depended on consumption lagged by one period [e.g. (Abel 1990)], Garcia et al. (2003) consider that reference levels react slowly to consumption. Assuming adaptive expectations, a change in the reference level is a function of the error when forming the reference level in the previous period, $\Delta X_{t+1} = \rho(C_t - X_t)$. Allowing for a constant a and iterating forward on $X_{t+1} = a + \rho C_t + (1-\rho)X_t$, we obtain

$$X_{t+1} = \frac{a}{\rho} + \rho \sum_{i=0}^{\infty} (1-\rho)^i C_{t-i}. \tag{6}$$

In this specification, which we refer to as "pure habit formation", the habit level thus depends on past realizations of consumption with declining weights.

In a second modeling strategy Garcia et al. (2003) assume that the investor uses some information available in $t+1$ when forming the reference level X_{t+1}. Specifically, they argue that the contemporaneous log-return of the market portfolio, r^m_{t+1}, qualifies as an important variable affecting the reference level.[7] This paper draws on this idea and extends it by arguing that the returns to human capital should be taken into account, too. This argument is backed by classic and recent literature. As emphasized for instance by Jagannathan and Wang (1996), aggregate wealth also contains a human capital component. The same argument has been put forth by Lettau and Ludvigson (2001b), who also estimate a "scaled human capital CAPM" (HCAPM) in their paper. Dittmar (2002) has argued for the importance of incorporating a measure of human capital into pricing kernels. Accordingly, we assume that the growth rate of the log reference level is determined by past log consumption growth, the log return on the market portfolio as well as the log return on human capital r^{hc}_{t+1}:

$$\Delta x_{t+1} = a_0 + \sum_{i=1}^{n} a_i \cdot \Delta c_{t+1-i} + b \cdot r^m_{t+1} + c \cdot r^{hc}_{t+1}. \tag{7}$$

We refer to this specification as the "human capital (HC) extended model". The economic intuition behind this specification is the following: when wealth increases in the economy (return on the market portfolio or the return on human capital move up), the investor adjusts her benchmark to a higher level. Garcia et al. (2003) assume that consumption growth equals the growth rate of the reference level plus noise. Hence, combining (7) and (4) at horizon one, one can write

[7] Throughout this paper we use lower case letters to denote natural logs of the respective variable.

$$\Delta c_{t+1} = a_0 + \sum_{i=1}^{n} a_i \cdot \Delta c_{t+1-i} + b \cdot r_{t+1}^{m} + c \cdot r_{t+1}^{hc} + \epsilon_{t+1}. \tag{8}$$

where ϵ_{t+1} is an orthogonal innovation, $E_t[\epsilon_{t+1}] = 0$, $E_t[\epsilon_{t+1} r_{t+1}^{m}] = 0$ and $E_t[\epsilon_{t+1} r_{t+1}^{hc}] = 0$. Reference level growth can then be written as

$$\frac{X_{t+1}}{X_t} = A \prod_{i=1}^{n} \left[\frac{C_{t+1-i}}{C_{t-i}}\right]^{a_i} \left(R_{t+1}^{m}\right)^{b} \left(R_{t+1}^{hc}\right)^{c}, \tag{9}$$

where $A = \exp(a_0)$. Plugging Equation (9) into (5), it follows that the SDF of the HC-extended model is given by

$$M_{t+1} = \delta A^{\gamma-\psi} \left[\frac{C_{t+1}}{C_t}\right]^{-\gamma} \prod_{i=1}^{n} \left[\frac{C_{t+1-i}}{C_{t-i}}\right]^{a_i(\gamma-\psi)} \left(R_{t+1}^{m}\right)^{b(\gamma-\psi)} \left(R_{t+1}^{hc}\right)^{c(\gamma-\psi)}. \tag{10}$$

Following Garcia et al. (2003), we define $\delta^* = \delta A^{\gamma-\psi}$ and $\kappa = b(\gamma - \psi)$. Equation (10) can be rewritten as

$$M_{t+1} = \delta^* \left[\frac{C_{t+1}}{C_t}\right]^{-\gamma} \prod_{i=1}^{n} \left[\frac{C_{t+1-i}}{C_{t-i}}\right]^{a_i(\gamma-\psi)} \left(R_{t+1}^{m}\right)^{\kappa} \left(R_{t+1}^{hc}\right)^{\frac{\kappa c}{b}}. \tag{11}$$

Garcia et al. (2003) show that the elasticity of intertemporal substitution implied by Equation (11) is given by $\sigma = \frac{1+b(\gamma-\psi)}{\gamma} = \frac{1+\kappa}{\gamma}$.[8] Hence, testing whether κ equals zero means testing whether the elasticity of intertemporal substitution is the inverse of the coefficient of relative risk aversion. This is one of the restrictive assumptions in the standard CCAPM with power-utility.

The SDF representation in Equation (11) is a general specification that nests various models proposed in the asset pricing literature as special cases. We turn to the estimation of these models in the next section where we also address additional assumptions for the empirical model as well as econometric issues.

[8] See Garcia et al. (2003) for further details on this result. They also show that a separation between risk aversion and intertemporal substitution is only possible when the reference level does not only depend on past but also on contemporaneous variables.

4 Results and Discussion

In this section we discuss the estimation results for consumption-based asset pricing models extended by a reference level. For the purpose of empirical performance comparisons we round up the usual suspects. Along with the inevitable CAPM, the CCAPM with power-utility serves as the natural benchmark, but we also present the results for empirically more successful models. The competitors include Lettau-Ludvigson´s scaled CCAPM and scaled CAPM, as well as the Fama-French three factor model which – on its own playing field (size and book-to-market sorted portfolios) – arguably represents the toughest challenge. Details about the respective stochastic discount factor specifications and empirical methodologies are delivered before discussing the respective empirical results. We report both first- and two-stage GMM estimation results. First-stage GMM, though less efficient, is preferable for model comparisons since the average pricing errors for the test assets are weighted identically across all compared models. We do not make use of instruments (managed portfolios). Instead we condition down and utilize the unconditional implications of the basic pricing equation (1) for GMM estimation of the parameters. Our basic test assets are the gross returns of the 25 Fama-French portfolios sorted by size and book-to-market. The estimation results for the sample period 1963:Q1-1998:Q3 used by Lettau and Ludvigson (2001) are reported in tables 2 through 7. In our discussion of the economic interpretation of the parameter estimates we focus on this sample period. The estimation results for the extended sample period (1952:Q2-2002:Q1) can be found in tables B.1 to B.6 of the appendix. Following recommended practice we assess the empirical model performances by average pricing error comparisons (Cochrane 2006). Figures 1 (sample period 1963:Q1-1998:Q3) and 2 (sample period 1952:Q2-2002:Q1) depict pricing error plots and report root mean squared average pricing errors for models of special interest. Additional pricing error plots can be found in figure 3 and in figure C.1 of the appendix.

CCAPM with power-utility Asset pricing with a reference level was motivated by the empirical weakness of the power utility CCAPM. Hence, this model serves as the natural benchmark for our comparisons. Estimation results for the model are reported in Table 2. We obtain the disturbing, yet familiar results. The GMM estimate of the RRA parameter γ is large, but also quite imprecise, and the estimate of the subjective discount factor is greater than one. The model is furthermore rejected by Hansen's (1982) J_T-test.[9] Figure 1 and 2 show that the model fails to account for the cross-sectional return variation of the 25 Fama-French portfolios. Overall, these results provide non-surprising further evidence for the apparent dismal empirical performance of the CCAPM with power utility.

Table 2: CCAPM: estimation results

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ	1.18	2.68	1.07	5.17
γ	39.96	0.46	17.15	0.44
J_T-Statistic	65.3		63.5	
p-value	0.0		0.0	

Note: The estimation was carried out using gross returns of the test assets. Sample: 1963:Q1-1998:Q3.

Pure Habit Formation Garcia et al. (2003) propose an asset pricing model in which the reference level is solely determined by past aggregate consumption levels ("pure habit formation"). Equation (5) implies that for the calculation of the model's SDF one needs habit growth data, $\{X_{t+1}/X_t\}$. These data are not directly observable. Garcia et al. (2003) suggest the following strategy to resolve this problem. Assuming that the reference level evolves according to the adaptive expectations hypothesis, habit can be expressed as a function of past consumption levels with declining weights over time. Furthermore, assuming that the reference level in $t+1$ is equal to the conditional expected consumption in $t+1$ one can

[9] It is well known that the J_T-test may be unreliable in that the large sample distribution of the test statistic under the null is not well approximated in small samples. Among others Hall and Horowitz (1996) and Altonji and Segal (1996) have pointed out that the J_T-test frequently over-rejects in small samples. Given the rejection of the overidentifying restrictions of all models considered in this paper, including the Fama-French three factor model itself (see tables 7 and B.6) we do not focus on the interpretation of the test of overidentifying restrictions.

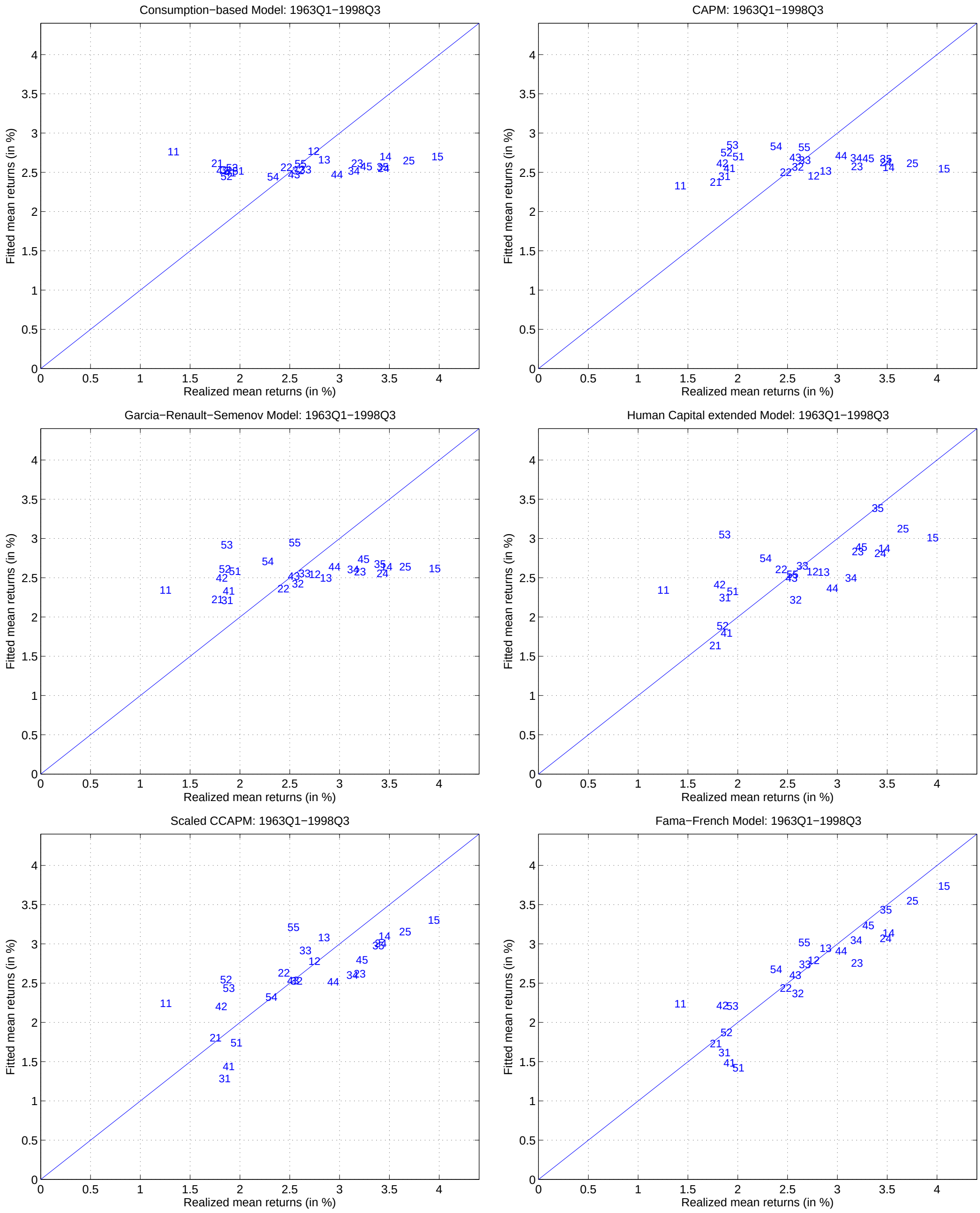

Figure 1: Consumption-based Asset Pricing Models and Benchmark Linear Factor Models: Fitted vs. Actual Mean Returns (in % per quarter). Sample period: 1963:Q1-1998:Q3.

Note: The graphs are based on first-stage GMM estimates using the 25 Fama-French portfolios as test assets. Realized mean returns are given on the horizontal axis, and the returns predicted by the model are provided on the vertical axis. The first digit represents the size quintiles (1=small, 5=big), whereas the second digit refers to the book-to-market quintiles (1=low, 5=big). The sample period is 1963:Q1-1998:Q3. The upper two graphs show results for the nonlinear consumption-based model with power utility [RMSE: 0.67%] and the Capital Asset Pricing Model [CAPM, RMSE: 0.69%]. Below we display the Garcia-Renault-Semenov model [RMSE: 0.65%] and its Human Capital Extension [RMSE: 0.51%]. At the bottom the plots for the scaled CCAPM by Lettau and Ludvigson (2001b) [RMSE: 0.45%] and the Fama-French model [RMSE, 0.31%] are shown.

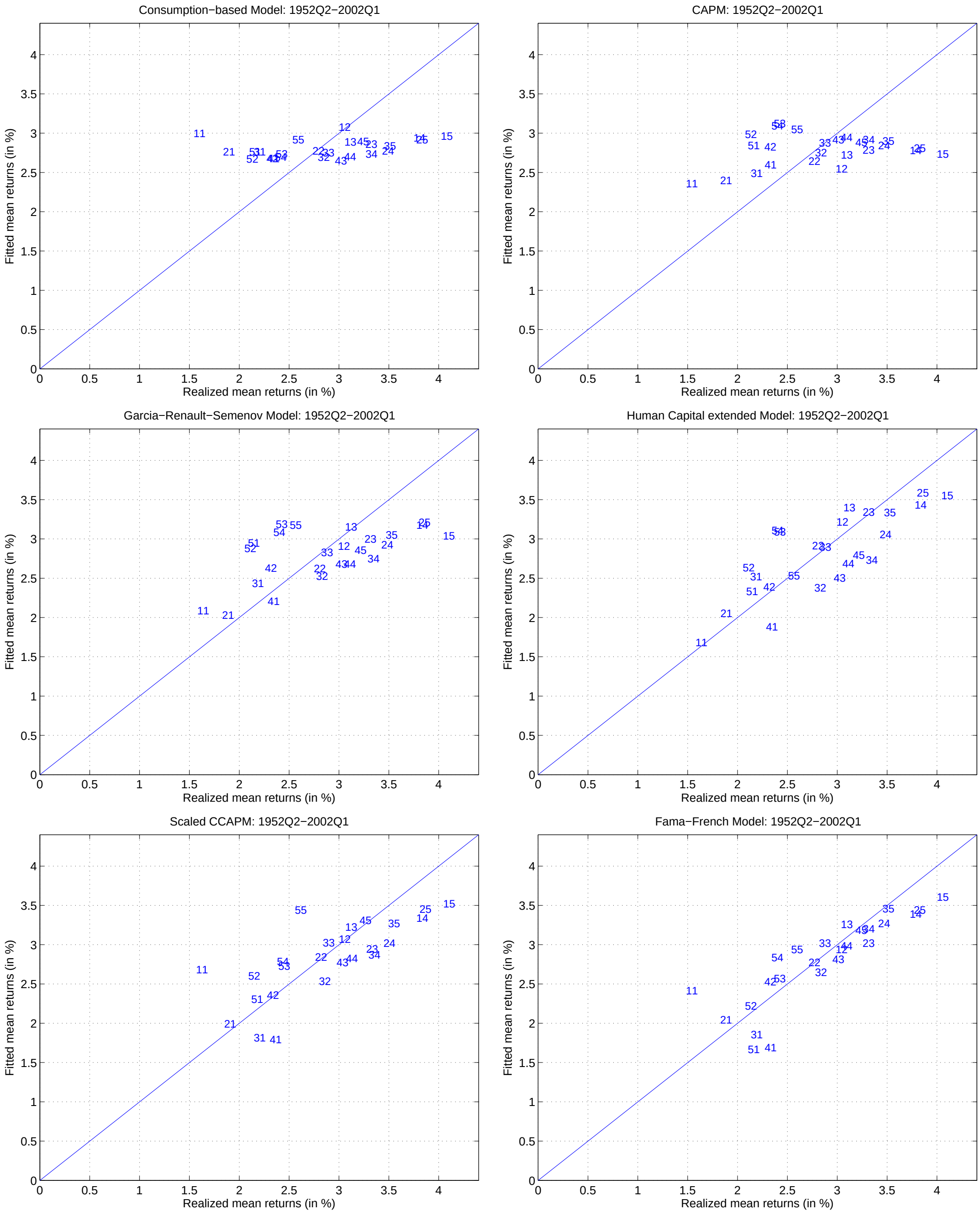

Figure 2: Consumption-based Asset Pricing Models and Benchmark Linear Factor Models: Fitted vs. Actual Mean Returns (in % per quarter). Sample period: 1952:Q2-2002:Q1.

Note: The graphs are based on first-stage GMM estimates using the 25 Fama-French portfolios as test assets. Realized mean returns are given on the horizontal axis, and the returns predicted by the model are provided on the vertical axis. The first digit represents the size quintiles (1=small, 5=big), whereas the second digit refers to the book-to-market quintiles (1=low, 5=big). The sample period is 1952:Q2-2002:Q1. The upper two graphs show results for the nonlinear consumption-based model with power utility [RMSE: 0.60%] and the Capital Asset Pricing Model [CAPM, RMSE: 0.60%]. Below we display the Garcia-Renault-Semenov model [RMSE: 0.51%] and its Human Capital Extension [RMSE: 0.38%]. At the bottom the plots for the scaled CCAPM by Lettau and Ludvigson (2001b) [RMSE: 0.41%] and the Fama-French model [RMSE, 0.32%] are shown.

write

$$C_{t+1} = \frac{a}{\rho} + \rho \sum_{i=0}^{\infty} (1-\rho)^i C_{t-i} + \epsilon_{t+1}, \tag{12}$$

where ϵ_{t+1} denotes an orthogonal innovation. A Koyck-transformation then leads to the following MA(1) representation:

$$\Delta C_{t+1} = a - (1-\rho)\epsilon_t + \epsilon_{t+1}. \tag{13}$$

We follow Garcia et al. (2003) and employ a two-step estimation procedure which entails estimation of the MA(1) parameters a and ρ in the first step. Using the estimated parameters one can then construct an estimated habit growth sequence $\{\hat{X}_{t+1}/\hat{X}_t\}$ which, in the second step, can then be used to estimate the SDF parameters by GMM as usual.

Table 3: Pure Habit Model: estimation results

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ	2.20	2.55	0.49	1.33
γ	-8.04	-0.08	59.57	1.13
ψ	304.97	2.22	-72.05	-0.62
$\gamma - \psi$	-313.01	-2.29	131.62	1.35
$1 - \psi$	-303.97	-2.21	73.05	0.63
J_T-Statistic	102.3		134.2	
p-value	0.0		0.0	

Note: We estimated an ARIMA(0,1,1)-model in order to obtain an estimate of habit as a function of past consumption levels. In the second step, we substituted habit growth in the stochastic discount factor by its estimate. The resulting moment conditions (1) were estimated by GMM. The sample period is 1963:Q1-1998:Q3. Standard errors of indirectly estimated parameters were calculated according to the Delta Method.

Estimation results for the pure habit formation model are provided in Table 3. The results are ambiguous and the empirical performance of the model is not encouraging. The first-stage GMM estimate of the subjective time discount factor is implausibly large while the second-stage GMM estimate is smaller than one. The first-stage GMM estimate of the RRA-coefficient is negative and not significantly different from zero, while the second-stage GMM estimate is again quite large. Based on the first-stage GMM results, the hypotheses $\psi = \gamma$ and $\psi = 1$, respectively, are both rejected at conventional significance levels. This could be interpreted as empirical evidence against the power-utility specification and against the hypothesis that the reference level does not directly affect the utility of the investor. However, based on the second-stage GMM results, both hypotheses cannot be rejected. The left panel of Figure 3 shows that the average pricing errors of the pure habit formation model are considerably smaller than those of the CCAPM with power utility. However, one should not be too much excited by this result. When the estimation is based on the extended sample period (1952:Q2-2002:Q1), the empirical performance of the pure habit formation model is quite poor.

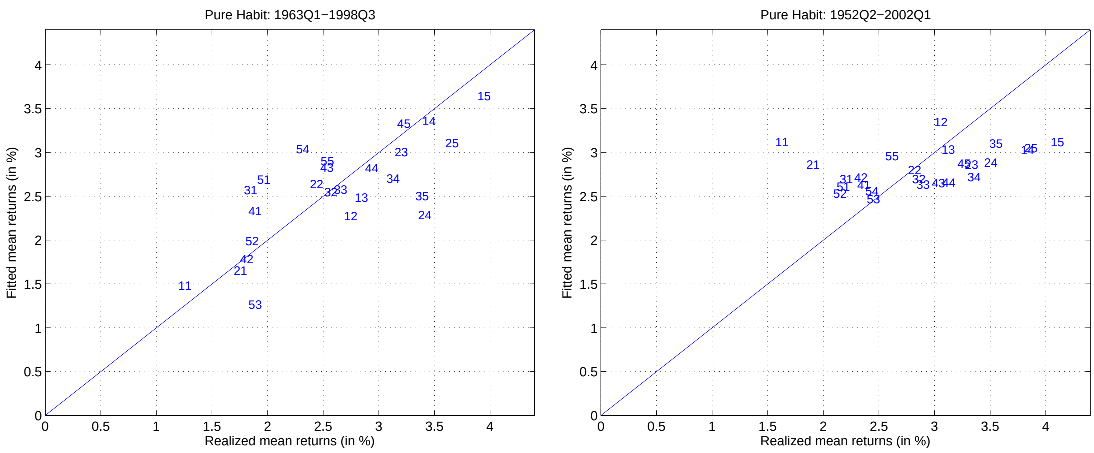

Figure 3: Pure Habit Model: Fitted vs. Actual Mean Returns (in % per quarter).

Note: The graphs are based on first-stage GMM estimates using the 25 Fama-French portfolios as test assets. Realized mean returns are given on the horizontal axis, and the returns predicted by the model are provided on the vertical axis. The first digit represents the size quintiles (1=small, 5=big), whereas the second digit refers to the book-to-market quintiles (1=low, 5=big). The sample periods are 1963:Q1-1998:Q3 and 1952:Q2-2002:Q1. The graphs show results for the pure habit formation model [RMSE: 0.47%, 0.57%].

Epstein-Zin Model Garcia et al. (2003) show that the class of asset pricing models with a reference level nests a specification of the SDF that is similar to the one that results from the assumption that investor's utility evolves recursively as in Epstein and Zin (1989). The SDF implied by their specification is obtained from equation (11) by imposing $a_i = 0$ $(i = 1, \ldots, n)$ and $c = 0$:

$$M_{t+1} = \delta^* \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(R^m_{t+1}\right)^{\kappa}, \tag{14}$$

where $\delta^* = \delta \cdot \exp[a_0(\gamma - \psi)]$ and $\kappa = b(\gamma - \psi)$. Conceiving the Epstein-Zin specification as a special case of an asset pricing model with a reference level one can write

$$\Delta c_{t+1} = a_0 + b \cdot r^m_{t+1} + \epsilon_{t+1}, \tag{15}$$

where r^m_{t+1} denotes the log return of the market portfolio proxy. For parameter estimation we follow Garcia et al. (2003) and include as instruments for the estimation of equation (15) the log market return and the log consumption growth lagged by two periods. The resulting moment conditions augment the standard moment conditions implied by the Epstein-Zin SDF such that all model parameters can be estimated simultaneously by GMM.

Table 4: Epstein-Zin Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	1.22	4.06	1.11	5.44
γ	57.56	1.02	35.10	0.95
κ	1.43	0.75	2.16	1.36
a_0	0.005	13.10	0.005	13.66
b	0.007	1.46	0.009	2.29
$\gamma - \psi$	210.85	0.77	241.38	1.32
ψ	-153.29	-0.50	-206.28	-1.04
$1 - \psi$	154.29	0.51	207.28	1.05
δ	0.42	0.65	0.32	0.96
σ	0.04	0.92	0.09	1.09
J_T-Statistic	86.9		82.5	
p-value	0.0		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (14). The moment conditions were then estimated jointly with the linear Equation (15). The sample period is 1963:Q1-1998:Q3. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

Estimation results for the Epstein-Zin model are reported in Table 4. As for the previous model the results are not convincing in terms of economic plausibility of the estimates. The empirical performance (RMSE) is also not encouraging. Both first- and second-stage GMM estimates of the RRA coefficient γ are quite large, but not different from zero at conventional levels of significance. The estimate of the subjective discount factor δ is implausibly small from an economic point of view. The hypothesis $\kappa = 0$ (or $\sigma = 1/\gamma$) cannot be rejected at conventional significance levels. Furthermore, neither the hypothesis $\psi = \gamma$, nor $\psi = 1$ can be rejected. Figure C.1 shows that the empirical performance of the Epstein-Zin specification in explaining the cross-sectional variation in stock returns remains unsatisfactory.

Garcia-Renault-Semenov Model In the following we consider a model in which the growth rate of the reference level is assumed to be a function of the current period market portfolio log return r^m_{t+1} and log consumption growth lagged by one period. This implies:

$$\Delta c_{t+1} = a_0 + a_1 \cdot \Delta c_t + b \cdot r^m_{t+1} + \epsilon_{t+1}. \tag{16}$$

The SDF of this specification results as a special case of equation (11) by setting $a_i \;=\; 0 \;\; (i = 2, \ldots, n);\;\; c = 0$:

$$M_{t+1} = \delta^* \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{C_t}{C_{t-1}}\right)^{\frac{\kappa a_1}{b}} \left(R^m_{t+1}\right)^{\kappa}, \tag{17}$$

where R^m denotes the market portfolio gross return. We refer to this specification as the Garcia-Renault-Semenov (GRS) model. The estimation strategy is analogous to the one pursued for the Epstein-Zin specification. The instruments to estimate the parameters in equation (16) are a constant and two period lagged consumption growth and log-market return.[10]

The estimation results for the GRS model are reported in Table 5. Again, the results do not make a strong case for asset pricing models with a reference level. We obtain the familiar CCAPM result that the first-stage estimate of the RRA

[10] Garcia et al. (2003) suggest to lag the instruments by two periods, arguing that Δc_t might be correlated with the innovation ϵ_{t+1}.

Table 5: Garcia-Renault-Semenov Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	1.34	2.75	1.40	4.17
γ	45.63	0.80	3.18	0.07
κ	0.53	0.19	-1.26	-1.41
a_0	0.003	0.23	0.002	2.13
a_1	0.488	0.22	0.606	4.09
b	-0.010	-0.34	0.010	2.20
$\gamma - \psi$	-55.54	-0.40	-125.47	-1.62
ψ	101.18	0.66	128.65	1.61
$1 - \psi$	-100.18	-0.66	-127.65	-1.60
δ	1.57	1.25	1.80	2.73
σ	0.03	0.56	-0.08	-0.01
J_T-Statistic	63.5		59.1	
p-value	0.0		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (17). The moment conditions were then estimated jointly with the linear Equation (16). The sample period is 1963:Q1-1998:Q3. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

coefficient is quite large, but the parameter is imprecisely estimated. The first-stage GMM estimate of the subjective discount factor δ is greater than one, which is implausible from an economic point of view. None of the power utility CCAPM model's implicit hypothesis can be rejected at conventional levels of significance. Neither can one reject hypothesis that the elasticity of intertemporal substitution is equal to $1/\gamma$ (i.e. whether $\kappa = 0$) nor the hypotheses that $\psi = \gamma$ or $\psi = 1$. The empirical performance in terms of average pricing errors is improved compared to the models considered so far, but the pricing error plots depicted in figure 1 and Figure C.1 show that the scaled CCAPM and the Fama-French model offer a much greater explanatory power. The performance in terms of pricing errors is improved when the GRS model is estimated on the extended sample period (see figure 2), but the parameter estimates are still not very plausible from an economic perspective.

Human Capital extended Model Let us now turn to the estimation results for a specification in which the returns on human capital are allowed to influence the reference level. This specification entails writing the consumption growth equation as

$$\Delta c_{t+1} = a_0 + a_1 \cdot \Delta c_t + b \cdot r^m_{t+1} + c \cdot r^{hc}_{t+1} + \epsilon_{t+1}. \tag{18}$$

where r^{hc}_{t+1} denotes the log return on human capital in period $t+1$. We follow Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b) and approximate r^{hc} by log labor income growth. Assuming that $a_i = 0$ $(i = 2, \ldots, n)$, the SDF in equation (11) reduces to

$$M_{t+1} = \delta^* \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{C_t}{C_{t-1}}\right)^{\frac{\kappa a_1}{b}} \left(R^m_{t+1}\right)^{\kappa} \left(R^{hc}_{t+1}\right)^{\frac{\kappa c}{b}}, \tag{19}$$

where R^m denotes the gross return on human capital. We refer to this specification as the human capital (HC-)extended model. Joint estimation of the parameters in equations (18) and (19) is performed analogously to the previously discussed two model specifications (Epstein-Zin and GRS).

Table 6: Human Capital extended Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	1.10	2.76	1.09	4.98
γ	159.34	1.47	145.72	2.65
κ	-0.82	-0.27	1.24	1.00
a_0	0.003	1.16	0.003	4.19
a_1	-0.330	-0.43	-0.108	-0.67
b	-0.006	-0.23	0.006	1.10
c	0.749	1.68	0.424	5.38
$\gamma - \psi$	130.08	1.21	223.56	3.68
ψ	29.26	0.40	-77.83	-1.69
$1 - \psi$	-28.26	-0.39	78.83	1.71
δ	0.77	2.92	0.51	3.37
σ	0.00	0.06	0.02	1.78
J_T-Statistic	90.4		63.6	
p-value	0.0		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (19). The moment conditions were then estimated jointly with the linear Equation (18). The sample period is 1963:Q1-1998:Q3. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

Compared with the models discussed so far, the estimation results for the HC-extended model provided in Table 6 are more sensible from an economic point of view. Furthermore, the empirical performance in terms of average pricing errors is encouraging. The estimates of the RRA coefficient are large and imprecise but we have got used to that in consumption-based asset pricing. Unlike for the other models discussed so far, the estimates of the subjective time discount factor are economically more plausible. As a matter of fact, the HC extended model is the first specification which rejects the restrictions of the CCAPM with power utility (which motivated the introduction of the model class in the first place). While the hypothesis that the elasticity of intertemporal substitution is equal to inverse of the RRA coefficient ($\kappa = 0$) cannot be rejected, the second-stage GMM estimates provide evidence against the CCAPM with power utility in that the hypothesis that $\psi = \gamma$ is rejected at conventional significance levels. As evinced by Figure 1, the HC-extended model accounts quite well for the cross-sectional variation in the returns of the 25 Fama-French portfolios. The specification delivers the smallest average pricing errors of all models with a nonlinear SDF. For the base sample period, the RMSE implied by the HC-extended model is about in the same range as the RMSE of the scaled CCAPM (scaled CAPM) by Lettau and Ludvigson. Estimated on the extended sample period, the HC-extension even outperforms these celebrated scaled factor models (see Figure 2) in terms of pricing errors. In fact, for this sample period, which overlaps the internet boom, the HC-extended model produces average pricing errors that come close to those of the Fama-French model estimated on its ”home turf”. These results can be regarded as an encouraging empirical success for consumption based asset pricing models with a reference level.

5 Conclusion

This paper investigated empirically, whether various types of reference-dependent asset pricing models are able to improve the performance of the consumption-based model in explaining the variation in cross-sectional returns. For the estimation of the models, we used a well-established cross-sectional data set: our test assets included the 25 Fama-French portfolios sorted by size and book-to-market. All estimations reported in this paper were conducted using a GMM approach.

We focussed our attention on different asset pricing models with a reference level of consumption proposed by Garcia et al. (2003). An important feature of their approach is to model the reference level also as a function of the contemporaneous return on the market portfolio. However, it is by now well established that, by doing so, the human capital component of aggregate wealth is neglected. A contribution of this paper is, therefore, to consider an extension of their model, where the benchmark level also depends on the return on human capital. As in the papers by Jagannathan and Wang (1996), Lettau and Ludvigson (2001b) and Dittmar (2002), the return on human capital was approximated by labor income growth.

Estimation of different types of the consumption-based model extended by a benchmark level of consumption revealed that the model extensions do a considerably better job than the original specification of the CCAPM with power utility in explaining cross-sectional variation of returns. The pricing errors of the model extensions are evidently smaller than those of the standard consumption-based model, which can be regarded as an important success of the new models. However, some parameter estimates remain problematic from an economic perspective: for instance, the models still require a high degree of risk-aversion as confirmed by our estimation results. Hence, the newly proposed models do not yet provide a true solution to the "equity premium puzzle", which originally motivated the new approaches. It must be pointed out, however, that this has not yet been accomplished by any model of this kind. Cochrane (2006, p.24) for instance concludes that "maybe we have to accept high risk aversion, at least for reconciling aggregate consumption with market returns in this style of model."

We also compared different models with a reference level to well-established linear factor models. The three-factor model by Fama and French (1993) has clearly the lowest pricing errors as visualized by the pricing error plots. This result is not too surprising given the fact that portfolio data are better measured than macroeconomic data. We found that our model extended by human capital rivals the scaled linearized CCAPM by Lettau and Ludvigson (2001b) in terms of pricing errors, which is the more appropriate benchmark since it is also based on macroeconomic data.

Essential for the empirical performance of the consumption-based asset pricing framework with a reference level, however, is the integration of human capital into the portfolio of aggregate wealth. Hence, our paper also corroborates the result by Dittmar (2002) who finds that the incorporation of human capital into the stochastic discount factor is very important for pricing the cross-section of stock returns. What is more, the central question of which macroeconomic risks drive risk premia remains essentially unanswered by portfolio-based models (Cochrane 2006, p.6). Representative agent models such as those presented in this paper ultimately seek such a deeper economic understanding. Our results can therefore be seen as a motivation that future research in the field of consumption-based asset pricing could lead us to a yet deeper understanding of the true economic forces behind the variation of expected returns across assets and over time.

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A Results: Linear Factor Models

Table 7: Linear Factor Models: estimation results

CCAPM	b_0			$b_{\Delta c}$	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.98			-83.13	64.82	0.00
t-Statistic	28.36			-0.86		
Second-Stage:						
Coefficient	1.00			-45.20	62.97	0.00
t-Statistic	50.76			-1.10		
Scaled CCAPM	b_0	b_{cay}	$b_{cay\cdot\Delta c}$	$b_{\Delta c}$	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.97	0.70	-170.85	-37.74	65.70	0.00
t-Statistic	11.72	1.38	-2.19	-0.26		
Second-Stage:						
Coefficient	1.00	-0.32	-14.29	24.26	52.67	0.02
t-Statistic	20.12	-1.13	-0.32	0.39		
CAPM	b_0			b_m	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.97			0.93	62.42	0.00
t-Statistic	85.78			0.53		
Second-Stage:						
Coefficient	0.97			1.07	61.49	0.00
t-Statistic	96.55			0.77		
Scaled CAPM	b_0	b_{cay}	$b_{cay\cdot m}$	b_m	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.96	1.10	-0.27	-1.76	45.74	0.14
t-Statistic	10.48	1.71	-0.06	-0.58		
Second-Stage:						
Coefficient	0.97	0.16	3.72	2.30	58.00	0.00
t-Statistic	22.63	0.75	1.59	1.33		
Fama-French	b_0	b_m	b_{SMB}	b_{HML}	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.98	-1.23	-1.18	-6.63	41.80	0.45
t-Statistic	31.73	-0.31	-0.39	-2.57		
Second-Stage:						
Coefficient	0.97	0.53	-2.97	-4.39	43.42	0.28
t-Statistic	40.47	0.21	-1.36	-2.24		

Note: This table reports the GMM estimation results for the benchmark linear factor models. The specification of the stochastic discount factor is a linear function of K factors $M_{t+1} = b_0 + b_1' f_{t+1}$. The models differ in their specification of the factors. The linearized CCAPM is a single-factor model, where log consumption growth is the only factor $f_{t+1} = \Delta c_{t+1}$. Lettau/Ludvigson's scaled CCAPM has three factors $f_{t+1} = [cay_t; cay_t \Delta c_{t+1}; \Delta c_{t+1}]'$. In the case of the CAPM $f_{t+1} = R^m_{t+1}$, whereas Lettau/Ludvigson's scaled CAPM uses $f_{t+1} = [cay_t; cay_t R^m_{t+1}; R^m_{t+1}]'$. The Fama-French model is specified as $f_{t+1} = [R^m_{t+1}; SMB_{t+1}; HML_{t+1}]'$. The sample period is 1963:Q1-1998:Q3.

B Tables: Extended Sample Period

Table B.1: CCAPM: estimation results

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ	1.28	4.22	1.13	5.32
γ	62.84	0.96	32.74	0.78
J_T-Statistic	71.6		72.4	
p-value	0.0		0.0	

Note: The estimation was carried out using gross returns of the test assets. Sample: 1952:Q2-2002:Q1.

Table B.2: Pure Habit Model: estimation results

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ	0.71	0.98	1.83	3.71
γ	64.11	1.12	3.36	0.05
ψ	-44.00	-0.25	184.49	2.51
$\gamma - \psi$	108.11	0.71	-181.13	-2.18
$1 - \psi$	45.00	0.25	-183.49	-2.49
J_T-Statistic	84.1		65.6	
p-value	0.0		0.0	

Note: We estimated an ARIMA(0,1,1)-model in order to obtain an estimate of habit as a function of past consumption levels. In the second step, we substituted habit growth in the stochastic discount factor by its estimate. The resulting moment conditions (1) were estimated by GMM. The sample period is 1952:Q2-2002:Q1. Standard errors of indirectly estimated parameters were calculated according to the Delta Method.

Table B.3: Epstein-Zin Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	1.24	4.60	1.10	5.51
γ	70.13	1.27	48.54	1.27
κ	1.76	1.00	3.39	2.14
a_0	0.005	14.96	0.005	15.58
b	0.008	2.11	0.007	2.26
$\gamma - \psi$	234.01	1.00	461.57	1.79
ψ	-163.88	-0.62	-413.04	-1.50
$1 - \psi$	164.88	0.63	414.04	1.50
δ	0.40	0.79	0.11	0.73
σ	0.04	1.08	0.09	1.32
J_T-Statistic	84.6		82.2	
p-value	0.0		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (14). The moment conditions were then estimated jointly with the linear Equation (15). The sample period is 1952:Q2-2002:Q1. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

Table B.4: Garcia-Renault-Semenov Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	0.58	1.90	0.60	2.91
γ	150.54	3.03	112.63	2.80
κ	5.93	1.87	5.40	2.68
a_0	0.003	2.61	0.003	5.68
a_1	0.394	1.97	0.313	3.25
b	0.015	4.17	0.012	3.90
$\gamma - \psi$	392.24	1.78	444.26	2.63
ψ	-241.70	-1.05	-331.63	-2.03
$1 - \psi$	242.70	1.06	332.63	2.04
δ	0.20	0.80	0.14	1.10
σ	0.05	1.91	0.06	2.93
J_T-Statistic	64.9		59.0	
p-value	0.0		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (17). The moment conditions were then estimated jointly with the linear Equation (16). The sample period is 1952:Q2-2002:Q1. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

Table B.5: Human Capital extended Model: estimation results.

	First-Stage GMM		*Two-stage GMM*	
	Estimate	t-Statistic	Estimate	t-Statistic
δ^*	0.37	1.76	0.77	3.27
γ	259.26	2.29	98.28	1.37
κ	7.83	2.51	1.79	1.36
a_0	-0.001	-0.22	0.002	2.90
a_1	0.693	2.26	0.211	1.45
b	0.028	5.16	0.009	2.36
c	0.270	1.09	0.233	5.97
$\gamma - \psi$	277.84	2.66	196.47	1.64
ψ	-18.58	-0.14	-98.19	-1.12
$1 - \psi$	19.58	0.15	99.19	1.13
δ	0.42	1.54	0.50	2.07
σ	0.03	2.10	0.03	2.31
J_T-Statistic	51.6		69.6	
p-value	0.1		0.0	

Note: We substituted the SDF in the moment conditions in (1) by the SDF in (19). The moment conditions were then estimated jointly with the linear Equation (18). The sample period is 1952:Q2-2002:Q1. Standard errors of indirectly estimated parameters were calculated by the Delta Method.

Table B.6: Linear Factor Models: estimation results

CCAPM	b_0			$b_{\Delta c}$	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.98			-127.03	61.80	0.00
t-Statistic	22.59			-1.27		
Second-Stage:						
Coefficient	1.00			-66.34	67.02	0.00
t-Statistic	42.13			-1.49		
Scaled CCAPM	b_0	b_{cay}	$b_{cay\cdot\Delta c}$	$b_{\Delta c}$	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.97	0.84	-128.35	-67.49	63.36	0.00
t-Statistic	16.48	1.82	-1.97	-0.53		
Second-Stage:						
Coefficient	0.97	0.55	-58.54	-29.78	67.04	0.00
t-Statistic	27.70	2.38	-1.40	-0.62		
CAPM	b_0			b_m	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.96			1.48	68.45	0.00
t-Statistic	76.65			0.92		
Second-Stage:						
Coefficient	0.96			2.11	66.38	0.00
t-Statistic	68.88			1.66		
Scaled CAPM	b_0	b_{cay}	$b_{cay\cdot m}$	b_m	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.97	1.17	-7.05	-3.79	37.71	1.40
t-Statistic	10.87	1.85	-1.52	-1.09		
Second-Stage:						
Coefficient	0.96	0.42	1.95	0.37	67.90	0.00
t-Statistic	22.93	1.97	0.97	0.19		
Fama-French	b_0	b_m	b_{SMB}	b_{HML}	J_T-Statistic	p-value (%)
First-Stage:						
Coefficient	0.96	2.78	-3.69	-2.83	53.32	0.01
t-Statistic	37.84	1.01	-1.84	-1.40		
Second-Stage:						
Coefficient	0.96	2.81	-4.32	-2.68	53.94	0.01
t-Statistic	40.30	1.36	-2.54	-1.58		

Note: This table reports the GMM estimation results for the benchmark linear factor models. The specification of the stochastic discount factor is a linear function of K factors $M_{t+1} = b_0 + b_1' f_{t+1}$. The models differ in their specification of the factors. The linearized CCAPM is a single-factor model, where log consumption growth is the only factor $f_{t+1} = \Delta c_{t+1}$. Lettau/Ludvigson's scaled CCAPM has three factors $f_{t+1} = [cay_t; cay_t \Delta c_{t+1}; \Delta c_{t+1}]'$. In the case of the CAPM $f_{t+1} = R^m_{t+1}$, whereas Lettau/Ludvigson's scaled CAPM uses $f_{t+1} = [cay_t; cay_t R^m_{t+1}; R^m_{t+1}]'$. The Fama-French model is specified as $f_{t+1} = [R^m_{t+1}; SMB_{t+1}; HML_{t+1}]'$. The sample period is 1952:Q2-2002:Q1.

C Additional Figures

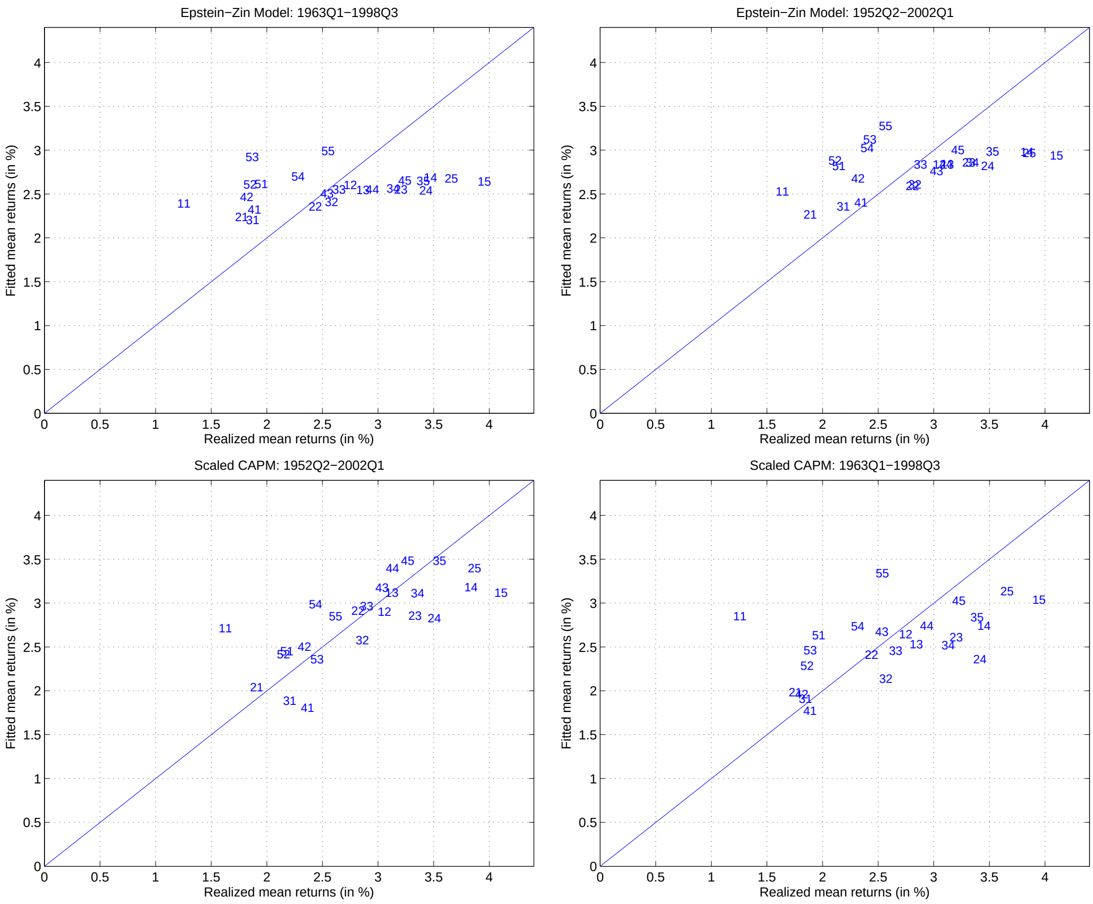

Figure C.1: Additional Plots: Fitted vs. Actual Mean Returns (in % per quarter).

Note: This figure displays plots for all models not shown in the main text. Realized mean returns are given on the horizontal axis, and the returns predicted by the model are provided on the vertical axis. The first digit represents the size quintiles (1=small, 5=big), whereas the second digit refers to the book-to-market quintiles (1=low,5=big). The sample periods are 1963:Q1-1998:Q3 and 1952:Q2-2002:Q1. The upper two graphs show results for the pure habit formation model [RMSE: 0.47%(1963:Q1-1998:Q3), 0.57%(1952:Q2-2002:Q1)]. Below we display the Epstein-Zin model [RMSE: 0.66%(1963:Q1-1998:Q3), 0.56%(1952:Q2-2002:Q1)]. At the bottom the plots for the scaled CAPM by Lettau and Ludvigson (2001b) [RMSE: 0.58%(1963:Q1-1998:Q3), 0.43%(1952:Q2-2002:Q1)] are shown.