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## Non–technical Summary

Recent reforms of the social security system in Germany will almost certainly lead to the merger of social benefits (Sozialhilfe) and unemployment assistance (Arbeitslosenhilfe) by the year 2005. When this reform takes effect, up to 1.7 million individuals and their families will obtain *new* needs-oriented social benefits (Arbeitslosengeld II) in addition to the over 2.3 million employable individuals currently receiving similar social benefits. In contrast to the current benefit system, the system of *new* social benefits is intended to provide stronger incentives to the unemployed to search for and accept new jobs (Hartz, 2002). However, gross need for social benefits will continue to be calculated on the basis of equivalence scales which determine the equivalent income between different demographic groups of households such that both types of households can achieve the same standard of living. The design of the equivalence scale scheme will therefore essentially drive the incentives for job seekers. This means that, finding appropriate values for the equivalence scales will assume even greater importance in the future.

This paper presents a comprehensive empirical study of the semiparametric estimation of consumption based equivalence scales. Equivalence scales for Germany are estimated by applying Wilke's (2003) estimator for the extended partially linear model suggested by Blundell et al. (1998) to the most recent version of the German income and consumption survey data (EVS 1998). For estimation purposes the data is segmented into homogenous groups of households conditional on employment status of the household head or the income level. The estimated consumption based equivalence scales are mostly lower than the equivalence scales of the German social benefits system.

It is difficult to infer policy recommendations from the results because of the large standard errors of the estimates and because of some degree of theoretical arbitrariness involved in the underlying modelling approach. However, the estimations provide some indications that on average the costs for additional persons in a household are *at least* covered by the standard rates of German social benefits. In the light of recent decisions of the Federal Constitutional Court (Bundesverfassungsgericht) concerning the costs of children and growing discussion of demographic transitions in Germany, it is not apparent from the estimation results that equivalence scales need to be increased for households with children.

# Semiparametric Estimation of Consumption Based Equivalence Scales - The Case of Germany

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**Abstract**

Consumption based equivalence scales are estimated by applying the extended partially linear model (EPLM) to the 1998 Income and Consumption Survey (EVS) carried out in Germany. In this model the equivalence scales are identified by virtue of nonlinearities in household demand. Therefore, the econometric framework should not impose any strong restrictions on the functional forms of household expenditure shares. The chosen semiparametric specification meets this requirement. It is flexible, it yields $\sqrt{N}-$consistent parameter estimates, and is consistent with consumer theory. Estimated equivalence scales are below or in the range of the expert equivalence scales of the German social benefits system.

**Keywords:** semiparametric estimation, wild bootstrapping, equivalence scales, social benefits

**JEL:** C14, C31, D12, H53

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# 1 Introduction

Recent reforms of the social security system in Germany will lead to the merger of social benefits (Sozialhilfe) and unemployment assistance (Arbeitslosenhilfe) by the year 2005. When this reform takes effect, up to 1.7 million individuals[1] and their families will obtain *new* needs-oriented social benefits (Arbeitslosengeld II) in addition to the over 2.3 million employable individuals currently receiving similar social benefits. In contrast to the social benefits system, the system of *new* social benefits is intended to provide stronger incentives to the unemployed to search for and accept new jobs (Hartz, 2002). However, gross needs for social benefits will still be calculated on the basis of equivalence scales which will essentially drive the incentive scheme. This means that finding appropriate values for the equivalence scales will assume even greater importance in the future. This paper provides estimates for this purpose by applying the semiparametric estimator of Wilke (2003) to the most recent version of the German income and consumption survey. It is the first comprehensive application of this estimator. Previous work for Germany was done with parametric demand systems which impose strong restrictions on the functional forms of the household expenditure shares.

Equivalence scales are often used in welfare systems to compute households' need for financial support. These scales determine whether and to what extent households are eligible for social benefit transfer payments. To make things more precise, let us state what is usually understood as an equivalence scale:

> *Equivalence scales deflate household money income [...] according to household type to "calculate the relative amounts of money two different types of households require in order to reach the same standard of living".* (Muellbauer, 1977)

The purpose of social benefit transfer payments is to ensure that all households enjoy a minimum standard of living. If equivalence scales are incorrectly codified, the standard rates for social benefits will not coincide with their intended values. If transfer payments are too high, the respective household may receive more money than it needs to reach the

[1] Note that the set of social benefits recipients and the set of unemployment assistance recipients are not disjoint.

minimum standard of living. On the other hand, if the value of the equivalence scales is too low, the respective household may not be able to achieve the minimum standard of living. The standard rates must therefore be determined with great care.

Many theoretical and empirical contributions have already examined the issue of how to find a *reasonable* equivalence scale for this purpose. In most cases one of the following three approaches has been adopted.[2]

In the first approach, "expert scales" are devised based on the opinion of social security experts. Table 1 presents two scales in this class. The standard rates of the German social benefits system and the so-called OECD scales. The average gross needs in table 1 are the empirical numbers for Germany. Rates calculated from these numbers deviate from the standard rates because they also consist of expenses for housing, heating and general supplementary costs. In the case of Germany the expert scales are supported by several example calculations. The main criticism of this approach is its lack of theoretical justification which means that the resulting equivalence scales appear to be ad hoc to a large extent.

The second approach uses data about the degree of satisfaction of a household with its income in order to determine subjective equivalence scales. One criticism of this method is that the results depend on subjective valuations. Other more objective criteria would be preferable. However, with this method equivalence scales can be estimated with sophisticated econometric methods. See Bellemare, Melenberg and van Soest (2002) for a comparison of different estimators using the German Socio Economic Panel (GSOEP).

[2]Alternative classifications can be found in Coulter, Cowell and Jenkins (1992). See Merz and Faik (1995) for further references.

Table 1: Comparison of existing equivalence scales schemes

| | German social benefits | | | OECD (1982) |
|---|---|---|---|---|
| | Standard rates* | Average gross needs** | | |
| | | West | East | |
| *Single households* | | | | |
| without children (S0) | 1.00 | 1.00 | 1.00 | 1.00 |
| with one child (S1) | 1.65 | 1.64*** | 1.68*** | 1.50 |
| *Couple households* | | | | |
| without children (C0) | 1.80 | 1.58 | 1.62 | 1.70 |
| with 1 child (C1) | 2.45 | 2.04 | 2.11 | 2.20 |
| with 2 children (C2) | 3.10 | 2.47 | 2.58 | 2.70 |
| with 3 children (C3) | 3.75 | 2.92 | 3.03 | 3.20 |

* Federal Law for Social Benefits (BSGH), children of age 7-13

** Reporting date 1/JAN/2003, source: Federal Ministry of Health and Social Security

*** Child aged $< 7$

The third approach -consumption based equivalence scales- is based on consumer theory. These scales are determined on the basis of households consumption behavior and can be estimated by using comprehensive cross section consumption data at the household level. This paper aims at estimating consumption based equivalence scales for Germany by using the semiparametric estimator of Wilke (2003) and the 1998 income and consumption survey of Germany. In the past this class of equivalence scales was mainly estimated with parametric linear demand systems. See for example Blundell and Lewbel (1991) for Britain, and Merz and Faik (1995) for Germany. Empirical evidence, however, has shown that in many cases households have nonlinear demand functions. (See, for example, Blundell, Paschardes and Weber,1993). An extension to nonlinear parametric or partially linear expenditure systems is straight and accounts for this misspecification. New developments in consumer theory show that this model choice may also be inappropriate when demographic variation is taken into account (Blundell, Duncan and Penkadur, 1998). In this light the parametric quadratic specification of Kohn and Missong (2003) for Germany appears to be crucial. Blundell et al. (1998) introduced a semiparametric approach, the so-called extended partially linear model (EPLM) which is based on the work of Pendakur (1999). In the EPLM the equivalence scales are identified from the non-linearities in the demand functions. It is well known that linear and quadratic parametric demand functions heavily restrict or do not even allow for nonlinearities. The EPLM by contrast, is more flexible because it allows for fully nonparametric expenditure shares. Wilke (2003) developed an implementable estimator for the EPLM and derived its theoretical properties. A small-scale application with the British Family expenditure survey indicates that this model is empirically supported. In this paper the EPLM and the same estimator are applied to the 1998 income and consumption survey (EVS) carried out in Germany. Compared to the previous versions a new design of the 1998 EVS survey is one step towards less measurement errors and more representative data. In this paper, a flexible estimator is thus applied to the best data currently available. The model specification appears to be appropriate for a variety of estimations which are not only conditional on demographics. It also segments the data according to the lowest or highest quartile of the household net income. Some of the estimated average equivalence scales are below and some are in the range of the expert scales used for the calculation of the gross needs for social benefits in Germany. However, the underlying approach also involves a degree of arbitrariness (Pollak and Wales, 1992, Kohn and Missong, 2003) which is mainly due to assumptions that cannot be tested and lack of data. Therefore, precise policy recommendations cannot be derived. Furthermore, some estimation results may be biased due to different compositions of the samples that are compared. In addition, severe standard errors lead to impreciseness in some cases. Possible measurement errors due to commodity aggregation and misreporting of the households may also affect the results. Nevertheless, we can postulate that this paper provides the first comprehensive empirical result for this class of models and most of criticism mentioned in this paper is aimed at the majority of contributions in this field. The underlying model is based on consumer theory, and the results are obtained from comprehensive data set. These are the striking advantages compared to the other approaches for the determination of equivalence scales.

The paper is structured as follows: Section 2 presents the theoretical framework of the underlying consumer theory. Section 3 sketches the system of social benefits in Germany and explains the importance of equivalence scales. Section 4 introduces the econometric model for the estimation of the extended partially linear model. Section 5 describes the data. The estimation results are presented in section 6. The last section concludes this paper and provides suggestions for further research.

# 2 Consumer Theory

This section presents the underlying static microeconomic framework for the econometric analysis. Since we consider cross section data which should be recorded at a given point of time we ignore calendar time variations. We denote $m(x, z, p)$ as the the vector of expenditure shares for commodities $j = 1, \ldots, J$, where $x$ is the log. of total expenditure, $z$ is a household specific finite dimensional vector of observable characteristics and $p$ is the $J \times 1$ vector of log prices. The equivalence scale between two groups $z_1$ and $z_0$ is defined as $exp(\alpha(z_1, p))$. It can be identified from the respective cost functions $c(p, u, z_0)$ and $c(p, u, z_1)$, which correspond to the minimum expenditures in order to achieve a specific utility level $u$. More specifically, we have

$$\alpha(z_1, p, u) = lnc(p, u, z_1) - lnc(p, u, z_0),$$

where $\alpha$ is the log. of the equivalence scale and $z_0$ denotes the reference group. Then household $z_1$ requires $exp(\alpha(z_1, p, u))$ of the reference household's income to reach the same utility level $u$. Cost functions and expenditure shares are directly related because we have $m(x, z, p) = \partial lnc(p, u, z)/\partial p$ from Shepard's lemma. This relationship suggest that the equivalence scales are identified from consumption data. However, the empirical approach to this method leads to the problem that $u$ is not observed, even though it is vital to know the utility level for welfare comparisons. This fundamental problem has not yet been solved (Pollak and Wales, 1992) and therefore further conditions are necessary which will ensure that the equivalence scale is independent of the utility level. Stronger assumptions such as the independence of the base utility, i.e. $\alpha(z, p, u) = \alpha(z, p)$, overcome this difficulty but they still lack empirical support. Kohn and Missong (2002) therefore conclude that "observed demand quantities do not suffice for a unique identification of equivalence scales – a fact that renders welfare comparisons impossible". Moreover, the utility arising from leisure is ignored by uniquely focusing on utility coming from consumption. Consequently, the leisure related part is not captured by a model that is solely estimated with consumption data. If we assume that utility can be separated into consumption and leisure, we would model and estimate solely the consumption-related utility element.

A variety of functional forms for expenditure shares are consistent with economic theory. A popular linear specification is the so-called Price-Independent Generalized Logarithmic (PIGLOG, see Muellbauer, 1976). It arises from indirect utility functions which are linear in the log. of total expenditure. Complete demand systems such as the AIDS (Deaton and Muellbauer, 1982) and the ELES (Lluch, 1973) are based on linear specifications of the expenditure shares. Today there is enough empirical evidence that this specification has to be generalized, because there is a nonlinear relationship for many goods. The partially linear model (PLM) is a generalization which includes the quadratic model as a special case. This class of models has attractive theoretical properties, and there is empirical evidence available for the quadratic specification (Blundell et al., 1993). More recently, Blundell et al. (1998) stated that if the expenditure share of one commodity is PIGLOG, then consumer theory induces the same property for all demand functions in a demand system. As a consequence the nature of the PLM can drastically restrict the functional forms for all expenditure shares in order to be still consistent with consumer theory if the demand function for one single good is linear. For this reason demand systems based on expenditure shares belonging to the class of PLMs involve crucial functional form restrictions for the estimation of equivalence scales. There is also some evidence that this is relevant when applying this method since, for example, the expenditure share for food is linear in Britain (Blundell et al., 1998). Inspired by Pendakur (1999) and the findings for Britain, Blundell et al. (1998) suggest an alternative system of expenditure shares that accounts for demographic decomposition, is nonlinear in log of total expenditure and consistent with consumer theory. In this case, however, we need to start off with the assumption that the equivalence scales are independent of the baseline utility. Given a smooth unknown function $g_j$, Blundell et al. (1998) state the following lemma for the extended partially linear model (EPLM):

*If expenditure shares have the EPLM form:*

$$m_j(x, z, p) = \frac{\partial \alpha(z, p)}{\partial p_j} + g_j(x - \alpha(z, p)), \tag{1}$$

*then if the reference share equations*

$$m_j(x, z_0, p) = g_j(x, p) \tag{2}$$

*are consistent with consumer theory and exp*$(\alpha(z, p))$ *is weakly concave and homogeneous of degree zero in exp*$(p)$*, expenditure shares given by (1) are also consistent with consumer theory.*

The derivation of the EPLM and further underlying theory can be found in Pendakur (1999) and in Blundell et al. (1998). It uses the main tools of dual theory and skillfully exploits the definition of baseline-utility-independent equivalence scales. Interestingly, the class of functionals in equation (1) belongs to the shape invariant models because we have simple vertical and horizontal (due to $\alpha(z, p)$) shifts of an unknown smooth function $g_j$. Apparently, the shape of the nonparametric function $g_j$ may differ across the commodities, whereby $\alpha(z, p)$ does not. The horizontal shift $\alpha(z, p)$ is of particular interest because its exponential transformation is the equivalence scale. The EPLM is therefore a general theoretical model for the estimation of equivalence scales that are independent of the base utility. It requires very mild assumptions on the functional form of the reference share equations (2) and it identifies the equivalence scales from the nonlinearities in the demand functions. It thus requires a flexible estimator in the empirical analysis that does not impose strong restrictions on the functional form of the derivatives of the expenditure shares.

# 3 Equivalence Scales for Social Benefits - the Case of Germany

In Germany most social benefits (and nowadays *new* social benefits) for more than 1.24 million households were calculated according to a method based on equivalence scales until

2004. See table 4 (appendix) for a descriptive overview of the year 2001. [3] Each household has a defined income requirement in order to achieve a minimum standard of living. The gross needs (Bruttobedarf) for social benefits should meet this amount. The calculation of the amount of gross needs is based on two parts: firstly, the standard rate (Regelsatz) and secondly, payments for housing, heating and other supplementary general costs. Standard rates are supposed to cover ongoing expenses for means of subsistence according to a fixed scheme that accounts for the demographic composition, i.e. the number of adults and the number of children living in the respective household. A civil servant at the social assistance office calculates the payments for housing, heating and other supplementary general costs on a case by case basis. The net entitlement in table 4 corresponds to the gross needs for social benefits minus the current income of the household. The net entitlement is the amount of money finally paid to the household.

The demographic composition of a household plays an essential role in determining the standard rates. For the latter the social planner computes the equivalent income between the demographic groups of households on the basis of an equivalence scale that is codified in the Federal Law of Social Benefits (BSHG). Table 1 presents the equivalence scales of the German social security system and the widely accepted "OECD (1982) scales". It also shows the demographic compositions that are subsequently considered for the estimations. It is evident that standard rates in Germany are higher than the OECD rates. If we look at the - empirically relevant - average gross needs, the opposite appears to be the case. The empirical scales computed from the average gross needs are below the OECD scales, with the exception of the scale between single person households and single parents with a child. This interesting observation has not been noted to date in the related empirical literature about Germany, e.g. Merz and Faik (1995) and Kohn and Missong (2003). One plausible explanation for this phenomenon is that the civil servants at the social assistance offices expect larger economies of scales in expenses for housing, heating and other supplementary costs of the household.

[3]Table 4 contains information about regular means of subsistence only. Households in specific circumstances, e.g. disabled persons who receive social benefits are not included because it is not possible to identify these households in the data.

# 4 Econometric Model

Economic theory suggests that the EPLM would be an appropriate framework for empirical analysis. The advantages of this semiparametric approach are also clear from the viewpoint of an econometrician: the risk of misspecification of the functional form of the expenditure shares is lower than for purely parametric models. Furthermore, the rate of convergence of the parameter of interest, e.g. of the equivalence scale parameter, is the same as in parametric frameworks $N^{1/2}$, where $N$ is the number of observations. Purely nonparametric estimators are ruled out as possible alternatives, as we intend to estimate a parameter of interest. In this paper we use the recently developed estimator of Wilke (2003) which is based on the work of Härdle and Marron (1990), which provides applicable solutions to the identification problems involved in this framework, and which has better finite sample properties.

We assume the availability of cross section data at a given point of time with given log. prices $p$. Define $m_j^0(x) = m_j(x, z_0, p)$ as the share equation of the reference household type $z_0$ and $m_j^1(x) = m_j(x, z_1, p)$ for any $z_1 \neq z_0$. According to the restrictions of the EPLM we may write equation (1) as (Blundell et al., 1998)

$$m_j^1(x) = a_j + m_j^0(x - \alpha), \tag{3}$$

where the function $m_j^1$ is a vertically and horizontally shifted translation of the reference function $m_j^0$. Our empirical focus is on the estimation of the parameter $\alpha$, which corresponds to the log. of the equivalence scale. The parameter $a_j$ reflects the elasticity of the equivalence scale with respect to the commodity price $j$. For the estimation of equation (3) we always compare two homogeneous subgroups of households. For each subgroup we have a sample of observations with different sample sizes $N_0$ and $N_1$. In order to identify the equivalence scale, we need a consistent estimate of $\alpha$. We therefore introduce the estimation model and the identification conditions as given by Wilke (2003).

Suppose we have samples $(Y_{ji}, X_i)_{i=1,...,N_0}$ and $(S_{ji}, W_i)_{i=1,...,N_1}$ with $j = 1, ..., J$. Let us assume the following functional relationships:

$$\begin{aligned} Y_{ji} &= m_j^0(X_i) + U_{ji} \\ S_{ji} &= m_j^1(W_i) + V_{ji} \quad \text{for } j = 1, \ldots, J \end{aligned}$$

with $E(U_{ji}|X_l) = E(V_{ji}|W_l) = 0$ for all $i, j$ and $l$. $U_{ji}$ and $V_{ji}$ have finite fourth moments and the pairs $U_{ji}, V_{ji}$ are mutually independent. $X_i \in \mathcal{X}_1$ and $W_i \in \mathcal{W}$ are i.i.d. random variables with realizations on compact sets with twice differentiable densities $f_x(x) > 0$ and $f_w(w) > 0$ for all $x$ and $w$. Furthermore, let the true parameter values $a_{j0}$ for $j = 1, \ldots, J$ and $\alpha_0$ be in the interior of open subsets in $\mathbb{R}$. Let us denote the set $\{x - \alpha\} = \mathcal{W}_\alpha$ for all $x \in \mathcal{X}_1$. The following assumptions ensure that the parameters can be identified: $\mathcal{W} \cap \mathcal{W}_\alpha$ is nonempty for all $\alpha$. This condition implies that the support of the two nonparametric functions overlaps for all $\alpha$. There exists a $j$ such that the function $m_j^0(x - \alpha)$ is not periodic on $\mathcal{W} \cap \mathcal{W}_\alpha$. This means that for at least one commodity there is no $\alpha \neq \alpha_0$ with $m_j^0(x - \alpha) = m_j^0(x - \alpha_0)$ for all $x - \alpha \in \mathcal{W} \cap \mathcal{W}_\alpha$. This is required for a unique solution in $\alpha$. Furthermore, $j$ is such that the function $m_j^0(x - \alpha)$ is nonlinear on $\mathcal{W} \cap \mathcal{W}_\alpha$ for all $\alpha$. This is required for the joint identification of $a_j$ and $\alpha$. Under several technical assumptions on the nonparametric estimates of $m^0$, $m^1$ and $f$, the solution to the problem

$$\min_{a,\alpha} L_{N_0,N_1}(a, \alpha) = \sum_{j=1}^{J} \frac{\int_{\mathcal{W} \cap \mathcal{W}_\alpha} [\hat{m}_j^1(x) - a_j - \hat{m}_j^\alpha(x)]^2 dx}{\int_{\mathcal{W} \cap \mathcal{W}_\alpha} \hat{f}_x(x) dx}, \tag{4}$$

yields consistent parameter estimates, where $\hat{m}_j^\alpha(x)$ denotes the nonparametric estimate of the function $m_j^0$ after shifting it horizontally by the parameter $\alpha$.[4] Under further technical conditions the parameter estimates converge at the rate $N^{1/2}$ and are normally distributed (Wilke, 2003). We use here the HM4SE[5] which is an improved version of the Härdle and Marron (1990) estimator.[6] The estimator is implemented as follows:

1. Estimate the nonparametric functions $m_j^0$ and $m_j^1$ for $j = 1, \ldots, J$. In our applications we use the Nadaraya-Watson estimator and the local linear smoother with constant bandwidths that are obtained with a plug-in method as given in Fan and Gijbels (1995).

[4]In fact the estimation objective function (4) does not involve the shape invariance restriction across all household types $z$ because it is restricted to the comparison of two household types only. The equivalence scales could be estimated for all groups simultaneously by using $\hat{m}_j^0(x) = a_j' z + \hat{m}_j(x - \alpha' z)$, where $a_j$ and $\alpha$ are column vectors of the length of the total amount of demographic groups and $z$ is a dummy vector of the same length.

[5]HM4SE is introduced by Wilke (2003).

[6]Alternative approaches can be found in Stengos and Wang (2002) and Pendakur (2004) who use a penalizing function in order to overcome the finite sample difficulties.

A parametric nonlinear least squares (NLS) estimator is also applied as a benchmark, which imposes a quadratic specification of the expenditure share.[7]

2. Estimate the parameters $a_j$ given $\alpha$ by least squares, i.e.

$$\min_{a_j} \int_{\mathcal{W}\cap\mathcal{W}_\alpha} (\hat{m}_j^1(x) - a_j - \hat{m}_j^\alpha(x))^2 dx$$

for any $\alpha$ and all $j$. Denote the estimate $\hat{a}_j^\alpha$.

3. Solve problem (4) numerically in $\alpha$ conditional on $\hat{a}_j^\alpha$ in order to obtain $\hat{\alpha}$. Denote the function $L_{N_0,N_1}(\alpha|a_\alpha)$ as the loss function in $\alpha$.

4. $\hat{a}_j = \hat{a}_j^{\hat{\alpha}}$ for $j = 1, \ldots, J$.

The least squares estimation in step three is not efficient, since the variance of the non-parametric estimators is a function depending on the location on the support where it is evaluated. This variance function might be estimated by (wild)-bootstrap and used for constructing weights in the least squares estimation.

The standard errors of the parameter estimates are computed from the empirical distribution of the parameter estimates obtained by wild bootstrapping. Wild bootstrapping in the EPLM is described in the appendix A I.

# 5 Data

The 1998 German Income and Consumption Survey (EVS) is used for the estimations. This survey data is based on $49,720$ households from both West and East Germany with more than 900 variables (demographic, consumption and income related). It is a quota sample with voluntary participation and is therefore not representative with regard to the whole population (Kühnen, 1999). Singles and blue-collar workers, for instance, have a lower rate of reply. The same is true for households with low or high incomes. Projection factors are available to generate representative results. The analysis in this paper does not use these factors as there is no obvious reason for doing so because the analysis is performed for

[7]This specification restricts the nonlinearities to a constant. The aim is to use it to verify whether results are sensitive with respect to this restriction.

homogenous demographic groups conditional on the level of household net income and/or on the employment status of the household head. We can only assume that the observed consumption behavior in each of the segments is nevertheless representative for the whole population segment. It is also important to mention that the sample and census design have significantly changed from previous EVSs used, for instance, by Merz and Faik (1995), and Kohn and Missong (2003). Due to the voluntary participation of the households and the generally long recording period of one year [8], attrition was too high in the past (Chlumsky and Ehling, 1997). For this reason the responsible Federal Statistical Office (Statistisches Bundesamt) reduced the recording period from one year to three months. In the author's view, this should also increase the quality of the observed variables. They should become more precise (due to higher motivation of the recording households). Moreover, the probability is greater that variables such as employment status, demographic decomposition and prices do not actually vary, as the former are recorded by interviews at the beginning and at the end of the recording period only. For estimations we only use observations that are recorded in the second or in the third quarter of the year, i.e. during summer time, in order to exclude calendar time effects on the consumption structure of the households. In terms of commodity aggregation we are confronted with the following trade-off: if we use all possible consumption items available in the data (several hundred) there will be insufficient observations and in many cases one commodity may substitute a very similar one. There is therefore no alternative but to work with several aggregated commodity groups. However, aggregation must be done carefully so as to avoid measurement errors which could seriously bias the estimation results. The econometric model is estimated using 12 aggregated commodity groups which are presented in table 2. These groups are directly taken from the 1998 EVS data. Since economic theory suggests that the equivalence scale does not depend on the specific consumption good, none of the original EVS groups is excluded from the estimations. In addition, a larger dimension of the expenditure shares vector provides more structure for the econometric model, since it incorporates more nonlinearities. The aggregation of the commodities is performed by the German Federal Statistical Office, and the commodity categories are created in such a way that each reflects a central need of the house-

[8] There are also, to some extent, records on a monthly basis (Feinaufschriebe).

Table 2: EVS commodity groups.

| $j$ | *Commodity group* | | |
|---|---|---|---|
| 1 | Food | 2 | Clothing |
| 3 | Housing | 4 | Energy |
| 5 | Interior decoration | 6 | Health care |
| 7 | Transport | 8 | Communication |
| 9 | Leisure and travelling | 10 | Education |
| 11 | Board and lodging | 12 | Other goods |

holds. This categorization is harmonized with international standards, i.e. COICOP 1998.[9] It seems therefore reasonable to adopt this categorization for our purposes. However, it is not clear to which extent measurement error might occur due to this aggregation. Another source of error is nonresponse of the households. A total of 8.5% of the $12 \times 49,720$ observed expenditures shares are zero. Zero entries in the data correspond to either zero expenditures or to missing values. When taking a closer look one finds that zero entries are clustered in commodity groups such as "education" or "board and lodging". In the following analysis the zero entries are treated as zero expenditures because it is likely that many households do indeed have zero expenditures for such goods as "education". This assumption is also substantiated by the fact that the households provide the information voluntarily. Since we are interested in transfer payments for regular means of subsistence, we should restrict the following analysis to expenses for non-durables. We thus need to modify the original commodity groups slightly because some groups contain expenses for durables, e.g. transport expenditures contain expenses for car purchases. Expenditures for durables[10] are therefore subtracted. As already mentioned, we only use observations that are recorded during the summer quarters of the year. The following demographic groups are used separately for

[9]There are some minor deviations from the international standard to allow for comparisons with older issues of the EVS-data.

[10]Furnishing, medical devices, purchases of or repair costs for cars, motor bikes or bicycles, purchases of leisure or electronic devices, musical instruments, jewelry, watches and precious metals. This expenditure amounts to $0 - 94\%$ of total household expenditure with a mean value of 9.5% and a median value of 4.3%. Expenditures for durables are lower for single households and do not increase with the number of children.

the estimations: (S0,C0), (S1,C1), (S0,S1), (C0,C1), (C1,C2), (C2,C3)[11] conditional on the status of the household head (full time employed or non-employed) and/or by distinguishing between the level of the household net income. These distinctions are made for the following reasons: distinctions with respect to the employment status are necessary because the income of households with a non-employed household head (retired, unemployed) typically depends to certain extent on social security transfer payments. It is interesting to see whether the consumption behavior of households which rely on a social transfer scheme is different from the consumption behavior of households with working income. Unfortunately, there are only a few households receiving exclusively social benefits (all other transfers schemes depend at least to some extent on the prior working income). For this reason the group of non-employed household heads is chosen with the drawback that a large number of these households has income related to prior working income. Nonetheless, this allows us to examine the consumption structure of different demographic, homogeneous household groups to each other with respect to the available leisure time. If leisure and consumption are perfectly separable and if the design of social transfers payment schemes does not affect the behavior of households, the results should be identical for the two leisure-groups. A distinction between different net income levels may provide a rough idea as to whether the assumption of the independence of the base utility, i.e. $\alpha(u) = \alpha$, has empirical evidence or not. The reason for this is that households with higher income level may attain a higher utility level from consumption. Therefore, estimations are also done separately for households in the lowest or in the highest quartile of the net income sample distribution. In the past Merz and Faik (1995) also considered potential effects of the income level on the equivalence scale.

The structure of the homogenous sub-samples revealed that the sample size in some cells decreased in such a way that reliable semiparametric estimations were no longer possible (see table 5 in the appendix). This is why the single household with one child (S1) group is not considered in two cases. The analysis in this paper does not explicitly consider the age of children as the German social security system does, where the equivalence scale increases when the children become older. This simplification ensures that enough observations are still considered in a sizeable proportion of the data segment (see table 5). The simplification

[11] Again, the notation of table 1 is used.

can also affect the estimation results if there is some variation in average age across the data segments. Particularly households with low net income typically have younger children than households with high net income (see table 7). In such a case estimation results between the data segments are not directly comparable if equivalence scales depend on the age of the children. In other cases, in particular for the non-employed, it is evident that sample composition of the compared demographic groups differs substantially (see tables 6 and 7). These composition issues can affect the estimation results and one should restrict attention to samples of similar composition. Since the size of the available data is not large enough it is impossible to overcome this difficulty with the underlying estimation approach.[12]

# 6 Estimation Results

The estimations reveal the general appropriateness of the model specification (tables 9 and 11 in the appendix). The estimated change in the equivalent income for additional adults or additional children is always within an economically plausible range $(0 - 100\%)$. However, in the case of the first child the estimated equivalence scale is often below this range, i.e. it is negative (table 10). If we turn our attention back to all the cases again, the shifted nonparametric functions appear to fit in well at first glance. In most cases the loss function possesses a unique minimum for plausible values of the equivalence scales, i.e. $c \in [0, ln2]$.[13] Tables 9-11 in the appendix report the detailed estimation results and present an extended coefficient of determination for the parametric part of the EPLM, $\mathbf{eR}^2$ which is introduced in appendix A.II. Since $\mathbf{eR}^2$ is mostly within the range $0.3 - 0.6$ it is evident that the simple transformation with two parameters yields a convincing fit for survey consumer data. This clearly indicates that large standard errors of the parameter estimates are driven by the variance of the first stage estimates. Therefore, it seems that the model (4) is appropriately

[12]Other characteristics such as sex of the household head are not considered for the same reason, nor is the disability of household members observed. A skillful extension of the semiparametric approach that accounts for a variety of regressors might be the subject of future research. Chen, Blundell and Kristensen (2003) move in this direction; however, their model identification conditions are subjected to hardly any practical verification.

[13]Figures which illustrates this are available upon request from the author.

| | *Additional adult* | | *Additional child* | |
|---|---|---|---|---|
| | C0/S0 | C1/S1 | C2/C1 | C3/C2 |
| *German social benefits* | | | | |
| Standard rates† | 80% | 49% | 27% | 21% |
| Average gross needs 2003 (West-Germany) | 58% | 24% | 21% | 18% |
| OECD (1982) | 70% | 47% | 23% | 19% |
| Consumption based, EVS 1998 | | | | |
| *Semiparametric estimation results* | | | | |
| Full sample | **48** – **51**% | **24** – **28**% | **12**% | 12 – 15% |
| lowest income quartile | – | – | **9** – **13**% | 14 – 19% |
| highest income quartile | **47** – **54**% | – | **1** – **7**% | – |
| Employed | **54** – **59**% | 14 – 21% | **11**% | **10** – 12% |
| lowest income quartile | 70 – 73% | – | **12** – **13**% | 17 – 22% |
| highest income quartile | 73 – 78% | – | **3** – **6**% | – |
| Other results | | | | |
| Merz and Faik (1995) | 54% | 43% | 7% | 6% |
| Kohn and Missong (2003) | 66% | 30% | 11% | 8% |
| Bellemare et al. (2002) | 29 – 44%* | 5 – 33%* | – | – |

Table 3: Increase of equivalent income: comparison of selected point estimates (tables 9 and 11) to policy rules of table 1. † BSHG, children of age 7-13. * This is the range of point estimates spanned by the various methods, bold items: value significantly (at the 10% level) below the standard rate.

specified for the EVS 1998. The reported standard errors are computed from the empirical distribution of 500 wild-bootstrap estimates. Note that each 12.000 Nadaraya-Watson and local linear smoothing estimates are computed in the bootstrap estimation of one standard error. In some cases the estimated parameters possess large standard errors and therefore have to be considered to be of limited reliability. The unreliability of the first stage estimates is mainly driven by small sample sizes or parts of the support with low data density. Furthermore, it is due to different sample compositions within and across data segments that cannot be captured by the model. The chosen bandwidths are obtained by using the plug-in method suggested in Fan and Gijbels (1995). The resulting bandwidths are mostly within the range of $0.2 - 0.5$.[14] The choice of the bandwidth and the support for the nonparametric estimation affects the results. However, the sensitivity of the results was controlled by slightly varying the boundaries of the support of the nonparametric functions. Thus, general insensitivity is found in cases of small standard errors.

A selection of the most reliable point estimates is presented in table 3. This compares them with the expert equivalence scales of the German social benefits system, the OECD equivalence scales, the empirically evident values computed from gross needs and with the estimation results obtained by Merz and Faik (1995), Kohn and Missong (2003), and Bellemare et al. (2002). The reported ranges of the estimation results are based on the two semiparametric point estimates obtained (see tables 9-11) and are therefore not based on distributional information. Bold numbers indicate, however, that point estimates are significantly below standard rates. It is apparent that some results are (significantly) below the value suggested by the expert equivalence scales of the German social security system or by the OECD and that some results are in the range of the expert scales. Intuitively, the estimated equivalence scales correspond to the increase in household income such that an average household is able to maintain the same standard of living if one more member (adult or child) is added. The word *average* refers to the empirical mean of all households in the respective data segments. It is therefore an estimate of the mean equivalence scale. The

[14] In an earlier version of this paper the bandwidth was chosen to be three times the optimal bandwidth. This high degree of oversmoothing was conducted in order to reduce the variance of the first step nonparametric estimates which was much greater when including expenditures for durables. As a result the parameter estimates based on the two nonparametric estimators diverged to a greater extent.

estimates cannot provide any information about a reasonable absolute figure that indicates the gross need for social benefits of a single person household (S0).

Let us turn now to a more detailed discussion of the estimation results which are detailed in tables 9-11.

**Choice of the first stage estimator** The choice of the first stage estimator affects results more strongly if it belongs to a different class of estimators. In many cases results for the two nonparametric estimators are pretty close, while results for the parametric first stage estimator rather differ. In 17 cases the parametric estimate benchmark is smaller than the results obtained with the semiparametric specification, in 6 cases it is in between and in 9 cases it is greater. Looking at the latter examples one finds that this is mainly to be found in the case of a household with a first child (6 of 9 cases fall into this segment). As discussed below, the estimation results are probably biased in the first child case due to issues in data composition. If we therefore restrict the attention to the other cases one can conclude that the parametric approach often yields a smaller estimate of the equivalence scale. This is some evidence for a systematic estimation bias of the parametric benchmark estimator due to the strong restrictions on the nonlinearities of the expenditure shares. Estimated wild bootstrap standard errors are often similar for the two nonparametric first stage estimates and there is no clear result pattern in the differences. In the parametric case, standard errors are often similar or smaller but we do not observe a clear advantage of the parametric estimator.

**Employment status** Results for employed and non-employed household heads are rather different. Since the groups compared are quite heterogeneous, one cannot directly infer that the employment status is the driving force behind the differences in the results. Results for the employed are less likely affected by compositional effects and appear more reliable. In contrast, the large standard errors and the low $\mathbf{eR}^2$ in the non-employment cases are probably due to different compositions of samples taken from the data segments under comparison (tables 6-7). Results for the non-employed are therefore not presented in table 3.

**Additional adult** An additional adult in a household without children requires an increase in equivalent income by $50 - 75\%$ and by up to 30% for households with one child, if one ignores in both cases the results for the non-employed which have large standard errors. Results with small standard errors provide estimated equivalence scales weakly below the standard rates. Standard rates are significantly above the estimation results for the full sample and for the employed in the C0/S0 comparison. Full sample C1/S1 estimation results are significantly below the standard rates and in the range of the empirical numbers for social benefits.

**Additional child** The EPLM is very well specified for the comparison of C2 and C1 with an $\mathbf{eR}^2$ of up to 0.85 and small standard errors. For the full sample and the employed, estimated equivalence scales are significantly below standard rates. For the non-employment group, standard errors are again quite large and also the $\mathbf{eR}^2$ goes down, reaching a value of 0.4. For the C3/C2 comparison we obtain that lower bounds are below (with a general lack of statistical significance) and upper bounds are in the range of standard rates. The greater standard errors are probably due to the smaller size of the C3 segments. In the case of an additional child, the compositions of samples in terms of average age of the household head and average age of the children in the household are quite similar. For this reason, we expect only little heterogeneity across the groups. This explains the generally good fit of the model in the case of an additional child.[15]

**First child** Results for the first child in a household appear implausible, since estimates are sometimes negative. Several factors explain this. If we think about preferences, the decision to have a child can be considered as a permanent decision against a high level of consumption. The model does not control for this heterogeneity in preferences and omitted variables. It also does not control for the employment status of the second adult in the household, which often changes with the arrival of the first child. Different compositions with respect to other variables such as the age of the household heads compared may also be a reason. All this can be a source of bias in the estimation for the case of a first child.

[15] Additional segmentation with respect to the children's age (<7, >12 or in between) did not improve the fit of the model and did not yield a clear result pattern.

For this reason, they are not presented in table 3. Another popular explanation could be that households often decrease their consumption expenditures after the birth of the first child. Here, this seems to be rather unlikely since the average age of the first child is quite high (9 – 16 years ) in the underlying data segments. Again, the estimation results for the non-employed have larger standard errors and differ from those obtained for the other groups.

**Income level** When considering the full set of estimation results, there is a weak tendency towards higher equivalence scales for households with lower net incomes. The income pattern is clearest in the case of additional children where we have the most precise estimates. The results presented therefore indicate that the level of household net income may have empirical relevance in determining equivalence scales. However, only better data and an improved model structure will answer the question as to whether the equivalence scale is independent of the income level or not.

# 7 Summary and Outlook

This paper presents a comprehensive empirical study on the semiparametric estimation of consumption-based equivalence scales. Equivalence scales for Germany are estimated by applying Wilke's (2003) estimator for the extended partially linear model suggested by Blundell et al. (1998) to cross-section EVS survey data for 1998 with almost 50.000 observations. The model identifies the equivalence scales from the nonlinearities in households' expenditure shares. The econometric framework accounts for that by keeping the expenditures shares nonparametric. This appears to be important because a comparison with a parametric benchmark estimator reveals rather systematic differences in the results. For estimation purposes, the data is segmented into homogenous groups of households according to the household head's employment status or net household income. The curse of dimensionality, heterogeneity with respect to ignored variables and the demanding computational approach are drawbacks in the framework adopted by this study. However, it is found that estimated consumption-based equivalence scales are weakly below the equivalence scales used by the German social benefits system. In some cases, estimates appear to be precise and in other cases they suffer from large standard errors. In these cases, the household groups compared differ considerably in their composition which probably has an impact on the estimation results. Another source of bias may be the measurement error due to commodity aggregation and misreporting of behaviour shown by the households interviewed. Moreover, several assumptions of the underlying economic theory are hardly verified in an application. The adoption of a more general model structure that incorporates dynamic consumer theory may also have an impact on results. These are several reasons why it is difficult to derive policy recommendations. However, most of these weaknesses apply to a broad range of contributions in this research field.

The estimation results do not provide indications that on average the standard rates of German social benefits do not *at least* cover the costs of additional persons in a household. In the light of recent decisions of the Federal Constitutional Court (Bundesverfassungsgericht) concerning the costs of children and increasing discussion about demographic transitions in Germany, it is not apparent from the estimation results that equivalence scales need to be increased for households with children. Before attempting to infer more precise policy recommendations from this class of models, an answer to the assumption that equivalence scales do not depend on household income should be found. The results of this paper point to an income pattern but they lack of statistical significance. Moreover, improving the model specification may help to overcome the evident sample selection issues in the first child case and they may help to reduce the noise in the data. Conditioning on the type of region of residence (urban, rural, etc.) or on the geographical factors West and East Germany did not improve the model fit, nor did it yield clear result patterns. We have already examined this issue. While segmenting the data we are directly faced with the curse of dimensionality, i.e. the problem of running into data cells with low frequency. It is therefore desirable to extend the approach to an estimation framework that overcomes some of these difficulties and, in addition, accounts for the endogeneity of household expenditure. The recent paper by Blundell, Chen and Kristensen (2003) presents some interesting developments that may soon contribute to the literature applied in this field.

# Appendix:

## A I: Wild bootstrapping in the EPLM

The idea of bootstrapping is to resample the observations several times and estimate the unknown regression functions and the unknown coefficients for each resample. This yields an empirical distribution for the parameter estimates of interest. However, naive resampling does not work in the EPLM because the conditions $E(U|X = x) = E(V|W = w) = 0$ would not be imposed. Therefore, wild bootstrapping is performed which induces the required conditions.

Let $Q$ be a random variable with a two-point probability distribution $H$:

$$Q = (1-\sqrt{5})U/2 \quad \text{with probability}(1+\sqrt{5})/2\sqrt{5}$$

and

$$Q = (1+\sqrt{5})U/2 \quad \text{with probability} \left(1-(1+\sqrt{5})\right)/2\sqrt{5}$$

This implies $E(Q|H) = 0$, $E(Q^2|H) = U^2$ and $E(Q^3|H) = U^3$.

Compute the residuals of the first step nonparametric estimation, i.e. $\hat{U}_{ji} = Y_{ji} - \hat{m}_j^0(X_i)$ and $\hat{V}_{ji} = S_{ji} - \hat{m}_j^1(W_i)$. Then carry out wild bootstrapping as follows:

1. Compute $U^*_{ji} = Q\hat{U}_{ji}$ and $V^*_{ji} = Q\hat{V}_{ji}$ for all $i$ and $j$.
2. Compute $Y^*_{ji} = \hat{m}_j^0(X_i) + U^*_{ji}$ and $S^*_{ji} = \hat{m}_j^1(W_i) + V^*_{ji}$ for all $i$ and $j$.
3. Estimate $m_j^{0*}$ and $m_j^{1*}$ using the samples $(Y^*_{ij}, X_i)$ and $(S^*_{ij}, W_i)$ for all $j$.
4. Obtain bootstrap parameter estimates $\hat{a}^*$ and $\hat{\alpha}^*$.
5. Repeat steps one to four in order to get finitely many realizations of $\hat{a}^*$ and $\hat{\alpha}^*$.

The empirical distribution of $\hat{a}^*$ and $\hat{\alpha}^*$ is used to approximate the distribution of $\hat{a}$ and $\hat{\alpha}$. For further details concerning the wild bootstrap method see Härdle and Mammen (1993). Härdle and Mammen (1993) suggest choosing a larger bandwidth for the pilot nonparametric estimates and an optimal bandwidth for the bootstrap estimates. In this paper, the same bandwidth is used for the estimation of $m_j$ and $m_j^*$. This is done for the simple reason of computational feasibility.

## A II: Second stage $\mathbf{R}^2$ in the EPLM

This appendix introduces the extended coefficient of determination for the parametric transformation in the EPLM, the $\mathbf{eR}^2$. It determines how well the differences between the two sets of nonparametric functions $\hat{m}_0^j$ and $\hat{m}_1^j$ are explained by the parametric part of the model. However, it only incorporates the point estimates and ignores information about higher moments of the distribution of $\hat{m}_0^j$ and $\hat{m}_0^j$. Since a large part of the estimated coefficients' variance is due to the variance of the first stage nonparametric estimates, the suggested $\mathbf{eR}^2$ cannot be seen as a general goodness of fit measure for the EPLM.

Let us denote $\bar{m}_0^j = \sum_i \hat{m}_0^j(x_i)$ and $\bar{m}_1^j = \sum_i \hat{m}_1^j(x_i)$ as the mean expenditure shares for commodity $j$. Then the coefficient of determination for commodity $j$ is given by

$$\mathbf{R}_j^2 = \frac{\left[\sum_i \left(\hat{m}_1^j(x_i) - \bar{m}_1^j\right)\left(\hat{m}_\alpha^j(x_i) - \bar{m}_c^j\right)\right]^2}{\left[\sum_i \left(\hat{m}_1^j(x_i) - \bar{m}_1^j\right)^2\right]\left[\sum_i \left(\hat{m}_\alpha^j(x_i) - \bar{m}_\alpha^j\right)^2\right]}$$

which has the standard properties of the $\mathbf{R}^2$, i.e. it is the squared correlation between the nonparametric function $\hat{m}_1^j$ and its predicted value $\hat{m}_\alpha^j$, both evaluated at the observations. Note that the constant $\hat{a}_j$ cancels out. The $\mathbf{eR}^2$ is simply an average over the $\mathbf{R}_j^2$, i.e.

$$\mathbf{eR}^2 = \frac{1}{J}\sum_j \mathbf{R}_j^2.$$

## A III: Tables

| Demographic group | # | average gross needs | average net entitlement | total gross needs | total net entitlements |
|---|---|---|---|---|---|
| *All households* | 1,235,326 | 860 | 394 | 1,062,663 294 | 486,718,444 |
| *Single households* | | | | | |
| without children | 605,020 | 581 | 326 | 351,705,194 | 197,312,874 |
| with 1 child (< 18 years) | 190,696 | 919 | 429 | 175,219,574 | 81,790,457 |
| with 2 children (< 18 years) | 106,664 | 1,214 | 470 | 129,541,414 | 50,116,993 |
| *Couple households* | | | | | |
| without children | 120,819 | 893 | 407 | 107,836,320 | 49,117,174 |
| with 1 child (< 18 years) | 67,016 | 1,101 | 464 | 73,778,288 | 31,103,827 |
| with 2 children (< 18 years) | 52 523 | 1,336 | 489 | 70,181,561 | 25,691,999 |
| more than 2 children (< 18 years) | 44,365 | 1,756 | 559 | 77,905,957 | 24,804,852 |

Table 4: Some facts about German "Sozialhilfe" in 2001: regular means of subsistence. Gross needs and net entitlements are per month in euros. Source: Federal Statistical Office (2001), own calculations.

Table 5: Sample Size

| | S0 | S1 | C0 | C1 | C2 | C3 |
|---|---|---|---|---|---|---|
| Full sample | 5,714 | 882 | 7,727 | 3,531 | 4,589 | 1,321 |
| quartile | 1,429 | 221 | 1,932 | 883 | 1,147 | 330 |
| Employed | 2,460 | 380 | 3,349 | 2,886 | 3,968 | 1,091 |
| quartile | 615 | –‡ | 838 | 722 | 992 | 273 |
| Non-employed | 3,254 | 502 | 4,378 | 645 | 621 | 230 |

‡ not considered, too few observations

Table 6: Average age of household head

| | S0 | S1 | C0 | C1 | C2 | C3 |
|---|---|---|---|---|---|---|
| Full sample | 50.7 | 41.5 | 56.7 | 42.7 | 41.1 | 41.7 |
| lowest income quartile | 48.9 | 38.4 | 60.6 | 39.0 | 41.1 | 41.7 |
| highest income quartile | 52.5 | 45.6 | 53.9 | 47.0 | 44.9 | 44.3 |
| Employed | 38.8 | 42.3 | 45.0 | 41.7 | 40.8 | 41.4 |
| lowest income quartile | 34.1 | –‡ | 43.2 | 37.8 | 37.7 | 39.0 |
| highest income quartile | 44.4 | –‡ | 47.7 | 46.0 | 44.4 | 43.9 |
| Nonemployed | 59.7 | 40.9 | 65.6 | 46.9 | 43.2 | 43.2 |

‡ not considered, too few observations

Table 7: Average age of children

| | S0 | S1 | C0 | C1 | C2 | C3 |
|---|---|---|---|---|---|---|
| Full sample | – | 13.4 | – | 11.7 | 10.8 | 10.5 |
| lowest income quartile | – | 10.6 | – | 9.0 | 9.0 | 9.3 |
| highest income quartile | – | 16.0 | – | 14.2 | 13.1 | 12.1 |
| Employed | – | 15.6 | – | 11.4 | 10.7 | 10.4 |
| lowest income quartile | – | –‡ | – | 8.5 | 8.9 | 9.2 |
| highest income quartile | – | –‡ | – | 14.2 | 12.8 | 12.0 |
| Non-employed | – | 11.7 | – | 13.0 | 11.6 | 10.9 |

‡ not considered, too few observations

Table 8: Employment status of 2nd adult in household

| | S0 | S1 | C0 | C1 | C2 | C3 |
|---|---|---|---|---|---|---|
| Full sample | | | | | | |
| full-time employed | – | – | 21% | 17% | 14% | 6% |
| part-time, other | – | – | 14% | 47% | 49% | 43% |
| non-employed | – | – | 65% | 36% | 37% | 51% |
| Employed | | | | | | |
| full-time employed | – | – | 43% | 19% | 14% | 5% |
| part-time, other | – | – | 21% | 49% | 50% | 43% |
| non-employed | – | – | 36% | 32% | 36% | 52% |

Table 9: Estimation results of the log of the consumption-based equivalence scale, part I

| *Additional adult* | | C0/S0 | | | C1/S1 | | |
|---|---|---|---|---|---|---|---|
| | | NW[a] | LLS[b] | NLS[c] | NW[a] | LLS[b] | NLS[c] |
| Full sample | $\hat{\alpha}$ | 0.3915 (0.1036)[†] | 0.4093 (0.0764)[†] | 0.3444 (0.0412) | 0.2136 (0.1098) | 0.2492 (0.0809) | 0.0740 (0.1291) |
| | $eR^2$ | 0.26 | 0.27 | 0.57 | 0.32 | 0.33 | 0.57 |
| low income | $\hat{\alpha}$ | –[§] | –[§] | 0.4627 (0.1042) | –[§] | –[§] | −0.0917 (0.2624) |
| | $eR^2$ | | | 0.47 | | | 0.39 |
| high income | $\hat{\alpha}$ | 0.3829 (0.0760) | 0.4285 (0.0831) | 0.4248 (0.0682) | –[§] | –[§] | 0.2006 (0.1334) |
| | $eR^2$ | 0.29 | 0.30 | 0.64 | | | 0.50 |
| Employed | $\hat{\alpha}$ | 0.4323 (0.0533) | 0.4611 (0.0557) | 0.4563 (0.0579) | 0.1321 (0.2058) | 0.1865 (0.1482) | 0.0634 (0.1672) |
| | $eR^2$ | 0.30 | 0.31 | 0.61 | 0.24 | 0.29 | 0.58 |
| low income | $\hat{\alpha}$ | 0.5305 (0.0978) | 0.5459 (0.0904) | 0.4269 (0.1242) | –[‡] | –[‡] | –[‡] |
| | $eR^2$ | 0.25 | 0.26 | 0.36 | | | |
| high income | $\hat{\alpha}$ | 0.5465 (0.0639) | 0.5769 (0.1114) | 0.5425 (0.1085) | –[‡] | –[‡] | –[‡] |
| | $eR^2$ | 0.39 | 0.41 | 0.65 | | | |
| Non-employed | $\hat{\alpha}$ | 0.2551 (0.2488) | 0.2551 (0.2044) | 0.1968 (0.1025) | 0.3945 (0.2180) | 0.3861 (0.1746) | 0.2589 (0.1679) |
| | $eR^2$ | 0.31 | 0.33 | 0.50 | 0.28 | 0.30 | 0.49 |

Wild bootstrap standard errors of $\hat{\alpha}$ in brackets

[a] Nadaraya-Watson 1st stage nonparametric estimate

[b] Local linear smoother 1st stage nonparametric estimate

[c] quadratic parametric 1st stage estimate

[§] no result available (boundary solution, misshaped objective function)

[†] results based on a 75% random sample (C0), otherwise too many observations

Table 10: Estimation results of the log of the consumption based equivalence scale, part II

| *First child* | | S1/S0 | | | C1/C0 | | |
|---|---|---|---|---|---|---|---|
| | | NW[a] | LLS[b] | NLS[c] | NW[a] | LLS[b] | NLS[c] |
| Full sample | $\hat{\alpha}$ | 0.0445 (0.1035) | 0.0668 (0.0840) | 0.1574 (0.0885) | −0.0540 (0.0435)[†] | −0.0432 (0.0386)[†] | −0.0408 (0.0387) |
| | eR$^2$ | 0.27 | 0.29 | 0.60 | 0.28 | 0.29 | 0.27 |
| low income | $\hat{\alpha}$ | –[§] | –[§] | 0.3973 (0.1701) | −0.0431 (0.0686) | 0.0000 (0.0762) | −0.0673 (0.0742) |
| | eR$^2$ | | | 0.45 | 0.38 | 0.38 | 0.58 |
| high income | $\hat{\alpha}$ | 0.0080 (0.1083) | −0.0080 (0.0935) | 0.1631 (0.1301) | 0.0121 (0.0629) | −0.0483 (0.0810) | −0.0342 (0.0635) |
| | eR$^2$ | 0.34 | 0.35 | 0.51 | 0.59 | 0.51 | 0.64 |
| Employed | $\hat{\alpha}$ | 0.2384 (0.1867) | 0.2831 (0.1672) | 0.3574 (0.1473) | −0.1113 (0.0552) | −0.1113 (0.0549) | −0.0799 (0.0466) |
| | eR$^2$ | 0.18 | 0.22 | 0.41 | 0.57 | 0.55 | 0.63 |
| low income | $\hat{\alpha}$ | –[‡] | –[‡] | –[‡] | −0.1136 (0.3210) | 0.0852 (0.2479) | −0.0317 (0.0733) |
| | eR$^2$ | | | | 0.25 | 0.38 | 0.69 |
| high income | $\hat{\alpha}$ | –[‡] | –[‡] | –[‡] | 0.0582 (0.0943) | 0.0582 (0.0992) | −0.1251 (0.0921) |
| | eR$^2$ | | | | 0.39 | 0.42 | 0.63 |
| Nonemployed | $\hat{\alpha}$ | −0.1970 (0.2813) | −0.1244 (0.2367) | −0.0900 (0.1730) | 0.1850 (0.1028) | 0.2081 (0.1048) | 0.1020 (0.0786) |
| | eR$^2$ | 0.21 | 0.22 | 0.48 | 0.29 | 0.32 | 0.54 |

Wild bootstrap standard errors of $\hat{\alpha}$ in brackets

[a] Nadaraya-Watson 1st stage nonparametric estimate

[b] Local linear smoother 1st stage nonparametric estimate

[c] quadratic parametric 1st stage estimate

[§] no result available (boundary solution, misshaped objective function)

[†] results based on a 75% random sample (C0), otherwise too many observations

Table 11: Estimation results of the log of the consumption based equivalence scale, part III

| *Additional child* | | C2/C1 | | | C3/C2 | | |
|---|---|---|---|---|---|---|---|
| | | NW[a] | LLS[b] | NLS[c] | NW[a] | LLS[b] | NLS[c] |
| Full sample | $\hat{\alpha}$ | 0.1102 (0.0350) | 0.1102 (0.0352) | 0.0683 (0.0357) | 0.1392 (0.0663) | 0.1160 (0.0557) | 0.1142 (0.0512) |
| | $eR^2$ | 0.81 | 0.81 | 0.84 | 0.57 | 0.62 | 0.75 |
| low income | $\hat{\alpha}$ | 0.1234 (0.0592) | 0.0905 (0.0678) | 0.0414 (0.0531) | 0.1320 (0.1659) | 0.1775 (0.1384) | 0.1895 (0.0746) |
| | $eR^2$ | 0.58 | 0.59 | 0.76 | 0.27 | 0.28 | 0.64 |
| high income | $\hat{\alpha}$ | 0.0069 (0.0731) | 0.0691 (0.0598) | 0.0435 (0.0626) | –[§] | –[§] | 0.3339 (0.1682) |
| | $eR^2$ | 0.60 | 0.65 | 0.75 | | | 0.49 |
| Employed | $\hat{\alpha}$ | 0.1036 (0.0356) | 0.1036 (0.0369) | 0.0803 (0.0343) | 0.1168 (0.0543) | 0.0973 (0.0508) | 0.0764 (0.0484) |
| | $eR^2$ | 0.85 | 0.82 | 0.84 | 0.65 | 0.62 | 0.68 |
| low income | $\hat{\alpha}$ | 0.1155 (0.0520) | 0.1227 (0.0668) | 0.0414 (0.0627) | 0.1547 (0.1664) | 0.1957 (0.1370) | 0.2021 (0.0855) |
| | $eR^2$ | 0.58 | 0.60 | 0.77 | 0.29 | 0.29 | 0.67 |
| high income | $\hat{\alpha}$ | 0.0297 (0.0571) | 0.0535 (0.0527) | 0.0521 (0.0583) | –[§] | –[§] | 0.2151 (0.1386) |
| | $eR^2$ | 0.68 | 0.73 | 0.75 | | | 0.40 |
| Non-employed | $\hat{\alpha}$ | 0.2150 (0.2434) | 0.2398 (0.2314) | 0.1685 (0.1112) | 0.0463 (0.3957) | 0.0556 (0.2727) | 0.1583 (0.1527) |
| | $eR^2$ | 0.41 | 0.39 | 0.61 | 0.44 | 0.48 | 0.72 |

Wild bootstrap standard errors of $\hat{\alpha}$ in brackets

[a] Nadaraya-Watson 1st stage nonparametric estimate

[b] Local linear smoother 1st stage nonparametric estimate

[c] quadratic parametric 1st stage estimate

[§] no result available (boundary solution, misshaped objective function)

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