Discussion Paper No. 02-15

Tax Progressivity and the Trade Union's Fallback-Option

Stefan Boeters

ZEW

Zentrum für Europäische
Wirtschaftsforschung GmbH

Centre for European
Economic Research

Discussion Paper No. 02-15

Tax Progressivity and the Trade Union's Fallback-Option

Stefan Boeters

Download this ZEW Discussion Paper from our ftp server:

ftp://ftp.zew.de/pub/zew-docs/dp/dp0215.pdf

Die Discussion Papers dienen einer möglichst schnellen Verbreitung von neueren Forschungsarbeiten des ZEW. Die Beiträge liegen in alleiniger Verantwortung der Autoren und stellen nicht notwendigerweise die Meinung des ZEW dar.

Discussion Papers are intended to make results of ZEW research promptly available to other economists in order to encourage discussion and suggestions for revisions. The authors are solely responsible for the contents which do not necessarily represent the opinion of the ZEW.

Non-Technical Summary

In models of firm-union wage-bargaining, a higher degree of labour income tax progressivity can contribute to wage moderation and thereby reduce unemployment. This is because tax progressivity alters the wage-employment trade-off for the union. With a high marginal tax rate, even a small increase in the consumer wage causes much higher labour costs and therefore large employment losses. This may induce the union to moderate its wage claims.

While a qualitative effect of this sort is generally accepted in the literature, its quantitative impact and relative importance as compared to possible countervailing effects (endogenous hours of work and human capital formation) is an open question. This paper shows via a simple numerical model that minor changes in the formulation of the fallback option of the union can change the quantitative effects of tax progressivity drastically. The paper compares two modelling variants: In the first one, there are no alternative employment options for the union members in other sectors of the economy. In the second one, workers are mobile between sectors and the probability of a job in another sector equals the overall employment rate.

The paper shows that the second modelling variant is preferable on two grounds: (1) With no alternative employment options, the effects of tax progressivity are unrealistically high. In effect, even slight increases in tax progressivity are sufficient to attain full employment. (2) In the model with no alternative employment options, we are bound to the conclusion that tax rates in a realistic range are Laffer-inefficient. It would thus be possible to increase tax revenue by cutting tax rates.

The modelling variant with alternative employment options does not suffer from these unrealistic features. However, it contains a slight logical inconsistency. The paper shows that eliminating this inconsistency by explicitly modelling the dynamics of labour market states does not alter the numerical results significantly.

Tax Progressivity and the Trade Union's Fallback-Option

Stefan Boeters*

Centre for European Economic Research, Mannheim

November 2001

Abstract

When we analyse the labour market consequences of labour tax reforms in a model of firm-union wage bargaining, minor changes in the formulation of the union's fallback option can have drastic effects. This paper compares two variants of the model in which either workers have no reemployment opportunity or the probability of employment in another sector is determined by the overall unemployment rate. It is argued both on analytical and numerical grounds that the second alternative is the only plausible one. This conclusion is confirmed by an explicit integration of workers' inter-sectoral mobility into the model.

Keywords: labour taxation, tax progression, tax reform, trade unions, unemployment, labour mobility

JEL Code: H 20, J 51

*Centre for European Economic Research, P.O. Box 10 34 43, D-68034 Mannheim, Germany, e-mail: boeters@zew.de. I am grateful to Wolfram Richter, Christian Keuschnigg, Peter Sørensen, Martin Jensen and participants of the 3rd IZA Summerschool in Buch 2000 for helpful suggestions.

1 Introduction

The taxation of labour features prominently among the factors that are held responsible for the high and persistent unemployment rates in most European countries. Besides the overall tax level, the structure of labour taxation has also received interest. A number of theoretical studies from the 1990s have shown that a higher degree of progressivity of the labour tax can contribute to reducing unemployment (Koskela/Vilmunen 1996, Holm/Koskela 1996, Sørensen 1997). These findings have been backed up by several empirical investigations that broadly confirm that tax progressivity is good for employment (Lockwood/Manning 1993, Holmlund/Kolm 1995, Aronson/Wickström/Brännlund 1997, Schneider 2000).

The labour market effect of tax progressivity can be analysed in different models of unemployment (see the comparative studies of Pissarides, 1998, and Sørensen, 1999). In this paper, I concentrate on the union-firm wage-bargaining model, which is often thought to be best suited to many European labour markets. In such a bargaining setting, the effects of tax progressivity are quite intuitive. The parties of the bargain face a wage-employment trade-off, which can be altered by the tax rates. The higher the marginal tax rate (which, at a given average tax rate, amounts to higher progressivity) the more "expensive" a higher wage in terms of employment is. Higher tax progressivity thus leads to wage moderation and higher employment.

In the literature, there are different ways of modelling the wage bargaining setting, but the qualitative findings about the effects of tax progression in this context seem to be robust. Usually, the choice of a certain specification of the union-firm bargain is not explicitly motivated. The details of the bargaining model are generally thought not to matter much for its overall behaviour. In this paper, I point to a modelling detail that, opon inspection, makes this view untenable. The feature of the models I focus on is the formulation of the union member's fallback option. The uniformity in the qualitative findings hides important differences both in the mechanismen of adjustment and in the quantitative reaction of the endogenous variables in a numerical simulation setting. In this paper, I compare two different formulations of the union's fallback option. The first rests on the assumption that union members who are not employed in the home sector of their union receive unemployment benefits with certainty. This formulation will be called "NA" (for "No Alternative employment") throughout the paper. Alternatively, in the second formulation, it is assumed that there is a certain possibility for union members not employed in the union's home sector to get work in another sector. I call this variant "IM" (for "Inter-sectoral Mobility"). The difference between the two variants of the model seems to be a minor detail, but in fact results in two totally distinct mechanisms of adjustment to tax rate changes. This has severe consequences. The paper shows that the NA variant of the model (no alternative employment possibilities) can be characterised in the following ways: (i) It reacts by far more sensitively to shifts in the tax rates than the IM variant. (ii) There is no limit for the employment boosting effect of tax reforms other than full employment. In fact, very small tax rate changes are sufficient to bring about full employment. (iii) The standard assumption of Laffer-efficiency of the tax rates cannot be met with parameter values within a reasonable range.

Given these implausible results of the NA variant of the model, it comes as no surprise that in numerical simulations of tax rate changes (Pissarides 1998, Sørensen, 1999) the IM variant (inter-sectoral mobility) is used. But this is usually done without explicitly discussing alternatives and without justifying the choice of one of them. However, it is important to compare the alternatives explicitly for two reasons: First, strictly considered, the IM variant is inconsistent: It equates the unconditional and a conditional probability of unemployment (see Section 5). It must be shown that this inconsistency can be tolerated. Second, the NA variant is widely used in analytical models that study the effects of tax rate changes (Koskela/Vilmunen 1996, Holm/Koskela 1996, Koskela/Schöb/Sinn 1998, Fuest/Huber 1999, Boeters/Schneider 1999, Marsiliani/Renström 2000). Given the defects of the NA variant demonstrated in this paper, the findings of that strand of the literature should be reconsidered.

As both the NA and the IM variant of the model are unsatisfactory in some respects, I round off the analysis of this paper by drawing on an explicit model of inter-sectoral worker mobility proposed by Layard/Nickell (1990). The resulting third variant of the model shows both analytically and numerically much more similarity to the IM variant than to its rival. Thus, the IM variant is supported and the doubt about the NA variant is confirmed.

The paper is organised as follows. Section 2 sets up a simple general framework of wage bargaining, which follows Holmlund/Kolm (1995), Marsiliani/Renström (1997) and Sørensen (1999). Both variants of the union's fallback option are formalised within this general framework. Section 3 derives basic comparative static effects with respect to the tax parameters and analytically demonstrates the difference between the two modelling variants. Section 4 uses numerical calculations to close some gaps that could not be decided analytically. In Section 5, I add an explicit formulation of inter-sector worker mobility to the model and compare the outcome to the two extreme cases of Sections 3 and 4. Section 6 concludes with a strong statement in favour of the IM variant of the model.

2 A simple model of wage bargaining

Consider an economy with a large number of symmetrical small sectors that interact through monopolistic competition of the Dixit-Stiglitz (1977) type. In each sector there is one firm and one union that bargain over the wage. In the following, I describe one representative sector; therefore, sector indices have been dropped.

The firm faces an output demand function of constant elasticity,

$$p = BQ^{-\frac{1}{\sigma}}, \tag{1}$$

where p is the output price, Q is the quantity produced, σ is the elasticity of output demand, which is derived from the household maximisation of a CES utility function with elasticity of substitution of σ over all product varieties (see Dixit/Stiglitz 1977 for details), and B is a variable that depends on the specific features of the monoplistic interaction (number of sectors and degree of competition) as well as on aggregate demand. Each firm is small as compared to the whole economy and treats B as a constant[1]. The firm's production is described by a decreasing returns-to-scale function

$$Q = AL^{\alpha}, \quad 0 < \alpha < 1, \tag{2}$$

with labour input L and a productivity parameter[2] A. The firm's profits are given by

$$\pi = pQ - wL, \tag{3}$$

[1] There is a separate appendix to this paper that shows the exact derivation of B and the closure of the model through the assumption of symmetrical sectors. This appendix is available on request.

[2] This parameter only serves to normalise the wage at unity later in the numerical calculations.

which, maximised subject to the output demand function, results in a mark-up rule. The output price is set proportionally to the marginal costs of production (see Appendix 1):

$$p = \frac{\sigma}{\sigma - 1} \frac{L^{1-\alpha}}{\alpha A} w. \tag{4}$$

The mark-up factor $\sigma/(\sigma - 1)$ will be denoted by m in the following. With $\sigma > 1$ (to ensure an interior maximum of the firm's profit), we have $m > 1$. The mark-up rule (4) can be interpreted as a labour demand function when it is solved for L as a function of the real wage, w/p.

On the labour market, the firm and a union bargain over the wage. I choose this right-to-manage variant of the bargaining model, that is, firms are free to adjust employment given the outcome of the wage bargain. Alternatively, one could consider a so-called "efficient bargaining model" where the bargaining also covers employment (see e.g. Layard/Nickell 1990 for a comparision of the two models). I stick to the right-to-manage model because bargaining over employment is rare in Europe, and the two variants of the bargaining model do not differ significantly in the context of labour tax progressivity (Sørensen 1999). The outcome of bargaining over the wage is modelled as the solution to the Nash-bargaining problem

$$\max_{w,L} \quad \left[(U - \bar{U})L\right]^{\lambda} \pi^{1-\lambda}, \tag{5}$$

subject to the labour demand function (4), where λ measures the bargaining power of the union. The firm aims at maximising its profit, π, as given in (3). Its alternative option is not to produce at all, which means zero profits. The union aims at maximising the expected income of its representative member[3]. U is the utility of an

[3]Like Pissarides (1998) and Koskela/Schöb/Sinn (1998), I abstract from the disutility of work, as this would not add essential features to the model.

employed union member, which is assumed linear in the after tax wage. The wage tax schedule is linear progressive with a marginal tax rate t and a tax exemption[4] of a. (t and a will be referred to as the tax parameters. I generally assume $0 < t < 1$ and $0 < a < w$.) Thus we have

$$U = \tilde{w} = (1 - t)w + ta. \tag{6}$$

The alternative option of the union members, $\bar{U}$, which forms the threat point of the union in the bargaining problem (5), has two components. On the one hand, a worker can find work in another sector, when there is no work available in the home sector. By the symmetry assumption, she then receives a wage that is identical to the wage in the home sector, $\bar{w} = \tilde{w}$. If no alternative job can be found, the worker receives unemployment benefits, b. I assume a fixed replacement rate, c, which means b is a constant fraction of the net-of-tax wage[5], $b = c\tilde{w}$. The respective probabilities of the two outcomes are denoted by $1 - z(u)$ and $z(u)$, which yields

$$\bar{U} = (1 - z(u))\bar{w} + z(u)b. \tag{7}$$

A survey of papers on wage bargaining in a public finance context shows that they nearly exclusively use one of two popular specifications of $z(u)$. A series of papers (Holm/Koskela 1996, Koskela/Vilmunen 1996, Koskela/Schöb/Sinn 1998, Koskela/Schöb 1999, Fuest/Huber 1999, Boeters/Schneider 1999, Daveri/Tabellini

[4]One might also consider the case in which the tax exemption is a fixed proportion of the wage, $a = \bar{a}w$. But observe that this formulation would run into problems. In equation (8) below, w would cancel out and in the NA variant of the model an equilibrium would only be possible as an improbable coincidence. Because of this, the case $a = \bar{a}w$ is not suited for a comparison of the two variants of the model.

[5]For the comparison with the case of an unemployment benefit that is fixed in absolute terms, see Pissarides (1998) or Koskela/Schöb (1999).

2000) assume z simply to equal 1. This means that there are no alternative employment possibilities, irrespective of the overall unemployment rate. This is the NA ("no alternative employment") variant of the model. Another extreme assumption (by Pissarides 1998, Sørensen 1999, Michaelis/Pflüger 2000) is that $z(u) = u$; that is, the probabiliy of finding a job elsewhere if not employed in the home sector exactly equals the overall unemployment rate ("IM" variant, "inter-sector mobility"). The general formulation (7) covers these two extreme cases together with possible intermediate ones. A similar formulation can be found in Layard/Nickell/Jackmann 1991: 101.

3 Comparative statics of tax rate changes

3.1 Wage and employment reactions

Solving the Nash-bargaining problem unter the normalisation $L = 1-u$ and subject to the labour demand function (Appendix 2) results in the following wage equation

$$\beta \frac{(1-t)w}{(1-t)w + ta} = z(u)(1-c), \tag{8}$$

where β is a positive constant derived from the parameters of the model: $\beta = \lambda(m-\alpha)/(\alpha + \lambda(m-\alpha))$. The fraction term on the LHS is one minus the marginal tax rate divided by one minus the average tax rate. This expression is known as the "coefficient of residual income progression" (CRIP) and will be used later to study the effects of tax rate changes. Together with labour demand,

$$w = \gamma(1-u)^{\alpha-1}, \tag{9}$$

where γ is again a positive constant, $\gamma = A\alpha/m$, (8) forms a system in the endogenous variables w and u, which describes the economy. The comparative statics of this system crucially depends on the sign of the Jacobian matrix, which cannot unambiguously be determined in the general case. The crucial term to be signed is (see Appendix 3):

$$J = (1-c)\left[\frac{taz(1-\alpha)}{1-u} - z'(u)\left[w(1-t)+ta\right]\right]. \tag{10}$$

Given J, the effects of a partial variation of one of the tax parameters, t and a, on unemployment can be determined as (see Appendix 4):

$$\begin{aligned} \frac{du}{dt} &= J^{-1}\frac{az(1-c)}{(1-t)} \\ \frac{du}{da} &= J^{-1}zt(1-c) \end{aligned}$$

Let us consider the two extreme cases of the previous section in turn. In variant NA, when there are no alternative employment possibilities ($z = 1$, $z' = 0$), the second term in (10) drops out and J is positive, given that $a, t > 0$. This means that a rise in both t and a results in higher unemployment. This finding does not lend itself to an easy interpretation in terms of tax progressivity. If z is constant, then (8) consists only of constants, except for the CRIP, the fraction term on the LHS. The only remaining endogenous variable in (8), w, has to adjust when one of the tax parameters changes to keep CRIP constant[6]. So if we take CRIP as our measure of tax progressivity, all tax structures (combinations of a and t) exhibit the same degree of progressivity *in their respective equilibrium.* But if we keep w constant, a rise in both t and a will increase the degree of progressivity. Thus the

[6]If indirect progressivity of the tax schedule was generated by a fixed transfer, d, instead of a tax exemption (i.e. $\tilde{w} = (1-t)w + d$), the after-tax wage, $\tilde{w}$, would be fixed by (8). A fixed after-tax wage is an important benchmark result of Koskela/Schöb/Sinn (1998), which can only arise in variant (1) of the model.

entire tax schedule can be called "more progressive". Under a linear progressive tax schedule, progressivity decreases (CRIP increases) with income. Therefore, when tax parameter changes make the tax schedule more progressive, the gross-of-tax wage must rise for CRIP to remain constant. This, in turn, leads to higher unemployment.

As employment in the NA variant of the model is only detemined through the labour demand function, there is no reason why tax policy (higher degree of progressivity) should *not* be capable of attaining full employment. In fact, the numerical calculations in Section 4 will show that even small changes in the tax parameters can make unemployment completely disappear.

Note that it would be impossible to analyse the above effects if the tax schedule had been restricted at the outset to have constant CRIP (as in Holmlund/Kolm 1995 or Sørensen 1999). If this were the case, only one of the remaining schedules would be compatible with (8), and every variation of the tax system would result in a corner solution ($u = 1$ or $u = 0$)[7].

Consider, by contrast, variant IM of the model, where $z(u) = u$ and $z' = 1$. Then in (10) the second term on the RHS enters and renders the sign of J ambiguous. However, for usual calibration values of α and u, we can determine the sign of J. In the IM variant, we have

$$J = (1-c)\left[ta\frac{u-\alpha}{1-u} - w(1-t)\right]. \tag{11}$$

In order for J to be negative, it is sufficient that $u < \alpha$, which can reasonably be assumed.

The change in the sign of J means that all comparative static effects are also reversed. In particular, both a higher t and a higher a in the IM variant lead to *lower*

[7]To show this rigorously, system (8,9) would have to be recast as a system of inequalities, of course.

unemployment. If we again adopt the interpretation of tax progressivity in terms of CRIP, we now have the well-known effects that higher progressivity (through higher t or a) boosts employment. This is because a second channel of transmission enters in (8) and dominates the first one, which governed the NA variant. Now u appears (through z) directly in (8), and a lower CRIP (given the change in w does not overcompensate) corresponds to a lower u.

This change in the transmission channel also has as a consequence that full employment cannot be attained by the tax policy under normal circumstances. With $z(u) = u$ full employment ($u = 0$) would mean that the RHS of (8) becomes zero. For (8) to hold, t would have to be set at 100% (given the restrictions on t and a and $w > 0$). This amounts to an unrealistically extreme tax schedule, where the government actually fixes the after-tax wage at a and confiscates any part of the wage that exceeds this amount. Besides being unrealistic, this would not make any sense in the bargaining context because it deprives the union of any incentive to bargain for higher wages.

3.2 Laffer efficiency

Laffer-efficiency of the respective tax rates is an important criterion to assess the overall plausibility of the two variants of the model. In fact, the assumption of Laffer-efficiency is often used to derive clear-cut analytical results (e.g. by Koskela/Vilmunen 1996: 75). If taxes were actually Laffer-inefficient, the prescriptions for tax policy would be blatantly simple: cut taxes, raise thereby tax revenue, and boost employment at the same time. This does not seem to be a realistic scenario.

However, Laffer-efficiency is an ambiguous concept if the government's budget depends on more than a single tax. It then has to be specified which measure of tax revenue is to be considered. For the model of this paper, the specification is important in two respects. First, there must be a decision on whether to focus on the revenue from labour taxation alone or to include the revenue from possible other taxes. In line with Sørensen (1999) or Fuest/Huber (1999) and the prescip-tions of optimal taxation literature, but contrary to Koskela/Vilmunen (1996) or Koskela/Schöb/Sinn (1998), I will assume that there is a tax on profits and that the question of the optimal design of distortionary taxes only arises if profits are fully taxed away. Second, there must be a decision on how the public expenses for the unemployment benefit are to be treated. Anticipating the welfare analysis of the following section, it is most reasonable to define government's tax revenue net of unemployment benefit payments, G, as

$$G = t(w - a)(1 - u) + \pi - uc\left[(1 - t)w + ta\right]. \tag{12}$$

G is that part of tax revenue that can be spent for publicly provided goods. A tax reform that is "revenue-neutral" in the sense of keeping G constant is natural as a basis for a welfare assessment because in this case the publicly provided goods can be ignored, and we do not have to bother about the relative preference of the household for these goods.

Analytical expressions for the impact of changes in the tax parameters, t and a, on G are supplied in Appendix 4. Only in some cases can the sign of these expressions be determined unambigously. In other cases, there are offsetting effects that have to be weighted numerically. What we can say qualitatively is that if a tax parameter change has large negative employment effects, tax revenue is likely to be reduced. Through its influence on the employment effects, both the sign and the value of the Jacobian determinant become crucial for assessing the Laffer-efficiency of the tax parameters, too. The numerical calculations of Section 4 actually show that the Laffer-properties of the marginal tax rate, t, are reversed in the IM variant, as compared to the NA variant.

3.3 Welfare

So far, the behaviour of the economy with respect to tax changes has only been described in terms of the wage, unemployment and tax revenue. However, to assess the desirability of a tax reform, it is also necessary to have a welfare measure. As utility of the workers is assumed linear in the after-tax wage, which amounts to risk-neutrality, the expected after-tax income of a worker is an appropriate indicator. On the one hand, tax changes will influence the after-tax wage both directly and indirectly through the labour costs, w. On the other hand, the rate of unemployment adjusts, which changes the weights that determine the value of the expected income. Expected income is

$$Y = [1 - u(1 - c)] [(1 - t)w + ta] . \tag{13}$$

Appendix 5 shows that the expressions we get for changes in expected income as a response to the variation of a single tax parameter are similar to those for the tax revenue. This means that we again have cases that can be determined analytically, while others need a numerical calculation.

What we are really interested in, however, is not the consequences of isolated tax parameter changes, but those of a revenue-neutral tax reform. Only if the amount of publicly provided goods remains constant (that is, if G, as defined in (12), is unchanged), can (13) be used as a welfare measure. In that case we have to weight the effects of the partial variation of the tax parameters with their respective revenue effects, which have been computed in Section 3.2:

$$\left.\frac{dY}{dt}\right|_{G=const} = \frac{dY}{dt} - \frac{dY}{da}\frac{dG}{dt}\bigg/\frac{dG}{da}. \tag{14}$$

In Appendix 5, it is shown that (14) can considerably be simplyfied to only depend qualitatively on du/da and dG/da. However, even the sign of this simplified version can not be determined analytically. Again, it is necessary to turn to a numerical specification of the model.

4 Numerical simulation

Both revenue and welfare effects of tax parameter changes can only partly be determined analytically. This section presents a numerical version of the model, which allows us to also get a feeling for the quantitative effects that are caused by tax policy. To generate results that are comparable to other numerical studies, I adopt the basic calibration parameters from Sørensen (1999): $m = 2.1$, $\alpha = 0.7$, $c = 0.6$, and for the initial equilibrium $u = 0.1$ at an average tax rate of $t^a = t(1 - \frac{a}{w}) = 0.55$. Whereas Sørensen calibrates the initial equilibrium for a proportional tax system ($a = 0$), I assume that there is already some indirect progression in the baseline situation[8] due to a tax exemption that equals one fifth of the gross wage: $a = 0.2 \cdot w$. Furthermore, I normalise the wage, w, at unity in the initial equilibrium.

We start with the NA case (no alternative employment possibilities, $z = 1$). Here, we get the following parameter values for the calibrated initial equilibrium: $\lambda = 0.679$, $B = 2.907$, $t = 0.688$ and $g = 2.268$. As suggested by the analytical calculations, tax rate changes in this case have drastic consequences. Table 1 reports

[8] Remember that with a proportional tax, the wage equation (8) in variant NA could only hold by chance.

the effects of a partial variation of one of the tax parameters on the wage, unemployment and the tax revenue. The qualitative results are already known from Section 3: A cut in t lowers unemployment, which is responsible for increasing tax revenue (Laffer-inefficiency of t, Table 1a). The same holds for a cut in the tax exemption, a: Unemployment goes down and tax revenue rises (Table 1b). But what strikes most in both panels of Table 1 are the quantitative reactions. Even very small changes in the tax parameters have enormous consequences for employment. A one percentage point reduction in unemployment can be brought about by either lowering t from 68,75 to 68,68 per cent, or a from 0.2 to 0.1993. Consequently, full employment[9] can be attained by changes in the tax parameters that are hardly noticeable: t at 68.07 per cent or a at 0.1938. Considering a revenue-neutral tax reform (Table 2), these extreme results are only slightly attenuated, because the tax changes now work in opposite directions. To keep tax revenue constant, a cut in t (which leads to lower u) must be complemented by a higher a (which raises u). Nevertheless, even in this case full employment can be attained at relatively small changes of the tax rates: $t = 54$ per cent and $a = 0.36$. These findings do not seem promising at all as a plausible simulation of a real economy. Let us turn now to the second extreme case and see whether it has advantages in this respect.

In the IM variant of the model, we have a probability of alternative employment of $1 - u$. The baseline calibration values result in the same initial equilibrium as in variant NA, except for λ, which is now 0.031. Table 3 shows the impact of a partial variation of t and a, respectively, on the wage, unemployment, and the tax revenue. We find the analytical results from Section 3 illustrated: An increase in

[9]The last rows of Tables 1 and 2 are calculated without taking into account the labour supply restriction $u \geq 0$. The values of the endogenous variables are set in parentheses. They show that the labour supply restriction must be explicitly added to the model in order to prevent negative unemployment rates.

t lowers unemployment and generates additional tax revenue. An increase in the tax exemption, a, also leads to lower unemployment, but now this means less tax revenue. Gross wage reactions are rather small, while unemployment is moderately affected by tax changes. The range of the quantitative reactions is now broadly in line with the results of similar simulation exercises by Pissarides (1998) and Sørensen (1999). To drive unemployment down by one percentage point, an increase in t from 68.8 to 74.9 per cent or an increase in a from 0.20 to 0.27 is necessary[10], both of which are significant changes.

Turning to the simulation of a revenue-neutral tax reform, we see that changes in both tax parameters reinforce one another (Table 4). A rise in t is now complemented by a rise in a to compensate for the loss in tax revenue. Both tax parameter changes, however, work in the direction of higher employment. Thus, the employment gains that can be achieved with a revenue neutral tax reform are higher than those attainable with a variation of only one tax parameter. Under these circumstances, a one percentage-point reduction in unemployment is brought about by raising t to 70.5 per cent and a to 0.25.

5 Modelling inter-sector mobility

Both variants of the model examined so far exhibit serious drawbacks when considered in more detail. This is obvious for the NA case (no alternative employment opportunities). Why should the workers be bound to stay in their home sector even if unemployment is very low and labour becomes scarce in the economy as a whole? This variant of the model only can be defended under the following interpretation:

[10]As the gross wage remains practically at unity, the absolute changes in a can also be read in terms of percentage points.

Think of the union as a national monopoly in a small open economy. This economy is fully integrated in international product markets, but labour is completely immobile. This interpretation of the model is only acceptable in countries with centralised wage bargaining, which is only the case for some of the European countries (see the classification of European countries according to their labour market regime in Daveri/Tabellini 2000). But even under this interpretation, we are left with the grossly implausible simulation results of Section 4.

The IM variant of the model, in which the probability of getting a job elsewhere equals the employment rate, seems more promising at first – but it suffers from a logical inconsistency. Setting $z = u$ amounts to equalising the *unconditional* probability of being employed and the *conditional* probability of getting a job, given that in the home sector no job is available. Elementary theorems of Probability Theory suffice to show that this equlity can only hold if no-one gets a job in their own sector[11]; but this does not make economic sense at all.

To avoid these defects of the two extreme cases, I draw on an explicit model of inter-sectoral worker mobility by Layard/Nickell (1990): Assume that the wage

[11] In formal terms, the probability of being unemployed (U) can be decomposed:

$$P(U) = u = P(U|HE)P(HE) + P(U|NHE)P(NHE),$$

where "HE" and "NHE" stand for "(No) Home Employment". We have $P(U|HE) = 0$ by definition; thus

$$u = P(U|NHE)P(NHE).$$

$P(U|NHE) = u$, which is assumed in the IM model, can then only hold if $P(NHE) = 1$. This argument is independent of whether we deal with small or large sectors.

The intuition behind the formal result is equally simple: If at least some of the jobs are assigned in a first round to home sector workers, the employment prospects of those who weren't successful in that first round must be worse than in average.

bargaining is repeated every period. There is an exogenous probability, s, that a working contract is terminated and the worker leaves the firm. Every worker that loses her job joins a homogenous unemployment pool where she faces an exit probability, a, which is endogenously determined by the steady state assumption. Given that an individual sector is small compared to the whole economy, it is almost certain that the next employment will be in another sector. Under these assumptions, we can formulate the following arbitrage equations for the value of a job in the home sector, V; in one of the other sectors, $\tilde{V}$; and of the state of unemployment, $\bar{V}$.

$$\begin{aligned} V_t &= (1+r)^{-1}\left[\tilde{w} + s\bar{V}_{t+1} + (1-s)V_{t+1}\right], \\ \tilde{V}_t &= (1+r)^{-1}\left[\bar{w} + s\bar{V}_{t+1} + (1-s)\tilde{V}_{t+1}\right], \\ \bar{V}_t &= (1+r)^{-1}\left[b + a\tilde{V}_{t+1} + (1-a)\bar{V}_{t+1}\right], \end{aligned}$$

where r is the interest rate for discounting. In the steady state, $a = s(1-u)/u$, $V_t = V_{t+1} = V$, $\bar{V}_t = \bar{V}_{t+1} = \bar{V}$, and $\tilde{V}_t = \tilde{V}_{t+1} = \tilde{V}$.[12] This allows us to compute the following steady state value of the union's bargaining position ($V - \bar{V}$ must be substituted for $U - \bar{U}$ in (5) when we change from the one-period to a many-periods context):

$$V - \bar{V} = \frac{1}{r+s}\left[\tilde{w} - (1 - z(u))\bar{w} - z(u)b\right], \quad \text{with} \quad z(u) = \frac{(r+s)u}{ru+s}.$$

We thus have determined a value for z that exceeds u (as $r + s > ru + s$) but is below unity. We actually have found an intermediate case. With which of the

[12] This is one of two cases Layard/Nickell (1990) consider. They label it "infinit bargain", i.e. the wage bargained is valid for all future periods. There is a second variant, "one period bargain", which assumes that the wage is bargained over in every period and the union cannot commit to a wage in the future. In this case, $\tilde{V}_t = \tilde{V}_{t+1} = V_{t+1}$. The consequences of that change are insignificant in the context of my paper.

extreme cases are we to expect this case to have more similarities? This depends on the specification of the two new parameters, r and s. Obviously, the Layard/Nickell ("LN") case will converge to the NA case when s is close to zero or r is very large. (In the first situation, there is hardly any fluctuation between employment and unemployment; in the second situation, the union does not care about employment in future periods.) Conversely, the LN case converges to the IM case ($z = u$) if $r \to 0$, in which case only the future employment in other sectors is relevant in the long run. If we take the time period to be one year, reasonable parameter values are $r = 0.05$ and $s = 0.2$. With an unemployment rate of $u = 0.1$ this results in a situation that is very close to the IM assumption $z = u$. We then have $z = 1.22 \cdot u$ and $z' = 1.19$, which is not significantly different from unity. While these are only loose plausibility considerations, a numerical simulation in line with Section 4 confirms that the LN variant of the bargaining model is very similar to the IM case. In fact, the differences between the numerical simulation of this case and Tables 3 and 4 are so small that I do not report them here. [But for comparison in the preliminary draft, see Tables 5 and 6.] The LN model thus strongly backs up the IM variant of the model while being in sharp contrast to the NA variant[13].

6 Summary and Conclusions

What we know about labour market effects of counterfactual tax structures will always be based on a combination of empirical estimations and numerical simulations. It is therefore important to know which factors determine the outcome of the models that are used to perform the numerical simulations. Even the simple models used

[13] This justifies Pissarides's (1998, 163) practice to use the IM variant "as an approximation and to avoid an explicitly dynamic model."

in papers such as Pissarides (1998) or Sørensen (1999) can be modified in so many respects that it is hardly possible to keep track of all the combinations: the number of factors of production, the specification of the production function, small open economy vs. closed economy, the parameterisation of the tax schedule, objective function of the union, right-to-manage vs. efficient bargaining, taxation of profits, inclusion of benefit payments in the government's budget. The best we can hope is that the model we actually use produces plausible results that are robust with respect to modifications of some of the model's key characteristics.

This paper focuses on one modelling detail that does not seem to be of much significance at first sight, since different specifications are chosen without justifying them against alternative models in different papers . However, it turns out that this detail dramatically alters the behaviour of the model, both analytically and numerically. The differences between the NA variant of the model (where there are no alternative employment options for the single worker) and the IM variant of the model (where the reemployment probability in another sector is given by the employment rate of the whole economy) are listed below.

- First and foremost, there are two different mechanisms of adjustment that govern the effect of the tax parameters on the endogenous variables of the model. In the NA variant, only the wage, not unemployment, appears in the wage equation. This means that the wage must adjust so that the progressivity of the tax system at the adjusted wage remains constant. The wage, in turn, determines employment and unemployment through the labour demand curve. In the IM variant, in contrast, there is a second mechanism of adjustment which dominates the first. Here unemployment directly appears in the wage equation because it influences the alternative option of the workers. Adjustment now takes place mainly through unemployment as the weight of the unemployment benefit in the alternative option.

- These different mechanisms analytically result in vastly diverging labour market reactions of the model, which most prominently are indicated by the change in sign of the Jacobian matrix.

- This, in turn, strongly bears on the labour market and tax revenue effects of tax parameter changes. When plausible parameter values are chosen in the NA variant, the marginal tax rate is bound to be Laffer-inefficient, and full employment can easily be attained by tax reforms that only slightly alter the tax parameters. In the IM variant, however, taxes are Laffer-efficient and even large variations in the tax parameters can only partly remove unemployment.

While there is no analytical proof that would force us to dismiss the NA variant of the model, there are several plausibility considerations that, taken together, are strong grounds for favouring the IM variant:

- The NA variant exhibits labour market reactions, the implied optimism of which can hardly be defended against the experience with real tax reforms (very small tax rate changes are sufficient to bring about full employment).

- If we adopt the criterion for calibration that taxes must be Laffer-efficient, this forces us to use the IM variant, because the marginal tax rate is Laffer-inefficient in NA due to the large positive employment effects of tax rate cuts.

- An explicit model of inter-sector labour mobilitiy turns out to have a structure very similar to IM, while it differs sharply from NA.

Given these arguments in favour of a model that allows for labour mobility, it seems necessary to reconsider the results of papers that heavily rely on a model structure that includes the assumption of no alternative employment possibilities. Any model implying that hardly noticeable changes in the tax parameters can bring about full employment should not be taken seriously.

In any case, we have seen that minor variations in the formulation of the labour market model can significantly influence the employment effects of tax parameter changes. With regard to the alternative option of union members, the focus of this paper, we have encountered forceful arguments in favour of a model version that explicitly accounts for inter-sectoral labour mobility. Out empirical knowledge about the extent to which workers actually change between sectors may then well become an important determinant for the overall performance of models of unionised labour markets.

7 Appendix

7.1 Firm's profit maximisation

Firms maximise profits

$$\pi = pQ - wL,$$

where

$$p = BQ^{-\frac{1}{\sigma}} \tag{15}$$

and

$$Q = AL^{\alpha}. \tag{16}$$

This gives

$$\frac{d\pi}{dL} = BA^{\frac{\sigma-1}{\sigma}}\frac{\alpha(\sigma-1)}{\sigma}L^{\alpha\frac{\sigma-1}{\sigma}-1} - w = 0.$$

Substituting (15) and (16) results in (4) in the main text.

7.2 Solution to the Nash-bargaining problem

The Nash bargaining problem is

$$\begin{aligned}\max_{w,L} \quad & \left[(U-\bar{U})(1-u)\right]^{\lambda}\pi^{1-\lambda} \\ & = \left\{\left[(1-t)w + ta - \bar{U}\right](1-u)\right\}^{\lambda}\left[pQ - w(1-u)\right]^{1-\lambda} \\ & \text{s.t.} \quad w = BA^{\frac{1}{m}}\frac{\alpha}{m}(1-u)^{\frac{\alpha-m}{m}}.\end{aligned}$$

Taking logs on (17) and differentiating gives

$$\frac{\lambda(1-t)}{z(u)(1-c)\left[(1-t)w+ta\right]} - \frac{(1-\lambda)\alpha}{(m-\alpha)w} - \mu = 0, \tag{18}$$

$$\frac{\lambda}{(1-u)} - \mu\frac{m-\alpha}{m}\frac{w}{(1-u)} = 0, \tag{19}$$

where μ is the Lagrange multiplier of the restriction and use has been made of

$$U - \bar{U} = z(u)(1-c)\left[(1-t)w + ta\right]$$

and

$$\pi = \frac{m-\alpha}{\alpha}w(1-u).$$

Solving (19) for μ and inserting into (18) gives the wage equation (8) in the main text.

7.3 Comparative statics of tax parameter changes

The system (8), (9) is characterised by the following comparative statics:

$$\begin{bmatrix} -(1-t)\left[\beta - z(1-c)\right] & z'(1-c)\left[(1-t)w + ta\right] \\ 1 & \gamma(\alpha-1)(1-u)^{\alpha-2} \end{bmatrix} \begin{bmatrix} dw \\ du \end{bmatrix} =$$

$$= \begin{bmatrix} -\beta w + z(w-a)(1-c) & -zt(1-c) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} dt \\ da \end{bmatrix},$$

which, using (8) and (9), can be simplified to

$$\begin{bmatrix} -\frac{taz(1-c)}{w} & z'(1-c)\tilde{w} \\ 1 & -\frac{w(1-\alpha)}{1-u} \end{bmatrix} \begin{bmatrix} dw \\ du \end{bmatrix} = \begin{bmatrix} -\frac{az(1-c)}{1-t} & -zt(1-c) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} dt \\ da \end{bmatrix}.$$

The Jacobian determinant, J, of this equation is

$$J = (1-c)\left[\frac{taz(1-\alpha)}{1-u} - z'\left[w(1-t) + ta\right]\right] \tag{20}$$

as given in the main text. Its sign crucially depends on the specification of $z(u)$. But given the sign of J for a certain specification of z and certain values of the parameters, we have the following comparative static effects:

$$\begin{aligned} \frac{dw}{dt} &= J^{-1}\frac{az(1-c)w(1-\alpha)}{(1-t)(1-u)} \\ \frac{du}{dt} &= J^{-1}\frac{az(1-c)}{(1-t)} \\ \frac{dw}{da} &= J^{-1}\frac{zt(1-c)w(1-\alpha)}{(1-u)} \\ \frac{du}{da} &= J^{-1}zt(1-c) \end{aligned}$$

where all the terms on the RHSs besides J^{-1} are unambiguously positive.

7.4 Laffer-efficiency of taxes

Given full taxation of profits, the government's budget can be expressed as

$$G = \left(t^a \frac{\alpha}{m} + \frac{m-\alpha}{m}\right) pQ - ucw(1-t^a),$$

with t^a denoting the average tax rate $t^a = t(1-a/w)$. Substitution for p and Q gives

$$G = w\left[(t^a + \frac{m-\alpha}{\alpha})(1-u) - uc(1-t^a)\right]. \tag{21}$$

Totally differentiating with respect to w, u and t^a, and using the relation between $dw/d\tau$ and $du/d\tau$ that results from the labour demand function, we have for the tax parameters $\tau = a, t$:

$$\frac{dG}{d\tau} = w\left\{[1-u(1-c)]\left(\frac{\partial t^a}{\partial \tau} + \frac{\partial t^a}{\partial w}\frac{dw}{d\tau}\right) - \left[m - (1-t^a)\left(\alpha - c\frac{1-u\alpha}{1-u}\right)\right]\frac{du}{d\tau}\right\}. \tag{22}$$

The total revenue effect of a tax parameter change is decomposed into a direct effect, depending on $dt^a/d\tau = \partial t^a/\partial\tau + \partial t^a/\partial w \cdot dw/d\tau$, and an indirect effect that works through unemployment changes triggered by the tax policy. Rising unemployment reduces the revenue generated by a tax parameter change. (The term in the second square brackets in (22) is positive because $m > a$ and $0 < t^a < 1$.) Tax parameters are "Laffer-efficient" if $dG/dt > 0$ or $dG/da < 0$, respectively. For the two variants of the model we have

case	$dt^a/d\tau$	$-du/d\tau$	sum	efficiency
NA: a	0	< 0	< 0	L-efficient
NA: t	> 0	< 0	?	(L-inefficient)
IM: a	< 0	> 0	?	(L-efficient)
IM: t	(> 0)	> 0	> 0	L-efficient

In the cases **NA**: t and **IM**: a, the two partial effects of $dt^a/d\tau$ work in the same direction. However, $du/d\tau$ has the opposite sign so that the overall effect has to be determined numerically. (The entries in the "efficiency" column that are in parentheses are the results of the numerical calculations in Section 4.) In the cases **NA**: a and **IM**: t, in contrast, the two partial effects of $dt^a/d\tau$ offset one another. In **NA**: a they exactly cancel out; in **IM**: t, however, the overall effect depends on the calibration values of the parameters. The sign given in parentheses for that case depends on $(w-a)(1-t)(1-u) - atu(1-\alpha) > 0$, which holds for the parameter values used in the numerical calculations. The indirect effect has the same sign so that in these cases the overall effect can be determined analytically.

7.5 Welfare effects

Welfare is measured by the expected income of the representative worker:

$$Y = [1 - u(1-c)]\,(1-t^a)w.$$

We thus have for tax parameter $\tau = t, a$

$$\frac{dY}{d\tau} = [1-u(1-c)]\left((1-t^a)\frac{dw}{d\tau} - w\frac{dt^a}{d\tau}\right) - (1-c)(1-t^a)w\frac{du}{d\tau} \tag{23}$$

Using the results from Appendix 3, we can arrive at expressions similar to (22)

$$\frac{dY}{d\tau} = -w\left\{[1-u(1-c)]\left(\frac{\partial t^a}{\partial \tau} + \frac{\partial t^a}{\partial w}\frac{dw}{d\tau}\right) - \frac{1-t^a}{1-u}\,[c-\alpha+\alpha u(1-c)]\,\frac{du}{d\tau}\right\}. \tag{24}$$

The similarity between (22) and (24) can be used to evaluate (14). Tedious but straightforward calculations show that the welfare effect of a tax reform can be expressed as

$$\left.\frac{dY}{dt}\right|_{G=const} = mw^2\,[1-u(1-c)]\,\frac{1-t^a}{1-t}\frac{du/da}{dG/da}$$

We know that $dG/da < 0$ (a is Laffer-efficient) in both variants of the model. The welfare effect of a tax reform is then directly related to the partial effect of α on the unemployment rate. Appendix 3 shows that $du/da < 0$ holds in the IM variant of the model, but not in the NA variant. Thus in the first variant, but not in the second, a tax reform that raises the marginal tax rate has positive welfare effects. This analytical result is in line with the numerical calculations reported in Tables 2 and 4.

References

[1] Aronson, Thomas; Wickström, Magnus; Brännlund, Runar (1997): Wage determination under non-linear taxes: Estimation and an application to panel data, Oxford Economic Papers 49, 404-418

[2] Boeters, Stefan; Schneider Kerstin (1999): Government versus union. The structure of optimal taxation in a unionized labor market, Finanzarchiv 56, 174-187

[3] Daveri, Francesco; Tabellini, Guido (2000): Unemployment, growth and taxation in industrial countries, Economic Policy 28, 49-104

[4] Dixit, Avinash; Stiglitz, Joseph E. (1977): Monopolistic competition and optimum product diversity, American Economic Review 67, 297-308

[5] Fuest, Clemens; Huber, Bernd (1999): Tax coordination and unemployment, International Tax and Public Finance 6, 7-26

[6] Holm, Pasi; Koskela, Erkki (1996): Tax progression, Structure of Labour Taxation and Employment, Finanzarchiv 53, 28-46

[7] Holmlund, Bertil; Kolm, Ann-Sofie (1995): Progressive taxation, wage setting and unemployment: theory and Swedish evidence, Swedish Economic Policy Review, 38, 285-303

[8] Koskela, Erkki; Schöb, Ronnie (1999): Alleviating unemployment: The case for green tax reforms, European Economic Review 43, 1723-1746

[9] Koskela, Erkki; Schöb, Ronnie; Sinn, Hans-Werner (1998): Pollution, factor taxation and unemployment, International Tax and Public Finance 5, 379-396

[10] Koskela, Erkki; Vilmunen, Jouko (1996): Tax progression is good for employment in popular models of trade union behaviour, Labour Economics 3, 65-80

[11] Layard, Richard; Nickell, Stephen (1990): Is unemployment lower if unions bargain over employment? Quarterly Journal of Economics 105, 773-87

[12] Layard, Richard; Nickell, Stephen; Jackman, Richard (1991): Unemployment: Macroeconomic Performance and the labour market, Oxford

[13] Lockwood, Ben; Manning, Alan (1993): Wage setting and the tax system: theory and evidence for the United Kingdom, Journal of Public Economics 52, 1-29

[14] Marsiliani, Laura; Renström, Thomas I. (2000): Imperfect competition, labour market distortions and the double dividend hypothesis, Fondazione Eni Enrico Mattei, Nota di Lavoro 11/2000

[15] Michaelis, Jochen; Pflüger, Michael (2000): The impact of tax reforms on unemployment in a SMOPEC, Journal of Economics (Zeitschrift für Nationalökonomie) 72, 175-201

[16] Pissarides, Christopher A. (1998): The impact of employment tax cuts on unemployment and wages; The role of unemployment benefits and tax structure, European Economic Review 42, 155-183

[17] Richter, Wolfram F.; Schneider, Kerstin (2001): Taxing mobile capital with labor market imperfections, International Tax and Public Finance, forthcoming

[18] Schneider, Kerstin (2000): Union wage setting and progressive income taxation with heterogenous labour: Theory and evidence from the German income tax reforms 1986-1990. Working Paper, University of Dortmund

[19] Sørensen, Peter B. (1997): Public Finance Solutions to the European Unemployment Problem?, Economic Policy 25, 223-264

[20] Sørensen, Peter B. (1999): Optimal tax progressivity in imperfect labour markets, Labour Economics 6, 435-452

Tax rate	Unemployment	Wage	Tax revenue	CRIP
0.688	0.100	1.000	2.268	0.694
0.686	0.079	0.993	2.301	0.694
0.684	0.050	0.984	2.366	0.694
0.682	0.020	0.975	2.423	0.694
(0.680)	(-0.010)	(0.966)	(2.481)	(0.694)

Table 1a: Partial variation of t with a constant at 0.2
(no alternative employment opportunities, z = 1)

Tax exemption	Unemployment	Wage	Tax revenue	CRIP
0.200	0.100	1.000	2.268	0.694
0.198	0.069	0.990	2.331	0.694
0.196	0.037	0.980	2.396	0.694
0.194	0.004	0.970	2.463	0.694
(0.192)	(-0.031)	(0.960)	(2.532)	(0.694)

Table 1b: Partial variation of a with t constant at 0.688
(no alternative employment opportunities, z = 1)

Tax rate	Tax exemption	Unemployment	Wage	Welfare change
0.688	0.200	0.100	1.000	0.000
0.650	0.235	0.075	0.992	0.112
0.610	0.277	0.047	0.983	0.254
0.570	0.324	0.020	0.975	0.386
(0.530)	(0.377)	(-0.009)	(0.966)	(0.519)

Table 2: Revenue-neutral tax reform (g constant at 2.268)
(no alternative employment opportunities, z = 1)

Tax rate	Unemployment	Wage	Tax revenue	CRIP
0.688	0.100	1.000	2.268	0.694
0.700	0.098	0.999	2.281	0.682
0.800	0.080	0.993	2.395	0.554
0.900	0.051	0.984	2.530	0.353
0.990	0.007	0.971	2.686	0.047

Table 3a: Partial variation of t with a constant at 0.2
(alternative employment opportunities with $z = u$)

Tax exemption	Unemployment	Wage	Tax revenue	CRIP
0.200	0.100	1.000	2.268	0.694
0.300	0.087	0.996	2.229	0.601
0.400	0.076	0.992	2.182	0.530
0.500	0.068	0.990	2.131	0.473
0.600	0.061	0.988	2.078	0.428

Table 3b: Partial variation of a with t constant at 0.688
(alternative employment opportunities with $z = u$)

Tax rate	Tax exemption	Unemployment	Wage	Welfare change
0.688	0.200	0.100	1.000	0.000
0.700	0.236	0.093	0.998	0.035
0.800	0.432	0.052	0.985	0.230
0.900	0.554	0.024	0.976	0.367
0.990	0.631	0.002	0.970	0.468

Table 4: Revenue-neutral tax reform (g constant at 2.268)
(alternative employment opportunities with $z = u$)

Tax rate	Unemployment	Wage	Tax revenue	CRIP
0.688	0.100	1.000	2.268	0.694
0.700	0.098	0.999	2.281	0.682
0.800	0.079	0.993	2.396	0.554
0.900	0.050	0.984	2.531	0.353
0.990	0.007	0.971	2.686	0.047

Table 5a: Partial variation of t with a constant at 0.2
(explicit inter-sector mobility with $z = (r+s)u/(ru+s)$)

Tax exemption	Unemployment	Wage	Tax revenue	CRIP
0.200	0.100	1.000	2.268	0.694
0.300	0.086	0.995	2.229	0.601
0.400	0.076	0.992	2.183	0.530
0.500	0.068	0.989	2.132	0.474
0.600	0.061	0.987	2.078	0.428

Table 5b: Partial variation of a with t constant at 0.688
(explicit inter-sector mobility with $z = (r+s)u/(ru+s)$)

Tax rate	Tax exemption	Unemployment	Wage	Welfare change
0.688	0.200	0.100	1.000	0.000
0.700	0.237	0.093	0.998	0.036
0.800	0.434	0.051	0.984	0.234
0.900	0.555	0.023	0.976	0.369
0.990	0.631	0.002	0.970	0.468

Table 4: Revenue-neutral tax reform (g constant at 2.268)
(explicit inter-sector mobility with $z = (r+s)u/(ru+s)$)

General Equilibrium in the Monopolistic Competition Model

separate Appendix to
"Tax Progressivity and the Trade Union's Fallback Option"

Stefan Boeters, November 2000

One aggregate (or representative) household is characterised by the CES utility function

$$u^h = \left(\sum_{i=1}^{n} (x_i^h)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{1}$$

which it maximises subject to the budget constraint

$$\sum_{i=1}^{n} p_i x_i = I^h. \tag{2}$$

This maximisation results in the FOC

$$x_i^h = \frac{X^h}{n} \left(\frac{P}{p_i} \right)^{\eta}, \tag{3}$$

where X^h is the composite commodity

$$X^h = n^{\frac{1}{1-\eta}} \left(\sum_{i=1}^{n} (x_i^h)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{4}$$

and P is a price index

$$P = \left(\frac{1}{n} \sum_{i=1}^{n} p_i^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{5}$$

so that

$$X^h = \frac{I^h}{P}. \tag{6}$$

Similarly, for the government, it is assumed that it produces a public good, X^g, with a CES production technology

$$X^g = n^{\frac{1}{1-\eta}} \left(\sum_{i=1}^{n} (x_i^g)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{7}$$

Inputs are chosen so as to minimise costs

$$G = \sum_{i=1}^{n} p_i x_i \tag{8}$$

subject to (7). Here we have the same type of demand fuctions as for the households

$$x_i^g = \frac{X^g}{n}\left(\frac{P}{p_i}\right)^{\eta}. \tag{9}$$

Firm i thus faces the output demand function

$$x_i = x_i^h + x_i^g = \frac{X}{n}\left(\frac{P}{p_i}\right)^{\eta}, \tag{10}$$

where $X = X^h + X^g$. The firm maximises its profits

$$\pi_i = p_i x_i - w_i L_i \tag{11}$$

in p_i, x_i and L_i s. t. (10) and the production function

$$x_i = A L_i^{\alpha}, \tag{12}$$

treating X, w and P as beyond its control. The maximisation results in the FOC

$$(1 - \frac{1}{\eta})p_i = \frac{1}{\alpha}\frac{w_i L_i}{x_i}, \tag{13}$$

which takes the form of a mark-up rule as the RHS are the marginal costs of an additional output unit. At the same time (13) fixes the income shares at

$$s_L = \frac{w_i L_i}{p_i x_i} = \alpha(1 - \frac{1}{\eta}) \tag{14}$$

and $s_\pi = 1 - s_L$. Substituting (10) and (12) into (13) gives labour demand, which depends only on the sector-specific wage and the overall economic variables, X and P :

$$L_i = \left[\left(\frac{X}{n}\right)^{\frac{1}{\eta}} \frac{\alpha(1-\frac{1}{\eta})P}{w_i} A^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\alpha(1-\frac{1}{\eta})}}. \tag{15}$$

The elasticity of labour demand is

$$\varepsilon_{L_i w_i} = -\frac{d\log L_i}{d\log w_i} = \frac{1}{1-\alpha(1-\frac{1}{\eta})}.$$

We now consider the symmetrical general equilibrium. It is caracterised by (a) $p_i = P$, (b) $w_i = w$, (c) $x_i = \frac{X}{n}$ and $L_i = \frac{L}{n}$, for all i. This means, from (13),

$$(1 - \frac{1}{\eta})P = \frac{1}{\alpha}\frac{wL}{X},$$

which, inserted into (15), gives aggregate employment in terms of the real wage

$$L = \left[n\alpha(1 - \frac{1}{\eta})A\frac{P}{w} \right]^{\frac{1}{(1-\alpha)(1-\frac{1}{\eta})}},$$

from which aggregate production immediately follows by the production function. As only the real wage is relevant, we can normalise the aggregate price level at this stage: $P \equiv 1$.