A TIME-VARYING NAIRU
Preliminary Draft

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1 Introduction

It is not deniable that the unemployment in Germany has stepwise increased for the last 30 years. That is also true for other European countries. Within the whole period from 1960 to 2000, the German unemployment rate is instationary. In opposite to the unemployment, the inflation rate appears to be rather stationary. That is true for three possible inflation measures: GDP deflator, consumption deflator and gross wages rate. The Phillips curve for Germany, that accounts for such a combination of stationary and instationary time series, will not suit the traditional model. The hypothesis of a NAIRU being constant over time has to be rejected. The time varying character of the NAIRU could be explained by structural factors that affect the labor and goods markets. These factors may have shifted the NAIRU permanently. Many ways of estimating a time-varying NAIRU are discussed in the recent literature. But because the long-term unemployment rate is unobserved, we are faced to the following methodical problem: some restrictions have to be imposed to interpret the estimation result as the NAIRU. In this paper we propose an estimation of a time-varying NAIRU for Germany with a Kalman filter. In order to interpret the result of the filter as the NAIRU, we impose the restriction that the NAIRU depends on exogenous variables and that its volatility is lower than the one of the unemployment rate.

2 Theory

The original Phillips curve identifyed a long term non linear trade-off between (nominal wage) inflation rate and unemployment rate. In his paper, Phillips (1958) explained the non-linear trade-off—among others—with the fact that firms have to bid in times of higher growth higher wages to attract workers. Few years after the publication of the Phillips paper, Samuelson and Solow (1960) applied the method to the USA. They replaced the change of money wage rates with the price inflation rate and observed a stable relation between this inflation rate and the unemployment rate. However they gave no explanation why this relation should be also valid for the long run. At the end of the 60's, Friedman (1968) revised the Phillips curve in such a way that this relation could not hold in the long-run. The main feature of his revision is that the behavior of economic agents cannot be influenced in the long-run by monetary changes. On the contrary, people learn to adapt their decisions to changing prices. As soon as they did, the trade-off between inflation and unemployment disappears. The Phillips curve is then vertical in the long-run. The long-run equilibrium does not imply a unemployment rate of zero. In the long run the unemployment rate can not fall short of the so-called "natural rate of unemployment". "The »natural rate of unemployment« (...) is the level which would be ground out by the Walrassian system of general equilibrium equations, provided that there is embedded in them the actual structural characteristics of labor and commodity markets, including market imperfections, stochastic variability in demand and supplies, the cost of gathering information about job vacancies and labor availibilities, the costs of mobility, and so on" (Friedman 1968, p.8). Friedman does not treat this natural rate as a constant: "I do not mean to suggest that it is immutable and unchangeable" (Friedman 1968, p.9). Because the main sources of influence lie in political and human decisions, the natural rate should change with them. Friedman takes as examples the level of minimum wage, the power of unions or the flexibility of the labor market. An increase of these variables should raise the natural rate for the first two and lower it for the last one. In other words a variable natural rate is possible. As Friedman pointed out it is difficult to list precisely all factors that may influence the natural rate. However, according to Friedman's point of view monetary policy cannot affect the level of the natural rate. For Friedman and for the Monetarists after him, monetary and real spheres are strictly separated.

The discussion so far refers to the conditions of the US economy. However, we want to test if the analysis still holds for the German economy. In Germany the unemployment rate has increased since the early 70's. It is an instationary series whereas the inflation stayed approximately stable, as the following graphs show.

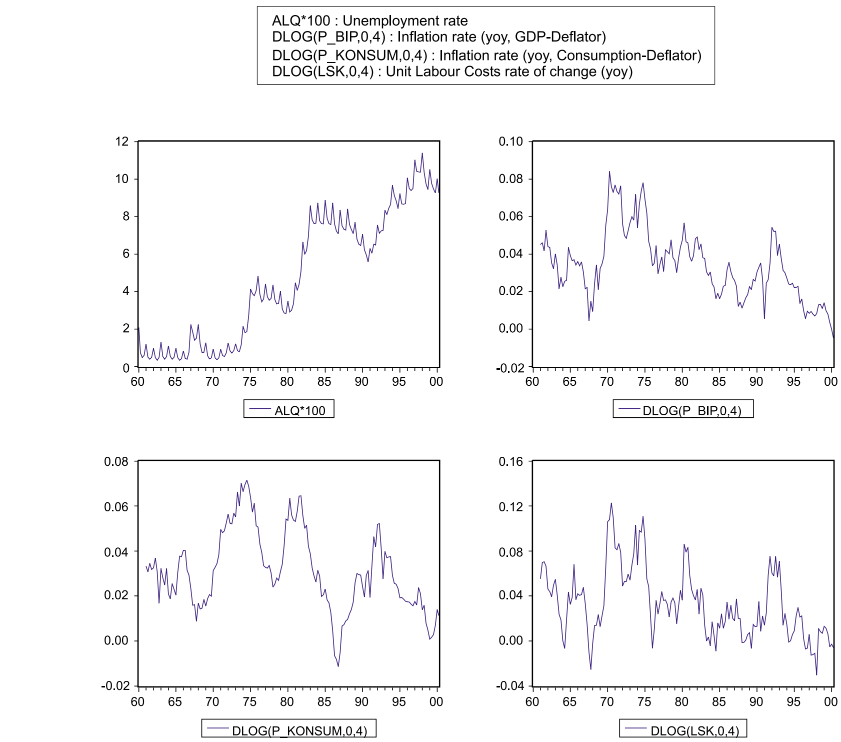

Figure 1: Unemployment rate and different kinds of inflation measures for Germany (1960-2000, quarterly data, seasonally unadjusted)

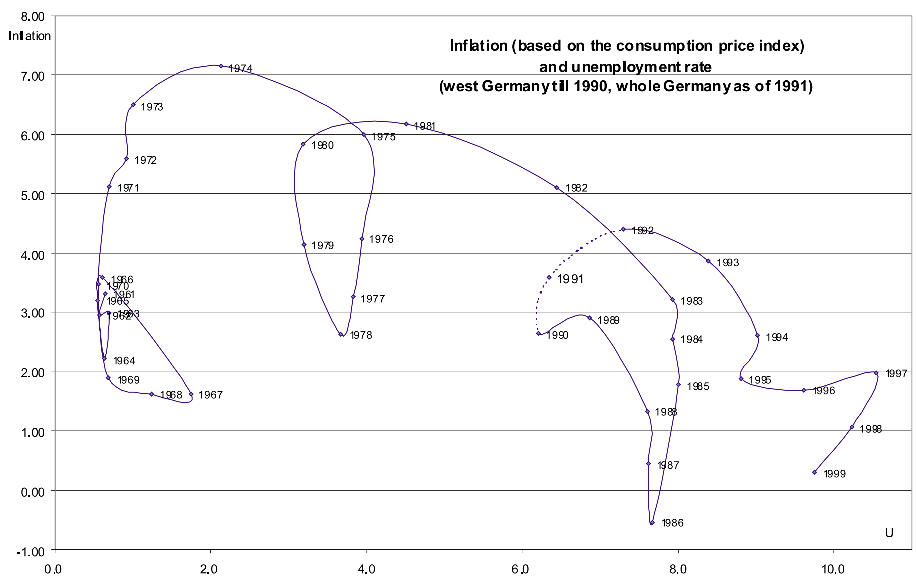

Figure 2: Inflation rate plotted against unemployment rate for Germany (1960-1999, yearly data)

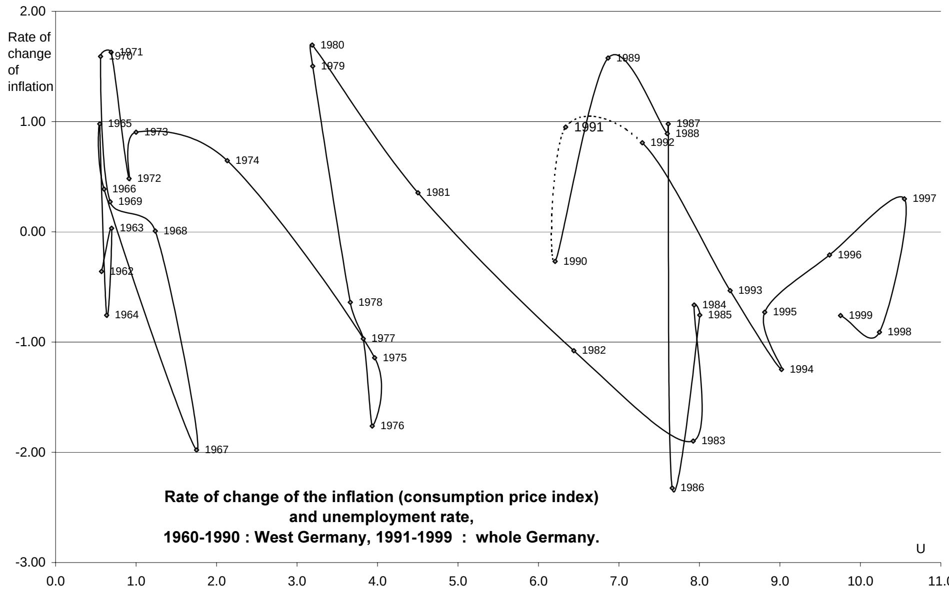

Figure 3: Changes of inflation rate plotted against unemployment rate for Germany (1960-1999, yearly data)

This fact let us assume that the natural rate increased also during this period. There exist two different views to explain this increase in the natural rate. The first explanation strategy is in accordance with Friedman's point of view. The high unemployment rate and the following high natural rate (in comparison with the 60's) can only be caused by structural factors. There are mainly three macroeconomic shocks that can be treated as structural shocks: the oil price increases in 1973 and 1979 and the reunification in 1990. The effects of these shocks were reinforced by the unability of the labor market to overcome the new economic conditions that arose. This point of view is shared by the German Council of Economic Experts (Sachverständigenrat, SVR) in their new annual report on the German Economy (Sachverständigenrat 2000). The SVR points out the lack of flexibility of the German labor market as a cause of the enduring labor crisis in Germany (§437-§439). The OECD makes similar statements in its Economic Outlook 65-1999 (OECD 1999, p.140-161). The second view takes additional factors into account. The first view may have well explained the German data until the first half of the 80's. This period was dominated by the two macroeconomic shocks that affected the aggregate supply. However, since the second half of the 80's and the 90's, labor market deregulation has begun: new forms of labor relations grew up (more part-time jobs), institutional regulation become more flexible (laws on hiring, temporary contracts, unemployment insurance). For a complete historical overview for Europe, see the survey of Cadiou and Guichard (1999). The persistence of unemployment in this period cannot be explained solely by the lack of flexibility. The notion of persistence is then introduced as the key element of the analysis. Econometrically a persistent time series is modelled by an autoregressive process with a AR-coefficient very close to one. It differs from hysteresis in the sense that this coefficient is not equal to one (the process doesn't follow a random walk) but just close to one. A persistent process will then be affected by shocks for a long time but their effects will disappear after a while. Standard unit root tests cannot make the difference between the two processes, and persistent processes will be estimated to be random walk. Only an non econometrically judgement can discern if we face a random walk or a persistent process. In the case of the unemployment rate, because this is a rate bounded between 0 and 1, it is a persistent process. The persistent path of unemployment rate has been studied a lot. We summarize here in three points the causes of persistence : unemployment devaluates the human capital of the unemployed person so that it will more and more difficult to find a job, unemployed people are often negatively stigmatized by employers so that it is more difficult to get a job and unemployed person may be discouraged after a too long time of unemployment, so that their efforts to find a job diminish and so their chances to be reemployed (see Layard et al. 1991, p.258-259). The limit between structural (or natural) and cyclical unemployment is therefore not that strict anymore. There exists transmission from the one to the other. This property of persistency is essential : economic policy have then an indirect influence on the natural rate. If a slow-down is not treated in time or is underestimate in a sense that the economy sticks too long in depression, the unemployment rate will raise and through the persistent effect its structural component too. The natural rate may therefore be affected for a long period by economic policy measures. Besides the mentioned structural factors, the natural rate could be affected by fiscal and monetary policy variables. Today, the term NAIRU is more used as the one of natural rate. "In fact, the main advantage of using the somewhat ugly term NAIRU instead of its more euphonious synonym, the natural rate, is that each time we use the term we are reminded of its meaning. The NAIRU is defined as the non-accelerating inflation rate of unemployment; that is, the rate of unemployment consistent with an unchanging inflation rate. [...] I think of the theory behind the NAIRU essentially as a description of how the economy behaves out of equilibrium" (Stiglitz 1997). Like Stiglitz we consider in this paper the two terms as equivalent and no difference will be made between them.

2.1 Our Model

We derive our model within the framework of the Layard-Jackman-Nickell approach (Layard et al. 1991). First the workers and firms bargain about the nominal wage. After they have signed an agreement, the firms decide about the product prices and their demand for labor.

2.1.1 The wage equation

To define their bargaining position, the workers take into consideration their expectations of future prices. We make the simplifying assumption that this expectation is given by the previous inflation rate π. Furthermore, the workers take into account the changes of productivity Δa.

However, their bargaining position depends crucially on their bargaining power. Usually, one assumes that this power is decreasing in the rate of unemployment (u_t). To take this into account, we use as proxy the deviation of the actual unemployment rate from the long run rate ($\tilde{u}_t^*$). With respect to this reasoning, we should have $\beta_1 < 0$

$$\Delta w_t = \pi_{t-1} + \beta_1(u_t - \tilde{u}_t^*) + \beta_3 \Delta a_{t-1} + \epsilon_{ws,t} \ . \tag{1}$$

Here, $\epsilon_{ws,t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_{ws}})$.

2.1.2 The price equation

We assume that the firms charge a markup over marginal cost. This is a plausible assumption for markets with imperfect competition. In that case, we have

$$\frac{(1+\delta)W_t}{P_t} = F_{L,t} \tag{2}$$

where P denotes the price for goods, W is the nominal wage, F_L is the marginal product of labor and $\delta \geqslant 0$ denotes the markup factor. The marginal product depends on the cost of other inputs and on the technology. After taking logarithms and calculating the differences through time, one derives with the auxiliary assumption that the right hand side is a linear function of the exogenous variables $x_{j,t}$ with $j \geqslant 4$

$$\pi_t = \Delta w_t + \beta_2 \Delta a_t + \sum_{j=4}^{J} \beta_j x_{j,t} + \epsilon_{ps,t} \ . \tag{3}$$

The exogenous variables are Δa with different lags, changes in import prices and industrial bonds yields . Furthermore we have $\epsilon_{ps,t} \sim \mathcal{N}(0, \sigma^2_{\epsilon_{ps}})$.

We obtain the Phillips curve if we put the wage equation (1) into the price equation (3)

$$\pi_t = \pi_{t-1} + \beta_1(u_t - \tilde{u}_t^*) + \beta_2 \Delta a_t + \beta_3 \Delta a_{t-1} + \sum_{j=4}^{J} \beta_j x_{j,t} + \varepsilon_t \ . \tag{4}$$

This equation resembles the usual textbook form (see for example Romer 1996, p.228). However, contrary to the usual form—with $\beta_j = 0$ for $j > 1$—we consider additional exogenous variables. So, our equation is quite similar to the one in Staiger et al. 1997. (4) makes clear the acronym NAIRU:

without shocks, a unemployment rate equal to $\tilde{u}_t^*$ means a constant inflation rate. In that case we have $\pi_t = \pi_{t-1} = \pi^*$. Thus, there exists a *tradeoff* "between output and the *change* in inflation, but no permanent tradeoff between output and inflation. For inflation to be held steady at any level, output must equal the natural rate. Any level of inflation is sustainable" (Romer 1996, S.229).

2.2 The unobservable NAIRU

We assume, that the long run rate of unemployment is given by the following process

$$\tilde{u}_t^* = \tilde{u}_{t-1}^* + \tilde{\boldsymbol{\gamma}}^T \mathbf{s}_t + \tilde{\nu}_t \tag{5}$$

where $\tilde{\nu}_t \sim \mathrm{N}(0, \sigma_{\tilde{\nu}}^2)$. The "natural unemployment rate" is an instationary process, which is influenced by the exogenous variables $\mathbf{s}_t$. To interpret the estimated rate $\tilde{u}_t^*$ as the natural rate, its behavior must be explained by the exogenous variables. Thus, we have two criteria at hand to asses if the estimated $\{\tilde{u}_t^*\}$ is the natural rate

- the variance of the process σ_ν^2 is "small"
- structural variables (first view) and—additionally—cyclical variables (second view) $\{\mathbf{s}_t\}_{t=1}^T$ explain—to some extend—this process.

The exogenous variables could be the number of participants in programs for vocational training, dummies for the oil price shocks (second quarter 1973 and 1979), for the effects of the German reunification (first quarter 1991), an indicator for the persistency of unemployment (rate of long lasting unemployment) and some indicators for fiscal and monetary policy.

However, $\tilde{u}^*$ is only identifiable—if ever—up to a multiple scale factor. On the grounds of this fact, we omit the tilde on the terms in (5) if they are implicitly multiplied with β_1.

2.3 State Space Form

The state space form (SSF) is given as (with *s*tate and *m*easurement)

$$\boldsymbol{\alpha}_t = \mathbf{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{\varepsilon}_t^s \tag{6a}$$

$$y_t = \mathbf{z}_t^T \boldsymbol{\alpha}_t + \varepsilon_t^m \; . \tag{6b}$$

with $\boldsymbol{\varepsilon}_t^s \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$ and $\varepsilon_t^m \sim \mathcal{N}(0, H_t)$ (the notation follows mainly Harvey 1989 p.100f). The disturbance vectors are distributed independently.

The unemployment rate is the first element in $\mathbf{x}_t$. We obtain for our model

$$\text{(7a)} \qquad \boldsymbol{\alpha}_t \equiv \begin{bmatrix} u_t^* \\ \boldsymbol{\gamma} \\ \boldsymbol{\beta} \end{bmatrix} , \quad \mathbf{T}_t \equiv \left[\begin{array}{c:c:c} 1 & \mathbf{s}_t^T & \mathbf{0}_{1\times K_\beta} \\ \hdashline \mathbf{0}_{K\times 1} & \multicolumn{2}{c}{\mathbf{I}_K} \end{array}\right] , \quad \boldsymbol{\varepsilon}_t^s \equiv \begin{bmatrix} \nu_t \\ \mathbf{0}_{K\times 1} \end{bmatrix}$$

$$\text{(7b)} \qquad y_t \equiv \Delta\pi_t \ , \quad \mathbf{z}_t \equiv \begin{bmatrix} -1 & \mathbf{0}_{1\times K_\gamma} & \mathbf{x}_t \end{bmatrix} , \quad \boldsymbol{\varepsilon}_t^m \equiv \varepsilon_t \ .$$

where K_γ denote the number of exogenous variables in the state equation, and K_β is the number of exogenous variables in the measurement equation. Let denote $K = K_\gamma + K_\beta$ the number of fixed state variables and let $S = K + 1$ denote the total number of state variables.

If we would know all parameters of the SSF (7), we could use the Kalman smoother to figure out the state vectors. However, in our model the parameters σ_ν^2 and σ_ϵ^2 are unknown. We stack these two parameters in the vector $\boldsymbol{\psi}$. Furthermore, we assume that the noise terms are distributed accordingly to $\boldsymbol{\alpha}_0 \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Due to this assumption the log likelihood function of our SSF is given (up to a constant (cf. Wu et al. 1996)) as

$$\text{(8)} \qquad \begin{aligned} \ln L(\boldsymbol{\psi}) = &-\frac{1}{2}\ln|\boldsymbol{\Sigma}| - \frac{1}{2}\boldsymbol{\varepsilon}_0^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}_0 \\ &- \frac{T}{2}\ln|\sigma_\nu^2| - \frac{1}{2}\sum_{t=1}^{T}\left(\frac{\nu_t}{\sigma_\nu}\right)^2 \\ &- \frac{T}{2}\ln|\sigma_\varepsilon^2| - \frac{1}{2}\sum_{t=1}^{T}\left(\frac{\varepsilon_t}{\sigma_\varepsilon}\right)^2 \end{aligned}$$

with $\boldsymbol{\varepsilon}_0 = \boldsymbol{\alpha}_0 - \boldsymbol{\mu}$, $\nu_t = \mathbf{e}_1^T(\boldsymbol{\alpha}_t - \mathbf{T}_t\boldsymbol{\alpha}_{t-1})$ and $\varepsilon_t = y_t - \mathbf{z}_t^T\boldsymbol{\alpha}_t$. The first element of the $(S \times 1)$ vector $\mathbf{e}_1$ is 1 and all other elements are 0. So, if we multiply this vector with a matrix, it picks up only the first row. The EM algorithm works in the following way: first, the expectation of the log likelihood is calculated (this is the expectation step). In the second step, the expected likelihood is maximized with respect to the unknown coefficients $\boldsymbol{\psi}$ (this is the maximization step). After that, the expected likelihood is recalculated with the estimated coefficients and the new expected likelihood is maximized. This procedure continues until the increase of the log likelihood function (8) is below some prescribed level (Shumway and Stoffer 1982, 2000 and Engle and Watson 1983). We calculate the log likelihood—given all coefficients—with the help of the Kalman filter (see for example (Harvey 1989, p.126).

To derive the expectation of (8) we define for $t \leqslant T$

$$\text{(9a)} \qquad \mathbf{a}_{t|T} \equiv E_T[\boldsymbol{\alpha}_t]$$

$$\mathbf{P}_{t|T} \equiv E_T[(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})^T] \tag{9b}$$

$$\mathbf{P}_{t,t-1|T} \equiv E_T[(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})(\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t-1|T})^T \ . \tag{9c}$$

We can calculate these expressions with the Kalman smoother. If we write

$$\boldsymbol{\varepsilon}_0 = (\boldsymbol{\alpha}_0 - \mathbf{a}_{0|T}) + (\mathbf{a}_{0|T} - \boldsymbol{\mu}) \ ,$$

$$\nu_t = \mathbf{e}_1^T((\boldsymbol{\alpha}_t - \mathbf{a}_{t|T}) - \mathbf{T}_t(\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t-1|T}) + (\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T}))$$

and

$$\varepsilon_t = (y_t - \mathbf{z}_t^T\mathbf{a}_{t|T}) + \mathbf{z}_t^T(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T}) \ ,$$

we obtain with $E[\boldsymbol{\varepsilon}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}] = \mathrm{tr}\{\boldsymbol{\Omega}^{-1}E[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T]\}$ for (8)

$$\begin{aligned} E_T[\ln L(\boldsymbol{\psi})] = &-\frac{1}{2}\ln|\boldsymbol{\Sigma}| - \frac{1}{2}\mathrm{tr}\big\{\boldsymbol{\Sigma}^{-1}(\mathbf{P}_{0|T} + (\mathbf{a}_{0|T} - \boldsymbol{\mu})(\mathbf{a}_{0|T} - \boldsymbol{\mu})^T)\big\} \\ &-\frac{T}{2}\ln|\sigma_\nu^2| - \frac{1}{2\sigma_\nu^2}\sum_{t=1}^T \mathbf{e}_1^T\mathbf{S}_t\mathbf{e}_1 - \frac{T}{2}\ln|\sigma_\varepsilon^2| \\ &-\frac{1}{2\sigma_\varepsilon^2}\sum_{t=1}^T M_t \end{aligned} \tag{10}$$

with

$$\begin{aligned} \mathbf{S}_t \equiv\ & \mathbf{P}_{t|T} - \mathbf{P}_{t,t-1|T}\mathbf{T}_t^T - \mathbf{T}_t\mathbf{P}_{t,t-1|T} + \mathbf{T}_t\mathbf{P}_{t-1|T}\mathbf{T}_t^T \\ &+ (\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T})(\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T})^T \end{aligned}$$

and

$$M_t \equiv \mathbf{z}_t^T\mathbf{P}_{t|T}\mathbf{z}_t + (y_t - \mathbf{z}_t^T\mathbf{a}_{t|T})(y_t - \mathbf{z}_t^T\mathbf{a}_{t|T})^T \ .$$

It is easy to see that other parameters of our model are unknown. As we have done above, we collect these parameters in $\boldsymbol{\psi}$. In addition to the variances, these parameters are $\boldsymbol{\mu}$ and vech$\boldsymbol{\Sigma}$. We have to choose these parameters in such a manner that the value of the expected likelihood (10) is maximized. It is obvious that $\hat{\boldsymbol{\mu}} = \mathbf{a}_{0|T}$ and that there is no way to derive an optimal choice of vech$\hat{\boldsymbol{\Sigma}}$. Furthermore, we obtain with the help of the first order conditions

$$\hat{\sigma}_\nu^2 = \frac{1}{T}\sum_{t=1}^T \mathbf{e}_1^T\mathbf{S}_t\mathbf{e}_1 \tag{11a}$$

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{T}\sum_{t=1}^T M_t \ . \tag{11b}$$

Both variances are non-negative because $\mathbf{S}_t$ and M_t are covariance matrices.

3 Methode und Daten

3.1 The data

The data come essentially from the quarterly national accounts calculated by the German Institute of Economic Research (DIW, Berlin). From 1960.Q1 to 1990.Q4, the data concern West Germany and up to 1991.Q1 whole Germany. Indexes are based on the year 1995.

Description of the variables		
Name	Source	Description
ALO	DIW	Unemployed persons (national definition)
ALQ	DIW	Unemployment rate (national definition)
APROD	DIW	Labor productivity (national definition, real GDP/Employment volume (in hours) * employment/employees
OFFEN	DIW	Degree of openness, (real import + real export)/real GDP
P_IMP	DIW	Import Price Index
P_KONSUM	DIW	Consumption Price Index
TOT	DIW	Terms of Trade, index of export prices/index of import prices
W_TOT_STD	DIW	Gross wage per hour (incl. employer's social contributions), labor costs/paid hours of employees
WEDGEW	DIW	Wedge between hourly gross wage and hourly netto wage, gross wage per hour/netto wage per hour
ZINS_ALL	BuBa	Yields on bonds outstanding issued by residents (industrial bonds). (=end of month) % p.a.
DISKONT	BuBa	Discount rate of the Bundesbank (till 1999) and Base rate of the ECB (after 2000).
ABM	IAB	Number of persons taking part of the active labor policies (Arbeitsbeschaffungsmaßnahmen.)
WBILD	IAB	Number of persons taking part of vocational training programmes.
LEg	IAB	Number of persons who benefit from the unemployment assurance (Arbeitslosengeld).
BLpKg	IAB	Gross benefit per head from the unemployment assurance.

LEh	IAB	Number of persons who benefit from the unemployment assistance (Arbeitslosenhilfe).
BLpKh	IAB	Gross benefit per head from the unemployment assistance.
LANGZEITALO	IAB	Long lasting unemployement
DIW = quarterly National Accounts calculated by the German Institute of Economic Research (DIW Berlin). **BuBa** = Database of the central bank of Federal Republik of Germany (deutsche Bundesbank). **IAB** = database of the Institute for Labor and Employment research (Institut für Arbeits- und Berufsforschung).		

The series are tested for unit root with the standard Augmented Dickey Fuller (ADF) test and the ADF test with structural break developed by Perron (1989) to take account of the reunification of Germany.

Order of integration

Variables	Integration order	
Consumption price index	I(1)	*
Unemployment rate	I(1)	***
Import price index	I(1)	***
Terms of Trade	I(0)	*
Returns of credit for companies	I(1)	***
Labor productivity	I(1)	***
Gross wage per hour	I(1)	***
Discount rate	I(0)	***
Wage wedge	I(1)	***
ABM participants	I(1)	***
Number of participants to a vocational training	I(1)	***
Receivers of unemployment insurance and assistance	I(1)	**
Gross benefits of the unemployment assurance/assistance per head	I(1)	*
Long lasting unemployment	I(1)	***
* (**) (***) significant at 10% (5%) (1%)		

3.2 Estimation

We use seasonally adjusted data for the estimation. The season adjustment follows the Berliner method BV4. We first begin with an OLS estimation of the Phillips curve (4). With this we just estimate the »middle« NAIRU. The OLS results and this middle NAIRU will be our stating values for the Beta-coefficients and for the variance of the residuals in the Phillips equation. Unfortunately there is no possibility to obtain an estimator of the Gamma-coefficients. We have to set those arbitrarily, as well as for the variance of residuals of the state equation. Altogether we use these coefficients to implement the matrices $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.

4 Results

In the table on the next page we present the results of this OLS estimation. It is worth to note that all the variables enters the estimation. The coefficient of the unemployment rate is not significant at all. This is not surprising since the unemployment rate is an integrated process of order one and the difference of inflation is surely a stationary one. The contemporaneous hourly wage (DLOG(W_TOT_STD)), the terms of trade (LOG(TOT)), the productivity (DLOG(APROD)) and the bonds yields (D(ZINS_ALL)) have all right signs; an increase in wages or the bond yields increases the prices. Furthermore, an increase in the terms of trade or in the productivity decreases the prices. The coefficient for the contemporaneous import prices is not significant. One explanation for this fact could be that the effects of an increase in the import prices takes time—one quarter—to affect significantly the consumption price index. The sign of the coefficient for the first lag is positive, that is just what we have expected.

Results of the OLS estimation

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000777	0.000789	0.985327	0.326300
ALQ*100	-0.000101	0.000079	-1.272387	0.205500
DLOG(P_KONSUM(-1),2)	-0.611708	0.067120	-9.113714	0.000000
DLOG(P_KONSUM(-2),2)	-0.228548	0.065103	-3.510541	0.000600
DLOG(P_KONSUM(-4),2)	-0.109442	0.055316	-1.978501	0.049900
DLOG(W_TOT_STD)	0.047398	0.026087	1.816926	0.071500
DLOG(W_TOT_STD(-4))	-0.054848	0.011953	-4.588568	0.000000
DLOG(P_IMP(-1))	0.059404	0.022573	2.631591	0.009500
DLOG(P_IMP(-4))	-0.036919	0.011639	-3.172100	0.001900
LOG(TOT)	-0.074115	0.016646	-4.452482	0.000000
LOG(TOT(-1))	0.152400	0.024535	6.211495	0.000000
LOG(TOT(-3))	-0.099754	0.026917	-3.705958	0.000300
LOG(TOT(-4))	0.024930	0.018209	1.369109	0.173300
DLOG(OFFEN)	0.038018	0.013113	2.899235	0.004400
DLOG(OFFEN(-1))	-0.025673	0.013421	-1.912857	0.057900
DLOG(OFFEN(-4))	0.032394	0.013922	2.326875	0.021500
DLOG(APROD)	-0.056005	0.020947	-2.673616	0.008400
DLOG(APROD(-1))	0.035627	0.008938	3.986195	0.000100
DLOG(APROD(-2))	-0.027860	0.007717	-3.610079	0.000400
DLOG(APROD(-3))	-0.034843	0.007973	-4.370223	0.000000
D(ZINS_ALL)	0.000973	0.000388	2.507346	0.013400
DW92_1	-0.012866	0.002247	-5.725119	0.000000
DW93_1	0.007715	0.002356	3.275195	0.001300
R-squared	0.721444	Mean dependent var		-3.32E-05
Adjusted R-squared	0.675367	S.D. dependent var		0.003641
S.E. of regression	0.002074	Akaike info criterion		-9.383017
Sum squared resid	0.000572	Schwarz criterion		-8.933359
Log likelihood	754.8753	F-statistic		15.65741
Durbin-Watson stat	2.051095	Prob(F-statistic)		0.00000

Our EM algorithm delivers results that are puzzling. First of all, with the OLS estimates and $\boldsymbol{\gamma} = \mathbf{0}$ as start values the log likelihood value (8) is 575.38. The EM algorithm brings the likelihood value in eight iterations up to 841.26. In every of the eight steps, the likelihood increases. In the last iteration, the relative change of the log likelihood is below the prescribed level of 0.001. The estimated standard deviation of the measurement equation is 0.0015 and thus quite similar to the estimated coefficient of the OLS regression. However, the estimated NAIRU process is in effect constant and zero. If we calculate the mean of this process for all quarters the result is similar to the constant of the OLS regression. Furthermore, the estimated coefficient β_1 is once again insignificant at all. So, the differences between the results of the EM algorithm and the OLS estimation are negligible. We have to conclude that our model is unable to identify the NAIRU. Instead of producing an instationary process that makes $\beta_1(u_t - u_t^*)$ stationary, the best fit is to eliminate the instationary unemployment rate and to produce a stationary—but insignificant—NAIRU.

In the next time, we have to check our estimation procedure. However, we are convinced that it works well. Furthermore, we want to estimate some differently specified NAIRU equations (for example: with the inflation rate as endogenous variable instead of the changes in this rate). Perhaps, we should compare our results with an estimation in which we use some sort of filter to generate a NAIRU from the observed unemployment rate. The series of the difference between the unemployment rate and the NAIRU generated in this way could be used as a regressor in the Phillips equation.

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