A Matching Model of Skills-Biased Technological Change in Dual Labour Markets

Louise Grogan

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Abstract

In this paper, a matching model containing both formal and informal sector employment opportunities and an unemployment pool is developed for the purpose of relating the impact of skills-biased technological shocks to labour market policies. The paper builds on a 2-state matching model of Mortensen and Pissarides (1998), who relate unemployment and wage dispersion responses to technological change to labour market policies. In the 3-state framework proposed here, it is possible to test the explanatory power of such an explanation in the case where workers have additional unprotected employment options in an informal sector. The addition of a second employment state provides a possible explanation for the very low elasticity of unemployment durations with respect to benefits changes which are observed in some OECD countries which also have large informal sectors. When labour markets are perfectly segmented by skill, the model provides testable predictions regarding the skills composition of each of the three labour market states. The model could be used to generate conditions under which a government would be financially justified in providing a stable environment for an unprotected sector to coexist with the formal sector. As well, because it identifies skills types who are likely to receive particularly poor labour market outcomes, the model provides an implicit link between technological change and poverty dynamics. The model can be readily estimated using longitudinal worker flow data as contained in household panels, with supplementary information about unemployment benefits and hiring and firing costs.

Keywords: matching models, skills-biased technological change, informal sector, unemployment, labour market policies

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Mailing address: Canadian International Labour Network (CILN), Department of Economics, McMaster University, Hamilton, ON, Canada L8S 4M4. Tel: (905) 525-9140 x 23221, Fax: (905) 521-8232. grogan@labour.mcmaster.ca

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1 Introduction

There is considerable debate currently about how technological advances are affecting labour markets, and about how labour market policies may enhance or mitigate these effects. Economists in developing countries and international financial institutions have become increasingly interested in documenting how technological aspects of the so-called *new economy* may be affecting inter and intra-national distributions of wealth via the labour market. One possible avenue of explanation of the emerging facts regarding labour and technology is that of a structural econometric framework which models both worker labour market transitions and firm personnel practises as dependent on technological change. In this paper, I introduce an empirically testable model which consists of a formal sector, in which workers bargain over their wages with employers, an informal sector, in which workers receive their marginal products but lack employment protection, and a search unemployment pool. I build on the seminal contributions of the many articles of Mortensen and Pissarides (see for example Mortensen and Pissarides (a), Mortensen and Pissarides (b), and Mortensen and Pissarides (c)) which link unemployment and wage distributions to labour market policies using a matching framework.

Mortensen and Pissarides (1998) use a 2-state search and matching framework to explain unemployment differentials between Europe and the USA in terms of differences in unemployment benefits and employment protection measures. Under the assumption that the US and Europe have been affected by the same rates of technological changes in the post-war period, a calibrated version of the MP98 model fits the stylised facts that higher unemployment and worker protection measures in Europe co-existed with low unemployment in the 50's and 60's, but with relatively high unemployment in recent years. Thus the MP98 model allows an interpretation of the trend of increasing US wage dispersion as a response to technological shocks in the absence of strong policy measures, and the trend of increasing unemployment in Europe as the response given high unemployment benefits and employment protection.

In this paper, I attempt to integrate the aspects of labour market insertion in the new economy included in MP98 (technological change, unemployment benefits, and employment protection policies) and the existence of an unprotected sector with a heterogeneous composition of workers. Given the existence of large and growing informal sectors in most countries of the world, such a framework may capture the outside option to formal sector employment more accurately than a 2-state model. I extend the MP98-type matching framework to a 3-state setting in which individuals move between a formal labour market sector, an unprotected informal sector, and search unemployment. In the present model, labour sub-markets are segmented by worker productivity, and are subject to stochastic technological shocks whose parameters may also depend on the particular skill sub-market of the worker.This 3-state model nests the MP98 model as a special case, and thus allows an assessment of the extent to which the original MP98 results hold when an unprotected sector and differential effects of technological shocks across the productivity distribution are introduced.

1.1 Background

In many countries of the world, very low unemployment benefits co-exist with relatively high levels of worker protection in the formal sector, and a large unprotected (informal) work sector. Even in OECD countries, such as Italy, unemployment benefits may be virtually non-existent even though formal-sector firms have strong disincentives to fire workers. Whether or not unprotected jobs exist may depend not only on the effectiveness of legal authorities in enforcing taxation payments from firms, but also the presence of entrepreneurial opportunities in general, and credit access constraints. Individuals who are self-employed will generally avoid costs of recruitment, training, and severance penalties, paid by firms. If these individuals are engaged in informal sector activities, they will also avoid paying regular taxes. As well, whether this self-employment is formalised or not, the impact of technological change on entrepreneurs may be very different from the impact on larger firms, due to differential capital needs and access to credit for new equipment.

In the early 1980's, the International Labour Office (ILO) recognised to the informal sector as a potential solution to unemployment in LDCs (see Sethuraman (1981)). Although the ILO vision of this sector was as an array of survival strategies constrained by undercapitalisation, low worker skills, and the small size of production, it was believed that that the the informal sector would be able to absorb employment if these negative conditions could be overcome. However, evidence from the 1990's suggests that the informal sector in many LDC's has expanded without raising living standards amongst it's members, and in spite of substantial deregulation of formal sector labour markets under structural adjustment-type programmes. The extent to which these effects are related to the self-selection of workers into the informal sector, perverse effects of deregulation, and assymetric effects of technological change across the sectors has not been widely examined.

By definition, the types of regulations imposed on labour in the formal sector, such as open recruitment policies, wage taxes, compulsory training, and severance penalties, are absent from informal sector work. The costs of regulations to registered firms may be minimised through capital investment (such as computers) and efficient administrative practises in larger firms, but on the small scale such regulations would be stiffling. Whereas formal sector firms may strive to grow large enough to capture economies of scale in the adherence to labour market regulations, informal sector firms may strive to remain as undercapitalised (and so flexible) as possible, in order to avoid detection, and to be more able to absorb technological or price shocks. Thus the formal/informal sector duality of the labour market may be strongly related to access to technology and capital. This duality may also help explain why informal sector activity is predominantly undertaken by women and the poor, since these groups are less likely to have access to the amount of capital necessary to make a formalised small enterprise economically viable (see Rakowsky (1994)).

OECD surveys of workers have consistently shown that perceptions of job instability are far more pronounced among less-educated workers than amongst the highly skilled (see Bowers and Martin (2000)), and have been so for decades. OECD firms which offer their workers greater amounts of skill training also tend to have lower levels of labour turnover, and longer observed job tenures. On average in the OECD, employers support about three quarters of ongoing vocational training obtained by adult workers, with more skilled workers also receiving more training. In might be expected that the level of on-the-job training invested in a worker by an employer would have an effect on the likelihood of that worker being dismissed in the event of an adverse technological shock. With more skilled workers receiving more on-the-job training, this implies that the impact of technological shocks in the formal labour market is skill-specific. In contrast, in an informal sector in which workers obtain their marginal product, and training is not undertaken (by definition), the impact of technological shocks may at best be only weakly related to worker skill levels.

While it would be very difficult to endogenise the emergence unprotected sector employment opportunities in matching model of technological change, there are insights to be gained from modeling this sector as a second employment option. First, it is possible to explore how well the MP98 explanation of European vs. US responses to technological shocks holds up in the case where informal sector employment is an alternative to unemployment in the event of adverse technological shocks in the formal sector. Second, a model in which unemployment is not the outside option to formal sector work over all policy-parameter combinations may explain why the responsiveness of unemployment rates to benefits changes are empirically marginal in many OECD countries, particularly among certain skill types. Third, the presence of this unprotected sector may explain some stylised facts about the skills types of individuals predominantly found in entrepreneurship, as well as support the MP98 prediction of a convex relationship between unemployment rates and worker productivity. The 3-state model presented here generates greater disparities in labour market tightness across skill groups than the 2-state model of MP98, due to the fact that high productivity individuals now may earn their marginal product in informal sector employment. An implication of this result is that the existence of an informal sector option will lower the mean skill quality of individuals in the unemployment pool. Given the facts that those engaged in informal sector activities are, by definition, not available for formal sector employment, and that not all productive enterprises experience hiring and firing costs, changes in labour market policies in the formal sector may have only small, and sometimes perverse, impacts.

The model to be presented here relates the dualistic nature of employment in many developing countries to both technological change and labour market policies.In modeling the existence of dual labour markets, one protected and one informal, it is possible to look at the effects of formal sector labour policies on the composition of the informal (and perhaps more volatile) sector. The model may be able to explain how technological shocks have not only had differential impacts on job security and job tenure across the worker productivity distribution, but also on the relationship between remuneration and productivity of individuals in the labour market. In this way, effects of technological change on relative poverty incidence and duration may be predicted. This model is testable using household survey data and Generalised Method of Moments (GMM) econometric estimators [1].

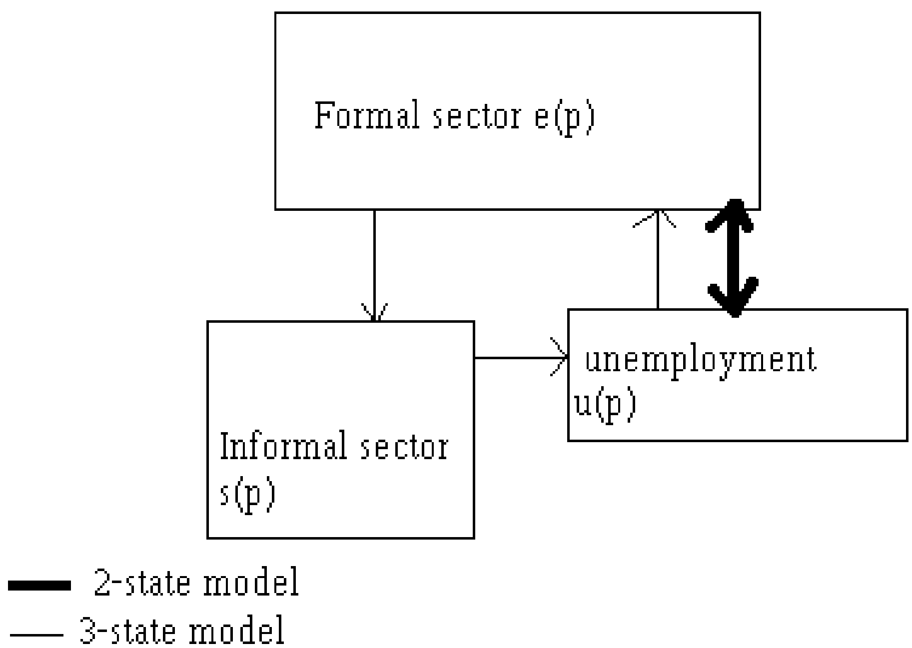
2 Model

Workers may be employed either in the formal sector, or in an unprotected, informal sector. In what follows, this second sector can be considered as a type of self-employment, in which workers receive their marginal product (p), independent of the state of technology. Depending on the productivity level of the individual, the frequency and magnitude of technological shocks, the levels of unemployment benefits, recruiting, training, and firing costs, and the volatility returns in the unprotected sector, individuals will have different preference orderings over the three sectors. For example, for some states of technology some very high productivity individuals may prefer work in the unprotected sector in which they receive their full marginal product to work in the protected sector, in which they must split the match surplus with firms. Given that both firms and workers understand the nature of the technological change process, worker transitions and match dissolutions are identifiable. There is a defined threshold skill level of workers below which they will always choose the secure unemployment benefit instead of self-employment, as well as one below which a formal sector job match will never be profitable. Worker flows in this model may be described as in figure 1, below:

In the unprotected, informal sector workers need not find a firm nor technology to match with. They will choose this sector with certainty when confronted with a choice between unprotected work and unemployment if the flow value of self-employment is higher than that in unemployment. To obtain a job in the formal sector, workers must

[1] Both this and a calibration exercise will be included in a finished version of the paper.

Figure 1:Labour market flows in the 3-state model



search from the unemployment pool. No transitions directly from the informal to the formal sector are allowed. They may lose this job if a skills-biased technological shock is sufficiently negative that both firm and worker agree to match dissolution. Jobs in the informal sector are also destroyed by a stochastic process of job destruction, which may be a technological one, but could perhaps be better described as one of product market destruction. When this negative shock arrives, a worker engaged in informal sector activity is automatically sent into the unemployment pool, and the option of informal sector employment is destroyed. The worker must first go through unemployment and a formal sector job before returning to the self-employed sector. As workers receive their marginal products in the informal sector, their remuneration from engaging in these activities is independent of the technological state of the world.

As is standard in matching models of the labour market, the flow of new matches to the formal sector submarket for each skill-type is determined by an identical, increasing, concave, and linearly-homogeneous matching function:

$$M\left[v(p), u(p)\right] = M\left[\theta(p), 1\right] u(p) = m\left[\theta(p)\right] u(p) \tag{1}$$

Here p comes from the distribution of skill types in the economy $K(p) \in p, \bar{p}$, job vacancies are denoted as $v(p)$, and unemployed workers are quantified by $u(p)$. Note that this search unemployment, $u(p)$, does not include self-employed workers. While the introduction of an unprotected sector does not directly affect the matching or job destruction processes of the formal sector, both equilibrium formal sector labour market tightness and reservation products of matches will be affected by the addition of an additional labour market state.

Labour market tightness, which is endogeneously determined within the model, is

$$\theta(p) = \frac{v(p)}{u(p)} \tag{2}$$

and is thus decreasing in the unemployment rate, and increasing in the vacancy rate in the submarket for each skill type.

As in MP98, I define $\tilde{V}(p, q)$ as the value to a firm who requires skill level q of opening a vacancy in the skill sub-market including unemployed workers of type p. The expected present value of future profits to a firm when a worker with skill-type p is hired to fill vacancy q is $\tilde{J}(p, q)$.

The flow value of unemployment to a worker is $\tilde{V}_u(p, q)$, and the value of market time to the worker is $\tilde{W}(p, q)$. As in MP98, both p and q belong to the distribution of worker skill types $K(p)$.

As in MP98, I assume that the opportunity cost of employment in the formal sector does not vary with worker skill, while training and recruiting costs of new workers do. The idea captured is that the value of non-market time (which may include unemployment compensation and the value of home production), varies less in worker skill than do hiring and training costs.

A key difference between the present model and that of MP98 is that in the MP98 model the opportunity cost of employment (b, unemployment benefits net of search costs) did not vary with worker skill. When this feature of their model was coupled with the fact that training and recruiting costs were assumed to be proportional to worker skill, a convex relation was generated between worker productivity and unemployment levels. In the present model, the outside option of a worker employed in the formal sector is either employment in the unprotected informal sector, in which the worker will p, or search unemployment. However, due to the fact that the individual must flow through the unemployment pool in order to regain employment in the formal sector, the outside option of a formal sector job will still vary less with worker skill than do hiring and training costs. Thus the present model will also allow for some degree of convexity of the unemployment-productivity relation.

2.1 The formal sector firm's value functions

Letting r denote the discount rate, the Bellman equations for the value of a vacancy requiring skill q in the submarket containing individuals of skill p may be described as:

$$r\tilde{V}(p,q) = \frac{m\left[\theta(p)\right]}{\theta(p)} max\left[J_0(q) - \tilde{V}(p,q) - qC, 0\right]\psi(p-q) - qc \tag{3}$$

As in MP98, $\psi(p-q)$ is an indicator function which takes the value 1 if its argument is greater than or equal to 0, and the value 0 otherwise. The value of a vacancy to a firm accounts for the possibility that a random meeting will not result in a match, and that the firm will continue to search for a worker. MP98 show that, under conditions which are also satisfied here, $p = q$ in equilibrium, and skill submarkets in the formal sector are perfectly segmented by skill.

The equation which describes the expected present discounted value of future profits to a formal sector firm is:

$$r\tilde{J}(p,x) = px - (1+\tau)w(p,x) + \lambda\int max\left[J(p,z), V(p) - pT\right] - J(p,x)dF(z) \tag{4}$$

The employer pays a payroll tax, τ. The arrival of a technological shock,λ, affects match product so that the match will dissolve if the match product falls below a skill-specific, endogeneously-determined reservation level $R(p)$. Both the value of a vacancy and the value of future profits to the formal sector firm are the same as in MP98.

2.2 The worker's value functions

The value of unemployment to a workless individual is:

$$r\tilde{V_u}(p,q) = m\left[\theta(p)\right] max\left[W_0(q) - \tilde{V_u}(p,q), 0\right] \psi(p-q) \tag{5}$$

Here, as in MP98, b is the opportunity cost of employment and $W_0(q)$ is the initial value in a job requiring skill level q to a worker who has skill level p such that $p >= q$. The self-selection requirement, such that workers will select and be selected into a submarket such that $p = q$ is shown in MP98 to hold, and will also hold in this extended framework.

The expected present discounted value of an new formal sector worker's wages will be determined, by:

$$r\tilde{W}_0(p) = w_0(p) + \lambda \int_{R(p)}^{\bar{x}} max\left[W(p,z), max[V_{se}(p), V_u(p)]\right] - W(p,x)dF(z) \tag{6}$$

In the case in which $V_{se}(p) < V_u(p)$, the model reduces to that of MP98. Otherwise, the outside option of an individual of productivity p subjected to match destruction is $V_{se}(p)$, the value of unprotected sector work. Thus, for given policy parameters and the stochastic shock processes facing the two labour markets, there will be some skill types who never engage in informal sector activities, and others who do.

The expected present discounted value of an employed worker's wages is:

$$r\tilde{W}(p,x) = w(p,x) + \lambda \int max\left[W(p,z), max[V_{se}(p), V_u(p)]\right] - W(p,x)dF(z) \tag{7}$$

Finally, we can define the value of a job in the unprotected sector. This is:

$$(r+\eta)V_{se} = p + \eta V_u(p,q) \tag{8}$$

Individuals engaged in the unprotected sector obtain the full value of their productivity, p, as their on-the-job wage. Exogeneous job-destroying shocks arrive at the rate η in this sector, and always result in the worker exiting to unemployment. By assumption, the worker may not return to self-employment from unemployment, and will search only for a formal-sector job.

2.3 Job creation

As in Pissarides (1990) and MP98, wages are determined by bilateral bargaining amongst risk-neutral agents with perfect information in each formal sector skill submarket. Vacancies will be created by firms until the expected present discounted value of creating an additional one is zero. From equation (1) above, it is easy to see that hazards of exit from unemployment in each skill submarket are $m\left[\theta(p)\right]$, where $m'\left[\theta(p)\right] > 0$, and $m''\left[\theta(p)\right] > 0$. Thus, by equation (2) unemployment rates will be decreasing in labour market tightness, $\theta(p)$.

The firm's free entry condition in the formal sector may be expressed as:

$$r\tilde{V}(p,q) = \frac{m\left[\theta(p)\right]}{\theta(p)} max\left[J_0(q) - \tilde{V}(p,q) - qC, 0\right]\psi(p-q) - qc = 0 \tag{9}$$

Here, $W_0(p)$, $V_u(p)$, $J_0(p)$, and $V(p)$ represent the respective values of wages, unemployment, profits, and vacancies, when all workers find jobs in the submarkets exactly at their skill level. The first order condition solving the Nash problem is:

$$(1+\tau)(1-\beta)\left[W_0(p) - max[V_{se}(p), V_u(p)]\right] = \beta\left[J_0(p) - pC - V(p)\right] \tag{10}$$

Given the Bellman equation for the value of a vacancy (above), and the assumption that firms will continue to enter a market until the value of a vacancy has been decreased to zero, we may write:

$$rV(p) = \frac{m\left[\theta(p)\right]}{\theta(p)}\left[J_0(p) - V(p) - pC\right] - pc = 0 \tag{11}$$

In equilibrium, the expected cost of filling a vacant job will te equal to the expected present value of future profit attributed to the filling of a job.

2.4 Evolution of the match product

The initial wage in the submarket for labour of skill-level p is determined by solving the Nash bargaining problem with threat points equal to the employer's and worker's respective values of continuing a worker and job searches. Denoting the bargaining power of the worker as β,

$$w_0(p) = argmax\beta(ln\left[W_0(p) - max[V_{se}(p), V_u(p)]\right]) + (1-\beta)ln\left[J_0(p) - pC - V(p)\right] \tag{12}$$

Here pC represents the initial training cost to the firm of breaking in a new worker.

As in MP98, the bargaining positions of firms and workers will change over time, due to the arrival of idiosyncratic shocks to the match product px. The values of the match to both the firm and the worker evolve over time, and so do the wages of the worker achieved in bargaining. The ongoing solution to the problem of the division of the match product describes the continuing wage received by the worker $w(p,x)$ and is:

$$w(p,x) = argmax\beta(ln\left[W(p,x)(p) - max[V_{se}(p), V_u(p)]\right]) + (1-\beta)ln\left[J(p,x) + pT - V(p)\right] \tag{13}$$

Here, as in MP98, pT represents the cost of separation paid by the employer. Using the value functions described above, we can solve for the initial and ongoing wages of the worker $w_0(p)$ and $w(p,x)$. For the case of $V_{se} > V_u$, the formal sector worker's wages are:

$$w(p,x) = \frac{\beta}{(1+\tau)}px - r\left[V(p) - pT\right] + r(1-\beta)max[V_{se}(p), V_u(p)] \tag{14}$$

and

$$w_0(p) = \frac{\beta}{(1+\tau)}p - (r+\lambda)pC - \lambda pT - rV(p) + r(1-\beta)max[V_{se}(p), V_u(p)] \tag{15}$$

It is evident that, in the event that $V_{se} < V_u$, the expressions result in values of $w(p,x)$ and $w_0(p)$ which are strictly greater than those which would result under the same unemployment benefit and employment protection parameters in a model without an unprotected sector.

2.5 Solving the workers' Bellman equations

As in MP98, we can use the Nash bargaining conditions and the Bellman equations above to solve for the respective values of unemployment, self-employment, and a formal sector continuing job:

$$V_u(p) = \frac{W_0(p) + b/m\left[\theta(p)\right]}{(r/m\left[\theta(p)\right]) + 1} \tag{16}$$

Substituting for $W_0(p)$ I obtain:

$$V_u(p) = \frac{\frac{b}{m[\theta(p)]} + \frac{p}{(r+\eta)} - \frac{\beta pc\theta(p)}{m[\theta(p)](1+\tau)(1-\beta)}}{\frac{r}{m[\theta(p)]} + 1 - \eta/r + \eta} \tag{17}$$

Here the value of unemployment $V_u(p)$, as well as the value of V_{se} and the value to the worker of a job in the formal sector will depend on $m\left[\theta(p)\right]$, the value of the matching function.

3 Search equilibrium

Given that this labour market is fully segmented by worker productivity levels, as demonstrated in MP98, firms will always hire workers of productivity $p = q$ and workers will search only in the submarket specific to their productivity levels. Search equilibrium in the formal sector will be characterised by a reservation product $R(p)$ and labour market tightness $\theta(p)$ pair which satisfies a free entry job condition (see below) and the job destruction condition (see above),

The equilibrium job creation condition is:

$$\frac{c\theta(p)}{m\left[\theta(p)\right]} = (1-\beta)\left[\frac{1-R(p)}{r+\lambda} - C - T\right] \tag{18}$$

This condition follows the same form as in the 2 state model of MP98. This is due to the fact that the parameters governing the value of a vacancy have not been altered. In the case where $V_{se}(p) > V_u(p)$, the value of a vacancy is greater than the value of unemployment, and the reservation product of a formal sector match, $R(p)$ will still be the point at which $J\left[p, R(p)\right] + pT - V(p) = 0$. The equilibrium job creation relationship is monotonically decreasing in $\theta(p)$.

The equilibrium job destruction relation is substantially different from the case of the 2-state model with no unprotected sector in the case of $V_{se}(p) > V_u(p)$. To determine the job destruction relation between $R(p)$ and $\theta(p)$, I use the sharing rule from the continuing Nash product (above), and the fact that we know that match values in the formal sector increase in x for both workers and employers. The reservation value of the match is implicitly defined by the equation

$$W\left[p, R(p)\right] - V_{se}(p) = J\left[p, R(p)\right] + pT - V(p) = 0 \tag{19}$$

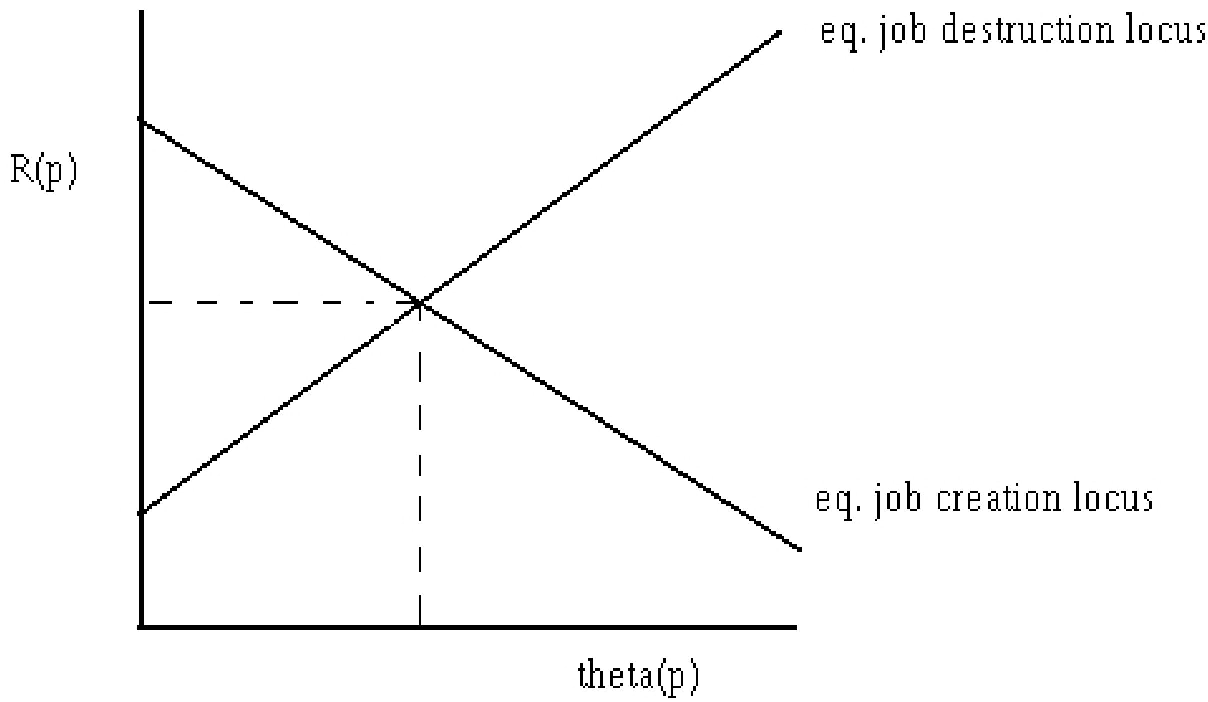
Using what is known about the component value functions in this relation I obtain the following relation between the reservation value and labour market tightness.

The equilibrium job destruction condition is:

$$R(p) + \frac{\lambda}{r+\lambda}\int_{R(p)}^{\bar{x}} \left[z - R(p)\right] dF(z) = (\frac{r}{p}V_{se} - rT)(1+\tau) \tag{20}$$

We can substitute for V_{se} using equations (5) and (12) above. The equilibrium job destruction relation will be monotonically increasing in $\theta(p)$. The solution to the model may be described in the space $(\theta(p), R(p))$, at the point where the job creation loci intersects the job destruction loci. As in MP98, the equilibrium job creation condition requires that the level of labour market tightness will be decreasing in the reservation produce, whereas the job destruction condition requires a positive relation between the reservation product and market tightness. The equilibrium solution to the model will only make economic sense if the equilibrium level of labour market tightness $(\theta(p) = \frac{v(p)}{u(p)})$ is positive. As long as the value of $R(p)$ at $\theta(p) = 0$ for the job creation condition is greater than the value of of $R(p)$ at $\theta(p) = 0$ for the job destruction condition, the loci of these 2 conditions will meet at some positive level of labour market tightness. The equilibrium conditions can be described as in figure 2, below

Figure 2: Search equilibrium in the formal sector



3.1 additional parameter restrictions

While the model above can be solved without specifying the functional form governing technological shocks in the formal sector, the simple way in which the informal sector was introduced has the potential to give rise to some counterintuitive results. In particular, the fact that the arrival of a job-destroying shock in the informal sector, η, unilaterally destroys the informal job possibility, leads to the following relations:

$$\frac{\partial V_{se}}{\partial V_u(p)} = \frac{\eta}{(r+\eta)} \tag{21}$$

, and

$$\frac{\partial V_{se}}{\partial \eta} = \frac{rV_u(p) - p}{(r+\eta)^2} \tag{22}$$

Also, because p and r enter into the respective values of unemployment, Vu, and informal sector employment, V_{se}, in similar ways, these values will move together as we move up the skills distribution or as the interest rate rises. The one parameter which is present in V_{se} but not in V_u is η. Given that η describes a Poisson arrival process, it is naturally restricted to being $\in (0,1]$. It would make economic sense to restrict $\frac{\partial V_{se}}{\partial \eta} < 0$, which implies that rises in the rate of informal sector job destruction devalue participation in this sector. As can be seen from the above, such a restriction would imply that $rV_u(p) - p < 0$. After substituting in the known value of V_u, we have that

$$\frac{\frac{rb}{m[\theta(p)]} + p(\frac{r}{r+\eta} - \frac{\beta c\theta(p)}{m\theta(p)(1+\tau)(1-\beta)})}{(\frac{r}{m[\theta(p)]} + 1 - \frac{\eta}{r+\eta})} < p \tag{23}$$

It turns out that the values of η which satisfy this restriction for a worker of median skill level normalised to $p = 1$, and for policy parameters in ranges close to those examined in MP98 for the 2-state model, are not overly restrictive.[2]

Another possible restriction that might prove important relates to skills levels below which individuals would never chose informal sector work when faced with a job loss, say p^*. In order to generate a positive mass of individuals in the informal sector at any one time, it should be the case that $p*$ lies below the upper bound of the productivity distribution in the economy, $K(p)$ under reasonable parameter values and labour market tightness.

For a given level of b in an economy, we would expect to see that workers below a certain corresponding productivity, say p^* would be in the 2-state world, while those above the threshold would be in the 3-state world. Individuals with productivities $p > p^*$ would engage in informal sector work when laid-off from formal sector jobs by a sufficiently adverse technological shock. Below $b^*(p)$, increases in unemployment duration and incidence

[2]In such matching frameworks, the conjecture that $\beta \in (0,1]$, $\tau \in (0,1]$, $\Omega \in (0,1]$,$r \in (0,1]$,$\lambda \in (0,1]$, and $m[\theta(p)] \in (0,1]$ is standard.

of workers of productivity p resulting from benefits increases will be extremely marginal. Given the assumption that the government sets b independently of worker skill, the model will predict that the informal sector will be composed of individuals with $p > p^*$, and that the unemployed sector will contain a higher concentration of low p individuals than would be the case in the 2-state world. As long as $\bar{p} > p*$, there will be some positive quantity of workers in the informal sector, and these workers will be relatively highly-skilled.

By definition, $p*$ is defined such that $V_u(p^*) = V_{se}(p^*)$ can be written implicitly as:

$$V_u(p^*) = \frac{(r+\eta)V_u(p^*) - p^*}{\eta} \tag{24}$$

After substituting for the known values of unemployment and informal sector employment, the following is found:

$$p^* = \frac{b}{1 - \frac{\beta c\theta(p)}{(1+\tau)(1-\beta)}} \tag{25}$$

For the parameter combinations used in the comparative statics exercise which follow, such a p^* would give rise to an observation of about 12% of the workforce in unprotected work.

With respect to changes in benefits (b), it should at least be feasible that p^* be rising in unemployment benefits levels.

Differentiating the above expression, I obtain

$$\frac{\partial p^*}{\partial b} = \frac{1}{1 - \frac{\beta c\theta(p)}{(1+\tau)(1-\beta)}} \tag{26}$$

which will hold under feasible parameter values.

3.2 Comparative statics

The comparative statics of changes in policy parameters will have substantially different effects when $V_{se}(p) > V_u(p)$ than otherwise. Recall that if $V_{se}(p) < V_u(p)$, that is the value of unemployment is sufficiently high, the model effectively reduces to the 2-state model of MP98. In this case $R(p)$ will be defined by $W(p, R(p))$=$V_u(p)$, and the worker will not enter into self-employment. Before we reach this threshold level of b, say $b^*(p)$ at which unemployment becomes the outside option of the match for a worker of skill level p, changes in b will have only an indirect effect on the job destruction relation. Note that $b^*(p)$ is a function of the productivity level of the individual, p. Since $V_{se}(p)$ is productivity-specific, the b value at which $V_u = V_{se}(p)$ will also be. The job destruction relation will be shifted slightly upwards in $(\theta(p), R(p))$ space due to the change in the outside option of an informal sector job (V_u), but not by nearly as much for a given change in b as when unemployment becomes the outside option of a formal sector match. Given that the equilibrium job creation condition does not contain b as an argument, the job creation locus will not be impacted by a change in unemployment benefits. To summarise, for $b < b^*(p)$, increases in b will have only second-order effects in decreasing equilibrium labour market tightness, but above b^*, the effects will be first-order. Figures 3 and 4 (below) illustrate.

Generally, $b^*(p)$, the threshold at which unemployment becomes the outside option of a formal sector job, will increase monotonically in worker productivity.

Changes in the severance tax or payroll tax rate facing formal sector firms result in very similar comparative statics to the MP98 model. T, for example, reduce the incentives of employers to create jobs, so that the job creation locus shifts down and to the left. However, job destruction is affected in the other direction, since increasing the costs of layoffs will mean that a technological shock must be more adverse in order to warrant match dissolution. Therefore the net impact of an increase in severance taxes on labour market tightness is ambiguous, but the equilibrium $R(p)$ (and so equilibrium unemployment incidence) will decrease. Due to the fact that workers are now allocated across 3 labour market states, the elasticities of formal sector employment and unemployment with respect to these policy changes will be different.

3.3 Flow conditions

In the stationary labour market equilibrium of this model, flows into and out of each of the three sectors of the economy will be equalised. This information allows us to deduce the following equilibrium quantities of individuals in each labour market state:

The quantity of individuals in the informal sector, $s(p)$ will be:

$$s(p) = \frac{m\left[\theta(p)\right]}{\eta + \frac{\eta m[\theta(p)]}{\lambda F(R(p))} + m\left[\theta(p)\right]} \tag{27}$$

, the quantity of unemployed individuals, $u(p)$ will be:

$$u(p) = \frac{\eta\lambda F(R(p))}{\eta\lambda F(R(p)) + \eta m\left[\theta(p)\right] + \lambda F(R(p))m\left[\theta(p)\right]} \tag{28}$$

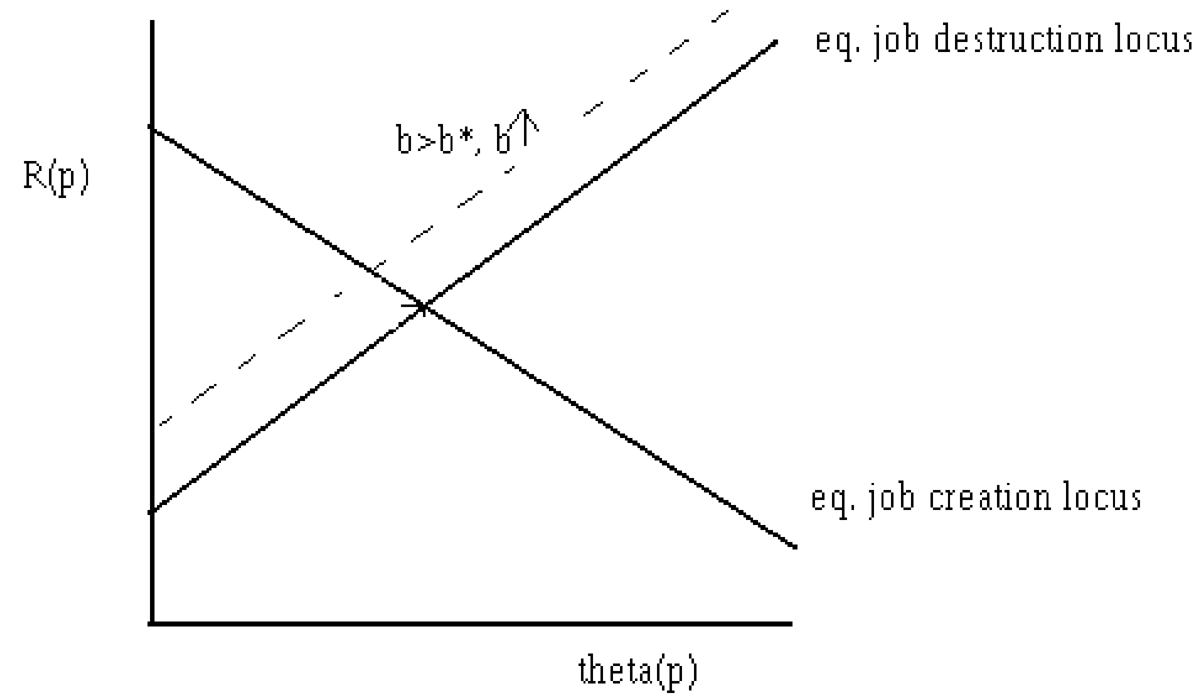
, and the quantity of individuals employed in the formal sector, $e(p)$ will be:

$$e(p) = \frac{\eta m\left[\theta(p)\right]}{\eta\lambda F(R(p)) + \eta m\left[\theta(p)\right] + \lambda F(R(p))m\left[\theta(p)\right]} \tag{29}$$

4 Calibration Exercises

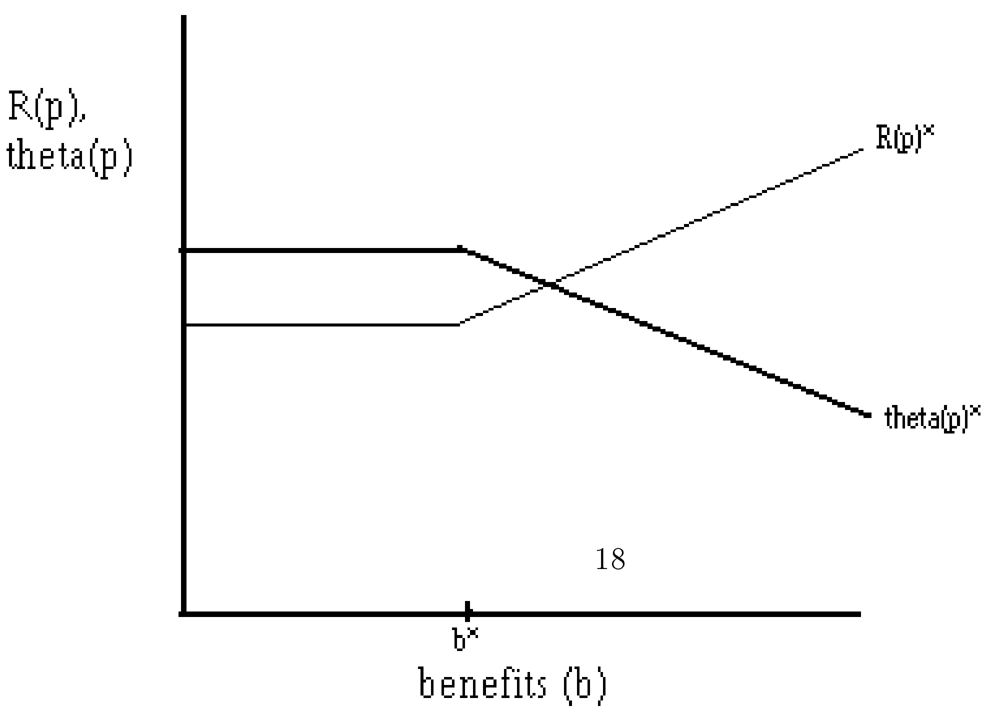
Calibration exercises are a convenient way of looking at the comparative statics results laid out informally in the previous section. The first calibration exercise uses the baseline parameter values used by MP98 in their calibration of the 2-state matching model. As

Figure 3: Comparative statics of an increase in unemployment benefits



In the 3-state model, there will be virtually no effect of benefits increases on eq. conditions until unemployment becomes the outside option. Above b*(p), an increase in benefits will shift the eq. job destruction relation up.

Figure 4 : The relation between benefits (b) changes and eq. conditions (R(p),theta(p))



Note: The relationship is drawn for a given worker skill level, p.

such, it is possible to analyse the effects of the introduction of an unprotected sector on the model's equilibrium. In this baseline case, the only values of η which will result in $\frac{\partial V_{se}}{\eta} < 0$ for the median productivity individual ($p = 1$) are in the range $0.5 - 0.7$, which implies that the informal sector is extremely volatile. This problem is mitigated when different values of the matching elasticity with respect to vacancies are chosen. As is done in MP98, it is assumed for simplicity that technological shocks in the formal sector are not skill-specific, and that they follow a uniform distribution with minimum match product γ. Possible skill-specific and economically-meaningful variations on this parametrisation are discussed in an appendix, but are left at this point in order to focus on the new implications of adding an unprotected sector.

While MP98 use a matching elasticity with respect to vacancies, Ω, of 0.5, studies which have estimated unemployment outflows into jobs have found far lower elasticities. For the USA Blanchard and Blanchard (1989) find $\Omega = 0.2$ using monthly data with decreasing returns to scale in matching, while Burda and Wyplosz (1994) find elasticities in the $0.2 - 0.3$ range for France, Germany, Spain, and the UK using monthly data and a constant returns to scale matching function. In the earliest study known to the author, Pissarides (1986) found a matching elasticity with respect to vacancies of 0.3 for the UK using monthly data and constant returns to scale. With Ω assigned in the $0.2 - 0.3$ range, the η values which result in $\frac{\partial V_{se}}{\eta} < 0$ cover a wider range than when $\Omega = 0.5$. and lie between 0.2 and 0.4 across the productivity range considered. The results of this calibration exercise, using $\eta = 0.2$ and $\Omega = 0.2$ are summarised in figure 5 below:

Figure 5: The unemployment-skills profile

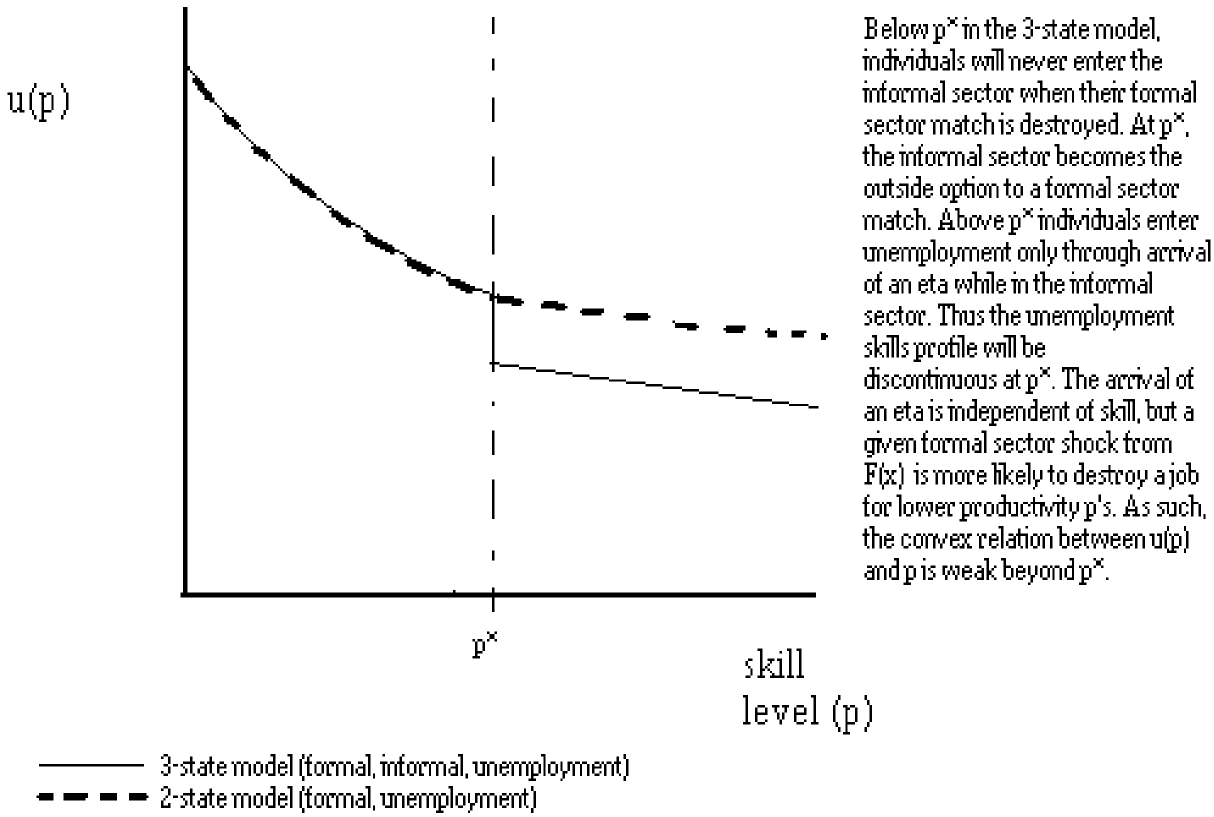


Table 1: baseline parameter values

MP98 parameters	
discount rate	r=0.02 per quarter
matching elasticity	Ω=0.5
recruiting cost	$\frac{c\theta(p)}{m[\theta(p)]}$
training cost	C=.3 per worker
technological shock frequency (formal sector)	λ=.1
minimum (formal sector) match product	γ=.64 per quarter
value of leisure	l=.28 per quarter
worker's share of (formal sector) match surplus	β=.5
unemployment benefit replacement ratio	ρ=.2
payroll tax rate	τ=.16
severance tax	T=.1 per worker
new parameters	
rate of exogeneous match destruction (informal sector)	η=.1

5 Conclusions

In this paper I developed a matching model in which both protected and unprotected jobs exist and are subject to stochastic shocks. The model nests the two-state matching model of MP98 as a special case. An important feature of the model is that changes in policy parameters such as severance penalties and unemployment benefits will have differential effects on individuals of a given productivity depending on whether unemployment or self-employment is (or becomes) their outside option to a formal sector match. Thus elasticities of unemployment and employment with respect to policy changes, which were not uniform across p in MP98, will have an additional source of heterogeneity. Policy impacts will generally be dampened by the fact that unemployment is not necessarily the outside option to a job. This result may help explain why changes in unemployment benefits in the 1990's in many European countries had far less of an influence on unemployment incidences an durations than would be predicted using the MP98 model.

The unprotected sector has been modeled very simply as paying individuals their marginal product, and being subjected to job-destroying shocks at the rate η. Still, if the rate of informal sector job destruction could be partially controlled by governments, perhaps by providing microcredits, securing access to inputs, or preventing the dumping of goods, the government would have an additional lever with which to influence the skills composition of the unemployment pool, unemployment durations, and incidences. As well, providing individuals with a third labour market option could mitigate the productivity-sapping effects of unemployment, and perhaps even more than compensate for the lack of tax revenue generated in this sector.

5.1 Appendix: Modeling technological shocks

Thusfar, the nature of the technological shocks which change the productivity of matches in the formal sector has not been discussed. While the model can be solved without making distributional assumptions, I must specify the stochastic process governing technological change in order to calibrate and estimate the model. MP98 calibrate a version of the model in which $F(x)$ is uniformly-distributed with some lower bound corresponding to a minimum match product. However, given the stylised fact that the type of capital employed by workers varies widely in p, and the demonstration in MP98 that skill submarkets will be perfectly segmented in equilibrium of such a matching framework, it seems reasonable to allow distributions of technological shocks to be specific to each $p = q$ submarket. While technological shocks may be allowed to occur at the same rate for all workers with productivity $p \in K(p)$, the impact of the arrival of such a shock on the match product px may vary. Here I consider calibrations of the model using two simple transformations of a uniform distribution.

First, I assume that the density of is described by some transformation of the uniform density function which has zero slope for the median productivity value, and whose slope is increasing in productivity. Specifically, if the uniform density for the median productivity individual $p = 1$ is:

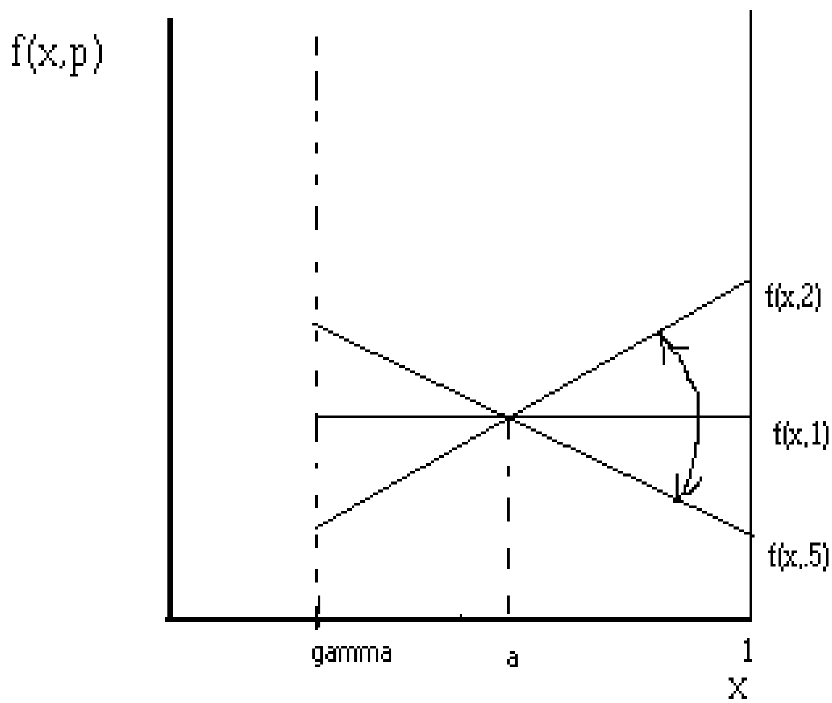
$$F(x,p) = \frac{(x-\gamma)}{(1-\gamma)} \tag{30}$$

$\forall$ x in $[0, 1]$, where γ may be interpreted as a universal minimum match product.

This distribution implies that there is a positive relation between worker skills and the probability that a given technological product is positive. High-productivity individuals are relatively likely to improve their match products with a technological shock, and unskilled workers are relatively likely to experience a decline in match product. Thus, if the impact of technological shocks such as the introduction of information and communications technology (ICT) did depend on worker skill in this way, we would expect that a given shock ended a relatively large amount of low-skilled matches, and preserved (or improved) a relatively large fraction of high-skilled matches. This relation could be observed despite the fact that $R'(p)$ is monotonically increasing in p for all p¿p* in this model. In a sense, this is a simple way of allowing for skills obsolesence amongst the low-skilled amidst general improvements in match products in the formal sector. This distribution is illustrated in the diagram below:

The second distribution of technological shocks used in the calibration exercise also involves a simple transformation of the above uniform distribution with minimum match product γ. Here I use the following power transformation:

Figure 6: Probability density function transformation around a point x=a



Different skill levels of individuals (p) imply shifts in the density of the formal sector technological change function, and imply a quadratic c.d.f.

$$F(u) = u \tag{31}$$

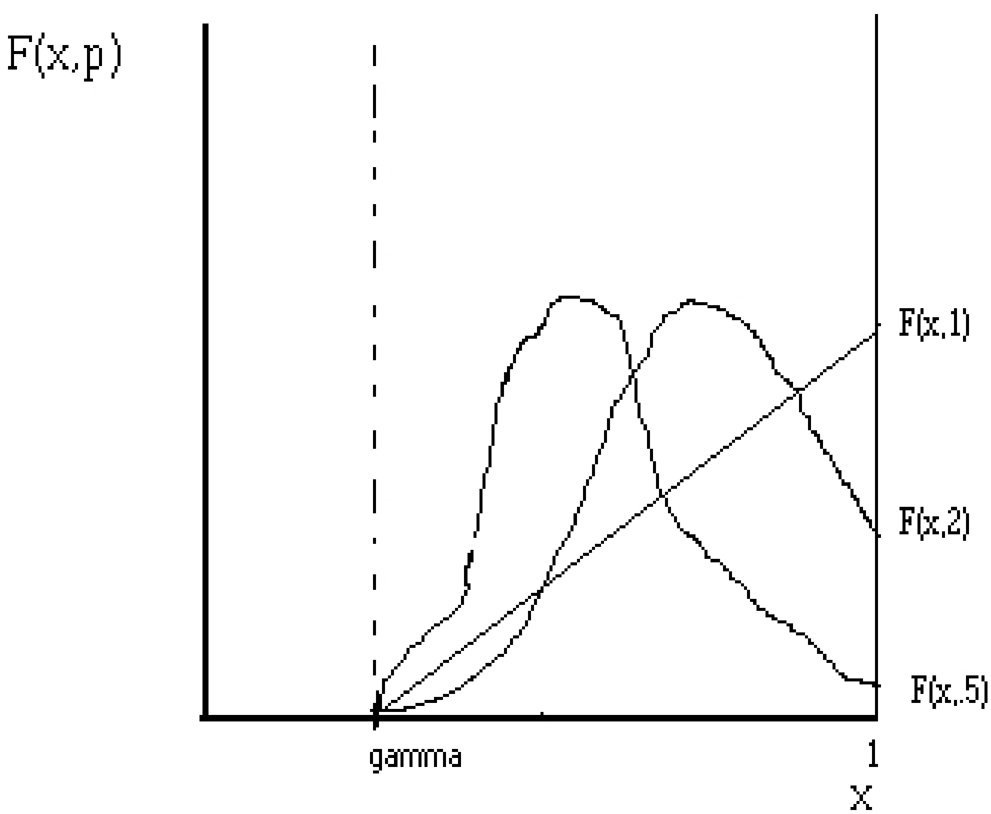
$$F(x) = Pr(u^{\alpha} < x) \tag{32}$$

$$F(x) = Pr(u < x^{\frac{1}{\alpha}}) \tag{33}$$

$$\tag{34}$$

This leads to the *cdf* $F(x) = x^{\frac{1}{\alpha}}$, and the *pdf* $f(x) = \frac{1}{\alpha}x^{\frac{1}{\alpha}-1}$. Letting $\alpha = \frac{1}{p}$, the mass of the distribution will be shifted to the right, when $p > 1$, and to the left when $p < 1$. Thus the same basic economic reasoning as in the first process will be used, and it will be possible to see whether or not the labour market implications are robust to the particularities of the technological shock specification. The power transformation will result in a *cdf* something like shown in figure 7 below:

Figure 7 : Power transformation of the uniform distribution of technological shocks



References

Blanchard, P. and O. Blanchard (1989). The beveridge curve. *Brookings Papers on Economic Activity 1*, 1–76.

Bowers, N. and J. P. Martin (2000). Going mobile? jobs in the new economy. *OECD Observer, Forum 2001 (www.oecdobserver.org)*.

Burda, M. and C. Wyplosz (1994). Gross worker and job flows in europe. *European Economic Review 38*, 1287–1315.

Mortensen, D. and C. Pissarides. Job creation and job destruction and the theory of unemployment. *Review of Economic Studies 61*.

Mortensen, D. and C. Pissarides. Technological progress, job creation and job destruction. *Journal of Economic Dynamics 1*.

Mortensen, D. and C. Pissarides. Unemployment responses to skills-biased technological shocks: The role of labour market policy. *The Economic Journal 109*.

Pissarides, C. (1986). Unemployment and vacancies in Britain. *Economic Policy 3*, 499–559.

Pissarides, C. (1990). *Equilibrium Unemployment Theory*. Oxford: Basil Blackwell.

Rakowsky, C. A. (1994). Convergence and divergence in the informal sector debate: A focus on Latin America. *World Development 22*, 501–516.

Sethuraman, S. (1981). The urban informal sector in developing countries: Employment, poverty and environment. *International Labour Office, Geneva*.