On the Fair Division of Greenhouse Gas Abatement Cost

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Downloadable Appendix

(ftp://ftp.zew.de/pub/zew-docs/div/fair.pdf)

Appendix A: Algebraic Model Summary

This section provides an algebraic summary of equilibrium conditions for an intertemporal multi-region Ramsey model designed to investigate the economic implications of carbon abatement strategies for the world economy. The following key assumptions apply:

- Output and factor prices are fully flexible and markets are perfectly competitive.
- Labor force productivity increases at an exogenous growth rate (Harrod-neutral technological progress).
- In equilibrium there is a period-by-period balance between exports from each region and global demand for those goods. The model adopts the Armington assumption for export and import markets of a non-energy macro good to differentiate between commodities produced for the domestic market, the export market and the import market. Fossil fuels are treated as perfect substitutes on international markets.
- In each region, a representative consumer (respectively the social planner) maximizes the present value of lifetime utility subject to (i) an intertemporal balance of payments constraint, (ii) the constraint that the output per period is either consumed (incl. intermediate demand and exports) or invested, and (iii) the equation of motion for the capital stock, i.e. capital stocks evolve through depreciation and new investment. This renders the optimal level of consumption and investment over time.
- The agents have an infinite horizon, and their expectations are forward looking and rational. To approximate an infinite horizon model with a finite horizon model we assume that the representative consumer purchases capital in the model's post-horizon period at a price which is consistent with steady-state equilibrium growth (terminal condition).
- In each region, there are backstop technologies for producing the industrial energy aggregate and the household energy aggregate. The backstop technology defines the price for a carbon-free energy source in infinite supply (e.g. photovoltaic, fuel cells) and provides an upper limit on the marginal costs of reducing carbon emissions. The backstops are produced in each region through the employment of the region's non-energy macro good.

The model is formulated as a system of nonlinear inequalities using GAMS/MPSGE (Rutherford 1999) and solved using PATH (Dirkse and Ferris 1995). The inequalities correspond to the three classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero-profit) conditions for constant returns to scale producers, (ii) market clearance for all goods and factors, and (iii) income balance for the representative consumers in each region. The fundamental unknowns of the system are three vectors: activity levels (production indices), non-negative prices, and consumer incomes. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint, a commodity price to a market clearance condition, and a consumer income variable to an income definition equation. An equilibrium allocation determines production, prices and incomes.

In the following algebraic exposition, the notation $\prod^X$ is used to denote the zero-profit function of activity *X*. Formally, all production activities exhibit constant returns to scale, hence differentiating $\prod^X$ with respect to input and output prices provides compensated demand and supply coefficients, which appear subsequently in the market-clearance conditions. All prices are expressed as present values.

Exhaustion of Product Conditions

Macro Good Production

Aggregate output in region *r* describes the supply of the non-energy macro good to the domestic market and export market. A separable nested constant elasticity of substitution (CES) cost function is employed to specify the substitution possibilities between capital (*K*), labor (*L*) and an energy composite (*E*). At the top level, a constant elasticity describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital. At the second level capital and labor trade off with a unitary elasticity of substitution. On the output side, production is split between goods produced for the domestic market and goods produced for the export market according to a constant elasticity of transformation. The (intra-period) zero-profit condition for the production of the macro good is:

$$\prod\nolimits_{rt}^{Y} = (\theta_r^X \; {p_{rt}^{X}}^{1-\eta_r} + (1-\theta_r^X)\, p_{rt}^{1-\eta_r})^{\frac{1}{1\text{-}\eta_r}} \text{ - } \left[\theta_r^{EY} \left(\frac{p_{rt}^{EY}}{\beta_{rt}}\right)^{1\text{-}\sigma_r^{KLE}} + (1-\theta_r^{EY})\left(w_{rt}^{\alpha_r} v_{rt}^{1-\alpha_r}\right)^{1\text{-}\sigma_r^{KLE}} \right]^{\frac{1}{1\text{-}\sigma_r^{KLE}}} = 0$$

where:

p_{rt}^X	output price of macro good produced in region *r* and period *t* for export market,
p_{rt}	output price of macro good produced in region *r* and period *t* for domestic market,
p_{rt}^{EY}	price of industrial energy aggregate for macro good production in region *r* and period *t,*
w_{rt}	wage rate in region *r* and period *t,*
v_{rt}	rental price of capital services in region *r* and period *t,*
θ_r^X	benchmark share of exports in macro good production of region *r,*

θ_r^{EY}	benchmark share of industrial energy aggregate in macro good production of region *r*,
α_r	benchmark share of labor in value-added of macro good production in region *r*,
η	elasticity of transformation between production for the domestic market and production for the export market,
σ_r^{KLE}	elasticity of substitution between the energy aggregate and value-added in production for region *r*,
β_{rt}	exogenous energy efficiency improvement index, which measures changes in technical efficiency for region *r* in period *t*,

and

Y_{rt}	associated dual variable which indicates the activity level of macro good production in region *r* and period *t*.

Fossil Fuel Production

The production of fuels requires inputs of domestic supply (macro good) and a fuel-specific factor which can be thought of as a sector-specific resource.[1] The zero-profit condition has the form:

$$\Pi_{rt,ff}^{F} = p_t^{ff} - \left[\theta_r^{ff} {q_{rt}^{ff}}^{1-\sigma_r^{ff}} + (1-\theta_r^{ff}) {p_{rt}^{A}}^{1-\sigma_r^{ff}}\right]^{\frac{1}{1-\sigma_r^{ff}}} = 0 \qquad \text{ff} \in \{\text{COA,OIL,GAS}\}$$

where:

p_t^{ff}	world market price of fossil fuel *ff* in period *t*,
q_{rt}^{ff}	price of fuel-specific resource for production of fossil fuel *ff* in region *r* and period *t*,
p_{rt}^{A}	Armington price of macro good in region *r* and period *t*,
θ_r^{ff}	benchmark share of fuel-specific resource for fossil fuel production in region *r*,
σ_r^{ff}	elasticity of substitution between the fuel-specific resource and non-energy inputs in fossil fuel production of region *r*,

and

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[1] A constant returns to scale production function with convex levelsets exhibits decreasing returns to scale in *remaining* factors when one or more inputs are in fixed supply. We exploit this result in representing a decreasing returns to scale function through a constant returns to scale activity which uses the fuel-specific factor.

F_{rt}^{ff} associated dual variable which indicates the activity level of fossil fuel production *ff* in region *r* and period *t*.

The value of the elasticity of substitution σ_r^{ff} between non-energy inputs and the fuel-specific resource determines the price elasticity of fossil fuel supply ε_r^{ff} at the reference point, according to the relation:

$$\varepsilon_r^{ff} = \sigma_r^{ff} \frac{\theta_r^{ff}}{1-\theta_r^{ff}} .$$

Armington Production

Inputs of the macro good into energy production, investment demand and final consumption are a composite of a domestic and imported variety which trade off with a constant elasticity of substitution. The corresponding zero profit condition for the production of the Armington good is given by:

$$\prod_{rt}^{A} = p_{rt}^{A} - \left[\theta_r^A \, p_{rt}^{1-\sigma_r^A} + (1-\theta_r^A) \left[\left(\sum_s \theta_{sr}^M \, p_{st}^{X^{1-\sigma_r^M}} \right)^{\frac{1}{1-\sigma_r^M}} \right]^{1-\sigma_r^A} \right]^{\frac{1}{1-\sigma_r^A}} = 0$$

where:

θ_r^A benchmark share of domestic macro input into Armington production in region *r*,

θ_{sr}^M benchmark share of imports from region *s* (aliased with index *r*) in total macro good imports of region *r*,

σ_r^A Armington elasticity of substitution between domestic macro good and imported macro good aggregate for region *r*,

σ_r^M elasticity of substituion between macro good imports for region *r*,

and

A_{rt} associated dual variable which indicates the activity level of Armington production in region *r* and period *t*.

Production of the Industrial Energy Aggregate

Energy inputs to the macro production are a nested separable CES aggregation of oil, gas and coal. Gas and oil trade off as relatively close substitutes in the lower nest of the energy composite; at the next level the oil and gas composite combines with coal at a lower rate. The zero-profit condition for the production of the industrial energy aggregate is:

$$\prod_{rt}^{EY} = p_{rt}^{EY} - \{\theta_r^{COA}(p_t^{COA} + pcarb_{rt}CO2_{COA})^{1-\sigma_r^{COA}} + (1-\theta_r^{COA})$$

$$[\theta_r^{OIL}(p_t^{OIL} + pcarb_{rt}CO2_{OIL})^{1-\sigma_r^{LQ}} + (1-\theta_r^{OIL})(p_t^{GAS} + pcarb_{rt}CO2_{GAS})^{1-\sigma_r^{LQ}}]^{\frac{1-\sigma_r^{COA}}{1-\sigma_r^{LQ}}}\}^{\frac{1}{1-\sigma_{COA}}} = 0$$

where:

$pcarb_{rt}$	carbon price in region *r* and period *t*,
$CO2_{ff}$	physical carbon coefficient for fossil fuels,
θ_r^{COA}	benchmark share of coal input into industrial energy aggregate of region *r*,
θ_r^{OIL}	benchmark share of the oil input into the gas and oil composite of industrial energy production in region *r*,
σ_r^{COA}	elasticity of substitution between coal and the gas and oil composite in industrial energy production of region *r*,
σ_r^{LQ}	elasticity of substitution between gas and oil in industrial energy production of region *r*,

and

EY_{rt}	associated dual variable which indicates the activity level of industrial energy aggregate production in region *r* and period *t*.

Production of the Household Energy Aggregate

Energy demanded by the household is a CES aggregate of fossil fuels. The zero-profit condition for the production of the household energy aggregate has the form:

$$\prod_{rt}^{EC} = p_{rt}^{EC} - \left(\sum_{ff}\theta_{r,ff}^{EC}(p_t^{ff} + pcarb_{rt}CO2_{ff})^{1-\sigma_r^{EC}}\right)^{\frac{1}{1-\sigma_r^{EC}}} = 0$$

where:

p_{rt}^{EC}	price of household energy aggregate for region *r* and period *t* ,
$\theta_{r,ff}^{EC}$	benchmark share of fossil fuel input *ff* in the household energy aggregate of region *r*,
σ_r^{EC}	elasticity of substitution between fossil fuel inputs within the household energy aggregate,

and

EC_{rt}	associated dual variable which indicates the activity level of household

energy aggregate production in region *r* and period *t*.

Production of the Household Consumption Aggregate

In final consumption demand the household energy aggregate trades off with the macro good at a constant elasticity of substitution:

$$\prod_{rt}^{C} = p_{rt}^{C} - \left(\theta_r^C \, {p_{rt}^{A}}^{1-\sigma_r^C} + (1-\theta_r^C)\, {p_{rt}^{EC}}^{1-\sigma_r^C} \right)^{\frac{1}{1-\sigma_r^C}} = 0$$

where:

p_{rt}^{C}	price of household consumption aggregate for region *r* and period *t*,
θ_r^C	benchmark share of macro good into aggregate household demand of region *r*,
σ_r^C	elasticity of substitution between macro good and energy aggregate in household consumption demand of region *r*,

and

C_{rt}	associated dual variable which indicates the activity level of household consumption in region *r* and period *t*.

Backstops for Industry and Household Energy Aggregate

For each region there is a carbon-free backstop for the industrial energy aggregate and the household aggregate. This backstop is available in infinite supply at a price which is calculated to be a multiple of the macro good price. Below, we take explicit account of the non-negativity constraint for backstop production:

$$\prod_{rt}^{\tau} = p_{rt}^{\tau} - a_r^{\tau} p_{rt}^{A} \le 0 \qquad \tau \in \{BC, BY\}$$

where:

p_{rt}^{τ}	price of energy backstop for industry (τ = BY) or household (τ = BC) ,
a_r^{τ}	multiplier of the macro good price index for industrial energy backstop (τ = BY) or household energy backstop (τ = BC),

and

BY_{rt} ,BC_{rt}	are the associated dual variables which indicate the activity levels of backstop energy production in region *r* and period *t* for industries or households.

Capital Stock Formation and Investment

An efficient allocation of capital, i.e. investment over time assures the following intertemporal zero-profit conditions which relates the cost of a unit of investment, the return to capital and the purchase price of a unit of capital stock in period *t:* [2]

$$P_t^K = v_{rt} + (1 - \delta)\, P_{t+1}^K$$

and

$$\Pi_{rt}^I = p_{rt}^I - p_{r,t+1}^K \geq 0$$

where:

P_{rt}^K	value (purchase price) of one unit of capital stock in region *r* and period *t,*
δ_r	depreciation rate in region r,
p_{rt}^I	cost of a unit of investment in period *t* which in our case equals p_{rt}^A,

and

K_{rt}	associated dual variable, which indicates the activity level of capital stock formation in region *r* and period *t,*
I_{rt}	associated dual variable, which indicates the activity level of aggregate investment in region *r* and period *t* [3].

Market Clearance Conditions

Labor

The supply-demand balance for labor is:

$$\bar{L}_{rt} = Y_{rt} \frac{\partial \Pi_{rt}^Y}{\partial w_{rt}}$$

where:

$\bar{L}_{rt}$	exogenous endowment of time in region *r* and period *t.*[4]

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[2] The optimality conditions for capital stock formation and investment are directly derived from the maximization of lifetime utility by the representative household taking into account its budget constraint, the equation of motion for the capital stock and the condition that output in each period is either invested or consumed. Note that in our algebraic exposition we assume an investment lag of one period.

[3] As written, we have taken explicit account of the non-negativity constraint for investment.

[4] Time endowment grows at a constant rate *g*, which determines the long-run (steady-state) growth rate of the economy.

Capital

The supply-demand balance for capital is:

$$K_{rt} = Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial v_{rt}}$$

Fuel-Specific Resources

The supply-demand balance for fuel-specific resources is:

$$\overline{Q}_{rt}^{ff} = F_{rt}^{ff} \frac{\partial \Pi_{rt,ff}^{F}}{\partial q_{rt}^{ff}} \qquad ff \in \{COA, OIL, GAS\}$$

where:

$\overline{Q}_{rt}^{ff}$ exogenous endowment with fuel-specific resource *ff* for region *r* and period *t.*

Fossil Fuels

The supply-demand balance for fossil fuels is:

$$\sum_{r} F_{rt}^{ff} = \left(\sum_{r} EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial (p_{t}^{ff} + pcarb_{rt} CO2_{ff})} + EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial (p_{t}^{ff} + pcarb_{rt} CO2_{ff})} \right) \quad ff \in \{COA, OIL, GAS\}$$

Macro Output for Domestic Markets

The market clearance condition for the macro good produced for the domestic market is:

$$Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial p_{rt}} = A_{rt} \frac{\partial \Pi_{rt}^{A}}{\partial p_{rt}}$$

Macro Output for Export Markets

The market clearance condition for the macro good produced for the export market is:

$$Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial p_{rt}^{X}} = \sum_{s} A_{st} \frac{\partial \Pi_{st}^{A}}{\partial p_{st}^{X}}$$

Industrial Energy Aggregate

The market clearance condition for the industrial energy aggregate is:

$$EY_{rt} + BY_{rt} = EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial p_{rt}^{EY}}$$

Household Energy Aggregate

The market clearance condition for the household energy aggregate is:

$$EC_{rt} + BC_{rt} = EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial\ p_{rt}^{EC}}$$

Armington Aggregate

The market clearance condition for Armington aggregate is:

$$A_{rt} = Y_{rt} \frac{\partial \Pi_{rt}^{Y}}{\partial\ \ p_{rt}^{A}} + C_{rt} \frac{\partial \Pi_{rt}^{C}}{\partial\ p_{rt}^{A}} + I_{r} \frac{\partial \Pi_{rt}^{I}}{\partial p_{rt}^{A}} + BY_{rt} \frac{\partial \Pi_{rt}^{BY}}{\partial p_{rt}^{A}} + BC_{rt} \frac{\partial \Pi_{rt}^{BC}}{\partial p_{rt}^{A}}$$

Household Consumption Aggregate

The market clearance condition for the household consumption aggregate is:

$$C_{rt} = D_{rt}$$

where:

D_{rt}	uncompensated final demand which is derived from maximization of lifetime utility (see below).

Income Balance of Households

Consumers choose to allocate lifetime income across consumption in different time periods in order to maximize lifetime utility. The representative agent in each period solves:

$$\text{Max} \sum_{t} \left(\frac{1}{1+\rho_{r}} \right)^{t} u_{r}(C_{rt})$$

$$\text{s.t.} \sum_{t} p_{rt}^{C} C_{rt} = M_{r}$$

where:

u_{r}	instantaneous utility function of representative agent in region *r,*
ρ_{r}	time preference rate of representative agent in region *r,*

and

M_{r}	lifetime income of representative agent in region *r*.

Lifetime income *M* is defined as:

$$M_r = p_{r0}^{K}\overline{K}_{r0} + \sum_t w_{rt}\overline{L}_{rt} + \sum_{ff} q_{rt}^{ff}\overline{Q}_{rt}^{ff}$$

$$+\sum_t \sum_{ff} pcarb_{rt} CO2_{ff}\left(EY_{rt}\frac{\partial \Pi_{rt}^{EY}}{\partial (p_t^{ff} + CO2_{ff}\, pcarb_{rt})} + EC_{rt}\frac{\partial \Pi_{rt}^{EC}}{\partial (p_t^{ff} + CO2_{ff}\, pcarb_{rt})}\right)$$

where:

$\overline{K}_{r0}$ initial capital stock in region *r*.

With isoelastic lifetime utility the instantaneous utility function is given as:

$$u_r(C_{rt}) = \frac{C_{rt}^{1-\frac{1}{\mu_r}}}{1-\frac{1}{\mu_r}}$$

where:

μ_r constant intertemporal elasticity of substitution.

The uncompensated final demand function D_{rt} is then derived as:

$$D_{rt}(p_{rt}^{C}, M) = \frac{(1+\rho_r)^{-t\mu_r}}{\sum_t (1+\rho_r)^{-t^{\mu_r}}\, p_{rt}^{C\,1-\mu_r}}\frac{M}{p_{rt}^{C\,\mu_r}}$$

Terminal Constraints

The finite horizon poses some problems with respect to capital accumulation. Without any terminal constraint, the capital stock at the end of the model's horizon would have no value and this would have significant repercussions for investment rates in the periods leading up to the end of the model horizon. In order to correct for this effect we define a terminal constraint which forces terminal investment to increase in proportion to final consumption demand:[5]

$$\frac{I_{Tr}}{I_{T-1,r}} = \frac{C_{Tr}}{C_{T-1,r}}.$$

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[5] This constraint imposes balanced growth in the terminal period but does not require that the model achieves steady-state growth (see Lau, Pahlke and Rutherford 2001).

Appendix B: Parameterization

Benchmark data are used to calibrate parameters of the functional forms from a given set of quantities, prices and elasticities (see Table 1 below). Data from four different sources are combined to yield a consistent benchmark data set:

- *GTAP database* (McDougall et al. 1998): GTAP includes detailed input-output tables for 45 regions and 50 production sectors as well as a world trade matrix with bilateral trade flows for all sectors and regions.
- *IEA energy balances and energy prices/taxes* (IEA 1996): IEA provides statistics on physical energy flows and energy prices for industrial and household demands.
- *IIASA/WEC* (IIASA 1998): IIASA/WEC makes projections on the future development of world GDP and fossil fuel production for the 21st century differentiated by countries.
- *World Population Prospects* (UN 1996): This source provides data on population growth till 2050 for 194 countries plus summary groups.

We replace GTAP's aggregate input-output monetary values for energy supply and demand with physical energy flows and energy prices as given in IEA's energy statistics to obtain a "bottom-up" calibration of energy demands and supplies.

Dynamic models in applied CGE analysis are often calibrated to a steady state growth path in which all physical quantities grow at exogenous rates.[6] In our analysis we incorporate the IIASA/WEC projections on non-uniform potential growth rates for GDP and fossil fuel production across countries. The exogenous assumptions on fossil fuel production for our business-as-usual (*BaU*) scenario imply a reference emission level for the world as a whole. At the country level, the *BaU* emission trajectory determines the extent to which restrictions of emission entitlements as prescribed by *COV* and *EPC* bind economies in the future.

Table B1 summarizes the central values for key elasticities underlying our core simulations. Our elasticities are based mainly on econometric evidence as summarized, e.g., by Burniaux et al. (1992), Jomini et al. (1991) Sawyer and Sprinkle (1999) and Dimaranan et al. (2001). With respect to σ^A, our setting represents the rather general finding that the elasticity of substitution between domestic and imported goods (Armington elasticity) is

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[6] The virtue of the steady state calibration is that the amount of exogenous information which goes beyond the explanatory scope of the model is kept at a minimum.

relatively low.[7] With respect to σ^M we follow Jomini et al. (1991) and Dimaranan et al. (2001) by setting this elasticity twice as high as the import-domestic elasticity (σ^A). Both σ^A and σ^M are lower than the values employed in the GTAP project (Dimaranan et al. 2001), but it should be noted that the latter reflect an upward adjustment of econometric evidence justified by "prior belief" of these authors. Moreover, the commodity disaggregation in GTAP is finer, implying higher elasticity values than in our framework (see Panagarya et al. 2001 for the relationship between elasticities and the level of aggregation). The other elasticities represent central values from the econometric evidence as summarized in the references mentioned above. Especially, our setting of $\sigma^{KLE} < 1$ is consistent with results from Kemfert and Welsch (2000) and Welsch and Ochsen (2005).

Table B1: Overview of key elasticities

Type of elasticity	Description	Central Value
Armington elasticity of substitution	Degree of substitutability	
	• Between macro imports from different regions	2
	• Between the import aggregate and the domestically produced macro good	1
Armington elasticity of transformation	Degree of substitutability between macro good produced for the domestic market and macro good destined for the export market	2
Price elasticity of fossil fuel supply	Degree of response of international fossil fuel supply to changes in fossil fuel price	1 (coal) 4 (gas) 8 (oil)
Elasticity of substitution between non-energy and energy composite in production and final demand	This value increases linearly over time between a short-run value of 0.2 and the long-run value of 0.8 to reflect empirical evidence on differences between short-run and long-run adjustment costs (Lindbeck, 1983)	0.2 (short run: 2000) 0.8 (long run: 2050)
Interfuel elasticity of substitution	Degree of substitutability between fossil fuels (fuel switching)	0.5 (final demand) 2[a],1[b] (industry)

[a] between oil and gas [b] between coal and the oil-gas aggregate

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[7] Armington elasticities have been estimated for several countries, e.g., Lächler (1985) for Germany; Corado and de Melo (1986) for Portugal; Shiells, Stern and Deardorff (1986), Reinert and Roland-Holst (1992), and Blonigan and Wilson (1999) for the U.S.; Kapuscinski and Warr (1999) for the Philippines; Panagarya, Shah, and Mishra (2001) for Bangladesh), Welsch (2001) for France, Germany, Italy and the U.K.

Appendix C: Implementation of Bounded Walrasian Solution

The constraint that imposes the upper utility bound in our model is:

$$U_r \leq \overline{U}_r^{BaU}$$

where:

$\overline{U}_r^{BaU}$ welfare of region *r* under business-as-usual (BaU), i.e. the reference situation without carbon emission constraints

and

m_r the associated dual variable for region *r* that adjusts lifetime income of region *r* to assure the upper bound utility (if a region's welfare increases above the level of the upper bound, the constraint binds which implies that m_r is no longer zero).

The lump-sum payments of regions with a binding upper bound are allocated across all other regions on an equal per capita basis (with populations being weighted over time). The respective constraint reads as:

$$trans_r \geq \frac{\sum_s m_s}{\sum_s \overline{pop}_s} \overline{pop}_r$$

where:

$trans_r$ the endogenous income transfer to region r

and

$\overline{pop}_r$ is the population index for region *r*.

Lifetime income M_r of the representative agent in region *r* adjusted for the bounded Walrasian solution is:

$$\begin{aligned} M_r = {} & p_{r0}^K \overline{K}_{r0} + \sum_t w_{rt} \overline{L}_{rt} + \sum_{ff} q_{rt}^{ff} \overline{Q}_{rt}^{ff} \\ & + \sum_t \sum_{ff} pcarb_{rt} CO2_{ff} \left(EY_{rt} \frac{\partial \Pi_{rt}^{EY}}{\partial (p_t^{ff} + CO2_{ff}\, pcarb_{rt})} + EC_{rt} \frac{\partial \Pi_{rt}^{EC}}{\partial (p_t^{ff} + CO2_{ff}\, pcarb_{rt})} \right) \\ & - m_r + trans_r \end{aligned}$$

Appendix D: Emission-based Bounded Walrasian Solution

The bounded Walrasian solution to an entitlement scheme (COV or EPC in our case) has been implemented by lump-sum wealth transfers between regions. The objective is to find an endowment vector of emission rights such that their unconstrained trading on competitive markets leads to the bounded Walrasian solution.

There are two conditions to assure equivalence. Firstly, the sum of transfers Δe_{rt} of emission rights across countries *r* must be zero for any period *t* along the timepath
to yield the same overall emission trajectory:

$$\sum_r \Delta e_{rt} = 0 \quad \forall t$$

Secondly, for each region, the discounted (present) value of emission rights transfers must be equal to the lump-sum transfer in lifetime income Δw_r as determined by the initial way to implement the bounded Walrasian solution. Using the carbon permit price $pcarb_t$ that emerges from the latter, the second condition can be written as:

$$\sum_t pcarb_t \, \Delta e_{rt} = \Delta w_r \quad \forall r$$

In principle, there are numerous emission transfer schemes that meet the above conditions. For reasons of practical (acceptable) policy making, we opt for a scheme that provides regions with their business-as-usual emissions in the first period and implies a linear course of regions' emission entitlement trajectories thereafter. We can solve for this using a simple least-square routine.

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