Environmental Tax Differentiation between Industries and Households - Implications for Efficiency and Employment

A Multi-Sector Intertemporal CGE Analysis for Germany

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Abstract

This paper investigates the economic impacts of environmental tax reforms designed to reach given emission reduction targets for the German economy. Our focus is on the efficiency and employment implications of alternative schemes for emission tax differentiation between the production sector and the household sector. We point out that strong tax discrimination in favor of the production sector may cause substantial excess costs. Differences in the emission tax base and the respective ease of emission mitigation across the production sector and the household sector are shown to play a crucial role for explaining our results.

Keywords: environmental taxes, taxing production vs. taxing consumption, environmental tax reforms, computable general equilibrium

JEL-classification: H21, D58, Q48
1. Introduction

In order to reduce their greenhouse gas emissions several OECD countries have meanwhile implemented some type of carbon or energy tax (OECD 2001). Nearly all tax schemes involve reduced tax rates or tax exemptions for industries that are energy intensive and/or export-oriented. The main motivation for this type of unequal treatment is to maintain competitiveness and sectoral employment in these industries as compared to trading partners who do not levy similar environmental taxes. In the same vein, concerns about adverse impacts of environmental taxes on employment motivate the recycling features of environmental tax schemes in various OECD countries: additional tax revenues are used to cut labor costs. Given persistently high unemployment rates in OECD countries, such a green tax reform is hoped to yield a "double dividend" in terms of both reduced emissions and increased employment (see e. g. Pearce 1991 or Repetto 1992). Moreover, as marginal tax rates on labor are high relative to other taxes in most OECD countries, the swap of green taxes for labor taxes suggests potential welfare gains from a more efficient tax system, sometimes referred to as the third (efficiency) dividend of a green tax reform.

In this paper we investigate the scope for a triple dividend putting special emphasis on the economic implications of emission tax differentiation between the production sector and the final consumption sector. The key insights gained from numerical simulations with a multi-sectoral intertemporal general equilibrium model for Germany are as follows:

- Even for relatively small emission reduction targets the hopes for a second or third dividend fail. The reason is that environmental taxes not only introduce new distortions of their own in intermediate or consumer goods markets but also cause market distortions similar to those of the replaced taxes.

- Given some exogenous emission reduction target, the economic implications of tax differentiation between the production sector and the household sector are relatively similar for a broader range of tax ratios. We may tax households at much higher rates than the production sector - and vice versa - without significant excess costs. However, as tax differentiation comes close to exempting the production, substantial excess costs of the environmental regulation are induced. In this case, large and cheap emission substitution possibilities in the production sector are given up which must be compensated by costly emission reduction in the household sector.

- If one had to choose between full exemption of the production sector versus full exemption of the consumption sector the latter would be much more preferable on efficiency and employment grounds. Due to the large emission base and better substitution possibilities of the production sector relative to the consumption sector, it is less distorting to drop emission taxes on consumption than on production.
Our findings contrast the results of some other studies which favor the tax exemption of the production sector based on either theoretical analysis (Richter and Schneider 2001) or numerical simulations (Wiegard and Ruocco 1997). The theoretical approach to the analysis of energy tax differentiation uses the framework of optimal taxation. It is of course insightful - as done in Richter and Schneider (2001) - to add some empirically relevant features in existing theoretical models and analyze how this will affect previous qualitative results. However, the analytical derivation of optimal taxes is already quite complex under very simplified assumptions. In other words, even for a very simplistic representation of the real economy, an analytical solution may not deliver any results for a sound economic interpretation. A complementary approach - followed here - is the simulation analysis of tax reforms on a numerical basis. Ruocco and Wiegard (1997) provide an earlier example of this approach for the policy problem of energy tax differentiation but employ a "toy" model with stylized data. The latter appears to be the major reason why they come to different conclusions. Unlike in the real world they assume the energy (emission) consumption of the production sector to be much lower than that of the household sector. This assumption joined with unrealistically high substitution elasticities between energy and other goods in final consumption compared to production implies that tax exemption of the production sector is potentially less costly than exemption of the household sector.

There have been several other studies on the economic and environmental effects of green tax reforms for Germany based on numerical large-scale models and real data (e.g. Conrad and Wang 1993, DIW 1994, Welsch and Hoster 1995, Böhringer et al. 1997, Böhringer and Rutherford 1997, Meyer et al. 1997, Welsch and Ehrenheim 2000). The evidence on employment and gross efficiency effects is mixed, partly due to differences in the concrete tax reform scenarios considered but more so due to differences in modeling assumptions with respect to existing tax distortions, foreign closure and labor market imperfections. None of these studies address the issue of tax differentiation in a structured way. Moreover, analyses of alternative tax policies often focus on the pure public finance aspects of green tax reforms neglecting the fact that an appropriate comparison requires to keep not only the level of public good provision constant but also the level of the environmental target. As illustrated by Böhringer, Ruocco and Wiegard (2001) this has important implications for the conclusions with respect to cost-effective tax setting.

The remainder of this paper is as follows. Section 2 entails a non-technical summary of the model used for our analysis. Section 3 lays out the policy scenarios and provides an interpretation of the simulation results. Section 4 concludes.

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1 This should not be construed as an argument against theoretical analysis which provides the base for understanding and explaining key economic relationships also in more complex numerical settings.

2 Böhringer et al. (1998) make a critical cross-comparison of studies linking the differences in results to differences in underlying model assumptions.
2. Model and Parameterization

This section provides a non-technical description of the intertemporal multi-sector, multi-region model designed for the analysis of (green) tax reforms in open economies. Our model combines several features that are required for an appropriate quantitative simulation of the effects induced by green tax reforms:

- At the sectoral level it incorporates sufficient detail on sector-specific differences in emission and factor intensities, degrees of input substitutability and price elasticities of demand in order to trace back the structural change in production induced by a policy shift.
- The German tax system is represented in sufficient detail to capture initial tax distortions which might give scope for efficiency gains from tax reforms.
- The phenomenon of persistent (equilibrium) unemployment is represented based on the established notion of a "wage curve", which can be derived from trade union wage models as well as from efficiency wage models (see e.g. Beißinger 1996 or Hutton and Ruocco 1999).
- Consumption and investment decisions are based on rational expectations of future prices (clairvoyance). This assures that the effects of policy interference on savings and investments are consistently taken into account.
- Apart from a complex nesting of constant-elasticity-of-substitution (CES) functions to capture the technological options for emission abatement via fuel-switching and energy savings, energy demands and energy supplies are based on actual data of physical energy flows and energy market prices. This bottom-up calibration increases the credibility of substitution possibilities associated with top-down CES cost and expenditure functions.
- Adjustment costs of capital stocks are incorporated via a putty-clay approach (Phelps 1963). This formulation accommodates premature retirement of extant capital due to larger policy shocks.
- Capital is internationally mobile with the rates of return determined by an exogenous international interest rate.

2.1. Basic Model Structure

The model contains a disaggregate representation of 10 industries. To account for different pollutant and energy intensities across energy goods, the model identifies 6 primary and secondary energy goods: hard coal (HCO), soft coal (SCO), gas (GAS), crude oil (CRU), refined oil products (OIL) and electricity (ELE). In addition, the model incorporates important carbon-intensive and energy-intensive industries which are most susceptible to the effects of carbon abatement policies: iron and steel (ORE), chemical products (CHM) and an aggregate of other energy intensive industries (EIS). The rest of the production side is then aggregated to one sector Y. Producer goods are directly demanded by government, investment and export. Producer goods for consumption are demanded only

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3 See Appendix for an algebraic model description.
indirectly because the model distinguishes 13 aggregate consumption categories which are produced by combining the outputs of the 10 industries in fixed proportions. Table 1 summarizes the classification of industry and consumer commodities.

**Production**

Competitive entrepreneurs minimize the cost of production and allocate investment across sectors in order to maximize the present value of firms. For each industry, an aggregate production function characterizes technology through transformation possibilities on the output side (between production for domestic and export markets) and substitution possibilities on the input side (between alternative combinations of inputs). On the output side, production is split between goods produced for the domestic market and goods produced for the export market subject to a constant elasticity of transformation. On the input side, nested separable CES functions describe the technological substitution possibilities in the domestic production between capital, labor, energy and material inputs. At the top level, material inputs are used in fixed proportions, together with an aggregate of energy and a value-added composite of labor and capital. The value-added composite is a CES function of labor and capital. The energy aggregate is produced with a CES function of a primary energy composite and electricity. The primary energy composite is then a CES composite of a coal aggregate (within which hard and soft coal are traded off at a constant elasticity of substitution) and a liquid fuel aggregate (within which oil and natural gas are traded off at a constant elasticity of substitution).

To reflect empirical evidence on differences between cost functions in the short run and in the long run the description of production is based on a partial putty-clay approach, which incorporates short run adjustment costs including the premature retirement of extant capital.\(^4\)

**Household Behavior**

Consumers choose to allocate lifetime income across consumption in different time periods. In each period the consumer faces the choice between current consumption (non-leisure consumption goods and leisure) and savings (future consumption). The pure rate of time preference determines the intertemporal allocation of consumption. We employ a separable intertemporal utility function where the intra-period utility from consumption is based on a nested CES function over leisure and non-leisure consumption commodities.

**Factors**

Primary factors of production are labor, sector-specific (extant) capital and capital which is freely mobile across sectors and domestic boundaries. Labor supply is elastic and inter-sectorally

\(^4\) In our baseline calculations we assume that 90\% of the initial capital operates as a fixed Leontief technology. Substitution between various forms of energy, material, capital and labor are possible only for 10\% of the capital stock in the initial period. Subsequent replacement of new capital results in an increasing elasticity of substitution between primary factors over time.
mobile within the home country. Total labor endowment increases with labor force efficiency along a steady-state growth rate of 1%. Capital stocks evolve through geometric depreciation and new investment. In the small open economy framework, the rates of return on mobile capital are determined by the international interest rate. With respect to capital we assume perfectly competitive factor markets in which factor prices adjust so that supply equals demand. Labor markets are treated as imperfectly competitive resulting in persistent unemployment. The latter is introduced through the specification of a wage curve, which postulates a negative relationship between the real wage rate and the rate of unemployment (see e.g. Böhringer, Ruocco, and Wiegard, 2001). The wage curve replaces the labor supply curve leading to an equilibrium wage rate above the market clearing wage rate, i.e. unemployment. We use a simple standard specification of the wage curve as a log-linear relationship between the real wage and the unemployment rate (which initially amounts to 10%). Unemployment benefits are assumed to be constant in real terms and are not taxed (see Koskela and Schöb, 1999, for a discussion of alternative forms of unemployment benefits). Welfare effects are also based on enforced leisure consumption.

Government Sector

The government distributes transfers and provides a public good (including public investment), which is produced with commodities purchased at market prices. Government expenditures are financed with tax revenues. The model incorporates the main features of the German tax and social transfer system: labor taxes including social insurance contributions, capital taxes (corporate and trade taxes), value-added taxes and other indirect taxes (e.g. mineral oil tax). In our tax policy simulations, we impose revenue-neutrality in the sense that the provision of the public good is kept constant. Any residual tax revenue is recycled lump-sum or through a reduction in existing taxes. Constant public good provision and separability between private and public good consumption simplifies the analysis of welfare effects induced by alternative tax reforms because we do not have to trade off welfare effects due to changes in public good provision.

The public budget is balanced on an intertemporal basis. Along the baseline growth path, public income and expenditures balance on a period-by-period basis.5

Investment and Savings

The level of savings is endogenously determined by households which maximize lifetime consumption over the time horizon. Firm owners choose investment in order to maximize the present value of the firm. In the simulations, new investment is assumed to have a 3 year gestation. Investors

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5 In the counterfactual scenarios, the application of environmental taxes might result in a temporary public deficit, as the anticipation of future tax revenue permits public expenditure to exceed government income during the initial periods. In all simulations the present value of public expenditure equals the present value of tax revenues.
and households compete for current consumption such that in equilibrium the marginal utilities of savings (future consumption) and demand (current consumption) are equalized.

Foreign Trade

Following the proposition of Armington (1969), domestic and foreign goods are distinguished by origin. This accommodates both imports and exports of the same commodity (crosshauling). Due to lack of more detailed data, domestic and imported varieties of the same good are aggregated with identical shares across all components of final and intermediate demand. Demand for imports stems from cost-minimizing producer behavior and utility maximization of households. On the export side, products destined for domestic and international markets are treated as imperfect substitutes (produced) subject to a constant elasticity of transformation. Germany is treated as small relative to the world market. The small country assumption implies that changes in the level of German exports and imports have no effect on the terms of trade - international prices are exogenously fixed in foreign currency, i.e. export demand and import supply functions are horizontal. International capital flows (borrowing and lending) are endogenous subject to an intertemporal balance of payments constraint, i.e. there is no change in net indebtedness over the model horizon. The imposition of an intertemporally balanced trade account is linked to a variable exchange rate which reconciles the present value of domestic import and foreign export demands.

2.2. Model Parameterization

As is customary in applied general equilibrium analysis, the model is based on economic transactions in a benchmark year, 1995 in our case. Benchmark data determines parameters of the functional forms from a given set of benchmark quantities, prices (expressed in present value), and elasticities. Data stem from the Statistical Federal Office of Germany which provides monetary input-output tables for 58 sectors as well as detailed information on physical energy flows to production sectors and final demand categories (StaBu 1996). We replace the aggregate input-output monetary values for energy supply and demand with physical energy flows supplemented by official energy prices for industry and households. This bottom-up calibration of energy demands and supplies yields sector-specific and energy-specific emission coefficients. The advantage is that marginal abatement cost curves and hence the cost evaluation of emission constraints are based on actual energy flows rather than aggregate monetary data, which strengthens the credibility of the quantitative results. Data on various tax payments and transfers are taken from the Statistical Yearbook (StaBu 1997).

Base year financial statistics indicate the value of payments to capital across sectors and the gross value of capital formation. Using this data, we infer three parameters of the intertemporal model in order to assure consistency with a balanced steady-state growth path. These parameters are the rate of time preference, the depreciation rate of capital, and the growth rate of labor in efficiency units. Assuming an exogenous labor growth rate of 1%, base year capital earnings and investment are
consistent with a time preference rate (interest rate) of 4.6% and a capital depreciation rate of 6.3%. Table 2 summarizes key elasticities.

3. Scenarios and Results

3.1. Scenarios

Within the Kyoto Protocol (UN 1997), Germany has committed itself to reduce its greenhouse gas emissions, most notably carbon emissions from the combustion of fossil fuels. In our policy simulations, we apply endogenous carbon taxes which are sufficiently high to effect a linear reduction of carbon emissions till 2010 and thereafter by 5% (scenario R05), 10% (scenario R10) and 20% (scenario R20) as compared to the respective business-as-usual values. The use of fixed reduction targets provides a meaningful basis for the cross-comparison of alternative carbon tax differentiation schemes. As to the differentiation of carbon taxes between the production and household sector, we consider various schemes between the two extremes of fully exempting the production as well as the household sector. We implement tax differentiation as the ratio between the emission tax on the household sector and the emission tax on the production sector. A tax ratio of 10, for example, indicates that the emission tax paid by households is ten times higher than that paid by the industry. Note that we can only specify the ratio of tax differentiation exogenously. The associated absolute tax rates are determined endogenously subject to the constraint that the exogenous emission reduction target will be met. With respect to the compensating adjustments in other taxes, we restrict ourselves to cuts in labor taxes which - for reasons of unemployment - are the primary candidate considered in actual policy making.

3.2. Economic Intuition

Before we enter interpretation of the concrete simulation results, let us develop the basic economic intuition on our tax policy problem. In simple formal terms, we are challenged to solve a joint environment-public goods optimal tax problem over some set of available tax instruments \( t_k \):

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\max \quad W(t) \quad \text{s.t} \quad E(t) = \overline{E} \quad G(t) = \overline{G}
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where:

- \( W \) is a welfare function,
- \( E \) is the level of pollution,
- \( G \) is the level of public good,
- \( \bar{G} \) is the lower bound for public good supply, and
- \( \bar{E} \) is the upper bound on permissible emissions.

The first order condition for \( t_k \) is then:

\[
\frac{\partial E}{\partial t_k} \lambda_E + \frac{\partial G}{\partial t_k} \lambda_G = \frac{\partial W}{\partial t_k}
\]

where:

- \( \lambda_E \) is the shadow price on the environmental constraint, and
- \( \lambda_G \) is the shadow price on the public goods constraint.

In the standard public finance setting - if the environmental constraint were non-binding, so \( \lambda_E = 0 \) - we obtain the usual optimality condition dictating equalization of the marginal costs of public funds (MCF) for all instruments:

\[
\lambda_G = \frac{\partial W / \partial t_k}{\partial G / \partial t_k} \quad \forall t_k
\]

This could be used in a numerical model setup for a simple recursive procedure to adjust the tax rates to their optimal values by increasing those taxes with a low MCF and decreasing those which have a high MCF.

Alternatively, we could omit the public goods constraint, so \( \lambda_G = 0 \), and focus on the environmental constraint which would yield the well-known Pigouvian tax as a non-discriminating uniform tax across all polluting sources. Reverting to our initial question of environmental tax differentiation, Figure 1 sketches the determinants of excess costs imposed by the deviation from uniform taxation. Given some overall emission reduction target, the optimal contribution of two different sectors - say industry on the one hand and households on the other hand - is given by the intersection of the respective marginal abatement costs. If we distort the optimal allocation of reduction contributions across sectors, the induced efficiency losses do not only depend on the magnitude but also on the direction of shifted emissions. In other words, the costs of differentiating taxes depend on the ratio of tax rates and the choice of sector to be favored or discriminated. In the simulation results below, we will see that the inferior emission substitution possibilities within the
household sector as compared to the industrial sector is the main reason why strong tax differentiation in favor of the industrial sector induces relatively high welfare and employment losses.

The preceding considerations have only focused on one constraint of the joint environment-public goods optimal tax problem. In the real world, which we try to mimic with our simulations, both constraints are binding and we must take into account the environmental impact as well as the impact of the public good provision. In this case, the MCFs do no longer provide sufficient information and it is not so easy to implement an iterative algorithm for finding the optimal tax structure. In our numerical calculations, we simply "brute force" the optimal solution by picking sufficiently small steps for tax differentiation.

3.3. Results

Following the traditional design of policy evaluation in general equilibrium analysis, the scenario evaluation is based on the comparison of alternative equilibria: the impacts of tax policy changes are reported with reference to a steady-state growth path, in which the baseline policy is maintained. Thus, all departures from the steady state can be attributed to the alternative policy. Our focus is on the long-run equilibrium efficiency and employment implications induced by alternative tax differentiation schemes. Hence, we do not discuss the adjustment paths of economic indicators towards the new steady-state but restrict our representation of results to the new steady-state equilibrium values.

Tables 3 to 5 summarize the impacts of the green tax reform on gross efficiency and unemployment rates for different exogenous reduction targets \( R_{0.5}, R_{1.0}, R_{2.0} \) and a large number of tax differentiation schemes (for a graphical representation of these results see Figures 2-4).

The more general finding is that hopes for a second and a third dividend do not materialize. As emission taxes have a small tax base compared to labor taxes the revenue-neutral swap of the former for the latter increases the excess burden of the tax system. Carbon taxes decrease the use of fossil fuels in production and lower the marginal productivity of labor. This downward pressure on the real wage is (even) not offset for the case of revenue-neutral adjustments of the labor tax. The fall in the real wage is associated with an increase in unemployment. Furthermore, we can observe that a move towards higher reduction targets gets increasingly more expensive. The higher the reduction target the further out is the economy on its marginal abatement cost function where additional units of energy savings or fuel switching get more difficult, i.e. more costly. The overproportional increase in marginal abatement costs is reflected in a corresponding rise of the inframarginal costs.

Our key insights refer to the implications of alternative tax differentiation schemes. First of all, our results highlight the importance of existing tax distortions for the cost-effective differentiation of carbon taxes. From a theoretical point of view the result is obvious if in addition to the carbon taxes,
which aim at the achievement of the carbon reduction target, first best (non-distortionary) taxes or transfers, are available in order to finance the public good or to refund carbon taxes to private households. In that case, energy consumption of the private and business sector should be taxed with equal rates (Pigouvian tax). Obviously, the existing German tax system is not first-best. One must, therefore, account for the effects of carbon taxes on the public policy objective of minimizing the excess burden of the available tax instruments. However, it is not possible to derive an analytical formula for our rather complex representation of the real economy which explicitly captures the trade-off between the environmental objective and the public policy objective. Our numerical results suggest that towards higher environmental targets the range for tax differentiation, which restricts excess costs as compared to the optimal carbon tax levels, shrinks. The economic explanation is that with stricter reduction guidelines the environmental objective dominates the public policy objective which implies rather equalization than differentiation of carbon emission taxes.

Let us consider the economic implications of tax differentiation in more detail. Starting with the extreme case of tax exemptions, we can see that full exemption of the production sector causes large excess costs of reaching some given environmental target. These excess costs increase drastically with the level of emission reduction. The large additional costs of exempting the productive use of emissions stem from foregone emission substitution possibilities in this sector: the production sector accounts for nearly 75% of total business-as-usual emissions and - ceteris paribus - has cheaper substitution possibilities (e.g. shifts from coal to gas in power production) compared to the final consumption sector. When we exempt the production sector, large emission taxes must be levied on households to comply with given reduction targets. From the point of view of cost-effectiveness, the contribution of the household sector to the overall reduction target is then much too high whereas the production sector, which is only indirectly affected through declining demand for goods, contributes by far too little. This result clearly contradicts suggestions for tax exemptions of intermediate emission use (see Ruocco and Wiegard 1997). In fact, we draw the opposite conclusion: looking at the other end of tax differentiation options, we find that full exemption of the household sector will cause only very small additional costs compared to the full exemption of the industry. While this scenario seems not particularly realistic given the relative small lobbying power of consumers it nevertheless bears important policy implications. Considering the strongly inferior efficiency properties of production exemptions compared to household exemptions, policy makers or industrial lobbyist will find it hard to push for the former.

Why would full exemption of the household sector hardly matter with respect to the efficiency and employment effects of targeted tax reforms? The emission base of the household sector is relatively small and substitution possibilities are rather expensive. Therefore, at economy-wide scale, the exemption of households causes a relatively small deviation of the resource allocation from the optimal adjustment pattern. The relative importance of taxes on industry as compared to taxes on households based on their uneven emission shares is reflected in the associated carbon tax rates. In real
terms, an exclusive carbon tax on industrial emission use is only slightly higher compared to some optimal level, whereas an exclusive carbon tax on household emission use amounts to a multiple of its optimal level.

We then turn to the issue of tax differentiation when the option of full exemption of either the production or the household sector is excluded. We see that tax differentiation within a broader scale has only negligible implications with respect to efficiency and gross employment. Even for very small tax ratios, i.e. very low emission taxes on the consumptive use of energy compared to the intermediate (industrial) use, the excess costs are limited. Although the real carbon tax rates for households fall sharply beyond their optimal levels, the small emission base of households assures that the compensating rise in the industrial carbon tax, which applies to a much larger base, is rather small and so are the induced deviations from the optimal adjustment to the exogenous emission reduction target. Excess costs of tax differentiation in favor of the industrial sector remain modest as long as they do not enforce a substantial deviation of the industrial carbon tax from its optimal level. In our simulations, this is still the case for tax ratios between 2 and 5: Due to the small emission base of the household sector the household carbon tax must rise steeply to offset additional emissions by the industrial sector. As a consequence, the absolute level of the industrial tax remains close to its optimal level even for tax ratios which are significantly beyond 2 (see also Figure 5). However, when the tax ratio is high enough to imply a substantial drop in the absolute level of the industrial carbon tax, tax differentiation in favor of the industry sector will cause significant excess costs. Unfortunately, current tax differentiation schemes reflect the latter constellation as they foresee effective tax rates for (energy-intensive) industries which are close to zero. We then find ourselves at the very bottom of Tables 1-3 with substantial additional costs due to the preferential treatment of industries.

4. Concluding Remarks

We have analyzed the economic impacts of environmental tax reforms designed to reach given emission reduction targets for the German economy. Our focus has been on the efficiency and employment implications of emission tax differentiation between the production sector and the household sector. Independent of the concrete differentiation scheme adopted our results indicate that - even for relatively low emission reduction targets - there are no prospects of efficiency gains and employment benefits due to a revenue-neutral swap of carbon taxes for labor taxes. As we measure the economic costs of green tax reforms abstracting from environmental benefits, this result can not be construed as an argument against environmental taxation. However, we may conclude that environmental tax reforms have to be justified on the basis of environmental benefits and cannot be considered a "no-regrets" strategy.

With regard to tax differentiation between the production and household sector, differences in the emission tax base and the respective ease of substitution play a crucial role. Under gross efficiency and employment considerations policy makers should abstain from full exemption of industries as this
causes costly deviations from a cost-efficient reduction strategy: the emission base in the production sector as well as carbon substitution possibilities are relatively high so that tax exemption foregoes cheap emission abatement in the production sector at the expense of considerably more costly additional abatement in the household sector. In comparison, full exemption of households induces much smaller excess costs due to the relative small emission tax base and lower substitution possibilities. There is a relatively large scope for tax differentiation between the industry sector and the household sector implying only small distortions with respect to the optimal contributions of both sectors to the overall reduction target. However, actual tax designs such as the current German environmental tax reform include extreme tax differentiation in favor of the (energy-intensive) industries which will cause substantial excess costs.

In our analysis we have not addressed the problem of carbon leakage. Unilateral carbon taxes may increase emissions by non-abating countries due to induced shifts in comparative advantage (Felder and Rutherford 1993). This might justify tax reductions for energy and trade-intensive sectors on global efficiency grounds (Hoel 1996). However, numerical analysis suggests that leakage rates are only of second-order magnitude when we employ empirical estimates for substitution elasticities of traded goods belonging to the same category (see e.g. Böhringer 1998).

References
Beißinger, Th. (1996): Inflation und Arbeitslosigkeit in der Bundesrepublik Deutschland, Marburg.
Meyer, B. et al. (1997): Was kostet eine Reduktion der CO₂-Emissionen, Beiträge des Instituts für empirische Wirtschaftsforschung, Universität Osnabrück, Beitrag Nr. 55
Richter, W.F. and K. Schneider (2001): Energy Taxation: Reasons for Discriminating in Favor of the Production Sector, working paper, Universität Dortmund
Table 1: Overview of producer goods and consumption categories

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<thead>
<tr>
<th>SIO</th>
<th>Classification of producer goods</th>
<th>Shortcut</th>
<th>SIO</th>
<th>Classification of consumer categories</th>
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<td>3</td>
<td>Electric power, steam and hot water</td>
<td>ELE</td>
<td>1</td>
<td>Food and beverages</td>
</tr>
<tr>
<td>4</td>
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<td>GAS</td>
<td>2</td>
<td>Tobacco</td>
</tr>
<tr>
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<td>Hard coal and hard coal products</td>
<td>HCO</td>
<td>3</td>
<td>Clothing</td>
</tr>
<tr>
<td>6*</td>
<td>Soft coal</td>
<td>SCO</td>
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<td>Shoes</td>
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<tr>
<td>8</td>
<td>Crude oil</td>
<td>CRU</td>
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<td>Rents</td>
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<td>9</td>
<td>Chemical products</td>
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<td>Energy (without fuel for traffic)</td>
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<td>Oil products</td>
<td>OIL</td>
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</tr>
<tr>
<td>16</td>
<td>Iron ore and steel</td>
<td>ORE</td>
<td>8</td>
<td>Health products</td>
</tr>
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<td>1,2,7,11-15,17-20,28,32,33</td>
<td>other energy-intensive products</td>
<td>EIS</td>
<td>9</td>
<td>Body care</td>
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<tr>
<td>5,21-31,34,42-58</td>
<td>rest of industry</td>
<td>ROI</td>
<td>10</td>
<td>Traffic (including fuels for traffic)</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>11</td>
<td>Telecommunication</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>12</td>
<td>Education</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>13</td>
<td>Other expenses</td>
</tr>
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* Classification according to SIO (System of Production Sectors for Input-Output-Computations)

** Additional sectors: hard coal and soft coal are subsectors of the aggregate coal sector (SIO: 6)

Table 2: Key elasticities underlying the core simulations

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
<th>Value</th>
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<td>$\sigma_{\text{KLEM}}$</td>
<td>elasticity of substitution between the material inputs and the composite of capital, labor and energy inputs</td>
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<td>$\sigma_{\text{KLE}}$</td>
<td>elasticity of substitution between energy inputs and value-added</td>
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<td>$\sigma_{\text{KL}}$</td>
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<td>elasticity of substitution between electricity and the non-electric energy aggregate of sectoral production</td>
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<td>$\sigma_{\text{LQD}}$</td>
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<tr>
<td>$\sigma_{\text{COA}}$</td>
<td>elasticity of substitution between hard and soft coal inputs in the coal aggregate of sectoral production</td>
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<tr>
<td>$\sigma_{\text{z}}$</td>
<td>elasticity of substitution between different demand categories in intra-period consumption</td>
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<td>$\sigma_{\text{DM}}$</td>
<td>elasticity of substitution between domestic goods and imports (Armington)</td>
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<td>$\sigma_{\text{DX}}$</td>
<td>elasticity of transformation between domestic goods and exports</td>
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<td>uncompensated labor supply elasticity</td>
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<td>$\sigma_{\text{t}}$</td>
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Table 3: Economic effects of tax differentiation for 5% reduction target (R05)

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<th>UR²</th>
<th>WAGE⁴</th>
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<th>CTAX_I⁶</th>
<th>CO2_H⁷</th>
<th>CO2_I⁸</th>
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Keys:

¹Tax ratio: Ratio between the emission tax on the household sector and the emission tax on the production sector (0:= tax exemption of the household sector, ∞:= tax exemption of the production sector)
²HEV: Hicksian equivalent variation in lifetime income ( % change from BaU)
³UR: Unemployment rate (BaU level is 0.1)
⁴WAGE: Real wage ( % change from BaU)
⁵CTAX_H: Carbon tax on household sector (DM95/t CO2)
⁶CTAX_I: Carbon tax on production sector (DM95/t CO2)
⁷CO2_H: Contribution of household emission cutbacks to overall emission reduction
⁸CO2_I: Contribution of industry emission cutbacks to overall emission reduction
Table 4: Economic effects of tax differentiation for 10% reduction target (R10)

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<th>Tax Ratio$^1$</th>
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<th>UR$^3$</th>
<th>WAGE$^4$</th>
<th>CTAX_H$^5$</th>
<th>CTAX_I$^6$</th>
<th>CO2_H$^7$</th>
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Keys:

$^1$ Tax ratio: Ratio between the emission tax on the household sector and the emission tax on the production sector (0:= tax exemption of the household sector, $\infty$: tax exemption of the production sector)

$^2$ HEV: Hicksian equivalent variation in lifetime income (% change from BaU)

$^3$ UR: Unemployment rate (BaU level is 0.1)

$^4$ WAGE: Real wage (% change from BaU)

$^5$ CTAX_H: Carbon tax on household sector (DM95/t CO2)

$^6$ CTAX_I: Carbon tax on production sector (DM95/t CO2)

$^7$ CO2_H: Contribution of household emission cutbacks to overall emission reduction

$^8$ CO2_I: Contribution of industry emission cutbacks to overall emission reduction
Table 5: Economic effects of tax differentiation for 20% reduction target (R20)

<table>
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<th>CTAX_H⁵</th>
<th>CTAX_I⁶</th>
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Keys:

¹Tax ratio: Ratio between the emission tax on the household sector and the emission tax on the production sector (0:= tax exemption of the household sector, ∞:= tax exemption of the production sector)

²HEV: Hicksian equivalent variation in lifetime income (% change from BaU)

³UR: Unemployment rate (BaU level is 0.1)

⁴WAGE: Real wage (% change from BaU)

⁵CTAX_H: Carbon tax on household sector (DM95/t CO2)

⁶CTAX_I: Carbon tax on production sector (DM95/t CO2)

⁷CO2_H: Contribution of household emission cutbacks to overall emission reduction

⁸CO2_I: Contribution of industry emission cutbacks to overall emission reduction
Figure 1: Welfare implications (% change of Hicksian equivalent variation (HEV) in lifetime income as compared to BaU) and implied carbon taxes rates for alternative tax differentiation ratios and scenario R20
Figure 2: Efficiency implications (HEV in lifetime income - % change from BAU)
Figure 3: Employment effects (UR - unemployment rate)
Figure 4: Percentage contribution of production sector to exogenous emission cutback
Figure 5: Welfare implications (% change of Hicksian equivalent variation (HEV) in lifetime income as compared to BaU) and implied carbon taxes rates for alternative tax differentiation ratios and scenario R20

Carbon Tax Rate
in DM$_{95}$/tCO$_2$
Appendix: Algebraic Model Description

A. Summary of the Generic Model

This section provides an algebraic summary of the equilibrium conditions for an intertemporal small open economy model designed to investigate the economic implications of an environmental tax reform. For the generic model with full employment (see section B for the specification of unemployment) the following basic assumptions apply:

(i) Commodity prices and factor prices are fully flexible within competitive markets.
(ii) Labor is intersectorally mobile but not mobile at the international level.
(iii) Capital is freely mobile across sectors and domestic boundaries.\(^9\)
(iv) Labor force productivity (efficiency) increases at an exogenous growth rate.
(v) Capital stocks evolve through constant geometric depreciation and new investment.
(vi) The level of investment in a given period is determined by competitive and individually rational entrepreneurs who allocate investment across sectors in order to maximize the present value of firms. Investors have no money illusion and issues such as debt-versus equity-financing are not considered.
(viii) The public budget is balanced on an intertemporal basis.
(viii) In international trade the domestic economy is considered as sufficiently small. Therefore the effects of exports and imports on international prices can be ignored.\(^10\) Within this small open economy framework, the model adopts the Armington assumption to differentiate between domestically produced commodities and foreign produced commodities in exports and imports. International capital flows (borrowing and lending) are endogenous, subject to an intertemporal balance of payments constraint, i.e. no change in net indebtedness over the model horizon.
(ix) Aggregate consumption and savings are derived from utility maximization of a representative household. To approximate an infinite horizon equilibrium with a finite model we assume that the representative consumer purchases capital in the post-horizon period at a price which is consistent with steady-state equilibrium growth (terminal condition).

\(^9\) The model version used for our simulations incorporates capital adjustment costs: We assume that 90% of the initial capital operates as a fixed-coefficient Leontief technology. Substitution possibilities between various forms of energy, labor, capital and material are possible only for 10% of the initial capital stock and new investment. This (partial) putty-clay formulation has the virtue of reflecting empirical evidence between lower short-run and higher long-run elasticities of input demands and accommodation premature retirement of extant capital.

\(^10\) Foreign export demand and import supply functions are horizontal and hence, can be omitted within the algebraic model formulation.
The model is formulated as a system of nonlinear inequalities. These inequalities correspond to three classes of equilibrium conditions: zero profit, market clearance, and income balance. The fundamental unknowns of the system are three vectors: activity levels, prices, and income levels. In equilibrium, each of these variables is linked to one inequality condition: an activity level to a zero-profit condition, a commodity price to a market-clearance condition, and an income variable to an income-definition equation.

In the following algebraic exposition, the notation \( \Pi_i^X \) is used to denote the profit function of sector \( i \), where \( X \) is the name assigned to this activity. Formally, all production sectors exhibit constant returns to scale (CRS), hence differentiating \( \Pi_i^X \) with respect to input and output prices provides compensated demand and supply coefficients, which appear subsequently in the market-clearance conditions. All prices are expressed as present values reflecting the assumed international interest rate and consumer's intertemporal preferences, i.e. the pure rate of time preference. In order to simplify the notation, time indices are omitted from those equations which are strictly intra-period.

**Zero profit conditions**

*Production*

In domestic production, nested, separable, constant elasticity of substitution (CES) cost functions are employed to specify the substitution possibilities between inputs of capital (\( K \)), labor (\( L \)), an energy composite (\( E \)) and a material composite (\( M \)). The energy composite is made up of the outputs of the energy industries. The materials consists of the outputs of the other non-energy industries. At the top level, the materials composite is employed in fixed proportions with an aggregate of energy, capital and labor. A constant elasticity describes the substitution possibilities between the energy aggregate and the aggregate of labor and capital at the second level. Finally, at the third level capital and labor trade off with a unitary elasticity of substitution. On the output side, production is split between goods produced for the domestic market and goods produced for the world market according to a constant elasticity of transformation. The resulting intra-period zero-profit condition for the production of good \( i \) is:

\[
\Pi_i^Y = \left( \theta_i^X (P_i^l (1 - t_i^l))^\sigma_{\alpha X} + (1 - \theta_i^X)(P_i^X (1 - t_i^X))^\sigma_{\alpha X} \right)^{1/(1+\sigma_{\alpha X})} - (1 - \theta_i^{KLE}) P_i^M
\]

\[
- \theta_i^{KLE} \left[ \theta_i^E P_i^E (1+\sigma_{\alpha E}) + (1 - \theta_i^E) \left( \left( w (1 + t_i^L) \right)^\sigma_{\alpha E} \left( r (1 + t_i^K) \right)^{1-\sigma_{\alpha E}} \right) \right]^{1/\sigma_{\alpha E}} = 0
\]

where:

\( \theta_i^X \) is the benchmark export value share in output of sector \( i \),
$P_i$ is the output price of good $i$,
$t_i^Y$ is the net production (output) tax on good $i$,
$\sigma^{DX}$ is the elasticity of transformation between production for the domestic market and production for the export market,
$PX_i$ is the export price of good $i$ (expressed in domestic currency)$^{12}$,
$\theta^{KLE}_i$ is the benchmark value share of capital, labor and energy inputs (KLE aggregate) in sector $i$,
$\theta^E_i$ is the energy input value share of the KLE aggregate in sector $i$,
$P_i^M$ stands for the composite price of the materials composite input into sector $i$,
$P_i^E$ stands for the composite price of the energy composite input into sector $i$,
$\sigma^{KLE}$ is the elasticity of substitution between the energy aggregate and the aggregate of capital and labor,
$w$ is the economy-wide gross wage rate (net of payroll taxes),
$t_i^L$ is the payroll tax rate in sector $i$,
$r$ is the uniform rate of return on mobile capital (net of capital taxes),
$t_i^K$ is the capital tax rate in sector $i$,
$\theta^L_i$ denotes the value shares of labor in the value added of sector $i$,
and
$Y_i$ is the associated dual variable, which indicates the activity level of production in sector $i$.

**Armington aggregation across imports and goods produced for the domestic market**

Each of the individual inputs which make up the energy and the materials composite is a composite of a domestic and an imported variety which trade off with a constant elasticity of substitution. The corresponding zero-profit condition for the production of the Armington good $i$ is given by:

$$
\Pi_i^A = P_i^A - \left[ \frac{\partial}{\partial \Pi_i^A} \left( \frac{(1 + t_i^M) PM_i}{1 - \theta^M_i} + (1 - \theta^M_i) P_i \frac{t_i^{OM}}{1 - \theta^{OM}_i} + t_i^{OM} \right) \right] = 0
$$

$^{11}$ Decreasing returns are accommodated in the CRTS framework through introduction of a specific factor under the assumption of perfect competition.

$^{12}$ The prices for exports $PX_i$ and imports $PM_i$ are expressed in domestic currency. Export prices $\overline{PX}_i$ and import prices $\overline{PM}_i$ in international currency (e.g. in $\text{US}$) are exogenous for the small open economy. The real exchange rate $\mu$ relates international prices to domestic prices, i.e. $PM_i \equiv \overline{PM}_i \mu$.  

where:

\[ P_i^A \] is the Armington price of the composite good \( i \),

\[ \theta_i^{IM} \] is the benchmark value share of imports in the Armington good \( i \),

\[ PM_i \] is the import price of good \( i \) (expressed in domestic currency),

\[ t_i^{IM} \] is the (ad-valorem) tariff rate on imported goods,

\[ \sigma_i^{DM} \] is the Armington elasticity of substitution between domestic goods and imports,

\[ t_i^{CO2} \] is the CO2 tax rate,

\[ a_i^{CO2} \] is the physical carbon emission coefficient for good \( i \),

and

\[ A_i \] is the associated dual variable, which indicates the activity level of Armington good production.

**Material composite**

The material composite is produced in fixed proportions (Leontief):

\[ \Pi_i^M = P_i^M - \sum_{j \neq EG} \theta_j^M P_j^A = 0 \]

where:

\[ \theta_j^M \] is the benchmark value share of the non-energy Armington good \( j \) (\( j \neq EG \)) in the materials composite of sector \( i \),

and

\[ M_i \] is the associated dual variable, which indicates the activity level of production of the materials composite for sector \( i \).

**Energy composite**

As to the formation of the energy aggregate, we employ several levels of nesting to represent differences in substitution possibilities between primary fossil fuel types as well as substitution between the primary fossil fuel composite and secondary energy, i.e. electricity. In the bottom nest, liquid fuels (refined oil (OIL), crude oil (CRU), gas (GAS)) trade off with a constant elasticity of substitution. At the same level, hard coal and soft coal are combined within a CES aggregate. The liquid fuel composite and the coal composite enter the next level with a constant elasticity of substitution. At the top level, the aggregate of non-electric energy combines with electricity at a constant elasticity of substitution\(^{13}\):

\(^{13}\) For the sake of brevity, we have dropped the explicit representation of the lowest nest and employ instead an artificial liquid fuel composite (price \( P_i^{COA} \)) and coal aggregate (price \( P_i^{LQ} \)).
\[
\Pi^E_i = P^E_i \cdot \left\{ \theta^E_i P^A_i \sigma^E \frac{1}{1 - \theta^E_i} + (1 - \theta^E_i) \theta^C_i P^C_i \sigma^C \frac{1}{1 - \theta^C_i} + (1 - \theta^C_i) \frac{1}{1 - \sigma^C} \right\} = 0
\]

where:

- \( P^E_i \) is the price of the energy composite good for sector \( i \),
- \( \theta^E_i \) is the benchmark value share of electricity in the energy aggregate of sector \( i \),
- \( \sigma^E \) is the elasticity of substitution between electricity and non-electric (fossil) energy,
- \( \theta^C_i \) is the benchmark value share of coal inputs within the fossil fuel aggregate input of sector \( i \),
- \( \sigma^C \) is the elasticity of substitution between the coal composite and the liquid fuel composite,

and

- \( E_i \) is the associated dual variable, which indicates the activity level of production of the energy composite for sector \( i \).

**Public output**

Public goods and services are produced in fixed proportions (Leontief aggregation) of commodity inputs which are composed as an Armington aggregate of domestic and imported commodities:

\[
\Pi^G = P^G - \sum_i \theta^G_i P^A_i = 0
\]

where:

- \( P^G \) is the composite price for government demand,
- \( \theta^G_i \) is the benchmark value share of Armington input \( i \) in public goods provision,

and

- \( G \) is the associated dual variable which indicates the activity level of public goods provision.

**Consumer good production**

Consumer goods are produced in fixed proportions of Armington goods (Z-Matrix transformation with fixed coefficients):

\[
\Pi^Z = P^Z - \sum_i \theta^Z_i P^A_i = 0
\]

where:

- \( P^Z \) is the price for consumption good \( z \) (net of value-added taxes),

28
\( \theta_{iz} \) is the benchmark value share of producer good \( i \) into the formation of consumer good \( z \),

and

\( Z_z \) is the associated dual variable which indicates the activity level of consumer good production.

**Non-leisure consumption composite**

Substitution patterns within the aggregate of (non-leisure) consumption goods are described by a Cobb-Douglas function. The zero-profit condition for the (non-leisure) consumption composite is given by:

\[
\Pi^{CG} = P^{CG} \cdot \prod_z \left( P_z (1 + t_z^{CG}) \right)^{\theta_z^{CG}} = 0
\]

where:

- \( P^{CG} \) is the price for the (non-leisure) consumption composite (gross of value-added taxes),
- \( \theta_z^{CG} \) is the benchmark value share of consumer commodity \( z \) in aggregate (non-leisure) household consumption
- \( t_z^{CG} \) is the value-added tax rate on inputs of consumption goods into aggregate consumption,

and

- \( CG \) is the associated dual variable which indicates the activity level of household consumption of commodities (excluding leisure).

**Full consumption**

Intra-period household demand is given as a separable nested CES function which describes the trade-off between leisure and consumption. The zero-profit condition reads as:

\[
\Pi^{C} = P^{C} \cdot \left[ \theta^{C} (w(1-t_w))^{1-\sigma^{C}} + (1-\theta^{C}) P^{CG} \right]^{\frac{1}{1-\sigma^{C}}} = 0
\]

where:

- \( P^{C} \) is the composite price index of aggregate leisure and goods consumption,
- \( \theta^{C} \) is the benchmark value share of leisure in intra-period household consumption,
- \( \sigma^{C} \) is the price of current leisure,
- \( \sigma^{F} \) is the elasticity of substitution between current leisure and commodity consumption (calibrated consistently to a given supply elasticity of labor with respect to the real wage),
- \( t^{w} \) is the labor income tax rate (applicable to the gross wage),
and
\[ C \] is the associated dual variable which indicates the activity level of aggregate household consumption (commodities and leisure).

**Capital formation and investment**

An efficient allocation of capital, i.e. investment over time assures the following intertemporal zero-profit conditions which relates the cost of a unit of investment, the return to capital and the purchase price of a unit of capital stock in period \( t \):\(^{14}\)

\[
\Pi^K_t = p^K_t - r^K_t - (1 - \delta) p^K_{t+1} = 0
\]

and

\[
\Pi^I_t = \sum \theta^I_i P^A_i - p^K_{t+1} \geq 0
\]

where:

- \( p^K_t \) is the value (purchase price) of one unit of capital stock in period \( t \),
- \( \delta \) is the capital depreciation rate,
- \( \theta^I_i \) is the benchmark value share of Armington input \( i \) in the homogeneous investment good,
- \( \sum \theta^I_i P^A_i \) is the cost of a unit of investment in period \( t \),\(^{15}\)

and

- \( K_t \) is the associated dual variable, which indicates the activity level of capital stock in period \( t \),
- \( I_t \) is the associated dual variable, which indicates the activity level of aggregate investment in period \( t \).\(^{16}\)

**Market Clearance**\(^{17}\)

*Domestic supply*

Producer goods produced for the domestic market enter Armington production:

\[
Y_t = \frac{\partial \Pi^Y_t}{\partial (P_t(1 - t^Y_t))} = A_t \frac{\partial \Pi^A_t}{\partial P_t}
\]

\(^{14}\) The optimality conditions for capital stock formation and investment are directly derived from the maximization of lifetime utility by the representative household taking into account its budget constraint, the equation of motion for the capital stock and the condition that output in each period is either invested or consumed. Note that in our algebraic exposition we assume an investment lag of one period.

\(^{15}\) The investment good is produced subject to a Leontief technology which combines Armington inputs in fixed proportions.

\(^{16}\) As written, we have taken explicit account of the non-negativity constraint for investment.

\(^{17}\) We exploit Shepard’s Lemma to provide a compact representation of compensated demand and supply functions.
Armington supply

Armington goods enter intermediate demand for the production of producer goods and consumer goods as well as government and investment demand:

\[ A_i = \sum Y_i \frac{\partial \Pi_i^Y}{\partial \Pi_i^A} + \sum Z \frac{\partial \Pi_i^Z}{\partial \Pi_i^A} + I \frac{\partial \Pi_i^I}{\partial \Pi_i^A} + G \frac{\partial \Pi_i^G}{\partial \Pi_i^A} \]

Intermediate energy supply

The sector-specific intermediate energy composite enters production:

\[ E_i = \sum Y_i \frac{\partial \Pi_i^Y}{\partial \Pi_i^E} \]

Intermediate material supply

The sector-specific intermediate material composite enters production:

\[ M_i = \sum Y_i \frac{\partial \Pi_i^Y}{\partial \Pi_i^M} \]

Consumer goods supply

Consumer goods enter final consumption demand:

\[ Z_i = CG \frac{\partial \Pi_i^{CG}}{\partial (P_i (1 + t_i^C))} \]

Non-leisure consumption

Non-leisure consumption enters the aggregate consumption (including leisure):

\[ CG = C \frac{\partial \Pi_i^C}{\partial P_{CG}} \]

Government provision

Public good provision increases at the exogenous (steady-state) growth rate of the economy:

\[ G_i = G_{i-1} (1 + gr) \]

where:

- \( gr \) is the exogenous growth rate, and
- \( G_0 \) is the base year level of public good provision.

Imports

The supply-demand balance for imported goods is:
\[ IM_i = A_i \frac{\partial \Pi^A_i}{\partial (PM_i (1 + t_i^{IM}))} \]

where:
\[ IM_i \] is the level of imports of good \( i \).

**Exports**

The supply-demand balance for exported goods is:
\[ EX_i = Y_i \frac{\partial \Pi^V_i}{\partial (PX_i (1 - t_i^V))} \]

where:
\[ EX_i \] is the level of exports of good \( i \).

**Foreign closure**

As to the trade balance with respect to the rest of the world a simple foreign closure rule is used: In the small open economy framework, CIF import prices and FOB export prices are exogenous and unaffected by the level of imports and exports. An intertemporal balance of payments constraint (trade closure) assures no change in net indebtedness over the model horizon\(^{18}\):
\[
\sum_{t=0}^{\tau} \sum_{i=1}^{n} PM_i IM_i = \sum_{t=0}^{\tau} \sum_{i=1}^{n} PX_i EX_i
\]

**Labor**

The intra-period supply-demand balance for labor is written:
\[
\bar{E} - \frac{\partial \Pi^C_i}{\partial (w(1 - t^w))} = \bar{E} - F = \sum_{i=1}^{n} Y_i \frac{\partial \Pi^V_i}{\partial (w(1 + t_i^L))}
\]

where:
\[ \bar{E} \] is the total endowment with time which grows at the exogenous (steady-state) rate \( gr \) of the economy,\(^{19}\) and
\[ F \] is the demand for leisure.

**Capital**

The supply-demand balance for capital services is written:
\[ rK = \sum_{i=1}^{n} Y_i \frac{\partial \Pi^V_i}{\partial (r(1 + t_i^K))} \]

where:

---

\(^{18}\) N.B.: In this framework, international flows of capital goods (borrowing and lending) are endogenous.

\(^{19}\) We represent growth in terms of Harrod-neutral technological progress in producing labor or leisure services per unit of actual time (efficiency growth).
$K$ is the aggregate capital stock for domestic production.

Current period's investment augments the capital stock in the next period. Capital stocks are updated as an intermediate calculation between periods. The stock-flow accounting relationship for capital (equation of motion for the capital stock) can be written as:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

**Income Balances**

**Household**

The representative household chooses to allocate lifetime income across consumption in different time periods:

$$\max U(\underline{u}(C_1, F_1), u(C_2, F_2), \ldots, u(C_T, F_T)) = \sum_{t=1}^{T} \rho^t u(C_t, F_t)$$

s.t.

$$\sum_{t=1}^{T} P_t C_t = \sum_{t=1}^{T} w_t (1 - \tau^t)(\bar{E}_t - F_t) + p_t^c \frac{1}{\rho} + \sum_{t=1}^{T} \tau_t K_t + \sum_{t=1}^{T} TR_t$$

where:

- $\rho^t$ is the pure rate of time preference which determines the intertemporal allocation of consumption, and
- $TR$ is an exogenous lump-sum transfer between the government and the household.

For the intra-period utility function we assume the functional form $u(C_t, F_t) = -\frac{1}{\alpha} \frac{C_t^{\alpha} F_t^{1-\alpha}}{C_t^{\alpha} F_t^{1-\alpha}}$ which is consistent with an intertemporal elasticity of substitution equal to 0.5.

**Government**

Government income consists of tax revenues from the representative household, which assures a balanced budget (fiscal closure). The intertemporal income balance of the government is given by (for the sake of brevity we omit production taxes and tariffs):

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20 Capital accumulation (i.e. the level of savings and investment) is determined by firms managers who allocate investment across sectors to maximize the present value of firms.

21 In our simulations, we assume that the capital stock is augmented by new investment with a three-year and depreciated at a constant geometric rate.

22 In our simulations with unemployment, $TR$ includes both, exogenous transfers as well as endogenous unemployment payments.
\[
\sum_{t=1}^{T} \sum_{t'} \left( P_t \cdot t'^{CG} \frac{\partial \Pi^{CG}}{\partial (P_{t'} (1 + t'^{CG}))} \right) + \sum_{t=1}^{T} w_t \cdot t'^{L} \frac{\partial \Pi^{L}}{\partial (w_t (1 + t'^{L}))} + \sum_{t=1}^{T} \sum_{i} t'_i \cdot w_i \cdot Y_{it} \frac{\partial \Pi_{it}^{Y}}{\partial (w_i (1 + t'_i))} + \sum_{t=1}^{T} \sum_{i} t'_i \cdot r_i \cdot Y_{it} \frac{\partial \Pi_{it}^{Y}}{\partial (r_i (1 + t'_i))} \\
+ \sum_{t=1}^{T} \sum_{i} t'_i \cdot P X_{it} \cdot E X_{it} + \sum_{t=1}^{T} \sum_{i} t'_i \cdot P M_{it} \cdot I M_{it} + \sum_{t=1}^{T} \sum_{i} t'_i \cdot P_{it} \cdot Y_{it} \frac{\partial \Pi_{it}^{Y}}{\partial (P_{t'} (1 + t'_i))} + \sum_{t=1}^{T} \sum_{i} t'_i \cdot \alpha_i^{CO2} \cdot A_{it} \right) \\
= \sum_{t=1}^{T} P_t^G \cdot G_t
\]

**Terminal Constraint**

The finite horizon poses some problems with respect to capital accumulation. Without any terminal constraint, the capital stock at the end of the model's horizon would have no value and this would have significant repercussions for investment rates in the periods leading up to the end of the model horizon. In order to correct for this effect we define a terminal constraint which forces terminal investment to increase in proportion to final consumption demand:

\[
\frac{I_T}{I_{T-1}} = \frac{C_T}{C_{T-1}}.
\]

**B. Unemployment**

We introduce unemployment through the specification of a wage curve, which postulates a negative relationship between the real wage rate (note that \(w\) below reflects the after-tax nominal wage, which corresponds to \(w \cdot (1 - \tau)\) in our previous description) and the rate of unemployment \(ur\):

\[
\frac{w}{P^{CG}} = g(\text{ur}) \quad \text{with} \quad g' < 0,
\]

where:

- \(P\) is the consumer goods price index (in our case \(P^{CG}\))
- \(\text{ur}\) is the unemployment rate.

This type of wage curve can be derived from trade union wage models as well as from efficiency wage models (see e.g. Beißinger, 1996 or Hutton and Ruocco, 1999). Figure 1 illustrates...
the wage curve in the traditional labor market diagram. The real wage rate \( w/P \) is measured on the vertical axis and the labor supplied and demanded are measured on the horizontal axis.

Full employment occurs with the real wage rate of \( (w/P)^0 \) at the intersection of the (inverse) labor demand function \( L \) and the labor supply function \( L^s \). The wage curve now replaces the labor supply curve. Consequently, the equilibrium wage rate \( (w/P)^1 \) lies above the market clearing wage rate. This causes unemployment at an amount of \( (L_s)^1 - (L)^1 \).

Our initial labor market clearance condition then becomes:

\[
\bar{E} = \frac{\partial \Pi^c}{\partial (w(1-t_w))} (1-ur) = \sum_i Y_i \frac{\partial \Pi^f_i}{\partial (w(1+t_i^e))}
\]

In our calculations, we assume that the unemployment benefit payments are constant in real terms and are not taxed. We can write the wage curve as a log-linear function:

\[
\log \left( \frac{w}{P} \right) = \gamma_0 + \gamma_1 \log(ur)
\]

where:

- \( \gamma_0 \) is a positive scale parameter which includes the real unemployment benefits, and
- \( \gamma_1 < 0 \) is the elasticity of the real wage with respect to the unemployment rate.