Access Price Regulation, Investment
and Entry in Telecommunications

Kaisa Kotakorpi

University of Tampere and FDPE
Department of Economics
FIN-33014 University of Tampere
Finland
E-mail: kaisa.kotakorpi@uta.fi

7 June 2004 (draft)

Abstract:

We consider a model with a vertically integrated monopolist telecommunications network provider who faces price taking rival operators in the retail market. We examine the network operator’s incentives to invest in a technology that increases demand, both when there is no regulation and when access prices are regulated. We find that investments are below the social optimum and access price regulation further reduces investment incentives. We show that this underinvestment problem may have negative effects also on the viability of competition as a low level of investment may force the rivals to exit the market. Further, in the presence of access price regulation, rivals are most likely to be foreclosed from the market precisely when they would bring highest benefits to consumers.

Keywords: access pricing, regulation, investment, foreclosure
JEL classification: L22, L43, L51, L96

* Financial support from the Yrjö Jahnsson Foundation and the Foundation for Economic Education is gratefully acknowledged. I would like to thank Mikko Leppämäki, Mikko Mustonen, Pekka Sääskilahti, Tuomas Takalo, Matti Tuomala and Juuso Välimäki for helpful comments. All remaining errors are mine.
1. Introduction

Introducing competition into local telecommunications has been an important goal of industrial policy in many European countries since the 1980s and the 1990s. Despite liberalisation, the market share of the incumbent telecommunications operator in the local telecommunications market in all European countries was above 80% and in most of them well above 90% at the end of the 1990s\(^1\). The question of determining the correct price at which one telecommunications operator can use the infrastructure of another, that is the access price, has been identified as a central issue in the attempt to make the liberalised telecommunications markets truly competitive. The principles of network access pricing in a static context have indeed been the subject of extensive research\(^2\). However, the important question of how access price regulation affects firms’ incentives for investment has so far received much less attention in the literature.

The aim of this paper is to analyse the effect of access price regulation on a network provider’s incentives to invest in its infrastructure. These infrastructure investments are assumed to increase customers’ willingness to pay for final telecommunications services. We consider a vertically integrated incumbent who provides network access to rival operators in the retail sector. The network is assumed to have natural monopoly characteristics and it is therefore an essential input in the production of telecommunications services. The assumption of a monopoly network can be justified by the current situation in many countries. Further, in a recent study by Faulhaber and Hogendorn (2000), it was found that the provision of access to broadband telecommunications networks is likely to remain a naturally monopolistic activity at least in less densely populated areas. We examine the incumbent’s investment decision both when there is no regulation and when the access price is set by a regulator. We assume that the regulator is unable to make a credible commitment to a particular access pricing regime prior to the incumbent’s investment decision, as investments in network infrastructure are typically irreversible and have an effect over a long period of time.

\(^1\) See for example a study prepared by Teligen Ltd for the European Commission on entry issues in EU telecommunications markets (http://europa.eu.int/ISPO/infosoc/telecompolicy/en/marketentry.html).

\(^2\) See for example Armstrong (2002) and Laffont and Tirole (2000) for recent surveys of the literature on access pricing.
We model the rival firms in the retail sector as a competitive fringe\(^3\). The assumption of price taking rivals in the retail market seems justifiable, as we assume that fixed costs in the retail market are negligible. Further, we assume that there is vertical differentiation between the incumbent and the fringe. Vertical differentiation can arise in our model for example due to switching costs that customers incur when they choose to subscribe to one of the fringe firms instead of the incumbent. This assumption together with the assumption that rivals are small makes the model suitable for analysing the effect of the incumbent’s investment decision on entry.

The main issues of interest that arise in our paper are two-fold. Firstly, we examine the effect of spillovers to rivals’ demand on the incumbent’s incentives for investment. Secondly, we draw conclusions on cases in which the incumbent takes actions to foreclose the rivals from the market. In particular, we are interested in the interaction between these two issues: on the one hand, the presence of rivals has an effect on investment incentives, and on the other hand, the incumbent’s investment decision has consequences for the viability of competition. The issues discussed in this paper are highly relevant to today’s regulators, who are much concerned about both encouraging investment for example in broadband telecommunications networks and ensuring competition in the provision of services over these networks\(^4\).

We find that spillovers have a negative effect on investment incentives when access prices are regulated: as the incumbent is not allowed to make a profit on access provision, investment incentives are lower than in the absence of regulation. Further, if rivals benefit more from the investment than does the incumbent, the disincentive effect is so strong that no investment will take place. We also find that the underinvestment problem may have negative effects also on the viability of competition: when also rivals benefit from the investment, a low investment level can force the rivals to exit the market. If investments were increased towards the socially optimal level, this would not only be beneficial in itself, but would also facilitate competition. Further, we find that in the presence of access price regulation, rivals are most likely to be foreclosed precisely when they would bring highest benefits to consumers.

\(^3\) For the theory of optimal one-way access pricing in a competitive fringe model without investment, see Armstrong (2002, Section 2).

\(^4\) See for example an outline of the EU Commission’s policies on promoting broadband access at http://europa.eu.int/information_society/eeurope/2005/all_about/broadband/index_en.htm
The remainder of this paper proceeds as follows. Some of the related literature is briefly discussed in section 2. Our model is introduced in section 3. In Section 4, we examine the case with no price regulation. In Section 5, the effects of access price regulation are considered. Section 6 concludes.

2. Related literature

The paper closest to ours is Foros (2004), who examines an access provider’s incentives to upgrade its network to broadband when it also acts as an internet service provider and faces a single competitor in the retail market. Our paper differs from that of Foros in that we assume that operators set prices, whereas Foros assumes Cournot competition between the ISPs in the retail market. Our paper is in this respect more closely related to the existing literature on access pricing and telecommunications competition, where the assumption of price competition has typically been made. Further, our specification differs from that of Foros due to our assumption of vertical differentiation between the operators. As in Foros, the investment examined in our paper can be interpreted for example as upgrading an existing (narrowband) telecommunications network to broadband. However, the framework is rather general and the investment can be thought of as any activity by the incumbent that increases customers’ willingness to pay for telecommunications services.

Foros (2004) emphasises that in the presence of access price regulation, the incumbent can use overinvestment (relative to the monopoly level) to deter entry when the incumbent’s ability to offer value added services is much higher than that of the rival. However, we demonstrate that the incumbent underinvests (relative to the socially optimal level) and when the rivals are relatively efficient in turning the investment into value added services, suboptimal investments can lead to foreclosure. As foreclosure due to underinvestment occurs precisely when entry would bring the highest benefits to consumers, this underinvestment problem is all the more detrimental to social welfare.

Other previous studies of the effects of access price regulation on investment incentives include Gans (2001), who considers a situation where different firms compete to become the
access provider for new types of network services. He shows that in the presence of such a race, a socially optimal level of investment can in fact be reached. Our focus is, however, on the incumbent’s incentives to upgrade its existing network, which is assumed to have natural monopoly characteristics. Pindyck (2004), Jorde, Sidak and Teece (2000) and Hausman and Sidak (1999) analyse the effects of the access pricing rule currently used in the US, namely total element long-run incremental cost (TELRIC) pricing, on the investment incentives of both incumbents and entrants. Sidak and Spulber (1997) emphasise the significance of the regulator’s commitment problem in a dynamic setting. Valletti and Cambini (2003) study competing network operators’ incentives to invest in facilities with different levels of quality.

Numerous authors have considered the question of how investment spillovers affect private incentives to perform R&D. These studies date back to Arrow (1962) and the issue has later been examined among others by Spence (1984), D’Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992). Investment incentives in an oligopoly with vertical product differentiation have been examined by Motta (1992) and Rosenkranz (1995). The case studied here is different from those models due to the vertical relationship between the incumbent and its rivals, that is, the fact that the incumbent acts as a supplier of an intermediate good to its rivals: even if the incumbent’s and the rival firms’ final services are substitutes, higher demand for the rivals’ services will not necessarily be detrimental to the incumbent, as it will also cause an increase in the rivals’ demand for access. The precise effect of access provision on investment incentives depends, of course, crucially on the access pricing regime.

We also examine the conditions under which the incumbent chooses an access price or an investment level that causes the rivals to exit the market and our paper is therefore related to the literature on foreclosure. A recent contribution in this field is Rey and Tirole (2003), which considers a bottleneck owner’s incentives for foreclosure in the absence of regulation. Other articles particularly relevant from the point of view of our paper include Weisman (1995, 1998, 2001), Reiffen (1998) and Beard et al (2001), which examine incentives to reduce the quality of the input sold to downstream rivals when there is price competition downstream. A major difference between our model and these analyses is that in those

---

5 See also Amir (2000) for a comparison between the different models and De Bondt (1996) for a survey and further references.
papers, discrimination takes the form of activities that raise rivals’ marginal costs. The bottleneck owner then has two possible instruments of discrimination, excessive access pricing and input quality degradation, that affect the rival in the same way (increase marginal cost) but the former has a direct positive effect on the incumbent’s profit while the latter may be costly for the incumbent (as for example in Weisman (1995)). Therefore it is not surprising that the incentives for quality degradation are shown to be especially strong under stringent access price regulation (see for example Beard et al (2001)). In our model, investment affects the incumbent as well as the rivals, and therefore cannot be used as an instrument of discrimination that could be targeted at the rivals only.

3. The Model

We model the rival firms in the retail sector as a competitive fringe, that is, the rivals are a group of small firms supplying an identical product. However, even though the fringe behaves competitively, it is assumed that the incumbent retains some level of market power and profits, as its end product is assumed to be vertically differentiated from that of the fringe. We assume that there are negligible fixed costs in the retail market and hence the assumption of price taking rivals in the retail market seems justifiable. Further, such a model allows us to concentrate on the effect of access prices on the incumbent’s investment decisions: we can abstract for example from strategic interaction between the incumbent and the rivals in the retail market, as the rivals’ retail price will always equal their marginal cost, plus the access charge. We also adopt the simplifying assumption of complete information and no uncertainty. The regulator is assumed to have full information about the incumbent’s technology and costs and there is no uncertainty about the returns on investment.

In the telecommunications market, consumers typically choose one operator and buy all their services from that single operator and consumers therefore face a discrete choice problem of choosing which operator to subscribe to. Unlike most previous studies of competition in telecommunications markets, we also allow for the possibility of partial participation in the market, so that some consumers can choose not to subscribe to any of the

---

6 See for example Pindyck (2004), Hausman (1999) and Hausman and Sidak (1999, 458-460) for a discussion of some effects of uncertainty on investment incentives under access price regulation.
firms. We assume that customers have unit demands, so that a customer who subscribes to firm \( i \) \((i = 1, 2)\) pays a price \( p_i \) for a single unit of telecommunications services.\(^7\) We use the index 1 to refer to the incumbent and 2 to the fringe.

We assume that consumers differ in their basic willingness to pay for telecommunications services. We denote a consumer’s basic valuation for the incumbent’s service by \( x \) and assume that \( x \) is uniformly distributed on \((0,1)\) with density 1. Further, the services of the incumbent are assumed to be vertically differentiated from those of the fringe so that a customer’s basic valuation for the fringe’s services is given by \( \gamma x \) with \( \gamma \in (0,1) \). This formulation can be justified for example by assuming that all customers have previously been served by the incumbent: the services of the fringe firms are assumed to only become available in the present period and a customer who chooses to use the services of one of the fringe firms instead of those of the incumbent will incur a switching cost equal to \((1 - \gamma)x\).\(^8\) This assumption together with the assumption that rivals are small makes the model suitable also for analysing the effect of investment on entry. Switching costs can arise for example as a result of the lack of number portability, customer loyalty or the costs of searching for information on prices and quality of different operators.

We investigate the incumbent’s incentives to invest in a technology that increases consumers’ willingness to pay for telecommunications services. We assume that the incumbent’s investment activity takes the following form\(^9\): an investment that causes all consumers’ valuation for the incumbent’s services to increase by an amount \( m \) costs the incumbent \( \frac{1}{2}\gamma m^2 \). We therefore make the simplifying assumption that the effect of the investment is the same for all consumers, independently of \( x \). However, we have also examined the implications of an alternative assumption, where the effect of the investment enters multiplicatively with \( x \), so that consumers with a higher \textit{ex ante} valuation for telecommunications services also value the investment more highly. This change in the

\(^7\) If the firms incur a constant marginal cost per customer (independently of call minutes), our assumption of unit demands is equivalent to assuming that the deals offered by the firms consist of an unlimited number of call minutes for a fixed charge. Such deals are commonly observed in the market for internet access.

\(^8\) Note that it is necessary to have switching costs differ continuously across individuals in order to avoid discontinuities in the demand functions. Here, the switching cost is proportional to the consumers’ basic valuation \( x \).

\(^9\) We model the innovation technology and spillovers as in D’Aspremont and Jacquemin (1988) (with the difference that their paper considers a cost reducing investment). Amir (2000) has pointed out some drawbacks in the D’Aspremont and Jacquemin model that occur when the number of innovating firms is large.
assumptions did not affect the qualitative results regarding the effects of access price regulation on investment incentives.

As the incumbent and its rivals use the same network, any demand enhancing investment by the incumbent in its infrastructure is likely to have a positive effect also on the demand for the rivals’ service. We denote the fraction of the benefits of the investment that spill over to the rivals by $\beta$, so that consumers’ valuation for the services provided by the fringe increases by $\beta m$. We therefore assume that the effects of the investment on the consumers’ valuation for the incumbent’s and the rivals’ products can differ in magnitude. The spillovers can be incomplete, in which case $\beta < 1$. However, we also allow for the possibility that the investment is more beneficial for the firms in the fringe than for the incumbent, so that $\beta > 1$. This latter case can occur for example if the rivals’ ability to use the new technology to provide new, value added services is greater than the incumbent’s. Indeed, our preferred interpretation of the spillover parameter $\beta$ is that it measures the rivals’ relative efficiency or ability to convert the infrastructure investment into new services valued by consumers. This seems to be the most appropriate interpretation in our model, as the incumbent and the fringe use the same physical network, yet consumers’ willingness to pay for the final services provided over the network can differ between the two.

A consumer with basic valuation $x$ chooses to buy from the incumbent rather than the fringe if $x + m - p_1 > \gamma x + \beta m - p_2$, that is, if $(1 - \gamma)x > p_1 - p_2 + (\beta - 1)m$. The consumer therefore compares the cost of switching to a rival with the price difference between the firms and the relative benefits from the investment. The person with the highest valuation buys from the incumbent if $1 + m - p_1 > \gamma + \beta m - p_2$. This condition is needed for it to be possible for both firms to be active in the market, and we assume for now that this condition holds. The market is then segmented in such a way that the incumbent serves those customers with the highest valuation for telecommunications services and the fringe those with an intermediate valuation, and customers with a very low valuation might not purchase at all. There are then two marginal consumers, a consumer (with valuation $x_c$) who is indifferent between buying from the incumbent and the fringe, and another (with valuation

---

10 A similar idea is incorporated into Foros’s (2004) model.
11 Shaked and Sutton (1982) show that such an equilibrium (where consumers with highest willingness to pay buy from the highest quality firm and so on) typically arises in models with vertical differentiation.
who is indifferent between buying from the fringe and not buying at all. The valuations of these marginal consumers are given by

\[ x_A = \frac{1}{1-\gamma} \left[ p_1 - p_2 + (\beta - 1) m \right] \quad \text{and} \quad x_B = \frac{1}{\gamma} \left[ p_2 - \beta m \right]. \]

Given our uniform distribution assumption, the demand functions for the firms’ final products are linear and they are given by \( q_1 = 1 - x_A \) and \( q_2 = x_A - x_B \), that is,

\[
q_1(p_1, p_2, m) = 1 + \frac{1}{1-\gamma} \left[ (1-\beta)m - p_1 + p_2 \right], \\
q_2(p_1, p_2, m) = \frac{1}{1-\gamma} \left[ \frac{\beta - \gamma}{\gamma} m + p_1 - \frac{1}{\gamma} p_2 \right].
\]

The corresponding inverse demands take a simple form are given by

\[
p_1(q_1, q_2, m) = 1 + m - q_1 - \gamma q_2, \\
p_2(q_1, q_2, m) = \gamma + \beta m - \gamma(q_1 + q_2).
\]

We assume that there is a fixed cost \( f \) of network operation but the fixed costs in the retail market are negligible. The incumbent incurs a marginal cost \( c \) per customer of originating and terminating calls on its network. Further, we assume that the incumbent and the fringe have equal marginal costs in the retail sector, and we normalise these costs to zero for simplicity. Because of zero marginal costs in the retail segment, the marginal cost of the fringe is simply equal to the access charge \( a \) and the marginal cost of the incumbent is \( c \). We make the assumption that \( c < \gamma \) so that the marginal cost of network provision is such both of the final products could be profitably supplied by a monopolist.

In equilibrium, the firms in the fringe set \( p_2 = a \) and make zero profit. The incumbent’s profits are therefore given by

\[
\pi(p_1, a, m) = \left[ 1 + \frac{1}{1-\gamma} \left[ (1-\beta)m - p_1 + a \right] \right] (p_1 - c) + \frac{1}{1-\gamma} \left[ \frac{\beta - \gamma}{\gamma} m + p_1 - \frac{1}{\gamma} a \right] (a - c) - \frac{1}{2} \gamma m^2 - f.
\]

\[ \text{Note that in order to avoid having to consider corner solutions, we need to make sure that } x_B > 0 \text{ in equilibrium (see footnotes 16 and 23 below).} \]
where the first term is the profit from retail sales to the incumbent’s own customers and the second term is the profit from selling access to the fringe.

4. Unregulated access charge

Let us first analyse the outcome of the model when the incumbent can set both its retail price and the access charge freely. The timing of the model is as follows. First, the incumbent chooses the level of investment and incurs the associated costs as explained above. Second, the incumbent sets the prices for its retail product and for access. The model is solved by backwards induction, that is, we start solving the model from the last stage.

4.1 Incumbent’s pricing decisions

In the last stage of the model, the incumbent chooses its own retail price and the access charge so as to maximise (3), taking the level of investment as given. The profit maximising levels of \( p_i \) and \( a \) as functions of the investment level are given by

\[
\begin{align*}
  p_i^*(m) &= \frac{1}{2} (1 + m + c) \\
  a^*(m) &= \frac{1}{2} (\gamma + \beta m + c)
\end{align*}
\]

Equations (4) show that in the present model, the incumbent sets the access charge and its own final price as if it was a monopolist supplying both its own retail product and the retail product of the rivals and facing the inverse demand functions given in equations (2). In the competitive fringe model, therefore, the rival firms resemble a subsidiary to the incumbent firm.

Inserting (4) into (1) yields the equilibrium quantities for the incumbent and the fringe as functions of the level of investment expenditures. These quantities are given by
\[ q_1^*(m) = \frac{1 - \gamma + (1 - \beta)m}{2(1 - \gamma)} \]
\[ q_2^*(m) = \frac{(\gamma - 1)c + (\beta - \gamma)m}{2\gamma(1 - \gamma)} \]  
(5)

The incumbent’s equilibrium output is increasing in the investment if \( \beta < 1 \) and the fringe’s output is increasing in the investment if \( \gamma < \beta \). There is therefore a range of parameter values \( \gamma < \beta < 1 \) where both quantities are increasing in the level of investment. It can also be seen from (5) that if the incumbent chooses not to invest at all, the rivals will not be active in the market\(^{13}\).

Further, there is another important result that we can infer from (5), namely that when there is no access price regulation, \( \gamma < \beta \) is a necessary condition for the rivals to be active in the market. This result is somewhat intuitive. Firstly, a low value of the parameter \( \gamma \) implies a high degree of vertical differentiation between the firms. Even though this implies that the rivals’ product is in this sense at a greater disadvantage as compared to the incumbent’s product, a high degree of differentiation implies that the incumbent can make a higher profit. In this case, the incumbent can effectively segment the market: it can set a relatively high price for its own service and thereby extract surplus from consumers with a high willingness to pay, whilst keeping the price of the rivals’ service relatively low and thus avoiding driving out consumers with a low willingness to pay. Secondly, a high value of the spillover parameter \( \beta \) implies that the incumbent’s investment has a large impact on customers’ willingness to pay for the rivals’ product. Therefore, the cases where spillovers or the degree of product differentiation are high (\( \gamma \) is low) are the cases where selling access to rivals is most profitable and foreclosure is least likely to occur\(^{14}\).

We have therefore obtained the result that even if the optimal investment level is positive, \( \gamma < \beta \) is a necessary condition for the rivals to be active in the market. If \( \gamma > \beta \), the incumbent sets the access charge at such a level that the rivals will be foreclosed from the

\(^{13}\) The conditions for when the rivals are active in the market are explored fully below. It should also be noted that if \( \beta \) is high, it may be possible that the incumbent itself will leave the retail market and act only as a network provider (a full analysis of this possibility is yet to be completed).

\(^{14}\) Reiffen (1998) obtains a related result that incentives to use non-price discrimination against rivals are lower when the rivals’ products are more differentiated from those of the network provider.
market regardless of the level of investment. From now on, we will therefore assume that the condition $\gamma < \beta$ holds and we are then able to consider the effect of the incumbent’s investment decision on the functioning of the market\(^{15}\). Indeed, as can be seen from equation (5), the condition $\gamma < \beta$ is not sufficient for the rivals to be active in the market, but the conditions under which competition is viable depend also on the incumbent’s investment decision.

### 4.2 Optimal investment level and effect of spillovers

The incumbent’s optimal level of investment can be calculated after substituting (4) into (3). The privately optimal level of investment is given by

$$m^* = \frac{(1-\gamma)(\gamma - \beta c)}{2\beta \gamma - \beta^2 - \gamma + 2\gamma \psi (1-\gamma)} \quad (6)$$

For the second order condition to be satisfied, we need to require that

$$2\beta \gamma - \beta^2 - \gamma + 2\gamma \psi (1-\gamma) > 0 \quad (7)$$

This is satisfied for example if the parameter $\psi$ is large enough, that is, if the investment cost function is convex enough. If $\psi$ were small and the above second order condition did not hold, it would be optimal for the incumbent to increase investment indefinitely. Therefore equation (7) is also the condition that is required for the existence of equilibrium\(^{16}\). It can be noted from (6) that the optimal investment level is positive as long as $(\gamma - \beta c) > 0$, that is, $\frac{\gamma}{\beta} > c$. The marginal cost of producing the extra demand should

---

\(^{15}\) If $\gamma > \beta$, the incumbent chooses the investment level that is optimal when it is a monopolist also in the retail market.

\(^{16}\) It can be shown that a sufficient (though not a necessary) condition to guarantee that $x_k > 0$ is

$$(2\beta \gamma - \beta^2 - \gamma + 2\gamma \psi (1-\gamma)) - \beta (1-\gamma) > 0.$$ That is, to avoid having to consider corner solutions, we need to impose a slightly stronger condition regarding the convexity of the investment cost function than condition (7). This does not affect the results of the paper.
therefore not be too high for investment to occur. We therefore also know that if \( \frac{\gamma}{\beta} < c \), the rivals will not be active in the market.

The comparative statics of the equilibrium investment level with respect to the level of spillovers are given by

\[
\frac{\partial m^*}{\partial \beta} = \frac{(1 - \gamma)[\beta^2 - 2\gamma(1 - \gamma)]}{[2\beta - \beta^2 - \gamma + 2\gamma(1 - \gamma)]}. \tag{8}
\]

It is shown in Appendix A that the expression in (8) is positive for the permissible range of parameter values. Hence we can say that in the absence of price regulation, spillovers have a positive effect on investment incentives. The fact that also rivals benefit from the investment is therefore not in itself detrimental to investment incentives. This finding is not surprising given that in the present model, the incumbent can earn monopoly profits also from the access market. A similar result is obtained by Foros (2004).

We can also compare the investment level \( m^* \) chosen by the incumbent to the investment level that would be socially optimal under the same circumstances (that is, assuming that the rest of the game is unaltered)\(^{17} \). We take social welfare to be the sum of consumer surplus and the incumbent’s profits (the profits of the fringe firms are zero). Given the inverse demand curves in (2), consumer surplus is given by

\[
CS(p_1, a, m) = (1 + m - p_1)\frac{q_1}{2} + (\gamma + \beta m - a)\frac{q_2}{2} \tag{9}
\]

and total welfare is then given by

\[
W(p_1, a, m) = CS(p_1, a, m) + \pi(p_1, a, m). \tag{10}
\]

In the present case total welfare is \( W(p^*_1(m), a^*(m), m) = CS(p^*_1(m), a^*(m), m) + \pi(p^*_1(m), a^*(m), m) \) and the total derivative of welfare with respect to investment is given by

\(^{17} \)In general, a monopolist can have either too high or too low incentives to invest in quality (Spence 1975).
The fourth and the fifth terms in this expression are zero by the envelope theorem. Setting this expression equal to zero and solving for the socially optimal investment level yields 18

\[
m^{so} = \frac{3(1 - \gamma)(\gamma - \beta c)}{3(2\beta\gamma - \beta^2 - \gamma) + 4\gamma\psi(1 - \gamma)}
\]

(11)

It is shown in Appendix B that \( m^{so} > m^* \) and we can therefore conclude that the incumbent underinvests compared to the social optimum. Our results regarding the incumbent’s investment decision are summarised in the following proposition:

**Proposition 1.** When there is no access price regulation, spillovers have a positive effect on investment. Nevertheless, investment is below the socially optimal level.

### 4.3 Foreclosure due to underinvestment

It was shown above that the incumbent’s optimal level of investment will be below the social optimum. In the present section, we will examine the consequences that the incumbent’s investment decision has on the firms’ outputs in equilibrium. Inserting (6) into (5) yields equations for the incumbent’s and the fringe’s equilibrium quantities as functions of the model parameters only. These quantities are given by

\[
q_1^* = \frac{\beta(\gamma - \beta + (\beta - 1)c) + 2\gamma\psi(1 - \gamma)}{2(2\beta\gamma - \beta^2 - \gamma + 2\gamma\psi(1 - \gamma))},
\]

\[
q_2^* = \frac{\beta - \gamma + (1 - \beta)c - 2c\psi(1 - \gamma)}{2(2\beta\gamma - \beta^2 - \gamma + 2\gamma\psi(1 - \gamma))}.
\]

(12)

We can now examine the necessary and sufficient conditions for the rivals to be active in the market. From (12) we can see that the rival’s output is positive if and only if

\[
\beta - \gamma + (1 - \beta)c - 2c\psi(1 - \gamma) > 0.
\]

This condition can be written as

\[
\text{18 The second order condition for the regulator’s problem is } 3(2\beta\gamma - \beta^2 - \gamma) + 4\gamma\psi(1 - \gamma) > 0 \text{ and we assume that it holds.}
\]
\[ \beta - \gamma + \left[1 - \beta - 2\psi(1 - \gamma)\right]c > 0. \] This entails that in order for the rivals to be active in the market, the model parameters have to satisfy the condition\(^{19}\)

\[ c < c' \equiv \frac{\beta - \gamma}{\beta - 1 + 2\psi(1 - \gamma)}. \] (13)

This can be interpreted as a condition concerning the marginal cost of production (\(c\)) or the degree of convexity of the cost of investment (measured by the parameter (\(\psi\))). Either of these should be sufficiently low for the rivals to be active in the market. Note also that the right hand side of (13) is lower than \(\frac{\gamma}{\beta}\) or \(\gamma\) and therefore (13) is stricter than our previous conditions concerning \(c\). We can now state the following proposition:

**Proposition 2.** When there is no access price regulation, condition (13) is necessary and sufficient for the rivals to be active in the market.

Further, as the right hand side of (13) is increasing in \(\beta\), we can make the following statement:

**Corollary 1.** When there is no access price regulation, foreclosure is most likely to occur when investment spillovers are low.

Therefore, even in the case where \(\gamma < \beta\), that is, when the situation is \textit{a priori} favourable for the rivals, the rivals will be foreclosed from the market if the marginal cost of production or the degree of convexity of the cost of investment is too high, that is, if condition (13) fails. The condition \(\gamma < \beta\) implies that the rivals’ output is increasing in the level of investment. Therefore a low level of output by the rivals (or foreclosure) in this case is a symptom of a low investment level. If the marginal cost of producing the extra demand is too high or if the cost of investment increases too rapidly, it is not profitable for the incumbent to engage in a high level of investment. Further, as was shown above, the incumbent chooses an investment level that is below the social optimum and we can therefore say that in the present model, foreclosure can be caused by underinvestment. If this

\(^{19}\) See Appendix C for a proof that \(\beta - 1 + 2\psi(1 - \gamma) > 0\).
underinvestment problem could be solved, this would not only be beneficial in itself, but it would also facilitate competition.

5. Regulated access charge

Let us next consider the outcome of the model when the access charge is optimally regulated. We assume that the regulator cannot commit to an access pricing rule prior to the investment stage. This is a reasonable assumption as we consider investments in telecommunications network infrastructure that are often irreversible in nature and have an effect over long periods of time. The timing of the model is then as follows. First, the incumbent chooses the level of investment. Second, the regulator sets the access charge. Third, the incumbent sets its retail price and outputs and profits are realised. The timing of the model is summarised in Figure 1. The model is again solved by backwards induction.

![Figure 1. Timing of the model.](image)

5.1 Incumbent’s pricing decision and access price regulation

In the last stage, the incumbent sets its retail price, taking the access charge and the level of investment as given. The incumbent’s problem is therefore to maximise (3) with respect to \( p_1 \). The optimal price as a function of the access charge and the level of investment is given by

---

\[ p_1 = \text{function of access charge and level of investment} \]

---

20 Foros (2004) makes the same assumption about timing as we do in the present paper. Valletti and Cambini (2003) argue for and use the alternative assumption that the regulator sets the access price prior to the investment stage.
\[ \begin{align*}
    p_i^*(a,m) &= \frac{1}{2} [1 - \gamma + 2a + (1 - \beta)m].
\end{align*} \] (14)

In the second stage, the regulator chooses the access charge so as to maximise total welfare, which is now given by \( W(p_i^*, a) = CS(p_i^*, a) + \pi(p_i^*, a) \), where the consumer surplus function is as in (9). The total derivative of total welfare with respect to the access charge is
\[
\frac{dW}{da} = \frac{\partial CS}{\partial P^*} \frac{dP^*}{da} + \frac{\partial \pi}{\partial P^*} \frac{dP^*}{da} + \frac{\partial \pi}{\partial a}
\]
and the third term of this expression is again zero by the envelope theorem. Setting this expression equal to zero and solving for the access charge yields the result that the socially optimal (regulated) access charge is in the present model equal to the marginal cost of network provision, that is, \( a_R = c \).

In the present model, the result \( a_R = c \) is not surprising: as the incumbent’s final price is not regulated, the problem of financing the incumbent’s fixed costs is not an issue for the regulator. From this point of view, if there are no other distortions present, optimal regulation indeed implies marginal cost pricing. However, our model involves distortions caused by incumbent market power. In models of access price regulation that take into account incumbent market power, there are usually two opposing considerations that call for an access charge different from marginal cost: on the one hand, an access charge below marginal cost would reduce the incumbent’s final price and therefore reduce the deadweight loss caused by market power. On the other hand, the need to rebalance consumers’ choice between the incumbent’s and the fringe’s final products would call for an access charge above marginal cost. In general, the overall effect of these two factors is ambiguous, and the optimal access charge can be either above or below marginal cost. However, Laffont and Tirole (1994) have shown that these effects cancel out when both the incumbent and the rival’s demand functions are linear, as we have assumed in the present model.

In the present model, optimal access pricing is independent of the investment decision taken by the incumbent firm. This feature is due to our assumption that the regulator cannot

---

21 In order to make the expression less cumbersome, we drop the arguments in the functions \( a = a(m) \)
\[ p_i^* = p_i^*(a(m), m), \quad q_i^* = q_i^*(p_i^*(a(m)), a(m)), \quad \text{and} \quad q_2^* = q_2^*(p_i^*(a(m)), a(m)). \]
22 We use the subscript \( R \) throughout to refer to outcomes in the presence of access price regulation.
commit to an access pricing policy before investment has taken place. After the investment decision has been taken, the regulator’s problem is essentially a static one.

The incumbent’s and the rivals’ equilibrium quantities in the model with access price regulation are given by

\[
q_{1,\text{r}}^*(m) = \frac{1 - \gamma + (1 - \beta)m}{2(1 - \gamma)} \\
q_{2,\text{r}}^*(m) = \frac{(1 - \gamma)(\gamma - 2c) + (2\beta - \gamma - \beta\gamma)m}{2\gamma(1 - \gamma)}.
\]  

(15)

It can be noted from (15) that the incumbent’s output (as a function of the level of investment) is the same as in the unregulated case. Access price regulation has lowered the price of the competitors’ product, which by (14) causes a fall also in the incumbent’s price. The effect of these price changes on the incumbent’s demand exactly offset each other in our linear model. The expression for the rivals’ output in (15) takes a more complicated form than in (5) and it is positive if \((1 - \gamma)(\gamma - 2c) + (2\beta - \gamma - \beta\gamma)m > 0\). Unlike in the unregulated case, we cannot identify any simple relation between the model parameters that would imply foreclosure independently of the level of investment.

In order to make this case comparable with the case of no regulation analysed above, we continue to assume that \(\beta > \gamma\). With this assumption, the rivals’ output is again increasing in \(m\) and \(c < \frac{\gamma}{2}\) would be a sufficient condition for the rivals to be active in the market regardless of the level of investment. In order to find a necessary condition, we need to examine the incumbent’s investment decision under access price regulation.

5.2 Optimal investment level and effect of spillovers

In the first stage of the model with regulation, the incumbent chooses the level of investment, anticipating the regulator’s decision on the access charge that will follow and its own pricing decision in the last stage. The incumbent’s objective function in the present
case is obtained from (3) by setting \( a_r = c \) and inserting the optimal price from (14). The incumbent’s optimal investment level is now given by

\[
m^*_R = \frac{(1 - \gamma)(1 - \beta)}{2\psi(1 - \gamma) - (1 - \beta)^2}.
\]  

(16)

The second order condition takes the form

\[
2\psi(1 - \gamma) - (1 - \beta)^2 > 0
\]

and it is satisfied, given \((7)\)\(^{23}\). We can observe from (16) that with optimal access price regulation, the level of investment is positive if and only if \( \beta < 1 \). In the presence of access price regulation, the comparative statics of the investment level with respect to the level of spillovers are given by

\[
\frac{\partial m^*_R}{\partial \beta} = \frac{(\gamma - 1)[2\psi(1 - \gamma) + (1 - \beta)^2]}{2\psi(1 - \gamma) - (1 - \beta)^2},
\]

(18)

which is always negative. These results have an intuitive interpretation: as the incumbent is not allowed to make a profit on access provision, the fact that part of the benefits of the incumbent’s investment efforts accrue on rivals has a negative effect on investment incentives. Further, if investment would benefit the fringe more than the incumbent (\( \beta > 1 \)), investment is not profitable for the incumbent and therefore no investment will take place: in this case, both the incumbent’s equilibrium price in (14) and its equilibrium quantity in (15) are decreasing in the level of investment. In Appendix D we show that access price regulation also lowers investment incentives relative to the unregulated case, so that \( m^* > m^*_R \).

\(^{23}\) In the case of access price regulation, a necessary condition to guarantee that \( x_g > 0 \) is

\[
c > c = \beta(1 - \beta)(1 - \gamma)[2\psi(1 - \gamma) - (1 - \beta)^2].
\]

That is, to avoid having to consider corner solutions, we need to impose a minimum level of \( c \). This does not affect the results of the paper (see footnote 27).
Foros (2004) obtains similar results, except that in his model investments are driven down to zero only when the competitor is twice as efficient as the incumbent in converting the investment into value added services ($\beta > 2$). Therefore the disincentive effects of investment spillovers under access price regulation are stronger in the present model than in Foros’s model.

Again, we can compare the investment level chosen by the incumbent in the presence of access price regulation, with the investment level that would be socially optimal under the same circumstances. The socially optimal investment level would be such that total welfare

$$W(p^*_i(m, a_R), a_R, m) = CS(p^*_i(m, a_R), a_R, m) + \pi(p^*_i(m, a_R), a_R, m)$$

is maximised. Again, taking the total derivative of total welfare with respect to the investment level and setting this expression to zero yields

$$m^{SO}_R = \frac{(1 - \gamma)(\beta \gamma + 3 \gamma - 4 \beta c)}{\beta (\beta \gamma - 4 \beta + 6 \gamma) - 3 \gamma + 4 \gamma \psi (1 - \gamma)}$$ (19)

It is shown in the Appendix E that $m^{SO}_R > m^*_R$, so that the socially optimal investment level would again be higher than the investment level chosen by the incumbent.

The results regarding the incumbent’s choice of investment level and the effect of spillovers in the case of access price regulation are summarised in the following proposition:

**Proposition 3.** When the access charge is optimally regulated, spillovers have a negative effect on investment. The level of investment is below the socially optimal level and it is also lower than in the absence of access price regulation. If competitors would benefit more from the investment than the incumbent ($\beta > 1$), there is no investment.

Comparing the cases with and without access price regulation we can say that the presence of competitors that also benefit from the incumbent’s investment efforts is not detrimental to investment incentives *per se*, but incentives are perversely affected by access provision only when the access charge is regulated.

---

24 We again assume that the second order condition $\beta (\beta \gamma - 4 \beta + 6 \gamma) - 3 \gamma + 4 \gamma \psi (1 - \gamma) > 0$ holds.
Throughout the analysis, we have made the simplifying assumption that the effect of the investment is the same for all consumers, independently of the consumer’s type \( (x) \). However, it should be noted that the qualitative results of the current as well as the next section are robust to a change in the model specification, where we allow the effect of the investment to vary according to the consumer’s type (that is, when consumers with a high initial valuation for telecommunications services also value the investment more highly than those consumers with a low initial valuation).

5.3 Foreclosure due to underinvestment

When the access price is regulated, the incumbent might want to use other instruments, namely its choice of the investment level, to deter entry\(^{25}\). However, as we continue to assume that \( \beta > \gamma \), the fringe’s output in (15) is increasing in the level of investment, and therefore using overinvestment to deter entry will not be possible\(^{26}\). On the other hand, entry deterrence by underinvestment is not a credible strategy: as noted by Fudenberg and Tirole (1984), the possibility of such strategic underinvestment arises only if the incumbent is not able to invest after entry.

However, as the incumbent’s optimal level of investment in the presence of competition is again below the socially optimal level, the model does exhibit an underinvestment problem. This can have similar consequences as those reported in section 4.3. Taking into account the level of investment given in (16), the equilibrium quantities of the incumbent and the fringe when the access charge is regulated are given by

\[
q^{*}_{1,R} = \frac{\psi (1 - \gamma)}{2\psi (1 - \gamma) - (1 - \beta)^2}
\]

\[
q^{*}_{2,R} = \frac{(\beta - 1)(\gamma - \beta + (\beta - 1)c) + \psi (1 - \gamma)(\gamma - 2c)}{\gamma [2\psi (1 - \gamma) - (1 - \beta)^2]}
\]

\(^{25}\) When the access price is not regulated, the incumbent will not have an incentive to distort its investment decision to deter entry, as raising the access price is a costless way of achieving the same result whenever it is optimal. This is related to the finding for example in Beard et al (2001) that the incentives for non-price discrimination increase when access price regulation becomes more stringent.

\(^{26}\) Note that the rivals’ output is increasing in investment if \( \beta > \frac{\gamma}{2 - \gamma} \), which is less strict than the condition \( \beta > \gamma \) that we have assumed.
As in the unregulated case, the incumbent’s output is always positive also with access price regulation. The rivals’ output will be positive if and only if 
\[
(\beta - 1)(y - \beta + (\beta - 1)c) + \psi(1 - \gamma)(y - 2c) > 0.
\]
This condition can be written as 
\[
[1 - \beta]^2 - 2\psi(1 - \gamma)c + (\beta - 1)(y - \beta) + \gamma\psi(1 - \gamma) > 0.
\]
We know by the second order condition (17) that the coefficient of \(c\) in this expression is negative. Therefore, when \(\beta < 1\) and the incumbent sets the level of investment given by (16), the necessary and sufficient condition for the rivals to be active in the market is

\[
c < c^f_k \equiv \frac{(1 - \beta)(y - \beta) + \gamma\psi(1 - \gamma)}{2\psi(1 - \gamma) - (1 - \beta)^2}.
\] (21)

The only assumption that we have made concerning \(c\) so far is \(c < \gamma\). It is shown in Appendix F that \(c^f_k < \gamma\) and therefore there is nothing in the model that would guarantee that (21) holds\(^{27}\). Therefore, access price regulation does not necessarily guarantee the viability of competition.

It should be stressed that condition (21) refers to the case where \(\beta < 1\). If spillovers were such that investment would benefit the rivals more than the incumbent \((\beta > 1)\), we know that the incumbent would undertake no investment and (21) would not be a sufficient condition for the rivals to be active in the market: in that case we would need the stricter condition \(c < \frac{\gamma}{2}\) to hold. The main results of this section are summarised in the following proposition:

**Proposition 4:** When the access price is optimally regulated and \(\beta < 1\), condition (21) is necessary and sufficient for the rivals to be active in the market. If \(\beta > 1\), \(c < \frac{\gamma}{2}\) is necessary and sufficient for the rivals to be active in the market.

\(^{27}\) Further, it can be shown that \(c < c^f_k\) (see footnote 21), so that there are parameter values for which the rivals are active in the market \((c < c^f_k)\) and parameter values for which they are foreclosed by insufficient investments \((c > c^f_k)\).
**Corollary 2.** When there is access price regulation, foreclosure is most likely to occur when investment spillovers are high.

It should be noted that the incumbent’s profit function is discontinuous at the point \( q_2^* = 0 \), as at this point some of the customers who have bought from the rivals will switch to buying from the incumbent. Therefore the incumbent might have an incentive to use underinvestment to foreclose the rivals from the market even for cost levels lower than those indicated by condition (21). However, as was noted above, such a strategy of entry deterrence would not be credible: if the rivals were to enter, the incumbent would raise investments to the level that is optimal in the presence of competition, that is, to the level given in (18). This level of investment leads to foreclosure, if condition (21) is violated. Hence, violation of condition (21) indicates the situations where foreclosure is both optimal and feasible for the incumbent.

Comparing the cases with and without access price regulation, it can be noted that in the case without regulation, the incumbent’s and the regulator’s interests are partially aligned in the following sense: without regulation, foreclosure is most likely to occur when the rivals are relatively inefficient in utilising the investment, that is, when spillovers are low. However, in the case with access price regulation, the opposite holds: foreclosure due to underinvestment is most likely to occur if the rivals are relatively efficient in turning the investment into services valued by consumers. Socially suboptimal investments now have three negative effects: the underinvestment problem is detrimental not only in itself, but it also causes problems for the viability of competition. Further, competition is most likely to be eliminated precisely when it would bring the highest benefits for consumers.

What are the public policy measures that could be used to solve the underinvestment problem found in our model? Increasing the access charge would naturally boost investment incentives, but this measure would in itself have negative effects on competition. Rather, it is likely that the problems caused by underinvestment could be alleviated if the regulator could in addition to access pricing use another instrument, namely an investment subsidy. A combination of two instruments would enable the regulator to achieve its two goals of promoting competition and providing optimal incentives for investment.
It should also be noted, however, that access price regulation may create incentives for the incumbent to engage in other types of activities that disadvantage the rivals. As was noted in the introduction, it may sometimes be possible for the incumbent to unilaterally reduce the quality of the input sold to the rival. Such a situation has been examined for example by Beard et al (2001), who find that the incentives for this type of discrimination typically increase when the access price is regulated.

6. Conclusions

In this paper we have analysed a vertically integrated monopolist telecommunications network provider’s incentives to invest in a technology that increases demand when it also provides network access to rivals in the retail market. We have found that the fact that also rivals benefit from the investment is not in itself detrimental for investment incentives: when the incumbent is free to choose the access charge, the level of investment is increasing in the degree of spillovers to rivals’ demand. However, access price regulation has a negative effect on investment incentives: the level of investment is in our model always lower with access price regulation than without it. Further, when the access price is optimally regulated, spillovers that benefit rivals have an unambiguously negative effect on investment incentives. If the rivals are more efficient in utilising the investment than is the incumbent, this disincentive effect is so strong that no investment will take place.

We have also shown that the incumbent’s optimal investment level is below the social optimum, regardless of whether access charges are regulated or not. We have emphasised three reasons for why this underinvestment problem deserves attention from regulators. Firstly, like any other market distortion, insufficient investments of course imply losses to overall welfare. Secondly, when spillovers are high and the rivals would therefore benefit considerably from the investment, the underinvestment problem will also have adverse effects on the viability of competition. In some cases the investment level can be so low that the rivals are foreclosed from the market completely. Thirdly, in the case of access price regulation, the underinvestment problem is most likely to lead to the exclusion of competitors precisely when they would bring the highest benefits for consumers. It was suggested that a combination of access price regulation and investment subsidies could be
employed to foster competition and to eliminate the problems associated with insufficient investments.

Throughout the paper we have assumed that the regulator cannot commit to an access pricing regime prior to the incumbent’s investment decision. This assumption is realistic in the present context, as investments in telecommunications infrastructure are typically irreversible and have an effect over long periods of time. The results of the analysis would most probably change if the regulator’s commitment problem could be solved. If the regulator were able to credibly commit to an access pricing regime prior to the incumbent’s investment decision, it would take into account the effect of access price regulation on investment incentives. The access charge would then be set at some level $a^g > c$ and the effect on investment incentives would be weaker than in the case analysed here. Indeed, the case $a^h = c$ is naturally the case where the negative effect of access price regulation on investment incentives is highest. On the other hand, we expect that the rule that would minimise the negative effect on investment incentives would be an ECPR-type access pricing rule, where the incumbent would be compensated for lost profit in the retail market due to entry. The optimal outcome would involve settling a trade-off between providing investment incentives and allocative efficiency and the outcome will be to set the access charge somewhere between the above mentioned extremes. Therefore, even if the commitment problem could be solved, the lessons of the present paper are likely not to lose their relevance entirely: some mitigating effect on investment incentives is likely to be observed as long as selling access is less profitable than selling telecommunications services directly to final customers.
References


Appendix

A. Proof of $\frac{\partial m^*}{\partial \beta} > 0$:

From equation (8), the sign of $\frac{\partial m^*}{\partial \beta}$ is given by the sign of

$$\left[\gamma - \beta^2 - 2\gamma \psi (1 - \gamma)\right] c + 2\gamma (\beta - \gamma).$$

The first part of this expression is negative by the second order condition (7) and by the assumption that $\beta > \gamma$. $\frac{\partial m^*}{\partial \beta}$ is therefore positive if the model parameters satisfy the condition

$$c < \frac{2\gamma (\beta - \gamma)}{\beta^2 - \gamma + 2\gamma \psi (1 - \gamma)}. \quad (\ast)$$

It is shown below that the rivals are active in the market only if condition (13) holds. Now, the difference between the RHS of (\ast) and the RHS of (13) is given by

$$\left(\beta - \gamma\right)\frac{2\beta \gamma - \beta^2 - \gamma + 2\gamma \psi (1 - \gamma)}{\left[\beta^2 - \gamma + 2\gamma \psi (1 - \gamma)\right]\left[\beta - 1 + 2\psi (1 - \gamma)\right]},$$

which is positive given the second order condition (7) and the assumption $\beta > \gamma$. Therefore the RHS of (\ast) is higher than the RHS of (13) and condition (\ast) holds for all permissible parameter values (that is, always when the rivals are active in the market). Hence $\frac{\partial m^*}{\partial \beta} > 0$.

B. Proof of $m^{so} > m^*$:

After some manipulations, we find that

$$m^{so} - m^* = \frac{2\psi \psi \gamma - \beta \psi \gamma (1 - \gamma)^2}{\left[2\beta \gamma - \beta^2 - \gamma + 2\gamma \psi (1 - \gamma)\right]\left[2\beta \gamma - \beta^2 - \gamma + 4\gamma \psi (1 - \gamma)\right]}.$$
Both terms in the denominator are positive by the second order conditions and as we know that $\gamma - \beta c > 0$ has to hold for the optimal investment level to be positive. Therefore $m^* - m^* > 0$.

C. Proof of condition (13):

In order to show that condition (13) is the appropriate condition for the rivals’ output to be positive we need to show that that $\beta - 1 + 2\psi(1 - \gamma) > 0$. Let us first assume that the reverse holds, that is, $\beta - 1 + 2\psi(1 - \gamma) < 0$. This together with our previous condition $\beta > \gamma$ would in fact yield a sufficient condition for the rivals to be active in the market. However, in order for those two conditions to possibly be consistent, we would need to have that $\psi < \frac{1}{2}$. With $\psi < \frac{1}{2}$, the expression associated with the second order condition in (7) is smaller than $2\beta\gamma - \beta^2 - \gamma + \gamma(1 - \gamma)$, which is equal to $2\beta\gamma - \beta^2 - \gamma^2 = -(\beta - \gamma)^2 < 0$. Therefore $\psi < \frac{1}{2}$ implies a failure of the second order condition for all values of $\beta$ and $\gamma$. Therefore, we need to have $\psi > \frac{1}{2}$ for the second order condition to be satisfied and this together with $\beta > \gamma$ implies that $\beta - 1 + 2\psi(1 - \gamma) > 0$. Hence the appropriate condition for the rivals’ output to be positive in the case of no regulation is given by (13).

D. Proof of $m^* > m^*_R$:

After some manipulations, we find that

$$m^* - m^*_R = \frac{\beta(1 - \gamma)(1 - \beta)(\beta - \gamma + c(1 - \beta)) + 2\psi(\gamma - c)(1 - \gamma)}{2\beta\gamma - \beta^2 - \gamma + 2\gamma\psi(1 - \gamma)[2\psi(1 - \gamma) - (1 - \beta)^2]}.$$ 

Both terms in the denominator are positive by the second order conditions and the denominator is positive given our assumptions $c < \gamma < \beta < 1$. Hence $m^* - m^*_R > 0$. 
E. Proof of $m_R^{SO} > m_R^*$:

After some manipulations we find that

$$m_R^{SO} - m_R^* = \frac{2(1-\gamma)\{2\beta(1-\beta)(\beta - \gamma + c(1-\beta)) + \psi(1-\gamma)(\gamma + 3\beta\gamma - 4\beta c)\}}{2\psi(1-\gamma) - (1-\beta)^2\beta(\beta\gamma - 4\beta + 6\gamma) - 3\gamma + 4\gamma\psi(1-\gamma)}.$$  

Both terms in the denominator are positive by the second order conditions and therefore the sign of $m_R^{SO} - m_R^*$ is given by the sign of

$$2\beta(1-\beta)(\beta - \gamma + (1-\beta)c) + (1-\gamma)(\gamma + 3\beta\gamma - 4\beta c).$$

The first half of this expression is positive, given our assumptions $\gamma < \beta < 1$. Therefore a sufficient (even though not a necessary) condition for the entire expression to be positive is $\gamma + 3\beta\gamma - 4\beta c > 0$. Further, given our assumption that $c < \gamma$, we can make the following comparisons:

$$\gamma + 3\beta\gamma - 4\beta c > \gamma + 3\beta\gamma - 4\beta\gamma = \gamma(1-\beta) > 0.$$  

Hence $m_R^{SO} - m_R^* > 0$.

F. Proof of $c_R^f < \gamma$:

After some manipulations we find that $\gamma - c_R^f = \frac{(1-\gamma)(\beta - 1)\psi}{2\psi(1-\gamma) - (1-\beta)^2}$, which is larger than zero if $\psi > \frac{(1-\beta)\beta}{\gamma} \in \left(0, \frac{\gamma}{4}\right)$. We also know that $\frac{\gamma}{4} \in \left(0, \frac{1}{4}\right)$ and (from Appendix C) that $\psi > \frac{1}{2}$. Therefore, $c_R^f < \gamma$.