Mobile Termination with Asymmetric Networks∗

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Abstract

This paper examines mobile termination fees and their regulation when networks are asymmetric in size. It is demonstrated that with consumer ignorance about the exact termination rates (a) a mobile network’s termination rate is the higher the smaller the network’s size (as measured through its subscriber base) and (b) asymmetric regulation of only the larger operators in a market will, ceteris paribus, induce the smaller operators to increase their termination rates. The results are supported by empirical evidence using data on mobile termination rates from 48 European mobile operators from 2001 to 2003.

Keywords: mobile termination, telecommunications, consumer ignorance, price regulation

JEL Classification: L13, L51, L96.

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1 Introduction

While in many mobile telecommunications markets across the world competition has long been left without much regulatory intervention, recently some aspects have come under close scrutiny by regulatory authorities. Apart from mobile number portability and national and international roaming, one of the key areas under investigation are mobile termination charges (see, e.g., European Commission, 2002, Gans and King, 1999).

While mobile termination rates are already regulated in some countries (such as the UK), they are not regulated in others (such as Germany). In some other countries again (such as the Netherlands), only the termination rates of the larger mobile operators (which are supposed to be dominant or to enjoy significant market power) are regulated. Hence, operators are regulated in an asymmetric fashion in the latter case, with some termination rates being regulated while others are set by unregulated firms.

Two policy questions arise, given these different institutional frameworks governing mobile termination: First, what termination rates do emerge if prices are left unregulated? And secondly, how are these rates affected by regulation?

Gans and King (2000) have addressed exactly these questions. Their finding is that mobile termination rates may even exceed monopoly prices due to a negative pricing externality, which results from consumer ignorance regarding prices. Consumer ignorance is a particular problem of mobile telephony as customers are often not able to identify which specific network they are calling. This is because consumers may not know which operator is associated with each particular number. As a consequence, consumers are often ignorant about the price that they actually have to pay for a mobile call if prices differ between different networks (see Gans and King, 2000; Wright, 2002). In addition, mobile number portability is likely to exacerbate this problem as mobile prefixes will no longer identify networks (see Buehler and Haucap, 2003). Hence, as Gans and King (2000) have pointed out consumers are likely to base their calling decisions on average prices. This will be the case if either carriers are unable to set different prices for different mobile networks anyway or if consumers cannot determine ex ante which mobile network they are actually ringing when placing a call, i.e. if callers suffer from consumer ignorance.

If consumers are not aware of the correct prices and base their demand on the average price, a negative pricing externality arises as the price of one firm will not only affect its own demand, but also that of its rivals. This induces firms to increase their termination rates to inefficiently high levels as they do not account for the effect that their own price has on the average price perception and, thereby, their rivals’ demand. This externality problem comes on top of any monopoly and associated double marginalization problems.
If market shares are endogenous and termination rates are set prior to other prices, termination rates may even be set so high that they "choke" off the demand for mobile termination altogether (see Gans and King, 2000, p. 323). Consequently, demand for termination services will increase with any downward regulation of termination rates.

We build on these research and extend it into three directions: Firstly, we will introduce network asymmetry into the model and consider mobile networks of different sizes (in terms of their subscriber bases). While Gans and King (2000) analyze a symmetric duopoly, we will provide a model with four asymmetric mobile network operators. Secondly, we will analyze the effects of asymmetric regulation in this framework, as asymmetric regulation is a common feature of many European telecommunications markets, which has been largely neglected so far. And thirdly, we will provide empirical evidence for our model.

The main results of our paper are, firstly, that smaller mobile operators will charge higher termination rates than larger operators, as a small operator's impact on the weighted average price is relatively small, so that smaller operators can increase their prices significantly without a major reduction in the quantity demanded. In contrast, a large operator also has a larger impact on the weighted average price so that the firm more is more constrained in its pricing policy. Secondly, asymmetric regulation of the larger operators will, ceteris paribus, induce the small operators to increase their termination rates even further. These results are supported by our empirical findings.

The remainder of the paper is organized as follows. In Section 2 we introduce the model and present the key results of our analysis. In Section 3, we provide empirical evidence to test the model’s hypotheses. Finally, section 4 discusses policy implications and concludes.

2 The Model

There are four mobile networks \( i = 1, 2, 3, 4 \), which differ in the size of their subscriber base. We assume that the four mobile networks’ market shares do not depend on the respective termination charges, i.e. consumers do not base their subscription decision on the price for being called. More precisely, we assume that the mobile networks’ market shares are already given when termination rates are set, so that we can treat them as exogenous.\(^1\) Let us also assume that there are two large and two small mobile networks, which is a fairly typical market structure for many European telecommunications markets.

\(^1\)Note that this assumption is also employed by Gans and King (2000) in much of their analysis. In addition, it has proven extremely difficult to analyze termination rates with endogenous market shares, as the optimization problem is no longer supermodular (see, e.g., Buehler, 2002).
markets. The two large networks \( i = 1, 2 \) have a subscriber base of \( x_1 = x_2 = x_L \) customers, while the small networks’ subscriber base is denoted by \( x_3 = x_4 = x_S \). We also assume that each individual subscriber has a linear inverse demand for mobile telephone calls, which is given by

\[
q_j = a - bp,
\]

where \( p \) denotes the perceived price for mobile-to-mobile calls. We follow the European Commission (2003) and regard the market for mobile termination services as a relevant market in its own. Furthermore, we assume that the marginal cost of terminating a mobile call is negligible and that prices for mobile-to-mobile calls are effectively determined through the respective termination charges. That is, we abstract from any double mark-up problem which may result if operators would compete in linear tariffs. As is well known from the literature (see, e.g., Laffont and Tirole, 1998, or Wright, 2002), the double mark-up problem vanishes if operators set two-part tariffs consisting of a fixed (monthly) fee and a price per calling minute. Given that mobile operators usually set multi-part tariffs we abstract from potential double marginalization problems and assume that the price for a call from, say, mobile network 1 to mobile network 2 is given by \( p_{12} = t_2 \) where \( t_2 \) is the termination rate set by operator B.

If consumers have perfect knowledge and are not ignorant about a network’s identity, the price for a calling unit from mobile network \( i \) to mobile network \( j \) will simply equal the monopoly mobile termination rate that network \( j \) will set, i.e. \( p_{ij} = t_j^M = a / (2b) \) for \( j = 1, 2, 3, 4 \).

To capture the idea of consumer ignorance, we now follow Gans and King (2000) and assume that it is the average price which determines consumer demand for calls to other mobile networks. To focus on the termination market we restrict the analysis to off-net calls and ignore on-net calls which are calls that originate and terminate within the same mobile network. Hence, we only consider the demand for calls which originate in one network and are terminated in another network, i.e. calls from network 1 to networks 2, 3 and 4, from network 2 to networks 1, 3 and 4, and so on. The according demand for calls from network 1 to network 2, 3 and 4 is given by

\[
q_1 = x_L(x_L + 2x_S)(a - b(\frac{x_L}{x_L + 2x_S}p_{12} + \frac{x_S}{x_L + 2x_S}p_{13} + \frac{x_S}{x_L + 2x_S}p_{14})),
\]

where \( p_{12} = t_2, p_{13} = t_3 \) and \( p_{14} = t_4 \). Hence, the demand for off-net calls from network 1 into other networks depends on the size of its subscriber base \( (x_L) \), the aggregate size of the other networks’ subscriber base \( (x_L + 2x_S) \) and the weighted average termination rate charged by the three other networks. Similarly, we can express the demand for off-net calls form the three other networks. Since the two small networks have a subscriber base
of size \( x_S \), making off-net calls to a total of \((2x_L + x_S)\) subscribers on the three other networks, we can write firm 3’s demand for off-net calls as

\[
q_3 = x_S(2x_L + x_S)(a - b(\frac{x_L}{2x_L + x_S}p_{31} + \frac{x_L}{2x_L + x_S}p_{32} + \frac{x_S}{2x_L + x_S}p_{34})),
\]

where again \( p_{3j} = t_j \) for \( j = 1, 2, 4 \).

The profit that an operator \( i \) generates from termination depends on its termination rate, \( t_i \), and the number of incoming calls from other networks. Assuming balanced calling patterns, operator 1’s profit is now given by

\[
\pi_1 = t_1\left(\frac{x_L}{x_L + 2x_S}q_2 + \frac{x_L}{2x_L + x_S}q_3 + \frac{x_L}{2x_L + x_S}q_4\right).
\]

Similarly, operator 3’s profit (as a small operator) is given by

\[
\pi_3 = t_3\left(\frac{x_S}{x_L + 2x_S}q_1 + \frac{x_S}{x_L + 2x_S}q_2 + \frac{x_S}{2x_L + x_S}q_4\right).
\]

Maximizing with respect to \( t \) and taking into account both the symmetry between the two large networks (1 and 2) and between the two small networks (3 and 4) we obtain the following best response functions

\[
t_L = \frac{1}{4x_L} \frac{a 4x_L^3 + 12x_Lx_S^2 + 9x_Lx_S^2 + 2x_L^3}{b} - \frac{x_S x_L^2 + x_Lx_S + x_S^2}{x_L 3x_S^2 + 2x_Lx_S + x_S^2} t_L,
\]

\[
t_S = \frac{1}{4x_S} \frac{a 2x_S^3 + 12x_Lx_S^2 + 9x_Lx_S^2 + 2x_S^3}{b} - \frac{x_L x_S^2 + x_S + x_S^2}{x_S 3x_S^2 + 2x_Lx_S + x_S^2} t_L.
\]

Note that, even though operators do not set quantities but prices, the networks’ prices are strategic substitutes as \( \partial t_1/\partial t_J < 0 \). This contrasts with price setting under Bertrand competition and with vertically related markets with double marginalization problems where prices are strategic complements. This leads us to our first observation:

**Remark 1.** As termination rates under consumer ignorance are strategic substitutes, downward regulation of the large operators’ termination rates will, ceteris paribus, lead to an increase in the small operators’ termination rates (as \( \partial t_1/\partial t_J < 0 \)).

Solving the best response functions given above, we obtain the following equilibrium termination rates:

\[
t_L = \frac{1}{4x_L} \frac{a 2x_S^5 + 9x_Lx_S^4 + 22x_L^2x_S^3 + 31x_L^3x_S^2 + 15x_L^4x_S + 2x_S^5}{b} - \frac{2x_L^4 + 6x_Lx_S + 11x_L^2x_S^2 + 6x_Lx_S^3 + 2x_S^4}{2x_L^4 + 6x_Lx_S + 11x_L^2x_S^2 + 6x_Lx_S^3 + 2x_S^4} t_L,
\]

\[
t_S = \frac{1}{4x_S} \frac{a 2x_S^5 + 9x_Lx_S^4 + 22x_L^2x_S^3 + 31x_L^3x_S^2 + 15x_L^4x_S + 2x_S^5}{b} - \frac{2x_L^4 + 6x_Lx_S + 11x_L^2x_S^2 + 6x_Lx_S^3 + 2x_S^4}{2x_L^4 + 6x_Lx_S + 11x_L^2x_S^2 + 6x_Lx_S^3 + 2x_S^4} t_L.
\]

Comparing \( t_L \) and \( t_S \) we can state the following result:
Proposition. The small operators’ termination rate, $t_S$, is strictly larger than the large operators’ rate, $t_L$, ($t_S > t_L$) if $x_L > x_S$.

Proof. See Appendix.

The intuition for this result is that the small operators only have a relatively small impact on the average price, which determines demand. Hence, if a small operator increases its termination rate the demand for off-net calls will only be reduced by a relatively small amount as the increase in the average termination price will be relatively small. In contrast a large operator has a relatively large affect on the average price so that the incentive to increase the termination rate will be lower than with a small operator.

To illustrate the relationship between network size and termination rates, let us assume that we can express the small networks’ size as a fraction of the large networks’ subscriber base, i.e. $x_S = g x_L$ with $0 \leq g \leq 1$. In this case, the firms’ termination rates are given by

$$t_L = \frac{a}{4b} \frac{2 + 15g + 31g^2 + 22g^3 + 9g^4 + 2g^5}{2 + 6g + 11g^2 + 6g^3 + 2g^4},$$

$$t_S = \frac{a}{4bg} \frac{2 + 9g + 22g^2 + 31g^3 + 15g^4 + 2g^5}{2 + 6g + 11g^2 + 6g^3 + 2g^4}.$$

Note that the large operators’ termination rates are increasing in $g$ for $0 \leq g \leq 1$ (i.e. $\partial t_L / \partial g \geq 0$), while the small operators’ termination rates are decreasing over this range (i.e. $\partial t_S / \partial g \leq 0$). Figure 1 depicts the termination rates given above for $a = b = 1$.

Comparing these termination rates with the monopoly price in the absence of consumer ignorance, $t_i^M = a/(2b)$, we find that the small operators’ termination rate, $t_S$, will always exceed $t_i^M$, while the large operators’ rate, $t_L$, will only exceed the monopoly benchmark for $g \geq g^* \approx 0.29783$. Otherwise, the negative pricing externality created by the small operator is so large that the large operators’ price will be constrained even below the monopoly level.

In summary, as can be seen from the Proposition and Remark 1, our theoretical model suggests (a) that a network’s termination rate is the higher the smaller the network’s size (as measured through its subscriber base) and vice versa and (b) that asymmetric regulation of only the larger operators in a market will, ceteris paribus, induce the smaller operators to increase their termination rates. In the following section, we will provide some empirical evidence to test these two hypotheses.
3 Empirical Evidence

3.1 Data

To test our model’s hypotheses empirically, we have assembled data on mobile termination rates and the subscriber base of 48 different mobile operators from 17 European countries.\textsuperscript{2} Data on the networks’ subscriber base has been gathered from Mobile Communications, while the termination rates have been obtained from various issues of the Cullen Report, published by Cullen International. Information on regulatory regimes has also been obtained from this source and also from various regulatory authorities. Our earliest observations are from February 2001 and our most recent one from February 2003. Hence, our data set includes regulated and unregulated termination rates. While we use monthly data in principle, there are missing observations for several months due to limited data availability.\textsuperscript{3} Therefore, we cannot conduct a panel data analysis, but

\textsuperscript{2}These are the 15 EU countries plus Norway and Switzerland.

have to confine our analysis to pooled estimations.

The endogenous variable of our analysis is the operators’ termination rate. Since termination rates differ in their structure across countries and at times even across firms,\(^4\) we have calculated termination rates for a two minute call. We have also restricted the analysis to peak-time tariffs. As exogenous variables we have used (apart from a constant) market shares (based on subscriber numbers), the Herfindahl Index (HHI), market size (based on total subscriber numbers) and two dummy variables describing the regulatory framework in place. The variable RC is set to one if any mobile termination rate in a specific country is regulated, while RC is zero if none of the mobile operators’ termination rates is regulated. Furthermore, the variable RF is set to one if a specific firm’s termination rate is regulated, while RF is zero if the firm’s termination rate is not regulated. Using two dummy variables is necessary because in some countries all mobile termination rates are regulated, in others only some termination rates (usually those of large operators) are regulated, and in others again none are regulated. Hence, \(RC = RF = 1\) if all firms are regulated in a country, \(RC = RF = 0\) if none is regulated, and \(RC = 1\) and \(RF \in [0,1]\) if some firms, but not all are regulated in a country. We have also used dummy variables indicating the respective year and country to control for eventual time trends and country-specific effects.

Let us also briefly provide some descriptive statistics of our variables to shed some light on price trends and regulatory practice in Europe. The (unweighted) average termination rate across all 48 operators decreases from 54.2 Eurocents in 2001 to 38.5 Eurocents in 2002 and 37.8 Eurocents in February 2003. While the maximum rate in 2001 has been 80 Eurocents and the minimum 37 Eurocents, in 2003 the maximum rate has been 54 Eurocents and the minimum 19.7 Eurocents. Over this period the regulated firms’ average termination rate has been 42.3 Eurocents, while it has been 45.3 Eurocents for unregulated firms. Looking at the different countries, the average termination rate in regulated countries has been 44.4 Eurocents, while it has only been 42.8 Eurocents in unregulated countries. This indicates that termination rates have been higher on average in regulated countries. In this context, it may be interesting to note that only 14 operators had been regulated in February 2001 while there have been 26 regulated firms in February 2003.

The observed firms’ average market share has been steadily around 33 percent, ranging from less than 2 per cent for the smallest operator (Italy’s Blu in February 2001) and more than 75 percent for the largest operator (Norway’s Telenor also in February 2001).
Finally, market size obviously varies considerably between countries ranging from 304,000 subscribers in Luxembourg in February 2001 up to more than 57 million subscribers in Germany in February 2003. More detailed descriptive statistics can be found in Table 1 in the Appendix.

### 3.2 Empirical Results

Table 2 reports estimation results of price levels and logarithms of the price (in that case also using logarithms of the market size).

In the regressions we have also calculated Moulton (1990) corrected $t$-statistics in order to correct for potential biases that arise if aggregated variables are used to measure effects on micro units. Since the assumption of independent disturbances is usually violated with aggregated exogenous variables, using ordinary least squares can lead to standard errors that are seriously biased downwards. Hence, it is important to bear in mind the data’s group structure as suggested by Moulton (1990). As can be observed from Table 1, the corrected $t$-statistics, given in brackets, are usually lower than the uncorrected $t$-statistics.

The analysis reveals that an operator’s market share tends to have a statistically significant impact on its termination rate with the sign as predicted by our model, i.e. smaller operators tend to have significantly higher mobile termination rates. This indicates that it seems to be especially the smaller operators who can exploit consumers’ ignorance and set relatively high termination rates as they only have small effects on average prices.

In contrast, the Herfindahl Index does not appear to be statistically significant for explaining termination rates. Hence, market concentration is apparently less of an issue for the determination of termination rates than the operators’ size or, more precisely, its smallness. Admittedly, we would expect that concentration had a significant impact on termination rates as well, since under consumer ignorance large asymmetries should lead the small operators to charge higher termination rates as Figure 1 illustrates. However, market shares and concentration are usually highly correlated, which may reduce the explanatory power of the HHI in combination with market shares due to their collinearity.

While market size is not statistically significant, we find statistically significant effects for the regulatory framework. On the one hand, firm-specific regulation tends to lower the regulated firm’s termination rate, even though there is some ambiguity regarding the significance of RF. On the other hand, termination rates in regulated countries tend to be higher.
Table 2: Linear and log-linear models

<table>
<thead>
<tr>
<th>Variable</th>
<th>I levels</th>
<th>II levels</th>
<th>III logs</th>
<th>IV logs</th>
</tr>
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<td>24.3259</td>
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<tr>
<td></td>
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<td>[8.81]</td>
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<td>(0.04)</td>
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<td>[0.14]</td>
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<tr>
<td>adj. $R^2$</td>
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Heteroscedasticity robust t-statistics are given in parenthesis, heteroscedasticity robust t-statistics using Moulton correction in brackets.
Note, however, that since RF captures the effects on the regulated firms’ termination rates, RC only indicates the impact of regulation on the unregulated firm within a regulated market! This supports our hypothesis formulated in Remark 1, i.e. that downward regulation of competitors’ termination rates leads, ceteris paribus, to an increase in the unregulated firms’ termination rate, as termination rates are strategic substitutes if consumers are ignorant.

Overall, our empirical analysis tends to support the hypotheses derived from our theoretical model. Firstly, smaller mobile operators tend to have higher termination rates than their larger competitors. Secondly, downward regulation of the large operators’ rates tends to have a positive effect on the termination rates of unregulated operators. In our view these results may be helpful for regulatory authorities that analyze mobile termination rates and their regulation.

4 Conclusion

In this paper, a simple theoretical model has been developed to show (a) that a mobile network’s termination rate is the higher the smaller the network’s size (as measured through its subscriber base) and vice versa and (b) that asymmetric regulation of only the larger operators in a market will, ceteris paribus, induce the smaller operators to increase their termination rates. These results are due to consumers’ ignorance and the resulting pricing externalities. Empirical evidence from 48 European mobile operators supports these results. In all our regressions market share has a statistically significant and negative impact on firms’ termination rates, as predicted by the model. Furthermore, unregulated firms in regulated markets tend to have higher termination rates than firms in unregulated markets.

We believe that these findings may be helpful for regulatory authorities that analyze mobile termination rates and their regulation. One should keep in mind, however, that we have adopted a very simple model of termination rate setting. In particular, we have abstracted from the challenging issue of endogenous market shares and ignored the possibility of further entry into mobile telecommunications markets. Future research into these directions might prove to be instructive for theorists and practitioners alike.

References

Buehler, S. and J. Haucap (2003), Mobile number portability, Discussion Paper 0303, University of Zurich.


Appendix

Proof of the Proposition.

We have to compare $t_L$ and $t_S$ as given in (2). Obviously, $t_L < t_S$ iff $t_L - t_S < 0$, which can also be written as

$$\frac{1}{2} a \frac{8 x_S^8 + 30 x_S^7 x_L + 43 x_S^6 x_L^2 + 35 x_S^5 x_L^3 - 35 x_L^5 x_S^3 - 43 x_L^6 x_S^2 - 30 x_S x_L^7 - 8 x_L^8}{x_L b(16 x_S^6 + 105 x_L^4 x_S^2 + 48 x_S x_L + 105 x_S x_L^2 + 148 x_S^3 x_L + 16 x_S^6 + 48 x_L x_S)} < 0.$$ 

The sign of this expression is determined by the sign of

$$8(x_S^8 - x_L^8) + 30(x_S^7 x_L - x_S x_L^7) + 43(x_S^6 x_L^2 - x_S^6 x_L^2) + 35(x_L^5 x_S^3 - x_L^5 x_S^3).$$

Close inspection reveals that this term is always negative for $x_S < x_L$, so that $t_L < t_S$ always holds for $x_S < x_L$.

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<td>Range</td>
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<td>Max</td>
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<td>S.D.</td>
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