Price strategies and compatibility in digital networks

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Abstract

We analyze competition between two horizontally differentiated network providers, and whether they will implement personalization technologies that allow them to use price discrimination. Demand depends on the degree of compatibility between the two networks. The degree of one-way compatibility may be lower when at least one of the firms uses first-degree price discrimination compared with the case where both use linear pricing. Furthermore, if we allow the firms to choose their pricing strategy before they take compatibility decisions, we find that in equilibrium they choose price discrimination. The game resembles the Prisoner’s Dilemma game.

Keywords: Compatibility, Network Effects, Price Discrimination, Internet, Competition

JEL classification: L13; L96; L41
1 Introduction

Personalization technologies allow firms to use new types of price and product customization strategies. These technologies help the provider to collect consumer-specific information, and in principle these tools may be used to practice first-degree price discrimination (Vulkan, 2003, Dewan, Jing and Seidmann, 2003, and Daripa and Kapur, 2001). Within e-commerce personalization technologies are already widely used, and the potential for such technologies will probably be even higher in proprietary digital communication networks such as cellular networks.¹

We analyze the incentives to use personalization technologies to practice price discrimination in an oligopolistic market with network effects. The network effects imply that the customers’ utility is increasing in the number of other users and the degree of compatibility between networks. Network effects may obviously be present for communication services such as voice telephony, SMS, e-mail, videoconferences, and instant messaging services. Communication services will probably be the “killer-services” in new digital networks such as 3rd generation mobile systems too (Odlyzko, 2001). Moreover, file-sharing programs, such as Kazaa, may imply that network effects are present also for conventional content services (e.g. movies and music) distributed by digital networks.

We combine elements from the literature on spatial competition and the choice of compatibility (Farrell and Saloner, 1992) and the literature on spatial competition and price discrimination (Thisse and Vives, 1988).² Farrell and Saloner (1992) show that market shares matter when the degree of compatibility is imperfect. Consequently, firms compete more fiercely for market shares the less compatible the networks are. Thisse and Vives (1988) show that discriminatory pricing may sharpen competition. This is because price discrimination implies that firms compete for each individual customer, and not just set the same price for marginal and infra-marginal customers.

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² In the traditional Hotelling (1929) the customers pay the transportation cost plus a linear price (the mill price). The constraint of linear pricing was first removed by Hoover (1937), and he allowed firms to use discriminatory pricing.
We analyze the following three-stage sequential game between two competing providers: At stage 1 the providers choose pricing strategy. If they implement personalization technologies they have freedom to choose between linear pricing (LP) and first-degree price discrimination (PD). Otherwise, they commit to using LP. At stage 2 the providers simultaneously decide on the degree of one-way compatibility, and at stage 3 there is price competition à la Hotelling. In all cases we are looking for pure sub-game perfect Nash equilibria. In concluding remarks we justify the assumption that the choice of pricing strategy is made before the compatibility choice. Moreover, in the appendix we show that this assumption is not crucial for our main results.

Throughout we assume that the degree of horizontal differentiation is sufficiently high compared to the network effects such that we have market sharing also when the networks are incompatible. As Liebowitz and Margolis (2002) note we regularly observe market sharing even if there is low degree of compatibility between the competing networks. In order to remove non-strategic incentives to degrade compatibility we assume that it is costless to improve compatibility.

Our first main result is that the choice of one-way compatibility depends on the choice of pricing strategy (LP or PD). The firms choose complete one-way compatibility if both use LP. If just one of the firms uses PD, then the PD-firm sets a low degree of one-way compatibility while the LP-firm sets perfect one-way compatibility. If both use PD, we have multiple equilibria. Higher one-way compatibility increases the rival’s profit, but has no affect on its own profit. The literature on network effects and compatibility has mainly focused on compatibility decisions in a setting where a large firm faces competition from a small firm. The seminal paper is Katz and Shapiro (1985), who in a Cournot framework show that the smaller firm has higher incentives to become compatible than the larger rival. In the present situation, there are no a priori differences in the firms’ size.

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3 One example, in Japan we have three providers of mobile Internet services even if the services are incompatible and the largest one (DoCoMo) has a large advantage in size.
Our second result is that there exists a unique sub-game perfect Nash equilibrium where both firms choose PD, and that this is a Prisoner’s Dilemma situation where the firms would have been better off if they had both chosen LP. The outcome is robust to different assumptions with respect to the compatibility choice (one-way and two-way compatibility). This result resembles the outcome without network effects shown by Thisse and Vives (1988).

Our third result relates to welfare considerations. Since price competition is tougher with discriminatory pricing than linear pricing (mill pricing), Thisse and Vives (1988) show that customers are worse off if regulation prevents price discrimination. In our analysis the choice of pricing strategy affects total welfare due to two forces. First, total welfare is reduced when the degree of compatibility is reduced. Second, when the firms use asymmetric pricing strategies, the PD-firm has more than half of the market. Then, the average transportation costs are higher than when the firms share the market equally. However, even if the degree of one-way compatibility is low when both use PD, customers are still better off when both use PD than when both use LP.

The majority of the literature focuses on two-way compatibility⁴ - e.g. Katz and Shapiro (1985) and Farrell and Saloner (1992).⁵ In the basic model, however, we assume one-way compatibility. Thereafter, we compare the outcome in the basic model with the outcome with two-way compatibility. Throughout we assume proprietary interfaces such that if one firm has the incentives to block or reduce compatibility, it has the ability to do so. We may classify four types of compatibility: First, two-way compatibility with proprietary interfaces where the quality (the compatibility) is the same in both directions in the interface between the networks, and the degree of two-way compatibility is chosen by the firm that values compatibility the least. Second, two-way compatibility without proprietary interfaces

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⁴ Shy (2000) and Manenti and Somma (2002) are exceptions. In contrast to the present paper these analyses do not consider price discrimination and they do not consider spatial competition.

⁵ Katz and Shapiro (1985) assume two-way compatibility with proprietary interfaces. Farrell and Saloner (1992) assume that the customers may invest into a two-way converter that makes otherwise incompatible technologies partly compatible. That will be more in line with what we note two-way compatibility without proprietary interfaces.
where neither of the firms have a veto, and the degree of two-way compatibility is chosen by the firm that values compatibility the most. Third, *one-way compatibility with proprietary interfaces* (as in our basic model) where firms unilaterally decide to what extent the rival will have access to its customers/services. Put differently, the providers have the ability to establish a “walled garden” where the rival’s customers have no access. Fourth, *one-way compatibility without proprietary interfaces* where the firms have no ability to establish a “walled garden”.

It is easy to find examples of both one-way and two-way compatibility in the market for digital network services. The Internet and other digital networks are often described as layered networks, and in the bottom layer we have the physical infrastructure. Protocols for distribution of data connect the bottom layer with communication and content services at higher layers. Crémer, Rey and Tirole (2000) justify the use of two-way compatibility with the fact that “It takes two to tango”. That is probably true when the compatibility refers to physical interconnection between network providers in the bottom layer. However, even with respect to transmission capacity one of the providers may have the ability to practice one-sided degradation if it has the incentive to do so. In contrast, in the market for communication and content services, compatibility often refers to what extent a provider allows the rival’s customers to access premium network services. Then, one-way compatibility will be the rule rather than the exception, and incompatible proprietary systems may create a quality difference. One example is in the Japanese mobile market where each of the three providers is operating incompatible systems for premium services. The largest provider, DoCoMo, has success with its mobile Internet service I-Mode (25 million subscribers, August 2001), but the services and

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6 In the literature on vertical integration and non-price foreclosure it is conventionally assumed that the vertically integrated upstream firm has the ability to unilaterally degrade the quality of the input sold to the downstream rival. Even if this literature is motivated by telecommunications and other network industries, network effects are not formally incorporated into these models (see Economides, 1998, among others).
content available to I-Mode subscribers are not available to subscribers of the rival providers (see The Economist, October 13th, 2001).  

Our assumption on proprietary interfaces implies that firms have the ability to deny the rival access to technical specifications that are needed in order to establish a converter, or the firms can deny the rival access to some premium proprietary services. DoCoMo’s mobile Internet service I-mode is one example where a firm is able to deny rival’s customers access to high quality services. Nintendo is another example: Atari wanted to have one-way compatibility towards Nintendo by including an adapter into its own technology, but Atari lacked the intellectual property rights needed to include such an adapter (Shapiro and Varian, 1998: pp. 285). However, we see examples where firms try to break down the door to “walled gardens”. AOL’s Instant Messaging (IM) is a text-based service where the customers can “chat” in near real time. Each subscriber has a “buddy list” which displays what other “buddies” are online at a given time. In 1999 Yahoo! and Microsoft, among others, entered the market with competitive IM services, and the entrants established an adapter that made their customers compatible with AOL’s 30 millions subscribers. AOL tried to block this attempt, and during the summer 1999 several attempts to break down AOL’s “walled garden” were temporarily successful. However, in the end AOL managed to block the rival’s compatibility towards AOL’s IM (Faulhaber, 2002).

The article is organized as follows: In section 2 we present the basic model with one-way compatibility. In section 3 we introduce two extensions: First, we analyze the outcome with two-way compatibility. Second, we analyze an alternative for stage 3 competition where firms use asymmetric pricing strategies. When only one firm uses PD, Thisse and Vives (1988) show that there may not exist a pure strategy Nash equilibrium when firms simultaneously choose price schedules. In the basic model we follow Thisse and Vives (1988), and we assume that the LP-firm acts as a Stackelberg leader and sets its price before the PD-firm at stage 3. However, Ulph and Vulkan

\[7\] Rubinfeld and Singer (2001) analyze whether the merged AOL Time Warner has the incentive to use different types of “walled garden” strategies in the broadband access market. However, they do not formally model network effects and the choice of compatibility.

\[8\] In the specific case of two-way compatibility and linear pricing the present model is analogous to Foros and Hansen (2001).
(2000a) show that (without network effects) there may be a unique Nash equilibrium when they simultaneously choose price schedules at the price sub-game. In the extension we follow Ulph and Vulkan (2000a), but unfortunately the equilibrium is not robust to trembling. Finally, in section 4 we give some concluding remarks where we discuss some of our key assumptions.

2 The model

We start from a slightly modified version of Hotelling (1929) where firm $a$ and firm $b$ are located at the extremes of the unit line, such that $x_a = 0$ and $x_b = 1$. Each firm offers a single service, and we analyze the following three-stage game. At stage 1, firms choose pricing strategy (LP or PD). At stage 2, firms choose the degree of one-way compatibility. At stage 3, the firms choose price schedules.

The preferences of customers are assumed to be uniformly distributed on $[0,1]$. The location of preferences on the unit line indicates the most preferred network type for each customer. The reservation utility is assumed to be zero, and the net utility for a customer located at $x$ connected to supplier $a$ is (analogous for firm $b$):

$$U_a = v - tx + \beta(n_a + k_b n_b) - p_a(x)$$

The first two terms are similar to the conventional Hotelling model, the fixed advantage $v$ of being connected to the network, and the disutility from not consuming the most preferred network type given by the transportation cost $t$. The transportation costs are passed on to the customers. The third term, $\beta(n_a + k_b n_b)$, is a utility term that depends on the number of on-net and off-net customers. Firm $a$ serves the portion $n_a$ of the customers, while firm $b$ serves the portion $n_b$ of the customers. The parameter $\beta$ indicates how important the quality-adjusted network size is. The degree of one-way compatibility (the quality of communication with the rival’s customers) is given by $k_b \in [0,1]$, and $k_b$ is decided by firm $b$ at stage 2. The quality of communication with other customers in the same network is equal to one.

The fourth term is the price schedule. Under linear pricing (LP), each firm charges the same price $p_i$ ($i = a,b$) to all customers. Under price discrimination (PD), they
compete for each individual customer with the price schedule \( p_i(x) \). The marginal cost of producing one unit of the service is \( c \), and \( c \) is independent of the degree of compatibility. Throughout we make the following assumptions:

A1: \( n_a + n_b = 1 \)

A2: \( t - 2\beta \geq 0 \)

The assumptions A1 and A2 ensure market coverage and market sharing, respectively. Market coverage implies that \( \nu \) is sufficiently high such that each of the customers on the interval \([0,1]\) in equilibrium prefers to subscribe to a network. Market sharing implies that there exists one customer located at \( \tilde{x} \), where \( 0 < \tilde{x} < 1 \), who is indifferent to which of the two firms he receives the service from. A sufficient condition to ensure market sharing in all cases we analyze is that \( t - 2\beta \geq 0 \) (see appendix).

2.1 Stage 3

At stage 3 we have three possible combinations: (i) Both firms use LP, (ii) both firms use PD, and (iii) one firm uses LP and one firm uses PD.

Both use LP

From A1 and A2 it follows that there exists a customer \( \tilde{x} \in (0,1) \) who is indifferent between buying from firm \( a \) and firm \( b \). The location of the indifferent customer describes the demand. We can compute the demand by setting \( U_a = U_b \):

\[
n_a = \tilde{x} = \frac{t - \beta(1 - k_b) - (p_a - p_b)}{2t - \beta(2 - k_a - k_b)}
\]

Accordingly, the profit of firm \( a \) is \( \pi_a = (p_a - c)\tilde{x} \), and the first order conditions give the following stage 3 equilibrium price, quantity, and profit (analogous for firm b):

\[
p_{a,LP-LP} = c + t - \frac{\beta(3 - k_a - 2k_b)}{3}
\]
Both use PD

With linear pricing (LP) the firms compete only for the indifferent customer. In contrast, when firms use first-degree price discrimination (PD), firms compete for each customer. A customer located in $x$ buys from firm $a$ if and only if:

\[ p_a(x) \leq p_b(x) + r(1-2x) + \beta (n_a(1-k_a) - n_b(1-k_b)) \]

Firm $i$ will capture an extra revenue, $\beta (1-k_j)n_j$, from its $n_j$ customers when it captures one more customer. Both firms are willing to offer the marginal customer $\bar{x}$ the following price:

\[ p_j(x) = c - \beta (1-k_j)n_j. \]

Put differently, the term $c - \beta (1-k_j)n_j$ is firm $i$’s perceived marginal cost by serving the marginal customer. We find the market share equilibrium by inserting (6) into (5):

\[ n_a^{PD-PD} = n_b^{PD-PD} = \bar{x} = 0.5 \]

Hence, even if $k_a \neq k_b$, the firms share the market equally as long as the market sharing condition is fulfilled. The reason is that if e.g. $k_a < k_b$, firm $b$ will make up for the disadvantage by lowering its price schedule correspondingly. Using (7) we find that firm $a$ (analogous for firm $b$) offers the following price schedule to its customers:

\[ p_a^{PD-PD}(x) = c + r(1-2x) - \frac{\beta (1-k_b)}{2} \text{ where } x \in [0,0.5] \]
Firm $a$ then offers the customers served by firm $b$ the same price as the marginal customer served by firm $a$, i.e. $p^{PD-PD}_a(x) = c - \beta (1-k_b)/2$. Hoover (1937) argued that each firm charge the customer located at $x$ the delivery costs of its rival. However, in the present setting, firms also take into account advantages or disadvantages due to the network effects.

The stage 3 equilibrium profit is (analogous for firm $b$):

$$\pi^{PD-PD}_a = \frac{(p_a(0) - c)}{2} \hat{x}(c) - \frac{(c - p_a(\bar{x}))}{2} (\bar{x} - \hat{x}(c))$$

where $\bar{x} = 0.5$ and $\hat{x}(c)$ is the location of the customer served by firm $a$ at a price equal to marginal cost. Then, the profit can also be written as:

$$(8) \quad \pi^{PD-PD}_a = \frac{(2t - \beta (1-k_b))^2}{16t} - \frac{(\beta (1-k_b))^2}{16t}$$

The first term in (8) is the profit from the customers who pay a price above the marginal cost, and the second term in (8) is the loss from the customers who pay a price below the marginal cost. In figure 1 we illustrate the price schedules if firm $a$ sets $k_a < 1$ and firm $b$ sets $k_b = 1$. The solid line is the price schedule from firm $a$, while the dotted line is the price schedule from firm $b$. \(^{10}\)

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\(^9\) See Lederer and Hurter (1986) for a more recent analysis.

\(^{10}\) The handset subsidizing from mobile operators may be one example of a strategy where some customers are served with prices below the marginal costs.
Firm a uses PD and firm b uses LP

Firm a uses a per-customer price \( p_a(x) \) and firm b uses a linear price \( p_b \geq c \). Firm a still sets the price given by equation (6) to the marginal customer located at \( x_\sim \). It is reasonable to make the following assumption:

\[
(9) \quad p_a(x) \geq c - \beta (1-k_b)n_a^{PD-LP} \quad \text{when} \quad x \in (x_\sim, 1]
\]

The assumption in (9) simply implies that firm a offers a price schedule to the customers served by firm b, such that firm a has no loss if one of them chooses to buy from firm a. Without network effects, this assumption implies that firm a sets a price equal to or above the marginal cost to the customers served by the rival. Thisse and Vives (1988) show that there does not exist equilibrium in pure strategies if firms set their price schedules simultaneously at stage 3. Here we follow Thisse and Vives (1988), and we assume that firm b (which uses LP) acts as a Stackelberg leader at stage 3. A customer located at \( x \) buys from firm a if and only if:

\[
(10) \quad p_a(x) \leq p_b + \tau(1-2x) + \beta (n_a(1-k_a) - n_b(1-k_b))
\]
To find the location of the marginal customer $\tilde{x}$, we insert (9) into (10). Hence for a given $p_b$, firm $a$ serves all customers on $x \in [0, \tilde{x}]$ where:

$$\tilde{x}(p_b) = \frac{(p_b - c) + t - \beta(1 - k_b)}{2t - \beta(3 - k_a - 2k_b)}$$

The price schedule offered by firm $a$ for a given $p_b$:

$$p_a^{PD-LP}(x) = p_b + t(1 - 2x) + \beta(\tilde{x}(p_b)(1 - k_a) - (1 - \tilde{x}(p_b))(1 - k_b)) \text{ where } x \in [0, \tilde{x}(p_b)]$$

$$p_a^{PD-LP}(x) = c - \beta(1 - k_b)\tilde{x}(p_b) \text{ where } x = \tilde{x}(p_b)$$

$$p_a^{PD-LP}(x) \geq c - \beta(1 - k_b)\tilde{x}(p_b) \text{ where } (\tilde{x}(p_b), 1]$$

The profit to firm $a$ is:

$$\pi_a^{PD-LP} = \left(\frac{p_a(0) - c}{2}\right)\tilde{x}(c) - \frac{(c - p_a(\tilde{x}(p_b)))}{2}(\tilde{x}(p_b) - \tilde{x}(c))$$

where $\tilde{x}(c)$ is the location of the customer served by firm $a$ at a price equal to marginal cost. The profits to firm $b$:

$$\pi_b^{LP-PD} = (p_b - c)(1 - \tilde{x}(p_b))$$

The first order condition, $\partial \pi_b^{LP-PD} / \partial p_b = 0$, gives firm $b$’s stage 3 equilibrium price:

$$p_b^{LP-PD} = \frac{t + 2c - \beta(2k_a - k_b)}{2}$$

By inserting (14) into (11) we find the stage 3 equilibrium market shares:

$$n_a^{PD-LP} = \frac{3t - \beta(4 - k_a - 3k_b)}{2(2t - \beta(3k_a - 2k_b))}$$

$$n_b^{LP-PD} = 1 - \tilde{x}(p_b^{LP-PD})$$

Inserting for (15) into (12) and (13) yield the stage 3 equilibrium profits:
The first term in (16) is firm \( a \)'s profit from the customers charged a price above marginal costs, while the second term in (16) is the loss from the customers charged below the marginal cost.

### 2.2 Stage 2

**Both use LP**

On the one hand, we know from Farrell and Saloner (1992) that a higher degree of compatibility dampens price competition. If firm \( i \) reduces \( k_i \) below one, firm \( j \) reduces its price. On the other hand, firm \( i \) gains a competitive advantage from lowering \( k_i \) below one. Hence, there is a trade-off between the price effect and the market share effect. From equation (4) we find the first order condition with respect to \( k_a \):

\[
\frac{\partial \pi_a^{LP-PD}}{\partial k_a} = \frac{\partial p_a^{LP-PD}}{\partial k_a} \chi^{LP-PD} + (p_a^{LP-PD} - c) \frac{\partial x^{LP-PD}}{\partial k_a}
\]

The first term is the positive price effect, and the second term is the negative market share effect. As long as A1 and A2 are fulfilled, the positive price effect dominates the negative market share effect, and we have the following result:

**Proposition 1:** When both firms use LP, and market sharing conditions are fulfilled, firm \( i \) unilaterally chooses complete one-way compatibility, \( k_i = 1 \), for firm \( j \)'s customers.

**Proof.**

\[
\frac{\partial \pi_a^{LP-PD}}{\partial k_a} = \frac{\beta (3t - \beta (3 - k_a - 2k_b))(t - \beta (1 - k_a))}{9(2t - \beta (2 - k_a - k_b))} > 0 \quad \text{and} \quad \frac{\partial^2 \pi_a^{LP-PD}}{\partial k_a^2} > 0
\]
Stage 2 equilibrium profit is then similar to the conventional Hotelling model:

\[
\pi_a^{LP-LP} = \frac{t}{2}
\]

The result in Proposition 1 is different from Shy (2001: pp. 200) who analyzes banks’ choice of one-way compatibility in ATM networks. In contrast to us, he allows for vertical differentiation, but no horizontal differentiation. The customers’ utility is then increasing in the number of available ATMs, and Shy shows that the profit level of a bank declines when it makes its ATMs one-way compatible for the customers of a competing bank. Shy (2001) then explains the observation that banks often share their ATM networks by that the banking industry should be viewed as a cartel. In contrast, from Proposition 1 we have that the equilibrium outcome with horizontal differentiation is complete one-way compatibility.

Both use PD

Equation (8) may be rearranged

\[
\pi_a^{PD-PD} = \frac{t - \beta(1 - k_b)}{4}
\]

We can then see that \(\partial \pi_a^{PD-PD} / \partial k_a = 0\), such that we have multiple equilibria:

**Proposition 2:** When both firms use PD, and market sharing conditions are fulfilled, higher one-way compatibility from firm i increases the rival’s profit, \(\partial \pi_j^{PD-PD} / \partial k_i > 0\), but does not affect its own profit, \(\partial \pi_i^{PD-PD} / \partial k_i = 0\).

The equilibrium where both firm choose complete compatibility is the Pareto-dominating outcome, and in some circumstances the firms may have the ability to coordinate on high compatibility through some cooperative agreements. Below, we show that with two-way compatibility there is a unique equilibrium with perfect compatibility. Hence, one potential cooperative agreement is to commit to two-way compatibility if possible. The result in Proposition 2 rests on the assumption of no cost of compatibility. If the compatibility costs are small but positive, there will be an unique equilibrium where both choose to set low one-way compatibility.
*Firm a uses PD and firm b uses LP*

Let us start with the simplest question. How will firm b set $k_b$? From (14) we see that firm b’s price increases in $k_b$, i.e. $\frac{\partial p_{b}^{LP-PD}}{\partial k_b} = \frac{\beta}{2} > 0$. The intuition is that firm b by increasing $k_b$ makes firm a less aggressive in stage 3. Hence, firm b chooses to set $k_b = 1$. There is a trade-off for firm a with respect to $k_a$. First, from equation (14) we see that $\frac{\partial p_{b}}{\partial k_a} > 0$. An increase in $k_a$ makes firm b less aggressive at stage 3. Second, from (15) we have that $\frac{\partial x}{\partial k_a} < 0$. By increasing $k_a$ firm a increases the rival’s quality, and the market share of firm a then decreases for given prices. In contrast to the case where both firms use PD, firm a increases its market share by reducing $k_a$. The reason is that firm b will be less aggressive compared to the case where also firm b uses PD. When firm b uses LP, firm b does not set a price below the marginal cost. Hence, by reducing $k_a$, the price schedule offered by firm a shifts outwards as illustrated in figure 2.

![Figure 2: The dotted lines are the price schedules if $k_a=k_b=1$. The solid lines are the equilibrium price schedules where $k_a=0$ and $k_b=1$.](image)

We then have the following result:

**Proposition 3:** When firm a uses PD and firm b uses LP, and market sharing conditions are fulfilled, we show that: Firm a chooses low one-way compatibility,
\( k_a = 0 \), for firm b’s customers. Firm b chooses complete one-way compatibility, \( k_b = 1 \), for firm a’s customers.

**Proof.** From equation (16) and equation (17) we find that:

\[
\frac{\partial \pi_a^{PD-LP}}{\partial k_a} = -\frac{\beta (3t - \beta(4 - k_a - 3k_b))(2t - \beta(1 - k_b))^2}{2(2t - \beta (3 - k_a - 2k_b))^3} < 0
\]

\[
\frac{\partial \pi_b^{LP-PD}}{\partial k_b} = \frac{\beta (t - \beta(2 - k_a - k_b))(t - \beta(1 - k_b))}{2(2t - \beta (3 - k_a - 2k_b))^2} > 0
\]

Stage 2 equilibrium profits are now:

\[(20) \quad \pi_a^{PD-LP} = \frac{t}{4} \left( \frac{3t - \beta}{2t - \beta} \right)^2
\]

\[(21) \quad \pi_b^{LP-PD} = \frac{(t - \beta)^2}{4(2t - \beta)}
\]

### 2.3 Stage 1

At stage 1 the firms choose between LP and PD, and the stage 2 equilibrium profits are given by (18), (19), (20) and (21). Since we have multiple equilibria when both use PD, we do not specify the level of compatibility in that case (see equation (19)).

In Table 1 we give the normal-form representation of the game. Since \( \pi_i^{LP-LP} < \pi_i^{PD-LP} \) and \( \pi_i^{LP-PD} < \pi_i^{PD-PD} \) we have a unique sub-game perfect Nash-equilibrium in pure strategies where both firms choose PD.

<table>
<thead>
<tr>
<th>Firm A</th>
<th>LP</th>
<th>PD</th>
</tr>
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<tbody>
<tr>
<td>LP</td>
<td>( \pi_a^{LP-LP}, \pi_b^{LP-LP} )</td>
<td>( \pi_a^{LP-PD}, \pi_b^{PD-LP} )</td>
</tr>
<tr>
<td>PD</td>
<td>( \pi_a^{PD-LP}, \pi_b^{LP-PD} )</td>
<td>( \pi_a^{PD-PD}, \pi_b^{PD-PD} )</td>
</tr>
</tbody>
</table>
Table 1: Normal-form game when firm \(a\) and firm \(b\) choose between LP and PD at stage 1.

**Proposition 4:** We have a unique sub-game perfect Nash-equilibrium in pure strategies where both firms choose PD. Both firms would have been better off if they used LP and the game resembles the Prisoner’s Dilemma game.

**Proof.** From equation (18), (19), (20), and (21) we find that:

\[
\pi_i^{LP-LP} - \pi_i^{PD-LP} = \frac{t}{2} - \frac{t}{4} \left( \frac{3t - \beta}{2t - \beta} \right)^2 = -\frac{t(2t\beta + (t - \beta)(t + \beta))}{4(2t - \beta)^2} < 0
\]

\[
\pi_i^{LP-PD} - \pi_i^{PD-PD} = \frac{(t - \beta)^2}{4(2t - \beta)} - \frac{t - \beta(1-k_j)}{4} = \frac{(t - \beta)^2 - (t - \beta(1-k_j))(2t - \beta)}{4(2t - \beta)} < 0
\]

\[
\pi_i^{LP-LP} - \pi_i^{PD-PD} = \frac{t - \beta(1-k_j)}{4} = \frac{t + \beta(1-k_j)}{4} > 0
\]

Hence, (PD, PD) is a dominant strategy equilibrium, and the game has the structure of the Prisoner’s Dilemma. These results are analogous to the outcome without network effects analyzed by Thisse and Vives (1988). In the appendix we show that (PD,PD) is the outcome also when the choice of pricing strategy and the choice of compatibility are made simultaneously.

### 2.4 Welfare analysis

Since customers have inelastic demand, any variations in prices, for a given level of compatibility and a given distribution of market shares, just lead to pure transfers between firms and customers. In Thisse and Vives (1988) customers are worse off if a regulation that prevents price discrimination is enforced since the average price is lower with PD than when firms use LP. In the present analysis total welfare is affected by the choice pricing strategy due to two forces. First, the average transportation costs are \(\bar{t} \bar{x}/2\) and \(t(1-\bar{x})/2\) for customers buying from firm \(a\) and
firm \( b \), respectively. The average transportation costs are therefore minimized when \( \bar{x} = 0.5 \). When firms use symmetric pricing strategies they share the market equally, while with asymmetric pricing strategies, the PD firm has a market share advantage. Second, since the degree of one-way compatibility may be lower when both use PD than when both use LP, total welfare may be higher if the firms cannot use PD.

To show this, we use that, as long as consumer surplus and profits are weighted equally, total welfare is given by the gross consumer surplus:\(^{12}\)

\[
W = \bar{x} \left[ v - \bar{x} \left( k_a - c \right) \right] + (1 - \bar{x}) \left[ v - t(1 - \bar{x}) / 2 + \beta ((1 - \bar{x}) + \bar{x} k_a) - c \right]
\]

When both use PD or both use LP, we have from above that \( \bar{x} = 0.5 \). The average distance from the most preferred service is 0.25, and since \( k_a = k_b = 1 \) when both use LP we have:

\[
W_{LP-LP}^{LP-LP} = v - 0.25 t + \beta - c
\]

When both firms use PD we have multiple equilibria, and the welfare depends on the specific outcome:

\[
W_{k_a,k_b=[0,1]}^{PD-PD} = v - 0.25 t + 0.5 \beta (0.5 + 0.25 (k_a + k_b)) - c
\]

Therefore, total welfare is higher when price discrimination technologies are not available to the providers as long as at least one of the firms chooses to set the level of one-way compatibility below one.

If we consider consumer surplus, the average price offered to the customers is lower when both use PD than when both use LP; i.e. \( \bar{P}_{LP-LP}^{LP-LP} = \bar{c} + t \) and \( \bar{P}_{k_a,k_b=[0,1]}^{PD-PD} = \bar{c} + 0.5 t - \beta (0.5 - 0.25 (k_a + k_b)) \). As long as the market shares are not affected by the choice of pricing strategies, this is sufficient to ensure that the

\(^{11}\) Thise and Vives (1988) show that (PD, PD) will be the outcome also when pricing strategy and price schedules are chosen simultaneously.

\(^{12}\) Anderson, de Palma and Thise (1989) show that total welfare depends on pricing strategies given less restricted assumptions with respect to the firms location.
consumer surplus is higher with or without price discrimination for a given level of compatibility. The level of one-way compatibility may, however, be lower when price discrimination is an option. However, when both firms use PD, the consumer surplus is not affected by the firms’ choice of compatibility, since the reduction in price from firm \(i\), when the rival reduces the degree of one-way compatibility, is equivalent to the consumers reduction in utility:

\[
CS_{k_a=k_b=1}^{LP-LP} = v - 0.25t + \beta - \bar{p}_{k_a=k_b=1} = v - 1.25t + \beta - c
\]

\[
CS_{k_a,k_b \in [0,1]}^{PD-PD} = v - 0.25t + \beta (0.5 + 0.25(k_a + k_b)) - \bar{p}_{k_a=k_b=0}^{PD-PD} = v - 0.75t + \beta - c
\]

We summarize the welfare considerations in the following Proposition:

**Proposition 5:** As long as market sharing and market coverage assumptions are fulfilled, total welfare is higher if the firms cannot use PD, i.e. \(W_{k_a=k_b=1}^{LP-LP} > W_{k_a,k_b \in [0,1]}^{PD-PD}\). Consumer surplus is lower if the firms cannot use PD, i.e. \(CS_{k_a=k_b=1}^{LP-LP} < CS_{k_a,k_b \in [0,1]}^{PD-PD}\).

A consequence of Proposition 5 is that technological improvements that only give the providers the opportunity to use personalization technologies may imply a reduction in total welfare.

### 3 Extensions

#### 3.1 Two-way compatibility

Now we consider the case where the firms need to reciprocally agree on two-way compatibility, such that \(k_a = k_b = k\). The degree of two-way compatibility is chosen by the firm that values compatibility the least. When both firms use LP, stage 2 and 3 are analyzed by Foros and Hansen (2001), and the outcome is analogous to the basic model above, i.e. the firms want complete compatibility (\(k=1\)). In contrast, when both firms use PD, the stage 3 equilibrium profit to firm \(a\) when both use PD is given by \(\pi_a^{PD-PD} = (t - \beta (1 - k)) / 4\). Hence, we have complete two-way compatibility at stage 2 also when both use PD. In the basic model we had multiple equilibria when both firms used PD.
The outcome with respect to the two-way compatibility choice is different from the basic model when firms use different pricing strategies. Recall that in the basic model firm \( a \), which uses PD, sets \( k_a = 0 \), while firm \( b \), which uses LP, chooses \( k_b = 1 \). In contrast, when the firms need to agree upon a two-way compatibility parameter \( k \), we now end up with complete two-way compatibility \( k = 1 \). To see this, we now have the following stage 3 equilibrium profit to firm \( a \) and firm \( b \), respectively (analogous to (16) and (17) in the basic model):

\[
\pi_a^{PD-LP} = \frac{1}{16t}\left(\frac{(3t-4\beta(1-k))(2t-\beta(1-k))}{2t-3\beta(1-k)}\right)^2 - \left(\frac{\beta(1-k)(3t-4\beta(1-k))}{2t-3\beta(1-k)}\right)^2
\]

\[
\pi_b^{LP-PD} = \frac{(t-2\beta(1-k))^2}{4(2t-3\beta(1-k))}
\]

It is straightforward to show that \( \partial\pi_a^{PD-LP}/\partial k > 0 \) and \( \partial\pi_b^{LP-PD}/\partial k > 0 \), such that \( k = 1 \). Hence, we see that the outcome with two-way compatibility is independent of whether the LP-firm or the PD-firm has a veto with respect to the degree of compatibility. At stage 1, when the firms choose between LP and PD, we have the same outcome as in the basic model. Both firms choose PD, and this is a Prisoner’s Dilemma since both firms would have been better off if both had chosen LP.

3.2 Sequential (Stackelberg) or simultaneous moves at stage 3?

When firms use asymmetric pricing strategies, and price schedules are set simultaneously, pure-strategy equilibrium does not exist. We have followed Thisse and Vives (1988), and assumed that the LP-firm is a Stackelberg leader at stage 3. From Table 1 we can see that there exists a unique sub-game perfect equilibrium where both use PD. The reason is that it is profitable to deviate from LP since \( \pi_i^{LP-LP} < \pi_i^{PD-LP} \). There will, however, be a question whether the deviation incentives are exaggerated. The firm that uses PD has a “double” advantage compared to the LP-firm. First, it is obviously an advantage to price discriminate. Second, the PD-firm is a Stackelberg follower. Let us now analyze to what extent the deviation incentives are driven from the second force (the follower advantage).
Without network effects Ulph and Vulkan (2001a) show that there exists an equilibrium also when the PD firm and the LP firm set their price schedules simultaneously. The only candidate for Nash equilibrium in pure strategies is when firm \( a \) offers the price schedule illustrated in Figure 3a. Firm \( b \), which uses LP, is then forced to set its linear price equal to the marginal cost. The solid lines indicate the price schedule to the customers served by firm \( a \), while the dotted lines indicate the price schedule firm \( a \) offers to customers served by firm \( b \). When this price schedule (the dotted line) is offered by firm \( a \) to the customer served by firm \( b \), the assumption in (9) above will not be fulfilled. Thus, let us now assume that firm \( a \) may offer a price schedule where \( p_a(x) < c - \beta(1-k_a)n_{PD-LP}^{PD-LP} \) to the customers served by firm \( b \). Then, analogous to the analysis without network effects by Ulph and Vulkan (2000a) we have the following unique Nash equilibrium where the firms set their price schedules simultaneously at stage 3:

\[
\begin{align*}
\text{Stage 3:} & \\
\text{PD-LP:} & \\
p_a^{PD-LP}(x) &= c + t(1 - 2x) + \beta(\tilde{x}(c)(1-k_a) - (1-\tilde{x}(c))(1-k_b)) \\
\text{where } x &\in [0, \tilde{x}(c)] \\
p_a^{PD-LP}(x) &= c + t(1 - 2x) + \beta(\tilde{x}(c)(1-k_a) - (1-\tilde{x}(c))(1-k_b)) + \epsilon \\
\text{where } x &\in (\tilde{x}(c), 1] \\
p_b^{LP-PD} &= c \text{ where } x \in [0, 1]
\end{align*}
\]

The parameter \( \epsilon \) is positive (but arbitrary small), and \( \epsilon \) ensures that firm \( b \) serves the customers \( x \in (\tilde{x}(c), 1] \). The price schedules are then as in Figure 3b where the solid line is the price schedule to the customers served by firm \( a \), and the dotted line is the price schedule offered by firm \( a \) to the customers served by firm \( b \) (the proof is analogous to Ulph and Vulkan , 2000a).

\[\text{Ulph and Vulkan (2000a) show that whether it is profitable to use first-degree price discrimination or not is depending of the distribution of customers’ loyalty (the transportation costs). First-degree price discrimination becomes more profitable relative to linear pricing when the transportation costs become more convex.}\]
When \( p_b^{LP-PD} = c \) for \( x \in [0,1] \), firm \( b \) may choose \( k_b = 0 \) at stage 2. Furthermore, since firm \( b \) sets \( p_b = c \) regardless of the degree of compatibility chosen by firm \( a \), firm \( a \) obviously chooses \( k_a = 0 \) at stage 2. In the basic model, firm \( a \) needs to take into account that if \( k_a \) is reduced, then firm \( b \) lowers its price. Now firm \( b \) always chooses \( p_b = c \). A qualitative difference from our basic model is that \( \pi_a^{PD-LP} \) is lower in the present case. Given \( p_b = c \), \( k_b = 0 \) and \( k_a = 0 \) the profit to firm \( a \) at stage 1 is now given by:

\[
\pi_a^{PD-LP} = \frac{(t - \beta)^3}{(2t - 3\beta)^2}
\]

It is now straightforward to show that \( \pi_a^{PD-LP} < \pi_a^{LP-LP} \) (see appendix). Hence, in contrast to the basic model, we now have two Nash-equilibria at stage 1. In addition to the equilibrium in the basic model where both use PD, the outcome where both use LP and set complete compatibility is an equilibrium. The asymmetric equilibrium with simultaneous moves considered here is not very appealing for several reasons. In particular the equilibrium in (22) is not trembling hand perfect.

4 Concluding remarks

We have developed a simple model to analyze competition between horizontally differentiated network providers that choose pricing strategy (price discrimination or
not) and compatibility endogenously. Most of the existing literature assumes two-way compatibility and linear pricing, and analyze whether larger firms have incentives to reduce compatibility with products offered by smaller rivals. In contrast, in the basic model we assume one-way compatibility where each firm can unilaterally establish a “walled garden” to deny the rival’s customers access to its own customers. Then discriminatory pricing is a unique sub-game perfect Nash equilibrium of the three-stage game in which firms first choose pricing strategy, second the degree of one-way compatibility, and finally, the price schedules. There are multiple equilibria with respect to the degree of one-way compatibility. With two-way compatibility perfect compatibility is a unique equilibrium. Analogous to the analysis without network effects (Thisse and Vives, 1988), the outcome is a Prisoner’s Dilemma both with one-way and two-way compatibility. The focus is interesting and important since there are many network markets where technological development has allowed firms to use personalization technologies and to price discriminate between different types of customers.

Since the model is stylized, let us make some comments on some other key assumptions. First, we assume that the choice of pricing strategy is made before the choice of compatibility. The motivation behind this assumption is that firms can have the option to using price discrimination if personalization technologies are implemented. If not, they commit to use linear pricing. The compatibility parameter, in particular one-sided compatibility such as access to premium proprietary content, will probably be much easier to change. Put differently, the compatible choice we focus on is much more a choice of a commercial agreement within existing technology. Moreover, it will be difficult to write complete contracts that cover all dimensions of compatibility. In consequence, even if the opportunity to be compatible is included into the technology standard, this opportunity does not prevent \textit{ex-post} degradation of compatibility. The choice of pricing strategy is more directly related to implementation of a new technology. One example is the mobile providers’ choice of investing into 3\textsuperscript{rd} generation mobile systems (3G). Compared to the current 2G

\textsuperscript{14} The choice of pricing strategy is a choice of implementing personalization technologies or not, and this choice has similarities with respect to timing with the choice of flexible manufacturing techniques analyzed e.g. by Norman and Thissé (1999).
systems (GSM), 3G gives the providers more accurate customer specific information (e.g. with respect to customers’ location at any time). Therefore, the investment in 3G may also be seen as an opportunity to improve the ability to use price discrimination.

Second, we use several conventional assumptions in the network literature. For instance, we assume that the customers are identical in their valuation of the network size, that the value to the customers when one more joins a network is independent of who the new customer is, and that the customers’ utility of the network component is linear in the number of compatible users. As argued by Liebowitz and Margolis (2002) these assumptions lean in direction of tipping to a single-network equilibrium. However, this is not a severe problem in the present paper since we only consider market sharing equilibria. Hence, we have limited the value of the network component.

Third, horizontal differentiation may be given several interpretations. If we consider broadband Internet connectivity different access technologies may give rise to horizontal differentiation. The customers may choose between cable-tv access, fixed-line copper access (through DSL) from the telecommunication incumbent, and, in the future, mobile access through 3G. The cable-tv provider may probably offer higher incoming access than the alternatives, and the cable-tv provider has more experience with tv-centric services. Hence, customers interested in new tv-centric entertainment services may prefer the service offered by the cable-tv-provider. The telecommunication provider has more experience with switching technologies, and may be the preferred provider for customers interested in two-way communications. Customers with preferences for mobility may choose 3G even if the capacity is lower than the fixed-line alternatives. A second interpretation is that customers are heterogeneous with respect to what extent they use price-comparison services. Digital networks offer price-comparison services to customers, but not all customers use

---

15 This assumption is made in all the papers on compatibility mentioned above. In contrast, Julien (2001) analyze the case where the customers’ preferences are heterogeneous with respect to network size and who the compatible users are. He analyzes the interaction between an incumbent with an advantage in reputation compared to an entrant. The entrant may overcome its disadvantage by offering lower prices to targeted groups of customers. In contrast to the present paper, Julien (2001) does not analyze endogenously set pricing strategies and the possibility that only one of the firm uses discriminatory pricing.
them. Moreover, the providers have the opportunity to identify the customers that use price-comparison services and separate them from the customers that access them directly (see Daripa and Kapur, 2001). The former type of customers are typically more price-sensitive, and the firms may then charge a lower price to customers that use price-comparison technologies compared to the price that is charged to the customers that access a given provider directly.

Fourth, like Thisse and Vives (1988) and Ulph and Vulkan (2000a) we only study the choice between linear pricing and first-degree price discrimination. A potential extension will be to analyze whether the ability to offer personalized services to each customer (mass-customization) will change the compatibility choice. In Ulph and Vulkan (2000b) firms have the ability to locate in an interval of the Hotelling line, and Chen and Iyer (2002) analyze price discrimination and addressability.\textsuperscript{16} Dewan et al. (2003) analyze product customization. In contrast to the present paper, these papers do not formally analyze network effects and the choice of compatibility.

**Appendix**

**A2: Market sharing:** Assume that almost all customers along the line buy from \( b \), such that \( n_a = 0 \) and \( n_b = 1 \).

**Both firm use LP:** The customer with the longest traveling distance to \( b \) is located in \( x=0 \). He buys from \( a \) iff \( v - p_a \geq v - t + \beta - p_b \). For any given \( p_b \geq c \), firm \( a \) will pick up the customer located in \( x = 0 \) by a price \( p_a \geq c \) as long as this condition holds. Hence a sufficient condition to ensure market sharing when both use LP is \( t - \beta > 0 \).

**Both use PD:** Firm \( b \) is willing to set the price \( p_b(0) = c - \beta(1 - k_a) \) in order to capture the customer located in \( 0 \), while firm \( a \) is willing to set \( p_a(0) = c \). The customer located at \( x = 0 \) buys from \( a \) iff \( v - c \geq v - t + 2\beta - c \). Hence, when both use PD a sufficient condition that ensures market sharing is \( t - 2\beta > 0 \). It is straightforward to

\textsuperscript{16}Addressability implies that a firm may undertake an investment that gives the firm the ability to use e.g. price discrimination towards groups of customers otherwise served with linear pricing.
show that the same condition ensures market sharing when they use different pricing strategies.

**Simultaneous choice of pricing strategy and one-way compatibility**

In contrast to the basic model we now assume that pricing strategy and one-way compatibility are chosen simultaneously. The providers will set either $k_i = 1$ and $k_i = 0$ ($i = a,b$), and, hence, we have four possible strategies for each firm, and from the payoff matrix we find four Nash equilibria: $(PD(0,0), PD(0,0)); (PD(1,1), PD(1,1)); (PD(1,0), PD(0,1)); (PD(0,1), PD(1,0))$ where $(PD(k_a,k_b), PD(k_b,k_a))$.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>$LP, k_a = 1$</th>
<th>$LP, k_a = 0$</th>
<th>$PD, k_a = 1$</th>
<th>$PD, k_a = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LP, k_a = 1$</td>
<td>$\frac{t}{2} \cdot \frac{t}{2}$</td>
<td>$\frac{(3t-2)\beta^2}{9(2t-\beta)} \cdot \frac{(3t-2)\beta^3}{9(2t-\beta)}$</td>
<td>$\frac{t}{4} \cdot \frac{t}{4}$</td>
<td>$\frac{(t-\beta)^5}{4(2t-\beta)} \cdot \frac{(t-\beta)^5}{4(2t-\beta)}$</td>
<td></td>
</tr>
<tr>
<td>$LP, k_a = 0$</td>
<td>$\frac{(3t-2)\beta^2}{9(2t-\beta)} \cdot \frac{(3t-2)\beta^3}{9(2t-\beta)}$</td>
<td>$\frac{(t-\beta)^5}{8} \cdot \frac{(t-\beta)^5}{16}$</td>
<td>$\frac{t}{4} \cdot \frac{t}{4}$</td>
<td>$\frac{(t-3)\beta^5}{4(2t-3\beta)} \cdot \frac{(t-3)\beta^5}{4(2t-3\beta)}$</td>
<td></td>
</tr>
<tr>
<td>$PD, k_a = 1$</td>
<td>$\frac{9t-\beta^3}{16}$</td>
<td>$\frac{9t-\beta^3}{16}$</td>
<td>$\frac{t}{4} \cdot \frac{t}{4}$</td>
<td>$\frac{t-\beta}{4} \cdot \frac{t-\beta}{4}$</td>
<td></td>
</tr>
<tr>
<td>$PD, k_a = 0$</td>
<td>$\frac{(3t-2)\beta^2}{4(2t-\beta)} \cdot \frac{(3t-2)\beta^3}{4(2t-\beta)}$</td>
<td>$\frac{(t-2)\beta^5}{4(2t-3\beta)} \cdot \frac{(t-2)\beta^5}{4(2t-3\beta)}$</td>
<td>$\frac{t}{4} \cdot \frac{t}{4}$</td>
<td>$\frac{t-\beta}{4} \cdot \frac{t-\beta}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

We have assumed no cost of compatibility. If we have a positive but arbitrary small cost of compatibility, $\varepsilon$, we have that $(PD(0,0), PD(0,0))$ is a unique sub-game perfect Nash equilibrium as in the basic model.

**Asymmetric equilibrium at stage 3:** $\pi_a^{LP-IP} - \pi_a^{PD-IP} \geq 0$ with simultaneous moves

It is straightforward to show that $\frac{\partial \pi_a^{PD-IP}}{\partial \beta} > 0$. Hence to show that $\pi_a^{LP-IP} - \pi_a^{PD-IP} \geq 0$ we insert the highest possible value for $\beta$ which is $t=2\beta$. Then we have $\pi_a^{LP-IP} - \pi_a^{PD-IP} = 0$. 


References


Ulph, D. and N. Vulkan. 2000a. Electronic Commerce and Competitive First-Degree Price Discrimination, Mimeo