Empirical TIPs

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Second Draft
January 30, 2003

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Acknowledgement

The author is indebted to Barclays Capital for making the TIPs data available and to Kevin Lee for outstanding research assistance.

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Abstract

U.S. Treasury Inflation Protection Securities (TIPs) were first issued in January 1997. Through the end of 2002, eleven TIPs have been issued with maturities ranging from a few years through thirty years. One TIP bond has already matured. Returns on TIPs have been positively correlated with returns on nominal bonds and negatively correlated with equity returns over the past five years. TIPs real durations are longer than nominal bonds because their real yields are low. However, their effective nominal durations are much shorter because they are not as sensitive to changes in expected inflation. TIPs volatility has displayed marked variation over time. It was relatively low during 1999-2000 and considerably higher during 2001-2002. This suggests that real interest rate volatility has increased recently. TIPs can be used to estimate the real yield curve. The real and nominal yield curves can then be combined to estimate the term structure of anticipated inflation. Because of their taxation, TIPs yields may not be entirely independent of inflation. Given plausible assumptions about future expected returns, an investment portfolio diversified across equities and nominal bonds would be improved by the addition of TIPs.

I. TIPs, a new asset class.

In January 1997, the U.S. Treasury issued a unique new security; a bond with ten years to maturity and with payments linked to the Consumer Price Index[1] (CPI). This bond is officially termed a "Marketable Treasury Inflation-Indexed Security" but it is commonly called a TIP, for Treasury Inflation Protection security. Through August of 2002, nine different TIPs had been issued with maturity dates ranging from July 2002 through April 2029. See Table 1.

The inflation linkage of U.S. TIPs adheres to the model for indexed bonds issued previously by the government of Canada. Under this structure, the nominal principal is accreted daily based on an extrapolation of inflation during the most recent reporting period for the price index. The bond's coupons are a fixed percentage of the accreted principal, so they too are effectively linked to the CPI. For American TIPs, the Treasury has established a floor in the event of deflation (which no longer seems such an unlikely event.) At maturity, TIPs will be redeemed at the greater of their inflation-adjusted principal or their par amount at original issue. During the lifetime of a TIP bond, its accreted principal could decline with deflation below the original par, so the coupons could decline below their originally stated dollar amounts, but the principal will eventually be redeemed at no less than the original face amount.

In the secondary market, TIPs prices are quoted as percentages of par. Consequently, the settlement amount includes the trade price multiplied by an inflation accrual factor established officially by the Treasury for every day of the month and all outstanding TIPs. Accrued interest, which depends on the same accrual factor, is added in to obtain the total settlement payment. More details are given in the Appendix, "Calculation of Nominal Holding-Period Returns on TIPs."

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[1] The index employed for TIPs is, more specifically, the U.S. city average all items consumer price index for all urban consumers (CPI-U), published monthly by the Bureau of Labor Statistics.

More than five years of daily TIPs trading experience have accumulated, so sample sizes are more than adequate to establish some empirical facts about TIPs behavior. This paper's modest goal is to report those facts. Among other matters, we investigate how TIPs are related to nominal bonds and equities. We measure the effective empirical durations of TIPs relative to nominal bond yields[2] and to changes in the shape of the nominal term structure of interest rates. TIPs volatilities are reported and shown to be remarkably time varying over the available history. TIPs and nominal bonds are combined to derive empirical estimates of the inflation term structure. A theoretical conjecture involving taxes and some supporting evidence are offered to explain the dramatic recent decline in real yields. Finally, TIPs are assessed as a component of a balanced and diversified investment portfolio.

II. Data.

Table 1 lists the nine TIPs, five nominal bonds, and three equity indexes used in this study. The sample period begins with the first available TIPs return, on July 22, 1997, and it ends on August 16, 2002, when I stopped collecting data and began writing. All outstanding TIPs are included here but only the January 2007 (maturity) issue has data covering the entire sample period. [3]

To develop a sample of nominal bond returns, constant maturity yields were downloaded from the U.S. Treasury web site for five different maturities. On August 16 of this year, the latest available yield was for July 26, so this is the terminal date of the nominal bond data. Approximate holding period returns were computed from yield changes by the following formula: $R_t = \frac{Y_{t-j}}{365} + \frac{N(Y_{t-j} - Y_t)}{100}$, where Y_t is the continuously-compounded nominal yield on date t (in percent per annum) and N is the number of years to maturity; most often j=1 so that the yield change is from day t-1 to day t, but j>1 when the yield change is across a holiday or weekend.

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[2] The real yields and real durations of TIPs can be calculated analytically in the usual fashion.

[3] Two additional TIPs bonds were issued in 2002, but there were no data available for them at the time this paper was written.

Equity returns were measured by three broad indexes, the value-weighted and equal-weighted CRSP[4] indexes for all major American exchanges and the S&P500 index. In each case, dividends were reinvested to obtain total index returns. Table 1 provides lists the data availability period for each of the seventeen assets.

III. Empirical Characteristics of TIPs, Nominal Bonds, and Equities.

III.A. Daily Nominal Holding-Period Returns.

Using all available observations, Table 2 reports descriptive statistics for daily nominal returns. Means and standard deviations are given in percent per day. Note that the number of observations used in computing each pair-wise correlation differs across asset pairs because of the availability of joint observations.

During the sample period, TIPs experienced exceptional average returns. The January 2007 issue, which has been around the longest, had a mean daily return of 0.0265%; this translates into 6.71%/annum based on 5.563 calendar years and 1409 trading days from January 22, 1997 through August 16, 2002. The latter part of the sample period was particularly favorable for TIPs. For example, the January 2012 TIP bond, issued in January 2002, had an annualized mean return of more than 17% during its first seven months of existence. Real yields on TIPs have declined during the sample period from above 3.5% to the neighborhood of 1.5% to 2.5% depending on maturity.

Nominal bonds have also done well. The thirty-year bond had an annualized average sample return of about 14.6% during the five years prior to its retirement in February 2002. On average, equities fared rather poorly by comparison. The S&P500 experienced an annualized average return of only 5.24% over the entire sample period and, of course, its return was strongly negative in 2001-02. The equal-weighted return appears to be

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[4] Center for Research in Securities Prices of the University of Chicago.

quite high on average, but this sample mean is severely biased upward by the bid/ask bounce and daily rebalancing (to equal weights.)[5]

In comparing sample volatilities, TIPs returns have lower volatilities than nominal bonds with similar maturities, as one would expect. For example, the January 2010 bond, whose maturity ranged from ten years to somewhat over eight years during the sample, had a daily standard deviation of 0.193%. This is only one-third of the volatility of the ten-year nominal bond and only about two-thirds the volatility of the five-year nominal bond. The largest volatility of any asset in Table 2 was for the thirty-year nominal bond, 1.47% per day or about 22.7% annualized (assuming intertemporal independence.) By contrast, the two TIPs with original maturities of thirty years, the April 2028 and April 2029 issues, had annualized return standard deviations of 5.14% and 5.35%, respectively. Changes in nominal yields are much more volatile than changes in real yields, for the obvious reason that the former include shocks in expected inflation.

TIPs returns are strongly correlated with each other, particularly for adjacent maturities. The April 2028 and April 2029 TIPs are almost perfectly correlated; a coefficient of 0.998. The six TIPs that had ten years to maturity at issuance, those with maturities from January 2007 through January 2012, all have correlations in the upper 90% range. Their correlations are, however, somewhat lower with the longest term TIPs (the two original thirty-year issues), but are still in the upper 70% area. Only the single short-term TIP bond, which matured in July 2002, displayed lower correlation with other TIPs and even this was the case only when it was close to maturity.[6]

Long-term nominal bonds are positively correlated with long-term TIPs. Being mainly in the range of 0.5 to 0.7 these correlations are not as large as those between adjacent TIPs. Shorter-term nominal bonds such as the three-month and one-year are more weakly correlated with both TIPs and with longer nominal bonds. Interestingly, the shortest

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[5] See Canina, Michaely, Thaler and Womack [1998].

[6] This can be deduced from the fact that the correlation between the July 2002 TIP and the January 2007 TIP was 0.757, sample size 1262, while the July 2002 and January 2012 correlation was only 0.183, sample size 124. The latter correlation was computed from observations close to the short TIP's maturity date.

nominal bond and the shortest TIPs are not all that correlated, a coefficient of only 0.229. This seems to suggest that shocks in expected inflation represent the predominant source of nominal interest rate volatility during the overall sample period.

One of the most striking patterns in Table 2 involves the negative correlations between equities and bonds, both TIPs and nominal. There is evidence that these correlations have become increasingly negative lately since more recently issued TIPs display larger (in absolute value) correlations with equities. Correlations between nominal bonds and equities are similar to correlations between equities and TIPs that were outstanding during most of the sample.

Over longer historical periods, equities have been positively correlated with nominal bonds,[7] so the sample period of this paper is somewhat unusual.

Daily TIPs returns have significant first-order serial correlation, Table 3, but virtually no autocorrelation at longer lags. The same is true for nominal bonds with one-, five-, and ten-year maturities. Thirty-year nominal bonds have none. The three-month nominal bond return has quite a bit of serial dependence even for lags as long as five days. For TIPs and for longer nominal bonds, the serial dependence may be statistically significant but is probably not economically relevant. Notice that the total explanatory power, (adjusted R-square) of the autocorrelation function is 1% to 3% in most cases. For the three-month nominal bond return, however, the explanatory power is almost 10%. Value-weighted equities and the S&P500 are not very autocorrelated but the equal-weighted return is, perhaps spuriously and attributable to daily rebalancing.

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[7] Using monthly returns, the S&P500 had correlations of 0.180, 0.136, and 0.0847 with, respectively, Long-Term, Intermediate-Term, and One-Year Treasury bonds from January 1926 through December 1996; (based on data from Ibbotson Associates.)

III.B. Real Yields, Real Durations, Effective Nominal Durations and Sensitivities to Nominal Term Structure Shape.

Like a nominal Treasury bond, a TIPs yields and durations can be computed directly from prices and promised cash payments. To calculate a real yield, inflation can be ignored since the transaction price and coupons are stated as percentages of the accrued face amount. Thus, the real yield is simply the internal rate of return equating the current price+(accrued real interest) to the discounted future real payments. Similarly, a real duration can be calculated in the usual way, ignoring inflation.

Figure 1 presents TIPs real yields over the available sample.[8] It is rather difficult to distinguish yields for the individual issues but they are almost always ranked higher for successively longer maturities. The April 2028 and April 2029 yields plot virtually on top of one another.

As the figure makes clear, TIPs yields of all maturities declined dramatically from the beginning of 2000 through mid-2002. From well over four percent per annum, yields fell to less than two percent for the shorter issues and less than three percent for the longest.[9] Yields for maturities out to 2012 were below 2.5% at the end of the sample.

Figure 2 shows real durations. The individual cases are easily discerned here because, understandably, they rank monotonically from longest to shortest maturity. They are unremarkable except for the slight jump on each ex-coupon date. Otherwise, they are very smoothly declining; only the two longest TIPs exhibit perceptible reactions to yield changes. The real durations of these two issues have not declined at all over the past several years (because yields have fallen enough to more than offset aging.)

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[8] These are bond-equivalent yields ("B.E."); i.e., semi-annually compounded.

[9] The shortest TIP bond exhibited rather bizarre price behavior as it neared maturity in July 2002. For quite a few trading days, its real yield was even negative. This might have been due to measurement error in the recorded trade price; when a bond approaches maturity, even slight errors are magnified as the internal rate of return is annualized. Consequently, this particular bond's yields are not plotted in Figure 1 during its final year of existence.

The smoothly declining durations for shorter issues is explained by the relatively low coupons of TIPs. Low coupons imply that the final principal repayment represents the preponderance of present value and thus Macauley's duration is relatively insensitive to yield changes. (Recall that a zero coupon bond, whose entire value depends on the single final payment, always has Macauley duration equal to maturity, entirely independent of yield.)

Real durations measure TIPs' return response to infinitesimal changes in the real yield. Because real yields and coupons on TIPs are generally lower than nominal yields and coupons on otherwise similar nominal Treasury bonds, TIPs real durations are longer than nominal bond durations. But such comparisons mix apples and oranges since real and nominal yields do not have the same volatility. If nominal yields are essentially real yields plus expected inflation, nominal yields must have larger volatilities unless real yields and inflation are quite negatively correlated.[10]

Consequently, it is of considerable practical interest to estimate TIPs durations with respect to changes in nominal yields, thereby placing TIPs and nominal bonds on a comparable footing. This cannot be done analytically because there is no obvious connection between nominal yields and TIPs prices. It can, however, be accomplished empirically by regressing TIPs returns on concurrent changes in nominal yields. The negative of the slope coefficient in such a regression is an estimate of modified duration.[11]

Table 4 reports such empirical nominal durations for TIPs where the regressor is a simple average of the constant maturity five- and ten-year nominal yield change. Of course, any nominal maturity could have been employed and the resulting duration estimates would have differed. Those reported in Table 4 should be taken as indications of effective TIPs durations compared to the average duration of five- and ten-year nominal Treasuries. At

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[10] Specifically, unless $\rho<-\frac{1}{2}\sigma_I/\sigma_r$ where σ_I (σ_r) is the standard deviation of anticipated inflation (real yields) and ρ is their correlation.

[11] Modified duration is Macauley's duration divided by (1+Y/k) where Y is the yield and k is the compounding frequency. For discrete compounding, modified duration's units are not exactly years, but they're close.

yields of 4%, 5%, and 6%, the nominal five/ten-year average modified duration for bonds selling at par would be, respectively, 6.40, 6.16, and 5.94 (years.) As shown in Table 4, TIPs durations are considerably shorter, even when their maturities are much longer. For example, the April 2029 issue, whose maturity was at least 26 years throughout the sample, had an estimated duration of only 2.9 years. Except for the sample aberration represented by the relatively short period of availability for the January 2012 TIP bond, their estimated durations increase monotonically with maturity.

The second set of regressions in Table 4 reports how TIPs respond to more general changes in the shape of the nominal yield curve. Following Litterman and Scheinkman [1991], the nominal yield curve shape on each date was characterized by its level, slope, and curvature estimated by a simple model, using approximate durations for the constant maturity Treasury yields.

$$Y_{j,t} = \text{Level}_t + \text{Slope}_t X_{L,j} + \text{Curvature}_t X_{Q,j}, \qquad j=1,..,4$$

where $Y_{j,t}$ is the nominal yield for the j^{th} maturity on date t, $X_{L,j,t}=a_t+b_tD_{j,t}$ and $X_{Q,j,t} = -(3X_{L,j,t}^2 - 1)/2$ are, respectively, linear and quadratic Legendre transformations of $D_{j,t}$, the estimated duration on day t of nominal Treasury bond j.[12] The Legendre transformations were employed because they are approximately orthogonal over the range –1 to +1.[13] The transformation coefficients a_t and b_t are computed as $b_t=2/[\max(D_t)-\min(D_t)]$ and $a_t=1-b_t\max(D_t)$, which assures that the transformed durations span the required range. The estimated regression coefficients provide measures of the Level, Slope, and Curvature of the nominal term structure on each date t.

The next step takes first differences of the above term structure shape estimators, thereby generating a time series of term structure shape changes, or factors. The change in Level is the "Shift" in the term structure; the change in Slope is the "Tilt"; and the change in Curvature is the "Flex." Then, for each TIPs, the following regression is estimated using OLS:

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[12] The duration was estimated by assuming that the yield pertained to a par bond.

[13] They are exactly orthogonal if continuous from –1 to 1. Curvature is positive if the term structure is concave downward.

$$R_{i,t} = \beta_0+\beta_{Shift}(Level_t-Level_{t-1})+\beta_{Tilt}(Slope_t-Slope_{t-1})+\beta_{Flex}(Curvature_t-Curvature_{t-1}),$$

where $R_{i,t}$ is the daily return on the i^{th} TIP bond. The three slope coefficients and their corresponding t-statistics are given in the right-most three column of Table 4.

First notice that the $-\beta_{Shift}$ is similar in most cases to the estimated duration (reported in column 2.) This suggests that the five/ten-year nominal bond average is a reasonable indicator of term structure level. Second, long TIPs are significantly and negatively related to the Tilt factor while shorter TIPs are positively related, though only the shortest is significantly positive. This means that increases in the slope of the nominal term structure, tilting around an intermediate maturity, drive down (up) long (short) TIPs prices just as they do nominal bond prices. Finally, increases in curvature, or "Flex," affect positively all but the shortest bond, (whose coefficient is insignificant), and the significance increases with maturity. Greater curvature of the underlying real term structure, i.e., more concavity from below, coincides with lower long-term real yields, *ceteris paribus*.

The explanatory power of the regressions in Table 4 is lower than would generally be found for nominal U.S. Treasury bonds. Typically, a three-factor model explains well over 90 percent of the returns on such bonds. But for the TIP bond with the longest sample, (the January 2007 issue), the explanatory power is slightly over 27%. Notice, however, that the explanatory power seems to be quite a bit higher recently. For the January 2012 bond, whose sample includes only the most recent 134 trading days, the adjusted R-square is 51%.

Although the results in Table 4 are interesting and provide some insights into the behavior of TIPs, we know for sure that the regression is mis-specified. The most obvious reason is that each bond has aged during the sample period. Neither its actual duration nor its responses to the term structure shape factors are constants. Of course, the materiality of this maturation is likely to be minor for the very long TIPs of April 2028 and 2029 because they have not aged much as a fraction of their original maturities. At the other extreme, however, the shortest issue has already matured, so its duration must have fallen to zero during the sample.

A second problem is possible autocorrelation in the residuals. We have already seen in Table 3 that TIPs returns are significantly autocorrelated at the first daily lag. The Durbin-Watson statistics in the penultimate column of Table 4 similarly reveal a mild case of autocorrelation in the regression residuals. Finally, there is quite a bit of excess kurtosis in the residuals.[14] Excess kurtosis implies either thick tails or non-stationary probability distributions, which would, in turn, imply the econometric problem of heteroskedasticity.

There is an available econometric correction for autocorrelation and heteroskedasticity in regression residuals. Newey and West [1987] derive an asymptotically consistent covariance matrix for the estimated coefficients, which produces corrected standard errors and hence t-statistics. As a robustness check, the Newey/West estimator was calculated for the second regression of Table 4 involving the term structure shape factors and the results are reported in Table 4-bis. As shown there, most t-statistics are somewhat smaller using Newey/West rather than OLS. However, only a few are rendered insignificant after the Newey/West correction. Consequently, the mild degree of autocorrelation and the presence of heteroskedasticity have not produced seriously misleading OLS results.

To tackle the problem of bond maturation, additional time dependent regressors were added to the models of Table 4 in an effort to capture declining TIPs sensitivity coefficients with decreasing bond maturity. The basic idea is that sensitivities should approximately satisfy a decreasing linear function of time. For example, duration at time t would be $D_t=D_0(1-t/M)$, where M is the original maturity of the bond, t is an index of maturation that varies from t=0 at issuance to t=M at maturity, and D_0 is the bond's duration at issuance. The same function can be used for the shift, tilt and flex factors. Using the duration regression as an example, the model becomes

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[14] Excess kurtosis is the italicized number in the last column.

$$R_{i,t} = \alpha_i + \beta_{i,1}\Delta Y_t + \beta_{i,2}[(t/M)\Delta Y_t]$$

where ΔY_t is the nominal yield change (on the average of the five- and ten-year constant maturity nominal Treasury). The first slope coefficient should be duration at issuance and the second coefficient should be its negative. The results are provided in Table 5.

The first TIP bond, maturity July 2002, looks fairly sensible. Notice that the duration coefficient estimate is 0.816, which is larger than the duration from Table 4, 0.585. This should be the case since the duration coefficient in Table 5 pertains to the origination date while the duration estimate in Table 4 is essentially an average over the entire lifetime of the bond. The second coefficient (in the Table 5 column labeled "Duration*t") is negative -.409 and significant as it should be, though at it seems too small.[15] In the second Table 5 regression for this bond, we see similar plausible results. The significant origination date coefficients (shift and flex) have time dependent coefficients with opposite signs, though only flex*t is significant.

However, all the longer TIPs in Table 5 have completely unexpected and startling patterns of coefficients. Not one of the duration*t coefficients is negative and most are actually significantly positive. Since maturities of these bonds became shorter by as much as five years during the sample, this can only mean that real interest rate volatility increased dramatically in the latter part of the sample. Additional supporting evidence for an increase in real rate volatility is provided by the time varying coefficient Shift*t. They would all be positive if volatility were constant. Instead, most are negative and many are significant. Notice that the most recently issued TIP bond, maturity January 2012, for which all sample observations are in the latest year, has extremely large time dependent coefficients. For this bond, the adjusted R-square is 51.4%, but many coefficients are insignificant. Consequently, one more item, multicollinearity, should probably be added to the list of econometric difficulties.

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[15] Its absolute value should be approximately the same as the coefficient labeled "duration."

III.C. Time-Varying Volatilities of TIPs Returns.

A formal model of time varying volatility for TIPs and other asset returns, the pleasant GARCH(1,1), is reported in Table 6. Everything is significant for TIPs and, indeed, for all assets. There is strong evidence of persistence since the lagged conditional variance has a coefficient in excess of 0.8 in most cases. This persistence seems to have been a bit higher in the latter part of the sample, (as revealed by slightly larger coefficients for TIPs that are more recently issued.)

Figure 3 provides pictorial evidence by plotting 21-day (approximately one month = 21 trading days) volatilities of TIPs returns. As the figure makes obvious, volatility grows with TIPs maturity. The two longest TIPs, April 2028 and 2029, have almost identical volatilities. The originally ten-year TIPs, January 2007 through 2012, are generally ordered by maturity. The single short bond of July 2002, is the only issue whose volatility appears to be declining from the beginning of 2001 through its maturity. All the others have considerably more volatility in 2001-2002 than in the two preceding calendar years, 1999-2000. Volatility was also relatively high around the Russian debt crisis in the latter part of 1998.

Confirmatory visual evidence of non-constant volatility is presented in Figure 4 for the July 2007 issue, the bond outstanding for the longest time. Both a rolling 21-day volatility and the conditional volatility from GARCH(1,1) are plotted. The rolling volatility is positioned on the termination date of each 21-day period, which explains why it appears to lag slightly behind the GARCH conditional volatility. Three features stand out: First, there was a substantial decline in volatility during the first 20 months of this bond's existence. Since the July 2007 issue was the very first, one might reasonably surmise that some learning about TIPs trading characteristics was taking place in those early days. TIPs were unfamiliar instruments in the American fixed-income market at the time, so differences of opinion could have brought about volatility that slowly dissipated as traders learned more about them. Second, the late 1998 crisis dramatically increased volatility for roughly five months, September 1998 through January 1999. Then for two years, volatility was low, hovering around 0.10 percent per day. Finally, around the end of 2000, volatility jumped dramatically and has fluctuated around 0.15 to 0.20 percent per day ever since.

What is the source of the substantially higher volatility over the past two calendar years as compared to the previous two years? To deduce something about this, Figure 5 plots the log ratio of volatilities for the two periods, not only for TIPs but for the other assets as well. Notice that all but the very shortest TIPs have at least fifty percent higher volatility in 2001-2002 than in 1999-2000. This is matched by neither nominal bonds nor by equities. There is some increase in both latter asset classes, but it reaches forty percent only for the one-year nominal Treasury. Equities show at most a ten percent increase; even less for the value-weighted CRSP index. Curiously, though the volatility increase seems to grow with maturity for TIPs, it declines with maturities beyond one year for nominal bonds.

A story consistent with the above pattern involves the relative volatilities of real interest rates and expected inflation. If real interest volatility increases while inflation volatility declines or doesn't increase very much, one would expect TIPs volatilities to increase (and possibly increase more for longer maturities), while nominal bond return volatilities could increase less (and possibly be negatively associated with maturity.) Real rate volatility affects equities too, but since they are much more volatile in general and subject to other sources of risk, the relative impact of real interest volatility should be smaller for equities than for TIPs.

IV. The Term Structures of Expected Inflation and Real Yields

IV.A. The Historical Patterns.

Although only nine TIPs have thus far been issued by the U.S. Treasury, when any four of them are outstanding, it is possible to estimate the general shape of the term structure of real yields by exploiting the three factor model used in section III.B above; viz., by fitting the level, slope, and curvature of the real yield curve on each trading date. The difference between the nominal and real yield curves is the yield curve of anticipated inflation (plus the inflation risk premium, if any.) Hence, it too can be estimated for each date.

Following the development in section III.B, for each date t in the sample when at least four TIPs were outstanding[16], a cross sectional regression was fit in the following form

$$y_{j,t} = \text{Level}_t + \text{Slope}_t x_{L,j} + \text{Curvature}_t x_{Q,j}, \qquad j=1,..,N$$

where N is the number of TIPs available, $y_{j,t}$ is the (real) yield for the jth maturity on date t, $x_{L,j,t}=a_t+b_t d_{j,t}$ and $x_{Q,j,t} = -(3x_{L,j,t}^2 - 1)/2$ are, respectively, linear and quadratic Legendre transformations of $d_{j,t}$, the estimated real duration on day t of TIP bond j.[17] The estimated coefficients provide measures of the Level, Slope, and Curvature of the real term structure on each date t.

Figures 6-8 depict the results by plotting, respectively, the Level, Slope, and Curvature estimates for both the nominal and real term structures. The vertical difference between the nominal and real term structures provides the corresponding term structure of anticipated inflation plus any inflation risk premium. For clarity, this is also depicted in Figure 9.

The term structures in Figure 6 basically track nominal and real interest rate movements over time. There has indeed been a dramatic decrease in both nominal and real interest rates since the beginning of calendar year 2000. The decline in nominal rates has been larger because anticipated inflation also has declined. The inflation level is now very close to zero. This is shown by the relatively small and apparently decreasing gap between the nominal and real yield curve levels toward the right (latest) end of Figure 6.

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[16] I did not use the July 2002 bond in these calculations because its annualized yield became very unstable as it approached maturity.

[17] The Legendre polynomials are orthogonal over the range –1 to 1. The transformation coefficients a_t and b_t are computed as $b_t=2/[\max(d_t)-\min(d_t)]$ and $a_t=1-b_t\max(d_t)$, which assures that the transformed durations span the required range.

As a general rule, the nominal yield curve is more steeply sloped than the real yield curve (Figure 7.) This is, of course, consistent with an inflation risk premium that increases with maturity. Notice in Figure 7 or Figure 9 that a sharply positive slope is presently the dominant feature of the anticipated inflation term structure. This is consistent with a market consensus belief that inflation will not remain for long at its current low level.

There was a brief period during the latter half of 2000 when the nominal yield curve was less steeply sloped than the real yield curve. At the time, the market consensus must have been forecasting a decline in anticipated inflation, enough of a decline to outweigh any term dependent risk premium. After the fact, that forecast was proven accurate; the estimated expected inflation level did decline.

As for term structure curvature, Figure 8 shows it is usually less in absolute value for the real yield curve than for the nominal yield curve. The real curvature is mostly positive (concave downward) throughout the sample and has been increasing lately. In term structure theory, curvature is often associated with volatility, so the recent increase in TIPs return volatility is broadly consistent with the observed curvature trend. The nominal curvature has also been positive during most of the sample except for the last quarter of 2000 when inflation was expected to decrease.

An obvious caveat applies to these results. Only sparse points along the real yield curve are available, so estimation error is a potentially significant problem. Caution is urged particularly with respect to the term structure of anticipated inflation.

IV.B. A Possible Explanation for the Recent Decline in Real Yields.

Real interest rates have fluctuated more in the past two years than in the two years earlier. At the same time, inflation seems to have declined in both level and in volatility. TIPs returns have been extraordinarily high during this latter period and yields have declined precipitously. Evidently, TIPs returns can be high when inflation is low even though real rates have become more volatile. Is there a linkage between decreasing anticipated inflation and lower TIPs yields?

Since TIPs are very well linked to official inflation, one might think there should be no connection between changes in their yields and changes in expected inflation. In an earlier paper, (Roll [1996]), however, I argued that inflation and TIPs yields might be coupled because of their tax treatment, which essentially stipulates full taxation of the real yield <u>and</u> the inflation accrual. For a taxable investor, the anticipated after-tax real yield on TIPs is

$$\rho = r(1-\tau) - \tau I^e/(1+I^e),$$

where τ is the effective tax rate, r is the pre-tax real yield on TIPs, and I^e is the rate of anticipated inflation.

When anticipated inflation changes, the pre-tax yield on TIPs must respond in order to maintain the same level of taxable demand. In other words, to maintain a constant after-tax real yield, we must have

$$\partial\rho = 0 \Rightarrow \partial r/\partial I^e = \tau/[(1-\tau)(1+I^e)^2].$$

For example, if the effective tax rate is 30% and the expected inflation is three percent per annum, $\partial r/\partial I^e \approx .396$; i.e., a decrease in anticipated inflation of one percent would induce a reduction in the pre-tax yield on TIPs of nearly 40 basis points.

The results presented above agree with prevailing market opinion that expected inflation has declined during the past two years. There is plenty of tertiary evidence; e.g., the Treasury auctioned three-month (nominal) bills on December 23, 2002, at an average yield of less than 1.2% per annum. Even if the real interest rate were only 0.5% per annum, this latest bill yield implies an anticipated inflation over the next quarter considerably below one percent (per annum.)

Falling inflation could conceivably explain why TIPs real yields have fallen (and why holding period returns have been so high.) Moreover, the magnitudes seem to be in the right ballpark; inflation has fallen from perhaps 3.0% during 1999-2000 to less than one percent at the end of 2002. At a marginal tax rate of 30%, there should have been a concurrent reduction in TIPs real yields of roughly 80 to 100 basis points, and, indeed, this has been close to the observed decline.

There is, however, a hole in this interpretation of recent TIPs history. It presumes that the marginal TIPs investor actually pays taxes. If TIPs are held mainly by tax-exempts such as pension funds and 401k plans, the explanation is less compelling. At the moment, however, there seem to be few alternative explanations for the dramatic recent holding period returns of TIPs.

A tax-induced relation between TIPs real yields and anticipated inflation implies more than just a long-term trend. Even daily real yield fluctuations should be affected by concurrent changes in the market's consensus belief about future inflation. We have already constructed empirical estimates of the daily term structures of expected inflation reported in the previous section. These can be related to TIPs' real yields.

In a first attempt at uncovering a possible tax effect, Table 7 reports changes in the real yields of TIPs regressed on concurrent changes in the level of expected inflation (the latter being the estimated "level" of the inflation yield curve from the previous section.) The regression equation is

$$\Delta y_{j,t} = \alpha + \beta \Delta I_t^e + \varepsilon_{j,t}$$

where $\Delta y_{j,t}$ is the real yield change for TIP bond j from day t-1 to t and ΔI_t^e is the contemporaneous change in the estimated level of anticipated inflation. The columns labeled "One Factor" in Table 7 report the results for each of the nine TIP bonds. In every case, there is a strong positive relation between changes in estimated anticipated inflation and changes in real yields. Notice that this is all the more impressive in that changes in real yields are themselves negatively related to estimated changes in anticipated inflation because the latter equals the difference between the fitted levels of the nominal and real yield curves. If measurement error had been material, one might have found a spurious negative relation between real yield changes and anticipated inflation changes; but, as it turns out, there is no evidence of this at all.

The slope coefficients in the second column of Table 7 are estimates of $\tau/[(1-\tau)(1+I^e)^2]$ where τ is the effective tax rate and I^e is true expected inflation. These coefficients are all positive (and significant) ranging from about .1 to slightly over .2. Assuming an expected inflation of one percent,[18] the implied marginal tax rates range from about 19% for the shortest TIP bond to about 9% for the longest. This seem sensible in that tax exempt investors such as pension funds have long horizons and are likely to be attracted to the longer maturities.

The second regression in Table 7 reveals that a simple bivariate comparison of real yields and inflation is too simplistic. Obviously, tax-paying investors are concerned not only about current inflation, but, perhaps to a greater extent, about future inflation and implied future after-tax yields. Except for the shortest TIP bond, the slope and curvature of the inflation yield curve are probably more pertinent for after-tax yields. This seems to be particularly the case for the inflation term structure's slope. As the "three factor" regressions of Table 7 reveal, the estimated inflation yield curve slope has a positive and highly significant association with real yields for all TIPs bonds beyond the shortest maturity. Evidently, increases in anticipated future inflation (as measured by the inflation term structure's slope), induce immediate and large increases in pre-tax real yields on TIPs. Inflation term structure curvature also has an impact on current real yields, perhaps because it is associated with inflation volatility.

What explanations other than taxes could induce the very strong empirical relations between real yields and anticipated inflation documented in Table 7? One possibility might involve the joint response of real interest rates and inflation to business cycles. If, for example, inflation and real interest rates increase (decrease) during an expansion (recession), their mutual correlation would not necessarily imply causation. Future research may settle this important issue. At the present, we must remember simply that there is definitely a positive and strong relation between real yields and anticipated inflation, whatever the cause. TIP bond real yields are not independent of inflation.

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[18] This calculation is not sensitive to the inflation assumption.

V. TIPs in Investment Portfolios

Upon the appearance of any new asset, investors are anxious to know how it fits into well-diversified portfolios. In this section, I attempt a preliminary foray into this complicated terrain using just the three broad classes of assets already discussed in the paper, TIPs, nominal Treasury bonds, and equities. Of course, any real-world investment portfolio would not restrict itself to such broad asset classes, yet we still might gain some insights about how TIPs fit in to an overall investment strategy.

An optimized portfolio in the mean/variance sense depends on three inputs: expected returns, return volatilities, and correlations. We have already seen candidates for some of these ingredients, the sample values given in Table 2. However, a glance at this table shows that we should not just blindly accept such historical estimates in a forward-looking portfolio allocation problem; (this is always true of historical estimates.)

For one thing, historical mean returns, particularly ones computed over rather short sample periods such as five years, are completely unreliable as estimates of future <u>expected</u> returns. Indeed, they are almost nonsensical. Equities, for example, have had mean returns lower than most TIPs over the past five years.[19] Probably few non-behaviorists expect this to continue.

Sample estimation errors are not confined to historical means. Correlations are a critical input for portfolio optimization. The correlation matrix reported in Table 2 is not even positive definite, so it cannot be used directly because matrix inversion is required.[20] More practically, many of the TIPs are so highly correlated that it makes little sense to include them all; e.g., the .998 correlation between the April 2028 and 2029 TIPs suggests that either, but not both, could be included.

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[19] The S&P500 Index return averaged only .208 percent per day during the sample period. All TIPs did better, even the shortest term. Remember that the equal-weighted CRSP index, which appears to do quite well in Table 2, has a mean return that is strongly upward biased.

[20] A correlation matrix computed from simultaneous observations with no missing values will generally be positive definite and thus invertible unless there are more assets than sample periods or there is perfect linear dependence among some groups of variables. In the case of Table 2, however, there were large differences in the number of available observations across assets.

As a consequence, I decided rather arbitrarily to restrict this exploration to two individual TIPs that have long samples, the January 2007 and the April 2028 issues; two constant maturity nominal bonds, the one-year and the ten-year; and one equity index, the CRSP value-weighted.

Finally, we have already seen that volatilities of all asset classes have been fluctuating materially over the five+ years of TIPs history. So it would not be wise to use all available sample observations in computing return standard deviations and correlations. Since portfolio allocation is forward looking, one should employ recent estimates of inputs rather than long-term historical values. Thus, I decided to compute the covariance matrix (i.e., standard deviations and correlations) from observations in calendar years 2001 and 2002 only, from the beginning of January 2001 through the last day of the available sample for nominal bonds, July 26, 2002.

For the expected returns on nominal bonds, I simply used their current yields to maturity on the day of this calculation, December 13, 2002. For TIPs, I used their real yields on the same day plus an assumed rate of expected inflation (which will be varied in some of the results to be reported shortly.) For equities, I assumed an annual premium of four percent over the one-year nominal bond yield. This value could and should be debated. It is a bit high in my opinion, but this enhances the desirability of equities relative to TIPs and is, therefore, conservative from the perspective of advocating TIPs for a diversified portfolio.

Assuming an expected inflation of 0.4%/annum, the resulting efficient frontier is plotted in Figure 10. The ten-year nominal bond and the CRSP value-weighted equity index lie to the right off the chart because their volatilities are so high. Nonetheless, they are held in positive amounts in some of the portfolios along the efficient frontier. For example, the portfolio indicated by an arrow has an expected return equal to the yield on a five-year nominal bond, which on December 13, 2002 was very close to 3%/annum. This portfolio is fairly evenly distributed among the five assets, though it has slightly more than 20% in each of the two TIPs and slightly less than 20% in equities. Its composition is plotted as the second pillar from the left in Figure 11.

Figure 11 shows how the inflation assumption affects optimal portfolio composition. The impact is dramatic. Varying assumed inflation by only ten basis points significantly changes the relative allocation to TIPs and nominal bonds. If you think inflation will be higher than 0.4% next year, you would want to emphasize TIPs. Indeed, if your inflation expectation were 0.9% (still less than one percent per annum), you would want to put nearly ninety percent of your assets in TIPs! Conversely, if you think inflation will be very low, say 0.3% or less, nominal bonds represent your cup of tea.

It is not hard to understand these results – current nominal bond yields already embed expected inflation while TIPs yields do not. So, by adding inflation to TIPs real yields to obtain their nominal expected returns, the greater the inflation, the more favorable TIPs appear.

Several caveats are appropriate. First, no account was taken of the possible tax/inflation interaction with respect to TIPs real returns. Above, we argued that TIPs real returns could conceivably decline if inflation increases because of their tax treatment. Second, there was no impact of inflation on the equity premium; perhaps it should decrease with inflation, or perhaps increase. Who knows? Finally, and most important, the seeming knife-edged response of portfolio composition is partly attributable to the small number of assets. If we had used several hundred assets rather than only five, the acute sensitivity would have been attenuated to some extent. There would have been, nonetheless, a large change in the total allocation to inflation protected securities relative to nominal bonds with a change in anticipated inflation.

The bottom line is: TIPs probably belong in many well-diversified portfolios, even when anticipated inflation is rather low, such as 0.4%. TIPs are not strongly correlated with other asset classes and they have very low volatilities, much lower than nominal bonds with similar maturity. They seem to represent a new diversification opportunity to enhance returns and reduce risk.

VI. Conclusions

U.S. Treasury inflation protection securities (TIPs) have existed for about six years. At the point of this writing, nine different bonds have been issued and one has already matured. The paper presents an empirical survey of their trading characteristics.

TIPs principal is indexed daily to an extrapolated value of the most recent consumer price index; see the Appendix for details. Their coupons and accrued interest are calculated based on the accreted principal, so they are very well protected against inflation (at least the inflation measured by the CPI.) A daily real yield to maturity and a real duration can be computed in the usual way from the transaction price, real accrued interest, and future real payments, always ignoring inflation.

Daily nominal holding period returns can be calculated from the change in the trading prices properly adjusted for the inflation accrual and the accrued interest. Such nominal returns can then be compared to other nominal returns such as those from ordinary Treasury bonds or equities.

Evidence presented here indicates the following about TIPs daily returns over the period of their existence, from January 1997 until recently: First, TIPs nominal return volatility is less than the volatility of ordinary Treasuries of similar maturity. This is to be expected since TIPs are subject mainly to real interest rate variability while nominal bonds are influenced by both real interest and by anticipated inflation. As an example, TIPs with an original thirty-year maturity had a daily return standard deviation of around 0.33 percent during the sample while the thirty-year constant maturity Treasury bond had about 1.47 percent. At a ten-year maturity, the TIPs/Nominal Bond comparison was 0.160/0.575%.

TIPs are highly correlated with each other, particularly for adjacent maturities. The daily return correlation between the issues of April 2028 (maturity) and April 2029 was an extraordinary 0.998. Correlations with nominal bond are strictly positive, though lower in magnitude. For example, the ten-year TIPs/nominal correlation is about 0.5 and the thirty-year almost 0.6. Short-term (three-month) nominal bond returns have correlations with TIPs of all maturities less than 0.3 and many less than 0.2. The shortest TIPs/3-month correlation is 0.229.

During the period of TIPs existence, equities were negatively correlated with TIPs and with nominal bonds. However, the correlations were small in absolute magnitude, being lower than -.3 only for very recently issued TIPs (and hence based on observations from the latter part of the sample, mainly during calendar year 2002.)

Empirical nominal durations for TIPs were obtained by comparing their daily returns to changes in mid-term nominal yields. The results give average durations ranging from about .5 years for the shortest TIPs, (whose average maturity was around 2.5 years), to almost three years for the longest TIPs, (whose maturities are 27-28 years on average.) Hence, nominal effective durations are much lower for TIPs than for nominal bonds. TIPs also respond to changes in the shape of the nominal term structure. Their response to parallel shifts is congruent with their effective empirical durations. Longer (shorter) TIPs, they also do poorly (well) when the term structure tilts upward. Increasing nominal term structure curvature differentially impacts long and short TIPs (as with nominal bonds.)

There is strong evidence of time varying volatility in TIPs returns. After their original appearance, volatility declined materially until the financial crisis of late 1998, when it jumped dramatically. Then, volatility declined again and was quite low during the calendar years 1999-2000. During 2001-2002, however, it was almost double its level in the previous two years. Interestingly, the recent increase in volatility was accompanied by large declines in the real yields on TIPs. Their nominal holding period returns have consequently been very high (and higher than returns on both equities and nominal bonds of similar maturity.)

Even though just a few issues are outstanding, it is possible to use TIPs to estimate the term structure of real yields. Then, by comparing the real and nominal yield curves, one can derive an estimate of the term structure of expected inflation.[21] The results reveal that short-term anticipated inflation is very low currently; probably well below one percent per annum. However, the inflation yield curve is sharply upward sloped, indicating that investors do not expect inflation to remain long near its current level. The inflation yield curve has little curvature at present, though it was concave downward during much of the sample except for a brief period in late 2000.

Lower inflationary expectations possibly explain the recent decline in TIPs real yields. Although TIPs are well linked to the CPI, their tax treatment could render them indirectly subject to inflation because inflation accruals are fully taxed. Hence, a decline in anticipated inflation could conceivably induce a reduction in real yields. Supporting evidence is that daily changes in TIPs real yields are strongly and positively associated with daily changes in estimated inflation.

TIPs probably belong in most well diversified investment portfolios. Under plausible assumptions, TIPs enhance the risk/return characteristics available with other asset classes. Moreover, to the extent that inflation is expected to increase, they should be the dominant component of any portfolio that attempts to match the nominal return of a mid-maturity bond while minimizing risk.

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[21] Technically, it's the term structure of expected inflation plus the inflation risk premium, if any.

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Appendix

Calculation of Nominal Holding-Period Returns on TIPs

Definitions:

P_t	Price on day t as a percentage of accrued face value
A_t	Accrued interest on day t
F_t	Face (Accrued) value as of day t
C	Stated Annual Coupon
k	Number of coupon payments per year (k=2 for US TIPs)
CPI_m:	The consumer price index, CPI-U, the non-seasonally adjusted U.S. City Average All Items Index for All Urban Consumers, for calendar month m
CPI_B:	The CPI-U on the base (issue) date
A_t:	The accrual factor for day t within month m, $(1 \le t \le 31)$
N_m:	The number of days within month m
G_m:	The growth factor for days within calendar month m

Accrual of Face value:

The growth factor is fixed so that the accrual factor on the first day of month m corresponds to the CPI reported for month m-3. Thus, the accrual factor for the first day of month m+1 corresponds to the CPI reported for month m-2. For month m, we would have

$$G_m = \sqrt[N_m]{CPI_{m-2} / CPI_{m-3}} \, . \tag{1}$$

Then, for each day t within month m, the principal amount of the bond, along with the accrued interest and gross purchase cost, are adjusted by the inflation accrual factor; i.e.,

$$F_t = \frac{CPI_{m-3}}{CPI_B} G_m^{t-1} \, .$$

The seeming skip of one month is explained by the delay in publication of the CPI. For example, the CPI for May is only revealed sometime in early June, so the accrual factor has to depend on the latest available CPI, which, during the first days of June, is for April.

During the bond's first month, an analogous method is used to establish CPI_B corresponding to the issue date. If N_I is the number of days in the month of issue (month I) and t is the actual issue date within the month $(1 \le t \le N_I)$, then G_I is calculated from (1) and

$$CPI_B = CPI_{I-3} G_I^{t-1} \, .$$

Accrued Interest:

The stated coupon is C/k paid k times each year on the accrued face value. Consequently, if F_p is the face value as of coupon payment date p, then the actual amount paid is

$$F_pC/k.$$

Accrued interest is calculated between coupon payment dates by using the accrued face amount on the settlement date. If n is the next coupon payment date and λ is the previous coupon date, then on settlement date t, $\lambda<t<n$, the accrued interest is

$$A_t = F_t \frac{(t-\lambda)}{(n-\lambda)} \frac{C}{k}.$$

Return:

Settlement occurs within one business day. For a trade occurring on day t, the accrued interest and principal accrual are determined as of the next trading day. Hence, the one-day return for day t is

$$R_t = \frac{P_tF_{t+j} - P_{t-k}F_t + A_{t+j} - A_t}{P_{t-k}F_t + A_t},$$

where j is the number of days between the trading day and the next business day and k is the number of days between the trading day and the previous business day. Abstracting from holidays, j=1 when t=Monday,…,Thursday and j=3 when t=Friday. Similarly, k=1 when t=Tuesday,…,Friday and k=3 when t=Monday. For returns across holidays, j and k could also be 2 or 4.

Table 1

Assets and Sample Periods

U.S. Treasury Inflation Protection Securities (TIPs), US Treasury Constant Maturity Nominal Bonds (UST), and Equities, (vwretd is the CRSP NYSE+AMEX+NASDAQ value-weighted index with dividends reinvested, ewretd is the CRSP equal-weighted index with dividends reinvested and sprtrn is the S&P500 Index with dividends included.)

			Daily Returns	
			Begin	End
		TIPs		
Coupon	Issued	Maturity		
3.625	Jul-97	Jul-02	16-Jul-97	11-Jul-02
3.375	Jan-97	Jan-07	22-Jan-97	16-Aug-02
3.625	Jan-98	Jan-08	15-Jan-98	16-Aug-02
3.875	Jan-99	Jan-09	15-Jan-99	16-Aug-02
4.250	Jan-00	Jan-10	19-Jan-00	16-Aug-02
3.500	Jan-01	Jan-11	17-Jan-01	16-Aug-02
3.375	Jan-02	Jan-12	15-Jan-02	16-Aug-02
3.625	Apr-98	Apr-28	15-Apr-98	16-Aug-02
3.875	Apr-99	Apr-29	16-Apr-99	16-Aug-02
	US Treasury Constant Maturity Bonds			
	3 Month		22-Jan-97	26-Jul-02
	1 Year		22-Jan-97	26-Jul-02
	5 Year		22-Jan-97	26-Jul-02
	10 Year		22-Jan-97	26-Jul-02
	30 Year		22-Jan-97	15-Feb-02
		Equity Indexes		
	vwretd		22-Jan-97	16-Aug-02
	ewretd		22-Jan-97	16-Aug-02
	sprtrn		22-Jan-97	16-Aug-02

Table 2

Daily Returns; Means, Standard Deviations, and Correlations

for U.S. Treasury Inflation Protection Securities (TIPs), US Treasury Constant Maturity Nominal Bonds (UST), and Equities, (vwretd is the CRSP NYSE+AMEX+NASDAQ value-weighted index with dividends reinvested, ewretd is the CRSP equal-weighted index with dividends reinvested and sprtrn is the S&P500 Index with dividends included.) Individual TIPs are identified by maturity. The sample period varies by asset but begins at earliest in January 1997 and ends at latest in August 2002. Sigma is the standard deviation and "N" is the number of daily sample observations in the mean and in sigma. Correlation coefficients computed from all available joint observations are reported in the upper right triangle and the corresponding numbers of available observations are given in the lower triangle.

		Return (%/day)			Jul-02	Jan-07	Jan-08	Jan-09	Jan-10	Jan-11	Jan-12	Apr-28	Apr-29	3 Mon	1 Yr	5 Yr	10 Yr	30 Yr	vwretd	ewretd	sprtrn
		Mean	Sigma	N	Correlations																
TIPs	Jul-02	0.0235	0.0680	1262		0.757	0.722	0.626	0.604	0.543	0.183	0.486	0.392	0.229	0.426	0.412	0.385	0.339	-0.068	-0.125	-0.044
	Jan-07	0.0265	0.1600	1409	1262		0.977	0.969	0.960	0.949	0.964	0.753	0.739	0.161	0.468	0.536	0.523	0.455	-0.071	-0.114	-0.053
	Jan-08	0.0303	0.1610	1161	1135	1161		0.982	0.978	0.974	0.973	0.779	0.766	0.194	0.530	0.588	0.570	0.486	-0.140	-0.164	-0.130
	Jan-09	0.0341	0.1680	908	882	908	908		0.991	0.987	0.981	0.788	0.787	0.189	0.514	0.599	0.586	0.495	-0.166	-0.201	-0.155
	Jan-10	0.0455	0.1930	652	626	652	652	652		0.996	0.993	0.798	0.799	0.216	0.568	0.676	0.676	0.578	-0.188	-0.219	-0.176
	Jan-11	0.0411	0.2430	401	375	401	401	401	401		0.995	0.802	0.801	0.278	0.588	0.715	0.726	0.639	-0.202	-0.241	-0.194
	Jan-12	0.0663	0.2580	150	124	150	150	150	150	150		0.894	0.896	0.273	0.665	0.763	0.774	0.756	-0.340	-0.407	-0.325
	Apr-28	0.0375	0.3230	1100	1074	1100	1100	908	652	401	150		0.998	0.090	0.393	0.508	0.527	0.546	-0.095	-0.105	-0.087
	Apr-29	0.0453	0.3360	846	820	846	846	846	652	401	150	846		0.100	0.392	0.521	0.548	0.579	-0.121	-0.139	-0.112
UST	3 Mon	0.0189	0.0141	1322	1194	1322	1086	847	605	365	129	1028	787		0.539	0.318	0.255	0.200	-0.082	-0.101	-0.075
	1 Yr	0.0221	0.0477	1322	1194	1322	1086	847	605	365	129	1028	787	1322		0.797	0.711	0.571	-0.163	-0.207	-0.151
	5 Yr	0.0331	0.2990	1322	1194	1322	1086	847	605	365	129	1028	787	1322	1322		0.940	0.829	-0.151	-0.204	-0.136
	10 Yr	0.0404	0.5750	1322	1194	1322	1086	847	605	365	129	1028	787	1322	1322	1322		0.906	-0.117	-0.169	-0.103
	30 Yr	0.0611	1.4700	1215	1098	1215	979	740	498	258	22	921	680	1215	1215	1215	1215		-0.007	-0.053	0.004
Equities	vwretd	0.0240	1.2900	1401	1254	1401	1153	901	647	396	149	1092	839	1321	1321	1321	1321	1214		0.842	0.983
	ewretd	0.0735	0.9440	1401	1254	1401	1153	901	647	396	149	1092	839	1321	1321	1321	1321	1214	1401		0.769
	sprtrn	0.0208	1.3100	1401	1254	1401	1153	901	647	396	149	1092	839	1321	1321	1321	1321	1214	1401	1401	

Numbers of Sample Observations in Correlations

Table 3

Autocorrelations of Asset Returns

Partial autocorrelation coefficients from one through five daily lags are reported for daily returns on U.S. Treasury Inflation Protection Securities (TIPs), US Treasury Constant Maturity Nominal Bonds (UST), and Equities, (vwretd is the CRSP NYSE+AMEX+NASDAQ value-weighted index with dividends reinvested, ewretd is the CRSP equal-weighted index with dividends reinvested and sprtrn is the S&P500 Index with dividends included.) Individual TIPs are identified by maturity. N is the sample size.

	Lag→	1	2	3	4	5	
	Asset	Partial Autocorrelation Coefficient (T-statistic)					Adjusted R-Square
TIPs	Jul-02	0.1214	-0.0110	-0.0136	0.0340	-0.0097	0.0117
	N=1257	(4.295)	-(.387)	-(.477)	(1.195)	-(.341)	
	Jan-07	0.1490	-0.0284	-0.0486	0.0270	-0.0186	0.0213
	N=1404	(5.555)	-(1.048)	-(1.818)	(1.008)	-(.704)	
	Jan-08	0.1763	-0.0250	-0.0189	0.0060	-0.0212	0.0265
	N=1156	(5.954)	-(.831)	-(.627)	(.20)	-(.713)	
	Jan-09	0.1823	-0.0003	-0.0349	-0.0022	0.0049	0.0282
	N= 903	(5.427)	-(.008)	-(1.020)	-(.064)	(.144)	
	Jan-10	0.1696	0.0001	-0.0306	0.0011	-0.0145	0.0214
	N= 647	(4.256)	(.002)	-(.757)	(.028)	-(.360)	
	Jan-11	0.1843	-0.0116	-0.0475	-0.0071	0.0001	0.0225
	N= 396	(3.604)	-(.223)	-(.911)	-(.135)	(.001)	
	Jan-12	0.1599	-0.0014	-0.0392	-0.0960	0.0370	-0.0005
	N= 145	(1.848)	-(.016)	-(.444)	-(1.087)	(.416)	
	Apr-28	0.2245	-0.0337	-0.0044	-0.0318	0.0094	0.0446
	N=1095	(7.386)	-(1.079)	-(.141)	-(1.014)	(.305)	
	Apr-29	0.2122	-0.0208	-0.0070	-0.0278	0.0176	0.0384
	N= 841	(6.110)	-(.586)	-(.196)	-(.776)	(.498)	
UST	3 Month	0.1600	0.0598	-0.0636	0.1496	0.1642	0.0946
	N=1317	(5.874)	(2.193)	-(2.332)	(5.485)	(6.027)	
	1 Year	0.1131	-0.0066	-0.0546	0.0006	-0.0025	0.0120
	N=1317	(4.096)	-(.238)	-(1.968)	(.023)	-(.089)	
	5 Year	0.0755	-0.0265	-0.0480	-0.0072	-0.0112	0.0052
	N=1317	(2.734)	-(.954)	-(1.734)	-(.261)	-(.403)	
	10 Year	0.0679	-0.0150	-0.0466	-0.0149	-0.0098	0.0038
	N=1317	(2.460)	-(.542)	-(1.687)	-(.539)	-(.354)	
	30 Year	0.0477	0.0003	-0.0306	-0.0132	0.0099	-0.0006
	N=1210	(1.654)	(.011)	-(1.060)	-(.459)	(.344)	
Equities	vwretd	0.0486	-0.0620	-0.0250	0.0184	-0.0612	0.0068
	N=1396	(1.815)	-(2.313)	-(.930)	(.685)	-(2.279)	
	ewretd	0.2410	-0.0369	0.1090	0.0672	-0.0057	0.0758
	N=1396	(8.983)	-(1.339)	(3.973)	(2.435)	-(.214)	
	sprtrn	0.0089	-0.0599	-0.0484	0.0113	-0.0572	0.0053
	N=1396	(.334)	-(2.236)	-(1.799)	(.422)	-(2.129)	

Table 4

TIPs Empirical Durations and Factor Sensitivities

Empirical Duration Estimates are from a regression of US Treasury Inflation Protection Securities (TIPs) returns on changes in the simple average of yields on constant maturity US Treasury Five- and Ten-Year Bonds. Duration units are effective years. Sensitivities to term structure shape are from a regression of TIPs returns on changes in term structure shape factors, "Shift," "Tilt," and "Flex," which are described in the text. An individual TIPs is identified by its maturity month and N is its sample size. T-statistics are in parentheses. The numbers in percentages are adjusted R-squares. For the second regression, two additional diagnostics are reported in italics; the first is the Durbin-Watson statistic and the second is the excess kurtosis of the residuals. These diagnostics are very similar in the first regression, so are omitted to save space.

TIPs	Duration	Shift	Tilt	Flex
Jul-02 N=1247	0.585	-0.548	0.4664	-0.0061
	(18.34)	-(12.21)	(5.74)	-(.06)
	21.2%	18.2%	*1.826*	*5.087*
Jan-07 N=1379	1.667	-1.644	0.3023	0.3231
	(24.85)	-(17.45)	(1.74)	(1.43)
	30.9%	27.3%	*1.842*	*8.835*
Jan-08 N=1133	1.784	-1.793	0.3445	0.4837
	(26.25)	-(18.29)	(1.98)	(2.12)
	37.8%	32.9%	*1.738*	*7.537*
Jan-09 N=883	1.845	-1.892	0.1907	0.7384
	(23.80)	-(16.45)	(.98)	(2.81)
	39.1%	33.8%	*1.753*	*4.907*
Jan-10 N=631	2.212	-2.448	-0.1352	1.357
	(23.90)	-(17.13)	-(.62)	(4.35)
	47.5%	42.6%	*1.821*	*6.473*
Jan-11 N=381	2.577	-3.478	-0.5115	3.399
	(19.76)	-(15.17)	-(1.61)	(6.49)
	50.6%	48.1%	*1.733*	*3.162*
Jan-12 N=134	3.143	-4.568	-2.537	7.624
	(12.93)	-(7.09)	-(2.66)	(7.42)
	55.6%	51.0%	*1.823*	*0.784*
Apr-28 N=1072	2.800	-3.524	-2.969	3.302
	(18.36)	-(17.50)	-(8.36)	(7.07)
	23.9%	31.4%	*1.672*	*7.486*
Apr-29 N=821	2.914	-3.868	-3.49	4.246
	(16.45)	-(16.44)	-(8.83)	(7.88)
	24.7%	34.6%	*1.753*	*7.059*

Table 4-bis

Robustness Check for Heteroskedasticity and Serial Dependence

Regressions of US Treasury Inflation Protection Security (TIPs) returns on the shift, tilt and flex term structure shape factor changes reported in Table 4 are redone using the Newey/West heteroskedasticity and autocorrelation consistent covariance matrix of the estimated coefficients. The coefficients are unaltered by the Newey/West procedure; only the standard errors (and hence the t-statistics) are affected. In the table below, the OLS t-statistics from Table 4 are given along side the Newey/West (NW) t-statistics. TIPs are identified by maturity date. Sample sizes, coefficients, and adjusted R-squares are reported in Table 4.

	OLS	NW	OLS	NW	OLS	NW
TIPs	Shift		Tilt		Flex	
Jul-02	-(12.21)	-(7.95)	(5.74)	(3.64)	-(.06)	-(.03)
Jan-07	-(17.45)	-(11.41)	(1.74)	(.97)	(1.43)	(.70)
Jan-08	-(18.29)	-(12.11)	(1.98)	(1.20)	(2.12)	(.99)
Jan-09	-(16.45)	-(11.21)	(.98)	(.60)	(2.81)	(1.26)
Jan-10	-(17.13)	-(12.43)	-(.62)	-(.46)	(4.35)	(1.77)
Jan-11	-(15.17)	-(11.08)	-(1.61)	-(1.16)	(6.49)	(2.44)
Jan-12	-(7.09)	-(7.05)	-(2.66)	-(2.31)	(7.42)	(5.69)
Apr-28	-(17.50)	-(8.37)	-(8.36)	-(2.98)	(7.07)	(2.44)
Apr-29	-(16.44)	-(7.71)	-(8.83)	-(3.13)	(7.88)	(2.65)

Table 5

Intertemporal Changes in TIPs Empirical Durations and Factor Sensitivities

Empirical Duration Estimates are from a regression of US Treasury Inflation Protection Securities (TIPs) returns on changes in the simple average of yields on constant maturity US Treasury Five- and Ten-Year Bonds. Duration units are effective years. Sensitivities to term structure shape are from a regression of TIPs returns on changes in term structure shape factors, "Shift," "Tilt," and "Flex," which are described in the text. In both regressions, each regressor X_t was augmented with a composite time trend regressor X_t*t, where t is the fraction of the original maturity remaining in the TIPs' life. An individual TIPs is identified by its maturity month and N is its sample size. T-statistics are in parentheses. The numbers in percentages are adjusted R-squares. For the second regression, two additional diagnostics are reported in italics; the first is the Durbin-Watson statistic and the second is the excess kurtosis of the residuals. These diagnostics are very similar in the first regression, so are omitted to save space.

TIPs	Duration	Duration*t	Shift	Tilt	Flex	Shift*t	Tilt*t	Flex*t
Jul-02	0.816	-0.409	-0.684	0.321	-0.590	0.217	0.328	0.951
N=1247	(11.75)	-(3.74)	-(7.37)	(1.60)	-(2.36)	(1.35)	(1.08)	(2.35)
	22.0%		21.4%		*1.837*		*4.563*	
Jan-07	1.337	1.038	-0.997	-0.164	-1.418	-2.351	1.401	5.960
N=1379	(8.82)	(2.43)	-(4.95)	-(.36)	-(2.55)	-(3.78)	(1.16)	(3.74)
	31.2%		28.8%		*1.857*		*7.605*	
Jan-08	1.214	2.244	-0.868	0.859	-1.604	-4.178	-1.958	9.237
N=1133	(8.24)	(4.35)	-(4.31)	(2.04)	-(3.17)	-(5.37)	-(1.37)	(4.95)
	38.8%		34.6%		*1.775*		*6.947*	
Jan-09	0.414	6.998	0.270	0.0037	-2.396	-11.98	0.0561	18.80
N= 883	(2.37)	(9.04)	(1.08)	(.01)	-(4.20)	-(9.59)	(.02)	(6.82)
	44.2%		40.0%		*1.759*		*4.550*	
Jan-10	1.487	5.036	-1.199	0.423	-1.709	-11.13	-4.57	27.23
N= 631	(6.39)	(3.40)	-(3.67)	(.88)	-(2.95)	-(4.68)	-(1.36)	(6.26)
	48.4%		45.7%		*1.769*		*2.876*	
Jan-11	2.008	7.855	-2.339	1.135	-1.733	-16.64	-22.36	67.23
N= 381	(7.51)	(2.43)	-(5.23)	(1.66)	-(1.54)	-(2.73)	-(2.43)	(4.93)
	51.3%		51.2%		*1.714*		*1.497*	
Jan-12	3.040	3.934	-5.436	-1.149	9.225	54.77	-42.74	-138.7
N= 134	(6.11)	(.24)	-(4.47)	-(.66)	(4.58)	(1.19)	-(.72)	-(1.54)
	55.2%		51.4%		*1.859*		*1.303*	
Apr-28	1.565	15.64	-0.904	1.469	-3.182	-36.96	-54.03	88.8
N=1072	(4.85)	(4.33)	-(2.31)	(1.82)	-(3.31)	-(7.63)	-(6.11)	(7.74)
	25.1%		36.1%		*1.700*		*4.868*	
Apr-29	1.031	30.15	-0.573	0.377	-2.884	-61.04	-63.86	131.5
N= 821	(2.55)	(5.17)	-(1.18)	(.37)	-(2.51)	-(7.60)	-(4.28)	(7.26)
	27.0%		40.5%		*1.737*		*4.977*	

Table 6

Time Variation in Return Volatility Estimated by GARCH(1,1)

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \rho h_{t-1}$$

In the variance equation of GARCH(1,1), h is the conditional variance and ε is the innovation. GARCH(1,1) is estimated using daily returns on U.S. Treasury Inflation Protection Securities (TIPs), identified by its maturity month. US Treasury Constant Maturity Nominal Bonds (UST), and Equities, (vwretd is the CRSP value-weighted index with dividends reinvested, ewretd is the CRSP equal-weighted index with dividends reinvested and sprtrn is the S&P500 Index with dividends included.) Sample sizes are the same as in Table 2 (less one for the single lag.) T-statistics are in parentheses.

		Lagged Squared Innovation, α_1	Lagged Conditional Variance, ρ
TIPs	Jul-02	0.1039 (6.51)	0.8782 (53.23)
	Jan-07	0.1663 (7.72)	0.8327 (45.61)
	Jan-08	0.1627 (6.83)	0.8322 (39.64)
	Jan-09	0.1568 (5.71)	0.8237 (29.25)
	Jan-10	0.1671 (4.88)	0.8232 (25.04)
	Jan-11	0.1229 (2.86)	0.8025 (11.08)
	Jan-12	0.0978 (1.55)	0.8865 (8.17)
	Apr-28	0.2881 (7.50)	0.7425 (26.97)
	Apr-29	0.0394 (4.83)	0.9595 (117.30)
UST	3 Month	0.2463 (7.24)	0.7036 (20.72)
	1 Year	0.1453 (6.06)	0.7954 (25.45)
	5 Year	0.0781 (4.27)	0.8590 (24.60)
	10 Year	0.0615 (3.61)	0.8739 (23.13)
	30 Year	0.0482 (2.52)	0.8402 (12.09)
Equity	vwretd	0.1189 (5.57)	0.8371 (29.31)
	ewretd	0.2835 (8.58)	0.7106 (25.26)
	sprtrn	0.1067 (4.98)	0.8338 (24.90)

Table 7

Tests of the Tax Conjecture

Daily changes in real yields on TIP bonds are regressed on concurrent daily changes in estimates of the inflation term structure. Two regressions are reported; the first (labeled “One Factor”) uses only changes in the anticipated inflation level; i.e., the “shift” in inflation as the explanatory variable. The second regression (Three Factors) includes as explanatory variables inflation “shift”, “tilt”, and “flex”; i.e., changes in the level, slope, and curvature of the estimated inflation term structure. An individual TIP bond is identified by its maturity month and N is its sample size. Coefficients are listed first, followed by t-statistics in parentheses, then adjusted R-squares as percentages (followed by one decimal digit) and Durbin/Watson statistics in italics. For the first regression, the implied estimated marginal tax rate is computed based on an assumption of a one percent expected inflation rate. It is given as a percentage without a trailing decimal.

	Shift	Implied Tax Rate	Shift	Tilt	Flex
	One Factor		Three Factors		
Jul-02 N=774	0.233		0.136	0.030	0.189
	(5.20)	19%	(2.01)	(.28)	(1.46)
	3.3% *1.65*		3.5%		*1.66*
Jan-07 N=888	0.172		-0.005	0.249	0.229
	(9.46)	15%	-(.21)	(6.39)	(4.68)
	9.1% *1.64*		21.8%		*1.57*
Jan-08 N=888	0.160		0.010	0.259	0.166
	(9.29)	14%	(.41)	(7.00)	(3.57)
	8.8% *1.64*		20.8%		*1.59*
Jan-09 N=888	0.149		0.014	0.276	0.125
	(8.99)	13%	(.60)	(7.75)	(2.80)
	8.3% *1.64*		20.6%		*1.60*
Jan-10 N=641	0.184		0.047	0.296	0.100
	(9.05)	16%	(1.56)	(7.49)	(1.89)
	11.2% *1.65*		24.3%		*1.62*
Jan-11 N=392	0.191		-0.045	0.447	0.198
	(7.13)	16%	-(1.03)	(8.74)	(2.50)
	11.3% *1.63*		34.3%		*1.70*
Jan-12 N=147	0.132		-0.244	0.893	0.116
	(3.65)	12%	-(3.79)	(15.19)	(1.06)
	7.8% *1.69*		68.5%		*1.93*
Apr-28 N=888	0.097		0.005	0.202	0.079
	(7.71)	9%	(.26)	(7.37)	(2.30)
	6.2% *1.58*		17.2%		*1.59*
Apr-29 N=827	0.101		0.011	0.199	0.075
	(7.61)	9%	(.58)	(6.98)	(2.08)
	6.5% *1.58*		17.0%		*1.59*

Figure 1
Real Yields on TIPS, 1997-2002

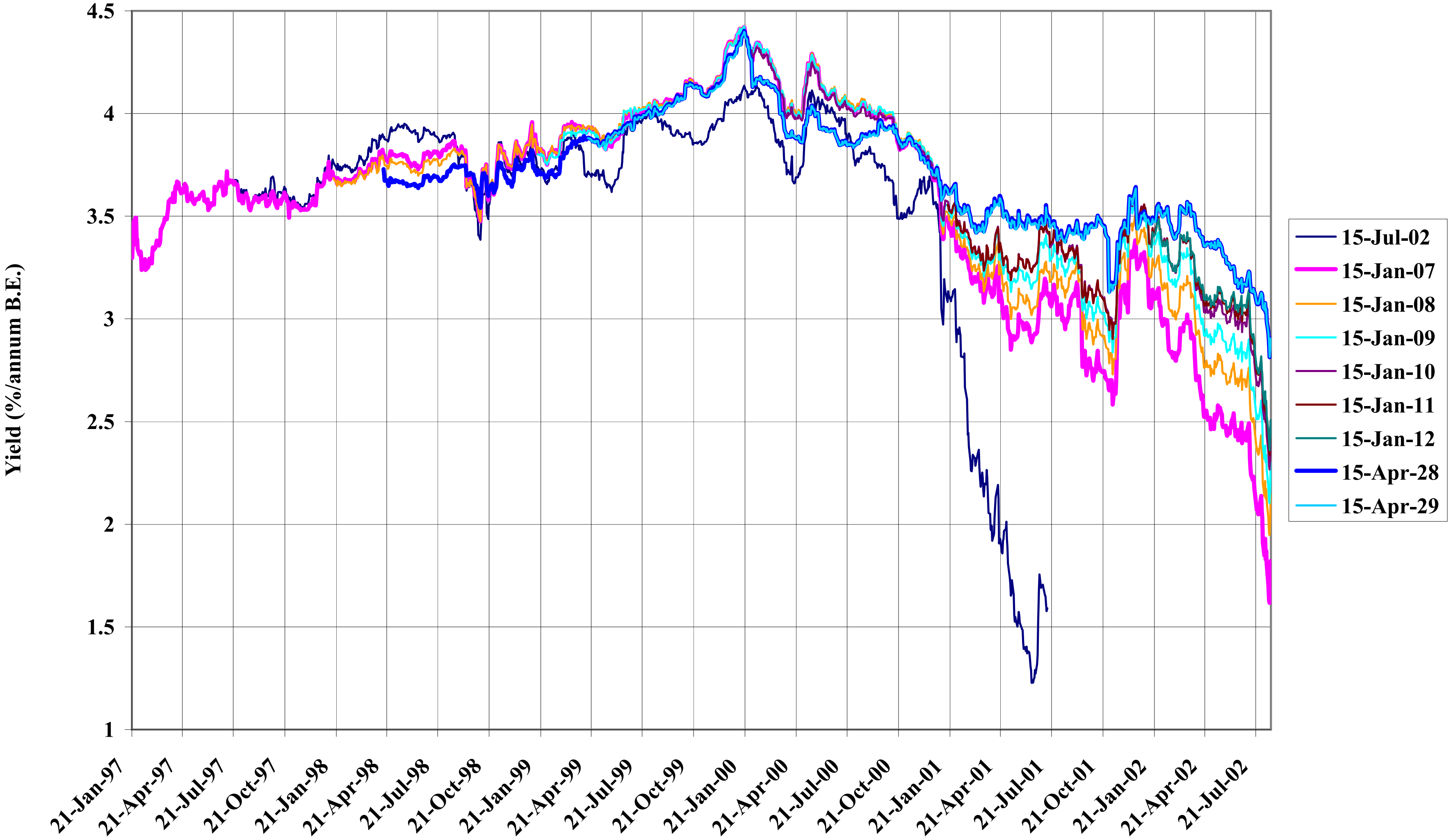

Figure 2
Real Durations of TIPS, 1997-2002

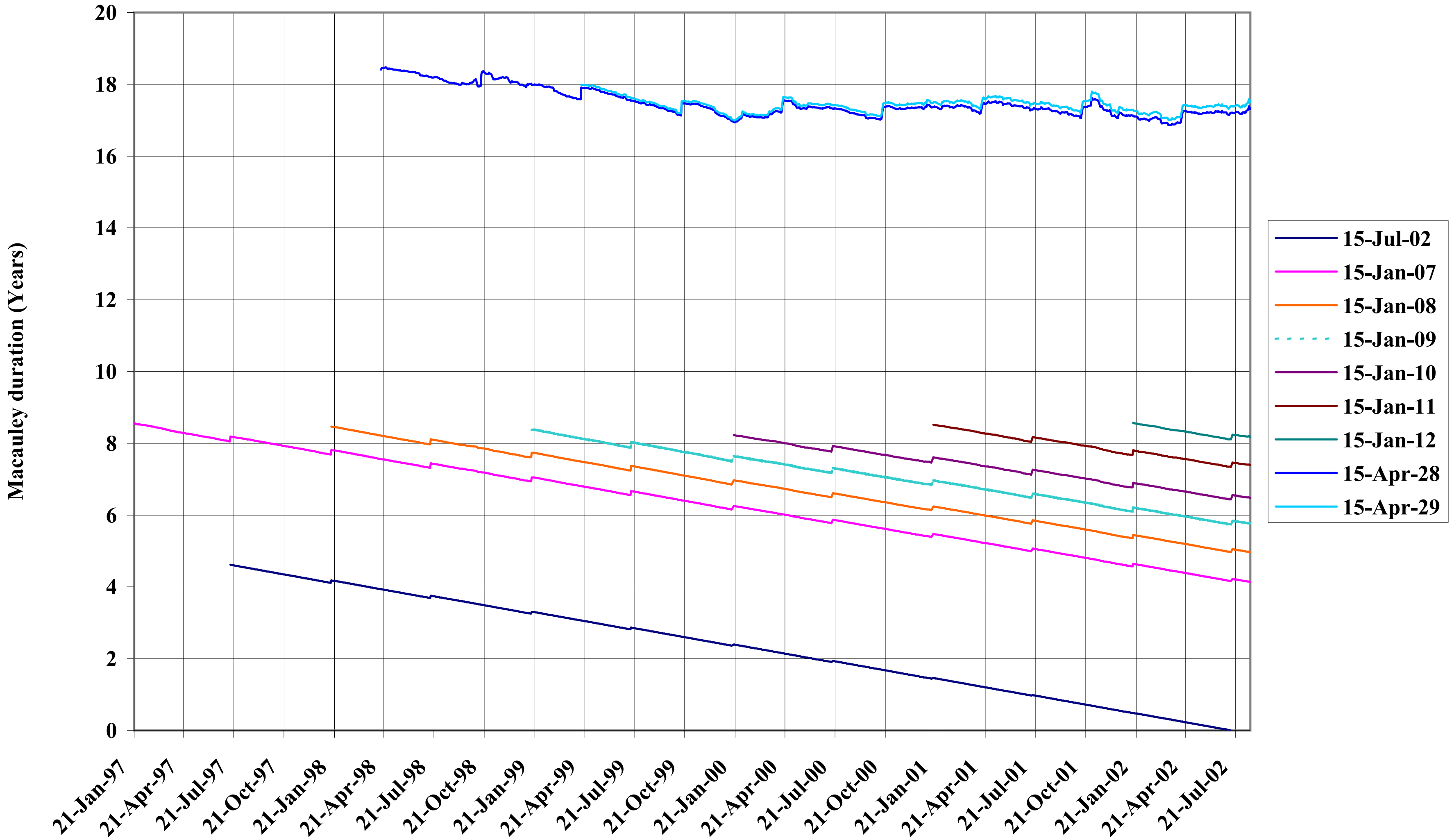

Figure 3
Rolling Volatilities of TIPS Returns
(approximate one-month overlapping windows)

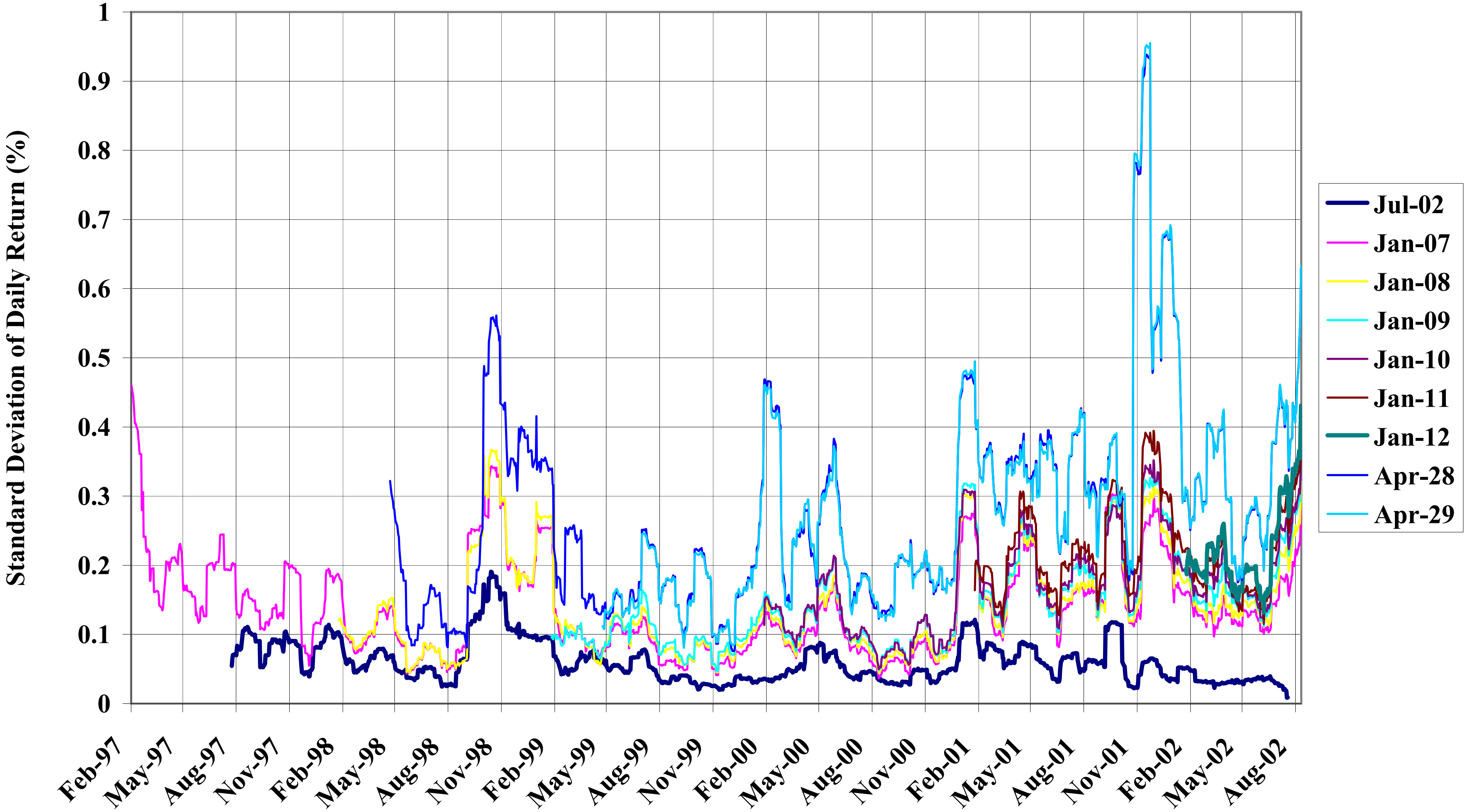

Figure 4
Jan07 TIPS Volatility Comparison
GARCH(1,1) vs. Rolling 21-day Window

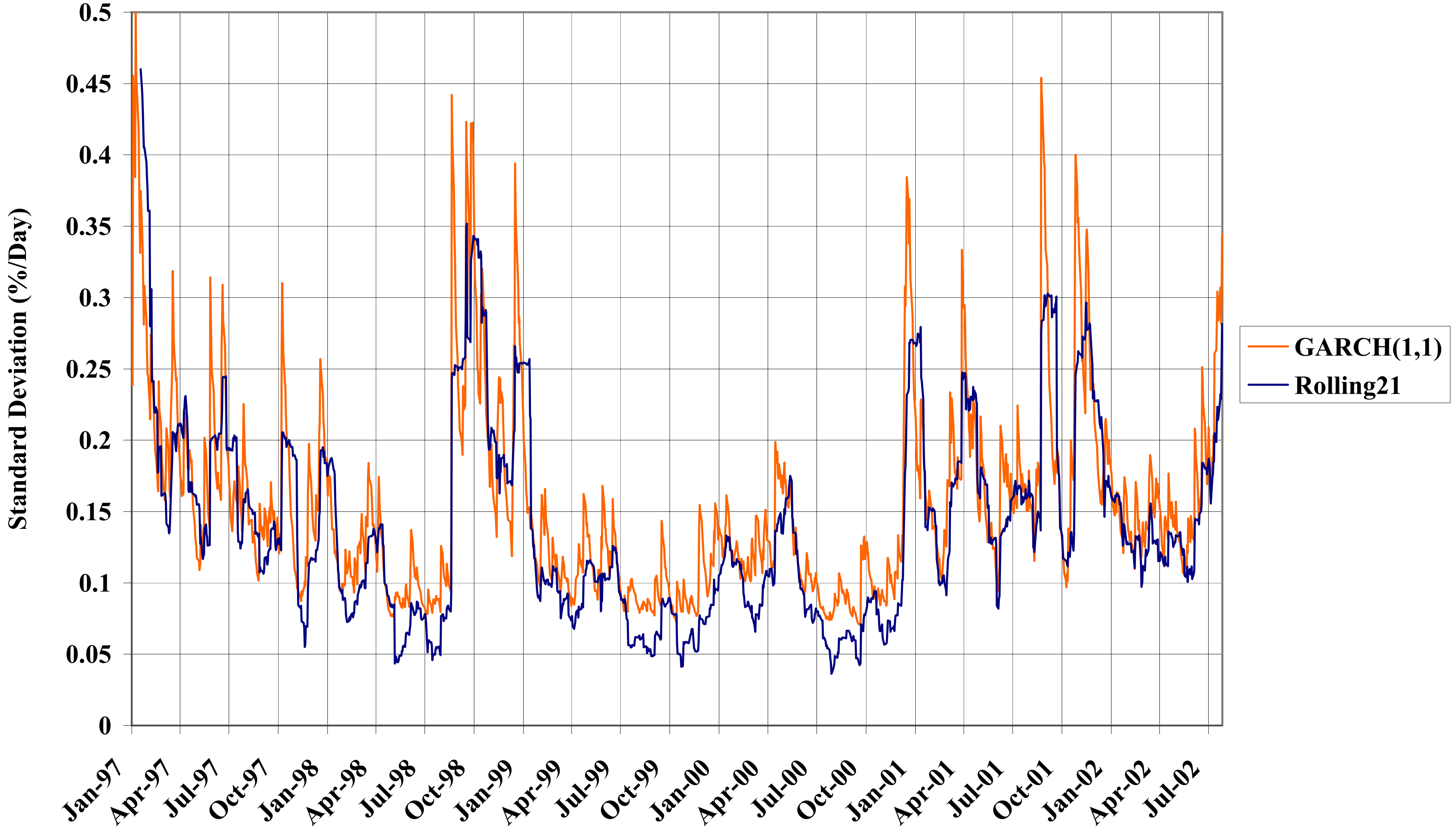

Figure 5
Log Ratio of Return Standard Deviations
Ln[Sigma(2001-02)/Sigma(1999-00)]

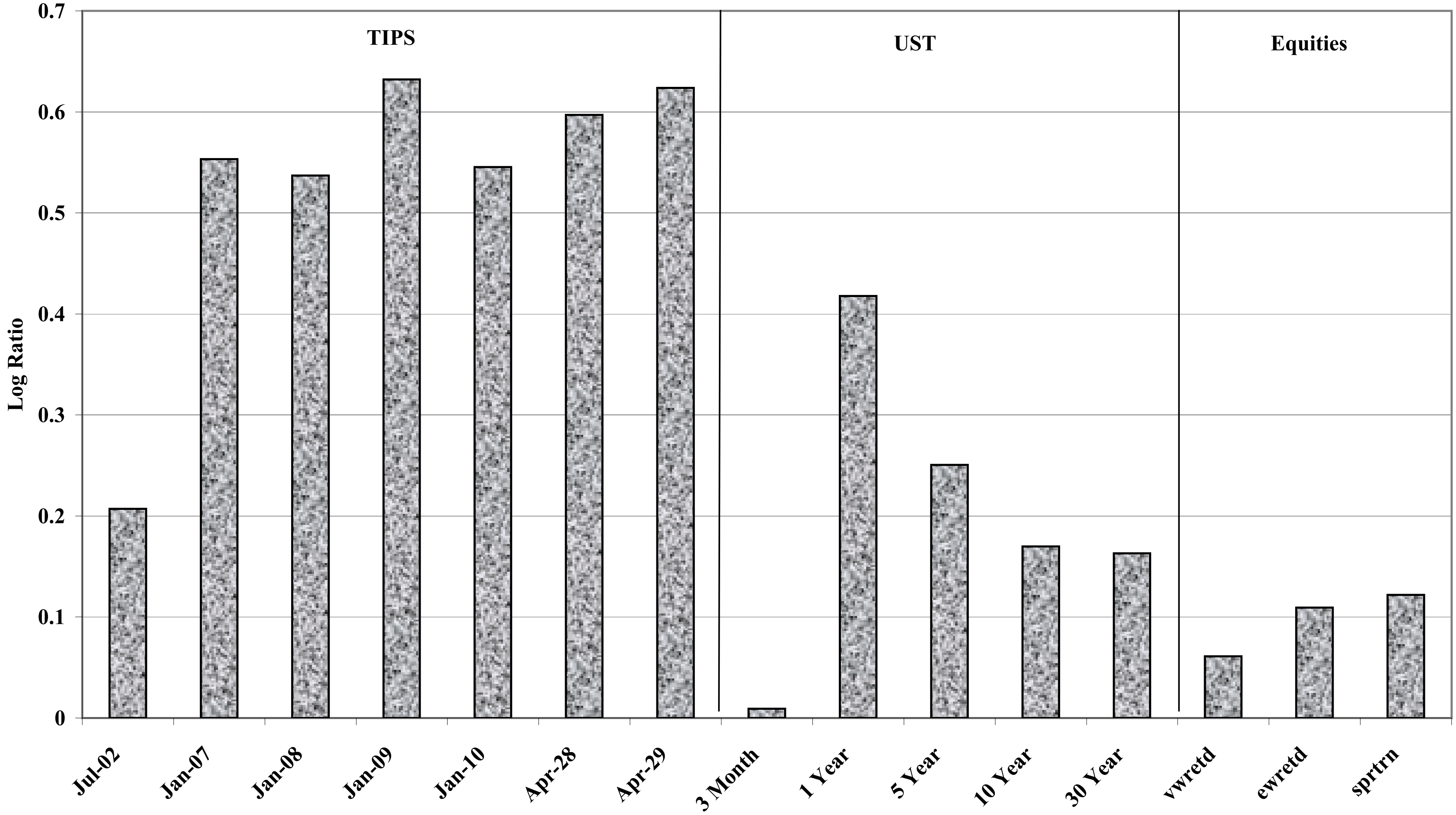

Figure 6
Term Structure Levels, Nominal and Real

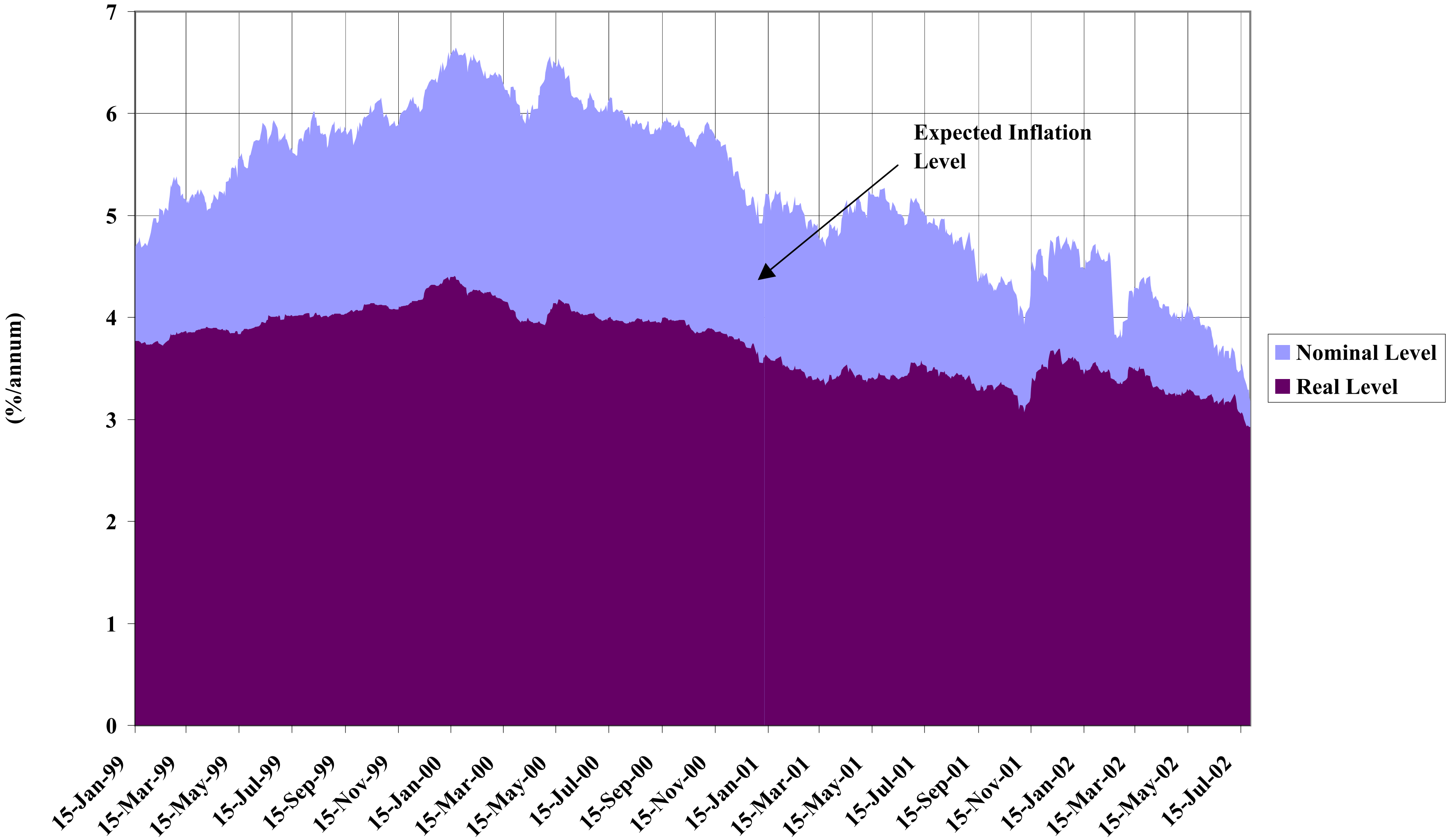

Figure 7
Term Structure Slope, Nominal and Real

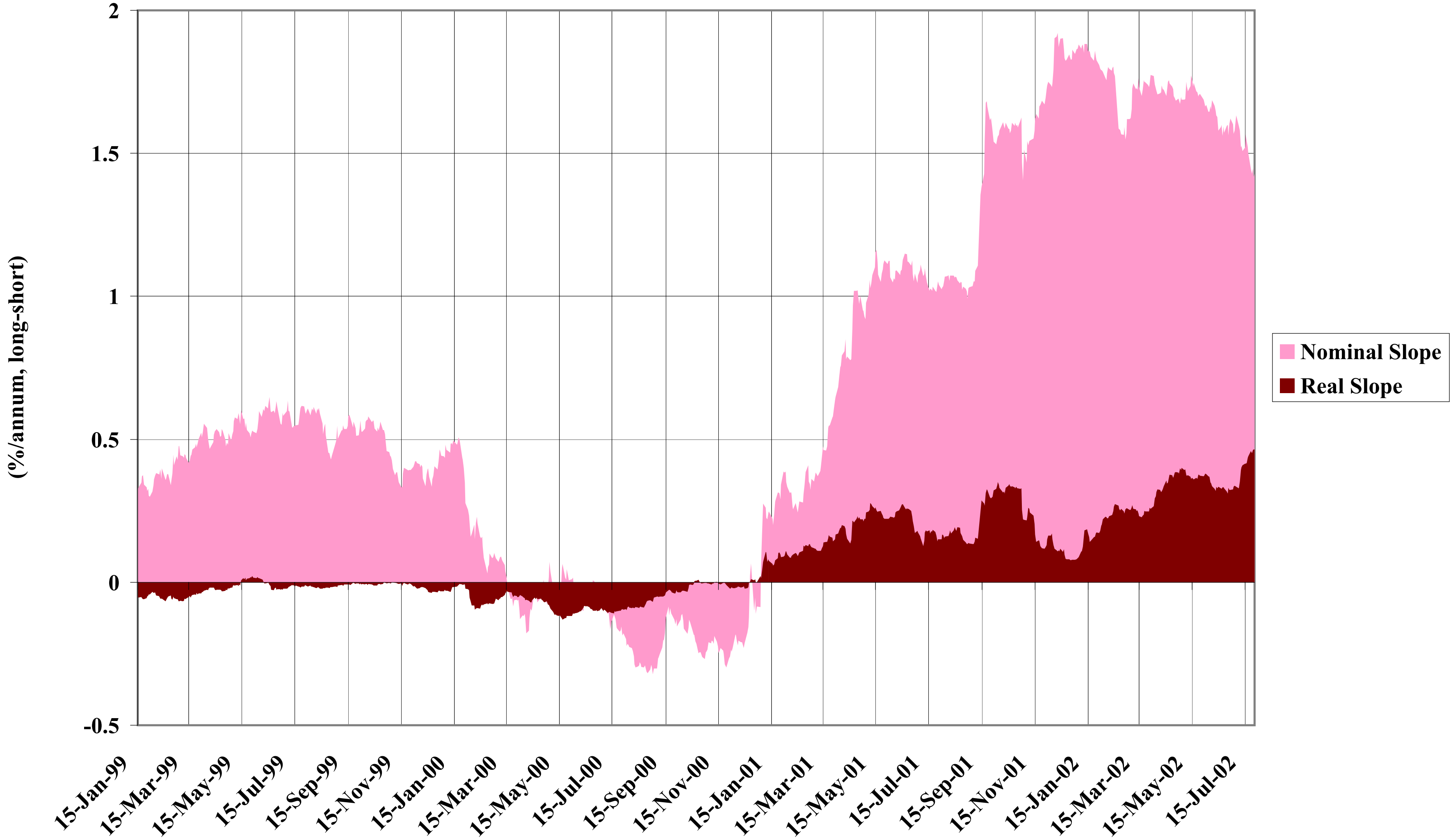

Figure 8
Term Structure Curvature, Nominal and Real

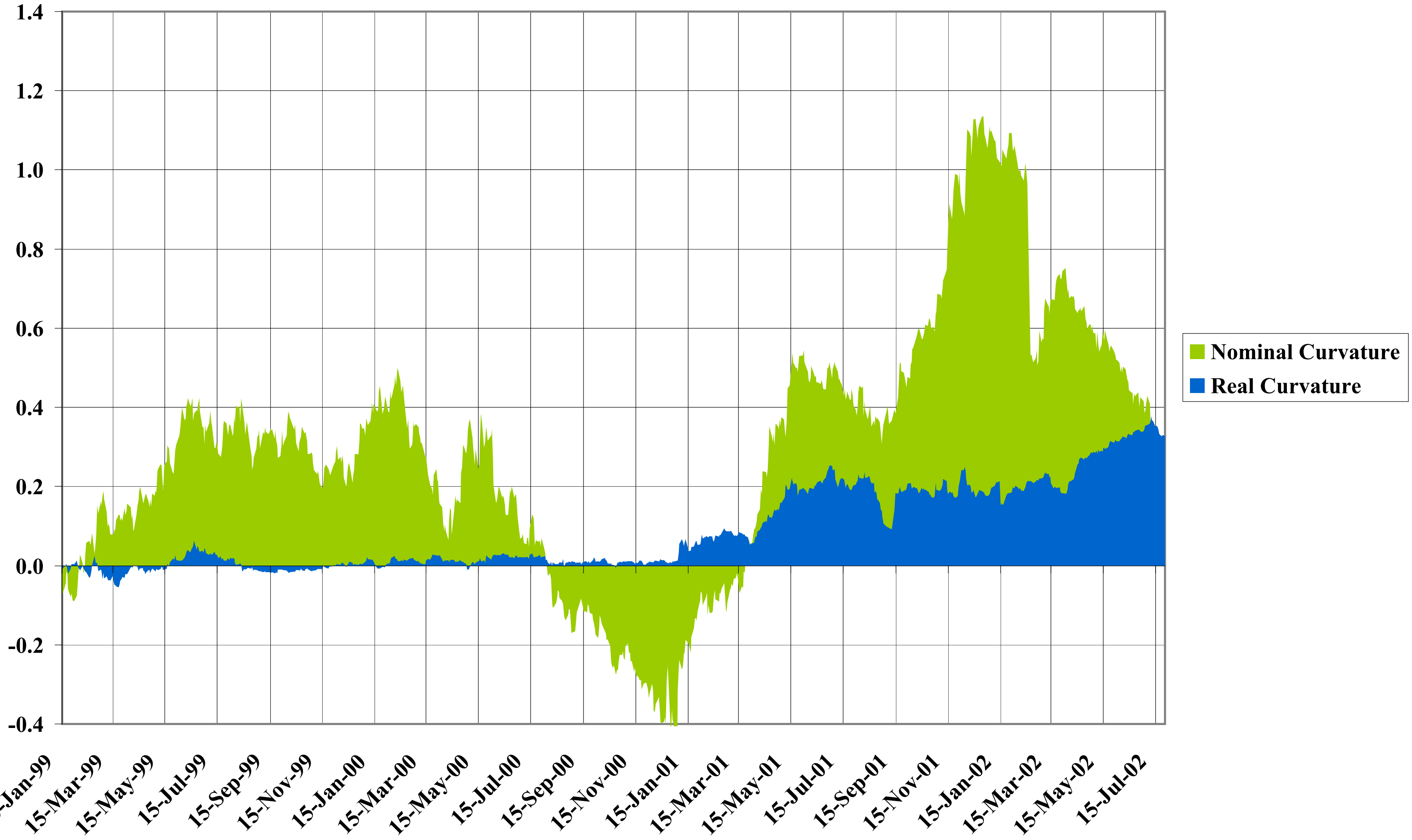

Figure 9
The Term Structure of Anticipated Inflation

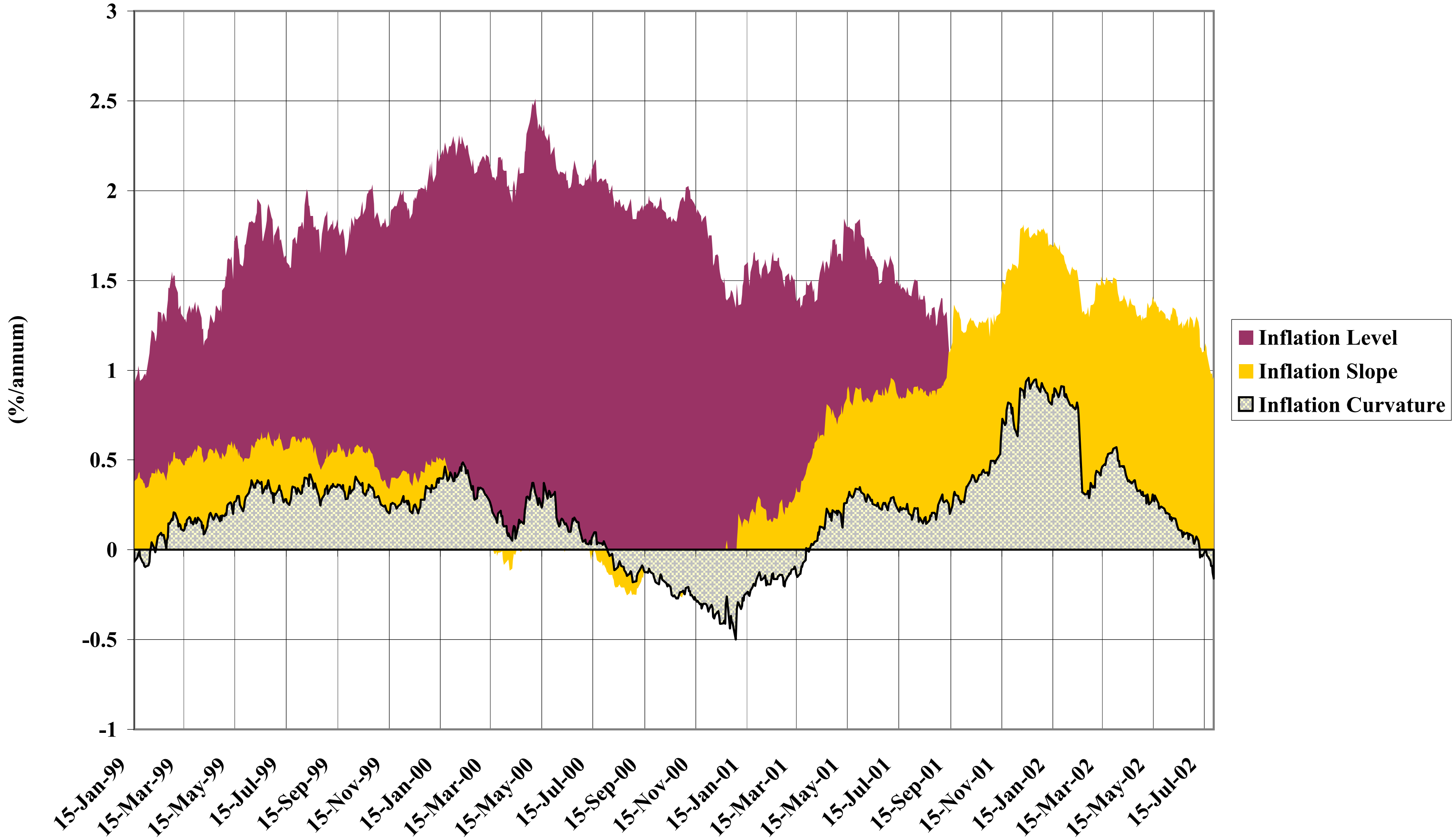

Figure 10

Efficient Frontier with Two TIPS, Two US Treasury Nominal Bonds, and Equities

Covariance Matrix from 2001-02 Data

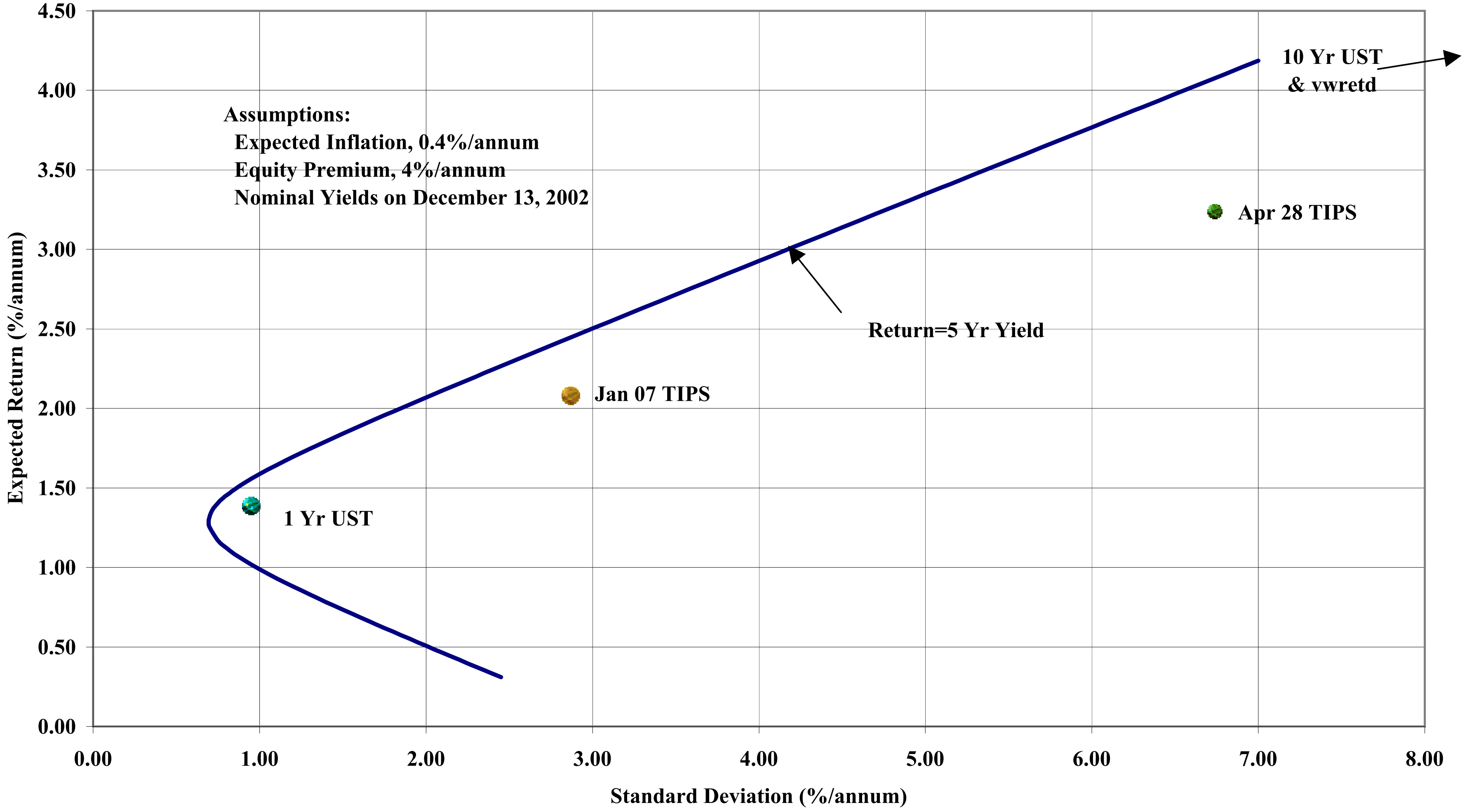

Figure 11
Composition of Optimum Portfolio to Match Five-Year Yield, Using two TIPS, two US Treasury Nominal Bonds, and Equities